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Progress Report on

PLASMA WAVES

Contract AT(04-3)767

with the U.S. Atomic Energy Commission

March 1973

Roy W. Gould, Principal Investigator

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I. RESEARCH TOPICS

During the period May 15, 1972 through February 15, 1973 covered by this report, the following problems were under investigation:

1. The effect of velocity-space diffusion on the degradation of plasma wave echoes and an experimental determination of the velocity-space diffusion coefficient. This study has been completed and results are summarized in Appendix I. A manuscript is being prepared for publication, and the PhD thesis of Dr. C. Moeller is available on request.
2. Thresholds for onset of parametric instabilities. The main result of this study has been to show that the thresholds for convective and absolute instabilities can be vastly different if the natural decay lengths of two parametrically coupled modes are different (Appendix II). These results are of importance for the parametric instabilities which arise in plasma heating (Appendix III).
3. Propagation and absorption of waves near the lower hybrid frequency. A review of prior work in this area and a theoretical study of various excitation schemes are both in progress. The "resonance cone" phenomenon studied extensively in this laboratory is of importance in determining the magnitude and spatial localization of energy absorbed. An experimental study of the propagation and absorption of energy in the lower hybrid frequency (and cyclotron harmonic) range is planned. It will use, in large part, the experimental

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C. Moeller*

J. Wilgen

K. Burrell

Two new students, R. Kleva and B. Levine, will join the project on about June 15, 1973.

III. PUBLICATIONS, REPORTS, TALKS

N. Singh and R. W. Gould, "Waves in a Hot Uniaxial Plasma Excited by a Current Source", Phys. Fluids 16, 75-81 (1973)

C. P. Moeller and R. S. Harp, "Saturation and Damping of a Second Order Plasma Wave Echo", paper GE15, presented at the 1972 Annual Meeting of the APS Division of Plasma Physics. Bull. APS Series II, 17, 1038 (1972)

B. D. Fried, G. Schmidt, R. W. Gould, "Spatial Growth Characteristics of Parametric and Backscattering Instabilities", paper presented at the 1972 Sherwood Theoretical Meeting, Austin, Texas, March 1973.

*Dr. Moeller accepted employment in the Gulf General Atomic fusion program on completion of his research in January 1973.

APPENDIX I

DAMPING AND SATURATION OF A SECOND ORDER

ELECTRON PLASMA WAVE ECHO

Introduction

In the original paper on plasma wave echoes by O'Neil and Gould [1] it was suggested that the echo could be a useful tool for investigating velocity space diffusion in a plasma. The damping of the electron plasma wave echo due to velocity space diffusion was calculated by O'Neil [2] and Su and Oberman [3].

The first experimental measurement of ion wave echo damping due to Coulomb collisions, by Wong and Baker [3] gave good agreement with theory, but because electrons of all velocities contributed to the echo, these results did not give information about the diffusion coefficient at any particular velocity. Guillemot [4] and Jensen [5] observing diffusion in velocity space due to broad band noise, were able to measure a value of the diffusion coefficient at a single velocity by using a resonant third order electron plasma wave echo.

Amplitude saturation of the echo, which can occur in a damping measurement and which depends on the velocity of the electrons producing the echo, was also observed by Guillemot [4] using a resonant echo.

This paper describes a measurement of the velocity dependence of the electron plasma wave echo damping due to Coulomb collisions and a measurement of the amplitude saturation of the echo as a function of the velocity of the electrons contributing to the echo.

Section I gives the results of the theories of the echo damping and saturation.

Section II gives the results of the echo damping measurements.

Section III gives the results of the echo saturation measurements.

I. THEORETICAL BACKGROUND

All the phenomena of interest here are governed by the one-dimensional Boltzmann equation with a Fokker-Planck collision term:

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial z} + \frac{e}{m} E \frac{\partial f}{\partial v} = - \frac{\partial(fD_z)}{\partial v} + \frac{\partial^2(fD_{zz})}{\partial v^2} \quad (1)$$

Even though the experiments are performed in a bounded plasma, a large uniform field in the z direction permits a one-dimensional analysis of the results.

