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A SELF-CONSISTENT TREATMENT OF PHONONS AND QUASIPARTICLES IN FINITE FERMION SYSTEMS AND APPLICATIONS TO RARE EARTH NUCLEI

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ABSTRACT

Using a Green's function formalism, the quasiparticle self-energy in a finite Fermi system is described in terms of the coupling of even- and odd-mass nuclear eigenmodes. The self-energy includes the usual QPC core polarization diagram, a "backwards" exchange diagram with ground state correlations, and higher order diagrams which correct for exclusion principle violations in the propagation of the quasiparticle and the microscopic phonon. Intermediate lines in the self-energy are described self-consistently, thereby including multiple-phonon core excitations. It is shown that the generalized Dyson's equation for the self-energy may be solved by matrix diagonalization rather than the usual dispersion relation. Renormalized phonons are calculated by taking into account splitting of the quasiparticle strength by QPC in the solution of the Bethe-Salpater equation. The quasiparticle self-energy and the phonon are calculated self-consistently.

The theory is used to calculate energy levels, transition probabilities, and one-nucleon transfer cross-sections for even- and odd-mass isotopes in the rare earth region. An "intruder" state is predicted in the Eu and Tb isotopes. The splitting of the $K_0 \pm 2$ states in ^{165}Ho and

167 ϵ_r is derived from higher order contributions to the self-energy.

The interference of pairing and quadrupole correlations in the ground state is discussed.

This dissertation includes an introduction to the applications of Green's function techniques in nuclear theory and an extensive review of the literature, in which the various theoretical attempts to describe low-energy properties of even- or odd-mass nuclei from the perspective of phonon and quasiparticle correlations are collated.

I. INTRODUCTION

The prime task of the nuclear physicist, engaged in the microscopic study of nuclear structure, is to describe the many-nucleon system in terms of a minimal number of elementary excitations and well-behaved interactions among them. Physicists approach this general end in various ways--their methods differing in degree of rigor, as well as intuitive clarity. On the one hand, a great deal of many-body theory is devoted to describing both qualitatively and quantitatively the effective interaction between nucleons in a nucleus. According to this school, it makes little sense to calculate nuclear properties without using the best effective interaction as determined from the NN potential, and then, of course, it makes no sense at all to work with an effective interaction which is not at least "good" by many-body standards. Quite on the other extreme are those whose aim is the development of a model scheme of excitations and interactions. The character of these schematic models is determined by the unholy matrimony of mathematical simplicity and agreement with the experimental data trend. Nevertheless, when confronted by a task so difficult as the many-body description of the atomic nucleus, it is natural to test our physical intuitions on something less glorious than the full-scale problem. Somewhere in the middle is phenomenological many-body physics. Theorists in this school determine the analytic behavior of the effective interaction and then put a number in front which is empirically based. From this, they engage the entirety of nuclear physics data. The middle-of-the-road school must, nevertheless, make many approximations. Primarily, these are concerned with the separation of relevant degrees of nuclear freedom.

Actually, all these nuclear physics approaches have some hope of converging to one another. We are proceeding on the assumptions that sometime in the future, the effective interaction between nucleons will be accurately known from a convergent calculation and that the branch of single particle excitations, viz., quasiparticles, will be accurately described using some offspring of Brueckner, Hartree-Fock, or Bogoliubov theories coupled to a higher order and convergent approximation to Dyson's equation. Call these developments Stage I. Using them as inputs, the low energy states of all nuclei must then be calculated successfully. This includes energies, transition probabilities, reaction cross-sections for even and odd, spherical and deformed, normal and superfluid nuclear systems. Even this Stage II will require many elements of sophistication and many-body theory. For almost every system, it will require a self-consistent and Pauli Principle allowed treatment of quasibosons--rotons and phonons--and quasiparticles or quasibogolons. For deformed systems, a unified treatment of rotational and intrinsic motion will necessarily be described. For systems deformed by pairing, i.e., superfluid, the uncertainty of number nonconservation will have to be eliminated.

Until these happy events are enacted (and nuclear physicists retire to other branches of physics, college administration, or unemployment), theoretical work will be in progress at both stages. It is not at all improper that those people who are interested calculate nuclear properties from an effective nuclear interaction which is less than God-given. While they might be deluding themselves in thinking that they have calculated the "right answer", their contribution to our

understanding of analytic effects (e.g., sufficiently correct treatment of the Pauli Principle, number conservation, angular momentum conservation, renormalizing far-away configurations, etc.) is vital. Also this work tends to develop the theory and computer programs into which we can plug the "real" effective interaction and effective or "quasi" particles, once we know them. Towards this latter goal, the pairing-plus-quadrupole model will not make a great contribution---just because it is too simple. Yet such simplicity is a virtue in that it has and will often permit a convenient evaluation of the analytic problems already mentioned.

Towards these noble ends of theoretical nuclear physics, we have been engaged in a thesis program whose purpose is to elucidate a self-consistent theory of phonons and quasiparticles. The setting for whatever ideas we might develop is the rare-earth region. The reason for this setting is in part historical--we were originally involved in a program to study the "effects" of the "effective" interaction in this region (e.g., the surface and spin characteristics of gamma vibrations). The various 0^+ states were also fascinating enough to motivate investigation. There are also some sound theoretical reasons for continuing to use this region of the periodic table to study quasiparticle-phonon coupling. The phonons involve only a few configuration mixings, and hence demand that the Pauli Principle be properly taken into account. A large amount of ground state correlation, pairing fluctuation, etc. is also present. These sorts of things have exotic consequences on the ultimate analytic formulation of the theory. We approach the QPC problem from the viewpoint of Migdal's Fermi liquid theory. That is, the analytic

form of the effective interaction and the spacing of single particle states are assumed and then parameterized to give agreement with experiment. While we plan to raise some questions about these assumptions, such an approach basically awaits the output of Stage I of nuclear theory as we discussed earlier. Also it is assumed that number nonconservation and rotational motion do not effect the theory. These are limitations which make the theory less fundamental (and perhaps grossly inaccurate, e.g., in ^{152}Sm).

The presentation of our dissertation work is somewhat out-of-the-ordinary. Section II has the stated purpose of reviewing some relevant theoretical background (i.e., Green's functions). It is actually somewhat more than that, namely a review of many applications of the formalism. Not only do we hope to present the reader who is unfamiliar with this approach to many-body physics with some examples to get him started, but it is also our intention to show that the nuclear motions can be traced from the bare NN-interaction to particle-vibration coupling, all within the Green's function approach. The inspiration for such a tack came as we prepared several lectures for a graduate course in Nuclear Chemistry. In this spirit, we have assumed the reader is familiar with such topics as quantum mechanics and second quantization, Nilsson orbits and Bohr-Mottelson phenomenology.

The "review of the literature" part of the dissertation is contained in Appendix B. Logically, it might be included after Sec. II. However, we have tried to make such a review more self-contained and extensive than would be comfortable if it were sandwiched between Green's functions and the work which we have initiated.

Section III presents the theory of self-consistent quasiparticle-phonon coupling (SCQPC). As indicated earlier, we have sought to develop a theoretical framework within which to examine the consequences of the Pauli Exclusion Principle and certain higher order effects of the QPC coupling term. Section IV details the numerical methods employed, and Sec. V deals with the results of actual calculation.

We should mention in reference to Sec. V that we regard our results in a somewhat schematic way. Some of the theoretical improvements we suggest--and which are calculated to be important--actually worsen agreement with experiment. Although we think that our numerical results will be useful to experimentalists, fitting the energy level scheme of ¹⁶⁵Er has not been our foremost consideration.

Finally, a warning about notations (a tabulation of which appears in Appendix A). The reader should note that as we delineate certain elements of nuclear theory from the NN-interaction through elementary excitations in a finite superfluid nucleus, it is convenient to introduce Greens's functions corresponding to particles, quasiparticles, bogolons, etc. which are "undressed" and "dressed" by QPC self-energy terms. It is thus impossible to use only single or double lines in the Feynman diagrams corresponding to undressed or dressed single nucleon Green's functions, G. The figures will be labelled when the "particle-type" is relevant and/or ambiguous.

II. THEORY---BACKGROUND

In this and the following section, we shall be concerned with the description of a many-nucleon system in terms of the motions and correlations of the individual protons and neutrons. In other words, we seek a microscopic theory of the nucleus.

The purpose of this section is to review the Green's function approach to many-body systems. Though a great number of applications and important results of this theoretical approach are discussed insofar as they help to detail a more complete picture of the nucleus, completeness and mathematical rigor are not attempted. Such details are contained in the references to which we shall refer. We have written this section with the intentions of presenting an introduction for the beginner and collecting together the various theoretical concepts which are the foundation of the following section on self-consistent quasiparticle-phonon coupling.

In our opinion, the most elegant and universal framework within which to describe the nucleus is that of Green's functions. Within this framework, one can derive perturbation theory and calculate theoretically important quantities such as the effective interaction, single particle states, and effective charges. Indeed, most of the important work in these fundamental areas is carried out using the "perturbation theory and Feynman picture" approach. Moreover, utilization of this method coupled to an understanding of the analytical properties of Green's functions enables us to describe experimentally observable features (energy states, as well as photon and nucleon transition amplitudes) of the many-nucleon system. Such descriptions can be carried out to any order of perturbation theory we choose.

Quite literally, diagrams and Green's functions are a language in which very complicated theory can be outlined in a simple way. The prerequisites to its usage are the rules of grammar and a little vocabulary. It also helps to have a "feel" for the language. And it is to this task we first address ourselves.

A. Green's Functions

We begin with a most basic equation from quantum mechanics, the time-dependent Schrodinger equation:

$$i \frac{\partial \psi}{\partial t} = H\psi \quad (\text{II.1})$$

ψ is the wave function of the system in which we are interested. It contains as much information as is allowed by the Uncertainty Principle. H is the Hamiltonian or energy operator. If H is time-independent, the time-dependent Schrodinger equation may be integrated to give

$$\psi(t') = e^{-iH(t' - t)} \psi(t) \quad (\text{II.2})$$

This equation, which contains the unitary time-evolution operator, $e^{-iH(t' - t)}$, tells us how the wave function develops in time. To obtain the exact value of ψ at time t , we may use Eq. (II.2) along with an initial condition, e.g., $\psi(0)$ or $\psi(-\infty)$. Using the time-evolution operator, we may define the generalized Green's function

$$iG_{AB}(t' - t) \equiv \langle A | e^{-iH(t' - t)} | B \rangle, \quad (t' - t) > 0 \quad (\text{II.3a})$$

Generally, $|A\rangle$ and $|B\rangle$ are taken to be basis states, i.e., they are simple configurations of the many-nucleon system, but they are not eigenstates. $iG_{AB}(t' - t)$ gives the probability amplitude that if we create the system $|B\rangle$ at t , it will have evolved into $|A\rangle$ at t' .

In a time-dependent sense, the system propagates from a position B , in configuration or Hilbert space, at time t , to a new position A , at time t' . Thus, Green's functions are often called propagators. The term Green's function has been employed because the co-ordinate space representation of G , namely

$$G(x', t'; x, t) = \sum_{AB} \langle x' | A \rangle G_{AB} \langle B | x \rangle \quad (\text{II.3b})$$

obeys the usual identities of a Green's function kernel¹

$$\psi(x', t') = \int dx' G(x', t'; x, t) \psi(x, t) \quad (\text{II.4a})$$

and

$$(i \frac{\partial}{\partial t} - H(x, x')) G(x', t'; x, t) = \delta(x - x') \delta(t - t') \quad (\text{II.4b})$$

It is often convenient to work with the Fourier transform of the time-dependent Green's function. The transformation is defined:

$$G(\omega) = \int_{-\infty}^{\infty} G(t' - t) e^{i\omega(t' - t)} d(t' - t) \quad (\text{II.5})$$

$$G(t' - t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{-i\omega(t' - t)} d\omega$$

The reason for such a transformation is that many algebraic operations involving Green's functions are quite simple if they are carried out with respect to the conjugate variable, ω , instead of time differences. Convolution integrals, such as Eq. (II.6), are replaced by simple products, as in Eq. (II.7). The Fourier transformation is a homomorphism:² it relates two sets of functions which are one-to-one and onto. Hence, whatever physics can be easily carried out using the $G(\omega)$ functions is identical to what would be obtained in the time-dependent formalism. In particular, infinite order perturbation theory and Feynman diagrams can be derived more transparently in the time-dependent formalism, while numerous physics problems can be solved almost trivially in the conjugate variable, ω . It is common to label ω the energy variable.

Let us interpose a complete set of eigenstates of H ,

$\sum |S\rangle\langle S| = 1$, between the time-evolution operator and the ket $|B\rangle$. Since $e^{-iH(t' - t)}|S\rangle = |S\rangle e^{-iE_S(t' - t)}$, we have

$$iG_{AB}(t' - t) = \sum_S \langle A|S\rangle e^{-iE_S(t' - t)} \langle S|B\rangle \quad (\text{II.6a})$$

The Fourier transform with respect to the variable, $t' - t$, is

$$G_{AB}(\omega) = \sum_S \frac{\langle A|S\rangle\langle S|B\rangle}{\omega - E_S + i\delta} \quad (\text{II.6b})$$

Such an equation seems eminently more convenient than Eq. (II.3).

Not only do the poles of this function give the eigenenergies, E_S , but the amplitudes in the numerator tell us the wave function of the eigenstate,

S. Suppose $|A\rangle$ and $|B\rangle$ were all the ^{209}Pb basis states, i.e., $1p$, $2p-1h$, $3p-2h$, $4p-3h$, etc. If we knew all the poles and residues, we would know all the eigenenergies and the overlap of eigenstates with the basis states. In other words, we would know completely about each eigenstate.

However, it is already known how to obtain this information--one needs only to diagonalize a matrix with the Hamiltonian operating between all the basis states. How is the Green's function related to the dynamics of the many-body system; how can we get G directly from H ? For the answer to this question, we return to the time-dependent formalism. The equations which follow are derived formally in Baranger's Varenna notes.³ They will be given here with "pictorial proof". See Fig. 1 for the appropriate diagrams.

First, we divide the Hamiltonian into two parts

$$H = H_0 + H_1 \quad (\text{II.7})$$

The states $|A\rangle$ and $|B\rangle$ are taken to be eigenstates of H_0 . H_1 , however, induces transitions between these basis states. Again, $G_{AB}(t' - t)$ is the total probability that the system will evolve from $|B\rangle$ at t to $|A\rangle$ at t' . How could this be brought about? The simplest possibility is that the H_1 not act at all. There would be no transitions within our system and no changes. $|A\rangle$ would thus have to be the same as $|B\rangle$. On the other hand, these might be one or two or three, etc., transitions, as in Fig. 1b. Consider all the diagrams in Fig. 1b. They all begin with free propagation of B at time t to D at time t' , i.e., there is no perturbation during

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Fig. 1. Contributions to the propagator $G_{AB}(t' - t)$. Note that in the diagram, time flows upward. In the analytical equations, time flows from right to left.

this time. Hence, $B = D$. However, at t'' the H_1 part of the Hamiltonian causes B to change into C . From t'' to t' , C evolves into A in exactly the same manner as B at t evolves into A at time t' , that is, via zero, one, two, or more transitions. The diagrammatic equivalent is shown in Fig. 1c.

The analytical expression makes use of the "undressed" propagator

$$G_{AB}^0 \equiv \delta_{AB} \langle A | e^{-iH_0(t' - t)} | B \rangle = e^{-iE_A(t' - t)} \quad (\text{II.8})$$

A single line depicts such a propagator. The various numbers of interactions "dress" the propagator, so that it becomes a double line.

$$iG_{AB}(t' - t) = i\delta_{AB} e^{-iE_B(t' - t)} + i^2 \int dt'' \sum_C G_{AC}(t' - t) \quad (\text{II.9a})$$

$$\cdot \langle C | H_1 | B \rangle e^{-iE_B(t'' - t)}$$

The Fourier transform of Eq. (II.9a) is

$$G_{AB}(\omega) = \frac{\delta_{AB}}{\omega - E_B + i\delta} + \sum_C G_{AC}(\omega) \langle C | H_1 | B \rangle \frac{1}{\omega - E_B + i\delta} \quad (\text{II.9b})$$

Equation (II.9b) illustrates why we prefer using expressions in the energy variable in actual calculations, viz., the absence of integrations.

By substituting the analytical expression Eq. (II.6b) into the integral Eq. (II.9b), we obtain

$$\sum_s \frac{\langle A|S \rangle \langle S|B \rangle}{\omega - E_s + i\delta} = \frac{\delta_{AB}}{\omega - E_B + i\delta} + \sum_c \sum_s \frac{\langle A|S \rangle \langle S|C \rangle}{\omega - E_s + i\delta} \langle C|H_1|B \rangle \frac{1}{\omega - E_B + i\delta}$$

(II.10)

From this point, it is easy to obtain the usual matrix diagonalization problem. For a particular eigenvalue E_B , when $\omega \approx E_B$, the terms with small denominators dominate Eq. (II.10), i.e.,

$$\frac{\langle A|S \rangle \langle S|B \rangle}{\omega - E_s + i\delta} = \sum_c \frac{\langle A|S \rangle \langle S|C \rangle}{\omega - E_s + i\delta} \langle C|H_1|B \rangle \frac{1}{\omega - E_B + i\delta}$$

Multiplying through by

$$(\omega - E_B + i\delta)(\omega - E_B + i\delta) / \langle A|S \rangle \quad (II.11)$$

this transforms

$$(\omega - E_B + i\delta) \langle S|B \rangle = \sum_c \langle S|C \rangle \langle C|H_1|B \rangle$$

Except for the $i\delta$ which has no significance when it is in the numerator, this is the usual matrix equation of elementary quantum mechanics.

So far, Feynman diagrams are no more descriptive than matrix diagonalization or, in the quantum chemist's language, configuration interaction. Of course, configuration interaction with an infinite dimensional computer would provide a complete description if the physicist or chemist were clever enough to glean the basic physics from the vast numerical output. Suppose, however, we ask our configuration

interaction practitioner to calculate with an interaction Hamiltonian, for example a two-body potential, V , which has a hard core: $\langle ab|V|cd \rangle = \infty$. How can the computer work with all of these infinities? Not to mention, there is the difficulty in choosing a few important configurations to diagonalize - since no computer is infinite - and knowing how the neglected configurations would affect the answer.

We can answer any and all such difficult questions with the "perturbation theory and Feynman picture" approach, although it requires the development of a more detailed formalism which follows.

B. One- and Two-Fermion Green's Functions

To describe a many-fermion system, the creation and annihilation operator formalism is employed. $|0 \rangle$ is the particle-hole vacuum, e.g., the ^{208}Pb shell model configuration. Creation operators for particles will be denoted by capital letter C's. The shell-model or Hartree-Fock energies will be written as ϵ . If $\epsilon > \lambda$, the Fermi energy, the single-particle state is unoccupied in the particle-hole (ph) vacuum.

The Pauli Exclusion Principle is ensured by requiring that the fermion operators anticommute. We should also point out that the single-particle labels which specify the occupied and unoccupied levels are usually just those of shell model or Hartree-Fock orbitals.

As before, we are interested in the time evolution of a simple configuration. The easiest way to do this is to cast our creation and annihilation operators into the Heisenberg form, and then the one- and two-particle Green's functions are defined

$$iG_{12}(t_1, t_2) \equiv \langle \tilde{0} | T \{ c_1(t_1) c_2^+(t_2) \} | \tilde{0} \rangle \quad (\text{II.12})$$

$$K_{1234}(t_1, t_2, t_3, t_4) \equiv \langle \tilde{0} | T \{ c_1(t_1) c_2(t_2) c_3^+(t_3) c_4^+(t_4) \} | \tilde{0} \rangle \quad (\text{II.13})$$

We have introduced several new notations here. $|\tilde{0}\rangle$ is not the usual particle-hole vacuum. It is the exact ground state of the system we are describing. T is the time ordering operator

$$T \{ c_1(t_1) c_2^+(t_2) \} = \begin{cases} c_1(t_1) c_2^+(t_2), & t_1 > t_2 \\ -c_2^+(t_2) c_1(t_1), & t_1 \leq t_2 \end{cases} \quad (\text{II.14})$$

That is, the Heisenberg creation operators are time ordered with the latest times to the left. There is also an overall sign introduced because c_1 and c_2 are fermions and anticommute, so if in time ordering we make an odd number of interchanges, we get a minus sign.

When $t_1 > t_2$, $iG_{12}(t_1, t_2)$ is the probability that if at time t_2 a particle in single particle state, 2, is added to the interacting system in its ground state it will evolve into the ground state plus a particle in state, 1, at time t_1 . If $t_2 > t_1$ we obtain (-1) times the probability that a hole in single particle orbit, 1, will evolve into a hole, 2, (plus the interacting ground state) in the interval t_1 to t_2 .

