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COMPUTER SIMULATION OF PULSED-NEUTRON EXPERIMENTS FOR THE FORT ST. VRAIN HTGR

by

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MASTER

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ABSTRACT

Prior to startup of the 330 MW Fort St. Vrain HTGR, pulsed-neutron experiments will be performed to determine the reactivity for various sub-critical, control rod configurations. To determine the reactivity, the inhour method will be used. This method requires that the time response following the pulse exhibit the die-away of the fundamental prompt mode. For such a large reactor, it is important to carefully position the source and detector in order to minimize the contribution of the higher prompt modes to the time response. Moreover, for a power reactor, economics dictates that the optimum source and detector locations be determined before the experiments. This can be done by simulating the experiments on a computer. Such computer simulations have been performed for all ten control rod configurations for which pulsing is planned. The optimum source and detector locations have been determined as well as the expected time responses.

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1. INTRODUCTION

Prior to startup of the 330 MW Fort St. Vrain high-temperature, gas-cooled reactor (HTGR), pulsed-neutron experiments will be performed to determine the reactivity for various subcritical, control rod configurations. These experimentally determined reactivities will be compared to those previously calculated. This comparison will provide an important check of the nuclear design.

A pulsed-neutron experiment consists of repetitively pulsing a subcritical assembly with a fast neutron source and measuring with a thermal detector the neutron flux as a function of time following each pulse (Ref. 1, pp. 546-554). If the source and detector are properly positioned and the pulse rate is appropriate, then a time response similar to that shown in Fig. 1 will be observed. The methods by which the reactivity is obtained from such a time response are described below. First, however, we interpret the shape of the time response in Fig. 1.

In general, the neutron flux, $\phi(\vec{r}, E, t)$, following each pulse can be written as a sum of kinetic modes,*

$$\phi(\vec{r}, E, t) = \sum_{i=0}^{\infty} a_i \phi_i(\vec{r}, E) e^{\alpha_i t}. \quad (1)$$

* This expansion results from separating the time dependence from the space and energy dependence in either the neutron transport or diffusion equation. Although a proof that the kinetic modes, $\phi_i(\vec{r}, E)$, are complete has not been given in the general case (see Ref. 1, pp. 533-534 and Ref. 2, p. 408, p. 552), we will assume in the following that the expansion given by Eq. 1 is adequate.

Fig. 1. Schematic time response of thermal flux following pulse.

The decay constants, α_i , and the kinetic modes, $\phi_i(\vec{r}, E)$, depend only upon the core configuration, whereas the amplitudes, a_i , depend upon the source characteristics as well. In particular, for a square pulse (in time),

$$a_i = \frac{\Delta t}{\alpha_i / R} < \phi_i^+(\vec{r}, E), S(\vec{r}, E) >$$

$$= \frac{\Delta t}{\alpha_i / R} \int \int \phi_i^+(\vec{r}, E) S(\vec{r}, E) d\vec{r} dE \quad (2)$$

(see Ref. 3, Eqs. 4.22 and 4.24) where

Δt = pulse width,

R = pulse rate,

$\phi_i^+(\vec{r}, E)$ = normalized adjoint flux for i th mode,

$S(\vec{r}, E)$ = source distribution.

For a subcritical reactor, the decay constants, α_i , are all negative and fall into two categories,⁴ those which are quite close to the negative precursor decay constants, $-\lambda_j$, $j=1, \dots, J$, where J is the number of precursors, and those which are quite far away from the former, $|\alpha_i| \gg \lambda_j$. Those modes with α_i in the first category are called delayed modes, while those with α_i in the latter category are called prompt modes. If the prompt modes are ordered such that

$$|\alpha_0^p| < |\alpha_1^p| < |\alpha_2^p| < \dots,$$

then ϕ_0^p is called the fundamental prompt mode.

We can now interpret the shape of the time response in Fig. 1. This time response is characterized by three time intervals:

1. a short interval of time following the pulse during which the fast source neutrons slow down and the higher prompt modes (harmonics) die away,
2. an intermediate interval of time during which the neutron flux is decaying exponentially with the decay constant, α_0^P , characteristic of the fundamental prompt mode, and
3. a long interval of time up until the next pulse during which the neutron flux remains nearly constant (very slowly decaying), due to the presence of delayed neutrons.

