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Kilovolt Europium Capture Cross Sections

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Kilovolt Europium Capture Cross Sections

by

Donald W. Barr
and
Joseph J. Devaney

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ABSTRACT

Estimates of the europium radiative capture cross sections for neutrons are given for Eu-151, 152, 153, 154, and 155. These calculations utilize the slope of the total cross section, resonance parameters, known odd-even versus odd-odd behavior, and theory. They are compared with experimental values derived from the nuclear device "Cyclamen"¹ and from direct measurement.

¹Los Alamos Radiochemistry Group, Los Alamos Scientific Laboratory Report LA-DC-8103 (revised), (1966).

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It is a pleasure to acknowledge conversations with Dr. Conrad Longmire and Dr. J. E. Young of the Los Alamos Scientific Laboratory, Dr. R. C. Block of the Rensselaer Polytechnical Institute, Dr. J. H. Gibbons and Dr. J. A. Harvey of the Oak Ridge National Laboratory, and Dr. Murray D. Goldberg of the Brookhaven National Laboratory. One of us (DWB) wishes to acknowledge the assistance of Dr. William R. Daniels for chemical separation of europium from Cyclamen debris, and of Dr. Bruce J. Dropesky for performing mass separation on the europium sample with the IASL radiochemical facility.

We are indebted to Dr. George I. Bell for a preprint of his work on the same subject,² and for a critical review of our manuscript, to Dr. R. L. Block for a preprint of his and Dr. R. L. Macklin's measurements, and to Dr. Viktor E. Hampel of the Lawrence Radiation Laboratory for calling our attention to other work.

²George I. Bell, Los Alamos Scientific Laboratory Report, LA-3463-MS, (1966), available from the Clearing House for Federal Scientific and Technical Information, Springfield, Virginia.

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I. RESULTS

Our best estimates of the average europium radiative capture cross sections at 1 KeV are given in Table I. Probable errors are estimated from experimental error, theoretical uncertainty, scatter of experimental points, and experimental disagreement. Comparison of the various cross section estimates are given in Table II and graphs as a function of energy follow.

TABLE I
Results at 1 KeV Incident Neutron Energy

Isotope	Capture Cross Section
Europium (normal isotopic abundance):	$\sigma_{n\gamma} = 32.2b \pm 20\%$
Eu ¹⁵¹	$\sigma_{n\gamma} = 38b \pm 25\%$
Eu ¹⁵²	$\sigma_{n\gamma} = \begin{cases} 35 \text{ to } 84 \text{ barns} \\ \text{or } 56b \pm 45\% \end{cases}$
Eu ¹⁵³	$\sigma_{n\gamma} = 27b \pm 30\%$
Eu ¹⁵⁴	$\sigma_{n\gamma} = \begin{cases} 16 \text{ to } 28b \\ \text{or } 24b \pm 40\% \end{cases}$
Eu ¹⁵⁵	$\sigma_{n\gamma} = 30b \pm 70\%$

TABLE II
Capture Cross Section Comparisons at 1 keV
(all in barns)

One Isotope	Two Barr and Devaney Theoretical (Strength Function, S_0 from Slope σ_f and Reso- nance Parameters. Expt. comparisons of odd-odd vs odd-even from Lu and Ta.)	Three Barr and Devaney Theoretical (As second column except theor. odd-odd vs odd- even.)	Four Barr and Devaney Radiochemical (Derived from Cyclamen and measurements of Konks, Penin, and Shapiro.)	Five Konks and Penin ^{3,4} Experimental (Pb slowing down)	Six Block, Von der Lage, and Weston ⁵ Experimental (ORNL-fast Chopper)	Seven Macklin and Gibbons ⁶ Experimental (ORNL-liquid scintillator and pulsed Van de Graaf) (Our extrapolation)	Theoretical - from low energy resonance param- eters and theoretical odd-odd vs. odd-even parameters. (See our remarks on reli- ability in Part II.)		
							Eight Barr and Devaney	Nine Barr, Devaney, Garrison ⁷	Ten Bell and Garrison ^{2,7}
Eu	31.3			31, 35	36.7	31.6	25	27.6	23.8
Eu ¹⁵¹	35.5			39			28.1	31.6	28
Eu ¹⁵²	35.4	37.2	84				29.1	31.9	34
Eu ¹⁵³	27.9			26.5			22.2	24	20
Eu ¹⁵⁴	27.5	27.4	16				21.4	23.3	24
Eu ¹⁵⁵			30						15
Eu ¹⁵⁶			22 (er- ror about a factor 5)						17

³V. A. Konks and Yu. P. Penin, Joint Institute for Nuclear Research, Dubna, Report No. 1845, p. 100 (1964).
The authors are grateful for the exceeding kindness of Dr. H. D. Lemmel, Division of Research and Laboratories,
International Atomic Energy Agency, Vienna, for furnishing copies of this report.

⁴J. J. Schmidt and I. Siep, Kern Forschungszentrum, Karlsruhe, Report KFK 352 (1965) have graphs of the Konks-
Penin work as well as meticulous analyses of Eu, Sm, Gd, and Hf. cross sections. Their europium cross sections
are normalized to the older R. L. Macklin, J. H. Gibbons, and T. Inada (Phys. Rev. 129, 2695 (1963)) work, and
so are not compared (ref. 6).

⁵R. C. Block, F. C. Von der Lage, and L. W. Weston, Oak Ridge National Laboratory Report ORNL-3085 (1961) page 48.
Also given in J. H. Neiler, "Physics of Fast and Intermediate Reactors, I, p. 98, International Atomic Energy
Agency, Vienna (1962).

⁶Private communication J. H. Gibbons (1966).

⁷J. D. Garrison, Ann. of Phys. 30, 269 (1964).

Fig. 1. Europium cross sections versus energy.

($\text{Eu}^{151} \pm 25\%$ Probable Error, $\text{Eu} \pm 20\%$, $\text{Eu}^{153} \pm 30\%$, and $\text{Eu}^{155} \pm 70\%$, which is thus taken the same as Eu to this accuracy.)

Fig. 2. Odd-Odd europium cross sections versus energy.

II. THEORETICAL ESTIMATES

A. General Considerations

Lacking a complete and reliable theory of the nucleus, it is clear that one can only use theory to extrapolate from known data. With awareness of intermediate structure,⁸ the old standby of extrapolation from the known low-energy resonance parameters is expected to be somewhat less reliable than heretofore. Also, such extrapolations involve both the determination of average parameters from small samples and the selection of parameter distribution and correlation laws. For these reasons, such studies are accorded less weight; they appear in columns 8, 9, 10 of Table II.

The role of "doorway-states" leading to intermediate structure is briefly (and loosely!) sketched as follows: With two-body forces as the dominant form of nuclear interaction, one envisages a nuclear reaction as beginning with an incident nucleon in a very high energy state of the total system and the nucleons of the target in their ground state. In

⁸ B. Block and H. Feshback, Ann. of Phys. 23, 47 (1963).

A. K. Kerman, L. S. Rodberg, and J. E. Young, Phys. Rev. Lett. 11, 422 (1963).

A lucid summary is to be found in H. Feshback, page 257, of M. Neve de Mévergnies et al, "Nuclear Structure Study with Neutrons," North-Holland, Amsterdam (1966).

K. K. Seth, Phys. Lett. (Nethl.), 16, 306 (1965).

that configuration (called "one quasi-particle" because residual interactions will mix pure single particle states), the incident particle constructively interferes with itself in the collective attractive well formed by the target nucleons and we obtain a size or single-particle resonance or "echo."⁹ These "resonances" depend on the size of the nucleus and are not expected to vary greatly with mass number A , nor with energy. Indeed, their behavior is well calculated by use of various optical models, the latter delimiting the accuracy of our preceding statement.¹⁰ Such structure, therefore, does not preclude useful comparisons at different energy or with other nuclei of the same type.

