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Carnegie Institute of Technology

Department of Physics

STUDY OF THE DEPOLARIZATION OF NEGATIVE MUONS

IN LIQUID HELIUM

He

by

John Kane

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May 1964

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A thesis, based upon the work reported here, has been submitted to the Department of Physics, Carnegie Institute of Technology, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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1. ABSTRACT

The ground state polarization was determined for μ^- -mesons stopping in nuclear spin zero targets of liquid helium, liquid argon, and carbon. A precessional technique was employed, allowing $(\mu - e)$ decay anisotropy to be observed in terms of the sinusoidally modulated decay distribution

$$N_0 e^{-\lambda T} [1 + A \cos (\omega t - \bar{\phi})] + B_0$$

An analysis of the data over one or more precession cycles was made by means of a least-squares procedure, with the results being corrected for the finite geometry used. In contrast to carbon, which under different conditions yielded asymmetry coefficients of $a = -0.052 \pm .004$ and $a = -0.040 \pm .010$, both helium and argon demonstrated asymmetry values consistent with total depolarization. This indicates a deviation from the pattern of large asymmetry established in a number of other spin zero nuclei.

II. INTRODUCTION

By 1949 the weak processes of beta decay ($n \rightarrow p + e^- + \bar{\nu}$), muon decay ($\mu^- \rightarrow e^- + \nu + \bar{\nu}$), and muon capture ($\mu^- + p \rightarrow n + \nu$) were sufficiently identified to suggest that a common interaction mechanism was involved. Tiomno and Wheeler,¹ working from cloud chamber evidence of cosmic ray muons, were the first to formulate a relationship between the three with their hypothesis of a universal Fermi interaction (UFI). While successful in demonstrating a near equality of coupling strengths, this proposal was still little more than an extension of Fermi's original beta theory,² and as such was incapable of resolving the final form or structure of the interaction. Of course beta decay selection rules were in evidence, with some transitions being classified as Fermi (S,V) and others as Gamow-Teller (T,A) interactions. But for the most part further progress in unraveling the lepton-nucleon four fermion interactions was impeded by the overabundance of possible coupling schemes arising from combinations of Scalar, Vector, Tensor, Axial-vector, and Psuedoscalar interaction terms.

The first step toward resolving this matter was indirectly taken in 1957 by Lee and Yang³ whose review of reflection symmetry evidence led to the sudden demise of parity conservation for weak interaction theory, and subsequently to the restoration of the two component neutrino theory⁴. Feynman and Gell-Mann⁵, carrying this last idea further, considered a single vertex coupling of four fermions with interaction form:

$$H_{int} = \sum_i C_i (\bar{\psi}_1 O_i \psi_2) (\bar{\psi}_3 O_i \psi_4) \quad (1)$$

where not only the neutrino but all particle wave functions Ψ_k ($k = 1, 2, 3, 4$) are left hand selected by the helicity projection operator $a = \frac{1}{2}(1 + \gamma_5)$. As a result of this approach only the Vector and Axialvector members of the Dirac operator family O_i survive, these appearing with equal weight, but undetermined relative sign, in a pure leptonic process such as muon decay. Experiment has since removed this last ambiguity, yielding V-A in a study of electron-neutrino asymmetry about the spin direction of the beta active neutron.⁶ Thus with the exception of nucleon renormalization of the Axial-vector contribution,⁷ which provides $g^{(A)}/g^{(V)} \sim 1.2$ for both beta decay and muon capture, the three leptonic interactions are considered symmetrically accounted for by the forms:

$$H_{int} (\beta^- \text{ decay}) = g_\beta^{(V)} [\bar{\Psi}_e \gamma_\lambda (1 + \gamma_5) \Psi_\nu] [\bar{\Psi}_p \gamma_\lambda (1 + \frac{g_\beta^{(A)}}{g_\beta^{(V)}} \gamma_5) \Psi_n] \quad (2a)$$

$$H_{int} (\mu^- \text{ decay}) = g_\mu^{(V)} [\bar{\Psi}_e \gamma_\lambda (1 + \gamma_5) \Psi_\nu] [\bar{\Psi}_\nu \gamma_\lambda (1 + \gamma_5) \Psi_\mu] \quad (2b)$$

and,

$$H_{int} (\mu^- \text{ capture}) = g_\mu^{(V)} [\bar{\Psi}_n \gamma_\lambda (1 + \frac{g_\mu^{(A)}}{g_\mu^{(V)}} \gamma_5) \Psi_p] [\bar{\Psi}_\nu \gamma_\lambda (1 + \gamma_5) \Psi_\mu] \quad (2c)$$

these following directly from eq. (1) in accordance with the UFI hypothesis, parity nonconservation, and evidence for a V-A interaction.

A more descriptive approach to the above interaction forms involves the definition of a parity violating weak "current"⁵

$$J_\lambda = J_\lambda(p, n) + J_\lambda(\nu_e, e) + J_\lambda(\nu_\mu, \mu) \quad (3)$$

Interacting with itself, possibly through a charge carrying intermediate

vector boson, this "current" produces the hamiltonian

$$H_{int} = g J_{\lambda}^{\dagger} \cdot J_{\lambda} \quad (4)$$

From this general form comes the various modes of strangeness - conserving weak interactions, some as yet unobserved. Muon decay of eq. (2b), for example, can be recognized as the term

$$H_{int} (\mu^- \text{ decay}) = g J_{\lambda}^{\dagger} (\nu_e, e) \cdot J_{\lambda} (\nu_{\mu}, \mu) \quad (5)$$

controlling either diagram (a) or (b) below:

a) Direct Coupling

b) Boson Exchange Coupling

In keeping with recent high energy neutrino results⁸ we have made explicit in J_{λ} the distinction between muon neutrinos ν_{μ} and electron neutrinos ν_e . In this way the theory both assures the familiar conservation of leptons (μ^- , e^- , ν) and the more specific conservation of mu-leptons (μ^- , ν_{μ})⁹.

Enlisting the aid of the well known Fermi coupling constant $g_{\beta}^{(V)}$ of O^{14} decay, Feynman and Gell-Mann⁵ calculated the lifetime of the free muon in terms of their (V,A) prescription. The agreement with the experimental lifetime value was found to be of a high order, higher in fact than one should expect from the UFI theory considering the

likelihood of the renormalization of $g_{\beta}^{(V)}$ under strong forces. Electrodynamics has shown a capacity to maintain invariant its charge coupling e during the virtual decomposition process $p \rightleftharpoons n + \pi^+$. In kind then, we might imagine for weak nucleon interactions a transfer of weak vector current coupling from neutron to pion during the virtual process $n \rightleftharpoons p + \pi^-$. Such a conserved vector current (CVC) hypothesis receives strong support from the fact that the free pion exhibits beta activity in agreement with the predicted branching ratio,

$$R = \frac{\lambda_{\pi^+} (\pi^0 + e^+ + \nu)}{\lambda_{\pi^-} (\mu^+ + \nu)}$$

On the basis of 52 beta events Depommier, Heintze, Rubbia, and Soergel have obtained $R_{\text{exp.}} = (1.08 \pm 0.21) \times R_{\text{CVC}}^{10}$

Further tests of the CVC theory follow from a careful inspection of the pion's role in the interaction hamiltonians of beta decay and muon capture. Here weak activity of the virtual pion is expected to appear as higher order correction terms to the main vector and axial-vector contributions. Anticipated are the following:

(a) Vector Term

To the familiar process of direct vector coupling would be added a two pion intermediate state of the type diagrammed below for muon capture.

This would account for a conserved vector current at low momenta (beta decay), and exhibit a slight form factor dependence at higher momenta

(muon capture). Calculation indicates $g_u^{(V)} = .97 g_\beta^{(V)}$ assuming Universality.

(b) Effective Tensor Term

The same two pion state would also be responsible for an induced tensor coupling to the weak current $J^{(V)}_{(\mu^-, \nu)}$. Referred to as "weak magnetism" this has been carried over from electrodynamics as the weak interaction counterpart of anomalous magnetic moment coupling. Here calculation indicates $g_u^{(M)} = 3.7 g_u^{(V)}$.

(c) Axial-Vector Term

The main axial-vector contribution is almost unaffected by pion cloud correction. For the record, nevertheless, we note a possible three pion intermediate state for which calculation predicts $g_u^{(A)} = .999 g_\beta^{(A)}$.

(d) Effective Pseudoscalar Term

A final modification to the muon capture interaction is expected from the single pion intermediate state. Diagrammed below, this takes the form of an induced pseudoscalar term with coupling $g_u^{(P)} = 8 g_\beta^{(A)}$.¹¹

Since these pionic renormalizations are all of a higher order, a great deal of care and attention must be given to any experiment designed with them in mind. The best example of this was recently provided by the beta decay work of Lee, Mo, and Wu,¹² in which "weak magnetism" effects

were looked for as spectral distortions in the Kurie plots of B^{12} and N^{12} . Proposed by Gell-Mann¹³ and later calculated by Gell-Mann and Berman,¹⁴ this experiment gave convincing evidence of interference between the second-order vector current and the allowed term of the axial vector interaction.

