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HELICOPTER CLOSE-ORDER FORMATION KEEPING SYSTEM

REPORT No. COO-1471-1

RECOMMENDED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

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U.S. ATOMIC ENERGY COMMISSION

Industrial
Nucleonics / *AccuRay.*

650 ACKERMAN ROAD • COLUMBUS, OHIO 43202

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HELICOPTER CLOSE ORDER
FORMATION KEEPING SYSTEM

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Columbus, Ohio

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1. INTRODUCTION AND SUMMARY

The work reported in this document was performed under contract with the Atomic Energy Commission (Contract No. AT(11-1)-1471) to develop an isotopic gamma ray signaling system which will permit close-order formation flying in helicopters. This first report contains information on the progress to date.

Prior to the submission of the proposal (Helicopter Close-Order Formation Keeping Instrumentation, dated May 6, 1964) which preceded this contract, experimental work and studies were carried out by the U. S. Naval Air Development Center, Johnsville, Pennsylvania, under the direction of Mr. George Wilcox. Concurrently with the work at NADC, Industrial Nucleonics Corporation also performed self-funded studies on several systems. One approach concentrated on an operational concept of using a single radio-isotope source on the lead helicopter with directional detectors on each other helicopter in the formation. This "follow-the-leader" concept gave the following helicopters information on their position relative to the leader but no information on the other members of the formation. Another approach would allow each member of the formation to know the position of all other members. Both approaches were discussed in the proposal.

As the contract work began, it became necessary to reach an agreement as to which method of operation was preferred. Accordingly, Mr. John Dempsey,

AEC, held a conference, attended by representatives of all military services and the contractor, to determine the desired manner of operation. The results of this conference are discussed in detail later in this report, but in essence the decision was to proceed with the concept of a source and detector on each helicopter.

With the operational concept firmed, studies and experiments have continued in the following areas: Dynamic Range and Response Function, Angular Dependence of the Collimated Detector Aperture, Improvement in Measurement Capabilities of the System, Feasibility of Stationary Array Directional Detector, Choice of Sources and Detectors (including health hazard considerations), and Angular Dependence of Detector Efficiency. Information on the work performed in all these areas is contained in this report. The work performed leads to a high confidence in the feasibility of the system with significant progress in measurement accuracy and proper choice of emitter energy.

2. SYSTEM PARAMETERS

2.1 Conference on Operational Requirements

On July 7, 1965, a discussion was held at the Atomic Energy Commission, Washington, D. C., among the personnel from the AEC, Army, Navy, and Air Force, and Industrial Nucleonics Corporation for the purpose of defining operational requirements of the Helicopter Formation Keeping System.¹ It is generally agreed that the following operational parameters are to be evaluated in the research and development program on the close-order flight formation system:

1. The range for the close-order formation keeping system is from 50 feet to 500 feet.
2. Each helicopter is to be provided with a source and detector (emitter and receiver) so other helicopters may be located in the formation within the prescribed range.
3. The number of helicopters in the formation is restricted to four.
4. The formation of the four helicopters may vary in its geometry.

The analysis of system performance must include all the possible variables and their effects on the system.

-
1. The attendees from the government agencies were: J. C. Dempsey, A. E. C., Lt. M. J. Cole, Navy, BuWeps, G. E. Wilcox, Navy, USNADC, Maj. L. F. Baltzell, Army, USADC AVNA, Capt. L. D. McIntoch, Air Force, SEG/RTD (SEAGN), T. E. Petriken, Army, Electronics Laboratory. Industrial Nucleonics Corporation was represented by C. O. Badgett, H. J. Evans, L. K. Hahn, and E. D. Jernigan.

5. The period of one scanning is to be so optimized as to be compatible with the average reaction time of pilot and with the signal strength that can be practically maintained within the range from 50 feet to 500 feet.
6. Isotopic gamma ray, beta-excited X-ray (and/or bremsstrahlung) and electron beam-excited X-ray are to be considered for a source. They will be evaluated for their relative advantages and disadvantages with respect to feasibility and practicality for use in helicopters.
7. A helicopter may, during a formation flight, tilt as much as $\pm 30^\circ$. The configuration and the position of a source beam and a detector in the helicopter will be such that any error due to the tilt in this angular range may be kept at a tolerable minimum level.
8. All important design parameters are to be studied to determine requirements to achieve $\pm 10\%$ accuracy in range measurements. Proper "tradeoffs" are to be made between the desired performance characteristics and the inherent limitations imposed on the system.
9. Directional accuracy of detector is to be compatible with the range accuracy of $\pm 10\%$.
10. The weight of the source and detection system should be kept at minimum, preferably not greater than 10 pounds and 30 pounds respectively.