Basically two methods are available for obtaining the reactivity from the time response, although numerous variations have been developed.^{1,5} The first method is the inhour method. The basic equation of this method is

$$\rho = \Lambda \alpha_0^P + \beta \quad (3)$$

where

- Λ = neutron generation time,
 β = delayed neutron fraction.

The delayed neutron fraction, β , is nearly independent of the control rod configuration. For an HTGR, $\beta = 0.0065$. The generation time, Λ , however, varies significantly with the control rod configuration and must be calculated. To obtain the reactivity, ρ , the experimental data over the second interval of Fig. 1 are analyzed by least squares to obtain α_0^P . Using a calculated value for Λ , the reactivity is then obtained from Eq. 3.

The second method is the area-ratio method. The basic equation of this method is

$$\frac{\rho}{\beta} = - \frac{\text{prompt-neutron area}}{\text{delayed-neutron area}} \times F \quad (4)$$

where the appropriate areas are indicated in Fig. 1. The correction factor, F , which depends upon the source and detector locations, accounts for higher harmonics and kinetic distortion.* An expression for F is derived in Ref. 5 (Eq. 38). For a light water reactor, F is near 1. For an HTGR, it may be much larger, e.g. 10, because of the small absorption in the reflector.

Since this large correction factor must be calculated, we do not plan to use the area-ratio method for analyzing the Fort St. Vrain pulsed-neutron experiments. Instead the inhour method will be used. Although this method also requires a calculation, Λ is less sensitive to calculational errors than F .

Pulsed-neutron experiments were performed on the 40 MW Peach Bottom HTGR prior to its startup and were successfully interpreted using the inhour method.⁵ However, interpretation of the experiments on the much larger 330 MW Fort St. Vrain HTGR may be more difficult. This is because the decay constants of the higher harmonics will be much closer to that of the fundamental mode. Moreover, the core loading is not symmetric azimuthally. Consequently the source and detector locations must be carefully chosen in order to minimize the contribution of the higher prompt modes to the time response. In addition, the pulse rate must be low enough so that the fundamental mode is observed before the delayed neutron background is reached. Thus if the experiment is not carefully planned, the first and last intervals of Fig. 1 may merge together so that determination of α_0^P is impossible.

* Kinetic distortion is the space- and energy-dependent difference between the fundamental prompt and delayed modes.

Although determination of the optimum source location, detector location, and pulse rate could be made experimentally, for a large power reactor it is far more economical to perform this optimization beforehand with a computer simulation of the experiment. One technique for performing this simulation is to obtain $\phi(\vec{r}, E, t)$ by a direct numerical solution of the neutron kinetics equation. Another technique is to calculate the a_i , α_i , and $\phi_i(\vec{r}, E)$ for the first few modes of Eq. 1. This latter technique is preferred since it permits a systematic determination of the optimum source and detector locations. Accordingly, two two-dimensional computer codes, RTMODE (for r- θ geometry) and RZMODE (for r-z geometry),³ have been written to simulate pulsed-neutron experiments by calculating kinetic modes. An explicit, three-dimensional simulation, although desirable, would be prohibitively expensive. However, by synthesizing the results from the two two-dimensional codes, a three-dimensional time response to the neutron pulse can be obtained.

This report describes the results of computer simulations of the pulsed-neutron experiments planned for the Fort St. Vrain HTGR. The optimum source and detector locations have been determined as well as the expected time responses.

2. GEOMETRIC MODELS

The actual layout of the Fort St. Vrain core consists of layers of hexagonal fuel elements in groups of seven called patches. To model the core in r - θ geometry with RTMODE, the patches were approximated by sectors of rings as shown in Fig. 2. The patches adjacent to the reflector were separated into two rings to model the buffer fuel zone.

The r - z geometry model used for RZMODE and GAMBLE⁹ is shown in Fig. 3.

In neither the r - θ nor the r - z model were the borated reflectors included. This avoids some convergence difficulties in the eigenfunction expansion method used in RTMODE and RZMODE (see Section 5.3).

Fig. 2. The r - θ geometric model.

Fig. 3. The r - z geometric model.

3. SEQUENCE OF CALCULATIONS

Pulsed-neutron experiments are planned for ten control rod configurations. For most of these configurations, all of the calculations given in Table 1 were performed.