The first particle excitation in this picture occurs when the incident nucleon reacts with a target nucleon to form a three quasi-particle state composed of the incident nucleon, now at a lower energy, plus the excited target nucleon, plus the hole left by the latter. This state is called the "doorway" (state) into the compound system and it leads to a resonance type behavior called intermediate structure.⁸ The latter has typical resonance widths of the order of 15 to 160 KeV spaced a few hundred KeV apart, with peak-to-valley variations expected to be

⁹K. W. McVoy, Phys. Lett. 17, 42 (1965).
 A. M. Lane, R. G. Thomas, and E. P. Wigner, Phys. Rev. 98, 693 (1955).
 J. M. Peterson, Phys. Rev. 125, 955 (1962).
 K. W. McVoy, "Giant Resonances and Neutron-Nucleus Total Cross Sections," (1966), to be published in the Am. J. of Phys. 1967.

¹⁰A recent extensive optical model study, although only above 150 KeV., is that of F. P. Agee and L. Rosen, Los Alamos Scientific Laboratory Report, LA-3538-MS (1966), available from the Clearinghouse for Federal Scientific and Technical Information, Springfield, Virginia.

5% to 100% of the average.

These spacings are appreciably smaller than the order of 10 MeV expected between optical "resonances" and one has no assurance that a few KeV energy difference or the addition of two like nucleons may not modulate the low-energy compound state resonance parameters from an intermediate peak into an intermediate valley. Thus errors of possibly as much as 100% in extrapolation of low-energy resonance parameters can not be excluded. To complete our story, further pairwise interaction can lead to five quasi-particles, seven, etc. until a full compound state is achieved with, of course, reverse transitions possible at every step. The compound state with its greater complexity has a much longer lifetime, hence a much narrower resonance width (less than an electron-volt in a heavy nuclei at low energy).

It is all very well to point out the additional uncertainty of intermediate structure over and above the often rather large degree of uncertainty of averaging widely fluctuating widths, neutron widths especially,¹¹ but unless one can fasten on a superior method, the remark is of little practical value. Fortunately, we have the remarkable method of the slope of the total cross section^{12,13} which leads directly to the

¹¹The definitive work on fluctuations is that of C. E. Porter et al. in C. E. Porter, "Statistical Theories of Spectra: Fluctuations," Academic Press, New York (1965).

¹²H. Feshback, C. E. Porter, and V. F. Weisskopf, Phys. Rev., 96, 448 (1954).

¹³D. B. Gayther and K. P. Nicholson, Proc. Phys. Soc. Lond., 70A, 51 (1957).

strength function and neatly bypasses most of the foregoing difficulties. Here we must, however, use low energy resonance parameters to differentiate between isotopes, since the total cross section for the separated isotopes is not yet measured. To begin we note that including $l \neq 0$ waves is unnecessary. Using average parameters we find that the p-wave total capture cross section is about 1.4% and surely less than 13% of the cross section at 30 KeV neutron energy for a representative europium isotope (odd-even), and at 1 KeV the p-wave is about 0.04% and surely less than 0.4% of the capture cross section. Consequently all of the following computations need be made only for zero orbital angular momentum below 30 KeV.

A convenient starting place for our purposes are the isolated Breit-Wigner compound system resonance formulas which we take in the form (for the j^{th} resonance, $l = 0$)¹⁴

$$\sigma_{sc}^j = 4\pi\lambda^2 \left| \frac{\Gamma_n^j/2}{(E-E_j) + \frac{1}{2}\Gamma^j} + e^{ikR} \sin kR \right|^2, \quad (1)$$

$$\sigma_Y^j = \pi\lambda^2 \frac{\Gamma_n^j \Gamma_Y^j}{(E-E_j)^2 + \frac{1}{4}\Gamma^j{}^2}, \quad (2)$$

where

$$l = 2.71828 \dots$$

$$\sigma_T \equiv \sigma_{sc} + \sigma_Y \text{ is the total cross section}$$

¹⁴See, for example, J. M. Blatt and V. F. Weisskopf, "Theoretical Nuclear Physics," John Wiley, New York (1952), p. 328, 392, 393.

σ_{sc} is the scattering cross section

σ_Y is the radiative capture cross section

Γ_n is the neutron width, actually $2g\Gamma_n$ where $g \equiv \frac{1}{2} \left(\frac{2J+1}{2I+1} \right)$, but we must take an average $g = \frac{1}{2}$ because of lack of sufficient knowledge

Γ_Y is the radiative width

$\Gamma \equiv \Gamma_Y$ is the total width

E is the incident laboratory neutron energy

E_j is the resonance energy for the j^{th} resonance

k is the incident neutron wave number

R is a "radius" associated with potential scattering

$\lambda = 1/k$ is the incident neutron wavelength.

We neglect any net gain or loss through level-level interference, and we take all parameters constant over a single compound resonance. Moreover, all parameters are in the laboratory system; however, the mass factor $[(m+M)^2/M^2]$, being omitted from the experimental data analysis, is therefore best omitted from our computations.¹⁵

In order to obtain average cross sections we shall integrate Eqs.

(1) and (2) for a fixed total angular momentum, $J = I \pm \frac{1}{2}$. For $l = 0$

¹⁵A discussion of this point is to be found in either: J. J. Devaney and R. G. Fagan, "U²³⁸ Cross Sections," Los Alamos Scientific Laboratory Report, LA-2144, p. 8 (1958); or: J. J. Devaney, L. O. Bordwell, and M. J. Devaney, "Hafnium Cross Sections," LA-2763, p. 10 (1962), Clearinghouse for Federal Scientific and Technical Information, Springfield, Virginia.

there are but two possibilities and further, since I is $5/2$ or 3 and J is in any case unknown, we shall weight the spin possibilities equally in our average. The area is then

$$A_{sc}^{J \in J} = \frac{2\pi^2 \chi^2 \Gamma_n^2 j^2}{\Gamma_j} + 4\pi R^2 D^j - \pi \Gamma_n^2 j_R^2, \quad (3)$$

$$A_Y^{J \in J} = \frac{2\pi^2 \chi^2 \Gamma_n^2 j_T j_Y}{\Gamma_j}, \quad (4)$$

where we have integrated either from $-\infty$ to ∞ or from $\frac{-D^j}{2} + E_j$ to $+\frac{D^j}{2} + E_j$ as appropriate, the D^j being the average spacing of the j^{th} level to its neighbors of the same J ; where j 's now run over one fixed J as indicated by $j \in J$; where we continue to neglect minor variations with energy over the resonance; and where we replace $\sin^2 kR$ by $(kR)^2$, incurring at most an error of 3% (for $R = 8$ fm or $r_0 = 1.5$ fm, and 30 KeV).

Adding Eqs. (3) and (4) and averaging, we get the average contribution of the j^{th} resonance to the total area

$$\langle A_T \rangle_{j \in J} = 2\pi^2 \chi^2 \langle \Gamma_n^2 j \rangle + 4\pi R^2 \langle D^j \rangle - \pi^2 R^2 \langle \Gamma_n^2 j \rangle. \quad (5)$$

The average total cross section is then just the sum of these in an average interval, divided by that interval. Thus if there are N resonances in the interval, then

$$\langle \sigma_T \rangle_J = \frac{\sum_{j \in \Delta E J}^N \langle A_T \rangle_j}{\Delta E} = \frac{\langle A_T \rangle_J N}{\langle D_J \rangle N} = 2\pi^2 \chi^2 \frac{\langle \Gamma_n \rangle}{\langle D_J \rangle} + 4\pi R^2 - \pi^2 R^2 \frac{\langle \Gamma_n \rangle}{\langle D_J \rangle}. \quad (6)$$

If further we take the average $J = I + \frac{1}{2}$ and $J = I - \frac{1}{2}$ each taken equally likely as we indicated earlier, we obtain

$$\langle \sigma_T \rangle = 2\pi^2 \lambda^2 S_0 \sqrt{E} + 4\pi R^2 - \pi^2 R^2 S_0 \sqrt{E}, \quad (7)$$

$$\text{with } S \equiv \frac{\langle \Gamma_n \rangle}{\langle D_J \rangle} = S_0 E.$$

D_J is intended to be the spacing between levels of the same J ; on the average D_J is twice the observed spacing, $D_{\text{obs.}}$. The quantity S_0 is the (reduced) S-wave strength function.