For muon capture the experimental evidence regarding correction diagrams, while more extensive, is decidedly less convincing. Consider first the theoretically simple process of capture in hydrogen,

Here attempts to extract coupling correction information from the total capture probability have been arrested by the following discrepancy between calculation and experiment:

<u>Source</u>	<u>Capture Rate Λ_c</u>
Calculation of Λ_c (ortho $p \mu p$) (15)	560 sec^{-1}
Chicago (Bubble Chamber) (16)	428 ± 85
CERN (Bubble Chamber) (17)	450 ± 50
Columbia (Counters) (18)	476 ± 40

Given a better understanding of $p \mu p$ and $p \mu d$ molecular formation in liquid hydrogen, however, this difference should hopefully disappear, allowing a clear measure of Primakoff's effective coupling constants G_V , G_A , and G_P .¹⁹

In more complex nuclear systems the capture process, for the most part, holds little promise of fundamental interpretation due to large uncertainties in the nuclear wave functions (a perplexing number

of highly excited final states are energetically possible with $Q \sim \frac{1}{2}m_\mu$). Exceptions are available, nonetheless, in the form of captures leading to identifiable final states. Muon capture on helium-3 via an ionizing triton channel

is possibly the most important example of selective capture, having been carefully observed as a monoenergetic recoil and reliably calculated with the aid of the β -active triton. Gathered for comparison these results show the following agreement:

<u>Source</u>		<u>Capture Rate Λ_c ($\text{He}^3 \rightarrow \text{H}^3$)</u>
Calculation by Wolfenstein	(20)	$1450 \pm 10\% \text{ sec}^{-1}$
Dubna (Diffusion Cloud Chamber)	(21)	1410 ± 140
Berkeley (Gas Counter)	(22)	1520 ± 50
Carnegie Tech (Gas Counter)	(23)	1470 ± 67

A second example of some significance would be muon absorption in carbon-12,

where final state considerations are restricted to the β -active ground state of boron-12. Being a $0^+ \rightarrow 0^+$ transition this has been analyzed as a Gamow-Teller type interaction and found to favor an induced pseudoscalar correction of positive sign relative to the axial-vector constant $g_u^{(\Lambda)}$. In other words $g_u^{(P)} = -8 g_u^{(\Lambda)}$ appears unlikely.²⁴

Of more current interest is the coupling sensitive observation of parity violating effects in muon capture. These include angular distribution asymmetry of direct emission neutrons, circular polarization of

radiative capture internal brehmstrahlung and angular distribution asymmetry of the same capture-correlated radiation. For our purposes it will suffice to consider the first of these. Calculation²¹ based upon a V-A interaction with CVC renormalization predicts for the neutron of hydrogen muon capture an angular distribution of the form

$$W(\theta_{nu}) = 1 + P_{\mu} \alpha_H \cos \theta_{nu} \quad (9)$$

Here P_{μ} is the residual polarization of the muon at time of capture, α_H is the intrinsic asymmetry parameter for the hydrogen capture process, and θ_{nu} is the angle defined by the momentum direction of the neutron and the polarization of the muon. The physics of the situation is of course contained in the coefficient α_H which depends upon the coupling constants of the interaction in such a way as to provide a sensitive test for the pseudo-scalar contribution $g_u^{(P)}$. Unfortunately total depolarization of the muon occurs in hydrogen²⁵ due to the cooperative influences of (1) a ground state hyperfine structure and (2) an atomic shuffling of the muon among protons prior to capture. Among certain complex nuclei of zero spin, on the other hand, there is evidence for a 17% retention of muon polarization²⁶. For such systems the neutron distribution takes the form²⁷

$$W(\theta_{nu}) = 1 + P_{\mu} P_n P_{\gamma} \tilde{\beta} \alpha \cos \theta_{nu} \quad (10)$$

where allowance is made by P_n for the isotropic boil off of low energy neutrons from the compound nucleus, and by P_{γ} for the associated background of γ -rays. The factor $\tilde{\beta}$ accounts for a lessening of neutron asymmetry due to direct neutron-residual nucleus interaction and parameter α for an intrinsic asymmetry which differs only a few percent from the hydrogen value α_H . So far neutron anisotropy has been investigated for spin zero S,

Mg, and Ca. Of considerable interest is the fact that when combined these results specify an average asymmetry value of $\langle \alpha \rangle = -0.92 \pm 0.19$, more than twice the expected value of $\alpha_{\text{theor.}} = -0.41$. Attempts to explain this discrepancy have mostly dealt with coupling constant adjustments, typical of which is the suggested fit $g_{\mu}^{(P)} / g_{\mu}^{(A)} \geq 20$.

This interpretation, however, is but one possibility. Another approach would be to question our knowledge of the various nuclear factors entering eq. (10). As the simplest of spin zero systems, He^4 appears the perfect target for an accurate estimate of factors P_n , P_γ , and $\tilde{\beta}$. All that's required experimentally is sufficient residual muon polarization P_μ , on the order of magnitude of other zero spin nuclei. This is determined most conveniently in an asymmetry measurement of the muon's decay angular distribution

$$W(\theta_{e\mu}) = 1 + P_\mu a_e \cos \theta_{e\mu} \quad (11)$$

where $a_e = -1/3$ in an integration over the electron energy spectrum and $\theta_{e\mu}$ is the angle between electron trajectory and muon spin axis. It is assumed in this decay distribution as well as in neutron distribution forms (9) and (10) that muon beam polarization is 100%, this in accordance with arguments given in Section III. Such an electron asymmetry experiment has already been attempted in liquid helium, providing uncertain evidence for strong depolarization, namely $P_\mu = 7.3 \pm 5.5\%$.²⁸ Too small a value could jeopardize both the He^4 muon capture asymmetry experiment and a recently proposed He^3 capture, triton recoil asymmetry measurement²⁹ for which additional depolarization is expected because of hyperfine coupling. For this reason we have undertaken a careful determination of muon

depolarization in liquid helium-4 employing the standard muon precession technique to observe asymmetry. Other spin zero targets included in this study are carbon and liquid argon.

Before proceeding to experimental detail it would appear the better course to identify as early as possible the various atomic mechanisms commonly held responsible for the depolarization of the muon. Consider a typical negative muon arriving with reduced energy and unaffected total polarization in the vicinity of a target atom. The first disruption of spin orientation occurs with the muon's atomic capture via an Auger process to a highly excited Bohr orbit of $n \sim 15$. From here the particle begins a rapid cascade to the K shell initially favoring loss of energy through further Auger events ($\Delta n \simeq 1$, $\Delta \ell = 1$) and subsequently completing the descent in a series of radiative dipole transitions ($\Delta n = \max$, $\Delta \ell = 1$). Along the way the muon's spin is randomly flipped under magnetic influences arising from the spin-orbit coupling of the muon and the shell motion of neighboring electrons. This latter influence is actually of consequence only for the ground state level where the muon must await decay or nuclear capture subject to the characteristic disappearance rate, $\Lambda_T = \Lambda_D + \Lambda_C$. Calculations^{30,31} for spin zero nuclei lend support to this description of depolarization activity, matching within experimental limits the results obtained by Ignatenko et al.,²⁶ namely: $P_\mu = 15 \pm 4\%$ for C, O, and S; $P_\mu = 19 \pm 7\%$ for Mg, Zn, Cd, and Pb. For nuclei of spin $I = \frac{1}{2}$, such as H¹ and He³, the hyperfine interaction $\vec{I} \cdot \vec{S}_\mu$ should cut the residual polarization figure yet another factor of two, assuming a statistical population of hyperfine levels.³²

Although depolarization is also observed for positive muons stopping in condensed matter,³³ the preceding interpretation cannot apply. Simply put, the positive particle is excluded from the vicinity of the repulsive nucleus, and so must either remain free, subject only to stray magnetic fields, or else undergo hyperfine depolarization in the formation of muonium ($\mu^+ e^-$). Clear evidence for the first of these possibilities is provided by the asymmetry measurement of Bardon³⁴ in bromoform for which depolarization is absent; equally clear is the example of muonium formation in argon gas where almost 100% formation occurs. For most other materials, however, the description must take a more complicated form.

III. EXPERIMENT

A. General Method

In order to measure the angular distribution of decay electrons for μ^- -mesons stopping in He^4 , we applied the technique of spin precession. Figure 1 shows schematically the experimental configuration employed for this purpose. Focal point for beam activity centered on target volume 3, a liquid helium scintillation counter of cylindrical design and average sensitive thickness $\bar{\xi} = 0.6 \text{ g/cm}^2$. Polarized muons were momentum selected and channeled through a 4" by 4" aperture in a concrete shielding wall, on line with monitor counters 1 and 2. Moderated in a copper absorber and further defined by a lead collimator of $2\frac{1}{2}$ " by $2\frac{1}{2}$ " aperture, a fraction of these were brought to rest in helium. Each satisfied the muon-stop counter requirement (1234). Subject to a uniform magnetic field the muon arrived in the ground state where it began to process, its spin confined to the plane of the diagram. This motion is described by the equation

$$\frac{d\bar{S}}{dt} = \bar{\omega} \times \bar{S} = \bar{\mu} \times \bar{B}$$

where $\bar{S}$ represents the expectation value for the muon spin variable, $\bar{\omega}$ is the precession frequency, and $\bar{\mu}$ is the muon magnetic moment of gyromagnetic ratio $g = 2$ given by the expression $g \frac{e}{2m_\mu c} \bar{S}$. The counter cone (56) stood in the plane of precession awaiting the muon-associated electron. Helmholtz coils encircled the counter array, cancelling a cyclotron fringing field of approximately 6 gauss. In this way we insured a constant frequency of 1.22 cycles/ μ sec for either sense of the precession field.

A time interval measurement was begun with each muon stop pulse. A gate was opened by (1234) and closed either by a (56) event, if available, or by an artificial pulse generated 8 μ sec after the muon stop. Scaled with a 20 Mc/sec oscillator, each time interval result was stored in the memory of a 256 channel analyzer. For muons decaying asymmetrically in a precession field the time spectrum of electrons, as observed with a fixed counter telescope and displayed by the pulse height analyzer, should be given by the expression

$$N(t) = N_0 e^{-\lambda_T t} [1 + A \cos(\omega t - \bar{\phi})] + B_0$$

where λ_T is the total muon disappearance rate, A the observed asymmetry coefficient, ω the precession frequency, $\bar{\phi}$ an observed phase angle, and B_0 the random background level. In terms of this function a least-squares fit to the data was made and the unknown parameters obtained. The following sections will examine this procedure and consider the results in corrected form.