The first three calculations were used to produce two-group macroscopic cross sections. SCANAL⁶ is a three-dimensional, single channel synthesis, diffusion code. It was run to generate four-group fluxes, which the cross section codes, SLMAC⁷ and SIGSAG,⁸ used to produce two-group macroscopic cross sections. Next a two-dimensional diffusion code, GAMBLE,⁹ was run using these cross sections in r-z geometry to obtain k_{eff} . Although k_{eff} is also calculated by SCANAL, the two-group k_{eff} obtained by GAMBLE is more consistent with the two-group α_0^p which is calculated by RZMODE. RZMODE³ is a diffusion code which simulates the pulsed-neutron experiment in r-z geometry by calculating the first several prompt and delayed modes of Eq. 1.

In r-z geometry, azimuthal symmetry is necessarily assumed. However, the core loading is not symmetric azimuthally and this asymmetry is accentuated by control rod withdrawal in configurations 4 and 5. An angular correction factor can be obtained from two r- θ geometry calculations, though, using RTMODE.³ The first calculation constrains the modes to be angle-independent, while the second calculation gives detailed r- θ modes.*

* Both RTMODE calculations use axial bucklings obtained from RZMODE.

TABLE 1
SEQUENCE OF CALCULATIONS FOR EACH CONFIGURATION

1. SCANAL — Generate four-group fluxes.
2. SLMAC — Use fluxes to obtain two-group macroscopic cross sections for top and bottom slices of core.
3. SIGSAG — Use fluxes to obtain two-group macroscopic cross sections for top and bottom reflectors.
4. GAMBLE — Calculate k_{eff} in r-z geometry.
5. RZMODE — Calculate r-z kinetic modes.
6. RTMODE — Calculate average radial dependence of fundamental mode for use in determining angular correction factor. Use axial bucklings from 5.
7. RTMODE — Calculate r- θ kinetic modes using axial bucklings from 5. Verify that amplitudes of higher harmonics are small. Calculate angular correction factor.
8. RZPLOT — Make contour plots of r-z kinetic modes.
9. TILE — Make contour plots of r- θ kinetic modes.
10. RZMODE — Using angular correction factor (obtained from 6 and 7) and optimum source and detector locations (obtained from 8 and 9), calculate time response of thermal flux at detector.

The ratio of the fundamental mode flux from the second calculation to that from the first gives a space-dependent correction factor, $f(r,\theta)$. This correction factor can be used to synthesize the fundamental r - θ mode with the first few r - z modes by*

$$\phi^2(r,\theta,z,t) = f(r,\theta) \left(\sum_{i=0}^{N^p-1} a_i^p \phi_i^{2p}(r,z) e^{\alpha_i^p t} + \sum_{i=0}^{N^d-1} a_i^d \phi_i^{2d}(r,z) e^{\alpha_i^d t} \right) \quad (5)$$

(see Eq. 3.1 of Ref. 3). The superscript 2 stands for the thermal group. The higher r - θ harmonics have small amplitudes and hence are neglected.

Finally RZMODE can be run again to calculate this time response, but only for specified source and detector locations. These locations are determined by examining r - z and r - θ mode contour plots as discussed in the next section. The contour plots are generated by RZPLOT and TILE.¹⁰

* Further discussion of this synthesis is given in Section 5.1.

4. SUMMARY OF RESULTS

The calculated reactivity, ρ , and the fundamental prompt decay constant, α_o^p , for each configuration are given in Table 2. The reactivity,

$$\rho \equiv \frac{k_{eff}^{-1}}{k_{eff}} \quad (6)$$

was obtained from the k_{eff} calculated by GAMBLE. The decay constant, α_o^p , was calculated by RZMODE. Both codes used the same two-group cross sections. Also given in Table 2 is the inferred generation time,

$$\Lambda = \frac{\rho - \beta}{\alpha_o^p} \quad (\text{see Eq. 3}) \quad (7)$$

These generation times will be recalculated prior to the experiments using 7-group cross sections for the as-built fuel loading as discussed in Sections 5.4 and 5.7.

When Λ and α_o^p are plotted against k_{eff} (see Fig. 4), both exhibit a strong correlation as would be expected from Eq. 7. The limited scatter is due to the fact that for a given k_{eff} , the generation time depends to some extent upon the particular control rod configuration. The general increase in the generation time, Λ , with decreasing k_{eff} is due to an increasing number of neutrons spending most of their life in the reflector where absorption is small.