Here, $f(z,v,t)$ is the single particle distribution function of the electrons, $E(z,t)$ is the total electric field in the z direction, D_z is the coefficient of dynamical friction, and D_{zz} is the coefficient of diffusion in velocity space along the magnetic field lines.

The procedure of O'Neil [2] is to expand f in a perturbation series $f = f_0 + f_1 + f_2 + \dots$ in the applied amplitudes. The f_2 term then gives the second order echo.

If two exciters in the plasma produce sinusoidal electric fields $E(z, \omega_1)$ and $E(x, \omega_2)$ at $z = 0$ and $z = \ell$, as shown in Fig. 1, with spatial Fourier transforms $\tilde{E}(k, \omega_1)$ and $\tilde{E}(k, \omega_2)$, respectively, an echo is produced of frequency $\omega_3 = \omega_2 - \omega_1$, centered at $z = \ell' = \ell \omega_2 / \omega_3$. The spatial Fourier transform of the echo electric field is

Figure 1.

$$E(k, \omega_3) = \frac{\pi e}{k^2 \epsilon(k, \omega_3) \epsilon_0} \frac{e}{m} \tilde{E}^*(\omega_1/v, \omega_1) \times \left[\frac{\omega_1 \ell e}{mv} \tilde{E}(\omega_2/v, \omega_2) \frac{\partial f_0}{\partial v} \exp[-D_{zz} \omega_1^2 \omega_2 \ell^3 / 3v^5] \right] \Big|_{v=\omega_3/k} \quad (2)$$

Here $\epsilon(k, \omega_3)$ is the plasma dielectric function.

This result requires that the exciting fields be localized compared to ℓ and to the damping lengths associated with D_{zz} . The interesting features of this result are the ℓ^3 dependence on exciter separation of the exponential damping term, and the $v = \omega_3/k$ relationship between a spatial Fourier component of the echo of wave number k and the velocity v of electrons which contribute that component to the echo. This ℓ^3 behavior is only observable for any single Fourier component of the echo; because of the strong velocity dependence of the damping term, the damping of the total electric field (the inverse spatial transform of $E(k, \omega_3)$) in general has a weaker dependence on exciter separation ℓ .

A principal objective of this work was to measure the factor multiplying ℓ^3 in the exponential damping term, which will be called the damping exponential.

The damping theory mentioned above is only valid for small exciter amplitudes. To find the range of validity, one can consider in the case of such a small exciter separation ℓ that the collisions can be neglected. In that case, the collisionless Boltzmann equation can be solved, following Coste and Peyraud [6] and Gould and O'Neil [1], by examining the ballistic motion of the electrons to obtain an expression for the echo electric

field which is valid for larger applied amplitudes than the perturbation approach of O'Neil [2].

Starting with an electron which arrived at a point $z > \ell$ with velocity $v > 0$ at time t , one can calculate the change in velocity it underwent at the excitors at $z = \ell$ and $z = 0$ in transit, and therefore deduce its initial velocity v_0 at $z < 0$. Assuming the distribution function at $z < 0$ for $v > 0$ is $f_0(v)$, $f(z, v, t) = f_0(v_0(z, v, t))$ for $v > 0, z > \ell$.

Using this distribution function, the spatial Fourier transform of the echo electric field at frequency $\omega_3 = \omega_2 + \omega_1$ is

$$\tilde{E}(k, \omega_3) = \frac{2\pi e}{k^2 \epsilon(k, \omega_3) \epsilon_0} \frac{e}{m} \xi_1 J_1(\omega_1 \ell) \xi_2 / (mv^3) \left. \frac{\partial f}{\partial v} e^{i(\theta_2 - \theta_1)} \right|_{v=\omega_3/k} \quad (3)$$

where the notation is the same as (2) except

$$\xi_1 e^{i\theta_1} = \tilde{E}(\omega_1/v, \omega_1) \quad \text{and} \quad \xi_2 e^{i\theta_2} = \tilde{E}(\omega_2/v, \omega_2)$$

The only assumption about excitation amplitudes is that the error in using the unperturbed velocity in computing the change in velocity $\Delta\omega$ be small.