The physical and analytical significance of one- (and also two-) nucleon Green's functions is that the evolution of a particle (or hole, or many particles or holes) is related to the exact description of the systems with $N \pm 1$ nucleons. To obtain an analytical expression for the one

particle Green's function, when $t_1 > t_2$ a complete set of eigenstates of the $N+1$ nucleus is introduced. The Fourier transform is

$$G_{12}(\omega) = \sum_{S(N+1)} \frac{\langle \tilde{0} | c_1 | S \rangle \langle S | c_2^+ | \tilde{0} \rangle}{\omega - (E_{N+1}^{(S)} - E_N^{(0)}) + i\delta} \quad (\text{II.15a})$$

where δ is an infinitesimal positive number. If $t_1 \leq t_2$

$$G_{12}(\omega) = \sum_{S(N-1)} \frac{\langle \tilde{0} | c_2^+ | S \rangle \langle S | c_1 | \tilde{0} \rangle}{\omega + (E_{N-1}^{(S)} - E_N^{(0)}) - i\delta} \quad (\text{II.15b})$$

For the general case, we have that the one-particle Green's function contains eigenvalues and spectroscopic amplitudes of the $N \pm 1$ system in its energy (ω)-dependent expression.

$$G_{12}(\omega) = \sum_{S(N+1)} \frac{\langle \tilde{0} | c_1 | S \rangle \langle S | c_2^+ | \tilde{0} \rangle}{\omega - (E_{N+1}^{(S)} - E_N^{(0)}) + i\delta} + \sum_{S(N-1)} \frac{\langle \tilde{0} | c_2^+ | S \rangle \langle S | c_1 | \tilde{0} \rangle}{\omega + (E_{N-1}^{(S)} - E_N^{(0)}) - i\delta} \quad (\text{II.16})$$

The two-particle Green's function contains such information for the N -nucleon, as well as $N \pm 2$ systems. There are 2^4 different time orderings possible. They can be classified physically.

Particle-Particle Channel: $t_1 \approx t_2, t_3 \approx t_4$.

In all, there are 8 possibilities here $(t_1 \leq t_2) * (t_3 \geq t_4) * (t_1 \geq t_3)$. In other words, we may put two particles in at t_3 and t_4 and take them out at t_1 and t_2 , but since we are interested in particle-particle correlation, t_1 and t_2 are either both earlier or both later than t_3 and t_4 .

First Particle-Hole Channel: $t_1 \approx t_4, t_2 \approx t_3$.

The eight distinct time orderings are $(t_1 \leq t_4) * (t_2 \geq t_3) * (t_1 \geq t_3)$. Since t_1 and t_4 are earlier or later than t_2 and t_3 , the Green's function tells of the evolution of a particle-hole pair.

Second Particle-Hole Channel: $t_1 \approx t_3, t_2 \approx t_4$.

The remaining time orderings are $(t_1 \geq t_3) * (t_2 \geq t_4) * (t_1 \geq t_2)$.

The Fourier analysis of the three cases is carried out by time ordering the operators, interposing three complete sets of eigenstates, and Fourier transforming three of the time differences. The generalized result is⁴

$$\langle \tilde{0} | A^\dagger(t_A) B^\dagger(t_B) C^\dagger(t_C) D^\dagger(t_D) | \tilde{0} \rangle \longrightarrow \frac{i}{2\pi} \delta(\pm\omega_A \pm\omega_B \pm\omega_C \pm\omega_D)$$

$$\sum_{mns} \frac{A_{om}^\dagger B_{ms}^\dagger C_{sn}^\dagger D_{no}^\dagger}{(\pm\omega_D + \omega_{no} - i\delta)(\pm\omega_A - \omega_{mo} + i\delta)(\pm\omega_C \pm\omega_D - \omega_{so} + i\delta)} \quad (II.17)$$

where $\omega_{no} = E^{(n)} - E^{(o)}$, $E^{(n)}$ is the energy of the $N, N\pm 1$, or $N\pm 2$ system as the situation demands, and $D_{no}^\dagger = \langle n | D^\dagger | \tilde{0} \rangle$. Note, that each particle or hole has its own energy variable, although the process of Fourier transformation causes them to have a delta function relationship. This delta function is often referred to as "conservation of energy".

The one- and two-nucleon Green's functions as defined in Eq. (II.12) and (II.13) can be distinguished from the "generalized" Green's functions

of Subsec. II.A in several mathematically subtle ways. First, by considering the evolution of only one and two nucleon additions to the correlated ground state we are focussing attention to a few very special configurations. Secondly, we have switched from the global energy variable formalism in Subsec. II.A to the single-particle energy variable formalism. That is, instead of associating one Fourier conjugate variable, ω , with the evolution of the entire system, one has an ω for each particle, l , which is added or subtracted to the system. The energy variable, ω , is different from the single-particle energy, h , (eigenvalue of H_0 part of Hamiltonian). It physically corresponds to that part of the energy of the system which is being carried by the propagating particle or hole. The sum of such ω 's represents the total energy of the system. When particles and holes interact or evolve, the sum of these energy variables is therefore conserved. Poles of the Green's function in the energy variables correspond to stable excitations.

For the particle-particle case, we can obtain

$$K_{1234}(\omega_1, \omega_2, \omega_3, \omega_4) = \frac{i}{2\pi} \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4)$$

$$\cdot \left[\sum_{S(N+2)} \frac{p_{12}(\bar{\omega}, \omega') p_{34}(\bar{\omega}, \omega)}{\bar{\omega} - (E_{N+2}^{(S)} - E_N^{(0)}) + i\delta} - \sum_{S(N-2)} \frac{q_{12}(\bar{\omega}, \omega') q_{34}(\bar{\omega}, \omega')}{\bar{\omega} + (E_{N-2}^{(S)} - E_N^{(0)}) - i\delta} \right] \quad (\text{II.18})$$

$$\text{where } p_{12}(\bar{\omega}, \omega') = \sum_m \frac{(c_1^+)_{sm} (c_2^+)_{mo}}{\left(\frac{\bar{\omega} - \omega'}{2}\right) - \omega_{mo} + i\delta} - \frac{(c_2^+)_{sm} (c_1^+)_{mo}}{\left(\frac{\bar{\omega} + \omega'}{2}\right) - \omega_{mo} + i\delta}$$

$$q_{12}(\bar{\omega}, \omega') = - \sum_m \frac{(c_2)_{sm} (c_1)_{m0}}{\left(\frac{\bar{\omega} + \omega'}{2}\right) + \omega_{m0} - i\delta} + \frac{(c_1)_{sm} (c_2)_{m0}}{\left(\frac{\bar{\omega} - \omega'}{2}\right) + \omega_{m0} - i\delta}$$

$$\bar{\omega} = \omega_1 + \omega_2 = \omega_3 + \omega_4$$

$$\omega' = \omega_1 - \omega_2$$

$$\omega = \omega_3 - \omega_4$$

Note also that

$$\frac{i}{2\pi} \int_{-\infty}^{\infty} p_{12}(\bar{\omega}, \omega') d\omega' \equiv x_{12} = \langle S(N+2) | c_1^+ c_2^+ | \tilde{0} \rangle \quad (\text{II.19})$$

while

$$\frac{i}{2\pi} \int_{-\infty}^{\infty} q_{12}(\bar{\omega}, \omega') d\omega' \equiv y_{12} = \langle S(N-2) | c_2 c_1 | \tilde{0} \rangle$$

Though such a system of equations looks awesome at first, it says very simply that the particle-particle Green's function contains two-nucleon spectroscopic amplitudes (or generalized density matrix elements) in the numerator and eigenstate energies (measured relative to the N-nucleon correlated ground state) of the N±2 nucleon systems in the denominator. There are three independent energy variables. $\bar{\omega} = \omega_1 + \omega_2 = \omega_3 + \omega_4$ is the energy variable for the two-nucleon pair.

For the first particle-hole case, the equations are

$$K_{1234}(\omega_1, \omega_2, \omega_3, \omega_4) = \frac{-i}{2\pi} \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4)$$

$$\left[\sum_{S(N') \neq 0} \frac{f_{14}(\bar{\omega}, \omega') f_{32}(\bar{\omega}, \omega)}{\bar{\omega} - (E_{N'}^{(S)} - E_N^{(O)}) + i\delta} - \frac{g_{14}(\bar{\omega}, \omega') g_{32}(\bar{\omega}, \omega)}{\bar{\omega} + (E_{N'}^{(S)} - E_N^{(O)}) - i\delta} \right] \quad (\text{II.20})$$

$$\text{where } f_{14}(\bar{\omega}, \omega') = \sum_m \frac{(C_1^+)_{sm} (C_4)_{mo}}{\left(\frac{\omega' - \bar{\omega}}{2}\right) + \omega_{mo} - i\delta} + \frac{(C_4)_{sm} (C_1^+)_{mo}}{\left(\frac{\omega' + \bar{\omega}}{2}\right) - \omega_{mo} + i\delta}$$

$$g_{14}(\bar{\omega}, \omega') = \sum_m \frac{(C_4^+)_{sm} (C_1)_{mo}}{\left(\frac{\omega' + \bar{\omega}}{2}\right) + \omega_{mo} - i\delta} + \frac{(C_1)_{sm} (C_4^+)_{mo}}{\left(\frac{\omega' - \bar{\omega}}{2}\right) - \omega_{mo} + i\delta}$$

$$\bar{\omega} = \omega_1 - \omega_4 = \omega_3 - \omega_2$$

$$\omega' = \omega_1 + \omega_4$$

$$\omega = \omega_3 + \omega_2$$

Also

$$\frac{-i}{2\pi} \int_{-\infty}^{\infty} f_{14}(\bar{\omega}, \omega') d\omega' \equiv X_{14} = \langle S | C_1^+ C_4 | \tilde{0} \rangle \quad (\text{II.21})$$

while

$$\frac{-i}{2\pi} \int_{-\infty}^{\infty} g_{14}(\bar{\omega}, \omega') d\omega' \equiv Y_{14} = \langle S | C_4^+ C_1 | \tilde{0} \rangle$$

Note that N' may be a different nucleus than N , if $1 = \text{proton(neutron)}$, while $4 = \text{neutron(proton)}$. Thus, the particle-hole Green's function

contains spectroscopic amplitudes (i.e., generalized density matrix elements) of particle-hole pairs in the numerator and eigenvalues (again, measured relative to the correlated ground state) of the N-nucleon systems in the denominator. The particle-hole Green's function in the other channel is similar and will not be written down.

It is possible to define three-particle Green's functions in a similar manner to what has been done above. However, if we believe that nucleons correlate in pairs - not in triplets - then we can make the approximation that the three-particle Green's function is a simple product of one- and two-particle Green's functions. This is approximately the case in nuclei, as evidenced by the facts that

- i. The three-body correction to the nuclear matter binding energy of 16 MeV/nucleon is probably less than 2 MeV/nucleon.⁵
- ii. Most low-energy excited states in nuclei can be described as simple one-particle excitations or correlated particle-hole (or particle-particle) pair excitations.*

We mention the three-particle Green's function at this time because the formulation of dynamical equations for the one- and two-particle cases will reflect our prejudice that only two-body correlations are important in the nucleus.

In the next section, we summarize one systematic approach to the evaluation of "exact" one- and two-nucleon Green's functions.

* There are several notable exceptions here. Quartet states⁶ are the result of strong two-proton-two neutron correlations, or "alpha-particle clustering". Rotational states are the result of coherent motion of all the nucleons, but, in the adiabatic approximation,⁷ may be treated separately from the low-order particle-hole and particle-particle correlations.

C. Perturbation Theory

We shall concentrate our attention on the analytical techniques of perturbation theory, viz., the evaluation of Feynman graphs. Since only an overview of the development of diagrammatic techniques from the Schrodinger equation will be given, the reader may wish to consult one or several references.^{1,4,8,9} We follow closely the development of Nozières.⁴ One begins with the many-body time-dependent Schrodinger equation

$$i \frac{\partial \psi}{\partial t} = H\psi = (H_0 + H_1)\psi = [(T + U) + (V - U)]\psi \quad (II.22)$$

Where H_0 is time-dependent and "unperturbed" (by definition). U is a one-body potential. H_1 is the perturbation and contains the two-body interaction V . The generalized (H_1 may be time-dependent) time-evolution operator $U(t', t)$ is defined such that

$$\psi(t') = U(t', t) \psi(t) \quad (II.23)$$

By substituting Eq. (II.23) into Eq. (II.22) an integral equation is obtained

$$U(t', t) = 1 - i \int_t^{t'} H_1(t'') U(t'', t) dt'' \quad (II.24)$$

which is solved by iterative substitution:

$$U(t', t) = \sum_{p=0}^{\infty} U_p(t', t)$$

$$U_0(t', t) = 1 \quad (\text{II.25})$$

$$U_p(t', t) = \frac{(-i)^p}{p!} \int_t^{t'} dt_1 \dots \int_t^{t'} dt_p P(H_1(t_1) \dots H_1(t_p))$$

where P puts the latest operators on the left, this time without any sign change. $H_1(t)$ contains matrix elements and creation and annihilation operators in the interaction representation:

$$C_2(t) = e^{iH_0 t} C_2 e^{-iH_0 t} = e^{-iH_2 t} C_2 \quad (\text{II.26})$$

By paying careful attention to the anticommutation relations and the e^{-iHt} and $e^{-iH_0 t}$ factors that arise when we work in the interaction representation, and with the development of Wick's theorem which simplifies much of the algebra, we can calculate the time evolution of a (ph) vacuum, as well as the time evolution of one or two particles or holes added to that vacuum. This is done in terms of Feynman graphs. It is convenient (as well as tedious) to Fourier transform from time-dependent diagrams to energy-dependent diagrams. In the following paragraphs we outline the results.

The rules assume that a set of single particles states (levels or "orbits") has been obtained by diagonalizing H_0 , the "shell-model" Hamiltonian. The particle-hole vacuum is obtained by filling the lowest N orbits with particles. Thus, a Fermi surface, λ , is defined. We may associate with each orbit, l , a number, n_l , which is one or zero depending whether the state is occupied or unoccupied in the (ph) vacuum. The

relative energy ϵ_1 is determined from the shell model energy h_1 and λ .

Namely,

$$\begin{aligned}\epsilon_1 &\equiv \langle 1 | H_0 | 1 \rangle - \lambda = h_1 - \lambda & \text{if } h_1 > \lambda \\ \epsilon_1 &\equiv \lambda - h_1 & \text{if } \lambda > h_1\end{aligned}\quad (\text{II.27})$$

In other words, the relative energy (in exact analogy to the quasi-particle or bogolon energy in a superfluid) is always positive.

1. Draw all topologically different diagrams - more about what "topologically different" means later.
2. With every line, which represents the unperturbed* propagation of a particle or hole, we associate the unperturbed Green's function

$$G_1^0 = \frac{1 - n_1}{\omega_1 - \epsilon_1 + i\delta} + \frac{n_1}{\omega + \epsilon_1 - i\delta} \quad (\text{II.28})$$

Note that the G^0 's are diagonal in the unperturbed interaction, H_0 . We must label each line by an index of some sort and associate with the line its own energy variable, e.g., ω_1 . If it is a particle, the arrow on the line should point upward. If it is a hole, it should point down.

3. With every two-body interaction, we draw a small block with four corners, each of which attaches to a line. The matrix element associated with such a block is $\langle \text{left out right in} | v | \text{left in right in} \rangle$. For the time being, this two-body interaction is antisymmetric. Later, in the case of the irreducible particle-hole block, which is a collection of graphs connecting a particle and a hole, the interaction will not be antisymmetric; however, the "left out...right in" rule will still be employed. All matrix elements are multiplied by a factor $\left(\frac{1}{2\pi}\right) (\omega_{\text{left out}} + \omega_{\text{right out}} - \omega_{\text{left in}} - \omega_{\text{right in}})$.

* Later, when we work with "dressed" single particle propagators, this rule will remain unchanged as long as $G_{12}(\omega) = \delta_{12} G_1(\omega)$.

4. There are several kinds of one-body insertions we must consider - that is matrix elements that interrupt one line rather than two.
 - a. For a true one-body interaction, such as an external field or the $-U$ in the expansion $H = (T + U) + (V - U) = H_0 + H_1$, we simply insert $\langle \text{out} | -U | \text{in} \rangle \delta(\omega_{\text{in}} - \omega_{\text{out}} - \omega_{\text{field}})$, in which case the lines on each side have their own labels and energy indices. Unless the external field carries some energy, ω_{in} will equal ω_{out} after integration over ω_{out} .
 - b. For a two-body interaction with any one of the core particles, in which the core particle does not change orbits we associate the Hartree-Fock potential

$$\sum_3 n_3 \langle 13 | V | 23 \rangle \delta(\omega_{\text{in}} - \omega_{\text{out}})$$

The graph is shown in Fig. 2e.

5. Integrate over all intermediate energy variables and sum over all single particle orbit labels, 1, 2, etc.
6. Finally some "topological" considerations.
 - a. We are using labelled particle-hole diagrams. Graphs are topologically equivalent if we can move the interaction blocks up or down, without changing the direction of an arrow, and make them identical.
 - b. A closed loop is obtained by following a line until it comes upon itself again. The rule for crossing interaction blocks is that we exit on left side if we enter on left side. If c is the number of closed loops, we multiply by $(-1)^c$.
 - c. Identical lines are those which begin and end on the same interaction blocks and run in the same direction. If m is the number of such pairs, we multiply by $(\frac{1}{2})^m$.
 - d. Associated with the crossing of two particle or hole lines is a minus sign which results from the anticommutation relations, as illustrated in Fig. 2g.

After such a long winded presentation of the rules, we present

a few illustrations:

Fig. 2. Sample diagram rules. All lines are shell model particles (SM).

Fig. 2a)

This diagram is a contribution to the effective-particle-hole interaction. 1,2,3,4 are stubs not lines; if they were lines, we would have a contribution to the particle-hole Green's function. It illustrates rules 2,3,5 and 6b. Its value is

$$(-1) \left(\frac{i}{2\pi}\right)^2 \sum_{56} \int d\omega_5 d\omega_6 \frac{1}{\omega_5 - \epsilon_5 + i\delta} \frac{1}{\omega_6 + \epsilon_6 - i\delta} < 15 | V | 36 > < 64 | V | 32 > \\ \cdot \delta(\omega_1 + \omega_4 - \omega_2 - \omega_3) \delta(\omega_1 + \omega_5 - \omega_3 - \omega_6)$$

The (-1) comes from the closed loop, (56). Note the difference between the Green's functions for 5, a particle, and 6, a hole. There are no identical lines.

Fig. 2b)

Fig. 2c)

Fig. 2d)

Figures 2b-d are contributions to the ground state energy according to Goldstone's theorem. They are "unlinked clusters". In particular, they

illustrate the rules about "topologically different". Figure 2b and 2c are topologically equivalent because we can obtain Fig. 2b from Fig. 2c by moving the upper right-hand box. Figure 2d, however, is topologically different from these.

Fig. 2e)

This diagram illustrates one-body insertions. The lowest insertion is the Hartree-Fock insertion. The second insertion is the residual one-body interaction. The third insertion is an external field which carries off energy $\Delta\omega$. The dressed Green's function

$$G_{4,1}(\omega_4, \omega_1) = \sum_{23} \int d\omega_2 d\omega_3 \delta(\omega_1 - \omega_2) \delta(\omega_2 - \omega_3) \delta(\omega_3 - \omega_4 - \Delta\omega)$$

$$\cdot < 4|V|3 > G_3^0(\omega_3) < 3|U|2 > G_2^0(\omega_2)$$

$$\left[\sum_5 n_5 < 25|V|15 > \right]$$

Note that since delta functions have units 1/Energy, just as Green's functions, the overall expression has the right dimensions.

Fig. 2f)

This contribution to the effective particle-hole interaction has a pair of identical lines, 5 and 6.

Fig. 2g)

First minus sign is because V is antisymmetric, i.e., $\langle 12|V|34 \rangle = -\langle 21|V|34 \rangle$. The second minus sign counts the crossing of two particle lines.

Having examined some diagram examples, we shall add a few more "rules" which in some cases are only "helpful hints".

7. When we are done integrating a contribution to the effective interaction we should have $(\frac{i}{2\pi}) \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4)$ left over, because this is the factor we will carry along with the effective interaction when we insert it into another graph.
8. Contributions to the ground state energy, the unlinked clusters such as Figs. 2b-d, have one less delta function and one extra $(\frac{i}{2\pi})$ factor.^{3,4} Diagrams such as Figs. 5b-d illustrate ground state correlations.*

9.

$$\frac{i}{2\pi} \int \frac{f(\omega) d\omega}{\omega \mp \epsilon \pm i\delta} = \pm f(\pm \epsilon)$$

These last two equations will make contour integrations slightly easier.

* Since it will not be necessary in the theory developed later on to evaluate higher order contributions to the ground state energy, we will neglect the details of Goldstone's Theorem. However, it will be useful to remember that the real ground state energy is different from that of the (ph) vacuum by the inclusion of the "unlinked clusters".