B. Beam Selection

The availability of muon beams approaching 100% longitudinal polarization is one of the more familiar consequences of parity violation. Originating from the free decay of the pion according to the weak process $\pi^- \rightarrow \mu^- + \bar{\nu}$ ($\pi^+ \rightarrow \mu^+ + \nu$) the negative (positive) muon has been found to be a right- (left) handed particle in the pion rest system. Most experimental groups engaged in muon precession work have used those muons emitted forward along the beam direction since the pions can then be totally absorbed from the beam before the target. We chose instead muons emitted backward to the beam direction. The reason for doing so is suggested by

Fig. 2 which depicts monitor counting rate versus bending magnet current (or particle momentum). Muons of lowest momentum are well separated from the pion peak and clearly distinguished in terms of "plateau edge" for the large bending angle of 60° . The relative position of "plateau edge" to pion peak is kinematically consistent with such a description. Properly utilized a clean beam of backward polarized muons should be available.

Transforming this backward beam to the laboratory system reverses the helicity of the muon, providing left-handed μ^- and right-handed μ^+ at the target. This can be checked with the help of the phase angle $\bar{\phi}$ which by definition corresponds to the angle between beam polarization and the fixed decay telescope. This was done; results and the interpretation of the same are given in Section V.

A differential range curve of negative muons stopping in the helium counter is given by Fig. 3. As the thickness of copper absorber was varied the number of (1234) events was scaled for a fixed monitor count. Allowing for the additional moderation of the beam in the plastic counters 1 and 2, and the front wall of the target dewar, this stopping distribution shows that muons of 50 MeV were brought to rest in the helium.

The technique of beam stretching was employed to reduce the random background level associated with too high an instantaneous beam rate. In the course of the experiment this took two forms. The first of these saw service for the bulk of the run and involved a two target arrangement for the interception of each proton burst. A rotating target synchronized to the repetition rate of the cyclotron made a slow pass through the beam parked at the same radius. This required a time spanning

many orbits, and scattered protons evenly onto a fixed beryllium target which was located further down-stream, 1/2" below the midplane. Beam structure, as evidenced by the monitor counters, underwent a debunching which involved about 60% of the beam.

The second stretching device was developed by Dr. Arata Suzuki at this laboratory. It consisted of an auxiliary Dee structure positioned upstream from a fixed graphite target. As the beam expanded to pass within the sector, the machine RF was cut, leaving the protons just short of target radius. A noise modulated radio frequency signal was then applied to the device for the duration of the OFF signal. For protons of the proper phase this provided an acceleration to target radius. With careful adjustment, up to 70% of the beam reaching the experimental area appeared as stretched beam.

C. Target Assembly

1. Helium Counter

The feasibility of developing a liquid helium scintillation counter was demonstrated as recently as 1959.³⁶ Since that time the emphasis has remained upon development, with applications appearing few and far between³⁷. We undertook our own development program, believing that much in the way of further counter performance was possible. As reported in the Review of Scientific Instruments³⁸, this proved to be the case.

The experimental dewar was a modification of a standard model from Hoffman Cryogenics. It is shown schematically in Fig. 4 together with supporting vacuum network and equipment. The target was a particle-sensitive volume of $2\frac{1}{2}$ " diameter by 4" height centrally located in the tail section of a 1.0-liter capacity helium reservoir. Surrounding this reservoir across a vacuum gap, but removed well above the target tail, was a liquid nitrogen jacket. A removable heat shield of thin-walled construction extended liquid nitrogen cooling from this bath, down about the helium target. This design provided the necessary thermal insulation for liquid helium without introducing low Z nitrogen to the beam.

Scintillations occurring in the target volume were viewed from below by a photomultiplier through a 1/8 inch-thick sapphire window. It can be noted from Fig. 5, an enlargement of the area about the sensitive volume, that neither a light pipe nor a thermal window was used between the helium window and the phototube. Instead the tube was brought within 2 mm of the target window, and its temperature lowered to $\sim 150^{\circ}\text{K}$ through thermal contact with the copper nitrogen tail. A fine spun copper wool tightly wrapped in layers about the glass envelope of the phototube served as the conductive path. This arrangement benefited counter operation in three ways: (1) the light collection efficiency of the detector system was improved with the removal of intermediate optical devices, (2) the gain of the phototube increased 40% from room temperature level due to cooling (an unexpected bonus), and (3) radiant heat influx from below was sufficiently diminished to limit the helium loss rate to 150 cc/hr. The phototube capable of this cryogenic operation was an EMI 9558QA featuring tri-alkalide photosensitivity, venetian-blind dynode structure,

and a quartz seal at the photocathode. It stood on thermal insulating stilts of thin-walled stainless tubing, and operated in the vacuum common to the rest of the dewar. A vacuum seal led interdynode voltages through the bottom base plate from a resistor chain in the atmosphere below.

Access to the inside of the target cylinder was provided by an indium solder joint. Counter sensitivity was limited to a volume defined by a removable liner-lid combination of 2 mil thick, high polish aluminum, the interior of which received a spraying of 5 mg/cm^2 of Tygon-ZnO reflector paint followed by an evaporation coating of $100 \text{ } \mu\text{g/cm}^2$ of p, p' diphenylstilbene (DPS). This liner matched the inner contour of the helium tail and terminated at the sapphire window where $25 \text{ } \mu\text{g/cm}^2$ of DPS conditioned the inner surface. Use of such an organic fluorescent layer was essential to the counter's operation. Calibration tests had shown DPS unmatched by other materials in the task of waveshifting the hard ultraviolet of liquid helium emission to a frequency compatible with photomultiplier response. For the liner in question a pulse height nonuniformity of 34% was observed along the center axis from the window to the lid of the sensitive volume, a factor actually of little significance to the role of the counter in this experiment. The source probe used for this measurement was a 5.3 MeV Po^{210} alpha emitter. Rated at 100 cps, it was withdrawn to the top of the liner for the duration of the precession run.

So far only liquid helium has been mentioned as target material. Other materials used in the course of the experiment were: 1) liquid argon, 2) a cylinder of graphite, and 3) a thin graphite slab. The first of these required no modification of counter arrangement since argon, like helium, is a noble liquid scintillant. The other two, however,

rendered the target area insensitive and necessitated the operation of a 3' plastic counter just before the dewar wall. A muon stop was then spelled (1234). For calibration purposes the graphite cylinder was shaped to fill the empty liner, and the thin slab to match the helium in equivalent stopping thickness.

2. Dewar Support Apparatus

As depicted by Fig. 4 the dewar support system is essentially self-explanatory. Features worthy of comment, however, include the following:

1. Transfer of liquid N^2 was automatically accomplished with the aid of High-Low level sensing resistors.

2. Thermocouple readings were recorded sequentially by a Leeds and Northrup potentiometer from three locations within the dewar. This provided a continuous monitor of temperature at the helium window, the phototube seal, and the nitrogen tail.

3. During the course of the experiment the liquid helium level was probed from above with a thermal sensitive 1/10 watt, 56 Ω Allen-Bradley resistor. The loss rate determined in this manner was such that a transfer of helium was typically required every 10 hours.

4. A brief portion of the run involved a study of decay asymmetry from superfluid helium. A high-speed mechanical pump (Welch Model 1397) coupled with small impedance to the throat of the helium reservoir was used to effect this condition, while a Wallace and Tiernan precision dial manometer was used to monitor helium II temperatures.

3. Precession Solenoid

We sought to locate a uniform precession field over the entire volume of sensitive helium. A solenoid with an end correction layer was wound on the outer skin of the target dewar as shown in Fig. 5. The axial field for this configuration is given by the expression

$$B_z = \frac{\mu_0 I N}{2l} [\cos \theta_2 - \cos \theta_1] + \frac{\mu_0 I' N'}{2h} [\cos \varphi_2 - \cos \varphi_1 + \cos \varphi_2' - \cos \varphi_1']$$

with lengths and angles illustrated in Fig. 6. Taking $I' = 2I$ this should provide a field uniform to within 1/2% along the axis for 6 cm above and 6 cm below the target midplane. Measurements of field uniformity were conducted at room temperature using as magnetic probe a small flip coil of 4 mm diameter. Figure 6 is a plot of this center axis measurement showing the expected uniformity to within $\pm 1\%$, a limit imposed by galvanometer deflection sensitivity. Also of interest in this figure is the extension of large magnetic field strength into the region occupied by the phototube. This has caused tube gain to drop a factor of 20 from zero field conditions as determined by Po-alpha pulse height measurements in helium. Nevertheless, sufficient pulse height was available for much-stop counter operation.

A more realistic check of field homogeneity was made with the target tail and nitrogen tail properly positioned within the solenoid wall. This should not differ from corresponding measurements in Fig. 6 unless magnetic materials were introduced with the helium tail. Figure 7 shows the field pattern recorded within the target tail both on and off the axis. As suspected, large nonuniformity appeared in the neighborhood of the sapphire window seal (Kovar metal). By adjusting the vertical position of

the lead collimator in front of the dewar we attempted to confine muons to the region of uniformity (again $\pm 1\%$) defined in Fig. 7. This precaution has had the added advantage of preventing muons from stopping in the low Z oxygen of the synthetic sapphire (Al_2O_3).