Writing $\lambda^2 = \lambda_0^2 E^{-1}$ and differentiating Eq. (7) with respect to $E^{-\frac{1}{2}}$ we get:

$$\frac{\partial \langle \sigma_T \rangle}{\partial E^{-\frac{1}{2}}} = 2\pi^2 \lambda_0^2 S_0 + \pi^2 R^2 S_0 E, \quad (8)$$

where omission of the last term for europium at 30 KeV and 8 fm radius leads to an error of 4.6% (0.15% @ 1 KeV), so that measurement of the slope of the total cross section plotted time of flight or $E^{-\frac{1}{2}}$ is a direct method for determining S_0 .

Similar to the derivation of Eq. (7), we start with Eq. (4) alone to get

$$\langle \sigma_{n\gamma} \rangle = \left\langle \frac{\sum_{J \in \Delta E, J} \langle A_Y \rangle J}{\Delta E} \right\rangle = 2\pi^2 \lambda^2 \frac{1}{\langle D_J \rangle} \left\langle \frac{\Gamma_n \Gamma_Y}{\Gamma} \right\rangle. \quad (9)$$

We note that Eq. (9) directly yields the average cross section for formation of the compound nucleus if the γ decay channels are enlarged to include all channels, or $\Gamma_Y \rightarrow \Gamma$, giving

$$\langle \sigma_{fcn} \rangle = 2\pi^2 \lambda^2 \frac{\langle \Gamma_n \rangle}{\langle D_J \rangle} = 2\pi^2 \lambda^2 S, \quad (10)$$

so that the strength function, S , is a direct measure of the average cross section for formation of the compound nucleus.

In the usual case that the radiative width may be taken to be a constant, $\langle \Gamma_\gamma \rangle$, we can write Eq. (9) in the form:

$$\langle \sigma_{n\gamma} \rangle = 2\pi^2 \lambda^2 \frac{\langle \Gamma_\gamma \rangle}{\langle D_J \rangle} \cdot \left\langle \frac{\Gamma_n}{\Gamma} \right\rangle \quad (11a)$$

or

$$\langle \sigma_{n\gamma} \rangle = 2\pi^2 \lambda^2 S_0 \cdot \frac{\langle \Gamma_\gamma \rangle}{\langle \Gamma_n^0 \rangle} \left\langle \frac{\Gamma_n}{\Gamma} \right\rangle. \quad (11b)$$

This last form takes greater advantage of the direct measurement of S_0 through Eq. (8), the last term of Eq. (11b) being less sensitive to variations of Γ_γ and Γ_n^0 . Indeed, in the limit $\Gamma_\gamma \gg \Gamma_n$, that last factor approaches $\sqrt{E}$, (refer to Table III for relative sizes).

We estimate¹⁶ that for neutron energies from 1 eV to 30 KeV, the average level spacing, $\langle D_J \rangle$, varies about 4%; thus we can take the average level density to be constant over this range.

¹⁶We used the simple formula $\langle D_J \rangle = \text{const.} (AW)^2 \exp[-(4AW/11)^{1/2}]$ to make this calculation. H. W. Newson and J. H. Gibbons, p. 1660 of J. B. Marion and J. L. Fowler, "Fast Neutron Physics", Part II, Interscience, New York (1963).

B. Odd-even Calculations

We begin with Eqs. (8) and (11) and note that odd-even and odd-odd refer throughout this work to the target nucleus.

Europium strength functions from the slope of the total cross section are reported to be

$$S_0 = 2.7 \times 10^{-4} \text{ (ev)}^{-\frac{1}{2}} \text{ from 0.7 to 14 KeV}^{13},$$

$$S_0 = 2.51 \times 10^{-4} \text{ (ev)}^{-\frac{1}{2}} \text{ from 0.7 to 14 KeV}^{17},$$

$$S_0 = (3.3 \pm 0.5) \times 10^{-4} \text{ (ev)}^{-\frac{1}{2}} \text{ from 200 to 7000 ev}^{18},$$

from which, and from expectation that the total cross section from Eq. (7) ought to be in the range 43-44 barns at 1 KeV,^{13,19} we take as the best estimate

$$\text{Eu } S_0 = 2.95 \times 10^{-4} \text{ (ev)}^{-\frac{1}{2}} \pm 13\%.$$

The rest of the data we must take from the low energy resonance parameters. Using the Brookhaven Sigma Center Compilation¹⁹

$$\text{Eu}^{151} \langle \Gamma_Y \rangle = 8.75 \times 10^{-2} \text{ ev},$$

$$\text{Eu}^{153} \langle \Gamma_Y \rangle = 10.15 \times 10^{-2} \text{ ev}.$$

¹⁷Our evaluation from $\langle \sigma_T \rangle$ vs time of flight of Ref. 13.

¹⁸R. L. Zimmerman in Oak Ridge National Laboratory Progress Report, ORNL-2309, p. 10 (Nov. 1956), Clearinghouse for Federal Scientific and Technical Information, Springfield, Virginia.

¹⁹D. J. Hughes, B. A. Magurno, and M. K. Brussel, "Neutron Cross Sections," Brookhaven National Laboratory Report, BNL 325, 2d Ed., Supplement 1 (1960). Clearinghouse for Federal Scientific and Technical Information.

The large number of degrees of freedom in a Porter-Thomas distribution expected for a total radiative width led to acceptance of the above straight average of these γ -widths. The neutron width is quite another matter, as is the spacing of widths. For the latter we expect that the Wigner surmise,²⁰ more explicitly the "nearest-neighbor spacing distribution of Gaussian-orthogonal ensembles for 2×2 matrices"²¹ will govern the spacing distribution. The probability P of a spacing between D and $D + dD$ is given in terms of the average spacing $\langle D \rangle$:

$$P(D)dD = \frac{\pi D}{2\langle D \rangle^2} \exp \left[-\frac{\pi}{4} \frac{D^2}{\langle D \rangle^2} \right] dD \quad (12)$$

Note: For simplicity, $D \equiv D_J$ in Eq. (12) and hereafter. D may also represent D_{obs} in the especial case that $\langle D_J \rangle_1 \approx \langle D_J \rangle_2 \approx 2D_{\text{obs}}$, as we will take it later, for lack of better information.

The average number of levels per dD is, of course, $dN = N_0 P(D)dD$, N_0 being the total number; and the number greater than D , $N_{>D}$, is given by integration of this expression from D to ∞ as

$$\ln N_{>D} = \ln N_0 - \frac{\pi}{4} \frac{D^2}{\langle D \rangle^2}, \quad (13)$$

so if we plot $N_{>D}$ versus D^2 on semilog paper, the curve should be a

²⁰E. P. Wigner, p. 59 of ref. 18.

²¹C. E. Porter, p. 52, 55, and 56 of Ref. 11.
H. Feshbach, p. 662, in F. Ajzenberg-Selove, "Nuclear Spectroscopy," Part B, Academic Press, New York (1960).
J. E. Lynn, p. 132 of M. Neve de Mévergnies et al, "Nuclear Structure Study with Neutrons," North-Holland, Amsterdam (1966).
J. A. Farrell, E. G. Bilpuch, and H. W. Newson, Ann. Phy. (N.Y.), 37 367 (1966).

straight line, the slope of which yields $\langle D \rangle$ through the formula

$$\langle D \rangle = \frac{1}{2} \sqrt{\frac{\pi(D_2^2 - D_1^2)}{\ln N_{>D1} - \ln N_{>D2}}} \quad (14)$$

where 1 and 2 are any two points of the straight line. Achieving best accuracy lies in the art of rejecting unsatisfactory data; that is, in knowing when to cut off the experimental measurements.

For example, for Eu^{151} , and D representing observed spacings, cutting off at 8 eV neutron energy¹⁹ leads to $\langle D_{\text{obs}} \rangle \sim 0.67$ eV; and cutting off at 15 eV to $\langle D_{\text{obs}} \rangle \sim 0.82$ eV. Similarly for Eu^{153} . The distribution law Eq. (12) is well established,²¹ nor do we dispute it, but for estimation of $\langle D \rangle$ we find that the direct plot of the number of levels less than E vs E is more convenient to use. Such a plot is found in Fig. 3. Here the slope gives $\langle D_{\text{obs}} \rangle$, and one can directly estimate missing levels from the curvature. We must of course make use of the distribution law in algebraic manipulations.