In addition to the above parameters to be evaluated, the following information is pertinent to the analysis of the close-order formation keeping problems:

1. The maximum speed in flight will be approximately 150 knots.
2. The length of individual blade on the helicopter is between 37 - 40 feet.
3. The altitude of the helicopters in flight will be approximately 200 feet above vegetation.
4. No boom is to be used for the source and detector.
5. For the position of the source and detector in the helicopter, the tail pylon and the small radome under the passenger seats respectively are available and should be considered. The minimum distance between the tail pylon and the personnel will be between 15 - 23 feet during flight.
6. For the prevention and minimization of any possible radiation exposure problems, all the practical safety precautions are to be studied.

2.2 Effect of Operational Requirements on System Parameters

The foregoing operational requirements of the close-order formation keeping study contains more facets than those treated in the proposal and elsewhere. It is clear that the program as now specified is more difficult than the "follow-the-leader" system. Instead of one source placed on the lead helicopter, the detector in each helicopter must respond to the range and direction of three different sources in all their different combinations.

Particular effects on directional accuracy and choice of source energy are discussed below.

2.2.1 Directional Accuracy

The directional accuracy equivalent to the range accuracy of $\pm 10\%$ need be determined. The range accuracy of $\pm 10\%$ is equivalent to a source at 500 feet from a detector measured with the uncertainty of ± 50 feet. In Figure 1, the half angle θ of a right cone of which the diameter is 50 feet and the height is 500 feet is

$$\theta_o = \tan^{-1} 0.1 = 5.71^\circ = 0.0997 \text{ radians} \quad (1)$$

The directional accuracy equivalent to the range accuracy of $\pm 10\%$ is $\pm 5.71^\circ$. In terms of the accuracy as defined, the maximum uncertainty in locating a source is given by a sphere whose radius is one-tenth the distance R between the source and detector.

The tilt angle of $\pm 30^\circ$ in conjunction with the definition of accuracy implies that the center of the sphere of uncertainty makes a circle of radius R (range) as shown in Figure 2. The significance of this tilt angle is that the overall efficiency of the detector for the incident radiations must not change by more than a certain tolerable amount--preferably about 1%--when the direction of incident radiation changes within this range of tilt angle. This is an important parameter in designing the detector.

2.2.2 Choice of Source Energy

The aforementioned definition of the helicopter formation keeping system renders it very difficult to use a high energy gamma source such as Cs-137 or

DIRECTION
OF HEADING

DIRECTION
OF HEADING

DIRECTIONAL ACCURACY

FIGURE 1

TILT ANGLE AND ACCURACY

FIGURE 2

Co-60, in combination with a scanning detector. If a source of identical strength fixed in each helicopter is continuously emitting monoenergetic gamma rays, the detector must have a shield of sufficient thickness to yield the necessary minimum signal-to-noise ratio. The minimum signal-to-noise ratio must occur when two sources can be as close as 50 feet away from the detector, contributing to the background level, while the other source being looked at through the detector collimator is at approximately 500 feet. (It is presumed that there is sufficient shielding between the source and detector in each helicopter and, therefore, that the direct background noise is negligibly small.) Simple straightforward calculations show that for high energy gamma rays such as those from Cs-137, the weight of necessary shielding to provide the required signal-to-noise ratio is prohibitively large. This will be discussed in detail in a future progress report.

3. PRELIMINARY ANALYSIS OF SYSTEM PERFORMANCE

Preliminary analysis of factors affecting system performance are made in this section.

3.1 Dynamic Range and Response Function

If a point source emits S_o per second of monoenergetic gamma ray (or X-ray), and if a detector at a distance R from the source has an efficiency η and aperture $A(\theta)$, then the number of primary rays registered N_a by the detector is given by

$$N_a = S_o \eta A(\theta) \frac{e^{-\mu \rho R}}{4\pi R^2} \text{ (counts/second)} \quad (2)$$

where μ is the mass absorption coefficient for the gamma ray and ρ is the density of air.