The preceding test for nonuniformity cannot, of course, be considered an absolute calibration of the precession field. For a measure of $\langle B \rangle$ over the target volume and hence the value of ω to be used in the asymmetry analysis, we must look ahead to the experimental results. Here the large decay asymmetry from μ^+ mesons stopping in graphite allowed an accurate best-fit for frequency. In general there was evidence for $\omega = (7.760 \pm .034)$ radians/ μsec , a value consistent with the calculated field strength $B_z = 90.3$ gauss. Further comment upon these measurements is given in Section V.

D. Counter Geometry

Plastic scintillation counters used to define the muon stop and decay electron typically consisted of commercial scintillant viewed across a Lucite light pipe by an RCA-6810A photomultiplier tube. All were of square geometry with dimensions easily estimated from Fig. 1. Since we are essentially interested in an angular distribution, electron counter size plays a role in the interpretation of experimental asymmetry. This is easily appreciated from the following argument:

Consider, in connection with the diagram below, those muons which decay from target origin, their associated electrons passing through telescope element dx at an angle ϕ to beam polarization axis z .

For these electrons we can expect a precession distribution of the form

$$\frac{dN}{d\varphi} = N_0' e^{-\lambda_T t} [1 + a \cos (\omega t - \varphi)] + B_0'$$

where a denotes the asymmetry parameter for an ideal geometry, and N_0' and B_0' specify amplitude and background respectively. In order to account for the finite width of the telescope we simply integrate this expression over the full 2ξ angular spread, neglecting the small effect of a finite helium volume. This results in the expected distribution form (Section IIIA.) where the measured coefficient $A = \frac{\sin \xi}{\xi}$ and the phase angle $\bar{\varphi}$ is specified in the diagram above as an average angle. For our geometry counters 5 and 6 define the same solid angle at the target. Thus with $\xi = 34.6^\circ$ the parameter a is modified to the extent that $\frac{\sin \xi}{\xi} = 0.94$. In terms of computer calculations this factor is automatically folded out.

By beginning with the distribution form

$$N_0 e^{-\lambda_T t} [1 + A \cos (\omega t - \bar{\varphi})] + B_0$$

we have made a number of assumptions concerning the experimental conditions responsible for the observed (μ -e) time spectrum. This matter, while not appropriate here, is treated at length in Appendix A. An understanding of time interval precautions will be assumed.

E. Electronics

Previous studies of μ^- decay asymmetry have for the most part relied upon a fixed gate for the electron detector. Delayed relative to the muon stop pulse this gate was chosen either to sample the sinusoidal modulation as a function of precession field or to simply extract asymmetry from a ratio of peak-to-valley readings. The digitron technique used in the present experiment measured time interval by counting the number of cycles from a crystal oscillator required to fill a (μ -e) controlled timing gate. In this way a complete time spectrum of the process was simultaneously secured, including information on beam polarization axis, precession frequency, and contaminating background both random and time dependent. This timing procedure has a well developed history at this laboratory, appearing in studies of the muon lifetime in carbon (NYO 9279) and in light isotopes (NYO 9286).

Circuit operation will be covered in terms of the following classification: 1. Coincidence Circuits, 2. Confusion Elimination Circuitry, and 3. Basic Digitron System.

1. Coincidence Circuits

The block diagram for time interval circuitry is given by Fig. 9. Not shown but formed to the left of input positions μ and e were the preliminary coincidences: muon (1234) and electron (56). Both were products of tunnel diode circuitry, the first using discriminator modules developed by Whetstone and Kounosu³⁹ followed by a Fanzini-type coincidence circuit⁴⁰. For the electron coincidence a "first and last pulse detector", designed by Dr. Arata Suzuki at this laboratory, was used.

As shown in Fig. 8a and Fig. 8b this circuit is capable of fivefold operation and determines coincidence via a time-to-pulse height conversion.

2. Confusion Elimination Circuitry

Figure 9 shows the μ pulse executing a number of different operations on route as "open" pulse to the input of the fast gate circuit. Some of these preliminary measures were purely precautionary, aimed at eliminating ambiguity among the events recorded, and so will be considered here.

As an initial step only those muons from the stretched portion of the beam were accepted for time interval study. To this end a time delay generator was driven by the cyclotron timing pulse, adjusted in gate width and delay to overlap the debunched beam, and placed in coincidence with the incoming μ^- pulse. For the μ^+ beam, down in intensity by a factor of 4, this coincidence was removed in favor of a direct input.

Satisfying this gating condition (if applied) the muon pulse was shaped by a fast discriminator and directed to a deadtime circuit. Here a univibrator of 2.3 msec recovery time was triggered, inactivating the system to subsequent muons from the same beam pulse. A separate dead time circuit passed the e pulse to the exclusion of later (56) activity. In this case, however, the univibrator recovered in 8 μ sec (the maximum time interval analyzed). The delay of 0.4 μ sec preceding the e circuit permitted a study of "negative time" activity.

A third precautionary step was taken, this to eliminate a possible "second muon" problem. In effect this is present whenever more

than one muon per beam pulse comes to rest in the target. This enhances the probability for observing an electron, thereby distorting the time spectrum otherwise expected. For this reason we rejected all such events with the aid of the "disable" gate generator. Inputs to this circuit consisted of the muon pulses and a "stop gate" 8 μ sec wide, generated by the first muon into the pulse forming circuit. A coincidence formed at the input stage of the "disable" circuit led to the generation of a "disable gate" (15 volt negative, 140 μ sec wide). This in turn served to prevent the add -1 pulse from reaching the memory of the analyzer. Circuit diagrams of the "disable" gate generator and the pulse forming circuit appear in NYO - 9286 and NYO - 9279, respectively.

3. Basic Digitron System

The pulse forming circuit, working with the muon input, sent a univibrator signal as "open" pulse to the fast gate circuit. This univibrator in turn triggered a second univibrator, with a 8 μ sec recovery period, the trailing edge of which served both to generate an artificial "close" pulse and to activate the memory unit of the analyzer. The artificial pulse, coming 8 μ sec after the muon pulse, defined the maximum (μ -e) interval; the signal activating the analyzer memory started the 2 Mc advance of the internal address scaler. Completing this pulse forming operation the output of the e dead time circuit was shaped on a univibrator and directed as "close pulse" to the fast gate circuit.

The fast gate circuit, a copy of the Hewlett-Packard (Model 5275A) Time Interval Counter, is shown together with the mixer circuit in Fig. 10.

The "open" and "close" pulse defined a gate generated by a fast flip-flop. This was coupled with the signal from a free running 20 Mc/sec oscillator to form the inputs to a fast coincidence circuit. The output, a gated train of 20 Mc/sec pulses, entered the mixer section of the circuit ahead of a 2 Mc/sec train of pulses produced by the analyzer and shaped on a fast discriminator. The mixed train of pulses was then sent to a fast scaler in loop with the pulse height analyzer. The scaler was a Hewlett-Packard 100 Mc/sec Decade counter, modified to give an output every 200 counts. A detailed description of this scaler and the fast gate circuit for the 100 Mc condition is given by Filippas, CIT Thesis, Nov. 63 (unpublished). The overflow pulse from the fast scaler was directed to the scaler of the analyzer halting its 2 Mc/sec advance which had begun with the "start 2 Mc" pulse. Barring a "disable" gate, a count was then added to the "stop" channel. The order is thus established: a particular (μ -e) event passes N 20 Mc/sec pulses to the mixer circuit on route to the fast scale of 200. A complement of $(200-N)$ 2 Mc/sec pulses then travels the "digitron loop", accounting for the addition of a count in channel $(200-N)$.

The following tests were periodically made upon this system:

1. The 20 Mc/sec oscillator was checked for frequency drift. No shift exceeded 4 parts per 20,000.
2. A test for linearity proceeded as follows: uncorrelated events were fed as input pulses to the digitron. A high degree of linearity is reflected in a vanishing value for slope b of the distribution $N_0 (1 + bn)$, where N_0 is the amplitude and n is the channel number.

A list of such randoms checks is given below:

N_0	b ($\times 10^3$)	Channel Interval	Chi Square
6625	$-.43 \pm .45$	153	154
9134	$-.65 \pm .37$		146
11119	$+.10 \pm .36$		167
29275	$-.41 \pm .19$		120

A small preference for a negative slope may be in evidence here, but its effect upon a decay curve would be negligible.

F. Experimental Routine

As mentioned earlier the experiment involved three different target materials, carbon taking two forms for purposes of calibration. Both the μ^+ and the μ^- beam were used in the study of each. As a precaution against slow variations in the system the experiment was divided into a sequence of "runs", each lasting from 4 to 7 hours. "Run" time was determined either by counting rate or, as in the case of helium, by a need to replenish the liquid reservoir. The solenoid precession field was reversed at the conclusion of each "run", alternating from a condition of B (up) to B (down). This procedure divided the data naturally for purposes of analysis as the following section will explain.

A typical helium "run" saw the following pattern of operation:

1. Current leads to both the precession solenoid and the compensating coil were reversed and the currents checked for stability at values of 2.5 amp. and 5.0 amp. respectively.

2. The helium reservoir level was probed for an estimate of available running time. Should a transfer of helium be necessary, a 100 liter storage dewar (1 week's supply) was brought forward.

3. Data was accepted and stored in the memory unit of the analyzer. Tapes were printed at the conclusion of the run; time zero checked for channel position.

4. A note was made of running time and the following quantities scaled:

- a. number of monitor counts
- b. number of muon stops
- c. number of "stretched" muon stops, and
- d. number of (56) events

Table I gives these figures for an hour's study of the negative muon in each material and the positive muon in graphite. Included is an estimate taken from the analyzer of the total 8 μ sec count excluding artificial events. This will serve a purpose in Appendix A where the influence of random background is considered. Table II sections the experiment according to target, beam, precession field, and total running time. An outline of this type stresses the attention given to each topic. In particular it shows that 56% of the beam time was devoted to the primary study of negative muon depolarization in liquid helium.