TABLE 2
REACTIVITIES, DECAY CONSTANTS, AND GENERATION TIMES

No.	Configuration	k_{eff}	ρ	α^p (sec ⁻¹)	Λ (msec)
1	All temporary absorbers in, rods 2, 4, 6 in	0.858	-0.166	-150	1.15
2	All temporary absorbers in, rods 2, 4, 6 out	0.922	-0.085	-147	0.63
3	All rods in	0.895	-0.117	-137	0.91
4	Rod 30 out	0.947	-0.056	- 90	0.70
5	Rods 30, 31 out	0.991	-0.009	- 27	0.59
6	Rod group 3C out	0.934	-0.071	-126	0.62
7	Rod groups 3C, 2A out	0.968	-0.033	-103	0.39
8	Rod groups 3C, 2A, 4B out	0.976	-0.025	- 77	0.42
9	Rod groups 3C, 2A, 4B, 4E out	0.993	-0.007	- 32	0.44
10	Rods in normal sequence out to near critical	0.999	-0.001	- 19	0.42

Fig. 4. Generation time and decay constant vs. k_{eff} .

The optimum source location depends upon the shape of the fast adjoint flux. This follows from the expression for the amplitude of the i th mode given by Eq. 2. For two groups,

$$\begin{aligned}
 a_i &= \frac{\Delta t}{\alpha_i/R} \langle \phi_i^+, S \rangle \\
 &= \frac{\Delta t}{\alpha_i/R} \sum_{g=1}^2 \int \phi_i^{+,g}(\vec{r}') S^g(\vec{r}') d\vec{r}' .
 \end{aligned} \tag{8}$$

However, since the source produces only fast neutrons,

$$S^g(\vec{r}') = 0 \quad \text{unless } g=1 .$$

Thus

$$a_i = \frac{\Delta t}{\alpha_i/R} \int \phi_i^{+,1}(\vec{r}') S^1(\vec{r}') d\vec{r}' . \tag{9}$$

For the prompt modes,

$$|\alpha_i^P/R| \gg 1$$

for typical pulse rates ($R \approx 1 \text{ sec}^{-1}$), so that Eq. 9 simplifies to

$$a_i^P = \Delta t \int \phi_i^{+,1P}(\vec{r}') S^1(\vec{r}') d\vec{r}' . \tag{10}$$

Since $S^1(\vec{r}')$ is localized about the source position, $\vec{r}'$, the amplitude of the i th mode can be made to vanish by placing the source at a node (zero) of $\phi_i^{+,1}(\vec{r}')$.

The optimum detector location depends upon the shape of the thermal direct flux. This follows by noting that the response of a thermal detector at $\vec{r}$ is proportional to the thermal flux $\phi^2(\vec{r}, t)$, given by Eq. 1. Thus by placing the detector at a node of $\phi_1^2(\vec{r})$, the i th mode will not contribute to the thermal time response.

The above considerations permit us to position the source and detector so as to effectively eliminate the contributions of the first few prompt harmonics in Eq. 1. The other prompt harmonics are of little concern, since they decay very rapidly. The delayed harmonics are also of little concern, since they have very small amplitudes. Consequently, for simplicity we will use "harmonics" to mean "prompt harmonics" in the rest of this report.

The following rules have been used to determine the optimum source and detector locations:

1. Place the source in the central patch to eliminate the first few r - θ harmonics.*
2. Choose the axial position of the source to eliminate the first r - z harmonic.
3. Choose the (r, z) position of the detector on the nodal contour of the second r - z harmonic where the fundamental r - z mode is a maximum.*
4. Now that r is determined, choose the θ position of the detector where the fundamental r - θ mode is a maximum.*

* The source and detector are restricted to reserve shutdown holes, one per patch.