In Eu^{151} we estimate that one or two levels are missed at 8-10 eV, and none or one levels at about 14 eV. We are therefore led to put $\langle D \rangle = 0.74$ eV for Eu^{151} ; similarly we take no levels lost in Eu^{153} to 6.2 eV and then the loss of seven levels so that we estimate $\langle D \rangle = 0.86$ eV for Eu^{153} .

The distribution governing the reduced neutron widths is, of course, the Porter-Thomas chi-square distribution with one degree of freedom.²²

²²C. E. Porter, p. 74 of Ref. 12.
T. Fuketa and J. A. Harvey, Nucl. Inst. Meth. (Netherlands) 33, 107 (1965).
J. A. Farrel, E. G. Bilpuch, and H. W. Newson, Ann. Phy. (N.Y.), 37, 367 (1966).

Fig. 3. Determination of $\langle D \rangle$.

The number of reduced widths in an interval $d\Gamma_n^o$, if N_o is the total number, is

$$dN = N_o [2\pi\Gamma_n^o \langle \Gamma_n^o \rangle]^{-\frac{1}{2}} \exp[-\Gamma_n^o / 2\langle \Gamma_n^o \rangle] d\Gamma_n^o. \quad (15)$$

By defining $M_{>\Gamma_n^o} \equiv \sum_{\Gamma_n^o} \sqrt{\Gamma_n^o} dN$, one obtains upon integration:

$$\ln M_{>\Gamma_n^o} = \ln \sqrt{2\pi \langle \Gamma_n^o \rangle} - \frac{\Gamma_n^o}{2\langle \Gamma_n^o \rangle} + \ln N_o, \quad (16)$$

with $\langle \Gamma_n^o \rangle$ obtained from the slope of $M_{>\Gamma_n^o}$ versus Γ_n^o . But our data,¹⁹

while not contesting the details of the Porter-Thomas distribution, is not good enough to utilize it and we find a simple exponential distribution to be adequate.

Accordingly we use the distribution

$$dN = (N_o / \langle \Gamma_n^o \rangle) \exp[-\Gamma_n^o / \langle \Gamma_n^o \rangle] d\Gamma_n^o, \quad (17)$$

and we plot the number of levels with reduced widths greater than Γ_n^o , $N_{>\Gamma_n^o}$ versus width to obtain

$$N_{>\Gamma_n^o} = \int_{\Gamma_n^o}^{\infty} dN = N_o \exp[-\Gamma_n^o / \langle \Gamma_n^o \rangle],$$

$$\ln N_{>\Gamma_n^o} = \ln N_o - \frac{\Gamma_n^o}{\langle \Gamma_n^o \rangle}. \quad (18)$$

See Figs. 4 and 5 where we expect that on semilog paper the plot of $N_{>\Gamma_n^0}$ vs Γ_n^0 will be a straight line with slope $-\langle\Gamma_n^0\rangle^{-1}$ obtained from any two points 1 and 2 by

$$\langle\Gamma_n^0\rangle = \frac{\Gamma_{n2}^0 - \Gamma_{n1}^0}{\ln\left(\frac{N_{>\Gamma_{n1}^0}}{N_{>\Gamma_{n2}^0}}\right)}. \quad (19)$$

Such a procedure yields

$$\langle\Gamma_n^0\rangle_{151} = (0.395 \pm 0.005) \times 10^{-3} \text{ ev}^{\frac{1}{2}} \text{ where (see Fig. 3)}$$

we cut the levels off at 15 eV and add two (inconsequential) levels with $\Gamma_n^0 \sim 0.003 \times 10^{-3} \text{ ev}^{\frac{1}{2}}$ to account for missing levels. Our results lie within 1% of a simple average.

For Eu^{153} the same procedure up to an incident neutron energy of 6.2 eV leads to widths between 0.37 and 0.46 mV (see Fig. 5) with a straight average of 0.325 mV. We are led to adopt

$$\langle\Gamma_n^0\rangle = (0.35 \pm 0.05) \times 10^{-3} \text{ ev}^{\frac{1}{2}} \text{ for } \text{Eu}^{153}.$$

Table III compares our parameters with those of Garrison.⁷

Fig. 4. Eu-151, Determination of $\langle \Gamma_n^0 \rangle$.

Fig. 5. Eu-153, Determination of $\langle \Gamma_n^0 \rangle$.

TABLE III

Average Low Energy Resonance Parameters

Parameter	Barr and Devaney	Garrison ⁷	Straight average of Brookhaven Compilation ¹⁹ Data; no additions, exclusions, or cutoffs.
Eu¹⁵¹			
$\langle \Gamma_n^0 \rangle \text{ ev}^{\frac{1}{2}}$	3.95×10^{-4}	4.2×10^{-4}	3.97×10^{-4}
$\langle \Gamma_\gamma \rangle \text{ ev}$	8.75×10^{-2}	9.1×10^{-2}	8.75×10^{-2}
$\langle D_{\text{obs}} \rangle \text{ ev}$	0.74	0.70	0.848
Eu¹⁵³			
$\langle \Gamma_n^0 \rangle \text{ ev}^{\frac{1}{2}}$	3.5×10^{-4}	5.1×10^{-4}	5.65×10^{-4}
$\langle \Gamma_\gamma \rangle \text{ ev}$	10.15×10^{-2}	9.7×10^{-2}	10.15×10^{-2}
$\langle D_{\text{obs}} \rangle \text{ ev}$	0.86	1.1	1.339

Our parameters in Table III yield $(\langle \Gamma_n^0 \rangle / 2\langle D_{\text{obs}} \rangle) = 2.67 \times 10^{-4} (\text{ev})^{-\frac{1}{2}}$ for Eu¹⁵¹ and 2.03×10^{-4} for Eu¹⁵³. We must use the ratio of these numbers, for want of a better method, to extract the strength functions for the isotopes from that measured for the normal element. This is the most suspect action in this calculation; see the discussion in Part A. Using the abundances ²³Eu¹⁵¹ 47.82% and Eu¹⁵³ 52.18% and $S_0 \text{ Eu} = 2.95 \times 10^{-4} \text{ ev}^{-\frac{1}{2}}$,

²³A. O. Nier in "Nuclear Data Sheets," National Academy of Sciences - National Research Council (1958).

we find

$$S_{01} = 3.37 \times 10^{-4} \text{ (eV)}^{-\frac{1}{2}} \text{ Eu}^{151} \text{ (Reduced) Strength Function}$$

$$S_{03} = 2.56 \times 10^{-4} \text{ (eV)}^{-\frac{1}{2}} \text{ Eu}^{153} \text{ Strength Function.}$$

With S available Eq. (10) gives directly $\langle \sigma_{fcn} \rangle$. In order to evaluate $\langle \sigma_{ny} \rangle$ from Eq. (11) we will find it convenient to evaluate

$\left\langle \frac{\Gamma_Y}{\Gamma} \right\rangle$. The relative constancy²⁴ of Γ_Y entitles us to write

$$\left\langle \frac{\Gamma_Y}{\Gamma} \right\rangle = \langle \Gamma_Y \rangle \left\langle \frac{1}{\Gamma} \right\rangle \equiv B \quad (20)$$

defining the quantity B. We also define

$$a \equiv \langle \Gamma_Y \rangle / \langle \Gamma_n \rangle = \langle \Gamma_Y \rangle / \sqrt{E} \langle \Gamma_n^0 \rangle, \quad (21)$$

with

$$\langle \Gamma \rangle = \langle \Gamma_n \rangle + \langle \Gamma_Y \rangle. \quad (22)$$

Both for simplicity and because it represents our data best, we take the distribution shown in Eq. (17) to evaluate B,

$$B \equiv \frac{\langle \Gamma_Y \rangle}{N_0} \int_0^\infty \frac{dN}{\Gamma} = \frac{\langle \Gamma_Y \rangle}{\langle \Gamma_n^0 \rangle} \int_0^\infty \frac{\exp[-\Gamma_n^0 / \langle \Gamma_n^0 \rangle]}{\langle \Gamma_Y \rangle + \sqrt{E} \Gamma_n^0} d\Gamma_n^0. \quad (23)$$

With $x \equiv \frac{\Gamma_n^0}{\langle \Gamma_n^0 \rangle}$ this becomes

²⁴Also: Garrison (Ref. 7, p. 300), reports no correlation for $\Gamma_Y : \Gamma_Y$ and none for $\Gamma_Y : \Gamma_n^0$.

$$B(a) = a \int_0^{\infty} \frac{t^{-x} dx}{a+x} \quad (24)$$

Equation (24) is the LaPlace transform $\frac{a}{a+x}$ which is

$$B(z) = -a \mathcal{L}^a E_1(-a), \quad (25)$$

where $E_1(z) \equiv \int_{-\infty}^z \frac{t^t}{t} dt$ is the exponential integral and $B(a)$ is evaluated by series and asymptotic series and plotted in Fig. 6.