Assuming a field of frequency ω and width Δz , this requires

$$\Delta z \frac{\Delta v}{v^2} \ll \frac{2\pi}{\omega}$$

It is evident that the argument of the J_1 Bessel function must be $\ll 1$ for Eq. (2) to be valid. This depends on ℓ and v as well as ξ_2 . Because of this ℓ dependence, this nonlinear behavior could be experimentally confused with damping. The result (3) is also interesting in itself, because if one can distinguish the contribution to the echo of electrons of a particular velocity, it is possible to see the $|J_1|$ behavior

by varying E_2 for fixed ℓ .

In both of these cases, the correspondence between a particular wave number k and the velocity v of electrons which contribute that wave number to the echo, it is possible in both the damping and saturation measurements to observe the phenomena of interest as functions of velocity as will be described below.

II. EXPERIMENTAL ARRANGEMENT AND APPARATUS

The plasma source and RF electrodes are shown schematically in Fig. 1. Electrons from the 5 cm diameter planar cathode, accelerated to 400 eV energy by the anode, ionized the background argon gas in the 60 cm long chamber. A 1 kilogauss magnetic field confined the electron motion to the Z (axial) direction.

With the typical operating conditions of a 3 ma beam current and a 1.6 micron background argon pressure, an electron density of $10^8/\text{cm}^3$ and an electron temperature of .65 eV (measured by ion acoustic waves) were obtained. The advantages of this plasma source were that it was volume produced and that under proper operating conditions it was quiet. Being volume produced, there was no drift velocity and large shadows were not cast by the RF electrodes. Low frequency noise in the ion saturation current was less than .5%.

The excitation of the echo was by means of the two grids shown schematically in Fig. 1. The signal from the receiving probe, after filtering and amplification at frequency ω_3 , was mixed with a reference signal generated from the mixing of the two exciter signals and of frequency equal to the difference of the two exciter frequencies. The resultant detected signal was further amplified and detected by a lock-in amplifier.

During an experimental run, the receiving probe was moved at a constant speed in the axial direction through the region containing the echo electric field. The frequency components of the output of the lock-in amplifier were then proportional to echo wave number. This output was sampled at a fixed rate by a digital voltmeter, the digital output of which was recorded on paper tape.

This digital record of echo amplitude as a function of receiving probe position was then Fourier transformed on a computer. According to the discussion of Section I, a Fourier component of wave number k represents the contribution to the echo of electrons of velocity $v = \omega_3/k$.

For the damping measurements, a series of such runs were made for different exciter separations ℓ . From a given Fourier component of wave number k , one then obtains the echo damping as a function of ℓ for the corresponding velocity $v = \omega_3/k$.

For the saturation measurements, a series of such runs were made by a fixed ℓ , but with different amplitudes applied to the second exciter.

III. EXPERIMENTAL RESULTS

A. Damping Measurements

The main objective of this work was to measure the echo damping due to diffusion in velocity space as a function of velocity. In practice this could only be done over a limited range of velocity because of the nature of the excitation and because of the strong velocity dependence of the damping. The results presented here are for frequencies $\omega_1/2\pi = 350$ MHz and $\omega_2/2\pi = 600$ MHz.

Figure 2 shows the magnitude of the spatial Fourier transform of the echo amplitude slotted for various exciter separation ℓ . It is already evident from this that damping increases for decreasing velocity (increasing wave number). Here $\omega_3/k_1 = 9.93 \times 10^7$ cm/sec while $\omega_3/k_3 = 7.22 \times 10^7$ cm/sec.

The shape of the curves is predicted by Eq. (3), but to get good quantitative agreement, one must assume a 3 eV electron temperature for f_0 . This is not inconsistent with a .65 eV temperature measured by ion waves, though, because the electron velocities corresponding to Fig. 2 are in the tail of the distribution, to which the ion waves are extremely insensitive.

In reality, electron-neutral collisions are not completely negligible. However, they are large angle collisions so that an electron colliding with a neutral completely loses the effects of its perturbations. The damping of the contribution to the echo of electrons of velocity v should then be just $\exp[-\ell'/\lambda(v)]$ where $\ell' = \ell\omega_2/\omega_3$ is the total distance from the first exciter to the center of the echo. Knowing the neutral pressure, $\lambda(v)$ can be determined from known data (Brode [7]).