Besides the periodic "randoms" test described in the preceding subsection, care was taken to exclude any run for which a

shift in the time zero channel had occurred. Unlike a simple exponential for which time-shifted runs may be combined, an exponential with sinusoidal modulation requires a fixed specification of time zero. Fluctuations when present were found not to exceed $1/10$ of a channel.

IV. ANALYSIS OF DATA

Figure 11 clearly incorporates all the features of the time distribution function

$$N_0 e^{-\lambda_T t} [1 + A \cos (\omega t - \bar{\varphi})] + B_0$$

Taken from a u^+ run with graphite cylinder it qualitatively exhibits in semilog form a large number of precession cycles, modulating over an 8 μ sec interval a single exponential of long lifetime. The B_0 background level is specified by 8 channels of "negative" time at 0.05 μ sec/channel, while time zero is defined by a large one or two channel buildup of "scatter" events, mostly beam particles which, satisfying (1234), promptly scatter into (56). Such was the general form of the u^+ data. With a characteristic shift to smaller asymmetry and shorter lifetime, the u^- data was found to agree in substance (cf. fig. 12). Here however we note the possible addition of one or more short-lived decay components arising from muons which have stopped in the dewar walls of copper and stainless steel.

Appendix A has been set aside to consider in detail the question of the expected distribution shape. Starting from the various probabilities known to control such a process, an overall time spectrum is developed and shown to match the above function.

Appendix B then considers the various techniques of least-squares programming which were used to extract from the data the parameters of amplitude N_0 , disappearance rate λ_T , asymmetry a , frequency ω , and spin phase $\bar{\varphi}$. Basically three types of programs were involved in this effort. Briefly described they were programs of lifetime, asymmetry, and frequency.

A. Lifetime Program

In cases where the negative muon decayed from helium and argon, and the coefficient A appeared to vanish, a lifetime program of the form $N_0 e^{-\lambda_T t} + B_0$ was run to better estimate the disappearance rate λ_T . Other applications of this program calibrated the timing system in a study of the μ^+ lifetime. Here as elsewhere in our work the background B_0 was provided by an average over the "negative" time channels.

B. Asymmetry Program

The second program opened with a foreknowledge of the parameters λ_T , ω , and B_0 . A least-squares fit was made for amplitude N_0 and asymmetry a while phase $\bar{\phi}$, starting from an initial guess, was looked for in an iterative process. Where strong μ^- depolarization made this iterative method unfeasible, a fixed value for $\bar{\phi}$ was selected from the calibration study of graphite. All asymmetry values to be given in the following section are taken from this analysis.

C. Frequency Program

The third study calibrated the angular frequency which in program (B) had been calculated from the expression $\omega = g \frac{e B}{2m_\mu c}$. For this our information concerning μ^+ decays from graphite with its pronounced asymmetry was most helpful. A best fit of the data here used the value for phase determined in program (B). Amplitude N_0 and asymmetry a were looked for while the value of ω was hunted via an iterative loop.

The question of time dependent background from short-lived μ^- decays in copper and iron is not lightly brushed aside for this

experiment, especially for the case of a thin helium sample. Here the expected lifetime in helium is 2.20 μsec , while in copper $\tau_{\text{Cu}} = 0.16 \mu\text{sec}$ and in iron $\tau_{\text{Fe}} = 0.201 \mu\text{sec}$. For our dewar dimensions an estimate of stopping power places only 30% of the muons in helium, with 27% and 43% respectively coming to rest in backwall stainless steel and copper. Such a stopping distribution is also supported by the data in a simple estimate of decay curve amplitudes at time $t = 0$. This would indicate that of the decays observed in channel 20 ($t = 1.0 \mu\text{sec}$) only 1% result from the high Z walls. Programs were run with this in mind. Thus the variation of the asymmetry parameter a for all targets was studied as a function of the "initial" channel I5 for a given "end" channel I6. In fact all programs whether they be of asymmetry, lifetime, or frequency incorporated this method of selectively altering the data interval (I5, I6). More will be said on this matter in connection with the results of Section V.

In all previous gated telescope experiments the reversal of precession field has been basic to the formation of a peak-to-valley ratio, and hence to the observation of asymmetry. For us, however, periodic field reversal amounted merely to a test for consistency, especially where the phase parameter $\bar{\phi}$ was concerned. The two types of data, namely B(up) and B(down), are therefore treated independently and the results of analysis so classified.

V. RESULTS AND DISCUSSION

Tables III and IV itemize results which for the most part are peripheral to the main question of negative muon decay anisotropy. Thus calibration results involving the μ^+ beam and concerning the parameters of angular frequency ω and phase angle $\bar{\phi}$ are given in Table III, while lifetime values measured for μ^- (helium) and μ^+ (argon) are listed in Table IV. Table V then summarizes the asymmetry results obtained for μ^- decay in carbon, liquid argon, and liquid helium.

A. Measurement of Frequency

As shown in Table (IIIA) the frequency was studied during the course of the experiment under various conditions of μ^+ precession. The first four measurements correspond to the usual strong field condition and average with proper weighting to a value of $(7.760 \pm .034)$ radians/ μ sec. This exceeds by little more than 1% the expected value of 7.677 radians/ μ sec as calculated for a free muon in a field of 90.3 gauss. Even better agreement is shown for the reduced field measurements. When coupled with flip coil evidence for field homogeneity this argues for a well defined precession cycle. Here as elsewhere in the Tables the data interval analyzed and the computed chi square accompany each result. As described in Appendix B this provides an estimate of the goodness of fit.

B. Specification of Phase Angle

In Section (IIIB) we talked in terms of a "backward" muon beam. Adapted to our experimental technique this should result in the following

values for phase: $\bar{\varphi}(\mu^+) = 90^\circ$ and $\bar{\varphi}(\mu^-) = -90^\circ$, the first corresponding to a downstream orientation of the muon spin and the second corresponding to an upstream orientation. Implicit in this convention is the further choice of sign: ω positive for B(up) and ω negative for B(down). As it happened, however, deviations from such a pattern were much in evidence.

For the μ^+ runs of Table (IIIB) large asymmetry allowed a careful inspection of the phase parameter. Despite a change in targets and a reduction in the precession frequency all best fit estimates tend to cluster about a value of 39° , fluctuations in either direction corresponding more or less to one channel of strong field precession. While at variance with a longitudinal polarization this result is not considered kinematically improbable.

Phase estimates for the μ^- beam are listed in Table V. The vanishing asymmetry for μ^- decay in argon and helium restricted this analysis to the calibration samples of carbon, and hence to four separate runs. As recorded the first three of these would specify an improbable downstream orientation for μ^- spin while the last run favors the expected upstream direction. In terms of the analysis this amounts to a discrepancy in sign for $\bar{\varphi}$, a rather suspicious condition considering that a mistake in the sense of precession would send $-\bar{\varphi} \rightarrow +\bar{\varphi}$. Indeed the experimental log records an uncertainty as to field direction for these particular runs. In the light of such indirect evidence it is felt that field reversal occurred and that in fact upstream polarization was observed for the negative muon. The μ^- data for helium and argon is analyzed accordingly.

C. Measurement of Lifetime

The μ^+ mean life $\tau(\mu^+)$ provides an experimental standard of high precision. With this in mind we calibrated our system for positive muons which stopped in the argon counter, obtaining a value of $(2.196 \pm .013) \text{ usec}$. This is consistent with the most recent value by Lundy⁴¹ $(2.203 \pm 0.004 \text{ usec})$. The measurement was accomplished under a strong field condition for which muonium precession was expected. This of course should eliminate any contribution to the lifetime due to asymmetry.

The μ^- mean life in argon has not been previously reported. Table II records our measurements. The latter value of $(0.571 \pm .004) \text{ usec}$ pertains to data combined from runs of zero field, B (up), and B (down). Again no contribution to the lifetime is expected because of precessing asymmetry. The value obtained is consistent with the low Z lifetime work of Sens,⁴² fitting naturally between his values for Cl and K. Referred to the Primakoff plot our result would place somewhat high on the scale of $\Lambda_{\text{CAP}} / \langle \rho \rangle$.

Mean life values for the negative muon in helium also appear in Table IV where they are listed according to a sixfold separation of the data. In the order given they correspond to the six asymmetry results of Table V. Weighting these values we obtain a mean of $(2.120 \pm .008) \text{ usec}$, a figure far below the expected helium mean life of 2.20 usec . The difference is attributed to the presence of low Z, time dependent background and thus can serve the purpose of estimating the same.