To summarize, we have presented the diagram rules for the single particle propagator formalism. The rules we have developed were for "undressed" particles and holes and the bare interaction. However, they shall remain unchanged when we employ "dressed" or "quasi" particles and holes and effective interactions.

D. Applications of Green's Functions

In this section, we will apply Feynman graph techniques to a number of problems and famous equations of nuclear physics. Our purpose is several fold:

- i. to develop an overview of nuclear physics
- ii. to illustrate the power of graphical methods, as well as the ease (except possibly for phase conventions) of using them.
- iii. to preface, in particular, the work presented later in the theory, Sec. III.

Let us first see if we can summarize where we are going in the applications section. Since we believe that two-nucleon correlations dominate the nuclear structure, we shall first expand the armament of analytical equations to include Dyson's equation and the Bethe-Salpater equations. These equations summarize how to obtain exact one-nucleon Green's functions (which yield the nuclear structure of ^{209}Pb , ^{207}Pb , etc., if the ^{208}Pb ground state is taken as the correlated vacuum) as well as exact two-nucleon Green's functions (which lead in the particle-particle channel to the structure of ^{210}Pb , ^{206}Pb , and ^{210}Bi , etc. and in the particle-hole channel to ^{208}Bi , ^{208}Pb (excited), etc.).

In fact, these rather general equations and the complete calculations they imply can be approximated by regarding ^{208}Pb or ^{210}Pb as a

shell model calculation, with which we presume the reader is already familiar. So the applications considered will reflect a shell model prejudice. In particular, we would like an effective interaction which we can employ in shell model calculations. We would also like to be able to describe the shell model orbit in terms of the bare or effective nucleon-nucleon potential. The zeroth order picture of independent particles moving in an average nuclear field should also be examined. Where it is necessary to develop approximations, our philosophy is to choose the most important low-order graphs, or to carry out infinite summations of important classes of diagrams (as in Brueckner theory).

1. Dyson's Equation

In attempting to describe the dynamics of the one-particle Green's function, that is, how G is related to the one-body kinetic energy and two-body potential energy, we return to graphical methods. Suppose we put a neutron with the ^{208}Pb core, how will the system evolve? At any time, the system either will be in a simple shell-model orbit + core configuration or the extra nucleon will be interacting with the core. The graphical way of saying this is shown in Fig. 3a. As with generalized Green's functions one can easily obtain an integral equation with an effective one-body interaction connecting the "dressed" and "undressed" propagators. In the case of the one-particle Green's functions, this effective interaction is called the self-energy or mass operator. The integral equation is called Dyson's equation and may be written.

Fig. 3. The exact one-nucleon propagator in terms of the self-energy.

$$\uparrow\uparrow = \text{FDSM}, \quad \uparrow = \text{SM}$$

Fig. 3a)

$$\uparrow\uparrow = \uparrow + \begin{array}{c} \uparrow \\ \textcircled{\Sigma} \end{array} + \begin{array}{c} \textcircled{\Sigma} \\ \uparrow \\ \textcircled{\Sigma} \end{array} + \dots = \uparrow + \begin{array}{c} \uparrow\uparrow \\ \textcircled{\Sigma} \end{array}$$

Fig. 3b) A reducible graph

Fig. 3c)

A particle in core interacts but does not change orbits. The radius of the circle should be zero because there is only the unexcited core + extra nucleon both before and after. The core particles should be exactly described, FDSM.

Fig. 3d) The sum of all more complicated contributions to the self-energy.

The two upgoing lines and one downgoing line mean that we have a temporarily excited 2p-1h core, i.e., one particle + 1p-1h excitation of core.

$$G_{12}(\omega) = G_{12}^0(\omega) + \sum_{34} G_{13} \sum_{34} G_{42}^0 \quad (\text{II.29})$$

What does the mass operator look like? First, it is irreducible. This means, it cannot be separated into two parts by cutting a single-line (Fig. 3b). It is a "blob" which connects simple lines. Secondly, it is of zero-length in time only if the interaction is between the extra nucleon and the core but does not stir up the core, i.e., the core nucleon which it collides with does not change orbits (Fig. 3c). Thirdly, if the core is excited, the transition back to one particle + excited core must take place in an amount of time ($t > 0$), and the sum of all such processes connecting four lines may be abbreviated, Γ . This contribution is written suggestively in Fig. 3d.

$$\Sigma_{34}(\omega) = \sum_5 \langle 35|V|4 \rangle + \sum_{\substack{567 \\ 5'6'7'}} \int d\omega_5 d\omega_6 d\omega_7 \left(\frac{i}{2\pi}\right)^2 (-1) \quad (\text{II.30})$$

$$\delta(\omega + \omega_7 - \omega_5 - \omega_6) < 37' | \Gamma(\omega, \omega_7, \omega_5, \omega_6) | 5'6' > < 56 | V | 37 > G_{5'5}(\omega_5) \cdot G_{6'6}(\omega_6) G_{7'7}(\omega_7)$$

This equation is derived analytically in Refs. 1 and 4.

2. The Bethe-Salpater Equations for the Generalized Two-Nucleon Scattering Amplitude, Γ

The Γ we have introduced as "the sum of all connections between four lines" is an essential ingredient of the two-particle Green's

function. In fact, K is the sum of an antisymmetric uncorrelated part and a correlated part.

$$\begin{aligned}
 K_{1234}(\omega_1, \omega_2, \omega_3, \omega_4) &= G_{13}(\omega_1) G_{24}(\omega_2) - G_{14}(\omega_1) G_{23}(\omega_2) \\
 &+ \frac{i}{2\pi} \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) \sum_{1'2'3'4'} G_{11'}(\omega_1) G_{22'}(\omega_2) G_{3'3}(\omega_3) G_{4'4}(\omega_4) \\
 &\langle 1'2' | \Gamma(\omega_1, \omega_2, \omega_3, \omega_4) | 3'4' \rangle
 \end{aligned} \tag{II.31}$$

This is shown graphically in Fig. 4a.

To be more precise about K or Γ , we must decide whether we are interested in the particle-particle or particle-hole channel. Consider the particle-particle channel: the sum of all graphs connecting two particles is shown in Fig. 4b. We obtain the Bethe-Salpater equation*

$$\langle 12 | \Gamma | 34 \rangle = \langle 12 | J | 34 \rangle + \frac{1}{2} \sum_{55'66'} \langle 12 | J | 5'6' \rangle G_{5'5} G_{6'6} \langle 56 | \Gamma | 34 \rangle \tag{II.32}$$

J is the sum of all irreducible graphs in the particle-particle channel, i.e., those which cannot be separated by "cutting" two lines. Some examples are shown in Fig. 5a.

The sum of all graphs connecting a particle and a hole is shown in Fig. 4c. An alternative series is Fig. 4d, which corresponds to the second particle-hole channel. The corresponding Bethe-Salpater equations are

* We will henceforth not explicitly write down the energy variables.

Fig. 4a. Graphical equation for the two-particle Green's function. Γ is the reducible scattering amplitude. Lines correspond to exact one particle Green's functions (FDSM).

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Fig. 4b. Graphical Bethe-Salpater equation for Γ in the particle-particle channel. Lines are FDSM.

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Fig. 4c. Graphical Bethe-Salpater equation for Γ in the first particle-hole channel.

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Fig. 4d. Graphical Bethe-Salpater equation for Γ in the second particle-hole channel.

Fig. 5a. Some low-order contributions to irreducible block in (pp) channel. Lines are FDSM.

Fig. 5b. Some low-order contributions to irreducible block in first-(ph) channel.

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Fig. 5c. Some low-order contributions to irreducible block in second-(ph) channel.

$$\langle 12|\Gamma|34 \rangle = \langle 12|I_1|34 \rangle + \sum_{55'66'} \langle 16'|I_1|5'4 \rangle G_{5'5} G_{6'6} \langle 52|\Gamma|36 \rangle \quad (II.33a)$$

$$\langle 12|\Gamma|34 \rangle = \langle 12|I_2|34 \rangle + \sum_{55'66'} \langle 16'|I_2|35' \rangle G_{5'5} G_{6'6} \langle 52|\Gamma|64 \rangle \quad (II.33b)$$

I_1 and I_2 are the irreducible scattering amplitudes in the corresponding particle-hole channels.

Inspection of the Eq. (II.33) as well as the low-order contributions to I_1 and I_2 in Figs. 5b and 5c should convince one that I_1 and I_2 are simply related, viz.

$$\langle 12|I_1|34 \rangle = - \langle 12|I_2|43 \rangle \quad (II.34)$$

The above Bethe-Salpater (BS) equations illustrate several important points:

- i. The irreducible block or effective interaction between two nucleons in a many-nucleon system includes higher order contributions, e.g., polarization of a particle-hole pair in the core.
- ii. The effective particle-particle and particle-hole equations are different because they contain different higher order contributions.
- iii. Γ for ^{208}Pb contains information via its poles and residues about ^{210}Pb as well as ^{208}Bi . When the BS equation is constructed in a particular channel, a lot of information about other channels is hidden in the higher order, energy dependent contributions to the effective, or irreducible, block. For example, an attempt to describe the particle-particle correlations in ^{210}Pb includes three-particle-one-hole correlations within the block, J , as shown in Fig. 5a.

- iv. Note that some of the contributions to the I_2 block, which is not separable in the second particle-hole channel (sideways), are reducible in the first particle-hole channel (up and down).

3. The Hard-Core Solution

If we wish to do nuclear theory in a fundamental way, we must begin - at least, in the non-relativistic limit - with a nucleon-nucleon potential. There are innumerable local and non-local potentials which fit the nucleon-nucleon scattering phase shifts. Most of them have either hard cores ($V = \infty$ inside a certain radius) or have large (though finite) repulsive matrix elements between high momentum states. In either case, it is difficult to do only second or third order perturbation theory because the higher order terms one neglects are either infinite or at least large and non-convergent. This is in contrast to a number of phenomenological potentials used in many-body calculations which yield fairly good answers and converge nicely in perturbation series. The first task of diagrams is to describe how to eliminate the hard core. The solution was originally proposed by Brueckner¹⁰ and applied to nuclear matter. A reaction matrix G is defined as in Fig. 6a.

The reaction matrix treats the two-body problem to infinite order. It is a finite quantity as long as the intermediate states include all of the high momentum (or kinetic energy) states. This is because these are the two-nucleon configurations which "see" the hard-core. Thus, an infinite series of perturbation theory diagrams - each of which may itself be infinite - yields a finite sum. For purposes of further discussion, the G matrix will be defined in a model space.^{11,12} The orbitals or single-

Fig. 6a. Graphical equation for G-matrix. Ideally, intermediate lines should be exact one-nucleon Green's functions. In the usual approximation, they are unperturbed, SM.

Fig. 6b. In model space G-matrix calculation, intermediate pair contains at least one particle outside model space.

Fig. 6c. G-matrix in which lines enter at bottom, exit at top.

Fig. 6d. Simplified notation for Fig. 6c.

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particle states which have been taken as the solutions to the H_0 part of the Hamiltonian are classified as to whether they are inside or outside the model space. The occupied single particle levels and a roughly equal number of unoccupied levels closest to the Fermi level constitute this model space. The intermediate pair of lines in the particle-particle ladder in Fig. 6 may not both be in the model space.

We can derive a formal expression by applying the diagram rules:

$$\frac{i}{2\pi} \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) < 12|G|34 > =$$

$$\frac{i}{2\pi} \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) < 12|V|34 > + \left(\frac{i}{2\pi}\right)^2 \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4)$$

$$\frac{1}{2} \sum \int d\omega_5 d\omega_6 G_5^0(\omega_5) G_6^0(\omega_6) < 12|V|56 > < 56|G|34 > \quad (\text{II.35a})$$

This yields upon integration

$$< 12|G(\omega_1 + \omega_2)|34 > = < 12|V|34 > - \sum_{5>6} < 12|G(\omega_1 + \omega_2)|56 >$$

$$\frac{1}{\epsilon_5 + \epsilon_6 - (\omega_1 + \omega_2)} < 56|V|34 > \quad (\text{II.35b})$$

However, (II.35a) reminds us that the intermediate propagators are "undressed" Green's functions and that to eliminate the factor $\frac{1}{2}$ for identical lines, 5 and 6 we must avoid double counting intermediate configurations.

The idea of Hartree-Fock by the diagrammatic method is to choose U to exactly cancel the two-body "average field", i.e.,

$$\sum_{12}^{\text{HF}} = 0 \quad (\text{II.37})$$

Thus, we have a description of the shell model potential, and we do not have to include the first order corrections if there is interest in improving the single particle Green's function according to Dyson's Eq. (II.29).

Of course, this Hartree-Fock justification of the nuclear shell model is justified only if V can be treated with low orders of perturbation theory. We have just seen that V is not so well-behaved. However, the reaction matrix, G , which describes the effective interaction between two nucleons with a hard core, is finite, and, even if we insist on the bare NN potential having a singular behavior, the shell model can be justified. The Brueckner-Hartree-Fock (BHF) prescription is to calculate G using some form of approximate shell model starting energies, $\omega_1 + \omega_2 = \epsilon_1 + \epsilon_2$. Then the Hartree-Fock equations are solved using G instead of V . Since the ϵ_1 , ϵ_2 , and other "shell model" energies which are eigenvalues of the HF equation will be slightly different, the G matrix must be recalculated and the whole procedure iterated until self-consistency.

Given the BHF description of a finite nucleus, or, for that matter, nuclear matter, the total binding energy and nuclear density can be calculated. It is found⁵ that contributions from three- and four-body correlations to the total energy, as well as three-body forces and

relativistic correlations to the G matrix, are important - about 25% of the binding energy. However, if the G matrix is semiempirically corrected to account for such effects, the binding energy and radii of finite nuclei are quite well reproduced, especially in the local density approximation.¹³

It is of interest to examine why BHF yields a satisfactory explanation of the shell model. What we are asking is whether the hole orbits are really occupied enough to generate an average field seen by one proton or neutron. Since the hole itself is determined self-consistently, terms like have been included in the calculation of the single particle Green's function.

The G matrix is not instantaneous in time; its finite length depends on how long nucleons in model space orbits spend in collisions with high Fourier components, i.e., high lying particle states. Since it costs a lot of energy to be in such a configuration, by the uncertainty principle, At $\approx \hbar/\Delta E$, we deduce that this "time" should not be too long. In fact, the percentage non-occupation is calculated as κ , the defect integral, in Brueckner theory. It is of the order of 15%.¹⁴

In terms of Green's functions, if we add one nucleon to a nucleus in a certain orbit, there is an 85% chance that it will be there at a much later time.* There is the additional possibility that it may be in any of a number of states with high kinetic energies. These probabilities

* In a time-dependent formalism, there is a "dressing" time in which the system responds to the sudden addition of a new particle and settles down to equilibrium. After dressing, the additional particle is found 85% of the time in its original state.

reflect themselves in the residues and poles of the Green's function:

$$G_1(\omega) \approx \frac{.85}{\omega - \epsilon_1' + i\delta} + R_1(\omega) \quad (\text{II.38})$$

the so-called regular part, $R_1(\omega)$ has no low-lying (in energy, ω) singularities. ϵ_1' is, of course, somewhat different from the usual Hartree-Fock energy ϵ_1 , due to occupation probability corrections for the other states which generate the Hartree-Fock field.^{11,15}

As we have mentioned, HF or BHF (at low energies) is an independent particle approximation. It does not allow for long-range correlations to be taken into account. Such correlations may be included in order to obtain a more accurate picture of the ground state energy, density, etc. In particular, as was alluded in the section on rules, the ground state (GS) energy is the sum of unlinked clusters. Diagrams such as those in Fig. 7 denote ground state correlations.

Fig. 7. GS correlations. Lines are BHF. Blocks are reaction G-matrix.

5. Effective Interactions

Suppose that we have carried out a model-space G matrix calculation. We would like to know the two-body matrix elements of a more general scattering amplitude - namely the irreducible particle-particle and particle-hole blocks. These irreducible blocks would then constitute the effective interaction to be employed in calculations of $^{18}_0$ or $^{208}_{\text{Pb}}$, for

example, in which all the model space orbits would be active in the construction of the particle-particle or particle-hole configuration space. The lowest order contributions are shown in Fig. 8. The ingoing and outgoing lines - actually stubs, since the analytical expressions for these graphs will not include the ingoing and outgoing Green's functions - are inside the model space. Intermediate lines may be outside the model space except we are not allowed to calculate a graph in which two G matrices are connected by a pair of identical particle lines, when the pair is outside the model space (i.e., one or both lines is outside the space). Within the confines of the above logistical requirements, it is correct to use the energy-dependent G matrix as the "bare" interaction along with the usual diagram rules.

Figure 8b is the famous core-polarization correction introduced in the original work of Bertsch and of Kuo and Brown.¹⁶ Equation (II.39) is the formula which corresponds to diagram 8b; in the approximations that G, the single particle Green's function, is unperturbed, G^0 , and that the Brueckner matrix, G, is energy-independent.*

$$G_{3p-1h} = (-1) \frac{i}{2\pi} \sum_{56} d\omega_5 d\omega_6 \delta(\omega_3 + \omega_6 - \omega_1 - \omega_5) \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) \\ < 15 | G | 36 > \frac{1}{\omega_5 - \epsilon_5 + i\delta} \frac{1}{\omega_6 + \epsilon_6 - i\delta} < 62 | G | 54 > \quad (\text{II.39})$$

* Since the G matrix has a finite time-length, it is not energy independent. However, it is well-approximated by a density dependent delta function in space and time. This is because it describes nucleon-nucleon scattering at a radius which is less than the average distance between nucleons in a nucleus.

Fig. 8. Low-order (in G) contributions to (pp) irreducible amplitude. 1,2,3,4 are all inside model space. Intermediate propagators are FDSM, (shown below) and may be inside or outside model space.

Lowest order contributions to (ph) irreducible amplitude. (Shown below.)

The exchange diagrams of Fig. 8d, 8e. (Shown below.)

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* 8c will be double counted if we use the effective particle-particle amplitude in an RPA, rather than TDA calculations. For details, see later in this section.

The (-1) is due to the closed particle-hole loop. We neglect the energy dependence when integrating to obtain

$$\left(\frac{1}{2\pi}\right) \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) \sum_{56} \frac{\langle 15|G|36 \rangle \langle 62|G|54 \rangle}{\omega_3 - \omega_1 - \epsilon_5 - \epsilon_6}$$

Note that $\epsilon_5 + \epsilon_6 = (h_5 - \lambda) + (\lambda - h_6) = h_5 - h_6$, as in the more usual notation. (See the explanation accompanying Eq. (II.27).) It is common, for example, in an ^{18}O calculation, to approximate the denominator by $\epsilon_3 - \epsilon_1 - \epsilon_5 - \epsilon_6 \approx -2\hbar\omega$, where $\hbar\omega$ is the energy separating particles and holes.

Before going on, we note that if both of the G matrices are attractive - as they will, in general, turn out - the total contribution to the effect interaction is attractive, thereby giving more binding in ^{18}O . Figure 8e, the so-called "screening" correction to the effective particle-hole interaction, is repulsive, as may be shown with the same application of diagram rules and the same assumption for the signs of the G matrices.

This is perhaps the best time to warn that higher order particle-hole diagrams are not always antisymmetric. For example, the G matrix itself is antisymmetric, so Figs. 8d and 8h are identical except for a minus sign:

$$\langle 14|G|32 \rangle = - \langle 14|G|23 \rangle$$

The effect of exchanging lines 3 and 2 in Figs. 8e, f, or g is to produce a graph which is a contribution to the second particle-hole

channel and, therefore, not counted. Only if 3 and 2 are attached to the same G matrix, making the final vertex antisymmetric, is the whole diagram antisymmetric. Figure 9 shows some higher order contributions to the effective interactions, which are obtained when the (ph) pair is allowed to interact and the corner or "one-three" vertices are "dressed". There are also miscellaneous 3rd and higher order corrections. Such contributions are being evaluated by several groups,¹⁷⁻¹⁹ and it is only fair to state that it is not clear which higher order diagrams should be calculated or whether the higher orders converge at all. In addition, the more diagrams that are calculated, the worse agreement with experiment becomes.

Several recent efforts^{11,20,21} have been made to formulate a theory of folded diagrams for model space effective Hamiltonians in which the effective interaction is energy-independent and Hermitian. (The numerical feasibility and convergence of these techniques has not yet been investigated.) The formalism employed is not precisely that of Green's functions, although the language and diagrams are an alternative to Green's functions in the sense that they are a different way of transforming time-dependent perturbation theory.

6. Fermi Liquid Theory, Quasiparticles

The concept of "dressed" or "quasi" particles and holes and an interaction between them was originally proposed by Landau.²² The theory has been further developed in a superb text by Nozieres.⁴ The theory for finite systems has been given by Migdal.^{23,24}

We remind the reader that a Landau quasiparticle has nothing to do with superfluidity. The physical notion is that if we add a particle to a

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Fig. 2. Higher-order contributions to the particle-particle irreducible block.

system, it does not stay in its shell model state as it propagates through the system nor does it move independently of all the other nucleons (except for their average field). Rather it is "dressed" by a cloud of other nucleons which "correlate" their motion to that of our "quasiparticle".