If the detector scans with a field of view θ_o radians at the rate ω radians per second, the number of the primary rays registered during one scanning period is normally given by

$$N_b = N_a \frac{\theta}{\omega} = S_o \frac{\eta A(\theta) \theta_o}{4\pi \omega} \frac{e^{-\mu \rho R}}{R^2} \text{ (counts/scan)} \quad (3)$$

There exist, in general, rays of lower energy incident on the detector in addition to the primary radiation. In the process of attenuation in air, some of the primary gamma rays undergo one or more interactions with the medium. If the energy of the primary rays and the interacting medium are such that the Compton scattering is not negligibly small, then the low energy secondary rays incident on the detector may become significant, generally increasing with distance as the primary radiation decreases.

It is to be noted that the intensity of the secondary (scattered) rays also contains some useful information on the range, apart from the primary rays which are related to the range by Equation (3). The variation of the total intensity N_t with the range R from the point source S_o is then

$$N_t = S_o \left(\frac{\eta A(\theta) \theta_o}{4\pi\omega} \right) \frac{e^{-\mu\rho R}}{R^2} B(\mu\rho R) \quad (4)$$

where $B(\mu\rho R)$ is the so-called build-up factor due to scattered radiation. The function $B(\mu\rho R)$ can be expressed empirically in a power series of the form

$$B(\mu\rho R) = 1 + a(\mu\rho R) + b(\mu\rho R)^2 + \dots \quad (5)$$

where a , b , etc., are constants determined mainly by the energy of the primary radiations and also by the response function of the detector used.

Thus, when the constants, a , b , etc., are not negligibly small, the second and the third terms will become relatively significant with increasing values of R , as the intensity of the primary radiation is reduced to a level where the statistical fluctuation may cause a serious measurement error. It appears, therefore, advantageous to utilize the scattered radiation for range measurements in combination with the primary radiations within a certain upper region of the dynamic range. A proper use of the secondary radiations would result in the reduction of necessary source strength for a given statistical deviation allowed at a large distance.

3.2 Angular Dependence of Detector Aperture-Collimator

The effect of a collimator on the radiation detection may, in general, be divided into three different parts: (1) geometrical aperture, A_g , (2) penetrating aperture, A_p , and (3) scattering aperture, A_s . Aperture is defined as the difference between the flux of unidirectional radiation passing through a shielding material of finite thickness with and without a collimator hole, divided by the incident flux per unit area found at the center of the collimator hole.

The rigorous treatment of collimator effect on the radiation detection is difficult due to complex geometrical relations among such parameters as the angle of incidence, the shape of collimator, detector and source configurations, distance between the source and the detector, etc. The problem is further complicated by various interactions that the incident radiation undergoes in the collimator material.

Calculations using the geometrical aperture alone does not usually give a satisfactory result since it is based on the assumption that the collimator material is completely opaque and non-scattering. The deviation becomes especially significant when the energy of gamma radiation is highly penetrating. The deviation is corrected for by addition of the penetrating aperture and the scattering aperture to the geometrical aperture.

However, when the energy of radiation is low and the collimator material is of a high Z element, the geometrical aperture is often found to be a reasonable approximation for many practical engineering problems. In

particular, the scattering aperture is relatively small because of relatively small cross sections for the Compton scattering. Thus, the penetrating aperture is the only useful correction if it is needed. The sum aperture is, then,

$$A_s = A_g \left(1 + \frac{A_p}{A_g}\right) \quad (6)$$

3.2.1 Geometrical Aperture

It is assumed that the collimator is made of a non-scattering, perfectly opaque material. If the source emitting monoenergetic gamma ray (or X-ray) is placed at a great distance from the detector, then the incident radiation may be taken to be a parallel beam for all practical purposes.

The collimator under discussion is in the form of a slit having the width a , length b , and depth c , as shown in Figure 3. When the source is located along the z -axis, the maximum aperture of the detector is obtained. As the collimator rotates on y -axis, the detector area exposed to the incoming beam decreases, becoming finally nil when the angle of rotation reaches θ_o .