We begin by listing the various low Z materials in and about the helium target. Essentially there were two, oxygen ($\tau_u = 1.80 \text{ } \mu\text{sec}$) and aluminum ($\tau_u = 0.88 \text{ } \mu\text{sec}$). Against 0.6 g/cm^2 of helium we estimate in equivalent helium the following contributions: (1) from oxygen - 0.004 g/cm^2 (waverifter-reflector layer); 0.060 g/cm^2 (solenoid insulation), and (2) from aluminum - 0.011 g/cm^2 (liner wall). Although shielded by a lead collimator (cf Fig. 7) the sapphire (Al_2O_3) window cannot be overlooked as yet another source of long life, zero asymmetry background. As previously noted in Section IV and as shown by Fig. 13 (representing half the helium data), high Z decays have all but disappeared for $t \geq 1 \text{ } \mu\text{sec}$. This means that only decays from helium, oxygen, and aluminum will persist for longer times. With this in mind the helium data was fitted in a least-squares analysis to the distribution form,

$$N_{\text{He}}(t_i) e^{-(t-t_i)/\tau_{\text{He}}} + N_{\text{O}}(t_i) e^{-(t-t_i)/\tau_{\text{O}}} + N_{\text{Al}}(t_i) e^{-(t-t_i)/\tau_{\text{Al}}} + B_0$$

where t_i is the initial channel of the interval under study ($t_i > 1 \text{ } \mu\text{sec}$), and N_{He} , N_{O} , N_{Al} are the amplitudes to be fitted for time t_i . This was repeated for a number of different t_i , each calculation extrapolating amplitudes back to time zero for a relative estimate of the muon stopping distribution in He, O, and Al. The program was surprisingly successful, giving consistently the following results: $N_{\text{He}}(0) (81 \pm 4)\%$, $N_{\text{O}}(0) - (4 \pm 4)\%$, and $N_{\text{Al}}(0) - (15 \pm 4)\%$. This is not at variance with the earlier listing of low Z materials if the sapphire window is allowed to play a role in stopping muons. At $t_{15} = 1.25 \text{ } \mu\text{sec}$ where the asymmetry analysis begins for helium the relative amplitudes have become:

$N_{\text{He}} = (89 \pm 4)\%$, $N_{\text{O}} = (4 \pm 4)\%$, and $N_{\text{Al}} = (7 \pm 2)\%$. Rather than calculate the effect of this background upon asymmetry (thus compounding estimates), we carried out the carbon calibrations considered below.

D. μ^- Decay Asymmetry in C^{12} , He^4 , and A^{40}

1. Carbon Measurements

Carbon was included among the targets as a control sample, its influence upon μ^- decay anisotropy well known from calculation and experiment alike. Two target types were employed, one a cylinder of large stopping power (6.9 g/cm^2) providing a condition favorable to the observation of asymmetry, the other a thin sheet whose stopping power (0.6 g/cm^2) was matched to that of the helium volume. Table V summarizes the results corrected for both telescope geometry and the energy dependence of $(\mu-e)$ asymmetry. Weighting runs of opposite precession we obtain for the cylinder case an average asymmetry coefficient of $a = -0.052 \pm 0.004$. Here a reduction factor of 1.105 was used to bring observation ($E_e \geq 18 \text{ MeV}$) in line with the total electron energy spectrum. This shows excellent agreement with the calibration value of Prepost²⁸ (-0.055 ± 0.005), but exceeds in absolute value the carbon measurement of Ignatenko⁴³ (-0.045 ± 0.005). Based on this agreement we conclude that our approach to the problem of μ^- decay asymmetry is fundamentally sound. This measurement, matching or exceeding in magnitude others previously reported, argues as well for a high degree of beam polarization. Normally this is taken to be 100% based upon μ^+ precession evidence.

Noting the variation of data interval in Table V it might be asked: what effect does data interval have upon asymmetry value?

This question was put to all the data in a detailed study of parameter a . Fig. 14, an analysis plot of asymmetry versus the initial channel 15 for fixed final channel 16, gives the answer for the graphite run of Fig. 12. Within the errors specified asymmetry is shown to be independent of starting channel and, to a lesser extent, of end channel. The only exception to this occurs for $15 < 10$ where high Z decays contaminate the time spectrum and distort the calculation of chi square.

In an attempt to simulate the helium condition the thin graphite target was introduced to the μ^- beam, resulting in an average coefficient of $a = -0.040 \pm 0.010$. (Here as well as for helium the electron energy correction was negligible.) This figure is encouraging even though it falls two standard deviations below that of the carbon cylinder. It shows, unfortunately with poor definition, that large asymmetry can be observed with a "helium-equivalent" sample of carbon. Actually this serves more as a lower limit than a comparison since the carbon arrangement lacked the counter sensitivity of the helium volume. How much of a lower limit remains uncertain, however, a check on decay background showing a low Z stopping distribution subject to wide interpretation: $N_C(0) = (73 \pm 21)\%$, $N_O(0) = (10 \pm 10)\%$, and $N_{Al}(0) = (17 \pm 17)\%$.

2. Helium Measurements

Results for liquid helium appear in Table V in the form of six independent measurements, each corresponding to a grouping of seven "runs". Without exception they specify decay asymmetry consistent with strong, if not total, depolarization of the negative muon. Combining these measurements we obtain for the mean and its standard error the following: $a = +0.001 \pm 0.004$. No allowance has been made here for

the possible suppression of asymmetry due to a time dependent background. On the basis of the low Z amplitude study previously discussed this should not alter the above value beyond its statistical limit. Despite this systematic effect, however, the conclusion holds firm: the μ^- -meson in liquid helium is subject to strong depolarization, exceeding that reported for all other materials with zero nuclear magnetic moment. This tends to confirm the preliminary measurement of Prepost²⁸ (-0.024 ± 0.018), and shows in fact a preference for a lower assignment.

In Section II we briefly described the principal mechanisms involved in the depolarization of the negative muon. For materials of zero nuclear spin these include: (1) atomic capture, (2) fine structure influence in cascade, and (3) possibly muon-electron hyperfine coupling at the K level. Calculations by Mann and Rose,³¹ and Dzhrbashyan³⁰ have shown that (1) and (2) alone are sufficient to account for previous experimental evidence, typical of which is carbon with a ~ -0.05 . Hyperfine effect (3) would cut the asymmetry estimate yet another factor of two and so is considered inoperative due to a net compensation of the magnetic moment of the electron shell. Our result for liquid helium requires of course a closer look at such additional mechanisms.

In helium where atomic capture and a subsequent Auger transition account for the ejection of both electrons the question of a muon-shell electron interaction should not apply. (A search²⁸ at the electron's precession frequency has been made which tends to support this contention.) Energetically unable to avail itself of a neighbor's electron, the ionic system $(\text{He } \mu^-)^+$ will remain metastable with a lifetime characteristic of the muon. For such a free ionic state depolarization is not expected

to exceed the limit set by mechanisms (1) and (2) previously considered. For the stable complex ion $(\text{He } \mu^-)^+ \text{He}$, however, the process of molecular formation or recombination will lead to the rotational influence of excited molecular states. Depending on the lifetime of such excitations this mechanism could lead to substantial depolarization, or none at all. Hughes,⁴⁴ in a study of this system, has noted a short thermalization time. This would favor the latter possibility and thus eliminate the molecular system $(\text{He } \mu^-)^+ \text{He}$ as a source of additional depolarization.

Actually experimental evidence would tend to favor a more complex ionic structure in liquid helium. Studies of ionic mobility in the liquid have lead Atkin⁴⁵ and independently Careri et al.⁴⁶ to propose the formation of large clusters He_n^+ where $n \sim 50$. In these studies the ions resulted from α -particle ionization of the liquid. In our case the analogous cluster would be $(\text{He } \mu^-)^+ \text{He}_n$. What this means in terms of recombination and subsequent thermalization time is not known. One possibility is that such a cluster will form over a period spanning the muon lifetime, thereby accounting for strong molecular depolarization. Lacking a quantitative estimate of this system, however, little more can be said on this matter.

Toward the end of the helium run we devoted a short time to the study of muon depolarization in liquid helium II, this in the simple hope of finding a helium phase for which muon polarization is large. The final helium result of Table V provides the answer, showing for asymmetry, $a = 0.000 \pm 0.009$ in complete agreement with the first five results from normal helium. For this work the counter operated at

1.5°K. This study was prompted by an earlier observation of phase-dependent pulse height phenomena.⁴⁷ There it was found under a condition of high resolution (9% FWHM) that a sudden drop in Po-alpha pulse height occurred at the λ -point. As shown in Fig. 15 this effect is quite pronounced, continuing with excellent definition to our present limit of 1.35°K. An attempt to relate this to the sudden rise in ionic mobility observed below the λ -point⁴⁸ has been made. For muon depolarization, however, no connection was anticipated.

3. Argon Measurements

Fig. 16 displays a typical precession curve taken with liquid argon. Corrected results based upon three such studies are given in Table V, each corresponding to a different field condition. Combining values for B (up) and B (down) we obtain an average asymmetry of $a = -0.005 \pm 0.006$. This is consistent with the B (zero) null result, $a = -0.006 \pm 0.013$, which also appears in the Table. For these runs conditions favored a sensitive test of (μ -e) asymmetry, with large stopping power (6.6 g/cm^2) available for argon counter operation and the analysis carried out over 1.3 precession cycles.

Once again it seems we are faced with a total depolarization of the μ^- -meson in a substance with zero spin nuclei. To what extent this can be attributed to the noble liquid structure is not known. Possible interpretation would appear to depend upon either the electron hyperfine argument or the case for ionic-molecular structure. In the first instance the closed outer subshell of the argon atom, disrupted by Auger transitions, takes on an effective magnetic moment. Barring a compensation of this moment by the neighboring argon atoms, the

resulting hyperfine structure for the K orbit muon could provide an additional depolarization of 59% under our muon precession condition. In itself this would not account for the total depolarization observed. In fact according to a classification of target materials by Ignatenko⁴³ this influence is not even expected for argon.

The molecular mechanism, if anything, is more uncertain than the hyperfine influence. For simplicity we shall consider the chlorine-like ion $(A u^-)^+$. Granting this system metastable status on the basis of an ionization argument, the question then becomes one of formation and stability for the molecular structure $(A u^-)^+A$. Should it readily form, the argument for rotational level depolarization of the muon comes into play. Sidestepping the matter of thermalization time which might be expected to quench this mechanism altogether, it soon becomes evident that the associated magnetic fields (approximately 2 gauss per rotational quantum number) are much too small to be of any consequence.