For a given wave number k , the amplitude of the Fourier component is then of the form

$$\ell a(\omega_3/k) \exp[-k(\omega_3/k)\ell^3 - \ell'/\lambda(\omega_3/k)]$$

Since the objectives of this measurement were to find $b(v)$ and verify the ℓ^3 behavior, the known value of $\lambda(v)$ was used to correct the data for electron-neutral collisions. For a velocity ω_3/k_3 in Fig. 2, the effect of this correction is shown in Fig. 3. Here the log of the

Figure 2. FOURIER TRANSFORM OF ECHO AS A FUNCTION OF k FOR VARIOUS EXCITER SEPARATIONS ℓ ,
 $\omega_1 = 2\pi \times 350$ MHz, $\omega_2 = 2\pi \times 600$ MHz

Fig. 3. The effect on the damping data of the correction for electron neutral collisions

magnitude of the Fourier component (divided by ℓ) is shown plotted against ℓ uncorrected and corrected by multiplication by $\exp[\ell'/\lambda(v)]$. It is evident that although for small ℓ , electron-neutral collisions are significant, for large ℓ the Coulomb collisions dominate. A $-b\ell^3$ curve has been fitted to the corrected data.

To better verify the ℓ^3 behavior, the same normalized corrected data is plotted against $(\ell/\ell_0)^2$, $(\ell/\ell_0)^3$ and $(\ell/\ell_0)^4$ in Fig. 4. It is evident that the correct exponent lies between 2 and 4.

Assuming the ℓ^3 behavior, the slope of the line then gives the damping coefficient b . From a series of such lines for the various wave numbers of the echo, b can be obtained as a function of velocity.

This data is shown in Fig. 5. The solid theoretical curves for the indicated temperatures were obtained using a diffusion coefficient D_{zz} for an infinite magnetic field, as obtained in the appendix. The electron density used in D_{33} was measured using a resonance cone technique (Fisher and Gould [8]). Although the magnitude of the damping coefficient is close to the predicted value, the velocity dependence is weaker. This can be explained qualitatively by the non-Maxwellian character of the high energy tail of the distribution function, which is also consistent with the shape of the curves of Fig. 2. Without knowing the actual electron velocity distribution, it is not possible to make any better comparison.

B. Echo Saturation

In order to verify that echo saturation was not affecting the damping measurements, a series of runs were made with $\omega_1/2\pi = 300$ MHz, $\omega_2/2\pi = 550$ MHz, and exciter separation $\ell = 7.9$ cm, with varying amplitudes applied to the second grid.

LOG OF THE FOURIER TRANSFORM OF
THE ECHO AMPLITUDE FOR FIXED k

Fig. 5.. Damping exponent predicted from D_{33} for a plasma with infinite B_0 .

Plotting the magnitude of a given Fourier component of the echo against the relative amplitude applied in that run, one obtains a plot, an example of which is shown in Fig. 6. The solid curve is the magnitude of the J_1 Bessel function fitted to the initial slope and peak-height of the data.

The lowest amplitude applied here was over twice the amplitude used in the damping measurements, so that saturation did not affect the damping measurements. That the data fits J_1 as well as it does provides evidence that only electrons of a narrow range of velocity contribute to a given wave number, for this J_1 behavior is not at all evident in the untransformed data, to which electrons of a broad range of velocities contribute.

