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DESIGN OF MODIFIED QUADRUPOLE MAGNETS

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DESIGN OF MODIFIED QUADRUPOLE MAGNETS

Abstract

Modifications to the shape of ideal quadrupole magnets were investigated, using an electrolytic tank method. Magnets with straight and with exponential pole faces were considered with 1:1, 1:2, and 1:3 apertures. The additional field introduced by these modifications was expressed in an harmonic series and the coefficients of this series plotted against the parameters characterizing the modifications made.

Pure quadrupole fields can be generated by magnets of the general cross-sectional form:

where the shape of the pole tips is given in the first quadrant by:

$$\text{Region 1: } \frac{y}{b} - \frac{2}{1-t} = - \frac{1+t}{1-t} \left(\frac{x}{a} \right)^{\frac{1-t}{1+t}}$$

$$\text{Region 2: } \frac{x}{a} - \frac{2}{1-t} = - \frac{1+t}{1-t} \left(\frac{y}{b} \right)^{\frac{1-t}{1+t}}$$

(t is a parameter which usefully ranges from -1 to $+\infty$)

and in the other quadrants by obvious modifications of these.

A constant current density, J_0 (in emu), is maintained in the copper windings in the direction indicated. Thus, if $\hat{z}$ is a unit vector in the z direction, the current density $\vec{J}_0$ is given by:

$$\begin{aligned} \text{First quadrant} \quad \text{Region 1: } \vec{J}_0 &= J_0 \hat{z} \\ \text{Region 2: } \vec{J}_0 &= -J_0 \hat{z} \end{aligned}$$

The magnetic field can be written in terms of a scalar potential:

$$\vec{B} = +\nabla V \quad (B_z = 0 \text{ in these magnets.})$$

Edge effects are neglected)

The scalar potential within the aperture is then:

$$V = (1+t) \pi J_0 r^2 \sin 2\theta \quad \text{with the convention}$$

where $\vec{B}$ is in gauss, (J_0 in emu = units of 10 amps/cm²).

Two particular cases are those with the parameter t equal to zero and one. These are, respectively, the so-called Lozenge magnet and what I have called the Exponential magnet. The equations of the pole tips in these cases are as follows:

Lozenge Region 1: $\frac{y}{b} - 2 = -\frac{x}{a}$

 Region 2: $\frac{x}{a} - 2 = -\frac{y}{b}$

Exponential Region 1: $\frac{x}{a} = e^{1-y/b}$

 Region 2: $\frac{y}{b} = e^{1-x/a}$

These look like:

For two magnets with the same aperture and producing the same quadrupole field, the power required to maintain the field is a function of the parameter t . Specifically, if w_t is the power required to maintain a field in a magnet characterized by t and w is the power required to maintain the same field in a Lozenge magnet, then:

$$\frac{w_t}{w} = \frac{1}{1+t}$$

Thus, for example, the Exponential magnet requires half the power needed by a Lozenge magnet to produce a comparable field. The current density is also reduced by a factor of $1/(1+t)$. Therefore, it is desirable to consider the construction of curved pole faced magnets. However, as soon as the parameter t is increased considerably above zero, the magnet becomes very large compared to the aperture, and in the limit, as t tends to one, the magnet becomes infinitely large.

It was the purpose of this investigation to consider the effect of modifying the ideal pole shapes in order to make them practicable. The general modification considered was a cut of the form:

A useful list of relations between the parameters characterizing the cuts is appended. These cuts were always made in such a manner as to leave the total cross-sectional area of the copper windings unaltered. However, one experiment is appended in which this was not the case, the cut being of the form:

The cases of exponential ($t=1$) and straight ($t=0$) pole tips were investigated as being convenient for interpolation to intermediate values of t and, should it be required, extrapolation beyond $t=1$. Square and rectangular (1:2 and 1:3) apertures were considered.

It can be shown that the modification to the quadrupole field introduced by cuts in the pole faces is equivalent to the field produced by appropriate current sheets along the surfaces of the

cut. An equivalent electrostatic model can be formed in which current sheets are replaced by potential gradients (which turn out to be linear for the particular cuts considered) and iron boundaries by electrically conducting boundaries.* This electrostatic model can then be investigated by means of an electrolytic tank.** It is worth noting that the modifications to the quadrupole field due to cuts in the pole surfaces are measured directly by this method rather than as a difference between two measurements. An a.c. (400 cps) bridge was used, but balanced only with resistors. Potentials were measured at a fixed series of points and then analyzed in an harmonic expansion. With the convention for angles indicated previously and used consistently throughout this report it is seen that the symmetries of the magnets imply that the coefficients of all terms in $\cos(n\theta)$ vanish, as do those of the sine of all odd powers of θ . Moreover, for square apertures the coefficients of $\sin(4n\theta)$, n integral, vanish.

In calculating the coefficients of the harmonic expansion from measurements of the potential at different points it was assumed that they vanish beyond a certain power. In most cases, that was chosen to be the twelfth harmonic, occasionally the tenth. In the results, which are displayed in graphic form, the coefficients, K_{2n} , of the potential expansion of the field due to the cuts when the uncut magnet would generate an $(\frac{r}{a})^2 \sin(2\theta)$ potential of unit amplitude are plotted against a measure of the cut size. Thus, the total potential generated by the cut magnet is:

$$V = (1+K_2)(\frac{r}{a})^2 \sin 2\theta + K_4(\frac{r}{a})^4 \sin 4\theta + K_6(\frac{r}{a})^6 \sin 6\theta + \dots$$

* See Appendix 2.