On this perplexing note we have exhausted the normal channels of depolarization open to the muon at a spin zero site. Where the answer to strong depolarization lies we cannot say. If further work is to be done there should be some attention directed to the class of noble liquids. In particular the question of molecular depolarization must be tested experimentally in terms of an applied "quenching" field.

VI. TABLES

- I. Hourly Rates for Scalers and Analyzer
- II. Beam Time per Target
- III. μ^+ Precession Parameters
- IV. Lifetime Results
- V. μ^- Decay Asymmetry Results

TABLE I. - Hourly Rates for Scalers and Analyzer

Beam (Target)	Monitor (12)	Stops (123L)	Stretched Stops S(123L)	Telescope (56)	Analyzer Count
	$\times 10^6$	$\times 10^6$	$\times 10^6$	$\times 10^6$	
μ^- (Helium)	5.30	0.19	0.27	0.28	5850
μ^- (Argon)	5.55	0.90	0.62	0.19	10300
μ^- (Graphite Cylinder)	5.60	1.03	0.72	0.26	20700
μ^- (Graphite Sheet)	5.15	0.84	0.51	0.18	9300
μ^+ (Graphite Cylinder)	0.93	0.16		0.19	8450

TABLE II. - Beam Time per Target

Beam (Target)	Precession Field	Total Running Time (Hours)
μ^- (Helium)	B (up)	90
	B (down)	118
μ^- (Argon)	B (zero)	6
	B (up)	7
	B (down)	6
μ^- (Graphite) Cylinder	B (up)	4
	B (down)	6
μ^- (Graphite) Sheet	B (down)	28
μ^+ (Helium)	B (up)	22
	B (down)	31
μ^+ (Argon)	B (up)	13
μ^+ (Graphite) Cylinder	B (up)	13
	1/2 B (up)	3
	B (down)	5
	1/2 B (down)	5
μ^+ (Graphite) Sheet	B (up)	10
	B (down)	4

TABLE III. - μ^+ Precession ParametersA. Frequency

Precession Field	Data Interval (I5, I6)	Chi Square	ω Radians Per μ sec	ω_0 (Calculated)
(Graphite Target)				
B (up)	5,60	60 51	$7.726 \pm .011$ $7.819 \pm .060$	7.677
B (down)		61 63	$7.775 \pm .037$ $7.761 \pm .054$	
1/2 B (up)	5,100	98	$3.871 \pm .030$	3.839
1/2 B (down)		93	$3.813 \pm .030$	

B. Phase and Asymmetry

Precession Field	Data Interval (I5, I6)	Chi Square	Phase $\bar{\phi}$ (Degrees)	Asymmetry a
(Graphite Target)				
B (up)	2,60	61	17 ± 5	$0.15 \pm .01$
B (down)		67	57 ± 4	$0.16 \pm .01$
1/2 B (up)	2,100	105	29 ± 4	$0.18 \pm .01$
1/2 B (down)	3,100	93	47 ± 3	$0.17 \pm .01$
(Helium Target)				
B (up)	4,10	32	18 ± 7	$0.08 \pm .01$
B (down)		42	36 ± 6	$0.08 \pm .01$

TABLE IV. - Lifetime Results

Precession Field	Data Interval (I5, I6)	Chi Square	Mean Life τ (μ sec)	Amplitude N(I5)
<u>μ^-(Helium)</u>				
B(up)	35,150	119	$2.115 \pm .015$	1375
		124	$2.097 \pm .015$	1411
		130	$2.150 \pm .016$	1457
B(down)		118	$2.100 \pm .013$	1950
		120	$2.123 \pm .014$	1700
		105	$2.155 \pm .019$	748
<u>μ^-(Argon)</u>				
B(Zero)	8,30	21	$0.566 \pm .008$	1688
B(Mixed)		14	$0.571 \pm .004$	6361
<u>μ^+(Argon)</u>				
B(up)	10,150	148	$2.196 \pm .013$	1429

TABLE V. - μ^- Decay Asymmetry Results

Precession Field	Data Interval (I5,I6)	Chi Square	Asymmetry a	Amplitude N _o	Phase $\bar{\phi}$
(Graphite Cylinder)					
B (up)	10,60	38	$-.053 \pm .006$	2316	$89 \pm 6^\circ$
B (down)	11,60	44	$-.051 \pm .006$	2937	$155 \pm 6^\circ$
(Graphite Sheet)					
B (up)	21,40	23	$-.038 \pm .014$	1578	$72 \pm 22^\circ$
B (down)	25,40	14	$-.041 \pm .014$	1863	$-82 \pm 24^\circ$
(Liquid Helium)					
B (up)	25,75	74	$-.004 \pm .009$	3024	
		65	$+.009 \pm .008$	3114	
		61	$+.014 \pm .007$	3260	
B (down)		54	$-.007 \pm .006$	4300	
		66	$-.001 \pm .007$	3770	
		45	$.000 \pm .009$	1662	
(Liquid Argon)					
B (Zero)	8,30	20	$-.006 \pm .013$	3135	
B (up)		14	$+.003 \pm .009$	4475	
B (down)		13	$-.012 \pm .008$	4266	

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VIII APPENDIX A. Expected Form of the Time Distribution Function

Aside from the earlier attention given in Section (III D.) to the geometrical factor $\frac{\sin \xi}{\xi}$ and to the average phase $\bar{\varphi}$ little has been said to support the overall form of the time distribution function used in the analysis. With this appendix we hope to give the matter a full hearing, demonstrating under the experimental conditions of spin precession, electron detection efficiency, and random background that the form

$$N_D(t) = N_0 e^{-\lambda_T t} [1 + A \cos (\omega t - \bar{\varphi})] + B_0 \quad (A1)$$

is indeed to be expected, this in agreement with the spacial distribution $1 + A \cos \theta_{eu}$ of Section II.

To begin we direct attention to the fact that the distribution is controlled by the total disappearance rate λ_T . This is a result common to all two mode processes, following in the present case from the probability for muon decay at time t in interval dt , namely,

$$\frac{dN_D}{N_T} = - \lambda_D dt \quad (A2)$$

where N_T is the muon population at time t and λ_D is the rate of decay. With N_T entering this expression in a linear fashion, λ_T becomes the controlling rate.

It should be evident that the cosine argument $\omega t - \bar{\varphi}$ appearing in $N_D(t)$ is equivalent to the argument θ_{eu} . This follows by definition since $\omega t - \bar{\varphi}$ describes the instantaneous angle between spin orientation and the electron momentum vector. The larger matter of justifying the overall shape of a modulated exponential plus a constant background is not so easily disposed of, however, requiring instead an

examination of the various elements which go to make up the probability of observing an event at time t over interval dt . Such an event can either be a decay electron, correlated as expected to a gate initiating muon, or a random count coming from somewhere outside the target volume. We wish to consider in turn the probability for each alternative. Combined and simplified they lead to the form for $N_D(t)$ previously assumed.

1. Decay Event

The probability that a decay electron will satisfy the counter telescope at time t over time element dt is given as a product of three probabilities,

$$dP_e(t) = e^{-\lambda_T t} \cdot e^{-\lambda_R \tau} \cdot E(t) \lambda_D dt \quad (A3)$$

Here the exponentials represent in the order of their appearance the probability that the muon has not disappeared before time t and the probability that a random event has not triggered the "e" dead time circuit sometime over the interval $\tau(8 \text{ } \mu\text{sec})$ preceding time t . The rate λ_R introduced by the latter probability is the random background rate. The third factor present gives the probability of observing a decay electron at time t in interval dt . The detection efficiency for telescope (56), here time dependent because of a precessing decay anisotropy, is given by the expression

$$E(t) = E_0 [1 + A \cos(\omega t - \bar{\varphi})] \quad (A4)$$

where E_0 is a measure of the average efficiency. Thus given a relatively small background rate λ_R we have accounted for the modulated exponential of the decay distribution.

2. Random Event

Here we show that for all practical purposes the probability for observing a random event at time t over interval dt is the same for all t . Again the demonstration involves a product of three probabilities: (1) the probability that no random event preceded time t by less than $8\mu\text{sec}$, (2) the probability that the interval dt will see a random event, and (3) the probability that no decay count has occurred before time t . In the order presented they combine to form

$$dP_R(t) = e^{-\lambda_R t} \cdot \lambda_R dt \cdot [1 - \int_0^t dP_e(t)] \quad (A5)$$

where in unabridged form the decay count probability integrates to

$$\begin{aligned} \int_0^t dP_e(t) &= \frac{E_0 \lambda_D e^{-\lambda_R t}}{\lambda_T} [1 - e^{-\lambda_T t}] + \\ &\quad \frac{E_0 \lambda_D A e^{-\lambda_R t}}{\lambda_T^2 + \omega^2} \cos \bar{\varphi} [e^{-\lambda_T t} (\lambda_T \cos \omega t + \omega \sin \omega t) - \lambda_T] + \\ &\quad \frac{E_0 \lambda_D A e^{-\lambda_R t}}{\lambda_T^2 + \omega^2} \sin \bar{\varphi} [e^{-\lambda_T t} (\lambda_T \sin \omega t - \omega \cos \omega t) + \omega] \end{aligned} \quad (A6)$$

The appearance of sinusoidal functions in the last two terms of this expression is a matter of small concern. Both are down by at least an order of magnitude from the value of the first term, even for the case of large asymmetry. In turn the total expression (A5) is considered small when compared to the decay count probability (A3). This follows since the latter comparison is controlled by the rate ratio λ_R/λ_D . Even if λ_R is overestimated in terms of the maximum "telescope count" from Table I, the ratio does not exceed a value of 1.6×10^{-4} . With the random probability small, its time dependence loses significance. Therefore the linear combination of decay count and random count

probabilities will correspond to the expected distribution, the first providing a sinusoidal modulation for the decay curve and the second a "constant" background.

