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FORTAN SUBROUTINE RANGE:
CALCULATING THE RANGE-ENERGY RELATION
FOR CHARGED PARTICLES IN CHEMICAL ELEMENTS

Berkeley, California

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W. Peter Trower

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ABSTRACT

FORTTRAN subroutine RANGE calculates the differential energy loss and the range of charged particles, electrons and positrons excepted, with kinetic energies between 1 MeV and 500 BeV in any chemical element. With the results of the element calculations, the energy loss and range can be readily computed for chemical compounds and mixtures.

I. INTRODUCTION

A problem inseparable from any experimental particle physics is that of determining the energy loss of a charged particle as it passes through matter. We have constructed a computer program to easily give useful and accurate solutions to this problem. The program is constructed basically to evaluate the Bethe-Bloch equation for all charged particles except electrons and positrons. At low energies shell corrections are applied. At high energies density-effect corrections are applied. Since the Bethe-Bloch equation yields the differential energy loss of the particle in MeV/cm, we can obtain the range of the particle by direct integration, using low-energy proton experimental ranges to initiate the integration. The range of a particle is given as a function of energy or momentum in terms of three units: centimeters, gram-centimeters², and moles of electrons/centimeters². This calculation is made for any homogeneous target material that is a chemical element. Using the results of element calculations, we can readily compute the energy loss and range in chemical compounds and mixtures.

The results of representative calculations have been checked with experimental results and have been found to be in good agreement.

The casual user can save time by referring to Secs. III. A, III. C, and III. E.

II. THEORY

In our calculation we have considered only those processes by which a charged particle loses energy to the atomic electrons of a target material. Nuclear processes are excluded from consideration but must be taken into account by the user. For strongly interacting particles, an energy exists beyond which the range-energy relation has no meaning, because calculated ranges greatly exceed the mean free path for nuclear interactions. For protons this point is about $T_p \approx 1$ BeV but for muons $T_\mu \approx 11$ BeV. For weakly interacting particles there is also a range beyond which this calculation is invalid because pair production, bremsstrahlung, etc. become important.

The velocity of the charged particle is assumed to be very large compared to the velocity of the atomic electrons of the stopping material. This condition allows the calculation to be made with the atomic electron considered as stationary during the collision and the charged particle applying an impulse to it as it passes. At low particle energies this assumption is violated. Also at low energies electrons can be stripped from the stopping material. These two effects have defied precise theoretical calculations up to this time, so that we must rely on experimental data in this low-energy region.

It has also been assumed that the mass of the charged particle is much larger than that of the electron. The theory is therefore valid for all particles except electrons and positrons, for which the equations must be modified. We have chosen not to consider electrons and positrons.

The theory upon which this calculation is based has been worked out by a number of people,¹ and it manifests itself in an expression for the differential energy loss known as the Bethe-Bloch formula:

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4 n}{mv^2} \left\{ \ln \left(\frac{2mv^2}{I(1-\beta^2)} \right) - \beta^2 - \frac{\delta}{Z} - \frac{C}{Z} \right\}. \quad (1)$$

The quantities below are associated with the incident particle:

z - charge

v - velocity

P - momentum

M - mass

$T = E$ - kinetic energy

$$\beta = \frac{v}{c} = \frac{P}{T + M}$$

$$\gamma = (1 - \beta^2)^{-1/2} = \frac{T + M}{M}$$

$$\eta = \beta\gamma = \frac{P}{M}$$

The following quantities are associated with the target material:

- Z - atomic number
- A - atomic weight
- I - mean ionization potential
- n - number of electrons per cm³
- ρ - density
- C - shell corrections
- δ - density-effect correction
- x - distance traversed in the target

The following physical constants are also used:

- m - electron mass
- e - electron charge
- c - velocity of light
- N - Avagadro's number
- r_0 - classical radius of the electron
- h - Planck's constant.

The last two terms that appear in the brackets of Eq. (1) are correction terms whose effects are appreciable only over limited energy regions. The term δ represents the density-effect correction and is necessary because high-energy charged particles tend to polarize the stopping material.² Sternheimer formulates an expression for this effect as

$$\delta = 4.606X(\eta) + b + a[X_1 - X(\eta)]^g, \quad (2)$$

The terms X_0 , X_1 , a, g, and b are constants that are functions of the target material alone. The first four of these are determined by the fitting of Eq. (2) to experimental results.³ The term X is a kinematical factor and is given by

$$X = \log_{10} \eta.$$

If $X > X_1$, then $a = 0$, and if $X < X_0$, then $\delta = 0$. The term b, which involves the plasma frequency ν_p , is determined by

$$b = -[2 \ln \left(\frac{I}{h\nu_p} \right) + 1],$$

where

$$v_p = \left(\frac{ne^2}{\pi m} \right)^{1/2} \quad (4)$$

The term C is the shell-correction term and is appreciable only at low energies where the velocity of the incident particle is not great compared to the velocities of the orbital electrons. Barkas⁴ found that the best semiempirical fit to the data was obtained by a C that was a function of I and η :

$$C(I, \eta) = (0.422377\eta^{-2} + 0.0304043\eta^{-4} - 0.00038106\eta^{-6}) \times 10^{-6} I^2 \\ + (3.85019\eta^{-2} - 0.1667989\eta^{-4} + 0.00157955\eta^{-6}) \times 10^{-9} I^3. \quad (5)$$

We have used this expression.

The quantity called stopping power is defined as

$$\xi = - \frac{1}{\rho} \frac{dE}{dx} = \frac{4\pi z^2 N r_0^2 mc^2 Z}{A\beta^2} \left\{ \ln \left(\frac{2mc^2 \eta^2}{I} \right) - \beta^2 - \frac{\delta}{Z} - \frac{C}{Z} \right\}, \quad (6)$$

where the quantities of Eq. (1) have been expressed in a manner more susceptible to direct calculations.