One kind of correlation is the hard-core induced variety. We would expect the hard core to keep other nucleons away, giving rise to an "empty cloud". As a result of such anticorrelation, the nucleon has less (negative) potential energy; since the same effect (more positive total energy) could be accomplished by increasing slightly the kinetic energy $p^2/2m^*$, the effective mass, m^* , is less than the real mass, m . On the other hand, we can expect the longer range and attractive parts of the nuclear force to have the opposite effect. In particular, core polarization may result from the attractive interaction of the quasiparticle and the core. This manifests itself diagrammatically as the contribution to the self-energy shown in Fig. 10. The analytical expression is shown in Eq. (II.40), which is derived for the approximate case of C , the reaction matrix, being energy independent and the intermediate Green's functions being zeroth order.

$$\begin{aligned} \sum_{(\text{core pol.})} &= (-1) \left(\frac{i}{2\pi}\right)^2 \sum_{234} d\omega_2 d\omega_3 d\omega_4 \delta(\omega_1 + \omega_4 - \omega_2 - \omega_3) \delta(\omega_1' + \omega_4 - \omega_2 - \omega_3) \\ &\cdot \frac{1}{\omega_2 - \epsilon_2 + i\delta} \frac{1}{\omega_3 - \epsilon_3 + i\delta} \frac{1}{\omega_4 + \epsilon_4 - i\delta} < 14 | G | 23 > < 23 | G | 14 > \\ &= \sum_{234} \delta(\omega_1 - \omega_1') \frac{|< 14 | G | 23 >|^2}{\omega_1 - (\epsilon_2 + \epsilon_3 - \epsilon_4)} \end{aligned} \quad (\text{II.40})$$

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Fig. 10. Core polarization contribution to the self-energy. Intermediate lines are FDSM.

As a first order contribution to the one-body potential, core polarization is attractive and lowers the total energy. m^* is increased by core polarization because the cloud weighs more.

These various contributions to the self-energy not only shift the first order shell model energy. Because they reflect our quasiparticle correlating more and "shell-modelling" less, the single particle Green's function of Eq. (II.38) has even more residue strength distributed to other poles. Clearly, calculations of low-energy nuclear properties would be easier if we did not have to be concerned with that part of the Green's function whose analytic structure is remote in energy. This is the basic concept of the quasiparticle in Fermi liquid theory. Let $G_2^{(1)}(\omega_2)$ of a bare particle, 2, be given by

$$G_2^{(1)}(\omega_2) = \frac{z_2}{\omega_2 - \epsilon_2^{(1)} + i\delta} + R(\omega_2) \quad (\text{II.41})$$

$R(\omega_2)$, the regular part, is due to high energy hard-core processes discussed in a previous paragraph and also core polarization processes which do not seriously deplete the shell model strength, z_2 . Low-energy core polarization processes which do substantially redistribute the shell model strength will have to be calculated separately. The superscript (1) means that the zeroth order Green's function has been partially dressed. We shall assume that all further self-energy corrections such as core polarization affect only the analytic part and not the regular part. At this point, we have stated Landau's approximation in a form which is amenable to further approximation. The Fermi liquid theory^{22,23} has usually assumed

that all self-energy contributions only shift the dominant pole without seriously depleting the single particle strength,

$$\text{i.e., } G = G^1 \quad . \quad (\text{II.41a})$$

We shall refer to Eq. (II.41a) as the "quasiparticle approximation". In all of the Fermi liquid theory done so far, it has been assumed that the $\epsilon^{(1)}$, the quasiparticle energies, may be taken from a simple local one-body potential, such as the Nilsson potential. In this sense, Fermi liquid theory is a phenomenological theory.

The quasiparticle Green's function, g^0 , is given by

$$g_2^0 = \frac{1}{\omega_2 - \epsilon_2^{(1)} + i\delta} \quad (\text{II.42})$$

The product of two Green's functions

$$G_1^{(1)} G_2^{(1)} = z_1 z_2 g_1^{(0)} g_2^{(0)} + B \quad (\text{II.43a})$$

where $B = R_1 R_2 + z_1 g_1^0 R_2 + z_2 g_2^0 R_1$ is regular (no poles) in the energy variables, ω_1, ω_2 . (The + sign when 1 and 2 are both particles, the - sign when one is a particle, the other a hole.)

Equation (II.43a) must be supplemented by the equations relating to exact Green's function product GG to g^0 and B . At this point, therefore, we introduce several approximations which will be made throughout this thesis and have generally been made in the theory of finite Fermi systems.

$$\text{a. } G_1 G_2 = z_1 z_2 g_1 g_2 + B \quad (\text{II.43b})$$

- i.e., all further corrections to the self energy will not affect B. This is reasonable because B is only dependent on high energy processes while further corrections to the self energy will be low-energy in nature. Equation (II.43b) is a definition of the fully dressed quasiparticle. The implication of (II.43b) is that z_1 and z_2 are the residue strengths remaining after the Brueckner and other high energy processes. g_1 and g_2 , which will be calculated from the effective quasiparticle interaction discussed later, will, of course, contain more than one pole and the residues $z_1^{(i)}$ and $z_2^{(i)}$ will be, in effect, reduced residues to be multiplied in the end by the z_1 and z_2 factors (if we want GG and not gg).
- b. Further corrections to Σ will be diagonal in the single particle indices, i.e., $\Sigma_{12} = 0$ unless $1 = 2$. This implies g and G may be written g_1 and G_1 . This assumption is introduced mostly for convenience in writing equations. It can be removed in practice without much difficulty.
- c. All residues z_1 , z_2 , etc. appearing in (II.43) shall be set equal to z . The assumption is that the single particle strength which is depleted by high energy is always about the same ($\sim 15\%$).

Thus, g^0 and g are the Fourier transforms of the approximate and "exact" quasiparticle Green's function.⁴

$$g_{12}(t) = -i \langle \tilde{0} | T \{ c_1(t) c_2^\dagger(t) \} | \tilde{0} \rangle \quad (\text{II.44})$$

where c_1 and c_2^+ are the fermion destruction and creation operators of quasiparticle and quasiholes in states 1 and 2;

$$\begin{aligned} c_1^+ |\tilde{0}\rangle &= |\phi_1\rangle & 1 = \text{quasiparticle, or } n_1 = 0 \\ &= 0 & 1 = \text{quasi-hole, or } n_1 = 1 \end{aligned}$$

The quasishell-model states $|\phi_1\rangle$ are, as we have mentioned, the partially elementary dressed excitations which evolve as Brueckner and high-energy core polarization (as well as a host of other) graphs are included. The effective quasiparticle-particle interaction, Γ^ξ , is defined*

$$\Gamma^\xi = J + J B_{pp} \Gamma^\xi \quad (II.45a)$$

The effective quasiparticle-hole interaction, Γ^ω , is defined

$$\Gamma^\omega = I + I B_{ph} \Gamma^\omega \quad (II.45b)$$

To obtain the renormalized Bethe-Salpater equation in the particle-particle channel, one should multiply (II.32) to the left by

$$1 + \Gamma^\xi B_{pp}$$

to obtain

*The reader may note that we have gone over to an abbreviated notation, in which explicit indices and integrations over intermediate energies are deleted, similar to the convention for vector and matrix multiplication.

$$\Gamma = \Gamma^{\xi} + z^2 \Gamma^{\xi}(g g) \Gamma \quad . \quad (II.46a)$$

Similarly, in the particle-hole channel,

$$\Gamma = \Gamma^{\omega} + z^2 \Gamma^{\omega}(g g) \Gamma \quad (II.46b)$$

The products $z^2 \Gamma^{\xi}$, $z^2 \Gamma^{\omega}$ are usually abbreviated F^{ξ} and F^{ω} . Equations (II.46) are the basic equations of Fermi liquid theory. They express the fact that particles in a liquid - that is, where orbital motion is perturbed, unlike a gas - interact differently than they would in a vacuum. This is because of the surrounding medium, which is rather dense.

Aside from the fact that we must either calculate Γ^{ξ} and Γ^{ω} - a very difficult task, and one which has not ever been successfully completed - or parameterize them - a less satisfying, though more practical, approach - Eqs. (II.46) yield the same nuclear structure information as Eq. (II.32) and (II.33) except that the Green's functions are more simple. They do not contain any high energy contributions - though they still may have more than one pole, as we shall discuss later.

We shall conclude this discussion of Fermi liquid theory by noting that most calculations²⁵⁻²⁷ so far have assumed that $g = g^0$ and that Γ^{ξ} and Γ^{ω} = density dependent delta functions. That is, only one pole quasiparticle Green's functions have been employed and any distinctly long-range characteristics of the quasiparticle interactions have been ignored.

7. Configuration Space Truncation

Feynman pictures provide a simple way of truncating configuration spaces to manageable proportions. As shown in Fig. 11a, the active

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Fig. 11a. Configuration space truncation.

particle-hole space has already been truncated to a "model space" by use of Brueckner theory, and we have given well-defined equations for obtaining Γ^ξ and Γ^ω within this model space. It may, however, become necessary to calculate with a subset of orbitals from the model space. How are Γ^ξ and Γ^ω modified? In exactly the same way we obtained a model space G from a full space V , we can obtain configuration space Γ'' and Γ' from the model space Γ^ξ and Γ^ω . The graphical equation is shown in Fig. 11b, in which intermediate configurations are outside the configuration space (i.e., at least 5 or 6 has been truncated). The exact expressions for the renormalization are

$$\Gamma'' = \Gamma^\xi + z^2 \Gamma^\xi (g\alpha) \Gamma'' \quad (\text{II.47a})$$

$$\Gamma' = \Gamma^\omega + z^2 \Gamma^\omega (g\alpha) \Gamma' \quad (\text{II.47b})$$

The Bethe-Salpater equations for Γ'' and Γ' are

$$\Gamma = \Gamma'' + z^2 \Gamma'' (g\alpha) \Gamma \quad (\text{II.48a})$$

$$\Gamma = \Gamma' + z^2 \Gamma' (g\alpha) \Gamma \quad (\text{II.48b})$$

In a manner similar to F^ξ and F^ω , F'' and F' are equal to $z^2 \Gamma''$ and $z^2 \Gamma'$. $z^2 \Gamma$ is the reducible scattering amplitude between quasiparticles. This renormalization is not the same as the quasiparticle renormalization, which would have been neglected had the regular part, R , of the Green's function been zero. Also F'' and F' are energy-dependent as were F^ξ and F^ω , the configuration space truncation, however, contributing additionally to the energy dependence, or time retardation.

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Fig. 11b. Graphical equation for (pp) interaction in truncated configuration space. 1,2,3,4 are inside space. At least, 5 or 6 is outside configuration space but inside the model space. Intermediate lines are exact propagators with Landau renormalization, FDLQ.

8. Particle-Particle and Particle-Hole Vibrations, RPA

We have discussed so far the use of diagrams in the derivation of several quantities of theoretical importance, such as the effective interaction, the Brueckner-Hartree-Fock description of the ground state Slater determinant, etc. In further sections of this thesis, we will be interested in a microscopic description of phonon-quasiparticle coupling. As a simple application of Green's function techniques, let us analyze the RPA equations, which describe phonons to lowest order.

In the particle-particle channel, we have

$$(z^2\Gamma) = F'' + F''gg(z^2\Gamma) \quad (\text{II.48a})$$

F'' is the interaction between two quasiparticles inside the configuration space. We are interested in the evolution of two quasiparticles, we define k , the two quasiparticle Green's function in a manner similar to K , the two particle Green's function.

$$k_{1234} \equiv \langle \tilde{0} | c_1(t_1) c_2(t_2) c_3^+(t_3) c_4^+(t_4) | \tilde{0} \rangle \quad (\text{II.49})$$

Figure 12 presents a summary of what may happen to the two quasiparticles in the presence of one another. Recall that most higher order processes are already included in F'' . The 4p-2h type correlations shown by the backwards diagram are unique in that the string or "ladder" is still separable. The corresponding equation is

$$k_{1234} = g_{13}g_{24} - g_{14}g_{23} + \frac{1}{2} \sum_{565'6'} g_{15}g_{26'} F''_{5'6'56} k_{5634} \quad (\text{II.50})$$

The details of a more general derivation for quasibogolons are contained in subsection III.C.4. However, the main idea is that the Green's functions can be analytically expressed in terms of spectroscopic amplitudes and eigenenergies of the N12 nuclear systems, see Eqs. (II.18,19). And a system of matrix equations can be obtained relating ($E_{N\pm 2}^{(S)} - E_N^{(0)}$) and the generalized density matrices, X, Y.

$$\left. \begin{aligned} \frac{X_{12}}{A_{12}(\omega_S)} &= \sum_{34 > \epsilon_F} F''_{1234} X_{34} + \sum_{34 < \epsilon_F} F''_{1234} Y_{34} \\ \frac{Y_{12}}{A_{12}(-\omega_S)} &= \sum_{34 > \epsilon_F} F''_{1234} X_{34} + \sum_{34 < \epsilon_F} F''_{1234} Y_{34} \end{aligned} \right\} \begin{cases} 12 > \epsilon_F, N \pm 2 \\ 12 < \epsilon_F, N \pm 2 \end{cases} \quad (\text{II.51})$$

where

$$A_{12}(\omega) = \frac{i}{2\pi} \int_{-\infty}^{\infty} d\omega_1 d\omega_2 \delta(\omega - \omega_1 - \omega_2) g_1(\omega_1) g_2(\omega_2)$$

$\approx (\omega - \epsilon_1 - \epsilon_2)$ in the quasiparticle

approximation, $1,2 > \epsilon_F$.

$$A_{12}(-\omega) \approx -(\omega + \epsilon_1 + \epsilon_2) \quad 1,2 < \epsilon_F.$$

$$X_{12} = \langle S_{N+2} | c_1^+ c_2^+ | \tilde{0} \rangle \quad 1,2 > \epsilon_F, \omega > 0$$

$$Y_{12} = \langle S_{N+2} | c_1^+ c_2^+ | \tilde{0} \rangle \quad 1,2 < \epsilon_F, \omega > 0$$

$$X_{12} = \langle S_{N-2} | c_{\bar{2}} c_{\bar{1}} | \tilde{0} \rangle \quad 1,2 > \epsilon_F, \omega < 0$$

$$Y_{12} = \langle S_{N-2} | c_{\bar{2}} c_{\bar{1}} | \tilde{0} \rangle \quad 1,2 < \epsilon_F, \omega < 0$$

Note that the X's and Y's are the generalized two-quasiparticle density matrix. The quasiparticle is the dressed shell model excitation. If we make what is generally called the "quasiparticle approximation", one pole in the quasiparticle, $1/A(\omega)$ simplifies to a linear expression and we obtain the usual RPA equations for the particle-particle channel.²⁸

The matrix equations themselves are the product of an analytic argument. However, as shown in Fig. 12, there is a very concrete physical picture in diagrams of what the eigenstates, or phonons, are.

If we put two quasiparticles into the system, they will correlate and very quickly collide to yield a pair of quasiparticles in some different configuration. This process occurs repeatedly because the basis states - or two-quasiparticle configurations - are not eigenstates. The eigenstate is approximately a linear combination of such configurations - however, it is much easier to think of linear combinations as population probabilities in time-dependent perturbation theory.

The use of such a picture also enables us to include 4-particle 2-hole and higher configurations without diagonalizing them explicitly. The "backwards" diagram in Fig. 12a represents a linear combination of

$$\begin{aligned}
 K &= K_{\text{free}} + \text{[diagram of a single wavy line with arrows at both ends]} \\
 &= \text{[diagram of two parallel vertical lines]} + \text{[diagram of two crossed lines]} + \text{[diagram of two loops]} + \text{[diagram of two crossed loops]} + \text{[diagram of two stacked loops]} + \\
 &\quad \text{[diagram of a loop with an arrow]} + \dots + \text{[diagram of a long chain of loops]} \\
 &= \text{[diagram of two parallel vertical lines]} + \text{[diagram of two crossed lines]} + \text{[diagram of a loop with an arrow and a shaded region]} + \text{[diagram of a curved shaded region]}
 \end{aligned}$$

Fig. 12a. RPA diagrams. Intermediate lines are FDLQ. For convenience,

$$\text{[diagram of two crossed lines]} = \Gamma''$$

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Fig. 12b. Microscopic analysis of two simultaneous phonons seldom yields a Pauli exclusion principle violation.

$2n + 2$ particles - $2n$ hole configurations. The "backwards" part of the diagram is initiated by ground state correlations, or spontaneous decay of the bare vacuum into a $2p$ - $2h$ configuration. The two quasiparticles correlate independently as well as the two quasiholes. The quasiholes, however, are filled at a later time when they "collide" with two quasiparticles.

Another interesting point is that if the two quasiparticles can easily correlate, that is scatter and rescatter many times, then when we start out two pairs of quasiparticles and permit them to independently propagate, there will be very few Pauli principle violations, or simultaneous occupations of an orbit by a quasiparticle from both pairs, Fig. 12b. Thus each pair of quasiparticles is approximately a boson, obeying the usual boson commutation relation. These collective states are thus called phonons, or, in analogy to the harmonic oscillator bosons, vibrations. It should be pointed out that the addition of two quasiparticles or holes may evolve into many possible phonons. However, the lowest in energy (occasionally the highest) is generally most important, not only because it is most easily seen in experiment, but also because it is coherent and often has high transition probabilities in an external field. For example, it is well-known that the pairing vibrations around ^{208}Pb have high (p,t) cross sections.²⁹

The correlation of a quasiparticle and a quasihole often results in a high $E2$ or $E3$ γ -transition probability, hence the name quadrupole and octupole vibrations. The particle-hole RPA equations can be obtained similarly or by taking the appropriate limit of the quasibogolon RPA equations, Eq. (III.34).

E. Superfluids and Graphical Techniques

As a final application of Green's functions, we devote a section to the subject of superfluid nuclei. We do this for several reasons

1. To remind the reader of the difficulties away from closed shells. Are such nuclei really any different?
2. To indicate that Green's functions provide a very general framework within which to treat finite nucleus approximations - including the anomalous self-energy or gap.
3. To introduce the rules for calculating with bogolons.

Away from closed shells, let us take the neutrons in ^{120}Sn as an example, the shell-model vacuum is not even approximately a good ground state. This is easily seen by considering Fig. 13a which shows the 2 particle-2 hole admixture into the ground state wave function. In first order perturbation theory, this is just

$$< 12 | G | 34 > / (\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4) \quad (\text{II.52})$$

These and other admixtures are large for two reasons

1. Matrix elements of the type $< 1-1 | G | 3-3 >$ are rather large for the short-range effective nuclear interaction.
2. $\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4$ is small. It doesn't take much energy to promote two nucleons from occupied orbits to unoccupied ones.

If one were to diagonalize the effective particle-particle interaction in a space of such $2n$ -particles- $2n$ -hole states, he would find³⁰ that the ground state is heavily weighted toward excited configurations. A typical wave function is shown in Fig. 13b. Suppose we performed a (d,p) reaction on ^{120}Sn ; we would find the stripping cross section into the orbitals below the Fermi surface to be non-zero, since they are

Fig. 13a. Lowest order two-particle two-hole contribution to ground state wave function.

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Fig. 13b. Important configurations in ground state of nucleus away from closed shell.

partially empty. (It is usual to call the percentage "emptiness" in stripping reactions, u^2 .) Similarly, we would find (d,t) pick-up cross sections from orbitals above Fermi energy to be finite. (The percentage "fullness" of these orbits is called v^2 .)

There are several reasons why ^{120}Sn poses a difficulty. First, to describe the neutrons outside the $N = 50$ closed shell accurately would take a 20 nucleon Green's function - or at least we would need this to calculate Γ^ω and Γ^ξ accurately. Secondly, intelligent perturbation theory-making a simple approximation of the 20 nucleon Green's function - is difficult because it is so easy to become confused. In addition to these very practical difficulties, there are some formal difficulties.^{4,6} One is that in the rigorous "adiabatic" derivation of perturbation theory, the transition from bare shell-model vacuum to actual ground state will be accompanied by a phase transition - at least in an infinite system - and such a sudden change destroys the adiabatic boundary conditions.