Implementation of these rules is facilitated by examining contour plots of the kinetic modes in which the nodal contours are evident. As an example, contour plots of the first six $r-\theta$ modes and the first three $r-z$ modes for configuration 3 are shown in Figs. 5-13. Examination of Figs. 6-10 provides justification for rule 1. The nodal contours of the second, third, and fourth $r-\theta$ harmonics pass through the central patch, while the nodal contours of the first and fifth $r-\theta$ harmonics run along the boundary of the central patch. As a result, the amplitudes of the first five higher harmonics are considerably smaller than that of fundamental mode as shown in the third column of Table 4. Rule 2 and Fig. 12 show that the source should be just above the core midplane. Rule 3 and Figs. 11 and 13 show that the detector should be near the side reflector midway between the top core-reflector interface and the core midplane. Finally rule 4 and Fig. 5 show that the detector should be in patch 30 or 31. The optimum source and detector locations which have been determined in this manner are given for all configurations in Table 3. Since three detectors are available for the experiments, Table 3 contains the optimum as well as two near-optimum detector locations for each configuration.

Once the experimental arrangement is specified, RZMODE can be run again to give the time response of the thermal flux at the detector. The resulting time response is shown for each configuration in Fig. 14. As can be seen from the figure, an exponential die-away with the proper decay constant (see Table 2) is to be expected for each configuration when the source and detector are located as specified in Table 3. A pulse rate of 1 pulse/sec has been assumed for each configuration except 10, for which a rate of 1 pulse/10 sec was assumed. The pulse rate determines the level of the delayed neutron background (see Eq. 9). As shown in Fig. 14, for a fixed pulse rate this background level rises as the reactor approaches critical.

CONTOUR
VALUES
9.00-03 1
8.00-03 2
7.00-03 3
6.00-03 4
5.00-03 5
4.00-03 6
3.00-03 7
2.00-03 8
1.00-03 9
0.00 10
-1.00-03 11
-2.00-03 12
-3.00-03 13
-4.00-03 14
-5.00-03 15
-6.00-03 16
-7.00-03 17
-8.00-03 18
UNITS DIGIT
OF CONTOUR
PLOTTED

MODE 4
THERMAL FLUX
ALPHA -1.3429548+02
MAXIMUM (H) 6.61-03
MINIMUM (L) 1.07-08

Fig. 5. Fundamental $r-\theta$ mode, thermal flux for configuration 3.

CONTOUR
VALUES
1.80+03 1
1.60+03 2
1.40+03 3
1.20+03 4
1.00+03 5
8.00+02 6
6.00+02 7
4.00+02 8
2.00+02 9
0.00 10
-2.00+02 11
-4.00+02 12
-6.00+02 13
-8.00+02 14
-1.00+03 15
-1.20+03 16
-1.40+03 17
-1.60+03 18
-1.80+03 19
-2.00+03 20
-2.20+03 21
-2.40+03 22
-2.60+03 23
-2.80+03 24
UNITS DIGIT
OF CONTOUR
PLOTTED

MODE 1
FAST ADJ. FLUX
ALPHA -1.4278934+02
MAXIMUM (H) 1.28+03
MINIMUM (L) -1.07+03

Fig. 6. First r - θ harmonic, fast adjoint for configuration 3.

CONTOUR
VALUES

1.80+03	1
1.60+03	2
1.40+03	3
1.20+03	4
1.00+03	5
8.00+02	6
6.00+02	7
4.00+02	8
2.00+02	9
0.00	10
-2.00+02	11
-4.00+02	12
-6.00+02	13
-8.00+02	14
-1.00+03	15
-1.20+03	16
-1.40+03	17
-1.60+03	18
-1.80+03	19
-2.00+03	20
-2.20+03	21
-2.40+03	22
-2.60+03	23
-2.80+03	24

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MODE 3
FAST ADJ. FLUX
ALPHA -1.4455949+02
MAXIMUM (H) 1.31+03
MINIMUM (L) -7.17+02

Fig. 7. Second $r-\theta$ harmonic, fast adjoint for configuration 3.

CONTOUR
VALUES
1.80+03 1
1.60+03 2
1.40+03 3
1.20+03 4
1.00+03 5
8.00+02 6
6.00+02 7
4.00+02 8
2.00+02 9
0.00 10
-2.00+02 11
-4.00+02 12
-6.00+02 13
-8.00+02 14
-1.00+03 15
-1.20+03 16
-1.40+03 17
-1.60+03 18
-1.80+03 19
-2.00+03 20
-2.20+03 21
-2.40+03 22
-2.60+03 23
-2.80+03 24
UNITS DIGIT
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PLOTTED

MODE 2
FAST ADJ. FLUX
ALPHA -1.4819440+02
MAXIMUM (H) 7.91+02
MINIMUM (L) -1.37+03

Fig. 8. Third $r-\theta$ harmonic, fast adjoint for configuration 3.