In order to obtain the range of a particle in g/cm^2 , the following integration must be performed

$$R(T) = R(T_0) + \int_{T_0}^T \xi^{-1} dT', \quad (7)$$

where the value for $R(T_0)$ must be obtained from experiment because Eq. (1) does not hold for low energies. The most reliable and abundant experimental data to determine $R(T_0)$ are available for protons. We use these data as compiled by Barkas⁴ and in a manner similar to the one devised by Bichsel.⁵ We have chosen $T_0 = 8$ MeV for protons.

Since the fundamental calculation is made for protons and because $\frac{1}{Z} \frac{dE}{dx}$ is a universal function of particle velocity, it is possible simply to scale these results for particles other than protons. For a particle of mass M_i and charge z_i , the scaling factors are given by

$$R_i = R_p \left\{ \frac{M_i}{M_p} \frac{z_p^2}{z_i^2} \right\}, \quad T_i = T_p \left\{ \frac{M_i}{M_p} \right\}. \quad (8)$$

To calculate dE/dx in mixtures and compounds, we follow the approach of Thompson.⁶ As an example let us consider a molecule of mass M of two elements α and β with atomic weights A_α and A_β and molecular weight A_M . Suppose they each have, respectively, a and b atoms per molecule. Generating the differential energy loss dE/dx for each of the constituent elements, we can construct

$$\left(\frac{dE}{dx}\right)_M = \left(\frac{dE}{dx}\right)_\alpha a \frac{\rho_M A_\alpha}{\rho_\alpha A_M} + \left(\frac{dE}{dx}\right)_\beta b \frac{\rho_M A_\beta}{\rho_\beta A_M} \quad (9)$$

for the molecular substance. The generalization of this method to more complex structures is obvious.

III. PROGRAM

A. Calling Program

This is a five-card program that is used to designate the input (IN) and output (IO) tapes for the user's particular monitor system and to call the subroutine RANGE.

Example:

```
COMMON IN, IO
IN = 2
IO = 3
CALL RANGE
END
```

B. Subroutine RANGE

This subroutine is written in FORTRAN II language for the purpose of solving Eq. (2) of Sec. I. A glossary of the mnemonics is given in Appendix A and a listing of the subroutine appears in Appendix B. A flow chart is provided in Fig. 1 as a further guide.

C. Input of RANGE

There are four separate inputs to this subroutine. The contents of the first three will not concern the casual user.

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Fig. 1. Subroutine RANGE flow diagram.

LOW ENERGY DATA are provided to the user in the form of the listing displayed in Appendix C. The contents of the first 39 cards are based upon the results of experimentally determined ranges of protons in various materials. The experimental data were interpolated to determine the range for various I values (rows) at 1-MeV-energy intervals between 1 and 8 MeV (columns). A given row represents the ranges of protons for a given ionization potential and a column represents the ranges of protons for a given kinetic energy. The last four cards give the incrementing value of energy and the initialization energy for the various energy intervals that can be calculated.

DENSITY EFFECT DATA are shown in Appendix D. The 23 cards have the format

Word	1	2	3	4	5
Quantity	Z	X_0	X_1	a	g.

ELEMENT DATA are displayed in the listing in Appendix E. They are on a set of 46 cards, each with the format

Word	1	2	3	4	5	6	7	8
Quantity	Z	A_z	I_z	ρ_z	Z+1	A_{z+1}	I_{z+1}	ρ_{z+1}

These data are the best known to the author and were obtained from various sources.⁶

USER'S INPUT is one card of data, part of which he must specify and part of which he specifies only if he wishes to override the internally stored data with that of his own.

Word	1	2	3	4	5	6	7	8
Quantity	z	M	T_i	T_f	Z	A	I	ρ
Limits								
Min.	None	100	1	1	1	1	15	None
Max.	None	None	5×10^6	5×10^6	92	250	1100	None
Units	-	MeV	MeV	MeV	-	amu	eV	g/cm^3
Card location	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80

← -----Necessary----- -----Optional----- →

All data are to be entered as decimal (floating-point) numbers.

The user may stack as many data cards in the above format behind one another as he wishes. Each card will initiate a new pass through subroutine RANGE.

D. Output of RANGE

The output for each pass is printed in tabular form on serially numbered pages, with the variable values that were used in the calculation appearing on the first page. This is of special interest for the user who did not specify the optional data and wishes to know what values the program (see Appendix E) chose for him. Each page of output has a heading that identifies the columns of the generated table.

Three features of the output deserve mention. First, for particles other than protons, the proton results are scaled and interpolated so as to have integral values of kinetic energy. Second, dE/dx and dP/dx are not output for any particle with comparable proton kinetic energy of less than 8 MeV. This is because they are not calculated and the range is obtained in this region by a table look-up and interpolation. Third, if the calculation goes into an energy region where the density effect becomes important (calls for a range correction of 1% or more) and the Sternheimer constants are not available for this target material, then a statement to this effect will be printed.

E. Deck Setup and Running Instructions

Control Cards

Calling Program

Control Cards

Subroutine RANGE

DATA Card

LOW-ENERGY DATA	(43 cards)
DENSITY-EFFECT DATA	(23 cards)
ELEMENT DATA	(46 cards)
USER'S INPUT	

No sense switches or key settings are demanded by this subroutine.

Let us, as an example, consider protons incident on copper from 1 MeV to 105 MeV. We specify the $I = 322$ eV but let the stored data fill in the other two optional quantities. Our data card looks like this:

[illegible]

The output for this run is displayed in Appendix F.

IV. COMPARISON OF CALCULATION WITH EXPERIMENT

To test the accuracy of our calculation we compared our results with experimental results. A representative case is protons on copper. A number of data were available. They are displayed together with our calculated range at the energies specified in Table I. A pictorial display of this calculation is shown in Fig. 2.

