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THE CONCEPTUAL ROLE OF REDUNDANCY
IN VERIFICATION

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IN VERIFICATION

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THE CONCEPTUAL ROLE OF REDUNDANCY IN VERIFICATION

I. INTRODUCTION

The effectiveness of various elements of a safeguards system in maintaining adequate control over the location and disposition of nuclear materials has been the subject of both theoretical and practical studies over a period of years. Historically, containment and surveillance are the oldest means of insuring adequate protection for valuable materials. The records of ancient civilizations speak of treasuries which were sealed and guarded. Thus, it is not surprising that the initial measures for safeguarding nuclear material were based largely on physical security. Material accounting as a means of control also appeared very early, but has been primarily associated with the industrial progress of recent centuries. The extensive use of material balances based on the measurement and recording of flow and inventories has been traditionally associated with the process industries of the twentieth century, and from this background its introduction as a primary means for the safeguarding of nuclear materials in process was both natural and inevitable.

Studies of the effectiveness of material balance accounting as a control measure have been carried out more or less continuously since the mid-1940's, starting with the control of measurement quality and culminating in the recent emphasis on integral experiments. Due to the sensitive nature of the actual flows and inventories involved, much of the early U.S. work is contained only in secret reports. However, since the mid-1950's there has been a substantial volume of published material in this area, and the first complete study of the effectiveness of a

system was released in 1959,⁽¹⁾ although it received only limited circulation. The control of measurement quality received early attention⁽²⁾ and problems associated with inventory effects were studied intensively.⁽³⁾ The properties of the material imbalance as a measure, or index, of control effectiveness were studied.⁽⁴⁾ Inventory taking was examined extensively from a statistical viewpoint.^(5,6) Intensive work was carried out on measurement standards for critical materials,⁽⁷⁾ and handbooks of measurement methods with their associated uncertainties were prepared.⁽⁸⁾

Based on these and similar studies and applications, systems were evolved which produced, implicitly or explicitly, the required effectiveness. By and large, these systems were designed to be operated by an agency or institution directly involved for their own benefit, either as a matter of national policy or because of self-imposed technical or financial requirements. With the advent of international concern over the disposition and utilization of nuclear material, and the transfer of domestic materials to private ownership, the effectiveness of a given system has become a matter of community concern rather than a purely internal, or at most governmental, matter. Hence, we are faced with a new level of evaluation of effectiveness, differing from the former in at least two important respects:

- 1) The desire for effective control is not necessarily self-imposed, and hence internal motivation toward the necessary effectiveness by the operator or institution cannot be assumed. At best there may be a difference in the desired degree of effectiveness; at worst, a difference in objective.

- 2) The scope of activities or actions which can be properly utilized by an outside safeguards authority is normally (and properly) constrained. Although community concern will lead to a mutual abrogation of some individual freedoms, there will still be a "bill of rights" which protects the individual operator.

Broadly speaking, the term "verification" has been used to describe those activities carried out by an outside agency to determine the effectiveness (both in degree and kind) of an internal system. Because of the constraints imposed and possible differences in motivation, it becomes necessary to re-evaluate the feasibility of application and the contribution of the various elements normally associated with a safeguards system, and to examine possible alternate methods of achieving adequate verification.

Requirements for verification by a safeguards authority stem from two separate objectives.⁽⁹⁾ The first objective is the need of the safeguards authority to be assured that effective safeguards can and are being carried out at the operating level. The second objective stems from the potential threat of diversion at the operating level. The two objectives are not necessarily congruent, i.e., achievement of the first does not ensure achievement of the second. An operating component could give every indication of having excellent safeguards procedures and practices and yet be engaged in diversion. In this sense, verification of operating procedures and practices is only meaningful under the assumption that the operating component itself is not a diverter. For international safeguards, it is the second objective, that of detecting and thereby deterring diversion at the operating level which is of major concern. In considering strategies for verification it is this distinction in purpose rather than just the methods of verification which must be borne in mind.

To detect and deter diversion at the operating level most systems workers have chosen verified materials accounting as the main method with surveillance and containment playing supporting roles. Two general approaches are taken to verified materials accounting. One approach is to endeavor to verify all or some part of the operator's data. If no significant discrepancies are noted, then the operator's data are accepted and safeguards judgments are made on that data. The second approach is to endeavor to independently account for the material using measurements made in part by an agency and in part by operators.

This report examines some aspects of the use of technical correlations that complement the verification of materials accounting. In particular, it examines the advantage resulting to a safeguards authority because of its central position. As an overview international agency, the IAEA is the possessor of information from all parts of a given fuel cycle. In addition, its information base extends to similar facilities located throughout the world. Although an attempt was made to set forth the methods and conclusions in a broad sense for an unspecified safeguards authority, it is believed that many of them will be applicable to the IAEA even though specific items may require modification.

II. SUMMARY AND CONCLUSIONS

The main safeguards method chosen for application by the IAEA in carrying out its safeguards tasks under NPT is that of verified materials accounting. The other elements of safeguards - containment and surveillance - play supporting roles. They validate the measurement verification activities carried out at strategic points. A detailed examination was made of several key aspects of verified materials

accounting. These are:

1. The value of redundancy in a fuel cycle completely internal to a nation.
2. The value of technical cross-checks in providing a more robust system and in limiting the extent to which alternate inspection strategies could be defeated.
3. The contribution of these activities in quantifying the effectiveness of verification activities.

The results of these considerations provide a framework for allocating verification effort throughout an internal fuel cycle. The results also provide useful guidelines for relating effort to effectiveness. The concepts and principles involved are discussed in the context of an example fuel cycle. (Figure 2, Page 16). The fuel cycle which is internal to a nation includes low enriched U fabrication, light water reactors, chemical processing, U & Pu storage, Pu fabrication and U product conversion. The results of the study are discussed below for the specific topics of interest.

A. Redundancy

Redundancy per se is not sufficient for verification. Of itself it does not constitute independence. However, there are two sources of redundancy, other than repetition, which contribute to the effectiveness of verification even in an internal fuel cycle. These are:

1. Information which is repeated at various places in the fuel cycle or at various times in the verification process. Such information has a synergistic interaction with the verification process and acts as both a deterrent to deliberate falsification and an enhancement of verification activities. Information reported or recorded at a prior time,

fixes the pattern of any future falsifications about the same material. Successful falsification schemes to mask diversion require that the "defects" (on batch data) at successive measurement points match to avoid detection. Detection of such concomitant events would be positive evidence of collusion. In addition, previously set patterns validate many random inspection strategies. And finally, successive falsifications under verification increase the probability of detection.

2. Information which is related to the structure of the fuel cycle. The presence or absence of redundancy in the structure of an internal fuel cycle largely determines the scope and form of verification strategies. On the basis of structural redundancy, verification points can be classified as primary and secondary. Primary verification points are those points in the fuel cycle where a successful falsification (and equal diversion) results in a permanent defeat, i.e. there is no structural redundancy. Secondary verification points are points where successful falsification results in a temporary defeat, but the redundant structure of the system is such that eventual detection is possible. In the context of materials accounting, the primary points determine the material-to-be-accounted-for, whereas the quantities and locations of materials inside the system are determined at the secondary points. The primary and secondary points for the example fuel cycle are given in Table I, Page 18.

B. Value of Technical Cross-Checks

A different type of redundancy is provided by technical cross-checks. These include technical parameters which bound, corroborate, or provide additional estimates of the data to be verified. Of particular importance are the technical cross-checks which allow the safeguards authority to take

advantage of its unique position as a possessor of data from all parts of the fuel cycle and from many parts of the world. The cross-checks which serve in this capacity are the process capability, reactor predictions of end-of-life content, the Pu/U ratio method, isotopic correlations, constancy of isotopic compositions across the chemical processing operation, and the process inventory method using the step function technique. The principal value of these cross-checks in many cases lies in their ability to bolster and strengthen more conventional procedures. In this capacity many of the cross-checks have immediate value. Particularly important are the cross-checks relating to the verification of plutonium input to chemical processing. This is the prime verification point for plutonium. Because conventional verification procedures for this point are inherently dependent on plant processing and equipment, the "robustness" provided by the technical cross-checks is most desirable.

C. Estimates of Effectiveness

An ideal expression of the effectiveness of verified materials accounting would be based on the standard deviation of an independently determined MUF (Material Unaccounted For). Because of the conditional probability imposed by the range of possible adversary strategies, simple statements of effectiveness are normally not possible. Limitations in the verification process itself impose further restrictions. For internal use by the safeguards authority it would be desirable to establish upper bounds on the amounts which could have been diverted without detection. Both the magnitude of these bounds and the robustness of the procedures used to establish them can be affected by redundant information. And, confidence in the procedures used to obtain statements of effectiveness may be appreciably enhanced.

Performance can be expressed in terms of the verified facility MUF or the MUF of the nation. For an internal fuel cycle, national performance is reflected by the sum of facility MUF's and S/R (Shipper/Receiver) differences taking into account possible differences in the times in at which inventory holdings are verified, (See Page 26).

D. Allocation of Effort

The activities required at the primary and secondary points of verification along with the strategic quantities involved lead to useful guides for allocating verification effort. At primary points the safeguards authority verifies the inputs and outputs of the system. Secondary points are used to verify inventory holdings and internal flows. Here some trade-offs are possible between effort and timeliness. The principles involved are useful for ranking inspection priorities. A typical ranking of plutonium verification priorities is given on Page 28.