In Eqs. (11a) and (11b) the quantity appearing is $\left\langle \frac{\Gamma_n}{\Gamma} \right\rangle$, not B ; the following equation gives the relationship

$$\left\langle \frac{\Gamma_n}{\Gamma} \right\rangle = \left\langle \frac{\Gamma - \Gamma_Y}{\Gamma} \right\rangle = 1 - \left\langle \frac{\Gamma_Y}{\Gamma} \right\rangle = 1 - \left\langle \frac{\Gamma_Y}{\Gamma} \right\rangle = 1 - B. \quad (26)$$

Substituting in Eq. (11) we get

$$\langle \sigma_{nY} \rangle = 2\pi^2 \kappa^2 \frac{\langle \Gamma_Y \rangle}{\langle D_J \rangle} (1-B), \quad (27a)$$

and

$$\langle \sigma_{nY} \rangle = 2\pi^2 \kappa^2 S_0 \cdot \frac{\langle \Gamma_Y \rangle}{\langle \Gamma_n \rangle} (1-B). \quad (27b)$$

Using Table III we calculate A from Eq. (21), and B from Fig. 6, whence Eq. (27b) and our S_0 's derived from Eq. (8) and listed following Table III, lead to

Fig. 6. B versus a.

$$\langle \sigma_{ny} \rangle \text{Eu}^{151} = 35.6 \text{ barns},$$

$$\langle \sigma_{ny} \rangle \text{Eu}^{153} = 28.0 \text{ barns},$$

and

$$\langle \sigma_{ny} \rangle \text{Eu} = 31.6 \text{ barns},$$

all at 1 KeV.

Now the $S_o = \langle \Gamma_n^o \rangle / \langle D_J \rangle$ derived from Table III are:

$\text{Eu}^{151} S_o = 2.67 \times 10^{-4} \text{ eV}^{-\frac{1}{2}}$ and $\text{Eu}^{153} S_o = 2.03 \times 10^{-4} \text{ eV}^{-\frac{1}{2}}$, whereas those derived from Eq. (8) are $3.37 \times 10^{-4} \text{ eV}^{-\frac{1}{2}} (\text{Eu}^{151})$ and $2.56 \times 10^{-4} \text{ eV}^{-\frac{1}{2}} (\text{Eu}^{153})$. If it can be argued that the difference between the two sets is principally from $\langle \Gamma_n^o \rangle$ rather than $\langle D_J \rangle$, it is appropriate to adjust the $\langle \Gamma_n^o \rangle$ of Table III to conform to S_o from Eq. (8). Considering the wildly varying nature of Γ_n^o , such an argument is not unreasonable and the correction runs less than 3% in the cross section.

Thus we would adjust

$$\langle \Gamma_n^o \rangle \rightarrow \Gamma_n^o \text{ of Table III} \times S_o \text{ from Eq. (8)} / S_o \text{ from Table III}$$

so that

$$\langle \Gamma_n^o \rangle \rightarrow 4.99 \times 10^{-4} \text{ eV}^{\frac{1}{2}} \text{ for } \text{Eu}^{151},$$

and

$$\langle \Gamma_n^o \rangle \rightarrow 4.42 \times 10^{-4} \text{ eV}^{\frac{1}{2}} \text{ for } \text{Eu}^{153}.$$

Substituting these values, together with $\langle \Gamma_Y \rangle$ from Table III, S_o derived from Eqs. (8) and (21), and Fig. 6, into Eq. (27b) give

$$\langle \sigma_{n\gamma} \rangle \text{Eu}^{151} = 34.3 \text{ barns,}$$

$$\langle \sigma_{n\gamma} \rangle \text{Eu}^{153} = 27.4 \text{ barns,}$$

and

$$\langle \sigma_{n\gamma} \rangle \text{Eu} = 30.7 \text{ barns,} \quad \text{all at 1 KeV,}$$

which lie well within error of the previous results.

Between these two sets we choose the values

$$\langle \sigma_{n\gamma} \rangle \text{Eu}^{151} = 35.5 \text{ barns,}$$

$$\langle \sigma_{n\gamma} \rangle \text{Eu}^{153} = 27.5 \text{ barns,}$$

and

$$\langle \sigma_{n\gamma} \rangle \text{Eu} = 31.3 \text{ barns,} \quad \text{all at 1 KeV,}$$

to list in column two of Table II as our "slope of total cross-section" results.

Next we calculate the capture cross section from the average resonance parameters determined at low energy. We expect this procedure to be intrinsically less accurate because the correlations and distributions enter in a more direct way so that their choice is more sensitive, and the average parameters obtained may be those appropriate to an intermediate resonance maximum or minimum and therefore may not represent the true large scale average.

The quantities of Table III, using Fig. 6, substituted into Eq. (27a) yield the kilovolt cross sections:

$$\text{Eu}^{151}\langle\sigma_{n\gamma}\rangle = 28.1 \text{ barns},$$

$$\text{Eu}^{153}\langle\sigma_{n\gamma}\rangle = 22.2 \text{ barns},$$

and

$$\text{Eu}\langle\sigma_{n\gamma}\rangle = 25. \text{ barns}.$$

These "resonance parameter" results are listed in column eight of Table II.

If we use Garrison's parameters⁷ and data from Fig. 6 in Eq. (27a) we get at 1 keV

$$\langle\sigma_{n\gamma}\rangle\text{Eu}^{151} = 31.6 \text{ barns},$$

$$\langle\sigma_{n\gamma}\rangle\text{Eu}^{153} = 24 \text{ barns},$$

and

$$\langle\sigma_{n\gamma}\rangle\text{Eu} = 27.6 \text{ barns},$$

which are found in column nine of Table II.

Bell² has used a form of Eq. (9), the parameters of Garrison,⁷ and a form of averaging correction used by Lane and Lynn²⁵ to arrive at the values in column ten of Table II, which we include for comparison.

²⁵A. M. Lane and S. E. Lynn, Proc. Phys. Soc. (London) A70, 557 (1957). See also A. G. W. Cameron, N. H. Lazar, and H. W. Schmitt, pp. 1744 - 1747 of J. B. Marion and J. L. Fowler, Fast Neutron Physics, Part II, Interscience, New York (1963).

C. Odd-odd Calculations

In calculating the odd-odd (target) europium isotopes we have two choices: (1) We can compare the known experimental odd-odd resonance parameters with their adjacent odd-even neighbors, or (2) we can make theoretical extrapolations from odd-even to odd-odd behavior. Neither method, that is neither experiment nor theory, is in anything like a satisfactory state. In fact, it is the opinion of these authors that the systematics of the odd-odd nuclei are not yet known. With what accuracy we can make these pessimistic statements and contrapuntally calculate the odd-odd europium cross sections follows:

We start with experimental comparisons, as in (1) above, containing all the errors of averaging and siting of the intermediate resonances heretofore mentioned. We have only two cases in this region of the periodic table that are not close to magic numbers. These are ${}_{71}\text{Ta}^{176}_{105}$ vs Lu^{175}_{104} and ${}_{73}\text{Ta}^{180}_{107}$ vs Ta^{181}_{108} . Once again we go through the procedure of plotting the number of levels less than E versus E to find D and making a log plot of the number of levels of width greater than Γ_n^0 vs Γ_n^0 , excluding missed levels as indicated by the previous plot. We spare the reader the details, the method has been described for Eu^{151} and Eu^{153} , and the results are shown in Table IV^{19,26}.