Table I. Comparison of experimental with calculated values
for protons in copper ($I = 322$ eV).

T_p (MeV)	Calculation R_p (g/cm ²)	Exptl. R_p (g/cm ²)	Reference
9.938	0.21436	0.21860 ± 0.002	7
17.893	0.58788	0.59462 ± 0.005	7
73.0	6.83	6.91 ± 0.31	8
76.1	7.35	7.48 ± 0.34	8
79.1	7.86	8.01 ± 0.36	8
84.0	8.73	8.84 ± 0.40	8
86.9	9.26	9.41 ± 0.42	8
89.8	9.81	9.94 ± 0.45	8
96.2	11.06	11.21 ± 0.50	8
99.8	11.66	11.93 ± 0.53	8
102.1	12.27	12.52 ± 0.56	8
108.3	13.59	13.70 ± 0.61	8
111.3	14.24	14.37 ± 0.64	8
113.7	14.78	14.87 ± 0.66	8
337.9	91.49	91.84 ± 0.10	9
338.5	92.02	91.77 ± 0.11	9
339.7	93.07	92.69 ± 0.11	9
658	259.1	257.6 ± 0.9	10b

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Fig. 2. Graph of proton range in copper generated by program RANGE ($I = 322 \text{ eV}$).

Points plotted according to data of: Bloembergen and Van Heerden (Ref. 8) ($\bullet$); Bichsel (Ref. 7) ($\square$); Mather and Segrè (Ref. 9) ($\circ$); and Zrelov and Stoletov (Ref. 106) ($\blacksquare$). — Calculated from RANGE ($I = 322 \text{ eV}$).

ACKNOWLEDGMENTS

This program represents the efforts of several people. First of all, Professor Hans Bichsel of the University of Southern California wrote the original program to which this current version is heavily indebted. Miss Marylin Gode-von Aesch of this Laboratory extensively modified it under the direction of Dr. Walter Barkas. Finally, Mr. Howard Gordon of the University of Illinois provided the programming support necessary to round it into its final form. Mrs. Myrna Rice assisted with the modification to the original code which constitutes this revision.

This work was performed under the auspices of the U. S. Atomic Energy Commission.

APPENDICES

A. Glossary of Terms Used in Program

A(1-3) - coefficients for Eq. (5).

AT - A

B(1-3) - coefficients for Eq. (5)

BE - beginning energy (T_i)

BETA 2(1-800) - β^2

C1 - $\frac{e^2 N}{\pi m}$

C2 - m in eV

C3 - $4 \pi m r^2 N$

C4 - $\frac{1}{Z}$

CDEDX(1-800) - $\frac{4 \pi r^2 m c^2}{\beta^2}$

CON - $R(T_0)$

CONST - b

COR(1-800) - shell correction

DEDX(1-800) - $\frac{dE}{dx}$

DEL(1-11)-increment of energy

DENSE(1-800) - $\frac{\delta}{2}$

DPDX - $\frac{dP}{dx}$

E(1-800) - T

ED(1-11) - initialization energies

ENERGY(1-800)-temporary storage for TE(1-800)

EPM - I

ETASQ(1-800) - η^2

FE - final energy (T_f)

H - h

I - running index

IG - energy-increment counter

IK - index for highest energy required

IN - input-tape number

IO - output-tape number

IPAGE - page count

IR - counter

JL - index I + 2

L - initial I

M - final I

NASTY - counter used for 1 to 8 MeV

NOTE - = 0 if Sternheimer's data available; if $\neq 0$ it equals the index of the T where the error exceeds 1%

NZ - Z

P(1-800) - P

PCONST(1-100, 1-3) - A, I, and ρ for each element

PROD1 - coefficients for I^2 terms in shell correction

PROD2 - coefficients for I^3 terms in shell correction

Q1(1-800) - temporary storage for RC(1-800)

Q2(1-800) - temporary storage for RGPCM(1-800)

Q3(1-800) - temporary storage for RCM(1-800)

Q4(1-800) - temporary storage for DEDX(1-800)

$R(1-800) - \int_{T_0}^T \xi dT'$

R3(1-39, 1-8) proton ranges for $15 \leq I \leq 1100$ eV

RANGE(1-800)

RC(1-800) - range in moles of electrons/cm²

RCM(1-800) - range in centimeters

RGPCM(1-800) - range in grams/cm²

RHO - ρ

RMAS - M

ST(1-93, 1-4) - Sternheimer's constants: X_0 , X_1 , a, and m'

STX(1-800) - X

SUM1 - intermediate sum for shell correction (I^2 term)

SUM2 - intermediate sum for shell correction (I^3 term)

TAB - subroutine to interpolate tabular quantities

TE(1-8) - low-energy index

TI - floating representation of index I

TN - floating representation of index N

X(1-39) - low-energy data index

$$\text{XFN}(1-800) - \ln \left(\frac{2m \eta^2}{I} \right) - \beta^2$$

Z - Z

ZI - z

$$\text{ZETA} - \frac{T}{M}$$

B. LISTING OF SUBROUTINE RANGE

```
SUBROUTINE RANGE
COMMON IN, IO
DIMENSION A(3),B(3),BETA2(800),CDEDX(800),COR(800),DEDX(800),
1DEL(11),DENSE(800),DPDX(800),E(800),ED(11),ENERGY(800),ETASQ(800),
2P(800),PCONST(100,3),Q1(800),Q2(800),Q3(800),Q4(800),R(800),
3RC(800),RCM(800),RGPCM(800),R3(39,8),ST(93,4),STX(800),TE(8),
4X(39),XFN(800)
```