E. Effort Versus Effectiveness

The use of independent materials accounting through verification at strategic points requires that certain verifications be performed by the safeguards authority itself. In all cases these are 1) the verification of all non-redundant flows into and out from the system, and 2) verification of inventory holdings. The effort required to perform these essential activities defines a minimum verification effort. In addition, the safeguards authority may also have to verify certain international transfers where the technical reliability of the data is in doubt (e.g. spent fuels). Reductions below the minimum level of verification effort obviously decrease the reliability and/or timeliness of the safeguards system.

The impact of increases or decreases in the level of onsite inspection depends on whether primary or secondary points are involved. Inspection effort

at secondary points can be equated roughly to the frequency of inventory verification (internal flow verification for the purpose of converting verified flow to verified inventory is an exception). The impact of reductions in effort at secondary points is a decrease in timeliness.

Increases in on-site effort at primary points, beyond the point of assuring the validity of the verification process, result in only small gains in the effectiveness of independent materials accounting. Decreases in onsite effort which leave the validity of verification in doubt, seriously reduce effectiveness. In general, a detailed case by case study is necessary to determine the exact impact of a reduction in effort at primary points.

III. DISCUSSION

A. Introduction

The need for verification of materials accounting stems from the danger that diversion could be masked by falsifying records and reports. Presumably, diversion at the operating level in excess of measurement uncertainties would involve falsifying data in order to escape detection. Thus, for methods of verified materials accounting to have tangible effectiveness they must prevent, detect, or limit the falsification of information.

One function of inspection is to generate some level of independent source data. Some of the important activities involved are verifying key measurements, insuring that data sources are not falsified, obtaining information, and, in some cases, making independent measurements. The effort involved in carrying out these activities and the assumptions underlying their use have been considered in a number of previous studies. (10,11,12)

Verification is possible only through some degree of redundant effort. The nature of this effort will depend on the nature of the process and/or facility, the degree of natural redundancy in the fuel cycle, and the methods available to the safeguards authority. The various ways by which an outside agency can achieve this redundancy and their relative dependence on a central system of records and reports as opposed to periodic or continuous physical inspection is discussed below.

1. Participation. The safeguards authority can actually take part in the control functions performed by the plant. The extreme (an infeasible) case of this is a "man-to-man" defense by the safeguards authority. More valid examples of this activity are joint inspector-operator weighing of shipments, inspection and installation of seals, and participation in sampling and inventory taking.

2. Replication. The safeguards authority can actually reproduce a part of the measurement process. This can be done either directly, through independent sampling and analysis or independent instrumentation by the safeguards authority, or indirectly through third party measurements, such as "redundant" analyses by shipper and receiver.

3. Technical Cross-Checks. In addition to the basic measurement information, there is certain information and analyses which have inherent value to safeguards from the standpoint of potential means of independent verification. This includes:

- a. Analysis of shipper-receiver data
- b. Data on isotopic compositions
- c. Limiting parameters on the production of fissile materials

- d. Reactor data and information which may simplify certain verification procedures or will provide the basis for future verification methods
- e. Historical process information

4. Added Capability. The verifying agency may install locks, seals, or instruments specifically for the purpose of containment or surveillance which are not directly related to the process but establish identity and maintain physical security.

The emphasis placed on these four elements is dependent both on the reliability of verification and the relative magnitude of the components themselves. If throughput can only be verified to say 5-10 percent, then little importance should be placed on verifying waste discards, or reported MUFs. Only when verification approaches normal measurement capability do traditional treatments of waste discards and MUF become important. Similarly, the attention given to verification of inventory holdings must be balanced against the uncertainties in the verification of the other components of the balance. Studies have been carried out on the effectiveness and effort required to establish various levels of verified materials accounting about a facility or throughout a national fuel cycle. These studies have shown that the effort required to establish a given level of verification is more or less directly proportional to the number of independent measurements and/or verifications made by the safeguards authority. If the safeguards authority must rely entirely on its own measurements as a basis for decision, a very large effort is required to reach a sufficient level of verification. Consequently the safeguards authority should plan to the extent possible to take advantage of supporting measurements so that the level of direct agency effort can be minimized.

B. Safeguards Value of Redundant Information

In general, redundant information of potential safeguards value can be divided into several classes. These include:

1. Redundant information generated by the safeguards authority.
2. Redundant information generated by parties of clearly diverse interests.
3. Redundant information generated by parties who could have compelling motives for collusion.

Previous systems workers have treated information in Classes 1 and 2 as being 100% "trustworthy." The need for further redundancy was based on improving the technical reliability of information which was considered trustworthy. For example, if independent shipper or receiver data were comparable in technical quality to measurements made by the safeguards authority, then those data (S/R data) provided adequate verification. In case the quality was in doubt, further redundancy was provided in the form of "independent" measurements by the safeguards authority.

Redundant information in Class 3, e.g., potentially not trustworthy, has usually been judged to be of little safeguards value. In particular redundant information generated within a national system has been considered to be of little value in reducing the verification efforts to be applied to that nation, e.g., redundancy is not necessarily equivalent to independence. However, consideration of the extent and internal dependency of redundant information in an internal fuel cycle is most useful. It is important in determining inspection strategies for independent materials accounting. It is also important to examine redundant information for potential values which may in a positive way enhance the effectiveness of verification activities. Consideration of redundant information from these standpoints leads to the following observations:

1. Redundant information has a deterrent value which arises from the need to maintain internal consistency among such data.
2. Redundant information which is separated in time has a synergistic interaction with the verification process. Statements made at a prior time fix the pattern of any future falsifications of data about the same material. Successful falsification schemes for masking diversion by falsifying redundant batch data at connecting points in the fuel cycle requires that the "defects" be matched by the second party to the collusion. The pattern of "defects" thus established enables the safeguards authority to randomly sample only a portion of the total samples possible and make statements about the verification based on statistical arguments. Detection of such concomitant events would be positive evidence of collusion.
3. Falsification of data which will be verified at succeeding points in the fuel cycle (or redundantly in time) increases the probability of detection.
4. The presence or absence of redundancy in the structure of an internal fuel cycle affects the scope and form of the independent verification strategies.

The properties listed in 1, 2, and 3 above provide a deterrent quality or enhance the credibility of the results of verification. They do not in themselves imply independence; they simply reinforce existing verification. This is illustrated in Figure 1 which shows qualitatively the nature of the interaction between internal redundancy and the credibility of verification.

The presence or absence of redundancy in the structure of the fuel cycle plays a major role in determining verification activities. It provides the rationale for the kind of activities to be performed whereas the inherent

FIGURE 1. Synergistic Interaction Between Internal Redundancy and Credibility of Verification

Qualities of redundancy influence the manner in which they are carried out. This is best illustrated by considering an example. Consider the simplified internal fuel cycle shown in Figure 2. The cycle imports low enriched UF_6 for conversion and fabrication into UO_2 fuels for light water reactors. The spent fuels are processed at a single chemical processing facility. The recovered plutonium is split into shipments to plutonium fabrication for thermal recycle and plutonium storage. The uranium product stream is also split. Material of sufficiently high ^{235}U concentration is converted to UF_6 at a conversion facility and exported for re-enrichment. Depleted uranium is either stored or used in thermal recycle.

It is assumed that the safeguards authority endeavors to apply the concept of verified materials accounting to the example fuel cycle. To accomplish this objective, it maintains separate accounting records based on facility data whose integrity is established by verification activities. Containment and/or surveillance are used to assure the validity of verification procedures and to assure material integrity where materials accounting procedures are not applicable.

On the basis of structural redundancy we can classify verification points as primary and secondary. Primary verification points are those points in the fuel cycle where a successful falsification (and equal diversion) results in a permanent defeat of the safeguards system, i.e., there is no structural redundancy. Secondary points correspond to verification points where successful falsification results in a temporary defeat, but the redundant structure of the system is such that eventual detection is possible. The primary and secondary points of verification for the example are given in Table I.

FIGURE 2. Example Internal Fuel Cycle

Table I

PRIMARY AND SECONDARY VERIFICATION POINTS
FOR FUEL CYCLE SHOWN IN FIGURE 2

Points for Primary Verification

Plutonium

1. Input to chemical processing
2. Pu discards
3. Output from Pu fabrication

Uranium

1. Input to U fabrication
2. U waste discards
3. U exports

Points for Secondary Verification

1. Product from chemical processing	1. Input to reactors ⁽¹⁾
2. Input to Pu fabrication	2. Input to chemical processing.
3. Inventory at Pu storage	3. Input to U product conversion
4. Inventory at Pu fabrication	4. Input to U storage or Pu fabrication
5. Inventory at chemical processing	5. Inventory at U fuel fabrication
6. Reactor input and (Inventory)	6. Inventory at chemical processing
	7. Inventory at product conversion
	8. Inventory at U storage
	9. Inventory of U at Pu fabrication

(1) When the Pu/U ratio method is used, the uranium input to reactors becomes part of the primary verification of the Pu input to chemical processing.

The primary points listed for the example fuel cycle require explanation. The uranium input to fuel fabrication is an import and therefore would not necessarily require direct verification (due to international independence). The plutonium output from plutonium fabrication is not a true exit point since the plutonium does not leave the national system. However, because of technical limitations, the detection of a falsification of that flow may be difficult after irradiation.