The geometrical aperture A_g is found to be related to the angle of rotation by

$$A_g(\theta) = 2b(a - c \tan \theta) \cos \theta \quad (7)$$

The angle θ_o subtended at the center O by the width a of collimator is simply

$$\tan \theta_o = \frac{a}{c} \quad (8)$$

APERTURE OF A SLIT COLLIMATOR

FIGURE 3

Finally, substitution of Equation (8) into Equation (7) gives

$$A_g(\theta) = ab \left(\cos \theta - \frac{\sin \theta}{\tan \theta_o} \right) \quad (9)$$

Equation (9) shows that as θ_o decreases--i. e., as the slit becomes narrower for a given depth c --the slope also decreases. It means that the directional accuracy increases proportionally with the decrease in the geometrical aperture, as obviously expected. It will be shown later that a substantially higher directional accuracy may be obtained by a certain signal process rather than merely by the geometrical aperture.

Integrated geometrical aperture is

$$A_{gt} = 2 ab \int_0^{\tan^{-1} a/c} \left(\cos \theta - \frac{\sin \theta}{\tan \theta_o} \right) d\theta = \frac{2 ab}{\sin \theta_o} (1 - \cos \theta_o) \quad (10)$$

Finally, differentiation of Equation (9) with respect θ gives

$$\frac{d A_g(\theta)}{d\theta} = - ab \left(\sin \theta + \frac{\cos \theta}{\tan \theta_o} \right) \quad (11)$$

Equations (9) and (11) are plotted for $\theta_o = 5.71^\circ$ and shown in Figures 4 and 5 respectively. The significance of Equations (10) and (11) is discussed in Section 3.3 in connection with the signal process for range and direction information.

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APERTURE FUNCTION
FOR $\theta_0 = 5.71^\circ$

— GEOMETRICAL APERTURE (A_g)
--- APPROXIMATE SUM APERTURE ($A_g + A_p$)

FIGURE 4

DERIVATIVE OF APERTURE FUNCTION
FOR $\theta_0 = 5.71^\circ$

FIGURE 5

3.2.2 Penetrating Aperture

Only one collimator is analyzed for penetrating aperture. The analysis can be extended, when necessary, to the case of multiple parallel slits comprising the final collimator aperture.

The slit collimator has the width a , the length b and the depth c as before. The two identical slabs forming the slit have the thickness d and are made of a high density material to act as a radiation shield. The collimator is attached by some mechanical means to a rotating platform of which the center coincides with the axis of a fixed detector. The detector may be in the form of a cylinder or even a section of sphere, whose side is properly shielded with the exception of the collimator-aperture. The radius r of the rotating platform may be taken to be approximately equal to that of the detector.

It is assumed that the source is placed on the $(x \text{ } o \text{ } z)$ plane, and that the distance between the source and detector is so large that the radiation incident on the detector is virtually a parallel beam.

The problem is to determine the path length ℓ as a function of rotating angle θ' that a ray takes in traversing the slab. The primary ray of a given energy is then attenuated by a factor $e^{-\mu_c \rho_c \ell(\theta')}$, where μ_c and ρ_c are the mass absorption coefficient and the density of the slab material. Only the attenuated primary rays are considered to contribute to the penetrating aperture. With the reference to Figure 3 and the symbols designated therein, it can be shown that the following geometrical relations hold for rays passing through the center.

$$l(\theta') = \frac{c+r}{\cos \theta'} - \frac{a}{2 \tan \theta' \cos \theta'} \quad (\theta_1' \leq \theta' \leq \theta_2') \quad (12)$$

$$l(\theta') = \frac{d}{\sin \theta'} \quad (\theta_2' \leq \theta' \leq \theta_3') \quad (13)$$

$$l(\theta') = \frac{a+2d}{2 \tan \theta' \cos \theta'} - \frac{r}{\cos \theta'} \quad (\theta_3' \leq \theta' \leq \theta_4') \quad (14)$$

and also

$$\tan \theta_1' = \frac{a}{2(c+r)}, \quad \tan \theta_2' = \frac{a+2d}{2(c+r)} \quad (15) \text{ and } (16)$$

$$\tan \theta_3' = \frac{a}{2r}, \quad \tan \theta_4' = \frac{a+2d}{2r} \quad (17) \text{ and } (18)$$

For given constants a , c , and d , the path length $l(\theta')$ can be computed using the above relations. The path length l for a given angle θ' is then used as the magnitude of penetrating aperture in terms of $e^{-\mu_c \rho_c l}$.