CONSTANTS

```
C
C
C
C1=(6.02486*4.80286**2*10.)/(3.14159*9.1085)
C2=510976.0
C3=4.0*3.14159*0.510976*6.02486*7.94027E-03
A (1)=-.00038106
A (2)=+.0304043
A (3)=+.422377
B (1)=+.00157955
B (2)=-.1667989
B (3)=+3.858019
E(1)=0.2
H=4.1354
IPAGE=1
NOTE=0
DO 100 I=1,92
DO 100 J=1,4
100 ST(I,J)=+0.0
C
C
C
READ INPUT - STANDARD DATA DECK'S
READ INPUT TAPE IN,1,((R3(I,J),J=1,8),I=1,39)
2 FORMAT(8F10.5)
READ INPUT TAPE IN,2, (DEL(I),I=1,11)
READ INPUT TAPE IN,2,(ED(I),I=1,11)
2 FORMAT(8F10.0)
101 READ INPUT TAPE IN,3,I,(ST(I,J),J=1,4)
3 FORMAT(I2,8X,4F10.5)
IF(I-92) 101, 102, 102
102 READ INPUT TAPE IN,
14, I, (PCONST(I,J),J=1,3), K, (PCONST(K,L), L=1,3)
4 FORMAT( 2(I 2,8X, 3F10.5))
IF(I-91) 102, 103, 103
C
C
C
READ INPUT - USER'S SELECTED DATA
103 READ INPUT TAPE IN, 5, ZI, RMASS, BE, FE, Z, AT, EPM, RHO
5 FORMAT(8F10.5)
NZ=Z
IF(AT) 104, 104, 105
104 AT=PCONST(NZ,1)
105 IF(EPM) 106,106,107
106 EPM=PCONST(NZ,2)
107 IF(RHO) 108,108,109
108 RHO=PCONST(NZ,3)
109 IF(RMASS-100.0) 110,110,111
```



```

110 WRITE OUTPUT TAPE IO, 10, ZI, RMASS, IPAGE
10  FORMAT(42H1MASS IS TOO LOW.      CALCULATION FOR Z=, F 5.1, 15H AND
1  REST MASS=, F10.5, 9H DELETED., 39X, 5HPAGE I3)
    IPAGE=IPAGE+1
    GO TO 103
111 IF(EPM-15.0) 113, 115, 112
112 IF(EPM-1100.0) 115, 115, 114
113 WRITE OUTPUT TAPE IO, 11, EPM, IPAGE
11  FORMAT(27H1MEAN EXCITATION POTENTIAL=, F5.2, 41H  VALUE IS TOO LOW.
1  CALCULATION DELETED., 48X, 5HPAGE I3)
    IPAGE=IPAGE+1
    GO TO 103
114 WRITE OUTPUT TAPE IO, 12, EPM, IPAGE
12  FORMAT(27H1MEAN EXCITATION POTENTIAL=, F10.2, 41H  VALUE IS TOO HIGH
1  CALCULATION DELETED., 42X, 5HPAGE I3)
    IPAGE=IPAGE+1
    GO TO 103

C
C          CALCULATION OF COEFFICIENTS USED IN
C          R-E FORMULA FOR THE VARIOUS ENERGIES
C
115  IK=0
    IG=1
    FEP=FE*938.256/RMASS
    DO 119 I=1, 800
    ZETA=E(I)/938.256
    BETA2(I)=(ZETA*(2.0+ZETA))/((1.0+ZETA)**2)
    IF(BETA2(I)-1.0) 116, 120, 120
116  ETASQ(I)=BETA2(I)/(1.0-BETA2(I))
    CDBDX(I)=C3/BETA2(I)
    XFN(I)=LOGF(2.0*C2*ETASQ(I)/EPM)-BETA2(I)
    IF(E(I)-ED(IG)) 118, 117, 117
117  IG=IG+1
    IF(IG-12) 118, 120, 120
118  E(I+1)=E(I)+DEL(IG)*.CC00001
    IK=IK+1
    IF(E(I)-FE+.1) 119, 119, 120
119  CONTINUE

C          PGM FINDS VALUE OF EXCITATION POTENTIAL, MEAN (EPM) AND
C          CALCULATES RANGE FOR ENERGIES BETWEEN 1 AND 8 MILLION ELECTRON VOLTS
C
120  X(1)=15.0
    X(2)=17.5
    M=3
    DO 121 I=20, 100, 10
    TI=I
    X(M)=TI
121  M=M+1
    DO 122 I=120, 340, 20
    TI=I
    X(M)=TI
122  M=M+1
    DO 123 I=380, 500, 40
    TI=I
    X(M)=TI
123  M=M+1
    DO 124 I=550, 1100, 50
    TI=I
    X(M)=TI