APPENDIX B. Least-Squares Calculations

As described in Section IV the method of least-squares saw three different applications in connection with this experiment. Classified according to the parameter of interest as programs of life-time, asymmetry, or frequency, each begins with the formation of the positive-definite expression

$$\Psi = \sum_1 [F - y_1]^2 / \sigma_1^2 \quad (B1)$$

Here F is the parameter-dependent function under study, y_1 the data to be fitted, and σ_1 the statistical deviation of y_1 , Poisson-valued at $\sigma_1 = y_1^{1/2}$ for our calculations. With a form for function F which is linear in the unknown parameters, the expression Ψ is minimized in a standard procedure and the "best-fit" values of the parameters obtained.

For the simple function

$$F = N_0 e^{-\lambda_T t} + B_0 \quad (B2)$$

where background B_0 is known, linearization in the total rate λ_T is achieved in a familiar way. For the precession function

$$F = N_0 e^{-\lambda_T t} [1 + A \cos (\omega t - \bar{\varphi})] + B_0 \quad (B3)$$

on the other hand linearization can take several forms:

1. When a knowledge of the angle $\bar{\varphi}$ is sought together with the asymmetry coefficient A , the cosine expression in (B3) can be expanded and simplified using the substitution

$$\bar{\varphi} + \varphi_0 + \delta \quad (B4)$$

Here φ_0 is an initial estimate of the phase taken from a plot of the data, and δ is a small adjustment to this value. Thus with the help of the approximations $\sin \delta \sim \delta$ and $\cos \delta \sim 1$, (B3) becomes

$$F = N_0 e^{-\lambda_T t} + N_0 A \cos (\omega t - \varphi_0) e^{-\lambda_T t} + N_0 A \delta \sin (\omega t - \varphi_0) e^{-\lambda_T t} + B_0 \quad (B5)$$

Setting $R = N_0$, $S = N_0 A$, and $T = N_0 \Lambda \delta$ we have an expression linear in the parameters R , S , and T .

2. If instead the frequency ω is uncertain and the phase angle $\bar{\varphi}$ well known, the analogous substitution

$$\omega = \omega_0 + \gamma \quad (B6)$$

can be made. This time γ is the small correction and the expansion becomes

$$\cos(\omega t - \bar{\varphi}) = \cos \gamma t \cos(\omega_0 t - \bar{\varphi}) - \sin \gamma t \sin(\omega_0 t - \bar{\varphi}) \quad (B7)$$

For linearization the condition $\gamma t \ll 1$ must be met over the time interval analyzed. Within the first 3 μsec this is fairly well satisfied for the strong precession field used. Granting this, then

$$F = R e^{-\lambda_T t} + S \cos(\omega_0 t - \bar{\varphi}) e^{-\lambda_T t} - T \sin(\omega_0 t - \bar{\varphi}) t e^{-\lambda_T t} + B_0 \quad (B8)$$

where $R = N_0$, $S = N_0 A$, and $T = N_0 \Lambda \gamma$. It should be noted that the asymmetry parameter A will also be given by this procedure. The disappearance rate λ_T , however, remains the object of a separate lifetime determination.

Returning to the linearized form (B5) we continue to detail the steps of the asymmetry calculation. Constructing the expression Ψ with the help of (B5) and minimizing with respect to parameters R , S , and T , we obtain

$$\frac{\partial \Psi}{\partial R} = 0 = \sum_1 [R e^{-\lambda_T t} + S e^{-\lambda_T t} \cos(\omega t - \varphi_0) + T e^{-\lambda_T t} \sin(\omega t - \varphi_0) + B_0 - y_1] e^{-\lambda_T t} / \sigma_1^2 \quad (B9)$$

$$\begin{aligned} \frac{\partial \Psi}{\partial S} = 0 = \sum_1 [R e^{-\lambda_T t} \cos(\omega t - \varphi_0) + S e^{-\lambda_T t} \cos^2(\omega t - \varphi_0) + \\ T e^{-\lambda_T t} \sin(\omega t - \varphi_0) \cos(\omega t - \varphi_0) + (B_0 - y_1) \cos(\omega t - \varphi_0)] e^{-\lambda_T t} / \sigma_1^2 \end{aligned} \quad (B10)$$

$$\begin{aligned} \frac{\partial \Psi}{\partial T} = 0 = \sum_1 [R e^{-\lambda_T t} \sin(\omega t - \varphi_0) + S e^{-\lambda_T t} \cos(\omega t - \varphi_0) \sin(\omega t - \varphi_0) + \\ T e^{-\lambda_T t} \sin^2(\omega t - \varphi_0) + (B_0 - y_1) \sin(\omega t - \varphi_0)] e^{-\lambda_T t} / \sigma_1^2 \end{aligned} \quad (B11)$$

Solving for R , S , and T with a standard inversion,

$$R = \frac{1}{\Delta} [k_1 (D_{22} D_{33} - D_{23}^2) - k_2 (D_{12} D_{33} - D_{13} D_{32}) + k_3 (D_{12} D_{23} - D_{13} D_{22})]$$

$$S = \frac{1}{\Delta} [-k_1 (D_{21} D_{33} - D_{23} D_{31}) + k_2 (D_{11} D_{33} - D_{13}^2) - k_3 (D_{11} D_{23} - D_{13} D_{21})]$$

$$T = \frac{1}{\Delta} [k_1 (D_{21} D_{32} - D_{22} D_{31}) - k_2 (D_{11} D_{32} - D_{12} D_{31}) + k_3 (D_{11} D_{22} - D_{12}^2)]$$

where the determinant

$$\Delta = D_{11} (D_{22} D_{33} - D_{23}^2) - D_{12} (D_{21} D_{33} - D_{23} D_{31}) + D_{13} (D_{21} D_{32} - D_{22} D_{31})$$

and the quantities D_{ij} and k_i are the elements from (B9), (B10), and (B11). In particular,

$$D_{11} = \sum_1 e^{-2\lambda_T t} / \sigma_1^2$$

$$D_{12} = D_{21} = \sum_1 e^{-2\lambda_T t} \cos(\omega t - \varphi_0) / \sigma_1^2$$

$$D_{13} = D_{31} = \sum_1 e^{-2\lambda_T t} \sin(\omega t - \varphi_0) / \sigma_1^2$$

$$D_{22} = \sum_1 e^{-2\lambda_T t} \cos^2(\omega t - \varphi_0) / \sigma_1^2$$

$$D_{23} = D_{32} = \sum_1 e^{-2\lambda_T t} \sin(\omega t - \varphi_0) \cos(\omega t - \varphi_0) / \sigma_1^2$$

$$D_{33} = \sum_1 e^{-2\lambda_T t} \sin^2(\omega t - \varphi_0) / \sigma_1^2$$

$$k_1 = \sum_1 e^{-\lambda_T t} (y_1 - B_0) / \sigma_1^2$$

$$k_2 = \sum_1 e^{-\lambda_T t} (y_1 - B_0) \cos(\omega t - \varphi_0) / \sigma_1^2$$

$$k_3 = \sum_1 e^{-\lambda_T t} (y_1 - B_0) \sin(\omega t - \varphi_0) / \sigma_1^2$$

It follows from an earlier definition that parameters $N_0 = R$, $A = S/R$, and $\delta = T/S$. With this we obtain $\bar{\varphi} = \varphi_0 + \delta$, thus completing the basic calculation cycle. Actually the analysis is repeated (on the average three times), each step improving on the preceding with the substitution $\varphi_0 + \delta \rightarrow \varphi_0$. This iterative procedure ends only when $|\delta/\bar{\varphi}|$ becomes less than 10^{-4} .

The final values for the minimizing parameters N_0 , A , and $\bar{\varphi}$ are used to compute the chi square function

$$\chi^2 = \sum_1 [F_1 - y_1]^2 / y_1.$$

Valid for a Poisson distribution of the data this provides a quantitative estimate of the goodness of fit. Here the difference $F_1 - y_1$ should be little more than the statistical deviation from y_1 . As such, χ^2 can be compared to the number of independent classifications for the data (here essentially the number of channels analyzed, $n = 16-15$).

This comparison therefore has been made a standard supplement to the results given in the Tables. Fluctuations from the number of independent classifications should follow those of a normal distribution.

The error which accompanies each result represents one standard deviation. For the asymmetry parameter A this is given by the calculation

$$\sigma_A = \left[\sum_1 \left(\frac{\partial A}{\partial y_1} \right)^2 \sigma_1^2 \right]^{1/2}$$

where the channel variables σ_1 are taken to be completely uncorrelated.⁴⁷

A similar expression holds for the other parameters.

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X. FIGURES

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μ^- STOP = 1234
 DECAY e = 56

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

ALL TUNNEL DIODES (TD1...TD8) : 1N3118, 10 ma 1V

ALL TRANSISTORS EX. ONE (T1...T16) : 2N769, 600 Mc
TRANSISTOR, T17 : 2N700

DIODE, D11 : 1N100

ALL THE OTHER (D1...D10) : 26100

SW OPEN FOR COINCIDENCE

DISCRIMINATION LEVEL $TD7 < TD6$

FIRST AND LAST PULSE DETECTOR
TIME-TO-PULSE HEIGHT CONVERTER

FIG. 8b

FIG. 9

FIG. 10

FIG. 11

FIG. 12

FIG. 13

ASYMMETRY ANALYSIS FOR μ^- DECAYS IN GRAPHITE CYLINDER TARGET; B FIELD (DOWN)

FIG. 14

FIG. 15

FIG. 16