CONTOUR
VALUES
1.80+03 1
1.60+03 2
1.40+03 3
1.20+03 4
1.00+03 5
8.00+02 6
6.00+02 7
4.00+02 8
2.00+02 9
0.00 10
-2.00+02 11
-4.00+02 12
-6.00+02 13
-8.00+02 14
-1.00+03 15
-1.20+03 16
-1.40+03 17
-1.60+03 18
-1.80+03 19
-2.00+03 20
-2.20+03 21
-2.40+03 22
-2.60+03 23
-2.80+03 24
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MODE 5
FAST ADJ. FLUX
ALPHA -1.5012997+02
MAXIMUM (H) 1.28+03
MINIMUM (L) -9.72+02

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Fig. 9. Fourth r-θ harmonic, fast adjoint for configuration 3.

Fig. 10. Fifth r- θ harmonic, fast adjoint for configuration 3.

CONTOUR
 VALUES
 4.50-04 1
 4.00-04 2
 3.50-04 3
 3.00-04 4
 2.50-04 5
 2.00-04 6
 1.50-04 7
 1.00-04 8
 5.00-05 9
 0.00 10
 -5.00-05 11
 -1.00-04 12
 -1.50-04 13
 -2.00-04 14
 -2.50-04 15
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THERMAL FLUX
 ALPHA -1.3710911+02

MOD. 1
 MAXIMUM (H) 1.59-04
 MINIMUM (L) 0.00

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Fig. 11. Fundamental r-z mode, thermal flux
 for configuration 3.

FAST ADJ. FLUX
ALPHA -1.6455758+02

MODE 2
MAXIMUM (H) 2.73+01
MINIMUM (L) -3.73+01

CONTOUR
VALUES
9.00+01 1
8.00+01 2
7.00+01 3
6.00+01 4
5.00+01 5
4.00+01 6
3.00+01 7
2.00+01 8
1.00+01 9
0.00 10
-1.00+01 11
-2.00+01 12
-3.00+01 13
-4.00+01 14
-5.00+01 15
UNITS DIGIT
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Fig. 12. First r-z harmonic, fast adjoint for configuration 3.

Fig. 13. Second r-z harmonic, thermal flux for configuration 3.

TABLE 3
OPTIMUM SOURCE AND DETECTOR LOCATIONS

No.	Configuration	Source Location	Detector Locations			
		Patch 1 z(cm)	Patch 16 z(cm)	Patch 30 z(cm)	Patch 31 z(cm)	
1	All temporary absorbers in, rods 2, 4, 6 in	240	130	100	100*	
2	All temporary absorbers in, rods 2, 4, 6 out	280	240	80	80*	
3	All rods in	240	130	100	100*	
4	Rod 30 out	210	120	100*	100	
5	Rods 30, 31 out	200	120	100	100*	
6	Rod Group 3C out	220	190	100	100*	
7	Rod groups 3C, 2A out	260	150	300	300*	
8	Rod groups 3C, 2A, 4B out	230	240	340	340*	
9	Rod groups 3C, 2A, 4B, 4E out	210	240	80*	80	
10	Rods in normal sequence out to near critical	180	240	80*	80	

Note: Both source and detectors are restricted to reserve shutdown holes, one per patch. The distance z is measured downward from the top core-reflector interface. The optimum detector location is indicated by a *.

Fig. 14. Expected time response of thermal flux following pulse.

5. DISCUSSION

5.1 The Separability Approximation

In order to simplify the simulation, a number of approximations have been made in the analysis. The most important approximation results from our inability to calculate three-dimensional kinetic modes explicitly. Accordingly we make a separability approximation. In particular we assume that the three-dimensional time response of the thermal flux can be given by

$$\phi^2(r, \theta, z, t) = f(r, \theta) \left(\sum_{i=0}^{N^p-1} a_i^p \phi_i^{2p}(r, z) e^{\alpha_i^p t} + \sum_{i=0}^{N^d-1} a_i^d \phi_i^{2d}(r, z) e^{\alpha_i^d t} \right) \quad (5)$$

where $f(r, \theta)$, $\phi_i^{2p}(r, z)$, and $\phi_i^{2d}(r, z)$ are obtained from two-dimensional calculations. The angular correction factor, $f(r, \theta)$, is derived solely from the fundamental r - θ mode. Thus we assume that the higher r - θ harmonics are negligible and that the first few r - z modes have the same θ -dependence as the fundamental mode.