If a beam of parallel rays is incident on the detector at an angle θ , the slab casts a shadow on the detector window. The area that remains exposed comprises the geometrical aperture; the shadowed area becomes the penetrating aperture by virtue of the rays partially transmitted through the slab. The penetrating aperture will not, in general, be uniform even for a given angle of incidence because of the peculiar shape of the slab. It should be expected that that portion of the shadow cast by the upper edge of the slab will contribute more to the penetrating aperture than the other.

When the penetrating aperture is added to the geometrical aperture, it will result in the decrease of a slope of the aperture curves. This is approximately represented by a dotted line in Figure 4. The degree of this change depends on the energy of gamma rays, the mass absorption coefficient and the density of the slab material. Clearly, the larger the product $\mu_c \rho_c l(\theta')$, the greater the degree of collimation, for it will then depend mainly on the geometrical aperture. It is believed that A_p/A_g in Equation (6) could be made as small as 0.1 or less by using a low energy gamma ray of the order of 100 kev and a proper high density material for the slabs.

3.3 Improvement in Measurement Capabilities of the System

The results presented in the preceding section indicate some important possibilities of improving the measurement of range and direction. The following discussions are intended to be general and limited to the system of a fixed source and scanning detector.

3.3.1 Range Measurement

In Section 3.1, the total number of registered radiations emitted from a source in one scanning period is normally given by Equation (4). Two important modifications appear now possible: one is that the integrated sum aperture $A_{st} = A_{gt} + A_{pt}$ can be substituted for $A(\theta)$; the other is that the scanning period

$$T_o = \frac{\theta_o}{\omega} \quad (19)$$

can be lengthened by a factor of two--namely,

Registered only which detector is substantially aligned

$$T_i = 2 \frac{\theta_o}{\omega} \quad (20)^2$$

where T_i signifies the actual period of registering radiations from a source during one scanning motion.

Referring to Figure 4 physically, the aperture function is integrated from $\theta = -5.71^\circ$ to $\theta = +5.71^\circ$. If A_p/A_g is negligibly small, then the integrated sum aperture A_{st} is given approximately by Equation (10). For $\theta_o = 5.71^\circ$, it is found that

$$A_{st}(\theta_o) \simeq 0.1 \text{ ab} \quad (21)$$

as expected.

Equation (3) is now modified according to the preceding argument and rewritten simply in the form

$$N_2 = K A_s(\theta) \text{ counts/scan} \quad (22)$$

where K may be regarded as an amplitude relating to the range R during the period T_i -- i. e.,

$$K = S_o \frac{\eta \theta_o}{2\pi\omega} \frac{e^{-\mu\rho R}}{R^2} B(\mu\rho R), \quad (23)$$

-
2. In the actual electronic process to incorporate this method, only 90% of this integrating time period may be used.

and $A_s(\theta)$ is given by Equation (6). The total number N_t registered over the period T_i is

$$N_t = \frac{1}{2} K A_s(\theta_o)$$

In this method of measuring range, it is important that the background level be kept at minimum through a proper shielding and collimation of the detector. However, the background is expected to vary with the radiation received from two other "background" sources in one formation, whose distances from the detector may change from one moment to another. Should the background level be found undesirably high, it would then be necessary to continuously monitor by means of a suitable electronic process, its magnitude and then subtract from the total number N_t the average background obtained immediately preceding the range measuring process.

3.3.2 Direction Measurement

The direction of a source being measured by a scanning detector can be determined from the differentiation with respect to time of the number of counts registered per scan as in Equation (23).

$$\frac{d}{dt} N_2(\theta) = \frac{\partial}{\partial \theta} K A_s(\theta) \frac{d\theta}{dt} = S_o \frac{\eta \theta_o}{2\pi} \frac{e^{-\mu \rho R}}{R^2} B(\mu \rho R) \frac{\partial}{\partial \theta} \{A_g(\theta) + A_p(\theta)\} \quad (24)$$

If A_p/A_g is assumed to be negligibly small, Equation (24) can be expressed as

$$\frac{d}{dt} N_2(\theta) = - S_o \frac{\eta \theta_o}{2\pi} \frac{e^{-\mu \rho R}}{R^2} B(\mu \rho R) ab \left(\sin \theta + \frac{\cos \theta}{\tan \theta_o} \right) \quad (25)$$

in which Equation (11) is substituted. Equation (25) has a discontinuity at $\theta = 0$, or at the moment when the scanning collimator is aligned exactly with the source, as shown in Figure 5.