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- ²⁶
 Ta^{180} : Dr. J. A. Harvey, Oak Ridge National Laboratory, private communication, (December, 1965).
 Ta^{181} : K. Way et al, Nucl. Data, B1-1-22 (1966).
J. Rainwater et al, No. 95 of the "International Conference on the Study of Nuclear Structure with Neutrons," EANDC-50-S, (1965).

TABLE IV

Odd-odd and Adjacent Odd-even Resonance Parameters

Lu^{175}	$\langle D_{\text{obs}} \rangle = 3.32 \text{ eV};$	$\langle \Gamma_n^o \rangle = 1.1 \times 10^{-3} \text{ eV}^{\frac{1}{2}};$	$\langle \Gamma_Y \rangle = 8.58 \times 10^{-2} \text{ eV}$
Lu^{176}	$\langle D_{\text{obs}} \rangle = 2.18 \text{ eV};$	$\langle \Gamma_n^o \rangle = 0.821 \times 10^{-3} \text{ eV}^{\frac{1}{2}};$	$\langle \Gamma_Y \rangle = 5.8 \times 10^{-2} \text{ eV}$
Ta^{180}	$\langle D_{\text{obs}} \rangle = 1.6 \text{ eV};$	$\langle \Gamma_n^o \rangle = 0.37 \times 10^{-3} \text{ eV}^{\frac{1}{2}};$	$\langle \Gamma_Y \rangle = 5.9 \times 10^{-2} \text{ eV}$
Ta^{181}	$\langle D_{\text{obs}} \rangle = 4.08 \text{ eV};$	$\langle \Gamma_n^o \rangle = 1.49 \times 10^{-3} \text{ eV}^{\frac{1}{2}};$	$\langle \Gamma_Y \rangle = 5.66 \times 10^{-2} \text{ eV}$

These numbers indicate the ratios:

$$\text{Lu: } \langle \Gamma_n^o \rangle_{\text{oo}} / \langle \Gamma_n^o \rangle_{\text{oe}} = 0.746; \quad \langle D_J \rangle_{\text{oo}} / \langle D_J \rangle_{\text{oe}} = 0.657; \quad S_{\text{o,oo}} / S_{\text{o,oe}} = 1.135$$

$$\text{Ta: } \langle \Gamma_n^o \rangle_{\text{oo}} / \langle \Gamma_n^o \rangle_{\text{oe}} = 0.248; \quad \langle D_J \rangle_{\text{oo}} / \langle D_J \rangle_{\text{oe}} = 0.392; \quad S_{\text{o,oo}} / S_{\text{o,oe}} = 0.633$$

where $S_o = \langle \Gamma_n^o \rangle / \langle D_{\text{obs}} \rangle$.

We weight these numbers 6 to 1 (Lu to Ta) to account for the greater number of levels in Lu, (Lu^{175} vs Ta^{180}) to obtain:

$$\langle \Gamma_n^o \rangle_{\text{oo}} / \langle \Gamma_n^o \rangle_{\text{oe}} = 0.675; \quad \langle D_J \rangle_{\text{oo}} / \langle D_J \rangle_{\text{oe}} = 0.619; \quad S_{\text{o,oo}} / S_{\text{o,oe}} = 1.06. \quad (28)$$

The average capture width seems to follow a fairly smooth curve vs Z at 63 (data from Ref. 21), so we shall estimate $\langle \Gamma_Y \rangle$ from Eu^{151} and Eu^{153} directly (see Fig. 7).

We shall first apply our odd-odd to odd-even comparisons to Eq. (27b). For Eu^{152} we average the parameters from Eu^{151} and Eu^{153} and multiply by Eq. (28), (refer to Table III et seq.)

Fig. 7. $\langle \Gamma_\gamma \rangle$ vs Z (Ref. 21).

$$\text{Eu}^{152}: S_0 = 3.15 \times 10^{-4} \text{ eV}^{-\frac{1}{2}}; \langle \Gamma_n^0 \rangle = 2.51 \times 10^{-4} \text{ eV}^{\frac{1}{2}}; \langle D_J \rangle = 0.99 \text{ eV};$$

$$\langle \Gamma_Y \rangle = 9.45 \times 10^{-2} \text{ eV}$$

whence at 1 keV $a = 11.9$, $(1-B) = 0.073$ from Eq. (21) and Fig. 6.

We extrapolate to Eu^{154} by multiplying the parameters of Eu^{152} by the parameter ratio of Eu^{153} to Eu^{151} to find:

$$\text{Eu}^{154}: S_0 = 2.4 \times 10^{-4} \text{ eV}^{-\frac{1}{2}}; \langle \Gamma_n^0 \rangle = 2.23 \times 10^{-4} \text{ eV}^{\frac{1}{2}};$$

$$\langle D_J \rangle = 1.151 \text{ eV}.$$

However for $\langle \Gamma_Y \rangle$ we expect little variation from Eu^{153} and so have taken the value 102.0 mV. So at 1 keV, $A = 14.48$, $(1-B) = 0.0614$.

Substituting these values into Eq. (26) we obtain at 1 keV:

$$\text{Eu}^{152} \langle \sigma_{n\gamma} \rangle = 35.4 \text{ barns}$$

$$\text{Eu}^{154} \langle \sigma_{n\gamma} \rangle = 27.5 \text{ barns}.$$

These "slope of total" plus "odd-odd to odd-even comparison" results are listed in column two of Table II.

The second method available to us for estimating odd-odd cross sections is that of theory, also a very weak reed. The argument is as follows: Since $\langle \Gamma_n^0 \rangle$ is a measure of the average neutron decay probability, it must be proportional to the period of the nucleus, which in turn is proportional to $\langle D_J \rangle$,²⁷ the average level spacing, (spin J). Thus the

²⁷V. F. Weisskopf, *Helv. Phys. Acta* **23**, 187 (1950).

J. J. Devaney, *Phys. Rev.* **91**, 587 (1953).

Note that this statement does not contradict the lack of correlation of individual Γ_n to D, Ref. 7.

strength function, $\langle \Gamma_n^o \rangle / \langle D_J \rangle$, is independent of variations of $\langle D_J \rangle$. How we found it experimentally for Lu and Ta above (and for other nuclei) is not too encouraging, but the weighted average of the two, namely, 1.06: odd-odd to adjacent odd-evens, does indeed bear this remark out. Both theoretically and experimentally we expect $\langle \Gamma_Y \rangle$ to be nearly a constant, independent of D , ($\langle D \rangle$ proportionality of $\langle \Gamma_Y \rangle$ here essentially cancels with multiplication by the density of levels decayed to). So, if we can predict $\langle D_J \rangle$, the constancy of the strength function odd-even to odd-odd gives $\langle \Gamma_n^o \rangle$ and we can evaluate the odd-odd cross sections. The work of Gilbert and Cameron²⁸ provides a convenient way to estimate the ratio, $R \equiv D_{oe\text{-target}} / D_{oo\text{-target}} = \rho_{oe\text{-compound}} / \rho_{oo\text{-compound}}$, where ρ is the density of levels (of a given angular momentum). We use their prescription (p. 1475) to calculate, (at 1 keV plus binding energy of the incident neutron):

$$R_{151-152 \text{ target}} = 6.78 \text{ and } R_{153-154 \text{ target}} = 3.14 \quad (29)$$

where we take the europiums as deformed because of their large quadrupole moments.²⁹ Further we have taken the following parameters:

²⁸A. Gilbert and A. G. W. Cameron, *Canad. Jour. Phys.* 43, 1446 (1965).

²⁹G. H. Fuller and V. W. Cohen, "Nuclear Moments," *Appl. to Nucl. Data Sheets*, National Academy of Sciences (1965).