From the slope and peak-height of the data for various wave numbers, one can obtain the argument of the J_1 as a function of wave number, which gives information about the nature of the excitation at the second grid. The argument of J_1 in Eq. (3) written in terms of a potential ϕ_2 is

$$\frac{e}{m} \frac{\omega_1}{v^3} \phi_2 F(k)$$

where F is a form factor for the grids. In the simplest case, with no self-consistent fields at the exciters, $\Gamma(k) = \frac{\sin^2(hk/2)}{(hk/2)^2}$, where h is the grid spacing. Plotting the magnitude of a Fourier component of wave number k divided by k^2 against $1/v = k/\omega_3$, the points in Fig. 7 were obtained. $\sin^2(hk/2)$ has been fitted to the data. The period of this function gives a value for h , the grid spacing, equal to the actual grid spacing 2 mm. This demonstrates that self-consistent fields are not important at the exciters, as expected theoretically.

$$b(k) = \frac{e}{m} \frac{\hbar \omega_1}{v^3} F(k)$$

$$F(k) = \hbar k \frac{\sin^2(\hbar k/2)}{(\hbar k/2)^2} \text{ FOR SIMPLEST MODEL}$$

Figure 6

Figure 7. $v^2 \times \text{ARGUMENT OF } J_1$ AS A FUNCTION OF $1/v$ (WAVE NUMBER OF THE ECHO)

Conclusion

This paper described the experimental study of several plasma wave echo phenomena as a function of the velocity of the electrons contributing to the echo. It proved possible to get good agreement with theoretical predictions for the quantities whose predictions did not require a knowledge of the equilibrium electron velocity distribution.

Appendix for Appendix I

THE COEFFICIENT OF DIFFUSION IN VELOCITY SPACE

The diffusion tensor is defined as

$$D_{ij} = \lim_{\Delta t \rightarrow 0} \frac{1}{2} \frac{\langle \Delta v_i \Delta v_j \rangle}{\Delta t}$$

where Δv is the change in velocity an electron undergoes in time Δt due to the fluctuating electric field of the cloud of electrons surrounding it. In order to apply a continuous stochastic theory to what is microscopically a discontinuous process, Δt must actually remain large compared to the correlation time τ_c of the fluctuating fields. Simultaneously, it must be small compared to macroscopic times.

Calculating the Δv_i using the test particle's unperturbed orbit, and assuming $\Delta t \gg \tau_c$, one obtains for the coefficient of diffusion along the magnetic field lines

$$D_{zz} = \frac{1}{2} \left(\frac{e}{m}\right)^2 \int_{-\infty}^{\infty} \langle E_z(\vec{R}_0(t-t'), t-t') E_z(\vec{R}_0(t), t) \rangle dt'$$

Here, $\vec{R}_0$ is the unperturbed motion of the test particle in the static

magnetic field, and $\vec{E}(\vec{r}, t)$ is the fluctuating electric field.

Upon taking the spatial and temporal Fourier transform of the fluctuating fields, one obtains

$$D_{33} = \frac{1}{2} \left(\frac{e}{m}\right)^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle E_z^2 \rangle_{\omega, \vec{k}} \exp[i\vec{k} \cdot (\vec{R}_0(t) - \vec{R}_0(t-t')) - i\omega t'] \\ \times \frac{d^3 k \, d\omega \, dt'}{(2\pi)^4}$$

In the experimental case of interest, the electron Larmour radius is considerably smaller than a Debye length, so that the assumption of an infinite static magnetic field B_0 is a reasonable first approximation. Therefore,

$$\vec{R}_{0\perp}(t) \approx r_0 \quad \text{and} \quad \vec{R}_{0\parallel}(t) = z_0 + \vec{v}_{0\parallel} t,$$

where $\perp$ and $\parallel$ are perpendicular and parallel to B_0 , respectively, then

$$D_{33} = \frac{1}{2} \left(\frac{e}{m}\right)^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle E_z^2 \rangle_{\omega, \vec{k}} \exp[i(k_z v_{0\parallel} - \omega)t'] \frac{d^3 k \, d\omega \, dt'}{(2\pi)^4} \\ = \frac{1}{2} \left(\frac{e}{m}\right)^2 \int_{-\infty}^{\infty} \langle E_z^2 \rangle_{v_{0\parallel} k_z, \vec{k}} \frac{d^3 k}{(2\pi)^3}$$

From the fluctuation-dissipation theorem (Sitenko [9], p. 28), one has

$$\langle E_z^2 \rangle_{\omega, \vec{k}} = \frac{4\pi i \, kT}{\omega} \{ \Delta_{33}^{-1} - \Delta_{33}^{-1*} \}$$

where

$$\Delta_{ij}(\omega, \vec{k}) = \eta^2 \left(\frac{k_i k_j}{k^2} - \delta_{ij} \right) + \epsilon_{ij}(\omega, \vec{k})$$

and $\eta = kc/\omega \gg 1$. For an infinite B_{0z} , the nonzero elements of the plasma dielectric function are $\epsilon_{xx} = \epsilon_{yy} = 1$ and $\epsilon_{zz} = \epsilon_{\ell}(\omega, k_{\parallel})$ is the longitudinal hot plasma dielectric function.