It is seen that the directional accuracy obtainable with this method is far greater than the one given by the collimating angle θ_0 . However, the collimating angle θ_0 is still necessary in order to resolve two sources when the angle ~~is~~ subtended by the two sources at the detector is close to θ_0 .

Electronic circuits to perform simultaneously the integration for range measurement and the differentiation for direction measurement are not difficult to develop. The discussion of the signal process will be presented in detail in the future reports when some necessary parameters such as the source strength, the detector characteristics, the nature and magnitude of buildup, etc. are well determined.

There will be a time delay from radiation measurements to information display due to a time constant involved in the signal process. However, the time delay can be simply compensated for by a proper displacement of the scale graduation in the display used.

4. ANALYSIS OF STATIONARY ARRAY DIRECTIONAL DETECTOR (AN ALTERNATE SYSTEM APPROACH)

During the first phase of the contract an alternate system approach was discovered. The alternate system uses three or more detectors arranged in a symmetrical array. (See Figure 6) These are fixed (non-rotating) detectors and directional information is obtained by combining electronically the count rate signals from the detectors. This alternate system approach has the desirable features of:

- 1) the absence of motional scanning or of physical motion of any sort in the detector operation,
- 2) the virtual absence of heavy shielding or collimating material at the detector,
- 3) the electronic data can be processed either in digital or analog form, and
- 4) range information is obtained from the same signal.

An undesirable feature of the alternate system is that, at the present time, it will only work satisfactorily with the nearest source. The presence of other sources at greater distances cause errors. A number of techniques are being considered, however, which from the initial analysis appear to overcome this limitation and make the alternate system operationally suitable. (A discussion of these techniques will be given in future reports.)

4.1 Response Function for Range and Direction

Three equivalent detectors arranged at the corners of an equilateral triangle are sufficient to provide azimuthal direction. It would also be possible

to use a greater number, such as four, on the corners of a square but the analysis will consider only three. If the detectors are placed close together, two of them will partially shield the third from radiation over approximately one-half of 360° . By modifying their shape and spacing slightly, the response of a single detector can be made to approach closely to a sinusoidal function of the azimuth angle to the source. Several possibilities are sketched in Fig. 6.

The idealized responses of the three detectors are plotted in Fig. 7. By changing the shapes, more or less curvature can be obtained at the maxima and minima but the periodicity of the responses remains the same for a symmetric arrangement.

If sinusoidal variation is assumed for the purpose of analysis, the count rates N_1 , N_2 and N_3 can be represented as

$$N_1 = H \cos \theta + M \quad (26)$$

$$N_2 = H \cos \left(\theta - \frac{2\pi}{3} \right) + M = -\frac{1}{2} H \cos \theta + \frac{\sqrt{3}}{2} H \sin \theta + M \quad (27)$$

$$N_3 = H \cos \left(\theta + \frac{2\pi}{3} \right) + M = -\frac{1}{2} H \cos \theta - \frac{\sqrt{3}}{2} H \sin \theta + M \quad (28)$$

where H is the amplitude of the variation in N and M is the average value of N . The angle of incident ray θ is measured with respect to N_1 maximum taken as a reference axis, as shown in Fig. 8.

From these equations, it follows that

$$N_1 + N_2 + N_3 = 3M \quad (29)$$

a constant independent of angle, and also

$$N_1^2 + N_2^2 + N_3^2 = \frac{3}{2} N^2 + 3M^2 \quad (30)$$

POSSIBLE ARRAY DETECTORS

FIGURE 6

COUNT RATE VS INCIDENT DIRECTION

FIGURE 7

FIGURE 8

a constant.