The last column of Table 4 shows that the contributions of the higher r - θ harmonics for configuration 3 are indeed smaller than that of the fundamental mode, although that of the first harmonic is not strictly negligible even 20 msec after the pulse. The contributions of the second and higher harmonics, however, are quite small and decrease rapidly with increasing i . The sixth harmonic (not shown in Table 4) has an α_i of -207 sec^{-1} and a relative contribution of 4×10^{-6} .

TABLE 4

RELATIVE CONTRIBUTIONS OF r - θ KINETIC MODES
TO TIME RESPONSE FOR CONFIGURATION 3

i	α_i (sec^{-1})	$\frac{a_i}{a_0^p}$ ^a	$\frac{\phi_i^2(\vec{r})}{\phi_0^{2p}(\vec{r})}$ ^b	$\frac{e^{\alpha_i t}}{e^{\alpha_0^p t}}$ ^c	$\frac{a_i \phi_i^2(\vec{r}) e^{\alpha_i t}}{a_0 \phi_0^{2p}(\vec{r}) e^{\alpha_0^p t}}$
Prompt					
0	-134	1.00	1.00	1.00	1.00
1	-143	0.43	-0.65	0.84	-0.23
2	-145	0.22	-0.24	0.80	-0.04
3	-148	-0.03	0.41	0.76	-0.009
4	-150	0.02	-0.005	0.73	-0.00007
5	-156	-0.15	-0.02	0.64	0.002
Delayed					
0	-0.075	0.002	0.93	14.56	0.03

a For the source in the central patch.

b For the detector in patch 31.

c For $t=20$ msec.

For comparison, the contribution of the fundamental delayed mode is also given in Table 4. This contribution, of course, increases with time following the pulse.

Calculation of the relative contributions of the higher harmonics was the primary purpose of the relatively expensive r - θ calculation. The secondary purpose was the calculation of the correction factor, $f(r, \theta)$, for prediction of the flux level at the detector. Correction factors for each configuration are listed in Table 5.

TABLE 5
ANGULAR CORRECTION FACTORS

No.	Configuration	f(r,θ)		
		Patch 16	Patch 30	Patch 31
1	All temporary absorbers in, rods 2, 4, 6 in	2.17	2.47	3.09
2	All temporary absorbers in, rods 2, 4, 6 out	1.82	2.49	3.14
3	All rods in	1.60	2.27	2.85
4	Rod 30 out	1.52	3.78	3.80
5	Rods 30, 31 out	3.70	3.99	5.14
6	Rod group 3C out	1.60*	2.27*	2.85*
7	Rod groups 3C, 2A out	1.25	1.83	2.14
8	Rod groups 3C, 2A, 4B out	1.15	1.07	1.19
9	Rod groups 3C, 2A, 4B, 4E out	0.95	2.26	1.91
10	Rods in normal sequence out to near critical	0.95**	2.26**	1.91**

* Assumed to be the same as for configuration 3.

** Assumed to be the same as for configuration 9.

The assumption of the same θ -dependence for the higher r-z modes as for the fundamental mode is an approximation that cannot be checked without an explicit three-dimensional calculation. For an azimuthally symmetric core, of course, this approximation would be exact.

5.2 Axial Bucklings

In order to perform the r- θ calculations, it is necessary to assume some axial bucklings. One way to obtain these bucklings is to perform an α -search using SCANAL.* This has been done for one configuration, but is rather time-consuming. A simpler procedure is to obtain them from RZMODE. Unfortunately the two sets of bucklings differ significantly, although it is not known why. The SCANAL bucklings are probably more accurate. However, for simplicity we used the RZMODE bucklings in our RTMODE calculations as mentioned in Section 3. Since the RTMODE decay constants are quite sensitive to these bucklings, the agreement between the α_0^p 's calculated by RZMODE and RTMODE is poor, particularly near critical. However, this discrepancy should not affect our conclusion regarding the small contributions of the higher r- θ harmonics.