$\epsilon_{\ell}(\omega, k_z) = 1 - \frac{1}{a^2 k_z^2} Z'(\frac{\omega}{k_z v_a})$, where $a = \frac{v_e}{\sqrt{2} \omega_p}$ is the Debye length, ω_p is the electron plasma frequency, v_e is the electron thermal speed, and $Z(\zeta)$ is the plasma dispersion function (Fried and Conte[10]).

$$\Delta_{ij}^{-1} = \frac{1 - \eta^2 k^2 / k^2}{\epsilon_{\ell} - \eta^2 (k_{\parallel}^2 \epsilon_{\ell} + k_{\perp}^2) / k^2}$$

where $k_{\parallel} = k_z$, and $k_{\perp}^2 = k_x^2 + k_y^2$. For $\eta \gg 1$, this becomes

$$\Delta_{ij}^{-1} \cong k_{\parallel}^2 / (k_{\parallel}^2 \epsilon_{\ell} + k_{\perp}^2). \text{ Because } \epsilon_{\ell}(v_{0\parallel} k_{\parallel}, k_{\parallel}) = 1 - Z'(v_{0\parallel}/v_e) / (a^2 k_{\parallel}^2)$$

the integral in (1) is readily done to give

$$D_{33} = \frac{e^2}{2\pi m} \frac{\omega_p^2}{v_{0\parallel}} \left\{ \frac{\tan^{-1}(uA)}{u} + \frac{A}{2} \ln\left(\frac{1/A^2 + u^2}{u^2}\right) \right\} \Bigg|_{u=1/(Re z')}^{1/(ak_0)^2}$$

where $A = \text{Im } Z'(v_{0\parallel}/v_e)$ and $k_0 = \frac{mv_e^2}{e^2}$ is a cut-off of $k_{\parallel}$ at impact parameters for which binary collisions become important. In this experiment $\ln(ak_0) \approx 13$, so that the dominant term gives $D_{33} \approx \frac{e^2}{m} \frac{\omega_p^2}{v_{0\parallel}} \frac{A}{\pi} n(ak_0)$.

APPENDIX II

CONVECTIVE AND ABSOLUTE INSTABILITY IN DISSIPATIVE SYSTEMS

The equations for two coupled modes whose amplitudes a_1 and a_2 can be written

$$\begin{aligned} a_1(z,t) &= e^{i(k_0 z - \omega_0 t)} A_1(z,t) \\ a_2(z,t) &= e^{i(k_0 z - \omega_0 t)} A_2(z,t) \end{aligned} \quad (1)$$

where A_1 and A_2 satisfy the coupled differential equations

$$\begin{aligned} \frac{\partial A_1}{\partial t} + \gamma_1 A_1 + V_1 \frac{\partial A_1}{\partial z} + \kappa A_2 &= 0 \\ \mp \kappa^* A_1 + \frac{\partial A_2}{\partial t} + \gamma_2 A_2 + V_2 \frac{\partial A_2}{\partial z} &= 0 \end{aligned} \quad (2)$$

where V_1 and V_2 are the group velocities; γ_1 and γ_2 are the damping coefficients of the respective modes; ω_0 and k_0 are the frequency and wave number at which the undamped ($\gamma_1 = \gamma_2 = 0$) modes would be phase matched. κ is a coupling coefficient and the upper sign is to be taken if both modes are positive energy modes. The lower sign is to be taken if one mode is a positive energy mode, while the other is a negative energy mode. This form applies for the weak coupling of two modes, including parametric coupling which gives rise to the decay instability.