From Eq. (26) and Eq. (29)

$$3H \cos \theta = 2 N_1 - N_2 - N_3 \quad (31)$$

and from Eq. (27) and Eq. (28)

$$\sqrt{3} H \sin \theta = N_2 - N_3 \quad (32)$$

Thus,

$$\tan \theta = \frac{\sqrt{3} (N_2 - N_3)}{2 N_1 - N_2 - N_3} \quad (33)$$

and the desired θ can be formed from the three count rates. The quadrant is obtained in the usual manner--i. e., if numerator and denominator are both positive θ is quadrant 1; $\frac{+n}{-d}$ in 2; $\frac{-n}{-d}$ in 3; $\frac{-n}{+d}$ in 4.

From Eq. (33), it can be shown that the standard deviation in θ is bounded by the following expressions (details now shown):

$$\sigma_{\theta} \leq \frac{1}{\sqrt{H}} \left(\frac{2 M + H}{3 H} \right)^{1/2} \text{ radians} \quad (34)$$

$$\sigma_{\theta} \geq \frac{1}{\sqrt{H}} \left(\frac{2 M - H}{3 H} \right)^{1/2} \text{ radians} \quad (35)$$

calculated on the basis that the standard deviations of N_1 , N_2 and N_3 are $\sqrt{N_1}$, $\sqrt{N_2}$, and $\sqrt{N_3}$ respectively from Poisson statistics.

The difference between H and M will be small if background and leakage are small compared to M so that a rough estimate for σ_θ is given by

$$\frac{1}{\sqrt{3M}} < \sigma_\theta < \frac{1}{\sqrt{M}} \quad (36)$$

Thus, if the total count $(N_1 + N_2 + N_3)\tau$ in the averaging interval τ is 300, the σ_θ is less than 1/10 radian or 5.7° ; at 100 times this count, the error is 0.57° .

The above relations could be used to obtain angular information. However, a more direct and more elegant approach is believed possible. If three signals, $S_1 = N_1 - M$, $S_2 = N_2 - M$, and $S_3 = N_3 - M$ are formed and A. C. signals X_1 , X_2 , and X_3 are developed with amplitudes proportional to S_1 , S_2 and S_3 at some arbitrary constant frequency, these A. C. signals can be used to operate a Selsens-type, three pole motor and indicate the direction by a rotation of the shaft. Alternatively, a three pole magnetic yoke can be placed on a cathode ray tube and energized with three D. C. signals proportional to S_1 , S_2 and S_3 . The resulting magnetic field will deflect the electron beam in a direction corresponding to θ .

For range information, the sum $N = N_1 + N_2 + N_3 = 3M$ can be used in the usual way based on Equation (2).³

3. In this discussion, the buildup factor has been omitted for brevity.

This relation is essentially direction independent (see Equation (29)) if the response curves are sinusoidal. A correction can be made, with more complexity, to eliminate the effects of background by using the results of Equation (30). If the background is isotropic, the quantity H, obtained from Equations (29) and (30) to be

$$H = \sqrt{2} \left[\frac{N_1^2 + N_2^2 + N_3^2}{3} - \left(\frac{N_1 + N_2 + N_3}{3} \right)^2 \right] \quad (37)$$

is independent of background and thus a better measure of range.

5. SOME PRELIMINARY CONSIDERATION ON SOURCES AND DETECTORS

Some preliminary considerations are given to several aspects of sources and detectors in order to assess and evaluate their characteristics with respect to the helicopter formation keeping system. The results are by no means conclusive as some of the vitally important information is not yet available. The following discussion is intended rather to describe the current state of progress made to date on this subject.

5.1 Gamma Ray Versus Secondary X-Ray

A gamma ray or X-ray of around 100 kev is desirable for use with a scanning detector system. The ratio of the attenuated primary ray at 50 feet to that at 500 feet is about 390 for Cs-137, whereas it is 1550 for 100 kev, 1900 for 80 kev, and 2700 for 60 kev. On the other hand, the amount of detector shielding necessary for Cs-137 when used with a scanning detector ^{is} found to be quite heavy for the helicopter formation keeping system.

It is noted that, although a larger source strength is needed for a gamma ray of low energy in order to have the same flux as that of a high energy gamma ray at 500 feet, the flux to give the same dose rate is much higher for low energy than for high energy. For instance, 6×10^4 photons/cm²-sec of 60 kev gamma ray is equivalent to 5×10^3 photons/cm²-sec of 662 kev for about 0.3 r/40 hour dose rate. Thus, an increase in the source strength S_0 of a low energy gamma ray will result in no appreciable increase in dose rate so far as the personnel exposure to radiation is concerned.