** The possibility that the fields generated by the cuts might be measured directly was suggested by G. Lambertson of the Lawrence Radiation Laboratory, Berkeley (private conversation with P.F. Cooper, Jr.).

It might be noted at this point that within the copper windings the magnetic field must be represented by a vector potential. Nevertheless, the contribution due to cuts may still be represented by a scalar potential, and the coefficients $K_2, K_4, \dots$ are still appropriate for the expression of this potential.

The magnetic field within the aperture can be written:

$$\begin{aligned}\vec{B} &= \vec{\nabla} V = \vec{\nabla} \left\{ (1+K_2) \left(\frac{r}{a}\right)^2 \sin 2\theta + K_4 \left(\frac{r}{a}\right)^4 \sin 4\theta + K_6 \left(\frac{r}{a}\right)^6 \sin 6\theta + \dots \right\} \\ &= \vec{b} + \vec{\beta} \\ \text{where } \vec{b} &= \vec{\nabla} \left\{ (1+K_2) \left(\frac{r}{a}\right)^2 \sin 2\theta \right\} \\ \text{and } \vec{\beta} &= \vec{\nabla} \left\{ K_4 \left(\frac{r}{a}\right)^4 \sin 4\theta + K_6 \left(\frac{r}{a}\right)^6 \sin 6\theta + \dots \right\}\end{aligned}$$

Then the value of $\frac{|\beta|}{|B|}$ is a measure of the deviation from pure quadrupole field. This quantity was calculated on a computer for a rectangular mesh of points covering the aperture, these points being $a/5$ apart in the x direction and $b/5$ apart in the y direction. The maximum value of $|\beta|/|B|$ thus obtained is termed the "maximum error field" and is graphed together with the K coefficients (in dot-dash lines). Generally, the maximum computed value occurred at the corner of the aperture, (a,b) . Where this was not the case, the point at which the maximum value of $|\beta|/|B|$ was found is indicated on the graph using the standard notation: (x-coordinate, y-coordinate).

On each graph one quadrant of the magnet cross-section is drawn. The rest of the cross-section is obtained by reflections in the x and y axes. In the case of square apertures, the effects of cuts in both regions 1 and 2 (see fig. on page 1) were measured simultaneously. However, for rectangular apertures the cuts in these two regions are not of the same shape and consequently were considered separately. The procedure adopted was as follows: First, the minimum convenient cut was made in region 1 (in the

case of the Lozenge magnet, none at all) and the effects of varying sizes of cut in region 2 were investigated. No potential gradient was applied in region 1, so that the coefficients obtained in these measurements represent the field due to the cut in region 2 only. From these measurements the size of a cut in region 2 which gave rise to a maximum error field of less than 0.01 was determined, and this was then used in a second series of measurements in which size of cuts in region 1 was varied and their effects measured. In these second measurements, potential gradients were applied only to the cut in region 1, and consequently the coefficients plotted represented the field due to this cut alone. On each graph the cut which generates the coefficient shown is indicated on the diagram of the magnet cross-section by heavy lines. To get the coefficients which describe the field in the actual aperture, the coefficients appropriate to each of the two cuts are simply added.

The errors indicated on the graphs are estimates of the probable errors in the measurements.

No effort has been made to extend credits for all the ideas and suggestions on which this work was based, since very many people were involved. The work was carried out under the supervision of Professor E. M. Purcell.

APPENDIX 1

Below are listed some relations between the parameters characterising the cuts.

(See the figure at the bottom of page 2 for the meaning of these symbols.)

$$\begin{aligned} \text{Region 1:} \quad \delta &= \zeta + \frac{b}{2} (1+t) \left(\frac{\alpha}{a}\right)^{\frac{1-t}{1+t}} \\ &= \frac{2b}{1-t} - \frac{b}{2} \left(\frac{\alpha}{a}\right)^{\frac{1-t}{1+t}} \frac{(1+t)^a}{1-t} \quad \text{for } t \neq 1 \end{aligned}$$

$$\begin{aligned} \text{Region 2:} \quad \gamma &= \epsilon + \frac{a}{2} (1+t) \left(\frac{\beta}{b}\right)^{\frac{1-t}{1+t}} \\ &= \frac{2a}{1-t} - \frac{a}{2} \left(\frac{\beta}{b}\right)^{\frac{1-t}{1+t}} \frac{(1+t)^a}{1-t} \quad \text{for } t \neq 1 \end{aligned}$$

APPENDIX 2

The effect of cuts can be seen more fully using, for simplicity of representation, the example of a Lozenge magnet.

1. Without cuts, a pure quadrupole field is generated in the aperture and the magnetic field is everywhere perpendicular to the iron boundary.

2. The introduction of cuts would, if the quadrupole field remained unaltered, result in the field no longer being perpendicular to the iron boundaries of the cuts. In order to maintain the original quadrupole field current sheets would have to be introduced along the surfaces of the cuts, having the necessary strength to cancel the component of the magnetic field parallel to the cut faces. It turns out that for the cuts considered these current sheets are of uniform density. Thus, to maintain the original quadrupole field the following situation would be necessary:

3. The deviation from a pure quadrupole field is due to the failure to introduce such current sheets. That is, there must be added to the previous pure quadrupole field the field caused by the reverse of the currents which would have remedied the situation. The contribution to the actual field of the cuts alone is then:

4. The electrostatic equivalent of this situation is obtained by replacing all iron boundaries by electrically conducting surfaces and all (uniform) current sheets by (uniform) potential gradients:

The explanation given in no way depends on the magnet being of the Lozenge variety and holds as well for the general case. The condition for the current sheets to be of uniform density is that the surfaces of the cuts be parallel to one or other of the edges of the aperture.

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