TABLE V
Europium Spins and Binding Energies

Nucleus:	Eu ¹⁵¹	Eu ¹⁵²	Eu ¹⁵³	Eu ¹⁵⁴	Eu ¹⁵⁵
Spin J: ³⁰	5/2	3	5/2	3	5/2
Binding Energy Last Neutron (Mev): ³¹	7.933	6.291	8.544	6.385	8.186

The possible compound spins were averaged to achieve the results of Eq. (29). As before, the variation of D over 30 Kev ($\leq 4\%$) is negligible to our accuracy. For Eu¹⁵² then, we average the parameters from Eu¹⁵¹, and divide by the ratio $R_{151-152}$ of Eq. (29) to get:

TABLE VI
Average Low Energy Resonance Parameters
(Theoretical odd-odd to odd-even ratios)

Eu ¹⁵² :	$S_0 = 2.97 \times 10^{-4} \text{ eV}^{-\frac{1}{2}}$ $\langle \Gamma_n^o \rangle = 0.549 \times 10^{-4} \text{ eV}^{\frac{1}{2}}$ $\langle \Gamma_\gamma \rangle = 9.45 \times 10^{-2} \text{ eV}$ $\langle D_J \rangle = 0.236 \text{ eV}$
Eu ¹⁵⁴ :	$S_0 = 2.26 \times 10^{-4} \text{ eV}^{-\frac{1}{2}}$ $\langle \Gamma_n^o \rangle = 1.05 \times 10^{-4} \text{ eV}^{\frac{1}{2}}$ $\langle \Gamma_\gamma \rangle = 10.2 \times 10^{-2} \text{ eV}$ $\langle D_J \rangle = 0.594 \text{ eV}$

³⁰"Nuclear Data Sheets," National Academy of Sciences, (1963), (1964).

³¹J. H. E. Mattauch, W. Thiele, and A. H. Wapstra, Nucl. Phys. 67, 32 (1965).

Where for Eu^{154} we extrapolate from the average of Eu^{151} and Eu^{153} by the ratio of parameters of Eu^{153} to Eu^{151} , corrected in the case of neutron width and spacing by the ratio $R_{153-154}^{-1}$ of Eq. (29); again we keep $\langle \Gamma_Y \rangle = 0.102 \text{ eV}$.

Substitution of the parameters from Table VI into Eq. (21), using Fig. 6, and Eq. (27b) we obtain, at 1 keV

$$\text{Eu}^{152} \langle \sigma_{n\gamma} \rangle = 37.2 \text{ barns},$$

and

$$\text{Eu}^{154} \langle \sigma_{n\gamma} \rangle = 27.4 \text{ barns},$$

essentially the same as our previous estimate. These "slope of total cross section" with theoretical odd-odd to odd-even comparison are found in column three of Table II.

On the other hand substitution of the parameters of Table VI, except S_0 , into Eqs. (21) and (27a), using Fig. 6, give at 1 keV

$$\text{Eu}^{152} \langle \sigma_{n\gamma} \rangle = 29.1 \text{ barns}$$

and

$$\text{Eu}^{154} \langle \sigma_{n\gamma} \rangle = 21.4 \text{ barns}.$$

These "resonance parameter" cross sections are listed in column eight of Table II.

The same estimation, using Eq. (29), applied to Garrison's⁷ parameters for Eu^{151} and Eu^{153} lead to Table VII.

TABLE VII

Average Low Energy Resonance Parameters

(From Garrison⁷ with Barr and Devaney extrapolation)

Eu ¹⁵² :	$\langle \Gamma_n^0 \rangle = 0.685 \times 10^{-4} \text{ eV}^{\frac{1}{2}}$
	$\langle \Gamma_\gamma \rangle = 9.4 \times 10^{-2} \text{ eV}$
	$\langle D_J \rangle = 0.265 \text{ eV}$
Eu ¹⁵⁴ :	$\langle \Gamma_n^0 \rangle = 1.8 \times 10^{-4} \text{ eV}^{\frac{1}{2}}$
	$\langle \Gamma_\gamma \rangle = 0.1 \text{ eV}$
	$\langle D_J \rangle = 0.9 \text{ eV}$

These into Eqs. (21) and (27a), using Fig. 6, give

$$\text{Eu}^{152} \langle \sigma_{n\gamma} \rangle = 31.9 \text{ barns}$$

and

$$\text{Eu}^{154} \langle \sigma_{n\gamma} \rangle = 23.3 \text{ barns, both at 1 keV.}$$

These values are found in Table II, ninth column.

III. MEASUREMENTS AND CALCULATIONS FROM CYCLAMEN

A portion of the nuclear device, Cyclamen,¹ contained samples of natural europium and thallium. The device was detonated underground, and a fraction of the surrounding soil was recovered and radiochemically analyzed. The raw data, corrected to total amounts for the whole device, follows:

TABLE VIII

Atom Abundances from Cyclamen

Eu:	151 input	20.46×10^{22} atoms,	(N_1^0)
	152 measured	3.81×10^{22} atoms,	(N_2)
	153 input	22.32×10^{22} atoms,	(N_3^0)
	154 measured	10.61×10^{22} atoms,	(N_4)
	155 measured	2.13×10^{22} atoms,	(N_5)
	(corrected for fission yield)		
	156 measured	0.63×10^{22} atoms,	(N_6) .
	(corrected for fission yield)		

Measurements on thallium-204 lead to an integrated flux times cross section ((neutrons/volume) x velocity x time x cross section) of 0.044 for that portion of the device containing both europium and thallium.

In calculating cross sections from the foregoing data, a number of simplifying assumptions are both possible and effective. First, the flux is taken to be dominant at one energy. Second, with direction not relevant, radiative capture is the only significant process. Starting with only one isotope, the first, N_1^0 atoms originally, we will form N_1 atoms of the i^{th} isotope, with a capture cross section, σ_c^i , from a flux, x , at an effective energy, E . In Eqs. (30), (31), (32), and (33), the numbers refer only to the first isotope, 2d, etc. We shall also use the labels N_3 for N(Eu-153), where such notation is both unambiguous and useful. If we write $\sigma_i x \equiv u_i$ and $\Delta_{ij} \equiv u_i - u_j$, then the standard production formulas take the form:³²

$$N_1 = N_1^0 e^{-u_1} \quad (30-1)$$

$$N_2 = N_1^0 u_1 \frac{e^{-u_1} - e^{-u_2}}{\Delta_{21}} \quad (30-2)$$

$$N_3 = N_1^0 u_1 u_2 \cdot \left[\frac{e^{-u_1} - e^{-u_3}}{\Delta_{21} \Delta_{31}} + \frac{e^{-u_2} - e^{-u_3}}{\Delta_{12} \Delta_{32}} \right] \quad (30-3)$$

$$N_n = N_1^0 u_1 u_2 \cdots u_{n-1} \cdot \left[\frac{e^{-u_1} - e^{-u_n}}{\Delta_{21} \Delta_{31} \cdots \Delta_{n-1}} + \frac{e^{-u_2} - e^{-u_n}}{\Delta_{12} \Delta_{32} \cdots \Delta_{n2}} + \cdots + \frac{e^{-u_{n-1}} - e^{-u_n}}{\Delta_{1(n-1)} \Delta_{2(n-1)} \cdots \Delta_{(n-2)(n-1)} \Delta_{n(n-1)}} \right] \quad (30-n)$$

³²See, for example: J. J. Devaney, A. G. Petschek, and M. T. Menzel, "On the Production of Heavy Uranium Isotopes in a Very High Density Fast Neutron Flux," Los Alamos Scientific Laboratory Report, IAMS-2226 (1958).

As noted by G. A. Cowan and C. I. Browne,³² in the special case that, $\sigma_c^1 = \sigma_c^2 = \dots \sigma_c^n \equiv \sigma_c$, the Eq. (30) reduce to:

$$N_n = N_1^0 [u^{n-1}/(n-1)!] l^{-u} \quad (31)$$

where, of course, $u \equiv \sigma_c x$.