```

```

124 M=M+1
    DO 125 J=1,8
125 RC(J)=TAB(BPM,X,R3(I,J),I,1,5,39,SICK)*.001
    C4 = 1.0/Z
C
C          CALCULATE SHELL CORRECTION FOR EACH ENERGY
C
    DO 128 J=1,IK
    PROD1=0.0
    DO 126 I=1,3
    SUM1=A(I)+PROD1
126 PROD1=SUM1/ETASQ(J)
    PROD2=0.0
    DO 127 I=1,3
    SUM2=B(I)+PROD2
127 PROD2=SUM2/ETASQ(J)
128 COR(J)=((EPM**2)*(1.0E-6)*(PROD1))+((EPM**3)*(1.0E-9)*(PROD2))
C
C          PGM CALCULATES RANGE FOR ENERGIES GRBATER
C          THAN 8 MEV AND DENSITY EFFECT CORRECTION
C
    CONST=-1*(LOGF(EPM**2/(((H**2)*C1*Z*RHO)/AT)))+F.)
    DO 137 ID=I,IK
    STX(ID)=0.5*0.43429*LOGF(ETASQ(ID))
    IF(ST(NZ,1))131,129,131
129 IF(STX(ID)-0.34)130,132,132
130 NOTE=ID
    GO TO 132
131 IF(STX(ID)-ST(NZ,1)) 132,133,133
132 DENSE(ID)=0.0
    GO TO 136
133 IF(STX(ID)-ST(NZ,2)) 134,135,135
134 DENSE(ID)=(.5)*(4.606*STX(ID)+CONST+ST(NZ,3)*(ST(NZ,2)-STX(ID))**
1ST(NZ,4))
    GO TO 136
135 DENSE(ID)=(.5)*(4.606*STX(ID)+CONST)
136 DEDX(ID)=CDEDX(ID)*C4*(Z*XFN(ID)-COR(ID)-DENSE(ID)*Z)
137 CONTINUE
C
C          CALCULATE RANGE
C
    DO 141 I=3,IK,2
    R(I)=R(I-2)+((E(I)-E(I-2))/6.)*(1./DEDX(I-2)+4./DEDX(I-1)+
1 1.0/DEDX(I))
    IF(ABS(E(I)-8.0)-0.001) 139,139,138
138 IF(E(I)-8.0) 141,139,140
139 CON=RC(8)-R(I)
    JL=I+2
140 RC(I)=R(I)+CON
141 CONTINUE
C
C          OUTPUT RESULTS
C
    WRITE OUTPUT TAPE IO, 13, IPAGE
13 FORMAT(1H1,119X,5HPAGE,13)
    WRITE OUTPUT TAPE IO, 14
14 FORMAT(47X,38HRANGES AS A FUNCTION OF KINBTIC ENERGY/51X29HAND MBA
IN EXCITATION POTENTIAL//A//)
    WRITE OUTPUT TAPE IO, 15, ZI, RMASS, Z, AT, EPM, RHO

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15  FORMAT(1H ,28X,44HCALCULATED FOR A PARTICLE WHICH HAS CHARGE=,
      IF4.1,15H AND REST MASS=,F 8.3,4H MEV/,
      2      1H ,28X,54HINCIDENT ON STOPPING MATERIAL WHICH HAS ATOMIC N
      3UMBER=,F4.1,16H, ATOMIC WEIGHT=,F 8.4,1H,29X,26HMEAN EXCITATION P
      4OTENTIAL=,F 7.2,14H, AND DENSITY=,F 7.4,23H GRAMS/CUBIC CENTIMETER
      5//A)
      IUINE=ILINE+5
      WRITE OUTPUT TAPE IO, 16
16  FORMAT(5X,6HENERGY,11X,3(5HRANGE,14X),4HDBDX,14X,5HDP/DX,14X,
      18HMOMENTUM/7X,3HMEV,9X,11HCENTIMETERS,7X,13HGRAMS/SQ. CM.,2X, 2
      23HMOLES ELECTRONS/SQ. CM.,2X,9HMEV CM2/G,10X,10HMEV CM2/GC,14X,5HM
      3EV/C//)
      ILINE=15
      DO 144 I=1,8
      AI=I
      TE(I)=AI*RMASS/938.256
      IF(IE-TE(I)-.01)142,142,144
142  IF(TE(I)-FB) 143,143,177
143  P(I)=SQRTF(TE(I)**2*2.0*RMASS*TE(I))
      ENERGY(I)=TE(I)
      Q1(I)=RC(I)*RMASS/(ZI**2*938.256)
      Q2(I)=Q1(I)*(AT/Z)
      Q3(I)=Q2(I)/RHO
144  CONTINUE
      DO 148 I=JL,IK,2
      JRANG=(I-JL+2)/2+8
      ENERGY(JRANG)=E(I)*RMASS/938.256
      IF(IE-ENERGY(JRANG)-.1)146,146,148
146  IF(ENERGY(JRANG)-FE-.1)147,147,149
147  Q1(JRANG)=RC(I)*RMASS/(ZI**2*938.256)
      Q2(JRANG)=Q1(JRANG)*(AT/Z)
      Q3(JRANG)=Q2(JRANG)/RHO
      Q4(JRANG)=DBDX(I)*Z*ZI**2/AT
148  CONTINUE
      JP=JRANG
      GO TO 150
149  JP=JRANG-1
150  AE=BE
      IDBB=0
      IBE=1
151  IF(AE-1.0)152,152,157
152  RCM(IBE)=TAB(AE,ENERGY,Q3,1,1,5,10,SICK)
      RGPCM(IBE)=TAB(AE,ENERGY,Q2,1,1,5,10,SICK)
      RC(IBE)=TAB(AE,ENERGY,Q1,1,1,5,10,SICK)
      P(IBE)=SQRTF(AE**2*2.0*RMASS*AE)
      IF(AE-TE(8)-0.01) 153,153,155
153  WRITE OUTPUT TAPE IO,17,AE,RCM(IBE),RGPCM(IBE),RC(IBE),P(IBE)
      ILINE=ILINE+1
154  AB=AE+0.1
      IBE=IBE+1
      GO TO 151
155  DEDX(IBE)=TAB(AE,ENERGY(9),Q4(9),1,1,5,8,SICK)
      DPDX(IBE)=((AE*RMASS)/P(IBE))*DEDX(IBE)
      WRITE OUTPUT TAPE IO,19,AE,RCM(IBE),RGPCM(IBE),RC(IBE),DEDX(IBE),
      1DPDX(IBE),P(IBE)
      ILINE=ILINE+1
      IF(IDBB)154,156,154
156  IDBB=IBE
      GO TO 154