It is evident that internal redundancy is of little benefit in reducing the need for direct verification at primary points. In theory, some trade-offs in location are possible, i.e., verification of a primary flow could be done at alternate locations. However, the need for direct verification remains the same as noted previously, internationally independent shipper or receiver measurements could replace agency verification at primary points.

The rigor and frequency of verification at secondary points is determined by considerations of timeliness and strategic quantities. In essence, verification at secondary points is equivalent to establishing the magnitude and location of the inventory holdings. Internal flows are in concept inventory shifts, since they do not leave the national system. As a result of the above considerations, the safeguards authority does have possible trade-offs between timeliness and inspection effort. When the inspection effort is resource limited, the areas of secondary verification appear to offer the best opportunity for reasonable trade-offs.

C. Value of Technical Cross-Checks

Another kind of redundancy of potential safeguards value is the redundancy provided by technical cross-checks. In general, these include technical parameters which bound, corroborate, or provide additional estimates of the data to be verified. Some of the more commonly accepted cross checks are listed in Table II.

The process variance and the process capability are useful test indices which bound the extent to which certain process related data may be falsified without that falsification being suspect. Since the process variance may also be falsified, the safeguards agency usually estimates this index from its own measurements. The process capability is useful for testing the credibility of stated waste discards. This concept is based on the principle that a given process will sustain a certain minimum waste loss in producing a satisfactory product. It is the "best" the process can achieve. When the stated waste loss is near the process capability, the safeguards agency may accept the stated value without fear that it was greatly overstated to mask diversion.

Calculated values for the end-of-life content of spent fuels, the Pu/U ratio method, isotopic correlations, and the across-the-plant isotopic composition can all reinforce the verification of plutonium input to the chemical processing plant.

Calculated values for plutonium from reactors (when trustworthy) serve as a redundant limit or boundary on the extent to which chemical plant input measurement may be falsified. In the case where the reactors are in the same nation, the safeguards authority cannot assume independence. However, when the safeguards agency is verifying the input measurements at the chemical

TABLE II

TECHNICAL-CROSS CHECKS OF QUANTITATIVE
VALUE IN INDEPENDENT MATERIALS ACCOUNTING

<u>Technical Cross-Check</u>	<u>Application Points</u>	<u>Safeguards Value</u>	<u>Quantitative Range, % σ</u>
1. <u>Fuel Specification Measurements</u> Pellet Weight, Size, Density, Assay, Rod Dimensions, Fuel Zone	Fabrication Output	Check on Maximum Fuel Loading	1 - 3
2. Reactor Calculations	Chemical Plant Input, Reactor Inventory	Confirmation of Chemical Plant Input	Pu 2-10, ^{235}U 2-6, ⁽²⁾ Total U 0.2 - 0.8
(1) 3. Isotopic Correlations	Chemical Plant, Reactor, U Fabrication	Confirmation of Chemical Plant Input. Confirmation of Prior Information	<1.0 - 5.0
(1) 4. Chemical Isotopic Mixing	Chemical Plant in Process Inventory	Dynamic Inventory at Chemical Processing	1 - 10
(1) 5. Across Chemical Plant Isotopic Composition	Chemical Plant	Confirmation of Input Isotopic Compositions, Volume Falsifications, and Sample Substitution	<1.0
6. Process Capability	Chemical Processing, U Conversion-Fabrication	Tests Credibility of Stated Discards	—
7. Process Variance	All Facilities	Limits the Extent Certain Data Can Be Falsified Without Detection	Depends on Process Variance

(1) See Reference 14.

(2) Uncertainties in reactor calculations for end-of-life values which are based on exposure estimates and stated starting compositions.

plant it also verifies the reactor statements for plutonium concentration in the fuel. Here a useful interaction results between the verification process and redundancy in the national data. The prior statements by the reactor fix the pattern of any later large falsifications at the chemical plant, e.g., the "defects" must be matched. This fixation of the boundary condition validates random inspection procedures within that boundary.

The calculated weight of uranium at end-of-life is generally considered to be of high technical reliability provided that the initial weight is known. When this value is verified, it may be used as the basis for the Pu/U ratio method of verifying plutonium input at the chemical plant. In the internal fuel cycle shown in Figure 2, the safeguards authority must first verify the total uranium content of the fabricated fuel to use the method. For use of the Pu/U ratio method it must only protect against an "overload" of the rods. Underloading is self-defeating to an effort to understate plutonium input to the chemical processing plant. The safeguards authority has two redundant verification procedures for testing the case of a uranium overload. It may directly measure or verify the loading process. It also may verify the material balance of the fuel fabrication plant, which indirectly protects against an "overload." This of course does not protect against a "hidden" uranium feed supply to the fabrication plant.

The various cross-checks for verification of the plutonium input to the chemical plant, when used "in total," provide a "robust" verification method. The total uranium from fabrication corrected for reactor burnout, protects against volume falsification. That value together with Pu/U ratio measurements provide a means of estimating total Pu input which is largely independent of volume falsifications and simple changes in sample concentrations. This method is further strengthened when the across-the-plant isotopic compositions are monitored. Finally, the isotopic correlations provide a

further dimension to the verification of the Pu/U ratio and to the confirmation of prior information about the spent fuel. In addition, they can provide protection against the selective removal of plutonium prior to the accountability tank.

D. Estimates of Effectiveness

The effectiveness of verified materials accounting can be broadly described as the exactness with which a material balance can be verified by an "outside" safeguards agency. The long term effectiveness is more dependent on the exactness with which the inputs and outputs can be verified, whereas the short-term effectiveness is more dependent on the exactness with which the inventory holdings (including internal flows) can be verified. The safeguards authority's task of verification is complicated by its remoteness from the system to be verified and the possibility that the system could be operated by an adversary intent on defeating the verification process.

In previous studies by the authors (10) effectiveness was expressed by the standard deviation (σ) of the MUF of a conceptually independent balance formed about a particular facility. The concept was further illustrated through power curves which portrayed the beta risk as a function of a given alpha risk and the potential level of diversion. The concept of a complete and truly independent material balance can, at best, only be approached in practice. Seldom can "true" independence be achieved for all components of the material balance. In addition, completeness is often a difficult and costly undertaking. As a result of these limitations it is often necessary to qualify statements of effectiveness by recognizing the lack of completeness and degree of dependence on the plant generated data. Recognition of plant dependence has been expressed in a qualitative way by describing particular verification procedures as being robust or fragile. Robustness was used to

describe verification procedures which were not easy for the plant operator to defeat. Those which could be easily defeated were termed fragile. The lack of completeness, e.g., less than 100% replication by the safeguards authority has been addressed by several workers (11, 13) for fractional sampling plans. The fractional sample is tested for a consistent bias and the possibility of "gross defects" in the total population. Since the probability of detection is conditioned by the adversary's strategy, a simple statement of effectiveness is not possible. Some of the technical cross checks mentioned earlier are useful in this case for setting boundary conditions from which the maximum falsification of the total population can be estimated. For example, composite sampling, "boundary" measurements with non-destructive devices, and isotopic correlations are useful solutions in many cases.

Expressions of the effectiveness of materials balance accounting should also take into account differences between short term and accumulated effectiveness and also between that of individual facilities and the nation as a whole. These differences are illustrated by the plutonium material balances (MBA) areas shown in Figure 3.

For the two process MBA's, chemical processing and plutonium fabrication effectiveness can be expressed for each MBA as the σ of the verified MUF for a standard accounting period and also for the σ of the accumulated MUF. Similarly, performance can be shown by the observed individual and accumulated MUFs. The storage area which is a "zero MUF" area requires different treatment. Expressions of effectiveness would depend on the kind of verification process used.

When physical inventories are taken at the same time, all three balance areas could be treated as a single MBA. The performance of the nation could be stated as the algebraic sum of the observed MUFs and any S/R differences between the

FIGURE 3. Plutonium Material Balances In An Internal Fuel Cycle

chemical plant and the plutonium fabrication plant. Chemical plant input shipper-receiver differences need also to be included in the national performance index unless they have been resolved during the verification process. Here the best estimate of the Pu S/R difference is the total U difference multiplied by the weighted Pu/U ratio. This of course is applicable only when the total U from fabrication is verified by the safeguards authority.

The application of the concept of a national MUF may be troublesome when inventories are taken at different times at different facilities. In this case, a national MUF would necessarily include some facility inventories at book values. For verified materials accounting, it is important that shipper-receiver differences between connecting facilities be taken into account. Errors in between facility transfer measurements can result in the transference of either positive or negative MUF from one facility to another. As a result, shipper-receiver differences are properly a part of MUF evaluation or MUF explanation. Such differences either increase or decrease the combined MUF of connecting facilities. Since between facility transfers are made up of identifiable items (cans, bottles, cylinders), shipper-receiver differences can be equated to each material balance regardless of physical inventory times, e.g. removals at one facility and receipts at the other. One pragmatic approach to the problem of different facility inventory times, is to show national performance on a cumulative basis. In this portrayal, the cumulative MUF for each facility would be shown at the time of the last verified physical inventory and cumulative shipper-receiver differences would be brought forward to include all shipments from the last completed shippers balance. Such a portrayal would be "after the fact" and not a common point in time. However, from cumulative trends an extrapolation could be made to a common point in time to yield a national index of

performance. Such an index, although not sufficiently exact for accounting purposes, would likely be suitable for purposes of evaluation.