Does this mean we should abandon perturbation theory for nuclei away from closed shells? The way out of our dilemma was originally proposed by Bardeen, Cooper, and Schrieffer,³¹ and also by Valatin and Bogoliubov.³² It was later³³ realized that the pairing correlations are what distinguishes these nuclei. If there were only one pair, we could describe it as a boson (see previous subsection on RPA). That there are two or five doesn't matter because the pairing correlations are strong and the boson configuration $(p^+)^\dagger |0\rangle$ does not often disobey the Pauli Principle. If

$$p^+ = \sum_{\alpha} \phi_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}^{\dagger} \quad ,$$

we approximate the ground state as $e^{\mathcal{P}^+} |0\rangle$ and determine the ϕ_α such that the total occupation (which was called v^2 in (d,t) reactions) of all the orbits is equal to the number of particles in the ground state. Most of the damage of replacing $(p^+)^n$ by $e^{\mathcal{P}^+}$ is taken care of by optimizing the ϕ_α ; the one major drawback is that $e^{\mathcal{P}^+}$ will still contain $(p^+)^{n+1}$, etc., and therefore the particle-number will fluctuate. We must therefore add a Lagrange multiplier condition to the Hamiltonian. In particular, the Hamiltonian is partitioned

$$H' = H - \lambda N = H'_0 + H'_1$$

$$H'_0 = \sum_{\alpha\beta} t_{\alpha\beta} c_\alpha^\dagger c_\beta + U_{\alpha\beta} c_\alpha^\dagger c_\beta - \lambda c_\alpha^\dagger c_\beta + S_\beta \Delta_{\alpha\beta} c_\alpha^\dagger c_{-\beta}^\dagger + S_\beta \Delta_{\alpha\beta} c_\alpha c_{-\beta} = T + U + \Delta - \lambda N \quad (\text{II.53})$$

$$H'_1 = V - U - \Delta$$

where α and β label single particle states. H'_0 may be diagonalized by defining an "undressed" or "shell-model" bogolon

$$A_1 = \sum_\alpha (U_1^\alpha c_\alpha - S_\alpha V_1^\alpha c_{-\alpha}^\dagger) \quad (\text{II.54a})$$

This generalized Bogoliubov-Valatin transformation³² can be specialized if we assume

$$t_{\alpha\beta} + U_{\alpha\beta} = h_1 \delta_{\alpha\beta} \delta_{\alpha 1} \text{ and } \Delta_{\alpha\beta} = \Delta_1 \delta_{\alpha\beta} \delta_{\alpha 1}$$

that is that α and β label the shell model (or Brueckner-Hartree-Fock) states and that pairing does not change the average potential. This is a good assumption for heavy nuclei.³⁴

In the special Bogoliubov-Valatin transformation

$$A_1 = U_1 C_1 - S_1 V_1 C_{-1} \quad (\text{II.54b})$$

where

$$U_1 = \frac{1}{2} \left[1 + \frac{h_1 - \lambda}{E_1} \right]$$

$$U_1^2 + V_1^2 = 1$$

$$E_1^2 = (h_1 - \lambda)^2 + \Delta_1^2$$

$$N = \sum_{1>0} 2 V_1^2$$

The transformed Hamiltonian H_0 is

$$H_0 = U + \sum_{\pm 1} E_1 A_1^+ A_1 \quad (\text{II.55})$$

That is, it is analogous to the one-body shell-model Hamiltonian with the property that

$$(H_0 - U) A_1^+ A_2^+ \dots A_3^+ |BCS\rangle = (E_1 + E_2 + \dots + E_3) |BCS\rangle \quad (\text{II.56})$$

suggesting that 0, 1, 2, and 3-bogolon configurations will form a convenient set of basis states. The BCS vacuum is defined in terms of the bare particle vacuum.

$$|BCS\rangle = \prod_{l>0} (U_l + V_l S_l c_l^+ c_{-l}^+) |bare\ vacuum\rangle \quad (II.57)$$

The BCS vacuum is actually a vacuum of what we choose to call "undressed" "shell-model bogolons". *

$$A_1 |BCS\rangle = 0$$

The creation and annihilation operators for shell-model bogolons obey the same commutation relations as the normal operators:

$$[A_1, A_2^+] = \delta_{12} \quad [A_1, A_2] = 0 \quad (II.58)$$

We have introduced bogolons without the aid of Green's functions for several reasons. The BCS vacuum is just another starting point for perturbation theory. However, it is much better behaved. One-bogolon excitations are separated in energy by the gap, Δ . Hence, we can expect perturbation theory convergence to be much better since the denominators are of the order of several MeV. Secondly, the u^2 and v^2 occupation probabilities which are experimentally observed are very naturally explained.³⁵ It is possible to derive the pairing formalism with the aid of anomalous Green's functions.³⁶ The reader may wish to examine

* We will later call superfluid Landau Quasiparticles quasibogolons.

Refs. 37 and 38 for a complete summary. In general, one defines both the normal Green's function

$$G_{12} = {}_N \langle \tilde{0} | T \{ c_1(t_1) c_2^+(t_2) \} | \tilde{0} \rangle_N \quad (\text{II.59})$$

and the anomalous Green's functions

$$F_{12} = {}_{N+2} \langle \tilde{0} | T \{ c_1^+(t_1) c_2^+(t_2) \} | \tilde{0} \rangle_N$$

It is then possible to derive a coupled set of Dyson's equations. The zero-order solutions are the usual BCS or Hartree-Fock Bogoliubov equations.³⁹ Nevertheless, the formalism offers a systematic framework for a higher order treatment of pairing. Also, one can treat two- or three-particle correlations using G and F.

The main result of anomalous Green's functions is that one obtains (after Landau and configuration space renormalization) a so-called gap equation which is very similar to the Hartree-Fock equation for the one-body potential.

$$\Delta_1 = \sum_{2>0} \langle 1-1 | F^{\xi} | 2-2 \rangle \frac{\Delta_2}{E_2} \quad (\text{II.60})$$

Thus, both h_1 and Δ_1 are defined self-consistently so that we do not have to employ any one-body Hartree-Fock insertions in the single-particle propagator.

Using the anomalous Green's functions, it is possible to rederive the rules of perturbation theory and carry out the Brueckner procedure, as

well as Landau and configuration space renormalization. It is common to assume that all such high energy processes do not depend on the pairing field, which is much weaker than the shell-model or Brueckner-Hartree-Fock field.

Throughout the remainder of this work, we shall employ the bogolon formalism. In this method, the Green's functions are those of quasi-bogolons:

$$\tilde{e}_{\alpha\beta} = \langle \tilde{0} | T \{ a_{\alpha} a_{\beta}^{\dagger} \} | \tilde{0} \rangle \quad (\text{II.61})$$

and the interaction between quasibogolons is a linear combination of the irreducible particle-particle and particle-hole blocks. Let us state the assumptions and the rules of this formalism. First the assumptions,

1. F^E and F^W do not depend on F , the anomalous Green's function. Superfluidity is a local energy phenomenon at $\epsilon \approx \epsilon_F$. The renormalization process and higher order graphs should not be too sensitive to this region.
2. $g_{\alpha\beta}$ is diagonal in α and β , as before.
3. The effects of the anomalous self-energy are accurately included by the BCS u 's and v 's.
4. High-order contributions to the self-energy may be calculated in the bogolon formalism.
5. The average residue and regular part of the Green's function are, as before, z and $R(\omega)$.

The advantage of the bogolons is that we only have to worry about one kind of elementary excitation, which is a linear combination of a particle and a hole. The disadvantages are that we have restricted ourselves to a description of Δ which does not depend on core excitations or the analytic structure of the regular self-energy, Σ . Higher order

processes such as core polarization may never be incorporated into the pairing field without abandoning the bogolon formalism and going back to normal and anomalous Green's functions. Also the phonon exchange diagrams which are among the lowest order contributions to irreducible (pp) and (ph) blocks should be sensitive to the details of pairing close the Fermi surface, in particular the energy and wave functions of the $2+$ vibrations. We have assumed they are not, in order to approximate the block which is "irreducible" in the bogolon-bogolon channel as a linear combination of (pp) and (ph) blocks.

The rules for calculating graphs are:

1. Draw all topologically different graphs
2. With every line associate the quasibogolon Green's function, $\tilde{g}$.
3. With the interaction of bogolons, we associate a block from which exit four lines. The block has associated with it the factor $\frac{i}{2\pi} \delta(\omega_{in} - \omega_{out})$. The matrix element of the block is a linear combination of particle-particle and particle-hole interactions and the coefficients are the u 's and v 's which are associated with each of the four lines.

For example,

is the interaction between the bogolon configurations $|12\rangle$ and $|34\rangle$.

In the particle-particle channel we have the contribution

$$u_1 u_2 u_3 u_4 \langle 12 | P'' | 34 \rangle \text{ and in the hole-hole channel } v_1 v_2 v_3 v_4 \langle 12 | P'' | 34 \rangle.$$

In the second particle-hole channel, if 1 and 3 are particles and 2 and 4 holes, or vice versa, we get the contribution

$(u_1 u_3 v_2 v_4 + v_1 v_3 u_2 u_4) < 1\bar{4}|F'|2\bar{3} >$. For the case in which 1 and 4 are holes, we get $(u_2 u_3 v_1 v_4 + u_1 u_4 v_2 v_3) < 1\bar{3}|F'|2\bar{3} >$ times a minus sign, which comes because we switch (commute) 3 and 4 to get the particle-hole matrix element for both configurations having the particle on the left and the hole on the right. Thus we have, after configuration space truncation,

$$\begin{array}{c} 1 \uparrow \uparrow 2 \\ | \text{P} | \\ 3 \downarrow \downarrow 4 \end{array} = < 12|P|34 >$$

and

$$\begin{array}{c} \diagup \quad \diagdown \\ | \text{Q} | \\ \diagdown \quad \diagup \\ 1 \quad 2 \quad 3 \quad 4 \end{array} = < 12|Q|34 >$$

where

$$< 12|1\pm Q|34 > \xi_{12}^{\pm} \xi_{34}^{\pm} < 12|F''|34 > + \eta_{12}^{\pm} \eta_{34}^{\pm}$$

$$[S_2 S_4 < 1-4|F'|-2\ 3 > \mp S_2 S_3 < 1-3|F'|-2\ 4 >]^*$$

$$\xi_{12}^{\pm} = (u_1 u_2 \mp v_1 v_2) \quad (\text{II.62})$$

$$\eta_{12}^{\pm} = (u_1 v_2 \pm v_1 u_2)$$

For the transitions from one to three and three to one bogolons, we shall anticipate the discussion of another section and employ irreducible particle-particle or particle-hole blocks according to the arbitrary

*Note, we have separated (ph) diagrams in the second (ph) channel to agree with Migdal.

convention: if 1 and 2 are both particles (or holes) use F'' ; else F' .

In that case

$$\begin{array}{c} \uparrow 1 \\ \uparrow 3 \\ \boxed{R} \end{array} \begin{array}{c} \uparrow 2 \\ \diagup 4 \end{array} \equiv S_4 < 12 | R | 3-4 >
 \qquad
 \begin{array}{c} \uparrow 4 \\ \boxed{S} \\ \uparrow 1 \end{array} \begin{array}{c} \diagup 3 \\ \uparrow 2 \end{array} \equiv S_3 < 12 | S | 3-4 >$$

$$\begin{aligned}
 < 12 | R \pm S | 3-4 > = - \xi_{12}^{\pm} \eta_{34}^{\pm} < 12 | F'' | 3-4 > \\
 &+ \eta_{12}^{\pm} \xi_{34}^{\pm} [S_2 S_{-4} < 14 | F' | -23 > \mp S_2 S_3 < 1-3 | F' | -2-4 >]
 \end{aligned}
 \tag{II.63}$$

The symmetry properties of P, Q, R, S are

$$< 12 | P | 34 > = - < 12 | P | 43 >, \text{ similarly for Q }
 \tag{II.64}$$

and

$$S_4 < 12 | R | 3-4 > = - S_4 < 12 | S | -43 >
 \tag{II.65}$$

In the sections (II.D and II.E) on applications of Green's functions, the use of diagrams has been illustrated by a series of approximations to the nuclear many-body problem, starting from the bare N-N interaction and proceeding through Hartree-Fock, renormalized interactions, and RPA. We have neglected to show the ease with which graphical techniques may be used to describe quasiparticle-phonon coupling or the response of a nucleus to an external field (e.g., electromagnetic). This was to avoid repetition since Sec. III and Appendix B are devoted exclusively to these topics. The reader may wish to proceed to the Appendix before reading the next section.

III. THEORY---SELF-CONSISTENT QUASIPARTICLE-PHONON COUPLING (SCQPC)

In this section, we describe a self-consistent theory in which quasiparticles and phonons are coupled. We divide the discussion into several parts: first, the concept of vibration-particle coupling is reviewed with the help of Feynman diagrams, and some complications are discussed with an eye toward their physical implications; second, we analyze the equation for the quasiparticle self-energy with the aim of developing a comprehensive approximation; in the third subsection we present the equations of SCQPC; finally, a subsection is devoted to the calculations of transition amplitudes in a quasiparticle-phonon coupled system. The extensive literature of QPC is surveyed in the Appendix.

A. Concepts in Core-Particle Coupling

The usual classification of nuclear motion is into three categories: single-particle motion, vibrational motion, and, in many cases, nuclear rotation.⁷ In zeroth order, one can imagine these as independent, normal modes. Disregarding rotation and focussing our attention on vibrations and shell-model orbital motion, we envision a vibrating nuclear core and a proton or neutron orbiting around the core. See Fig. 14a. The amplitude and frequency of the core motion ought to be about the same as without the additional nucleon since there are already so many nucleons participating in the vibrational correlation. Conversely, the average field seen by the orbiting nucleon should not be much affected by the small amplitude motions of the core.

However, we know that the core and the particle are not completely independent because of the strongly attractive part of the nuclear

Fig. 14a. Orbiting particle and vibrating core.

Fig. 14b. A beta vibration is characterized by stretching along the Z-axis.

Fig. 14c. The $\frac{1}{2}[660]$ Nilsson orbital is very prolate, i.e., cigar-shaped along the Z-axis.

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interaction, and some polarization of the nucleus should be expected, much in the same manner as the moon polarizes the oceans on earth to produce tides. Indeed, one might anticipate certain favorable situations in which the path of the orbital motion is parallel to the collective movement. For example, in a deformed nucleus the beta vibration might be expected to couple strongly to motion of a nucleon in the $1/2[660]$ orbital, as shown in Figs. 14b,c. We can analyze this very fundamental nuclear process in a microscopic way, using diagrams. First, we recall that a shape vibration is characterized as a collective particle-hole state. This is shown in Fig. 15a.* One of the first questions we can ask is what happens when a phonon and a quasiparticle propagate simultaneously as in Fig. 15b. Several diagrammatic possibilities are shown in Figs. 15c, d,e.

Figure 15c, which is called the core polarization process, is important to the calculation of the quasiparticle self-energy because, when we put a shell-model particle into the nucleus, there is a good chance that it will scatter into a different configuration. This lowering of probability for stable single-particle propagation reflects itself in the distribution of residues of the Green's function. There is single-particle "strength" at several poles, or eigenenergies of the odd-mass

* In connection with the phonon propagators shown in Fig. 15 and elsewhere in this thesis, the following graphical rules apply:

1. $K(\omega) = 1/(\omega - E_S + i\delta)$. If an arrow is drawn, it always points upwards.
2. Phonon-particle vertices are calculated microscopically as in Sec. III.C.1 or parameterized with the units of energy. They are accompanied by the

usual $\frac{1}{2\pi} \delta(\sum \omega_{in} - \sum \omega_{out})$ factor.

nucleus, because single-particle and core-coupled states mix. This explains the satellite peaks in one-nucleon stripping and pickup reactions.⁴⁰

Figure 15d will be typical of what we will refer to as third order phonon-quasiparticle coupling. That is, first, the effective interaction acts to uncouple the phonon. Secondly, either the decoupled quasiparticle or hole interacts with the extra quasiparticle. Thirdly, the quasiparticle and hole recouple to a phonon excitation.

Figure 15e, in addition to its rather general property of recoupling particles and phonons, takes into account certain violations of the Pauli Principle. To see this we construct in Fig. 16a a partially labelled phonon + particle. This configuration might be present, for example, as part of the core polarization diagram in Fig. 15c. Note that the particle-hole configuration $|\alpha\beta\rangle$ cannot be excited while α is occupied. This impossibility---it is essentially a violation of the Pauli Principle---is not accounted for if we permit the phonon to be independent of the quasiparticle, α . However, the diagram shown in Fig. 16b (same as Fig. 15e) exactly cancels the illegal contribution to the independent phonon-plus-quasiparticle shown in Fig. 16a. The cancellation is exact because all the two-body matrix elements are the same, the only arithmetic difference between the two diagrams being that there is one less particle-hole closed loop in Fig. 16b, hence a -1 factor. We shall call it an "exchange" diagram. (The general prescription for exchange diagrams is shown in Fig. 16c.) The need for exchange diagrams occurs in the case of the $1/2[660]$ neutron and the β -vibration of ^{153}Sm . The

Fig. 15. For simplicity, we draw the particle-hole block as $\times$, as well as the QPC vertex. Lines may be taken to be LQ.

(a) particle-hole vibration.

(b) quasiparticle plus vibration.

(c) core-polarization.

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- (d) "Third-order coupling" of quasiparticle-phonon configurations.

- (e) "Second-order" recoupling (or EPV) diagram.

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Fig. 16a. Pauli exclusion principle violation in propagation of quasi-particle and rhomon.

(b) Exchange diagram (same as Fig. 15d) which corrects for exclusion principle violation (EPV) in Fig. 16a.

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(c) Prescription for exchange diagrams: interchange endpoints of lines.

Fig. 73. Diagrams of Hamamoto which give rise to splitting of ^{209}Bi multiplet. Lines are SM.

Fig. 74. Mixing of j (single particle) states by off-diagonal terms in the self-energy.

Fig. 75. This violation of the Pauli Principle is cancelled by Fig. 73c.

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Fig. 17a. Quasiparticle plus ground state correlation, in which EPV occurs.

Fig. 17b. "Backwards" diagram which cancels Fig. 17a.

Fig. 18. Two-phonon contributions to self-energy.

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Fig. 19. Renormalization of RPA by QPC.

Principle interference can raise the excited core-plus-quasiparticle energy, as well as the unexcited core-plus-quasiparticle energy. We mention briefly the possibility that after the propagating quasiparticle excites a core vibration, the quasiparticle-plus-core may polarize yet another core mode. If the second excitation de-excites first, we get the "nested" diagram shown in Fig. 18a. If the original core excitation disappears first we obtain the "staggered" diagram shown in Fig. 18b. Since core polarization fractionates single-particle strength, these higher order polarizations will further redistribute the pole strengths.

Clearly, the odd-mass nuclear energy levels (and wave functions) will be sensitive to the QPC process - just because the lowest energy states before coupling are undressed quasiparticles and phonon-plus-quasiparticle configurations. As we shall see in detail later, the even-even nucleus also depends on these processes. The phonon, itself a linear combination of particle-hole configurations, will be more stable or depressed in energy if the particle (and hole) can polarize the core. This is shown in Fig. 19a.

However, correcting for Pauli Principle violations of particle-hole pairs and ground-state clusters will increase the phonon energy. (See Fig. 19b.) It might also be thought that the higher order diagrams would have some effect on the phonon. It turns out - both numerically and for sound theoretical reasons - that this is not the case.

B. Dyson's Equation

In this subsection, we attempt to motivate quasiparticle-phonon coupling from the viewpoint of Dyson's equation. The "derivation" is not

in any sense exact; however, it illustrates the origin of "higher order terms" in the self-energy and also shows the necessity of employing "one-three blocks" (not equal to the (pp) or (ph) blocks) to connect the self-energy to the propagator line, in an approximate treatment. We carry out the analysis assuming FDSM propagators and irreducible blocks I and J. Landau renormalization should follow as usual.

It should be recalled from Sec. II.D.1 that the self-energy is given by Eq. (II.30), with account to be taken of generalized time ordering. The graphical equation in the particle-hole formalism is shown in Fig. 20.

In analogy, we may write an approximate equation for the self-energy in terms of the G matrix, which is the "bare" two-body interaction inside the model space. We take j, k, A to be inside this space, and so neglect certain complicated (non-ladder) diagrams in which intermediates are outside the model space. This should not be too serious an approximation. The model-space equation is graphed in Fig. 21. In Brueckner-Hartree-Fock, the loop (which should be a fully dressed propagator) in contribution (a) of Fig. 21 is approximated by the lowest order propagator. In renormalized BHF, some rearrangement terms are included.

At any rate, contributions (b) and (c), which are "residual" self-energy ($\sum_R$) terms, give rise to phonon-particle-coupling. We shall discuss (b), since (c) is just the "backwards" diagram and may be similarly analyzed. The new Dyson's equation, after BHF has been carried out, is

$$G = G_{\text{BHF}} + G_{\text{BHF}} \sum_R G \quad (\text{II.29'})$$

Fig. 20. Exact expression for self-energy. V is the bare NN-interaction. Lines are FDSM.

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Fig. 21. Approximate expression for self-energy using reaction matrix, G . Lines are FDSM and inside model space.

In Fig. 22, we write Fig. 21b with the total scattering amplitude, Γ , decomposed in the second particle-hole channel. This is very suggestive of the simple QPC contribution to the self-energy drawn earlier, in which a string of particle-hole bubbles was attached to a propagator line.