Our objective is to use Table VIII and Eq. (30) to derive cross sections. Upon actual calculation we find that the cross sections depend so critically on the values in Table VIII and elsewhere that the consequent errors are excessive. However, taking ratios of Eq. (30) leads to a less sensitive method of obtaining the cross sections, reducing the probable error markedly. Accordingly we list ratios of atom abundances derived from Eq. (30) as follows:

$$\frac{N_2}{N_1} = u_1 l^{u_1(1-\delta_{12})} \quad (32a)$$

$$\frac{N_3}{N_2} = u_2 \frac{\delta_{123}}{1-\delta_{12}} \quad (32b)$$

$$\frac{N_4}{N_3} = u_3 \frac{\delta_{1234}}{\delta_{123}} \quad (32c)$$

.....

$$\frac{N_{i+1}}{N_i} = u_i \frac{\delta_{123\dots i+1}}{\delta_{12\dots i}}, \quad i \geq 3 \quad (32d)$$

where

$$\delta_{12} = \frac{u_1+u_2}{2!} - \frac{u_1^2+u_1u_2+u_2^2}{3!} + \frac{u_1^3+u_1^2u_2+u_1u_2^2+u_2^3}{4!} + \dots \quad (33a)$$

$$\delta_{123} = \frac{1}{2!} - \frac{u_1+u_2+u_3}{3!} + \frac{u_1^2+u_2^2+u_3^2+u_1u_2+u_1u_3+u_2u_3}{4!} - \frac{u_1^3+\dots}{5!} + \dots \quad (33b)$$

$$\delta_{1234} = \frac{1}{3!} - \frac{u_1+u_2+u_3+u_4}{4!} + \frac{u_1^2+u_2^2+u_3^2+u_4^2+u_1u_2+u_1u_3+u_1u_4+u_2u_3+u_2u_4+u_3u_4}{5!} - \frac{u_1^3+\dots}{6!} + \dots \quad (33c)$$

$$\delta_{12345} = \frac{1}{4!} - \frac{u_1+u_2+u_3+u_4+u_5}{5!} + \frac{u_1^2+\dots}{6!} + \dots \quad (33d)$$

.....

For the $\text{Tl}^{203}\sigma_{n\gamma}$ we use the work of Konks and Shapiro,³³ and for Eu^{151} , $\text{Eu}^{153}\sigma_{n\gamma}$, for consistency, we remain with Konks and Fenin.³ Our method of calculation is to obtain first the effective neutron energy and thus from Ref. 3 the Eu^{151} and Eu^{153} cross sections from which by Eqs. (30) or (32), (33), and Table VIII we calculate the other cross sections. The detailed method is as follows:

1. We calculate using Eq. (30) and cross sections from Konks et al and the results from previous calculation of this series, the contribution to Eu^{154} , Eu^{155} and Eu^{156} from successive capture on Eu^{151} .

2. We calculate using Eq. (31) and an estimated average Eu cross section from the previous calculation, the contribution to $\text{Eu} \geq 157$ from Eu^{153} .

³³V. A. Konks and F. L. Shapiro, Sov. Phys. JETP, 20, 531 (1965).

3. To the sum $N_{154} + N_{155} + N_{156}$ of Table VIII we subtract the sum of the results of step 1 and add the (∞) sum of the results of 2, obtaining $N_{153,3}$. $N_{153,3}$ is that portion of N_{153} arising only from captures on Eu^{153} . $N_{153,0}$ is the original number of Eu^{153} atoms.

4. The product $\sigma_c^3 x = \ln[N_{153,0}/N_{153,3}]$ is then obtained, where $\sigma_c^3 \equiv \sigma_{n\gamma}(\text{Eu}^{153})$ here Eq. (30a).

5. We must have $\sigma_{n\gamma}(\text{Tl}^{203}) \cdot x = 0.044$. Using the results of 4 and the cross sections of Konks et al, we solve for a consistent x , and a consistent neutron energy E (two equations, two graphs, two unknowns).

6. E and Konks and Fenin give $\sigma_c^1 \equiv \sigma_{n\gamma}(\text{Eu}^{151})$. Multiplication by x gives u_1 , whence using Table VIII, values for Eu^{152} , Eu^{151} (i.e. N_2 and N_1^0) and Eq. (32a) give δ_{12} .

7. δ_{12} and u_1 give u_2 , and x give $\sigma_c^2 \equiv \sigma_{n\gamma}(\text{Eu}^{152})$.

8. Using the notation that N_{ab} is the number of atoms of Eu^{15a} obtained from successive capture on Eu^{15b} , then Table VIII gives

$$N_4 = N_{41} + N_{43} \text{ and } N_5 = N_{51} + N_{53}.$$

Eq. (32d) for $i = 4$ gives

$$\frac{N_{51}}{N_{41}} = u_4 \frac{\delta_{12345}}{\delta_{1234}} \quad (34)$$

and Eq. (32b) gives

$$\frac{N_{53}}{N_{43}} = u_4 \frac{\delta_{345}}{1 - \delta_{34}} \quad (35)$$

The factors $\delta_{12345}/\delta_{1234}$ and $\delta_{345}/(1-\delta_{34})$ being very nearly $\frac{1}{4}$ and $\frac{1}{2}$ respectively, (see Eq. (32)), enable us to write, very nearly:

$$N_5 = N_{51} + N_{53} \approx u_4 \left[\frac{N_{41}}{4} + \frac{N_{43}}{2} \right] = \frac{u_4}{2} [N_4 - N_{41}] \quad (36)$$

or

$$\sigma_c^4 \equiv \sigma_{n\gamma}(\text{Eu}^{154}) = \frac{u_4}{x} \approx \frac{2N_5}{x[N_4 - N_{41}]} \quad (37)$$

Accordingly we have an approximate value for σ_c^4 , using N_{41} from the preceding calculation, which we then correct by Eqs. (34) and (35) using Eq. (33) and cross sections from our previous calculation.

9. In an identical way, from N_5, N_6 , Eqs. (32d) and (33) we obtain

$$\sigma_c^5 \approx \frac{3N_6}{x \left[N_5 - \frac{2}{5} N_{51} \right]} \quad (38)$$

Again, N_{51} is obtained from an earlier calculation as are the correction factors arising from Eq. (33) and the equations

$$\frac{N_{61}}{N_{51}} = u_5 \frac{\delta_{123456}}{\delta_{12345}}, \quad (39)$$

and

$$\frac{N_{63}}{N_{53}} = u_5 \frac{\delta_{3456}}{\delta_{345}} \quad (40)$$

(both are just forms of Eq. (32d)).

10. We cannot apply the method of steps 8 and 9 to the calculation of σ_c^6 for we need the number of atoms of Eu^{157} , which is not available, but we may use the less accurate formula, Eq. (30d) with $n = 6$,

to calculate σ_{ny} (Eu^{156}). However, for early calculations this value is too wild to use in iteration so we replace it by an average value, and calculate only step 10 at the end.

11. Iterate steps 1-9 (10 only at the end) until the results are stable.

Our calculations according to this prescription lead to a flux of 0.045 barns^{-1} at an effective energy of 2 keV with cross sections given by Table IX, where we arbitrarily assigned all input data an error of $\pm 20\%$.

TABLE IX

Neutron Capture Cross Sections Associated with Cyclamen

Energy 2.04 keV

Isotope	Value (barns)	Error	Source or Derived
Eu^{151}	23.1	assumed $\pm 20\%$	Konks and Fenin ³
Eu^{152}	57.7	$\pm 32\%$	Derived
Eu^{153}	15.3	assumed $\pm 20\%$	Konks and Fenin ³
Eu^{154}	10.9	$\pm 52\%$	Derived
Eu^{155}	20.2	$\pm 52\%$	Derived
Eu^{156}	15	$\sim$ a factor 5	Derived

These values, adjusted theoretically to 1 keV, are found in column four of Table II.

IV. DISCUSSION

In the estimation of capture cross sections we believe the slope of the total cross section to be a most direct method of obtaining the strength function $\langle \Gamma_n \rangle / \langle D \rangle$, at once bypassing questions both of choice of distribution and of determination of average parameters. Further, the total cross section is the one most easily and accurately measured. Consequently, we encourage continued isotopic measurement of total cross sections.

It should be clear from Table II that the various estimates and the various measurements of the capture cross section hardly agree to any great accuracy. Our own Cyclamen results upon addition of two neutrons (Eu^{152} to Eu^{154}) change the cross section by a factor of over five. We find this change very hard to understand, and we are accordingly most suspicious of these results. We note that our phenomenological theory only allows a factor 1.3. We look forward with interest to further experimentation bearing on these cross sections.