5.3 Accuracy of Eigenfunction Expansion

Both RTMODE and RZMODE represent the two-dimensional kinetic modes as a series of homogeneous reactor modes, $W_v(\vec{r})$, e.g.

$$\phi_i^{2p}(\vec{r}) = \sum_{v=1}^N W_v(\vec{r}) (x_i^p)_v \quad (11)$$

where the series is truncated after N terms (see Eq. 3.10 of Ref. 3). Because the reactor is quite heterogeneous, a large number of terms is required. In RTMODE 80 terms are used, while in RZMODE 60 terms are used. More terms are required in r- θ than r-z geometry because the homogeneous expansion modes are doubly degenerate.

* The static mode bucklings (from a k-calculation) are quite different from the kinetic mode bucklings (from an α -search) and should not be used.

For the fundamental mode, the accuracy of Eq. 11 can be checked by comparison with an explicit finite difference solution obtained by an α -search. Thus for the fundamental r- θ mode, we can compare RTMODE and GAUGE,¹¹ while for the fundamental r-z mode, we can compare RZMODE and GAMBLE.

The latter comparison was performed for the first three configurations. In each case both RZMODE and GAMBLE gave the same α_0^P to within 0.5%. For configuration 3, the thermal flux shapes are compared in Fig. 15. As seen there, RZMODE underpredicts the flux peak in the reflector. The agreement in the core is very good, however, although this is not apparent from the figure.

A similar comparison between RTMODE and GAUGE was performed for configuration 5. Here the agreement was considerably poorer, even when the same axial bucklings were used. This could be due to the approximate geometric modelling in RTMODE (see Section 2) as well as to the limitation to 80 terms in the series expansion.

5.4 The Two-Group Model

All of the results presented in the previous section are based upon two-group calculations. To check the adequacy of using only two groups, two 7-group GAMBLE calculations were performed to obtain k_{eff} and α_0^P for configuration 3. The results of the 2- and 7-group calculations are compared in Table 6. The lack of any large disagreement between the two sets of results suggests that the source and detector locations can be adequately specified by a two-group analysis.

However, for use in interpreting the experiments by the inhour equation,

$$\rho = \Lambda \alpha_0^P + \beta , \quad (3)$$

Fig. 15. Comparison of fundamental r-z mode, thermal flux, as calculated by RZMODE and GAMBLE.

TABLE 6
COMPARISON OF 2- AND 7-GROUP RESULTS
FOR CONFIGURATION 3

	2-group	7-group
k_{eff}	0.895	0.905
ρ	-0.117	-0.105
α_0^p (sec ⁻¹)	-137	-129
Λ (msec)	0.91	0.87

an accurate value for Λ is required. Consequently, prior to the experiments, the Λ 's will be recalculated using 7-group cross sections for the as-built fuel loading (see Section 5.7).

5.5 Sensitivity Study

Because of the approximations made in the simulation model, it is desirable that the locations given in Table 3 be "broad" optima rather than "narrow" ones. This was investigated for configuration 3 by determining the sensitivity of the time response to various changes in the source and detector locations. In addition to the three cases corresponding to the detector locations listed in Table 3, time responses for eight other cases were simulated. In particular, the detector was also moved to patches 14, 15, 29, and 32. In addition, for the detector fixed in patch 31, the source and detector were separately moved up and down 50 cm. For all eleven cases, the maximum variation in α_0^p inferred from a least squares analysis of the time response curves was +7% and -3% about $\alpha_0 = -137 \text{ sec}^{-1}$.

5.6 Kinetic Distortion

Kinetic distortion is the space- and energy-dependent difference between the fundamental prompt and delayed modes. This distortion increases as the reactor becomes more subcritical. Preskitt et al.⁵

reported large distortion effects in the experiments performed on the Peach Bottom HTGR. Similar distortion effects are to be expected for the Fort St. Vrain HTGR. This is shown in Fig. 16 by comparing the fundamental prompt and delayed r-z modes for configuration 3. These distortion effects will have little effect on our analysis of the experiments, however, since the area-ratio method (see Section 1) will not be used.

5.7 Fuel Loading

The fuel loading used in all of the calculations described in this report was that in the design specifications. The loading which was actually fabricated, though, is somewhat different. Consequently, some of the simulations will be redone for the as-built loading to assess the effect of the loading difference on the optimum source and detector locations. As noted in Section 5.4, all of the Λ calculations will be redone.

Fig. 16. Comparison of fundamental prompt and delayed r-z mode, thermal flux for configuration 3.

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