If we make the following assumptions, viz. $I \approx G$ and energy-independent, then we obtain the approximation shown in Fig. 23. The particle-hole bubble string can be replaced by a sum over phonons of different energy and angular momentum and the QPC vertices (see Fig. 24) can be defined as

$$\langle \alpha | H_{\text{eff}} | \beta N \rangle = \sum_{\gamma \delta} X_{\gamma \delta}^N \langle \alpha \delta | G | \beta \gamma \rangle + Y_{\gamma \delta}^N \langle \alpha \bar{\gamma} | G | \beta \bar{\delta} \rangle \quad (\text{III.1})$$

In this instance, simple QPC would be the only contribution to the self-energy (coming from Fig. 21b). Suppose

$$\Sigma_{\alpha} = \sum_{\beta} \sum_N \frac{|\langle \alpha | H_{\text{eff}} | \beta N \rangle|^2}{\omega_{\alpha} - E_{\beta} - E_N} \quad (\text{III.2})$$

as in Fig. 23, along with the assumption $G_{\beta} \approx G_{\text{BHF}} \approx 1/(\omega_{\beta} - E_{\beta} + i\delta)$ the poles of the Green's function $G(\omega_{\alpha})$ are given by Eq. (II.29') which may be algebraically manipulated at a pole of G (i.e., G_0 neglected):

$$G(\omega_i) \approx G_0(\omega_i) \sum (\omega_i) G(\omega_i) \longrightarrow$$

$$\frac{Z_i}{\omega_i - E_i} = G_0 \sum (\omega_i) \frac{Z_i}{\omega - E_i} \longrightarrow$$

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Fig. 22. Separation of self-energy diagram in second particle-hole channel. Lines are FDSM. I is the irreducible (ph)-block.

Fig. 23. Simplest QPC approximation to self-energy diagram in Fig. 22.
 χ is the reaction matrix, lines are BHF.

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Fig. 24. Definition of QPC vertex as sum over RPA amplitudes times two-body matrix elements.

$$\frac{1}{G_0} = \omega_i - E_\alpha = \sum (\omega_i) = \sum_{\beta} \sum_N \frac{|\langle \alpha | H_{\text{eff}} | \beta N \rangle|^2}{\omega_i - E_\beta - E_N} \quad (\text{III.3a})$$

To obtain an equation for the residue Z_i we rearrange Eq. (II.29') to obtain

$$G = \frac{1}{\frac{1}{G_0} - \sum} \quad (\text{II.29''})$$

With the assumption $G_0 = \frac{1}{\omega - E_\alpha}$ and a residue theorem from complex variables analysis,⁴¹ viz. $\text{Res} \left(\frac{p(Z)}{q(Z)} \right)_{t=Z_0} = \frac{p(Z_0)}{q'(Z_0)}$, we obtain

$$Z_i = \frac{1}{1 - \frac{dG}{d\omega}} \Big|_{\omega = \omega_i} \quad (\text{III.3b})$$

It is possible to obtain the solutions ω_i and Z_i of Eq. (III.3) according to a graphical procedure shown in Fig. 25. Alternatively, we may employ an algebraic device.

If we define $C_\alpha \langle \alpha | H_{\text{eff}} | \beta N \rangle = (\omega_i - E_\beta - E_N) C_{\beta N}$, then Eq. (III.3a) reduces to a matrix diagonalization problem

$$C_\alpha (\omega_i - E_\alpha) = \sum_{\beta N} \langle \alpha | H_{\text{eff}} | \beta N \rangle C_{\beta N} \quad (\text{III.4a})$$

$$C_{\beta N} (\omega_i - E_\beta - E_N) = \langle \alpha | H_{\text{eff}} | \beta N \rangle C_\alpha$$

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Fig. 25. Graphical solution of Dyson's equation $\omega - E_\alpha = \Sigma(\omega)$. Intersections of the two curves occur at poles of Green's function. e.g., $\omega = E_\alpha'$.

If we adopt the usual matrix normalization

$$c_{\alpha}^2 + \sum_{\beta N} c_{\beta N}^2 = 1, \text{ then}$$

$$Z_i = \frac{1}{1 - \frac{\partial \Sigma}{\partial \omega}} = \frac{1}{1 + \sum_{\beta N} \frac{|\langle \alpha | H_{\text{eff}} | \beta N \rangle|^2}{(\omega_i - E_{\beta} - E_N)^2}}$$

$$= \frac{1}{1 + \sum_{\beta N} \frac{c_{\beta N}^2}{c_{\alpha}^2}} = c_{\alpha}^2 \quad (\text{III.4b})$$

Either Fig. 25 or the system of Eqs. (III.4) should convince us that QPC mixes single-particle states with phonon-quasiparticle configurations. Thus, several eigenstates will contain a percentage of the basis state $|\alpha\rangle$. This is why the single-particle strength in many (d,p) and (d,t) reactions is fragmented. Indeed, the simple approximation we have just described (often called the intermediate coupling model) has been very successful in nuclear physics.

Figure 25 also tells us why of all the particle-hole eigenstates we can obtain an accurate description using only the lowest one---which is generally the most collective and coherent.

1. Since the lowest energy phonon is well-separated from the other phonon energies, according to second order perturbation theory

arguments (i.e., large energy differences in the denominator except for phonon energy E_N), the lowest should be the main core excitation mode contribution to shifting E_α to E'_α .

2. Since the low- ω state is collective and $\langle \alpha | H_{\text{eff}} | \beta N \rangle \gg \langle \alpha | H_{\text{eff}} | \beta N' \rangle$, residues associated with the N' modes will be close to zero.

However, successful the approximation $I \approx G \approx \text{unretarded}$, turns out, we should investigate certain complications. The most important difficulty is that even if G is approximately energy independent, I is certainly not. In Fig. 26, we draw some contributions to I . It is our belief that these are very important: Fig. 26a and 26b will contribute to the low-energy structure of I (in second (ph) channel) whenever there are low-energy particle-hole or particle-particle vibrations. Ironically, we would like to calculate such vibrations but ignore these higher graphs. Figure 26c,d,e show yet a higher order process (not separable in any channel) in which the incoming particle and hole polarize the coherent core excitation. Figure 26f and 26g are not separable in any sense either. They are probably of considerable consequence, however, since the two-phonon energy is low enough to exert influence on nuclear structure studies - both theoretically and experimentally.

This considerable energy dependence of I poses a definite problem in finite nucleus calculations:

1. We must either calculate I exactly and substitute it into the graphical equation for the self-energy given in Fig. 22; or

Fig. 26. Important energy dependent contributions to irreducible (ph) scattering in the second (ph) channel. Lines are FDSM.

(a) separable in the first (ph) channel.

(b) separable in the particle-particle channel.

(c,d,e) diagrams in which one phonon correlation precedes absorption of the incoming hole.

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Fig. 26 (cont'd). Two-phonon intermediates contributing to irreducible scattering in second (ph) channel.

2. We must use some lower form of interaction, e.g., G , and calculate all the high order graphs explicitly. The latter approach will be taken in this thesis.

If we believe the contributions to I shown in Fig. 26 are important, let us substitute them into the uppermost I -block in Fig. 22 to obtain explicitly the higher order contributions to the self-energy which we think are important. In particular, we have chosen only the uppermost I -block because it connects one- and three-quasiparticle configurations. The other I -blocks play their normal role of scattering particle-hole pairs. What we are, in essence, proposing is that the phonon itself can be described by a phenomenological particle-hole amplitude, I , without too much regard for its energy dependent structure, or two-phonon admixtures. However, the peculiar role of the one-three blocks demands that we at least investigate the role of their energy dependence. Figure 27a-e correspond to Fig. 26a-e. Figure 27a is essentially a recoupling of the phonon + quasiparticle. The "independent" particle becomes the correlating (with the hole) particle, and vice versa. Figure 27b exhibits the recoupling of a particle-hole phonon plus particle to yield a particle-particle phonon and a hole. In Fig. 27c, 27d the spectator particle interacts with either the hole or particle "between correlations". In Fig. 27e, the same process precedes a recoupling. The two-phonon self-energy contributions which derive from Fig. 26f,g can be similarly drawn.

We shall henceforth limit consideration to one-phonon self-energy contributions. While it might be argued for energy reasons that the two-phonon contributions are less important, our primary motivation is

(a)

(b)

(c)

(d)

(e)

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Fig. 27. Self-energy diagrams obtained when graphs of Fig. 26 are substituted into Fig. 22's "upper most I-block". Lines are FDSM.

convenience, not only in the discussion which follows, but also in the practical matter of calculating the approximate mass-operator.

Werner⁴² has taken the QPC vertex to be I at all times without mentioning that in Dyson's equation, the phonon is initiated by something else. This is not rigorous.

In a phenomenological theory, these "one-three" blocks should have their own identity as well as their own parameters, spin and isospin dependence. The interface between theory and experiment is not clear on this point and the determination of " I_{13} " and " J_{13} " would be exceedingly arbitrary. The risk in simply using I for (ph)-correlated self-energy contributions and J for (pp)-correlated contributions is that the magnitude and the spin-dependences may be wrong. (For the spin uncorrelated modes typical of low-energy spectra in heavy nuclei, this will, fortunately, not be too serious.)

It is here that we must settle on an approximate treatment of Dyson's equation. We remind the reader again that in Fig. 22 the phonon or bubble string is initiated by a G and terminated by an I. Secondly, the uppermost I-block which is "one-three" in nature contains some important higher order processes (e.g., the graph which corrects for Exclusion Principle Violations in simultaneous phonon-quasiparticle propagation within its energy dependence). We attempt to write down these long-range contributions in an expansion, explicitly leaving behind a G-matrix interaction plus more complicated terms about which we shall not concern ourselves further. The approximation we shall make is to take these two "one-three" blocks (the G-matrix and the G-matrix + other things) to be I, where I will be parameterized as short-range.

Finally, our arguments above could have been carried out for (pp) phonons, and we would have arrived at the approximation:

"one-three blocks" $\rightarrow$ J.

Also, in a real Fermi liquid I and J will be replaced by F^E and F^W .

C. The SCQPC Equations

In previous sections, we have suggested a foundation upon which to base a theory of odd-mass nuclear excitations, as well as the fully dressed one-nucleon Green's function to be employed in the calculation of even mass nuclear excitations. This basis is predicated upon the quasiparticle approximation and the assumption that certain long-range core-polarization type diagrams dominate the analytical structure of the quasiparticle self-energy. In this section, we outline a specific theory for nuclear structure calculations subject to these approximations. Within the quasiparticle picture we have the following quantities:

$$g_{\alpha}^0(\omega_{\alpha}) = \frac{1}{\omega_{\alpha} - E_{\alpha} \pm i\delta} \quad \alpha \gtrless \text{Fermi surface}$$

(III.5)

$$g_{\alpha}(\omega_{\alpha}) = \sum_i \frac{Z^{(i)}}{\omega_{\alpha} - E_{\alpha}^{(i)} + i\delta} + \sum_j \frac{Z^{(j)}}{\omega_{\alpha} + E_{\alpha}^{(j)} - i\delta}$$

Equation (III.5) is the Landau renormalized version of Eq. (II.16). These are the partially and fully dressed Green's functions for quasiparticles and quasiholes. In a truncated space of particle-hole and particle-particle configurations, the effective or irreducible interactions are F' and F'' .

In the superfluid nucleus, the elementary excitations will be called quasibogolons. They are determined by the BCS procedure from quasiparticles and quasiholes. The quasibogolon Green's functions are

$$\tilde{g}_{\alpha}^0(\omega_{\alpha}) = \frac{1}{\omega_{\alpha} - E_{\alpha} + i\delta}$$

$$\tilde{g}_{\alpha}(\omega_{\alpha}) = \sum_i \frac{Z^{(i)}}{\omega_{\alpha} - E_{\alpha}^{(i)} + i\delta} + \sum_j \frac{Z^{(j)}}{\omega_{\alpha} + E_{\alpha}^{(j)} - i\delta}$$
(III.6)

Equation (III.6) alludes to the rule that in BCS only forward bogolons are allowed; however, when further ground state correlations are taken into account, backwards eigenvalues and residues (which should be small) also appear. We remind the reader that in the odd-(BCS) and even-mass (QRPA)⁴³ bogolon theories which are obtained when second-order self-energy diagrams are not included in the calculation, one always obtains a formal set of equations which have twice the dimensionality of the one- or two-bogolon basis and which yield a set of "nonphysical" eigenvalues such that for every $E_{\alpha}^{\text{physical}} > 0$, there is an $E_{\alpha}^{\text{nonphys.}} < 0$ and $E_{\alpha}^{\text{nonphys.}} = -E_{\alpha}^{\text{phys.}}$. This technicality is very important, because the final eigenstates are the best elementary excitations we can define, i.e., they are least affected by the quasiparticle interaction most of which has already been diagonalized away. The superfluid picture, however, should not yield more elementary excitations than would be possible in the usual particle-hole picture. Alternatively, this double set of eigenvalues suggests a very simple hamiltonian:

$$H = \sum_{\alpha} E_{\alpha} A_{\alpha}^+ A_{\alpha}$$

where A_{α}^{+} creates an elementary excitation and E_{α} is energy of this elementary excitation.

The interactions between quasibogolons are given in terms of F'' and F' in Eq. (II.62) and (II.63). We recall that these formulae imply a very serious approximation: namely that the irreducible interaction between bogolons depends only kinematically on the particle or hole character of the bogolon. Dynamical effects of superfluidity on I and J are ignored.

If the even-even nucleus contains certain low-lying collective excitations (called phonons), then these will dominate the analytical features of the dressed quasiparticle or bogolon Green's function, $\tilde{g}$, and diagrams shown in Figs. 15-18 should be calculated. Exact calculational devices will be developed in the following subsections:

1. QPC vertices
 2. H_{eff} between phonon-quasiparticle configurations.
 3. A diagonalization scheme for the self-consistent perturbation theory of $\tilde{g}$.
 4. The renormalized RPA equations for the two-bogolon Green's function, $\tilde{k}$.
 5. The renormalized gap equation.
1. Quasiparticle-Phonon Coupling Vertices

In this subsection we describe the H_{eff} between a quasibogolon and a quasibogolon-plus-phonon. In Sec. III.C.4, we will show that the RPA amplitudes $X_{\alpha\beta}$ and $Y_{\alpha\beta}$ may be obtained by self-consistent diagonalization where the off-diagonal matrix elements are equal to $\langle \alpha\beta | P | \gamma\delta \rangle$ or $\langle \alpha\beta | Q | \gamma\delta \rangle$.

$$(\alpha\beta|P\pm Q|\gamma\delta) = \xi_{\alpha\beta}^{\pm} \xi_{\gamma\delta}^{\pm} \langle \alpha\beta|F''|\gamma\delta \rangle_A \quad (II.62)$$

$$+ \eta_{\alpha\beta}^{\pm} \eta_{\gamma\delta}^{\pm} [S_{\beta}S_{\delta} \langle \alpha-\delta|F'|- \beta\gamma \rangle \mp S_{\beta}S_{\gamma} \langle \alpha-\gamma|F'|- \beta\delta \rangle]$$

Recalling that α , β , γ and δ are bogolons, we have the following graphical equivalences.

$$\begin{array}{c} \gamma \uparrow \delta \\ \hline \alpha \downarrow \beta \end{array} = P \qquad \begin{array}{c} \alpha \quad \gamma \\ \diagdown \quad \diagup \\ \hline \diagup \quad \diagdown \\ \beta \quad \delta \end{array} = Q$$

Also, we know that the vertices R and S defined pictorially

$$\begin{array}{c} \gamma \quad \alpha \\ \diagdown \quad \diagup \\ \hline \diagup \quad \diagdown \\ \chi \quad \beta \end{array} = S_{\gamma} \quad (\alpha\beta|R|\chi-\gamma)$$

and

$$\begin{array}{c} \gamma \quad \alpha \\ \diagdown \quad \diagup \\ \hline \diagup \quad \diagdown \\ \chi \quad \bar{\beta} \end{array} = S_{\gamma} (\alpha\beta|S|\chi-\gamma)$$

are analytically equivalent to

$$\begin{aligned} (\alpha\beta|R\pm S|\chi-\gamma) = & - \xi_{\alpha\beta}^{\pm} \eta_{\chi\gamma}^{\pm} \langle \alpha\beta|F''|\chi-\gamma \rangle \\ & + \eta_{\alpha\beta}^{\pm} \xi_{\chi\gamma}^{\pm} [S_{\beta}S_{-\delta} \langle \alpha\gamma|F'|- \beta\chi \rangle \mp S_{\beta}S_{\chi} \langle \alpha-\chi|F'|- \beta-\gamma \rangle] \end{aligned} \quad (II.63)$$

In the calculation of backwards diagrams, we need the vertex

. This is clearly just $S_Y(\alpha\beta|Q|\chi-\gamma)$.

Other vertices may be obtained from the symmetry properties (II.64,65).

In writing Eq. (II.63) we have already incorporated the assumptions about "one-three" blocks discussed in Sec. III.B.

We are actually seeking the QPC vertices shown in Fig. 28.

However, these may be written in terms of Eq. (II.62) and Eq. (II.63) if we recall

$$Q^+ = \sum_{(\alpha\beta)} X_{\alpha\beta} A_{\alpha}^+ A_{\beta}^+ - \sum_{(\alpha\beta)} Y_{\alpha\beta} A_{\beta} A_{\alpha} \quad (\text{III.7})$$

The graphical equations for the quasiparticle-phonon coupling vertices are shown in Fig. 28. The analytical expressions are given in Eq. (III.8) and the symmetry relations, Eq. (III.9), are easily derived.

$$P(\alpha\beta) = \sum_{\gamma\delta} \langle \alpha\beta|P|\gamma\delta \rangle X_{\gamma\delta} + \langle \alpha\beta|Q|\gamma\delta \rangle Y_{\gamma\delta} \quad (\text{III.8a})$$

$$Q(\alpha\beta) = \sum_{\gamma\delta} \langle \alpha\beta|P|\gamma\delta \rangle Y_{\gamma\delta} + \langle \alpha\beta|Q|\gamma\delta \rangle X_{\gamma\delta} \quad (\text{III.8b})$$

$$R(\chi-\gamma) = \sum_{\alpha\beta} \langle \alpha\beta|R|\chi-\gamma \rangle X_{\alpha\beta} + \langle \alpha\beta|S|\chi-\gamma \rangle Y_{\alpha\beta} \quad (\text{III.8c})$$

$$\alpha \begin{array}{c} \nearrow \\ \bullet \\ \searrow \\ \text{wavy} \end{array} \beta \equiv P(\alpha\beta) = \sum_{\gamma\delta} \begin{array}{c} \alpha \nearrow \\ \text{box } P \\ \searrow \beta \\ \text{wavy} \end{array} + \begin{array}{c} \alpha \nearrow \\ \text{box } Q \\ \searrow \beta \\ \text{wavy} \end{array}$$

$$\alpha \begin{array}{c} \nearrow \\ \bullet \\ \searrow \\ \text{wavy} \end{array} \beta \equiv Q(\alpha\beta) = \sum_{\gamma\delta} \begin{array}{c} \alpha \nearrow \\ \text{box } Q \\ \searrow \beta \\ \text{wavy} \end{array} + \begin{array}{c} \alpha \nearrow \\ \text{box } P \\ \searrow \beta \\ \text{wavy} \end{array}$$

$$\gamma \begin{array}{c} \nearrow \\ \bullet \\ \searrow \\ \text{wavy} \end{array} x \equiv S_x R(x-\gamma) = \sum_{\alpha\beta} \begin{array}{c} \gamma \nearrow \\ \text{box } R \\ \searrow \beta \\ \text{wavy} \end{array} + \begin{array}{c} \gamma \nearrow \\ \text{box } S \\ \searrow \beta \\ \text{wavy} \end{array}$$

$$\gamma \begin{array}{c} \nearrow \\ \bullet \\ \searrow \\ \text{wavy} \end{array} x \equiv S_x S(x-\gamma) = \sum_{\alpha\beta} \begin{array}{c} \gamma \nearrow \\ \text{box } S \\ \searrow \beta \\ \text{wavy} \end{array} + \begin{array}{c} \gamma \nearrow \\ \text{box } R \\ \searrow \beta \\ \text{wavy} \end{array}$$

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Fig. 28. QPC vertices in terms of RPA amplitudes and interactions between quasibogolons.

$$S(\chi-\gamma) = - \sum_{\alpha\beta} < \alpha\beta | R | \chi-\gamma > Y_{\alpha\beta} + < \alpha\beta | S | \chi-\gamma > X_{\alpha\beta} \quad . \quad (III.8d)$$

$$P(\alpha\beta) = -P(\beta\alpha) = P(-\alpha -\beta) \text{ and similarly for } Q \text{ while} \quad (III.9a)$$

$$R(-\gamma\chi) = S(\chi-\gamma) \quad . \quad (III.9b)$$

For $K = 0$ phonons, there are several modifications. The P and Q which are needed in RPA are defined

$$(\alpha\beta | P_0^{RPA} \pm Q_0^{RPA} | \gamma\delta) = \frac{1}{\sqrt{1 + \delta_{\alpha-\beta}}} \frac{1}{\sqrt{1 + \delta_{\gamma-\delta}}} (\alpha\beta | P \pm Q | (\gamma\delta) + (-\delta -\gamma)) \quad . \quad (III.10)$$

However, the probability that a phonon will resolve into the pair $\alpha\beta$ and not $-\beta -\alpha$ is

$$P_0^{QPC}(\alpha\beta) = \sqrt{\frac{1 + \delta_{\alpha-\beta}}{1 + \delta_{KO}}} \sum_{\gamma\delta} (\alpha\beta | P_0^{RPA} | \gamma\delta) X_{\gamma\delta} + < \alpha\beta | Q_0^{RPA} | \gamma\delta > Y_{\gamma\delta} \quad (III.11)$$

Similar equations are true for $Q_0^{QPC}(\alpha\beta)$, $R_0^{QPC}(\chi-\gamma)$, $S_0^{QPC}(\chi-\gamma)$.