```

```

157 JTRAN=JTRAN+IBE-2
    L=1
    LL=19
    DO 162 KK=IBE,IK
      IF(E(LL)-FE-.1)158,158,163
158 RCM(KK)=TAB(E(LL),ENERGY(L),Q3(L),1,1,5,JP,SICK)
      RGPCM(KK)=TAB(E(LL),ENERGY(L),Q2(L),1,1,5,JP,SICK)
      RC(KK)=TAB(E(LL),ENERGY(L),Q1(L),1,1,5,JP,SICK)
      IF(E(LL)-TE(8)-.01)161,161,159
159 IF(IDEB)161,160,161
160 IDEB=KK
      LPP=JP-IDEB+1
161 DEDX(KK)=TAB(E(LL),ENERGY(IDEB),Q4(IDEB),1,1,5,LPP,SICK)
      LL=LL+2
      IF(LL-1K)162,162,163
162 CONTINUE
163 J=19
      DO 176 I=IBE,IK
        IF(E(J)-BE)175,164,164
164 IF(E(J)-FE-.1)165,165,177
165 P(I) = SQRT(E(J)**2+2.0*RMAS*E(J))
        DPDX(I)=(E(J)+RMAS)/P(I)*DEX(I)
        NWD=I
        IF(NOTE-I) 166,167,166
166 IF(NOTE-I-1) 169,167,169
167 IF(ILINE-52) 168,168,173
168 WRITE OUTPUT TAPE IO, 18
18  FORMAT(2X,10(13H*****))//23X,85HAT THIS ENERGY, THE DENSITY
1  EFFECT BECOMES IMPORTANT, HOWEVER STERNHEIMER'S CONSTANTS/27X,71H
2  ARE NOT GIVEN, HENCE THE DATA WILL BE IN ERROR BY MORE THAN 1 PER
3  CENT./2X,10(13H*****))
      ILINE=ILINE+6
      NOTE=0
169 IF(I-IDEB)170,171,171
170 WRITE OUTPUT TAPE IO,17,E(J),RCM(I),RGPCM(I),RC(I),P(I)
17  FORMAT(2X,F9.2,6X,3(1PE12.4,7X),19X,19X,3X,OPF10.3)
      GO TO 172
171  CONTINUE
      WRITE OUTPUT TAPE IO, 19, E(J), RCM(I), RGPCM(I),RC(I), DEDX(I),
1  DPDX(I),P(I)
19  FORMAT(2X,F9.2,6X,5(1PE12.4,7X),3X,OPF10.3)
172  ILINE=ILINE+1
      IF(ILINE-56)175,174,174
173  NOTE=J
174  IPAGE=IPAGE+1
      WRITE OUTPUT TAPE IO, 20, IPAGE
20  FORMAT(1H1,119X,5HPAGE ,13)
      WRITE OUTPUT TAPE IO,16
      ILINE=1
175  J=J+2
      IF(J-1K) 176,176,177
176  CONTINUE
177  IPAGE=IPAGE+1
      IDEB=0
      GO TO 103
      END

```

G. LOW-ENERGY DATA

.77449	2.7486	5.7500	9.6951	14.529	20.210	26.706	33.995
.81110	2.8278	5.8785	9.8860	14.800	20.585	27.212	34.664
.84553	2.9036	6.0035	10.073	15.066	20.950	27.700	35.302
.96783	3.1804	6.4686	10.772	16.057	22.301	29.488	37.611
F.0738	3.4257	6.8872	F1.405	16.950	23.508	31.068	39.624
F.1694	3.6491	7.2710	F1.985	17.768	24.607	32.495	41.430
1.2575	3.8563	7.6280	12.525	18.526	25.622	33.808	43.082
1.3400	4.0508	7.9638	13.032	19.237	26.571	35.031	44.614
F.4180	4.2352	8.2821	13.513	19.909	27.465	36.179	46.049
F.4923	4.4111	8.5859	13.971	20.549	28.314	37.267	47.404
F.5636	4.5799	8.8773	14.410	21.161	29.125	38.303	48.691
F.6988	4.8999	9.4293	15.240	22.315	30.650	40.247	51.099
1.8259	5.2010	9.9480	16.018	23.394	32.071	42.052	53.328
1.9468	5.4869	10.440	16.755	24.413	33.410	43.747	55.415
2.0625	5.7605	10.910	17.457	25.382	34.679	45.351	57.385
2.1739	6.0236	11.361	18.130	26.309	35.892	46.879	59.256
2.2816	6.2778	11.797	18.779	27.200	37.055	48.342	61.045
2.3861	6.5243	12.218	19.405	28.060	38.174	49.747	62.760
2.4878	6.7640	12.628	20.013	28.892	39.256	51.104	64.413
2.5871	6.9976	13.026	20.603	29.700	40.305	52.415	66.009
2.6841	7.2258	13.415	21.179	30.486	41.323	53.688	67.555
2.7791	7.4492	13.795	21.740	31.252	42.315	54.925	69.056
2.8723	7.6681	14.167	22.289	31.999	43.281	56.129	70.515
3.0538	8.0942	14.890	23.354	33.447	45.149	58.451	73.324
3.2298	8.5065	15.588	24.380	34.838	46.939	60.673	76.007
3.4008	8.9071	16.265	25.372	36.181	48.664	62.808	78.580
3.5677	9.2973	16.923	26.335	37.481	50.331	64.869	81.057
3.7711	9.7724	17.722	27.503	39.055	52.345	67.351	84.038
3.9694	10.235	18.500	28.636	40.579	54.289	69.744	86.905
4.1633	10.687	19.258	29.738	42.057	56.173	72.058	89.672
4.3533	11.130	19.998	30.813	43.497	58.003	74.301	92.351
4.5397	11.564	20.723	31.863	44.901	59.785	76.483	94.951
4.7230	11.990	21.434	32.892	46.273	61.524	78.607	97.479
4.9035	12.410	22.133	33.900	47.617	63.224	80.681	99.944
5.0813	12.823	22.819	34.890	48.934	64.888	82.708	102.35
5.2567	13.230	23.495	35.863	50.227	66.519	84.692	104.70
5.46	13.6	24.35	37.1	51.4	68.1	86.4	107.0
5.62	13.9	24.99	38.8	52.3	69.2	87.8	109.9
5.78	14.25	25.65	38.99	53.2	70.0	88.9	111.0
0.1	0.5	1.0	2.5	5.0	50.0	100.0	250.0
500.0	10000.0	50000.0					
2.0	10.0	50.0	200.0	1000.0	1600.0	10000.0	50000.0
100000.0	500000.0	3000000.0					