The value of redundancy in structure is further developed on a mathematical basis in Appendix A. In the appendix the various material balance expressions for the three internal plutonium balances are developed for single accounting periods and for the sum of many accounting periods. The important conclusions developed in the appendix are:

1. Falsifications of inventory holdings or internal transfers represent only a delay in detection. To defeat the system permanently, falsification of internal data must be converted to a falsification of a primary flow.
2. A falsification of an identifiable item is retained until that item is converted to process. For example, if the amount of plutonium contained in a plutonium nitrate shipping container is overstated by 10% to mask diversion, the false statement does not leave the item until it is transferred to process. Thus the falsification or error, so to speak, is retained as long as item identification is retained. In this context, the falsification is not overshadowed by accumulated terminal measurement errors for the overall balance. The specific falsification could be detected several accounting periods later.
3. The long-term effectiveness is seen to involve only the beginning inventory (zero or the amount present at safeguards implementation) and the ending inventory and the accumulated terminal measurements. Inventory errors for all accounting periods between zero time and the time of last inventory cancel out.

It is apparent from the previous discussion that all falsifications of inventory holdings and internal flows are potentially subject to repetitive verification. By contrast, terminal flows have no redundancy or only a single opportunity for verification. Thus, verification effort could be apportioned in terms of completeness, e.g., 100% verification of main terminal flows and fractional sampling of internal flows and inventory holdings.

E. Allocation of Effort

One aim of the previous discussions has been to establish a framework for allocating inspection effort in an internal fuel cycle. A main factor for determining verification effort is the presence or absence of redundancy in the structure of the fuel cycle. Certain key measurement primary points, the safeguards authority must directly participate in the verification process. At all other points, the secondary points, redundancy in structure exists. Here some trade-offs between inspection intensity and timeliness are possible.

These concepts may be illustrated by considering the internal fuel cycle shown in Figure 2. The ranking of inspection priorities shown below suggest themselves as useful guides for allocating effort for plutonium verification in this example. These are listed immediately below and discussed subsequently. They are also shown in schematic form in Figure 4.

Ranking of Verification Priorities for Pu

1. Pu input to chemical processing
2. Pu output from Pu fabrication
3. Chemical processing output to Pu storage
4. Uranium input to reactors
5. Waste discards at chemical processing and Pu fabrication
6. Inventory at Pu storage
7. Inventory at Pu fabrication
8. Pu input to Pu fabrication
9. Pu inventory at chemical processing

FIGURE 4. Inspection Priorities For Plutonium Verification In An Internal Fuel Cycle

1. Priority #1 - Input to Chemical Processing

This is the prime verification point for plutonium. It establishes the plutonium to-be-accounted for. Successful falsification at this point, followed by diversion, permanently defeats the safeguards system. Only equivalent technical trade-offs appear possible, e.g., equivalent verification at a lower inspection level. However, because of its overriding importance, the authors feel that the verification process for chemical plant input needs to be strengthened and bolstered. It is our opinion that some technical cross-checks (isotopic correlation, Pu/U ratio, etc.) are necessary to provide adequate verification at this point. Simple replication appears to be too plant dependent or "fragile" to provide adequate assurance by itself.

2. Priority #2 - Output From Pu Fabrication

This is a primary verification point for plutonium. Uranium verification is also important here. Although plutonium does not leave the system, it appears that overstatements to mask diversion would be extremely difficult to detect after irradiation of the Pu-containing fuel. Trade-offs in location or direct technical trade-offs may be possible.

3. Priority #3 - Chemical Plant Output to Pu Storage

This is a secondary point of verification. However, because of practical necessity, it is treated with high priority to provide a basis for the later verification of plutonium product at Pu storage. It is assumed that the inspectors would verify the measurements, establish measurement values by non-destructive testing, and seal the containers prior to shipment to storage. Inventory verification at the storage facility is assumed to consist of seal checks, item identification and

non-destructive measurements. If new technology permits measurement at the storage facility with a comparable reliability to verification at the chemical plant, trade-offs in location are possible.

4. Priority #4 - Output from U Fabrication

This is a secondary point of uranium verification since the uranium does not leave the system. However, to protect the later verification of plutonium input to the chemical plant, verification must protect against a uranium "overload" (one-sided verification). This is also the product point for low enriched fabrication and "two sided" verification is required for uranium verification.

5. Priority #5 - Pu Waste Discards

The primary exit points for plutonium waste discards are at chemical processing and plutonium fabrication. The relative importance of waste discards depends on their magnitude in relation to the exactness with which primary flows can be measured. These are "bounded" risks in that the falsification cannot exceed the stated discards. Once process capabilities are established, it may be possible to reduce the level of effort.

6. Priority #6 - Inventory at Pu Storage

This is a secondary point of verification since inventory verification is a redundant activity. Possible trade-offs between inspection level and effectiveness are controlled by critical time considerations.

7. Priority #7 - Inventory at Pu Fabrication

This is also a secondary point of verification. The ranking order, which is below "Inventory at Pu Storage", assumes that the quantities in the storage facilities will exceed those at Pu fabrication, at least, for the near future. When most of the Pu product flows directly to Pu fabrication

then the priorities should be reversed.

8. Priority #8 - Input to Pu Fabrication

This is a secondary point of verification. Redundancy exists at the shipper points - chemical plant or Pu storage. Falsifications represent only a delaying action. Some random checks are necessary for quality evaluations and protection against inventory falsifications. This priority assumes all Pu input to the facility is internal. International imports would have to be evaluated for independence and technical reliability.

9. Priority #9 - Inventory at Chemical Processing

This is a secondary point of verification. It is listed as low priority on the assumption that the chemical plant will ordinarily reach a clean out status several times a year.

The ranking of priorities assumes sufficient resources are available to cover all nine activity areas. These rankings for plutonium verification are intended only to illustrate the principles involved. Similar rankings can be developed for uranium verification. The example assumes that sufficient containment and surveillance are applied at the reactors to insure the integrity of the fuel during irradiation, e.g., neither substitution nor direct diversion at reactor sites.

The distinction between primary and secondary points is made on the basis that a verified accounting system can be permanently defeated at the primary points, whereas the verification process at the secondary points is largely a matter of verification of inventory and internal flows. This assumes that the inspectorate can always "catch up" with inventory. It is

evident that there must be sufficient overall coverage to prevent the "flotation" of inventory from location to location as a way to mask diversion. Information which more completely characterizes material held in inventory such as chemical form and isotopic composition provides a basis for verification activities which protect against this threat.

F. Effort Versus Effectiveness of Detection

It is apparent that use of the main safeguards method, i.e., verified materials accounting through verification at strategic points, requires that certain essential verifications be performed by the safeguards authority. In all cases these are 1) the verification of non-redundant flows into and out from the system, and 2) the verification of inventory holdings. The effort required to perform these essential activities defines a minimum level of effort required of an agency. Reduction of that effort obviously decreases the reliability and/or timeliness of independent materials accounting.

The particular question at hand is how much does effectiveness change with effort? More specifically, how does it change with changes in the level of onsite inspection? First it is necessary to recognize the two components of effort-onsite inspection and the supporting offsite capabilities. Effectiveness also has two qualities-exactness of verification and timeliness. In general, timeliness can be related directly to the frequency of inventory verification. Hence, doubling the frequency doubles the onsite inventory effort and also proportionately increases the work load of the offsite functions.

The relationship between the level of onsite inspection and the exactness of verification is more complex. A critical factor is the role of the inspector

in the verification process. If the inspector's effort is mainly devoted to making independent measurements, changes in effort can be equated to increases or decreases in the number of such measurements. On the other hand, the function of the inspector can be to assure the validity of data sources, with the principal measurements being made offsite. In this case, an increase in onsite inspection beyond that needed to assure validity may represent only a small gain, whereas a decrease below that level may seriously reduce effectiveness. Although the guidelines are clear, a detailed case by case examination appears necessary to reach a quantitative basis for decision. In this context, it should be recalled that onsite inspection is only one aspect of verified materials accounting. The other functions, e.g., the accounting, data analysis, technical analysis, instrumentation, analytical chemistry, etc., all play a major role in effectiveness.

A further aspect of effort and effectiveness concerns the safeguards authority's advantage in having access to information from all parts of the fuel cycle. As was noted earlier, it may use its overview position to combine information from various parts of the cycle in its verification process. This concept was particularly stressed in illustrating the use of technical cross-checks for verification of the plutonium input to chemical processing. It is important that this advantage be fully exploited. Care should be used in accepting the hypothesis that simple replication, using the operator's procedure, should be agreed to as the only method of verification. Ideally, the verification process should be independent of what is being verified. Admittedly, this is often difficult in practice. However, when possible, the safeguards authority should use every means at its disposal and within its resources, to establish "robust" (not easily defeated) verification procedures. Thus a given verification procedure could consist of a number of different layers and dimensions of redundancy.

The need for "robustness" is particularly important at primary points. The case of the verification of plutonium input to a chemical processing plant is a good illustration of the principles involved. Of the three measurement components - volume, sampling, and analysis - only one, the analytical measurement can be done independently. If sample dilutions are also made by plant personnel with plant equipment, then that component also could be partially dependent. To provide a more robust verification, the safeguards authority can use the technical cross-checks to join information from other strategic points to strengthen these measurements. For the most part, these techniques are simply a matter of applying available data to the task.