2. Heff Between Quasibogolon-Phonon Configurations

In Sec. III.A, we have introduced the concept that a particle-plus-phonon are connected by second order graphs which account for exclusion principle violations and certain two-phonon correlations (see Fig. 16b,

18b). In this section, we write down these second order H_{eff} 's explicitly. The second and third order contributions are shown again in Figs. 29-31.* In the calculational part of this work, we will neglect the diagrammatic contributions of Figs. 30 and 31. The reasons are:

- a. The third order couplings shown in Fig. 30 involve, in addition to two QPC vertices, a sum over incoherent two-bogolon scattering amplitudes. By incoherent we mean that the values of $K = \Omega_\gamma + \Omega_\delta$ (or $\vec{J} = \vec{J}_\gamma + \vec{J}_\delta$ in a spherical nucleus) can vary. So in addition to the usual energy loss due to a higher order of perturbation theory ($\frac{\text{numerator}}{\text{denominator}} = \frac{\text{vertex}}{\text{energy differences}} < 1$), the sum is not coherent or phonon-like as in the case of the other vertices. (The incoherence of this sum also makes numerical calculations very tedious.)
- b. The backward second order couplings of Fig. 31 are unimportant because they involve large energy denominators (e.g., $E_\beta + E_\alpha + E_N + E_N'$) in the H_{eff} , and because they couple forward and backward configurations, which implies that the diagonal energies will differ by $\approx 2 \cdot (E_{\text{quasibogolon}} + E_{\text{phonon}})$.

It should also be noted that certain coherent fourth-order diagrams (viz., Fig. 32) are included by iterating (or, as we shall see later, diagonalizing) the second-order diagrams.

The Feynman graph expressions for Fig. 29a and 29l are

*Second "order" means there are two interactions connecting elementary excitations, namely quasibogolons or phonons. These interactions may be of the QPC or usual two-body varieties.

Fig. 29. Second-order H_{eff} between phonon-plus-quasibogolon configurations.

Fig. 30. Third-order H_{eff} between phonon-plus-quasibogolon configurations.

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Fig. 31. Backwards second-order H_{eff} .

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Fig. 32. Possible fourth- and fifth-order contributions to H_{eff} which are counted when second-order term is iterated.

$$\text{Fig. 29a: } \langle \alpha N | H_{\text{eff}} | \beta N' \rangle = \sum_{\gamma, K} \frac{P^{N'}(\alpha\gamma) P^N(\beta\gamma) Z_Y^K}{E_\alpha + E_\beta + E_Y^K - \omega} \quad (\text{III.12})$$

$$\text{Fig. 29b: } \langle \alpha N | H_{\text{eff}} | \beta N' \rangle = \sum_{\gamma, K} \frac{R^{N'}(\alpha-\gamma) R^N(\beta-\gamma) Z_Y^K}{\omega - E_Y - E_N - E_{N'}}$$

where $\omega = \omega_N + \omega_\alpha = \omega_{N'} + \omega_\beta$.

The Eqs. (III.12) show that, even if we make the approximation of energy-independent quasiparticle-phonon vertices, the effective coupling of phonon-quasiparticle configurations depends on the energy, ω , of the propagator-line in which the self-energy appears. In other words, each eigenstate which includes such configurations will have different matrix elements; this implies a self-consistency problem, knowing the poles in the energy variable ω in order to find them.

A reasonable approximation would be to set

$$\omega = \omega_{\text{ave}} = \frac{1}{2} (E_\alpha + E_N + E_\beta + E_{N'}) \quad (\text{III.13})$$

that is, take the average energy of the two configurations as the energy variable. The justification is that only when the two configurations are close in energy will the coupling matrix element be important anyway and, secondly, that final eigenvalues will only slightly depend on the energy difference $(\omega - \omega_{\text{ave}})$, if the $H_{\text{eff}} < \omega_{\text{ave}}$. This is the procedure which we shall adopt in Sec. V for the calculation of Fig. 29a.

An alternative procedure,⁴⁴ which seems quite workable when one is interested in the dressed energy of the quasiparticle and the main residue

is large (≈ 1), is to set $\omega = E_{\text{quasiparticle}}$. The relation of " H_{eff} " techniques to the general perturbation theory expansion is outlined in Ref. 45.

Type 1 (Fig. 29a) energy denominators are thus typically of the order of $3E_{\text{qp}} - 1E_{\text{phonon}} - 1E_{\text{qp}} \approx 2E_{\text{qp}} - E_{\text{phonon}} > 0$. Type 2 (Fig. 29b) energy denominators are typically

$1E_{\text{phonon}} + 1E_{\text{qp}} - 1E_{\text{qp}} - 2E_{\text{phonon}} \approx -E_{\text{phonon}} < 0$. The latter type, however, is subject to "infinities" when the intermediate quasifermion is much less energetic than the incoming and outgoing ones. In this case, the approximation $\omega_{\text{ave}} = E_Y$ insures a finite denominator and will yield approximately the right H_{eff} energy dependence for low-lying eigenstates.

3. Diagonalization

As has already been indicated, we are seeking the poles of the quasiparticle Green's function in order to describe odd-mass nuclei as well as renormalized even-mass nuclei. For the simple case of the forward core polarization contribution to the self-energy, it is possible to obtain, Eq. (III.4), a simple diagonalization scheme. We would like to develop a more general diagonalization scheme which can account for dressed intermediate propagators, backwards diagrams, and effective interactions between phonon-plus-quasiparticle configurations. There are two possibilities:

- a. Each row and column (except for "bare" quasibogolons, $\tilde{g}^0$) of the matrix corresponds to a quasibogolon-plus-phonon with several poles. In this case, H_{eff} is given by Eq. (III.8,12). Since the intermediate has several poles, the energy dependence must be treated in a self-consistent manner, i.e., iterative diagonalizations.

- b. Each row and column corresponds to a particular pole of a quasibogolon-plus-phonon. Here, the iterative matrix diagonalization is not necessary and the H_{eff} 's are multiplied by residue strengths at the pole in question.

Method b is more appealing, not only for numerical reasons, but because our "basis" becomes a set of "undressed" quasibogolons and exact odd-mass eigenstates (we shall call them spectrons) coupled to exact even-mass eigenstates (phonons). Both of these kinds of basis vector may be represented by a quasifermion acting on $|\tilde{0}\rangle$.

$$b, b^+ |\tilde{0}\rangle \equiv \text{our "basis"} \quad (\text{III.14})$$

$b_\alpha^+ \equiv a_\alpha^+$ where a_α^+ is the creation operator for quasibogolons (or partially dressed elementary excitations in a superfluid).
 $b_\beta^+ i_N = a_\beta^+ Q_N^+$ where Q_N^+ is the creation operator for the phonon (see II.74) and a_β^+ is the creation operator for the fully dressed quasifermion excitation which contains an appreciable fraction of the single particle strength, β . The various eigenstates containing such a fraction are labelled, i .

The various quasifermions will be labelled b_1^+ , b_2 , etc. The generalized quasifermion Green's functions are defined

$$\mathcal{L}_{12}^F(t) \equiv -i \langle \tilde{0} | T(b_1(t) b_2^+(0)) | \tilde{0} \rangle \quad (\text{III.15a})$$

$$\mathcal{L}_{12}^B(t) = -i \langle \tilde{0} | T(b_1(t) b_2(0)) | \tilde{0} \rangle \quad (\text{III.15b})$$

We shall take $|\tilde{0}\rangle$ to be the BCS vacuum plus correlations. The number of particles in the state $|\tilde{0}\rangle$ is determined by the BCS parameters Δ and λ , and may be even or odd. If we interpose a complete set of nuclear states $|k\rangle$ and fourier transform Eq. (III.15), we obtain the analytic structure for $L_{12}(\omega)$, namely

$$L_{12}^F(\omega) = \sum_k \frac{\langle \tilde{0} | b_1 | k \rangle \langle k | b_2^+ | \tilde{0} \rangle}{\omega - E_k + i\delta} + \sum_{k'} \frac{\langle \tilde{0} | b_2^+ | k' \rangle \langle k' | b_1 | \tilde{0} \rangle}{\omega + E_{k'} - i\delta} \quad (\text{III.16a})$$

$$L_{12}^B(\omega) = \sum_k \frac{\langle \tilde{0} | b_1 | k \rangle \langle k | b_2 | \tilde{0} \rangle}{\omega - E_k + i\delta} + \sum_{k'} \frac{\langle \tilde{0} | b_2 | k' \rangle \langle k' | b_1 | \tilde{0} \rangle}{\omega + E_{k'} - i\delta} \quad (\text{III.16b})$$

The derivation of Eq. (III.16) is similar to that of Eq. (II.6). Essentially, L^F is the probability amplitude that an elementary excitation, 2, placed into the nucleus at time, 0, will evolve into some other excitation, 1, at time t. L^B gives the probability amplitude that elementary excitations 1 and 2 may be removed from the correlated ground state at times t and 0 without changing the nucleus. It is non-zero only because of ground state correlations. We should point out that our generalized quasifermion Green's function is not as general as it could be: we might also include quasi-fermions such as

$$a_{\beta}^{\dagger} a_{N'}^{\dagger}, \quad N' \neq \text{lowest, most collective phonon.}$$

Later, we will normalize our equations by assuming that b_{α}^{+} and b_{β} are the only kind of elementary excitations. This is the most crucial assumption of SCQPC.

The graphical equations for $\mathcal{L}_{12}(\omega)$ can be easily constructed and are shown in Fig. 33. The analytical equivalents of Fig. 33 are

$$\begin{aligned} \mathcal{L}_{12}^F = \delta_{12} \mathcal{L}_{12}^O + \sum_3 \mathcal{L}_{13}^F < 2 | H_{\text{eff}} | 3 > \mathcal{L}_2^O \\ + \sum_3 \mathcal{L}_{1\bar{3}}^B < 2\bar{3} | H_{\text{eff}} | \tilde{0} > \mathcal{L}_2^O \end{aligned} \quad (\text{III.17a})$$

$$\mathcal{L}_{1\bar{2}}^B = \sum_3 \mathcal{L}_{13}^F < \tilde{0} | H_{\text{eff}} | \bar{2}3 > \mathcal{L}_{\bar{2}}^O + \sum_3 \mathcal{L}_{13}^B < \bar{3} | H_{\text{eff}} | \bar{2} > \mathcal{L}_2^O \quad (\text{III.17b})$$

H_{eff} is the effective interaction between quasifermions which has been discussed in previous paragraphs, plus one modification detailed below.

Incorporating Eq. (III.16a,b) into Eq. (III.17a), we may write

$$\begin{aligned} \sum_k \frac{< \tilde{0} | b_1 | k > < k | b_2^{+} | \tilde{0} >}{\omega - E_k + i\delta} + \sum_{k'} \frac{< \tilde{0} | b_2^{+} | k > < k | b_1 | 0 >}{\omega + E_{k'} - i\delta} \\ = \delta_{12} \mathcal{L}_{12}^O + \sum_3 \left[\sum_k \frac{< 0 | b_1 | k > < k | b_3^{+} | 0 >}{\omega - E_k + i\delta} + \sum_{k'} \frac{< 0 | b_3^{+} | k' > < k' | b_1 | \tilde{0} >}{\omega + E_{k'} - i\delta} \right] \end{aligned}$$

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Fig. 33. Graphical equations for quasifermion Green's functions, $\mathcal{L}_{12}^B$. Lines are QF, block is H_{eff} .

$$\begin{aligned}
 & \cdot \langle 2|H_{\text{eff}}(\omega)|3 \rangle \mathcal{L}_2^0 + \\
 & \sum_3 \left[\sum_k \frac{\langle \tilde{0}|b_1|k \rangle \langle k|b_3|\tilde{0} \rangle}{\omega - E_k + i\delta} + \sum_{k'} \frac{\langle \tilde{0}|b_3|k' \rangle \langle k'|b_1|\tilde{0} \rangle}{\omega + E_{k'} - i\delta} \right] \\
 & * \langle 2\bar{3}|H_{\text{eff}}(\omega)|\tilde{0} \rangle \mathcal{L}_2^0 \quad \text{(III.18)}
 \end{aligned}$$

At $\omega \approx E_k$, i.e., at poles of the first summation, one can disregard the finite terms and cancel away the common factor $\langle \tilde{0}|b_1|k \rangle / (\omega - E_k + i\delta)$

$$\begin{aligned}
 \langle k|b_2^+|\tilde{0} \rangle &= \sum_3 \mathcal{L}_2^0 \cdot \{ \langle 2|H_{\text{eff}}(\omega)|3 \rangle \langle k|b_3^+|\tilde{0} \rangle \\
 &+ \langle 2\bar{3}|H_{\text{eff}}(\omega)|\tilde{0} \rangle \langle k|b_3|\tilde{0} \rangle \} \quad \text{(III.19)}
 \end{aligned}$$

Now $\mathcal{L}_2^0$ is the propagator of either an undressed bogolon, $1/(\omega - E_\alpha + i\delta)$ or the propagator of a spectron-plus-phonon, $1/(\omega - E_\beta^{(j)} - E_N + i\delta)$. For the self-consistent definition of $\mathcal{L}^0$, see Eq. (III.30), later in this subsection. Note that in either case there is only one pole, say at E_2 , so we can multiply both sides by the energy denominator $\omega - E_2$. Also, we assumed $\omega = E_k$. Thus,

$$\begin{aligned}
 (E_k - E_2) \langle k|b_2^+|\tilde{0} \rangle &= \sum_3 \langle 2|H_{\text{eff}}(E_k)|3 \rangle \langle k|b_3^+|\tilde{0} \rangle \\
 &+ \sum_3 \langle 2\bar{3}|H_{\text{eff}}(E_k)|\tilde{0} \rangle \langle k|b_3|\tilde{0} \rangle \quad \text{(III.20a)}
 \end{aligned}$$

At the poles $\omega = -E_k$ in Eq. (III.18), we may neglect the finite terms to obtain

$$\begin{aligned}
 -(E_k + E_2) \langle k | b_2^\dagger | \tilde{0} \rangle &= \sum_3 \langle 2 | H_{\text{eff}} | \tilde{3} \rangle \langle k | b_3 | \tilde{0} \rangle \\
 &+ \sum_3 \langle 23 | H_{\text{eff}} | \tilde{0} \rangle \langle k | b_3 | \tilde{0} \rangle \quad (\text{III.20b})
 \end{aligned}$$

Equation (III.20) constitutes a matrix diagonalization problem for the quasifermion amplitudes $\langle k | b_2^\dagger | \tilde{0} \rangle$ and $\langle k | b_2 | \tilde{0} \rangle$ and the poles of the quasifermion Green's function. There are several points which are not yet clarified, however. First is the exact equations for the effective matrix elements of the Hamiltonian, after spectrons are introduced. Second is the normalization of the amplitudes, and third is the relation of the dressed bogolon Green's function, $\tilde{g}$, to the quasifermion Green's function, $\mathcal{L}$.

First, consider the matrix element $\langle 2 | H_{\text{eff}} | 3 \rangle$.

Case 1. 2 and 3 are bare quasibogolon $\langle 2 | H_{\text{eff}} | 3 \rangle = 0$.

Case 2. 2 (or 3) is a bare quasibogolon, α . 3 (or 2) is a spectron plus phonon, $\beta^\dagger N$. In effect, we are asking for one QPC vertex

$$= \sqrt{z_\beta^{(j)}} S_\beta R^N(\alpha - \beta) \quad (\text{III.21})$$

where R is defined in Eq. (III.8).

The amplitude $\sqrt{Z_{\beta}^{(j)}}$ is the probability amplitude that the eigenmode, j , propagates through the bare bogolon, β . In connection with this, we note that the various energies $E_{\beta}^{(j)}$ as well as the $Z_{\beta}^{(j)}$ are the object of this section as well as one of the tools. Hence, they are solved for self-consistently.

Case 3. 2 and 3 are eigenmodes $\alpha^{(i)}$ and $\beta^{(j)}$ coupled to phonons N and N' .

$$\begin{aligned} \langle \alpha^{(i)} N | H_{\text{eff}}(\omega) | \beta^{(j)} N' \rangle = & \sum_{\gamma, k} \frac{\sqrt{Z_{\alpha}^{(i)} Z_{\beta}^{(j)}} Z_{\gamma}^{(k)} P^{N'}(\alpha\gamma) P^N(\beta\gamma)}{E_{\alpha}^{(i)} + E_{\beta}^{(j)} + E_{\gamma}^{(k)} - \omega} \\ & + \sum_{\gamma, k} \frac{\sqrt{Z_{\alpha}^{(i)} Z_{\beta}^{(j)}} Z_{\gamma}^{(k)} R^{N'}(\alpha - \gamma) R^N(\beta - \gamma)}{\omega - E_N - E_{N'} - E_{\gamma}^{(k)}} \end{aligned} \quad (\text{III.22})$$

Equation (III.22) is identical to (III.12) except for the residue square roots $\sqrt{Z_{\alpha}^{(i)}}$ and $\sqrt{Z_{\beta}^{(j)}}$. These are, as in case 2, the wave function amplitudes that the eigenmodes i and j propagate through the bare bogolon excitations α and β , since it is only when this simple excitation is being populated that a simple quasiparticle-phonon coupling can occur. For the backward matrix elements $\langle 2\bar{3} | H_{\text{eff}} | \bar{0} \rangle$,

Case 1. 2 and 3 are bare bogolons $\langle 2\bar{3} | H_{\text{eff}} | \bar{0} \rangle = 0$

Case 2. 2 is a bogolon and $\bar{3}$ is a spectron-plus-phonon. This is just the vertex

$$= S_{\beta} \sqrt{Z_{\beta}^{(j)}} Q^N(\alpha - \beta) \quad (\text{III.23a})$$

In the vice versa case,

$$= -S_{\beta} \sqrt{Z_{\beta}^{(j)}} Q^N(\alpha - \beta) \quad (\text{III.23b})$$

The minus sign arises because in transforming α and $\overline{\beta N}$, to obtain $\overline{\alpha}$ and βN , time reversing a quasifermion twice yields a (-1) phase.

Case 3. 2 and 3 are spectron-plus-phonon configurations.

$$\langle 2\overline{3} | H_{\text{eff}} | \widetilde{0} \rangle = 0 \quad (\text{III.24})$$

according to the approximation discussed in connection with Fig. 31.

Except for the subtlety of energy dependent effective interactions, that is, basis states in our model space are connected by finite time-length configurations outside the model space, Eq. (III.20) constitutes a symmetrix matrix problem. The normalization condition derives from the anticommutation rules for quasifermions.

$$\sum_3 |\langle k | b_3^+ | \widetilde{0} \rangle|^2 + |\langle k | b_{\overline{3}} | \widetilde{0} \rangle|^2 = 1 \quad (\text{III.25})$$

Note that although we have a theory with backwards amplitudes very much like RPA, the forward-backward coupling matrix is asymmetric (in contrast

to RPA). Hence, the total matrix problem is symmetric (again in contrast to RPA). Associated with this is the normalization, Eq. (III.25), which adds forward and backwards amplitude squares, instead of subtracting them (as in RPA).