D. DENSITY-EFFECT DATA

1	.415	3.	.0569	6.224
2	2.21	3.	2.13	3.22
3	-.05	2.	.374	3.05
4	-.10	2.	.413	2.82
6	-.05	2.	.531	2.63
7	1.86	4.	.125	3.72
8	1.90	4.	.130	3.72
10	2.14	4.	.258	3.18
12	.10	3.	.0938	3.56
13	.05	3.	.0906	3.51
14	.10	3.	.0874	3.586
18	2.02	5.	.0255	4.36
26	.10	3.	.127	3.29
29	.20	3.	.109	3.39
32	.10	3.	.1666	3.136
36	2.12	5.	.0771	3.57
47	.20	3.	.251	2.88
50	.20	3.	.404	2.52
54	1.90	5.	.150	3.07
74	.30	4.	.0283	3.91
79	.30	4.	.0436	3.62
82	.40	3.	.355	2.64
92	.30	4.	.0652	3.37

E. ELEMENT DATA

1	1.00797	18.7	.0586	2	4.0026	42.0	2126
3	6.939	39.032	.534	4	9.0122	56.0	1.8
5	10.811		2.34	6	12.01115	78.0	2.25
7	14.0087	92.0	.808	8	15.9994	105.0	1.14
9	18.9984		1.11	10	20.183	130.016	1.2
11	22.9898		.971	12	24.312	156.4	1.74
13	26.9815	163.	2.699	14	28.086		2.42
15	30.9738		1.82	16	32.064		2.07
17	35.453		1.56	18	39.948	210.0	1.40
19	39.102		.87	20	40.08	200.	1.55
21	44.956		3.02	22	47.90	225.	4.5
23	50.942	254.	5.96	24	51.996		7.1
25	54.9380		7.20	26	55.847	273.	7.86
27	58.9332	298.	8.9	28	58.71	312.	8.90
29	63.54	322.	8.94	30	65.37	331.	7.14
31	69.72		5.91	32	72.59		5.36
33	74.9216		5.73	34	78.96		4.8
35	79.909		3.12	36	83.80	493.68	2.6
37	85.47		1.53	38	87.62		2.54
39	88.905		5.51	40	91.22		6.4
41	92.906	410.	8.4	42	95.94	420.	10.2
43	99.			44	101.07		12.2
45	102.905	450.	12.5	46	106.4	460.	12.16
47	107.870	487.	10.50	48	112.40	468.0	4.65
49	114.82	490.	7.28	50	118.69	500.	7.31
51	121.75		6.691	52	127.60		6.24
53	126.9044		4.93	54	131.30	757.52	3.52
55	132.905		1.873	56	137.34		3.5
57	138.91		6.155	58	140.12		3.92
59	140.907		6.5	60	144.24		6.95
61	147.			62	150.35		7.8
63	151.96		5.24	64	157.25		
65	158.924			66	162.50		8.56
67	164.930			68	167.26		4.77
69	168.934			70	173.04		
71	174.97			72	178.49		13.3
73	180.948	720.	16.6	74	183.85	740.	19.3
75	186.2		20.53	76	190.2		22.48
77	192.2	760.	22.42	78	195.09	777.	21.37
79	196.967	786.	19.32	80	200.59		13.546
81	204.37		11.85	82	207.19	818.	11.35
83	208.980	826.	9.747	84	210.		
85	210.			86	222.		9.73
87	223.			88	226.		
89	227.			90	232.038		11.3
91	231.			92	238.03	908.	18.68

F. SAMPLE OUTPUT
RANGES AS A FUNCTION OF KINETIC ENERGY
AND MEAN EXCITATION POTENTIAL

CALCULATED FOR A PARTICLE WHICH HAS CHARGE= 1.0 AND REST MASS= 938.256 MEV
INCIDENT ON STOPPING MATERIAL WHICH HAS ATOMIC NUMBER=29.0, ATOMIC WEIGHT= 63.5400,
MEAN EXCITATION POTENTIAL= 322.00, AND DENSITY= 8.9400 GRAMS/CUBIC CENTIMETER