An unfortunate emphasis has been given to technical cross-checks in that too often they are considered solely as replacements or substitutions for already existing procedures. The emphasis should be on bolstering and strengthening. Those techniques which allow the safeguards agency to use its unique overview position should be actively fostered and pursued to substantiate rather than replace, i.e., to enhance credibility.

It is possible and perhaps even likely that an agency such as the IAEA will be limited in verification capability. Such limitations could arise from a number of reasons. These could include constraints placed on inspection intensity by safeguards agreements and limitations in resources such as funding, manpower, and facilities. In this situation, the use of technical cross-checks may be the only possible method of verification at a particular point. In some cases, the application of technical cross-checks could be equivalent or perhaps superior to other methods of verification. But, in other cases, their use as the only method of verification could result in a reduced effectiveness. As a result of this latter possibility, the author much prefers to emphasize a concept of "addition to" rather than

"in place of" for the technical cross-checks described in this paper. In this context, a distinction should be made between technical cross-checks which have redundancy in dimension and the concept of redundancy in the structure of the fuel cycle. The technical cross-checks described in this paper are used to substantiate or increase the credibility of verification methods, whereas, redundancy in structure influences the scope and allocation of the verification effort.

Finally, it is important to distinguish between the effectiveness of a given level of effort in detecting a diversion and the need for that level of effort in view of an assumed probability that a diversion will be attempted. Care must be taken to guard against reducing the level of verification effort solely on the basis that a certain activity reduces the probability of diversion being attempted. Unfortunately, the relationship between the probability of diversion being attempted and the probability of detection is one-sided. Increasing the probability of detection discourages diversion or reduces the probability of diversion being attempted. However, reducing the probability of diversion being attempted, does not reduce the effort needed to detect that diversion, if attempted.

The reader will note that this paper has discussed effectiveness largely in the context of the safeguard authority's ability to detect diversions. In doing so the authors have ignored the other component of effectiveness, namely, the number of diversions attempted. If the safeguards authority could devise a system which guaranteed zero diversion attempts, they obviously would not require a detection capability. In addition to the deterrent effects of the safeguards system, a thorough analysis would require an evaluation of the capabilities and especially the motivation of the diverter. There is a hazard involved in decreasing

inspection effort, solely because the probability of diversion is very low. It should be made clear that such a reduction must take into account three related effects: 1) The reduction in the detection capability, 2) the probable increase in the threat of diversion, and 3) the reduction in the "robustness" (through redundancy) of the system. Thus, a reduction in inspection effort may have a greater adverse effect than might be at first realized.

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APPENDIX A - A Discussion of MUF's

1. Introduction

Ordinarily, when one talks of MUF, reference is made to a statistic which is based on estimates of material located in and flowing into and out of a given part of a facility's system defined as a MBA (material balance area), and which is used to estimate the amount of material in a MBA which is unaccounted for. Actually there are many types of MUF, that is, there are many levels at which MUF can be discussed. Some of these are as follows:

1.1 The True MUF

If all of the pertinent values, inputs, outputs, wastes and inventory were known without error, the true MUF for the i th MBA

$$MUF_i = R_i + \Delta I_i - O_i - W_i$$

would reveal the true amount of material which is unaccounted for where R_i , O_i , W_i , and ΔI_i are the true receipts, outputs, wastes and incremental inventory values. $\Delta I_{ij} = I_{ij} - I_{i,j+1}$ for the i th MBA, j th period.

1.2 The Observed MUF

In practice, of course, all of the parameters R_i , ΔI_i , O_i and W_i must be estimated. Denote an estimate of x by $\hat{x}$ so that

$$\hat{MUF}_i = \hat{R}_i + \hat{\Delta I}_i - \hat{O}_i - \hat{W}_i.$$

This statistic is what is usually meant when one talks about MUF, i.e., it is the estimate of MUF, the amount of material which is unaccounted for in the MBA. In practice, the estimate $\hat{MUF}$ is not capped because the context generally indicates that the estimated MUF is intended. For our purposes, it is advantageous to make a distinction between the two meanings.

1.3 $E(\hat{MUF})$, The Expected Value of $\hat{MUF}$

This is the expected value of $\hat{MUF}$ in the sense that it is the average value $\hat{MUF}$ would have if it were repeatedly calculated under independent but similar measurement or estimation procedures.

1.4 $\hat{t} = \hat{MUF}/\hat{\sigma}_{MUF}$, The Normalized MUF Value

In order to make a decision about whether $\hat{MUF}$ indicates that $E(\hat{MUF})$ represents a significant departure from zero, it is necessary to normalize $\hat{MUF}$ by dividing it by its estimated standard deviation. If the distribution of $\hat{MUF}$ were known, in the conceptual sense of repeated estimation under the same conditions, then critical values of $\hat{MUF}$ could be obtained independent of $\sigma_{\hat{MUF}}$. In the absence of such specific knowledge for a MBA, and by an appeal to the central limit theorem, however, one can argue that the density of $\hat{MUF}$ tends to normality unless a subset of the measurements which make up $\hat{MUF}$ are non-normal and are dominating error sources. Even this normalized $\hat{MUF}$ value can be further differentiated into different cases.

1.4.1 $\hat{t} = \hat{MUF}/\hat{\sigma}_{MUF}$, Under Random Errors

In this case if $\hat{t}$ is greater than $t_{1-\alpha}$ one rejects $H_0: E(\hat{MUF}) = 0$ in favor of the alternative $H_1: E(\hat{MUF}) > 0$. Then bias must be counted among the possible mechanisms which can cause the significant $\hat{MUF}$.

1.4.2 $\hat{l} = \hat{MUF}/\hat{\sigma}_{MUF}$, Under Random and Systematic Errors

Here the estimate of $\sigma_{\hat{MUF}}$ also includes estimates of systematic error components and a rejection of H_0 in favor of H_1 then indicates that there is a mechanism causing a significant MUF which is not caused by biases. All of this must be interpreted in a probabilistic sense,

of course, and just stating the situation is not meant to minimize some of the statistical problems inherent in case 1.4.2 such as the number of degrees of freedom to use in $t_{1-\alpha}$, as well as the appropriateness of various estimates of the variance components to use for the systematic error.

2. MUF in its Accountability Context

To make the arguments more specific the following discussion pertains to the Pu fuel cycle given in Figure 3. Let R_i , O_i , ΔI_i , and W_i denote, respectively, the receipts, outputs, incremental changes in the inventory, and the wastes for the i th MBA. To avoid possible misinterpretation the uncapped symbols are used to denote true amounts. For this cycle there are two output components from MBA_1 . Call $O_{1,s}$ and $O_{1,f}$ the outputs, respectively, which go to Pu storage and to Pu fabrication. The true MUF for the i th MBA, MUF_i , is

$$MUF_i = R_i - I_i - O_i - W_i.$$

There are constraints on these MUF relationships since

$$O_1 = O_{1,s} + O_{1,f}$$

$$R_2 = O_{1,s}$$

$$R_3 = O_{1,f} + O_2$$

Then the equations can be written

$$MUF_1 = R_1 + \Delta I_1 - O_{1,s} - O_{1,f} - W_1$$

$$MUF_2 = O_{1,s} + \Delta I_2 - O_2$$

$$MUF_3 = O_{1,f} + O_2 + \Delta I_3 - O_3 - W_3$$

and

$$\sum^3 \text{MUF}_i = R_1 + \Delta I_1 + \Delta I_2 + \Delta I_3 - O_3 - W_1 - W_3 .$$

The waste stream from MBA_2 is assumed to be 0.

If the three material balance areas as a whole are treated as one MBA then R_1 , O_3 , $(\Delta I_1 + \Delta I_2 + \Delta I_3)$, W_1 and W_3 , respectively, constitute the receipts, outputs, incremental inventory amounts and waste for the total material balance area. This indicates that the true MUF for the total material balance area is the same as the sum of the true MUF's for the three material balance areas.

If there are no misstatements and everything is perfectly estimated then MUF is equal to the true amount of material unaccounted for. If there is a total sink in the MBA then $\text{MUF} = \text{the amount of the sink}$.

For the purposes of seeing what happens over long-term MUF's a subscript is added so that MUF_{ij} denotes the MUF for the j th period and the i th MBA. Further $\text{MUF}^n = \sum_{i=1}^3 \sum_{j=1}^n \text{MUF}_{ij}$, denotes the long-term MUF which is the sum of MUF's over time and the individual MBA's. Algebraically then

$$\text{MUF}^n = \sum_{j=1}^n R_{1j} - \sum_{j=1}^n O_{3j} - \sum_{j=1}^n (W_{1j} + W_{3j}) + \sum_{i=1}^3 \sum_{j=1}^n \Delta I_{ij} .$$

Let I_{ij} denote the inventory in the i th MBA at the beginning of the j th period. Then

$$\begin{aligned} \sum_{i=1}^3 \sum_{j=1}^n \Delta I_{ij} &= \sum_{i=1}^3 (I_{i1} - I_{i,n+1}) \\ &= (I_{11} + I_{21} + I_{31}) - (I_{1,n+1} + I_{2,n+1} + I_{3,n+1}) \end{aligned}$$

$$= \left(\begin{array}{c} \text{Total MBA Beginning} \\ \text{Inventory} \end{array} \right) - \left(\begin{array}{c} \text{Total MBA Ending} \\ \text{Inventory} \end{array} \right)$$

and one sees that the vital parts of the long-term MUF for the total MBA are the receivals at MBA_1 , the shipments from MBA_3 , the wastes, and the initial and final total inventories. The internal material transfers over the whole MBA area and the internal inventory values over time cancel out.