The following energy conservation relation follows from (2)

$$\frac{\partial}{\partial t} (A_1 A_1^* \pm A_2 A_2^*) + 2\gamma_1 A_1 A_1^* \pm 2\gamma_2 A_2 A_2^* + \frac{\partial}{\partial z} (V_1 A_1 A_1^* \pm V_2 A_2 A_2^*) = 0 \quad (3)$$

We can identify $A_1 A_1^* \pm A_2 A_2^*$ as the energy density and $V_1 A_1 A_1^* \pm V_2 A_2 A_2^*$ as the energy flux.

We are primarily interested in the situation which can lead to absolute instability, namely

- a) oppositely directed group velocities ($V_1 V_2 < 0$)
- b) one negative energy mode (lower sign).

A parallel treatment of the other cases can readily be carried out.

Dispersion Equation

The assumption of a solution $\sim e^{ikz - i\omega t}$ yields the dispersion equation:

$$(\omega + i\gamma_1 - kV_1)(\omega + i\gamma_2 - kV_2) \mp |k|^2 = 0 \quad (4)$$

whose solutions are

$$k = \frac{1}{2} \left(\frac{\omega + i\gamma_1}{V_1} - \frac{\omega + i\gamma_2}{|V_2|} \right) \pm \sqrt{\left(\frac{\omega + i\gamma_1}{2V_1} + \frac{\omega + i\gamma_2}{2|V_2|} \right)^2 + \frac{|k|^2}{V_1 |V_2|}} \quad (5)$$

where we have taken $V_2 < 0$ and the lower sign. As $|k|^2 \rightarrow 0$, the solutions tend to

$$k_1 \rightarrow \frac{\omega + i\gamma_1}{V_1} \quad k_2 \rightarrow - \frac{\omega + i\gamma_2}{|V_2|} \quad (6)$$

i.e., damped waves in $+z$ and $-z$ directions, respectively. With increasing $|\kappa|^2$ the damping of each wave is decreased and the wave with the smallest spatial decay factor ($\gamma_1/V_1 = \Gamma_1$, since we suppose this to be wave number 1) is the first to achieve negative damping or spatial growth (see Figure 8). This occurs for $\omega = 0$, $\text{Re } k = 0$ and

$$|\kappa^2| = \gamma_1 \gamma_2 \quad (7)$$

This is the threshold for convective instability. The threshold for absolute instability is reached, according to Briggs, when the two solutions of (5) coalesce for some ω in the upper half plane. This occurs when

$$|\kappa^2| = \frac{(\gamma_1 |V_2| + \gamma_2 V_1)^2}{4V_1 V_2} \quad (8)$$

which is the threshold for absolute instability.

The threshold for convective instability is smaller than the threshold for absolute instability by the factor

$$\frac{|\kappa^2|_{\text{conv}}}{|\kappa^2|_{\text{abs}}} = \frac{4(\Gamma_1/\Gamma_2)}{(1 + \Gamma_1/\Gamma_2)^2} \quad (9)$$

where $\Gamma_1 = \gamma_1/V_1$ and $\Gamma_2 = \gamma_2/V_2$ are the spatial decay rates for the two modes. Note that the threshold for convective instability is less than that for absolute instability so long as $\Gamma_1 \neq \Gamma_2$. In particular, if $\Gamma_1 \ll \Gamma_2$ (or $\Gamma_2 \ll \Gamma_1$) the thresholds can be vastly different.

Green's Function (Solution for a Pulse)

The Fourier-Laplace transform of the solution of equations (2) with a source term $\delta(z) \delta(t)$ added to the right hand side of the second equation is

$$A_1 = \frac{-\kappa}{(s+\gamma_1 + ikV_1)(s+\gamma_2 - ik|V_2|) - |\kappa|^2} \quad (10a)$$

$$A_2 = \frac{s+\gamma_1 + ikV_1}{(s+\gamma_1 + ikV_1)(s+\gamma_2 - ik|V_2|) - |\kappa|^2} \quad (10b)$$

The inverse Fourier spatial transform can be carried out on the contour shown in Figure 9, yielding

$$A_1 = \frac{-\kappa}{V_1|V_2|} \begin{cases} \frac{e^{ik_2 z}}{i(k_2 - k_1)} & z > 0 \\ \frac{e^{ik_1 z}}{i(k_2 - k_1)} & z < 0 \end{cases} \quad (11)$$

where the previous definitions (Eq. 5 with $\omega = is$ and $\text{Re } s > 0$) apply.