ENERGY MEV	RANGE CENTIMETERS	RANGE GRAMS/SQ. CM.	RANGE MOLES ELECTRONS/SQ. CM.	DEDX MEV CM2/G	DP/DX MEV CM2/GC	MOMENTUM MEV/C
1.00	6.8341E-04	6.1097E-03	2.7885E-03			43.330
2.00	1.8311E-03	1.6370E-02	7.4713E-03			61.295
3.00	3.3901E-03	3.0308E-02	1.3833E-02			75.090
4.00	5.3417E-03	4.7755E-02	2.1795E-02			86.730
5.00	7.6778E-03	6.8640E-02	3.1328E-02			96.993
6.00	1.0395E-02	9.2928E-02	4.2413E-02			106.278
7.00	1.3491E-02	1.2061E-01	5.5047E-02			114.824
8.00	1.6961E-02	1.5163E-01	6.9204E-02			122.785
9.00	2.0629E-02	1.8442E-01	8.4172E-02	2.9258E 01	2.1275E 02	130.267
10.00	2.4605E-02	2.1997E-01	1.0039E-01	2.7105E 01	1.8713E 02	137.350
12.00	3.3451E-02	2.9905E-01	1.3649E-01	2.3718E 01	1.4972E 02	150.540
14.00	4.3455E-02	3.8849E-01	1.7731E-01	2.1166E 01	1.2389E 02	162.687
16.00	5.4579E-02	4.8793E-01	2.2270E-01	1.9166E 01	1.0510E 02	174.012
18.00	6.6791E-02	5.9711E-01	2.7252E-01	1.7551E 01	9.0883E 01	184.665
20.00	8.0064E-02	7.1577E-01	3.2668E-01	1.6216E 01	7.9789E 01	194.757
22.00	9.4374E-02	8.4371E-01	3.8507E-01	1.5093E 01	7.0918E 01	204.370
24.00	1.0970E-01	9.8073E-01	4.4761E-01	1.4134E 01	6.3682E 01	213.570
26.00	1.2602E-01	1.1267E 00	5.1421E-01	1.3304E 01	5.7679E 01	222.408
28.00	1.4333E-01	1.2813E 00	5.8481E-01	1.2578E 01	5.2629E 01	230.925
30.00	1.6159E-01	1.4446E 00	6.5934E-01	1.1937E 01	4.8328E 01	239.156
32.00	1.8080E-01	1.6164E 00	7.3773E-01	1.1367E 01	4.4627E 01	247.128
34.00	2.0095E-01	1.7965E 00	8.1993E-01	1.0856E 01	4.1412E 01	254.867
36.00	2.2202E-01	1.9848E 00	9.0588E-01	1.0395E 01	3.8597E 01	262.394
38.00	2.4399E-01	2.1813E 00	9.9553E-01	9.9776E 00	3.6113E 01	269.725
40.00	2.6685E-01	2.3857E 00	1.0888E 00	9.5972E 00	3.3909E 01	276.876
42.00	2.9060E-01	2.5980E 00	1.1857E 00	9.2490E 00	3.1940E 01	283.862
44.00	3.1523E-01	2.8181E 00	1.2862E 00	8.9291E 00	3.0172E 01	290.693
46.00	3.4071E-01	3.0459E 00	1.3902E 00	8.6341E 00	2.8577E 01	297.381
48.00	3.6704E-01	3.2814E 00	1.4976E 00	8.3611E 00	2.7131E 01	303.935
50.00	3.9422E-01	3.5243E 00	1.6085E 00	8.1077E 00	2.5816E 01	310.364
55.00	4.6578E-01	4.1640E 00	1.9005E 00	7.5469E 00	2.2999E 01	325.934
60.00	5.4239E-01	4.8490E 00	2.2131E 00	7.0708E 00	2.0707E 01	340.868
65.00	6.2393E-01	5.5779E 00	2.5458E 00	6.6613E 00	1.8812E 01	355.244
70.00	7.1027E-01	6.3498E 00	2.8981E 00	6.3050E 00	1.7222E 01	369.128
75.00	8.0130E-01	7.1636E 00	3.2695E 00	5.9921E 00	1.5870E 01	382.575
80.00	8.9691E-01	8.0184E 00	3.6596E 00	5.7151E 00	1.4709E 01	395.627
85.00	9.9699E-01	8.9131E 00	4.0680E 00	5.4679E 00	1.3702E 01	408.324
90.00	1.1014E 00	9.8469E 00	4.4942E 00	5.2459E 00	1.2822E 01	420.697
95.00	1.2102E 00	1.0819E 01	4.9379E 00	5.0455E 00	1.2046E 01	432.774
100.00	1.3231E 00	1.1829E 01	5.3987E 00	4.8635E 00	1.1358E 01	444.580
105.00	1.4401E 00	1.2875E 01	5.8762E 00	4.6976E 00	1.0744E 01	456.135

FOOTNOTES AND REFERENCES

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1. Two excellent review articles that cite the original papers and explicate the steps leading to Eq. (1) are: Passage of Radiation through Matter, H. A. Bethe and J. Ashkin, Part II of Experimental Nuclear Physics, E. Segrè, Ed., (John Wiley and Sons Inc., New York, 1952) Vol. I. E. A. Uehling. Penetrations of Heavy Charged Particles in Matter, Ann. Rev. Nucl. Sci. 4, 315-350 (1954).
2. The density effect is discussed in an article by R. M. Sternheimer, Phys. Rev. 88, 851 (1952).
3. The density-effect constants that we use are given in an article by R. M. Sternheimer, Phys. Rev. 103, 511 (1956).
4. W. H. Barkas, The Range Energy Function, UCRL-10292, August 1962.
5. H. Bichsel, Linear Accelerator Group, University of Southern California Technical Report No. 2. (1961) and Technical Report No. 3. (1961).
6. T. J. Thompson, Effective Chemical Structure on Stopping Powers for High Energy Protons, (Ph. D. Thesis), UCRL-1910, Aug. 1952.
7. H. Bichsel, R. F. Mozley, and W. Aron, Range of 6 to 18-MeV Protons in Be, Al, Cu, Ag and Au, Phys. Rev. 105, 1788 (1957).
8. N. Bloembergen and P. S. van Heerden, The Range and Straggling of Protons between 35 and 120 MeV, Phys. Rev. 83, 561 (1951).
9. R. Mather and E. Segrè, Range Energy Relation for 340-MeV Protons, Phys. Rev. 84, 191 (1951).
10. The values for A and ρ were obtained from Handbook of Chemistry and Physics (Chemical Rubber Company, Dayton, Ohio, 44th Edition. 1963). In some cases the density will vary because the element can have various states. In this case the one most likely to be encountered in practice was used (i. e., the ρ for carbon was chosen as that of graphite rather than diamonds).

The values for I were taken from the following references, which are listed in order of my preference:

- a. W. H. Barkas (private communication) and reference 4.
- b. V. P. Zrelov and G. D. Stoletov, Range Energy Relation for 660-MeV Protons, Soviet Physics--JETP 9, 461 (1959).
- c. Compilations that appear in references 5 and 3.

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