The conclusions here are as follows:

- In the absence of errors, only the receipts to MBA_1 , the outputs from MBA_3 , the waste stream amounts, and the incremental total inventory are needed to form a system MUF for an individual period.
- Measurements on the receipts to MBA_1 , the outputs from MBA_3 and wastes (when wastes are large enough) are absolutely essential in the sense that these are primary measurements whose values, when unmeasured (or measured badly), are forever lost and the inherent information provided at these primary points cannot be recovered. It is also essential to get an adequate estimate of the initial inventory which in many cases may be zero.
- The long-term MUF, $MUF^n = \sum_{i,j}^{3n} MUF_{ij}$, is dependent upon all of the receipts to MBA_1 , the outputs from MBA_3 , the waste measurements, the beginning total inventory (which may be zero), and the last total inventory.

In practice, of course, all of this is complicated by random and systematic error components for the measurement and estimation procedures. In addition, the internal measurements in the system provide information on the individual MBA's which can be used to make decisions and these internal measurements also provide information on the total system.

3. The Effect of the Falsification of Stated Values

Suppose that for MBA_i the receipts are falsified by an understatement of the amount $k_{i1}\delta_i$, the outputs are overstated by the amounts $k_{i2}\delta_i$, and the wastes by the amounts $k_{i3}\delta_i$, where $k_{i1} + k_{i2} + k_{i3} = 1$. Let primed values denote stated amounts and unprimed values denote true amounts. Then (assuming no measurement errors)

$$MUF'_1 = R_1 - k_{11}\delta_1 + \Delta I_1 - O_{1,s} - O_{1,f} - k_{12}\delta_1 - W_1 - k_{13}\delta_1,$$

$$MUF'_2 = R_2 - k_{21}\delta_2 + \Delta I_2 - O_2 - k_{22}\delta_2, \quad (k_{23} = 0)$$

$$MUF'_3 = O_2 + O_{1,f} - k_{31}\delta_3 + \Delta I_3 - O_3 - k_{32}\delta_3 - W_3 - k_{33}\delta_3$$

and

$$MUF' = MUF'_1 + MUF'_2 + MUF'_3 = R_1 - (\delta_1 + \delta_2 + \delta_3) - O_3 - W_1 - W_3 + \sum_{i=1}^3 \Delta I_i$$

If the individual MBA's are falsified under no diversion one has

$$\begin{aligned} MUF' &= R_1 + \sum_{i=1}^3 \Delta I_i - O_3 - W_1 - W_3 - (\delta_1 + \delta_2 + \delta_3) \\ &= MUF - (\delta_1 + \delta_2 + \delta_3) \\ &= 0 - (\delta_1 + \delta_2 + \delta_3) \end{aligned}$$

so that MUF' , the stated MUF, has the value $-(\delta_1 + \delta_2 + \delta_3)$. If the true amount diverted is Δ , then MUF, the true MUF, is Δ , and MUF' , the stated MUF, is $MUF' = \Delta - (\delta_1 + \delta_2 + \delta_3)$. If the diverter falsifies by the amount $(\delta_1 + \delta_2 + \delta_3)$ which is equal to his diversion Δ , then MUF' , the stated MUF, is zero.

If, however, the MBA_2 accepts MBA_1 's values, MBA_3 accepts MBA_1 's and MBA_2 's shipment values, and MBA_2 uses its receipt values on individual items for shipment values, then the stated MUF is

$$MUF' = R_1 - k_{11}\delta_1 - k_{13}\delta_1 - 0_3 - k_{32}\delta_3 - k_{33}\delta_3 + \sum_3 \Delta I_i - W_1 - W_3 .$$

This seems reasonable since it would not appear to be a good ploy in a national system for an operator-diverter to use values which display egregious contradiction. MUF' can be written

$$MUF' = MUF - \delta_1(k_{11} + k_{13}) - \delta_3(k_{32} + k_{33}) .$$

Then under this arrangement the reported MUF's low bias is induced only by the overstated wastes at MBA_1 and MBA_3 , the understated receipts at MBA_1 and the overstated shipments at MBA_3 .

The operator may attempt to conceal diversion by overstating the ending inventory but this is a self-defeating process. Suppose for example that for each time period the incremental inventory for the i th MBA is overstated by the amount ϵ_i in order to conceal diversion. That is, $I'_{i,j+1} - I'_{ij} = I_{i,j+1} - I_{ij} + \epsilon_i = -\Delta I'_{ij} = -\Delta I_{ij} + \epsilon_i$, so that

$$\begin{aligned} \sum_{ij}^n \Delta I'_{ij} &= \sum_{ij}^n (\Delta I_{ij} - \epsilon_i) = \sum_{ij}^n \Delta I_{ij} - n\epsilon_i \\ &= (I_{i1} - I_{i,n+1} - n\epsilon_i) . \end{aligned}$$

Let $I_{i1} = 0$. Then after n periods the operator is overstating his amount by $n\epsilon_i$ units for the i th MBA. Here not only is the operator getting more vulnerable with respect to total amounts being overstated but he is on record for falsifying the amount values by larger and larger amounts and these reported statements are subject to repeated verification. For the overall reported MUF one has

$$\begin{aligned}
MUF^{n'} &= \sum_{i,j}^{3n} MUF_{ij}^i \\
&= \sum_{i,j}^n R_{1j} - \sum_{i,j}^n O_{3j} + \sum_{i,j}^{3n} \Delta I_{ij} - n(\epsilon_1 + \epsilon_2 + \epsilon_3) - W_1 - W_3 \\
&= MUF^n - n(\epsilon_1 + \epsilon_2 + \epsilon_3)
\end{aligned}$$

More specifically the total inventory gets biased low by the amount $n(\epsilon_1 + \epsilon_2 + \epsilon_3)$. The discrepancy between the true inventory and the stated inventory gets larger with added periods and this increases the verification procedure's power to detect falsification. Thus, with the two effects of

1. understating inputs to MBA_1 , overstating outputs from MBA_3 and overstating wastes, and
2. overstating incremental inventories as well over time, one has

$$MUF^{n'} = MUF^n - n(\epsilon_1 + \epsilon_2 + \epsilon_3 + \delta_1 + \delta_2 + \delta_3)$$

or

$$MUF^{n'} = MUF^n - n(\epsilon_1 + \epsilon_2 + \epsilon_3 + k_{11}\delta_1 + k_{13}\delta_1 + k_{23}\delta_3 + k_{33}\delta_3)$$

where

MUF^n is the true amount diverted, and

$MUF^{n'}$ is the stated amount diverted.

The former or latter equation for $MUF^{n'}$ is appropriate according to whether the transfers inside the total MBA are not or are forced to be consistent.

The conclusions here are as follows:

- The operator in order to conceal diversion will attempt to make MUF' , the stated MUF value, decrease to zero by a system of understating receipts and overstating wastes and removals.

- In a closed system where the overstated outputs from certain parts of the MBA are accepted as inputs to other parts, the internal biases tend to cancel out leaving only those input understatements at MBA_1 , overstatements for shipments from MBA_3 and overstatements of wastes.
- Overstating incremental inventory amounts in order to mask diversion tends to become self-defeating in the cumulative MUF since then $I_{i,n+1}$, the inventory at the end of the n th period, becomes overstated by the amount ne_i which increases the probability of detection both the total amount overstatement and the individual item discrepancies.

4. The Effect of Measurement Errors

Sections I and II have discussed MUF's outputs, inputs, wastes and inventories as true values. In practice, these values depend on measurements which are subject to various kinds of random and systematic errors. Furthermore, when an item is shipped from one MBA to another two measurements, a shipper and a receiver measurement, are often made.

In practice, one could define MUF for the total fuel cycle as

$$\widehat{MUF} = \widehat{MUF}_1 + \widehat{MUF}_2 + \widehat{MUF}_3 \quad (1)$$

or

$$\widehat{MUF} = \widehat{R}_1 - \widehat{O}_3 + \sum_{i=1}^3 \widehat{\Delta I}_i - (\widehat{W}_1 + \widehat{W}_2) \quad (2)$$

(1) and (2) both have the same expected values but are not algebraically identical unless the shipment values from one area are accepted as receipt values to the next material balance area. For definition (1)

the components parts are uncorrelated when only random errors are considered, so that in this situation, care must be taken to define precisely what is meant by covariances which are not zero. The discussion herein is concerned with definition (1) because it is felt that there will be some tendency to use it in practice.

If $\hat{x}$ is an estimate of the parameter x , the estimated MUF for the i th MBA is

$$\widehat{MUF}_i = \hat{R}_i + \Delta \hat{I}_i - \hat{O}_i - \hat{W}_i$$

where $\Delta \hat{I}_i = \hat{I}_{i,1} - \hat{I}_{i,2}$ and $\hat{I}_{i,1}$ & $\hat{I}_{i,2}$ denote, respectively, the inventory estimates at the beginning and at the end of the period.