Noting that

$$ik_{2,1} = \frac{1}{2} \left(\frac{s+\gamma_2}{|V_2|} - \frac{s+\gamma_1}{V_1} \right) \mp \sqrt{\left(\frac{s+\gamma_2}{2|V_2|} + \frac{s+\gamma_1}{2V_1} \right)^2 - \frac{|\kappa|^2}{V_1|V_2|}}$$

can be written

$$ik_{2,1} = \frac{s'}{v} + \epsilon \mp \frac{\sqrt{s'^2 - k'^2}}{v}$$

with

$$\frac{1}{V} = \frac{1}{2} \left(\frac{1}{V_1} + \frac{1}{|V_2|} \right), \quad \frac{1}{v} = \frac{1}{|V_2|} - \frac{1}{V_1}, \quad \epsilon = \frac{\gamma_1 - \gamma_2}{V_1 + |V_2|}, \quad \kappa'^2 = \frac{\kappa^2 V^2}{V_1 |V_2|},$$

and
$$s' = s + \frac{V_1 \gamma_2 + V_2 \gamma_1}{V_1 + V_2} \equiv s + \alpha$$

Equation (11) can then be written:

$$\begin{aligned} A_1 &= - \frac{\kappa V}{2V_1 V_2} e^{-\alpha t + \epsilon z} \int_c e^{s'(t + \frac{z}{V})} \frac{e^{-\sqrt{s'^2 - \kappa'^2} \frac{z}{V}}}{\sqrt{s'^2 - \kappa'^2}} \frac{ds'}{2\pi i}, \quad z > 0 \\ &= - \frac{\kappa V}{2V_1 V_2} e^{-\alpha t + \epsilon z} I_0(\kappa' \sqrt{(t + \frac{z}{V})^2 - (\frac{z}{V})^2}), \quad t \geq \frac{z}{V_1} \end{aligned}$$

For $z = 0$

$$A_1 \sim e^{-\alpha t} I_0(\kappa' t)$$

so the disturbance grows or decays at $z = 0$ according to whether κ' is greater or less than α , which is the condition for absolute instability (Eq. 8).

Figure 9.

APPENDIX III

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SPATIAL GROWTH CHARACTERISTICS OF PARAMETRIC AND BACKSCATTERING INSTABILITIES

B. D. Fried and G. Schmidt, UCLA, and R. W. Gould, Caltech

Both the familiar electrostatic parametric instability (photon $\rightarrow$ phonon plus plasmon) and the backscattering instability (photon $\rightarrow$ photon plus plasmon or phonon) are usually considered in the context of initial value problems (real k , complex ω). However, some applications of the theory, including plasma heating and laser backscattering, involve steady state excitation necessitating consideration of spatial growth for real ω . When damping effects, collisional or collisionless, are neglected, the decay instabilities are always absolutely unstable (exponential growth in time at a given location). For the choice of k which allows exact frequency match, the pump power threshold for instability is zero; for other k , there is a non-zero threshold; and above it the instability is always absolute. A simple analysis, virtually identical with that used for beam instabilities, shows that when damping effects are included, there is a non-zero threshold, P_c , for convective instability, and only above a still higher threshold, $P_A > P_c$, does the instability become absolute, a behavior reminiscent of loss cone instabilities. If the decay length ($v_{\text{group}}/\gamma_{\text{decay}}$) for one produce wave greatly exceeds that of the other,

$P_c/P_A \ll 1$ so there is convective growth for a large range of pump powers, it follows that wave propagation, with the attendant refraction and change of k , may in many circumstances be more important for saturation than nonlinear effects. For the purely growing modes (oscillating two-stream instability in the parametric case, filamentation in the backscattering problem) there is no region of convective instability.

Appendix IV. Block Diagram of Real-Time Direct-Reading
Microwave Interferometer

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