It appears that three different kinds of sources will need to be evaluated for their relative advantages. They are: (1) isotopic gamma ray such as

Am-241 and Cs-137, (2) beta-excited X-ray using Kr-85 (or other beta emitters) with a high Z target, and (3) electron beam-excited X-ray. Considerations of half-life and price make the rest of low energy gamma ray sources almost impractical at the present time as far as their use in the helicopter formation keeping system is concerned.

(1) Am-241 is readily available and has found practical applications in industrial process controls. However, considerable self-absorption of 60 kev gamma rays in the high density source material and the size of the source may be a problem in view of a probable large source strength necessary in this application.

The size of the source and the amount of shielding around the detector for Cs-137 pose difficult problems for the scanning detector. It is these two factors which force us to give serious consideration to a multiple detector stationary system of more complex electronics.

(2) Beta-excited X-ray using Kr-85 (or other beta emitters) appears highly attractive for several reasons. It can be made into a source that may be turned on and off by changing the housing by some suitable means from a high Z (target) material for on-position to a low Z (shielding) material for off-position. The efficiency of producing X-rays is generally small. It is, however, possible to contain a high concentration of activity in a relatively small volume. The bremsstrahlung generated by beta rays interacting with the target and other surrounding materials may be also useful for the range and direction measurements. Further, Kr-85 is chemically inert gas and thus provides an inherent safety. For these

reasons, the possibility of beta-excited X-ray is receiving careful consideration with the help of John C. Dempsey of the Atomic Energy Commission, Washington, D. C.

(3) The conventional X-ray source is capable of producing a flux of a large magnitude, either in a pulsed or continuous mode. Secondly, it can be turned on and off easily.

The conventional X-ray sources have serious drawbacks in that they are invariably heavy as compared to isotopic X-ray sources, occupying a large space for power supply and other electronic control instruments. Generally, they also suffer from a limited life time, fluctuation in flux, etc.

It is reported that compact X-rays are now available from several manufacturers. Their performance characteristics and practicality are being evaluated with respect to the problems that are peculiar to the helicopter formation keeping system.

5.2 Angular Dependence of Detector Efficiency

A short experiment was carried out to determine the angular dependence of detector efficiency. Since a helicopter in flight formation may tilt as much as $\pm 30^\circ$, it is important for a detector to be of such a geometrical configuration that its efficiency changes not by more than an acceptable amount within this tilt angle.

The detector was a NaI(Tl) cylindrical crystal, two inches in diameter and two inches long, coupled to a photomultiplier. It was used in conjunction with a 512-channel spectrum analyzer (Nuclear Data Model 130A) and IBM typewriter ND-300. A Cs-137 of about 30 mc was placed at a distance 141.5 inches from the center of the crystal and had a collimating angle of 0.0054 steradian. This collimating angle is much larger than the one subtended by the detector crystal at the distance mentioned above. The detector was supported in such a manner that it could be rotated about y-axis through the center of the crystal in x-z plane and perpendicular to the direction of the radiation. (See Figure 9.)

Both the photopeak efficiency and overall efficiencies (for the entire spectrum) were measured as a function of the angles. The results are plotted in a polar coordinates and shown in Figure 10. The efficiency of the detector measured when the detector axis (z-axis in Figure 6) coincides with the radiation beam axis (i. e., z-axis) is normalized, and the other efficiency as a function of tilt angle is expressed in terms of a per cent change from it.

The results indicate that the efficiency of this particular detector changes appreciably with the angle of tilt. It is believed possible, however, to reduce the change in efficiency by using a crystal in which the ratio of the diameter to the length is small. Of course, a spherical crystal would be the most desirable in this regard, but deemed rather impractical in terms of engineering difficulties and expenses involved in the fabrication.

MEASUREMENT OF ANGULAR DEPENDENCY
OF DETECTOR EFFICIENCY

FIGURE 9

b

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b

6. FUTURE PLANS

The following is a list of major subjects to be investigated in the next reporting period:

- A. Continued evaluation of isotopic gamma ray, beta-excited X-ray and electron beam excited X-ray, leading toward the final selection of one for use in (the feasibility demonstration of) the helicopter formation keeping system.
- B. A preliminary design of detector and collimator-shield.
- C. A preliminary design of electronic signal process and display.