The constraints on the system now become

$$E(\hat{R}_2) = E(\hat{O}_{1,s})$$

$$E(\hat{O}_2) + E(\hat{O}_{1,f}) = E(\hat{R}_3)$$

$$\hat{O}_1 = \hat{O}_{1,s} + \hat{O}_{1,f}$$

where $E(x)$ denotes the expected value of x . Then $\widehat{MUF}$, the estimate of MUF for the total material balance area, becomes

$$\begin{aligned} \widehat{MUF} = & \hat{R}_1 + \Delta \hat{I}_1 - \hat{O}_{1,s} - \hat{O}_{1,f} - \hat{W}_1 + \hat{R}_2 + \Delta \hat{I}_2 - \hat{O}_2 + \hat{R}_3 + \Delta \hat{I}_3 \\ & - \hat{O}_3 - \hat{W}_3 \end{aligned}$$

which under expectations becomes

$$E(\widehat{MUF}) = E(\hat{R}_1) - E(\hat{O}_3) - E(\sum \Delta \hat{I}_i) - E(\hat{W}_1) - E(\hat{W}_3),$$

and

$$E(\widehat{MUF}) = \Delta$$

under no falsification and no measurement biases. If the operator

falsifies as indicated before with diversion and falsification, then

$$E(\widehat{MUF}) = \Delta - (\delta_1 + \delta_2 + \delta_3)$$

or

$$E(\widehat{MUF}) = \Delta - \delta_1(k_{11} + k_{13}) - \delta_3(k_{32} + k_{33})$$

The latter case applies when internal shipper-receiver differences must be consistent with the random errors of measurement.

Under diversion, the operator would like to shift $E(\widehat{MUF}) = \text{Diversion}$

Amount = MUF under no falsification back to $E(\widehat{MUF}) = MUF - (\delta_1 + \delta_2$

$+ \delta_3)$ under falsification. The variance of $\widehat{MUF}$, as given by $\widehat{MUF} = \widehat{MUF}_1 + \widehat{MUF}_2 + \widehat{MUF}_3$ is

$$\begin{aligned} \sigma_{\widehat{MUF}}^2 &= \sigma_{\widehat{MUF}_1}^2 + \sigma_{\widehat{MUF}_2}^2 + \sigma_{\widehat{MUF}_3}^2 \\ &\quad + 2\text{cov}(\widehat{MUF}_1, \widehat{MUF}_2) + 2\text{cov}(\widehat{MUF}_1, \widehat{MUF}_3) \\ &\quad + 2\text{cov}(\widehat{MUF}_2, \widehat{MUF}_3) \end{aligned}$$

(A definition and discussion of what $\sigma_{\widehat{MUF}}^2$ means is postponed for a few lines). Under the type of control provided by adequate containment and surveillance, the variance in $\widehat{MUF}$ for a facility stems from the process areas. Here the essential $\widehat{MUF}$ variance for a period in a facility comes from the feed material introduced into the process area during the period, the non-stagnant part of the process storage, (i.e. the part of the inventory at the time of the inventory taking which was not present in the previous period) and the product material shipped from the process area during the period. In this sense the variance of the incremental inventory is the variance of the difference in the non-stagnant part of the inventory at the beginning and end of the period. If the data are biased in the sense

that $E(\hat{x}_i) = x_i$, $x_i = R_i$, O_i , ΔI_i , $i = 1, 2, 3$, then the biases play the same role as the falsifications. For example, a bias on receipts which results in an understatement is equivalent "on the average" to falsifying by the amount of the bias. Unintentional biases, however, would not necessarily be to the operator's benefit since one would expect about half of them to be in the wrong direction.

The variance of $\hat{MUF}$ can be considered to consist of only random errors or to consist of random errors and biases. Let $E(\hat{MUF})$ denote the expected value of $\hat{MUF}$ under random errors. Write

$$\hat{MUF} - \Delta = [\hat{MUF} - E(\hat{MUF})] + [E(\hat{MUF}) - \Delta]$$

where Δ = the true amount of material unaccounted for.

Under random errors alone the variance of $\hat{MUF}$ is defined as

$$E[\hat{MUF} - E(\hat{MUF})]^2$$

and under random errors plus systematic errors as

$$\begin{aligned} E(\hat{MUF} - \Delta)^2 &= E[\hat{MUF} - E(\hat{MUF})]^2 + [E(\hat{MUF}) - \Delta]^2 \\ &= \left(\text{Variance due to Random Errors} \right) + (\text{Bias})^2 \end{aligned}$$

Then in the context that $\left(\sigma_{\hat{MUF}}^2 \right)$ represents the expected mean square of $\hat{MUF}$ from Δ , one has that

$$\left(\sigma_{\hat{MUF}}^2 \right) = \sigma_{\hat{MUF}}^2 + (\hat{MUF} \text{ bias})^2,$$

where $\sigma_{\hat{MUF}}^2$ is the variance due to random errors.

To see what the form of the bias of $\widehat{MUF}$ is write

$$\begin{aligned}
 E(\widehat{MUF}) &= E(\widehat{MUF}_1) + E(\widehat{MUF}_2) + E(\widehat{MUF}_3) \\
 &= \sum^3 [(R_i + B_i) - (O_i + C_i) + (\Delta I_i + D_i) - (W_i + E_i)] \\
 &= \sum^3 (R_i - O_i + \Delta I_i - W_i) + \sum^3 (B_i - C_i + D_i - E_i) \\
 &= MUF + \sum^3 (B_i - C_i + D_i - E_i) .
 \end{aligned}$$

Where B_i , C_i , D_i & E_i denote biases to the receipts, shipments, incremental inventories and waste streams, respectively. But

$$C_{1,s} = B_{2,C}, C_{1,f} + C_2 = B_3, E_2 = 0, \text{ so that}$$

$$E(\widehat{MUF}) = \Delta + (B_1 - C_3 - E_1 - E_3 + \sum^3 D_i) .$$

Thus, for the total $\widehat{MUF}$ values, bias is determined by the inputs to MBA_1 , the outputs from MBA_3 , in incremental inventories and the wastes.

$$\text{Then } [E(\widehat{MUF}) - \Delta]^2 = (B_1 - C_3 - E_1 - E_3 + \sum^3 D_i)^2$$

$$\text{and } E[\widehat{MUF} - \Delta]^2 = \sigma_{\widehat{MUF}}^2 \text{ (due to random errors) + bias}^2 .$$

Thus, it appears that as far as variances are concerned, where one is talking about the expected value of $(\widehat{MUF} - \Delta)^2$, the internal biases wash out and the controlling biases are in the receipts at MBA_1 , the shipments at MBA_3 , the wastes and the biases in the incremental inventories.

This is an added word about some possibilities for future development of estimators of MUF's by taking advantage of some of the inherent information in the total information system. Extra information is provided by 1) duplicate measurements of the same quantity as in shipper-receiver measurements and 2) natural constraints provided by the system. One procedure is as follows.

A model is provided for each measurement in terms of the total MBA system's parameters. The variance-covariance matrix of the measurements is given by Σ_y say. Then the system parameters can be estimated by the generalized least-squares procedure

$$\hat{\beta} = \left(X' \Sigma_y^{-1} X \right)^{-1} X' \Sigma_y^{-1} Y$$

where X is the so-called design matrix of the coefficients of the parameters in the models for the y observations. Then the estimates of the parameters β , are unbiased with minimum variance. In addition, a so-called estimable function of the $\hat{\beta}$'s, such as $\hat{MUF}$, will also be unbiased and have minimum variance. This provides a good area for future research.

The conclusions here are as follows:

- There are several estimates which are possible for the total MBA, which have different variances but which have the same expected value.
- For $\hat{MUF} = \hat{MUF}_1 + \hat{MUF}_2 + \hat{MUF}_3$, the internal biases tend to cancel out when calculating $(\sigma_{MUF}^2)^{-1}$, which is defined as $E(\hat{MUF} - \Delta)^2$.
- Under falsification and diversion, the similar properties of $\hat{MUF}$ and $E(\hat{MUF})$ are observed.
- Biases are dominating at the inputs to MBA_1 , and the outputs from MBA_3 . Wastes and incremental inventory biases also enter into the picture.

- A distinction should be made as to just what is meant by $\sigma_{\widehat{MUF}}^2$. Does it for example, include systematic error components or not?
- Further developments of estimating the MBA parameters in $\widehat{MUF}$ by making use of the extra information provided by the duplicate measurements which sometimes exist and due to the inherent constraints (which create redundant information) are possible.

5. The Power to Detect Diversion

The internal measurements $\hat{O}_1, \hat{R}_2, \hat{O}_2, \hat{R}_3$ are not used in the MUF form

$$\widehat{MUF} = \hat{R}_1 - \hat{O}_3 + (\Delta \hat{I}_1 + \Delta \hat{I}_2 + \Delta \hat{I}_3) - \hat{W}_1 - \hat{W}_3$$

for the total MBA. However, they are used in the individual MUF forms $\widehat{MUF}_1, \widehat{MUF}_2$, and $\widehat{MUF}_3$, which are used to estimate MUF_1, MUF_2 , and MUF_3 for the three material balance areas. In addition $\hat{t}_i = \widehat{MUF}_i / \sigma_{\widehat{MUF}_i}$, $i = 1, 2, 3$, the normalized $\widehat{MUF}$ values, are used to test for the statistical significance of the observed MUF values.

The advantages of the $\widehat{MUF}_i$ values, $i = 1, 2, 3$, are that when they are successful in detecting a diversion, they have also located the diversion with a proximity in time and location. Thus, there are several dimensions to the MUF problem. An individual MBA can be considered with regard to the $\widehat{MUF}$ of an individual period or according to the cumulative $\widehat{MUF}$ over a number of successive periods. The total $\widehat{MUF}$ may be regarded for an individual period or for the sum of a number of consecutive periods, the cumulative $\widehat{MUF}$. The question of which $\widehat{MUF}$ to use is related to the type of diversion one is attempting to detect.

Suppose that the $\widehat{MUF}_i$ values are not falsified, that $\widehat{MUF}_i$ is unbiased and that the variance of the various MBA components are known. For the purposes here waste variances are ignored. For simplicity in the arguments let

$$2\sigma^2 = \sigma_{\widehat{R}}^2 + \sigma_{\widehat{O}}^2, \quad \sigma_{\widehat{\Delta I}}^2 = k\sigma^2$$

for all material balance areas. The standard deviation for a short-term individual MUF is $\sigma\sqrt{2+k}$, the standard deviation for a cumulative MUF over n periods for an individual MBA is $\sigma\sqrt{2n+k}$. The standard deviation of a MUF over the three MBA's for an individual period is $\sigma\sqrt{2+3k}$, and for a cumulative period over the three MBA's is $\sigma\sqrt{2n+3k}$. The t value

$$t = MUF / \sigma_{\widehat{MUF}}$$

is used as an index of the power of the technique in question. Here MUF indicates the amount of diversion.

Case 1, One MBA, One Diversion

Suppose a diversion of the amount Δ is located in the i th MBA. Then various t values can be formed. They are

$$t_i = \Delta / \sigma_{\widehat{MUF}_i} = (\Delta/\sigma) / \sqrt{2+k}$$

$$t_i^n = \Delta / \sigma_{\widehat{MUF}_i^n} = (\Delta/\sigma) / \sqrt{2n+k}$$

$$t = \Delta / \sigma_{\widehat{MUF}} = (\Delta/\sigma) / \sqrt{2+3k}$$

$$t^n = \Delta / \sigma_{\widehat{MUF}^n} = (\Delta/\sigma) / \sqrt{2n+3k}$$

Since t_i , the t value for the individual time and location MUF is larger than t_i^n , t , and t^n , the t values for an individual diversion of

Δ for, respectively, a cumulative individual MBA's $\widehat{MUF}$, an individual period, overall $\widehat{MUF}$, and an overall cumulative $\widehat{MUF}$, then in this context individual time and location $\widehat{MUF}$'s have more power to detect individual time and location diversions.

Case 2. A Diversion of Δ from Each Material Balance Area

Here Δ units are taken for each MBA for the same time period.

The t values are

$$t_i = \Delta / \sigma_{\widehat{MUF}_i} = (\Delta / \sigma) / \sqrt{2 + k}$$

$$t_i^n = \Delta / \sigma_{\widehat{MUF}_i^n} = (\Delta / \sigma) / \sqrt{2n + k}$$

$$t = 3\Delta / \sigma_{\widehat{MUF}} = (3\Delta / \sigma) / \sqrt{2 + 3k}$$

$$t^n = 3\Delta / \sigma_{\widehat{MUF}^n} = (3\Delta / \sigma) / \sqrt{2n + 3k}$$

Here t , the $\widehat{MUF}$ over all the individual material balance areas for one period has the greatest value and hence the greatest power to detect diversion.

Case 3. A Diversion of Δ from Each Time Period for the i th MBA

Here Δ units are taken for each accountability period from the same MBA. The t values here are

$$t_i = \Delta / \sigma_{\widehat{MUF}_i} = (\Delta / \sigma) / \sqrt{2 + k}$$

$$t_i^n = n\Delta / \sigma_{\widehat{MUF}_i^n} = (n\Delta / \sigma) / \sqrt{2n + k}$$

$$t = \Delta / \sigma_{\widehat{MUF}} = (\Delta / \sigma) / \sqrt{n + 3k}$$

$$t^n = n\Delta / \sigma_{\widehat{MUF}^n} = (n\Delta / \sigma) / \sqrt{2n + 3k}$$

Here t_i^n , the cumulative $\widehat{MUF}$ for the individual MBA has the most power to discriminate.

Case 4. A Diversion of from Each MBA during each Time Period

Here an amount Δ is taken from each MBA for every accountability period. Then

$$t_i = \Delta/\sigma_{\widehat{MUF}_i} = (\Delta/\sigma)/\sqrt{2 + k}$$

$$t_i^n = n\Delta/\sigma_{\widehat{MUF}_i^n} = (n\Delta/\sigma)/\sqrt{2n + k}$$

$$t = 3\Delta/\sigma_{\widehat{MUF}} = (3\Delta/\sigma)/\sqrt{2 + 3k}$$

$$t^n = 3n\Delta/\sigma_{\widehat{MUF}^n} = (3n\Delta/\sigma)/\sqrt{2n + 3k}$$

Here t^n has the largest value and thus a MUF formed over all material balance areas over all time periods has the greatest power to detect diversion when the amount Δ is taken from each MBA during each time period.

Another aspect which is very important in any efficient or near-optimum allocation of resources to detect diversion is the control of the error rate. Type I errors, i.e. false alarms, will be made $100\alpha\%$ of the time on the average when H_0 is true. In the application of the MUF statistic H_0 , the null hypothesis, is that the true amount of material unaccounted for is zero. If n such tests are made when H_0 is true, there will be $n\alpha$ such false alarms, on the average. If n such tests are made the probability is $P = 1 - (1 - \alpha)^n$ that there will be one or more false alarms when H_0 is true.

Another aspect to be considered under systematic diversion is the relative power of the n individual tests to detect diversion in one or more cases against the power of the particular cumulative MUF to detect diversion. Of course n different cumulative MUF's could also be formed based on $1, 2, \dots, n$ periods.

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The answer here as to what statistic is best under systematic diversion depends upon the magnitude of the individual diversions. If a diversion is large enough so that its detection probability is large for the unit period, then the individual MUF is appropriate. There are cases, however, when the amount diverted would have little chance of being detected on an individual MUF basis but the cumulative effect of n such diversions in n periods would render the total diversion as a visible and statistically significant effect. In any event both types of MUF, the individual and the cumulative could be computed as tests against the different forms that diversion might take.

Let α_i denote the probability of a type I error for the i th test and let α' denote the probability of a falsely triggered alarm on one or more of the tests made over n periods. Then $\alpha' = 1 - \prod(1 - \alpha_i)$. If $\alpha_i = \alpha$ are the same for all cases $\alpha = 1 - (1 - \alpha')^{1/n}$ and one can then establish the α level to work at on the individual test as a function of the error rate which is desired for the pattern of tests. Bonferroni's inequality states that $\alpha' < \sum_{i=1}^n \alpha_i$ so that if one wishes to control the probability of the type I error at the α level or less one may accomplish this by working at the α/n level on the individual tests. As an example if 10 tests are conducted at the $0.05/10 = 0.005$ level, the error rate for type I errors, i.e. a falsely triggered alarm, at least once, is less than 0.05. In the case where the standard deviations are unknown and student's t is needed for an alarm level, some technique such as the batching factors given in (15) may be applicable.

In the case of the normal distribution in the one-sided case with known σ , the Bonferroni Inequality indicates that one should use the value $t_{1-\alpha/n}$ for the n tests. The following table indicates the consequences of this procedure for several cases.

α	n	$t_{1-\alpha}$	$t_{1-\alpha/n}$	$(t_{1-\alpha/n})/t_{1-\alpha}$	$\sqrt{n}$
0.05	1	1.645	1.645	1.00	1.00
	2	1.645	1.960	1.19	1.41
	5	1.645	2.326	1.41	2.24
	10	1.645	2.576	1.57	3.16
	50	1.645	3.090	1.88	7.07
0.10	1	1.282	1.282	1.00	1.00
	2	1.282	1.645	1.28	1.41
	5	1.282	2.054	1.60	2.24
	10	1.282	2.326	1.81	3.16
	100	1.282	3.090	2.41	10.00

If k is small then $t_1^n = (n\Delta/\sigma)/\sqrt{2n+k}$ tends to increase directly as $\sqrt{n}$. The value $t_{1-\alpha/n}/t_{1-\alpha}$ indicates the relative increase in the critical values needed to control the type I error. When k is small $\sqrt{n}$ indicates roughly the increase in the power to detect systematic diversion by the cumulative MUF as compared to the unit MUF.

There are other possibilities available to the inspectors in the sense that one also has the option that MUF's may be created for certain subsets of all the MBA's and time periods.

The conclusions are as follows:

- When the diversions are isolated it is a more powerful technique to detect diversion by using the individual time period and the individual material balance area.
- Systematic diversion, over the whole system, over successive time periods with the same material balance area has more power of detection by an appropriate form of cumulative MUF.

- The problem of the power of detection is somewhat more complicated, however, than the simple t forms suggest since in the situation when tests are made for diversion for each period and for each area, the safeguards authority has more than one chance to detect diversion.
- An optimum allocation of the resources to detect diversion requires some control of the error rate. The Bonferroni inequality is one possibility. There may be opportunities for research here into ways of enhancing the power to detect diversion while at the same time controlling the type I error rate.