Finally, we come to the very important question: what is the dressed bogolon Green's function in terms of the generalized quasifermion Green's function. Clearly, $\tilde{g}_{\alpha\beta} = \mathcal{L}_{\alpha\beta}$ where α and β are both bare quasibogolons, that is

$$\tilde{g}_{\alpha\beta} = \sum_k \frac{\langle \tilde{0} | a_{\alpha} | k \rangle \langle k | a_{\beta}^{\dagger} | \tilde{0} \rangle}{\omega - E_k + i\epsilon} + \sum_k \frac{\langle \tilde{0} | a_{\beta}^{\dagger} | k \rangle \langle k | a_{\alpha} | \tilde{0} \rangle}{\omega + E_k - i\delta} \quad (\text{III.26})$$

In the approximation that, $\tilde{g}_{\alpha\beta} = \delta_{\alpha\beta} \tilde{g}_{\alpha}$, the residues, $Z_{\alpha}^{(k)}$, of are just $|\langle k | a_{\alpha}^{\dagger} | \tilde{0} \rangle|^2$ or for backwards residues $Z_{\alpha}^{(k, \text{back})} = |\langle k | a_{\alpha} | \tilde{0} \rangle|^2$.

$$\tilde{g}_{\alpha\beta} = \delta_{\alpha\beta} \tilde{g}_{\alpha} = \sum_k \frac{Z_{\alpha}^{(k)}}{\omega - E_{\alpha}^{(k)} + i\delta} + \frac{Z_{\alpha}^{(k, \text{back})}}{\omega + E_{\alpha}^{(k)} - i\delta} \quad (\text{III.27})$$

The negative energy poles, which are opposite in sign but equal in magnitude to the positive energy poles, represent non-physical states. However, the residues associated with them denote a change in ground state occupation probabilities. In superfluid nuclei, these changes are generally negligible and a reasonable approximation to Eq. (III.27) is

$$\tilde{g}_{\alpha} \approx \sum_k \frac{Z_{\alpha}^{(k)}}{\omega - E_{\alpha}^{(k)} + i\delta} \quad (\text{III.28})$$

In normal nuclei, the backwards part of the problem is not a reflection of the forwards part. In fact, the diagonalizations couple the $N \pm 1$ nuclei, e.g., ^{209}Pb and ^{207}Pb or ^{41}Ca and ^{39}Ca . In this case, the backwards residues are physical and, in some cases, (e.g., the finite occupation probability of the $f_{7/2}$ orbit in ^{39}Ca) substantial. An approximation such as Eq. (III.28) is then dangerous.

$\tilde{g}$ represents a set of improved elementary excitations for fermions. If we put a quasibogolon into the system, it will evolve into another configuration because it is unstable, i.e., not an eigenmode. However, we have defined a further spectron excitation in terms of the old excitations. The Green's function for this even more "quasi" excitation is quite simple.

$$\tilde{g}_{\alpha}^{(i)}(\omega) = \frac{1}{\omega - E_{\alpha}^{(i)} + i\delta} \quad (\text{III.29})$$

It is the zeroth order description of a dressed excitation. In our formulation of self-consistent quasiparticle-phonon-coupling, it is this dressed eigenmode which we have coupled to the phonon, itself a dressed two-quasiparticle mode.

$$\begin{aligned} \mathcal{L}_{\alpha N}^{(i)}(\omega) &= \frac{i}{2\pi} \int d\omega_N d\omega_i \frac{1}{\omega_N - E_N + i\delta} \frac{1}{\omega_i - E_{\alpha}^{(i)} + i\delta} \delta(\omega - \omega_i - \omega_N) \\ &= \frac{1}{\omega - E_{\alpha}^{(i)} - E_N} \end{aligned} \quad (\text{III.30})$$

Hopefully, this explanation of a zeroth order excitation employed earlier is not too confusing. It is an output of the theory which we have sought, but the dressed quasifermion excitation is also a necessary input for a self-consistent theory.

4. The Renormalized RPA Equations

In this section, we present the derivation of the renormalized RPA equations for the superfluid.

We have already given the prescription for going from g , the quasiparticle Green's function, to the quasibogolon function, $\tilde{g}$, which contains the effects of the Brueckner renormalization and the pairing field. (Sec. II.D.4 and II.E.) The two-bogolon Green's functions will be defined

$$\begin{aligned}\tilde{k}^f &= \langle \tilde{0} | T \{ a_1(t_1) a_2(t_2) a_3^+(t_3) a_4^+(t_4) \} | \tilde{0} \rangle \text{ and} \\ \tilde{k}^b &= - \langle \tilde{0} | T \{ a_1(t_1) a_2(t_2) a_3(t_3) a_4(t_4) \} | \tilde{0} \rangle\end{aligned}\tag{III.31}$$

$\tilde{k}^f$ is the probability amplitude that if two quasibogolons are added to the interacting core they will evolve into two other quasibogolons. $\tilde{k}^b$ is the amplitude that the ground-state will be correlated, i.e., that the $|\tilde{0}\rangle$ contains 4-quasibogolon excitations. It is possible to derive the analytic structure of $\tilde{k}^f$ and $\tilde{k}^b$ in a manner similar to the two-particle Green's function (see Eq. (II.18)). The results are

$$\tilde{k}^f(\omega_1, \omega_2, \omega_3, \omega_4) = \frac{i}{2\pi} \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) \cdot \tag{III.32}$$

$$\left[\sum_s \frac{p_{12}(\bar{\omega}, \omega') p_{34}(\bar{\omega}, \omega)}{\bar{\omega} - (E^{(s)} - E^{(0)}) + i\delta} - \sum_s \frac{q_{12}(\bar{\omega}, \omega') q_{34}(\bar{\omega}, \omega)}{\bar{\omega} + (E^{(s)} - E^{(0)}) - i\delta} \right]$$

where $\bar{\omega} = \omega_1 + \omega_2 = \omega_3 + \omega_4$

$$\omega' = \omega_1 - \omega_2$$

$$\omega = \omega_3 - \omega_4$$

$$\frac{1}{2} \left(\frac{i}{2\pi} \right) \int_{-\infty}^{\infty} p_{12}(\bar{\omega}, \omega') d\omega' \equiv X_{12} = \langle s | a_1^+ a_2^+ | \tilde{0} \rangle$$

$$\frac{1}{2} \left(\frac{i}{2\pi} \right) \int_{-\infty}^{\infty} q_{12}(\bar{\omega}, \omega') d\omega' \equiv Y_{12} = \langle s | a_2 a_1 | \tilde{0} \rangle .$$

$$\hat{k}^b(\omega_1, \omega_2, \omega_3, \omega_4) = \frac{i}{2\pi} \delta(\omega_1 + \omega_2 + \omega_3 + \omega_4) .$$

$$\left[\sum_s \frac{p_{12}(\bar{\omega}, \omega') q_{34}(-\bar{\omega}, \omega)}{\bar{\omega} - (E^{(s)} - E^{(0)}) + i\delta} - \sum_s \frac{p_{34}(-\bar{\omega}, \omega) q_{12}(\bar{\omega}, \omega')}{\bar{\omega} + (E^{(s)} - E^{(0)}) - i\delta} \right]$$

$$\bar{\omega} = \omega_1 + \omega_2 = -\omega_3 - \omega_4$$

The main difficulty associated with the expansion (III.32) is the number non-conservation which accompanies the use of bogolons. The consequences and difficulties are discussed in several references.⁴⁶ We remark here that the most important phonons in the QPC scheme turn out to be the particle-hole shape vibrations. This (ph)-nature of the state restrains number non-conservation to a great extent. The two-bogolon Green's functions obey the graphical equations shown in Fig. 34. The analytical form of these Bethe-Salpater equations is:

$$\begin{aligned} \tilde{k}_{1234}^f &= \tilde{g}_1 \tilde{g}_2 \delta_{13} \delta_{24} - \tilde{g}_1 \tilde{g}_2 \delta_{14} \delta_{23} \\ &+ \frac{1}{2} \left(\frac{i}{2\pi} \right) \sum_{56} \int d\omega_5 d\omega_6 \delta(\omega_1 + \omega_2 - \omega_5 - \omega_6) \tilde{g}_3 \tilde{g}_4 < 34 | P | 56 > \tilde{k}_{1256}^f \\ &+ \frac{1}{2} \left(\frac{i}{2\pi} \right) \sum_{56} \int d\omega_5 d\omega_6 \delta(\omega_1 + \omega_2 + \omega_5 + \omega_6) \tilde{g}_3 \tilde{g}_4 < 34 | Q | 56 > \tilde{k}_{1256}^b \end{aligned} \quad (\text{III.33})$$

and a similar equation for $\tilde{k}_{1234}^b$. P and Q are the irreducible blocks in the bogolon-bogolon channel. We might calculate these from the G matrix using bogolons as the intermediates in the construction of the effective interaction. If we did so, Eq. (III.33) would provide an exact description of the two-bogolon Green's functions, and hence approximate specifications of the nuclear excited states (i.e., exact, except for number non-conservation). Rather than calculate P and Q, one generally parameterizes them in terms of F'' and F' (the quasiparticle-particle and quasiparticle-hole interactions in a truncated configuration space) and the BCS u and v

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Fig. 34. Graphical equations for k^F and k^B , the two-quasibogolon Green's functions.

ments. In reality, the actual figure is probably considerably higher, since the cutting of a second x-ray analysis sample from the same piece of metal produced a sample with no appreciable sign of the f.c.c. phase.

Pieces of 50-100 μg were cut from each piece and dissolved for copper spark spectrographic analysis; the results are shown in Table 7 together with the results of an analysis of the acid mixture used to dissolve the samples.

As mentioned above, the sample containers were capped with quartz balls which had been partially covered with Apiezon W wax. Considerable care is necessary in both the initial covering of the quartz ball by gently heating it and in the thermal softening of the wax when sealing the plug to the sample container as the wax evidently can produce paramagnetic impurities if overheated. This problem can usually be minimized if the wax is not heated so intensely that it flows freely. The effect of overheating is shown in Fig. X as the temperature dependence of the magnetic susceptibility at low temperature of the empty sample container used for metal sample #2. The susceptibility of the sample container used for metal sample #1 was not measured below 77°K since it showed no significant temperature dependence to that temperature and the effect observed later was not known. Comparison of the effect noted at low temperature in the empty (and later loaded) sample container for sample #2 with the low temperature deviation from temperature independence of sample #1 reveals striking similarities, and since no reasonable physical explanation would seem to account for such behavior of the metal, this deviation below 20°K has been attributed to the container rather than

$$= \tilde{g}_3 \tilde{g}_4 \left\{ \sum_{5 < 6} < 56 | P | 34 > X_{56} \right. \\ \left. + \sum_{5 < 6} < 56 | Q | 34 > Y_{56} \right\}$$

Finally, we integrate both sides by

$$\frac{i}{2\pi} \int d \left(\frac{\omega_3 - \omega_4}{2} \right)$$

to obtain

$$\frac{X_{34}}{A_{34}(\omega_s)} = \sum_{5 < 6} < 56 | P | 34 > X_{34} + < 56 | Q | 34 > Y_{34} \quad (\text{III.34a})$$

where

$$A_{34}(\omega_s) = \frac{i}{2\pi} \int d \left(\frac{\omega_3 - \omega_4}{2} \right) \sum_{i,j} \frac{Z_i}{\omega_3 - E_3^{(i)} + i\delta} \frac{Z_j}{\omega_4 - E_4^{(j)} + i\delta} \\ = \sum_{i,j} \frac{Z_i Z_j}{\omega_s - E_3^{(i)} - E_4^{(j)}}$$

To obtain a similar equation for the Y amplitudes, we consider pole terms

$$\bar{\omega} \approx - (E^{(s)} - E^{(o)}) \equiv -\omega_s:$$

$$\frac{Q_{12} Q_{34}}{\bar{\omega} + \omega_s} = \frac{i}{2\pi} \int d\omega_5 d\omega_6 \sum_{5 < 6} \frac{Q_{12} Q_{56}}{\bar{\omega} + \omega_s} < 56 | P | 34 > \tilde{g}_3 \tilde{g}_4 \delta(\bar{\omega} - \omega_5 - \omega_6)$$

$$+ \frac{i}{2\pi} \int d\omega_5 d\omega_6 \sum_{5 < 6} \frac{Q_{12} P_{56}}{\bar{\omega} + \omega_s} < 56 | Q | 34 > \tilde{g}_3 \tilde{g}_4 \delta(\bar{\omega} + \omega_5 + \omega_6)$$

Hence

$$Q_{34} = \tilde{g}_3 \tilde{g}_4 \left\{ \sum_{5 < 6} < 56 | Q | 34 > Y_{56} + \sum_{5 < 6} < 56 | Q | 34 > X_{56} \right\}$$

Finally,

$$\frac{Y_{34}}{A(-\omega_s)} = \sum_{5 < 6} < 56 | P | 34 > Y_{56} + < 56 | Q | 34 > X_{34} \quad (\text{III.34b})$$

Equation (III.34) constitutes the renormalized RPA for a superfluid nucleus. It is possible to obtain the renormalized RPA for particle-particle and particle-hole nuclei by expressing P and Q in terms of u, v, F', and F'' and taking the limits (u,v) = (0,1) or (1,0) depending on whether the single particle state is above or below the Fermi surface.

The normalization of the amplitudes X and Y is related to the inhomogeneous Eqs. (III.33), i.e., the homogeneous terms must be consistent with the choice $\tilde{g}_1 = \frac{N}{\omega - E_1 + i\delta}$ and N = 1. The expression has been derived by Migdal²³

$$\sum_{5 > 6} X_{56}^2 \frac{d}{d\omega} (A_{56}^{-1}(\omega))_{\omega=\omega_s} + Y_{56}^2 \frac{d}{d\omega} (A_{56}^{-1}(-\omega))_{\omega=\omega_s} = 1 \quad (\text{III.35})$$

In the usual RPA description,

$$A_{56}(\omega) = \frac{1}{\omega - E_5^{(0)} - E_6^{(0)}} \quad \text{and}$$

$$\sum_{5>6} x_{56}^2 - y_{56}^2 = 1 \quad .$$

The difference between the RPA and renormalized RPA is shown graphically in Fig. 35. It is essentially that the intermediate bogolons, while they may not interact with one another, may dress themselves by polarizing the core. We have discussed RPA in the particle-hole and particle-particle cases. For the superfluid, the situation is similar to the normal system. The poles of the Bethe-Salpater equation are the correlated eigenstates in the even-even nucleus. They are commonly called phonons because the lowest such eigenstates are generally very collective, having high transition probability or spectroscopic amplitudes. It is these elementary excitations, phonons, which are coupled in the odd-mass nucleus, or dressed one-particle Green's function.

A technical point which we have not mentioned thus far is that since bogolons do not conserve particle number there are certain spurious states whose generalized density matrices $\langle s | a_1^+ a_2^+ | \tilde{0} \rangle$ and $\langle s | a_2^- a_1^- | \tilde{0} \rangle$ correspond to the expectation value of the number operator in the ground state, as discussed in Ref. 43. In RPA, these states come to zero energy and are not mixed with the physical states under three conditions:

- a. $\tilde{g}_\alpha = g_\alpha^0$, i.e., one-pole quasibogolon Green's functions with $E_\alpha^{(0)}$ as determined from BCS.

Fig. 35a. Total quasibogolon scattering amplitude in usual RPA approximation. Lines are LQB, i.e., undressed.

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Fig. 35b. Additional contributions of renormalized RPA.

- b. $F''(\text{BCS}) = F''(\text{RPA})$ and all α in BCS problem included as $|\alpha\bar{\alpha} \uparrow \downarrow >$ RPA configurations.
- c. No two-quasibogolon pairs of the type $a_{\alpha}^{\dagger} a_{-\beta}^{\dagger} |\tilde{0} >$, where $\beta \neq \alpha$ but $\Omega_{\beta} = \Omega_{\alpha}$, which interact in the particle-particle channel.

In renormalized RPA, condition (a) is violated because the BCS bogolons are "dressed" by QPC. In deformed nuclei, condition (b) is difficult to satisfy within the limits of computer storage, while condition (c) is violated because there are many closeby Nilsson orbitals with the same Ω and π which are close in energy.

We maintain that it is convenient and proper to determine F'' by allowing the spurious state to come at approximately zero energy without orthogonalization. This is because we take the u's and v's from experiment and the equations for configuration space renormalization of $F''(\text{BCS})$ and $F''(\text{RPA})$ are identical, as long as $\langle \alpha\beta \neq \bar{\alpha} | F'' | \gamma\bar{\gamma} \rangle$ matrix elements are small.

5. Renormalization of Gap Equation

An additional self-consistency problem which should be considered is the effect of SCPQC on the pairing field. It is usually assumed that the pairing correlations predominate in nuclear structure and that long-range "phonon" correlations are weaker and do not affect the pairing. Hence, the non-self-consistent flow chart for nuclear calculations becomes

Before considering the equations for incorporating phonon correlations into the pairing field, let us consider the possible physical effects. The first is that since EPV diagrams such as Fig. 17b tend to raise the quasiparticle energies, $\epsilon_a = h_a - \lambda$ as well as the quasihole energies, $\epsilon_A = \lambda - h_A$, phonon correlations may interfere with pairing correlations. That is, if we calculate the phonon correlation first, the quasiparticle spectrum would develop a gap at the Fermi surface. In analogy to ^{208}Pb , where this shell model gap is so large that the pairing field is essentially zero, one might expect the "pushing up" effect of the EPV diagrams to weaken the amount of pairing. Conversely, the pushing down effect of the usual core-polarization diagram, Fig. 15c, might enhance pairing.

In a self-consistent calculation, we propose to correct for such effects by determining Δ from a "renormalized" gap equation:

$$\Delta_\alpha = \sum_{\beta, i} \langle \alpha \bar{\alpha} | F'' | \beta \bar{\beta} \rangle \frac{\Delta_\beta z_\beta^{(i)}}{E_\beta^{(i)}} \quad (\text{III.36})$$

That is, instead of using the formula $E_{\beta} = \sqrt{(h_{\beta} - \lambda)^2 + \Delta_{\beta}^2}$ one employs the fully dressed bogolon energies in the denominator. When the dressed quasibogolon energies $E_{\beta}^{(i)}$ are pushed up by backwards coupling, Δ will decrease. When the lowest quasibogolons are pushed down by QPC, Δ will increase. The origin of Eq. (III.36) is in the graphic equation for the anomalous self-energy shown in Fig. 36.²³ The matrix element F'' induces (pp) correlation while $\frac{Z_{\beta}^{(i)} \Delta_{\beta}}{E_{\beta}^{(i)}}$ is an "occupation probability". This "occupation" factor in Eq. (III.36) may be regarded as an ansatz, because we have not performed the rather arduous task of calculating exactly (from anomalous Green's functions) the renormalization of the pairing field. When such equations are explicitly written down, it is likely that the pairing field will also depend on which pole term we are considering, i.e., which eigenstate the nucleus is in. This complication will necessitate the use of state dependent u's and v's also. Short of carrying out what will surely be an extensive theoretical program, we cannot be sure that Eq. (III.36) is the best prescription for renormalizing the pairing field.

D. External Fields

So far we have focused our attention mainly on the determination of eigenstates* in the even and odd-mass nuclear systems. Although the

* In this context, we have actually determined the generalized density matrix elements connecting an eigenstate with a correlated reference state, whose composition is not explicit. Likewise, our eigenenergies are not absolute, rather they are energy differences. Nevertheless, the word "eigenstate" seems to us appropriate.

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Fig. 36. Graphical equation for the anomalous self-energy (c.f. Migdal²³).

energies of such eigenstates are quantities which can be measured from experiment, the other theoretical quantities, especially two-body matrix elements and Green's function amplitudes, are not so easily accessible. We are interested, therefore, in relating electromagnetic transition probabilities, and nucleon-transfer cross-sections to the various information we have available to us after calculating the eigenstates according to our approximate prescription.

An external field may be just what its name implies: a field applied to the nucleus externally, e.g., the Coulomb field of a nearby heavy ion. It could also be a term in the nuclear hamiltonian which is so weak that its only role in the nucleus is to connect states which are eigenstates of the strong part of the hamiltonian, e.g., the electromagnetic or weak interactions which induce γ - and β -decay.

The essential feature of an external field is that it promotes transitions between eigenstates. Hence, it is sensitive to the generalized density matrix elements or fractional parentage coefficients which relate eigenstates to one another. These are precisely the output of the theory of finite Fermi systems.

In particular, we would like to calculate, with the framework of SCQPC,

1. Electromagnetic transition probabilities between eigenstates, and
2. one-nucleon stripping cross-sections from even- to odd-mass nuclei.

1. Electromagnetic Transition Probabilities

We discuss in detail the first of these because the external field is weak (i.e., first order perturbation theory is applicable), and well-

understood theoretically. Hence, all discrepancies between theory and experiment are due to shortcomings in the nuclear structure theory, rather than inadequate treatment of the external field itself.

There are two common approaches to the evaluation of transition probabilities. Both treat the external field to first order in perturbation theory; first, the transition amplitude is calculated as $\langle f|v|i \rangle$ and the second order two-photon process (where i, f are initial and final states)

$$\int \sum_k \frac{\langle f|v|k \rangle \langle k|v|i \rangle}{E_i - \hbar\nu_1 - E_k} d\nu_1$$

is assumed negligible. The transition probability W_{fi} is obtained according to the Fermi Golden Rule.^{23,47}

$$W_{fi} = 2\pi \delta(\omega - (E_i - E_f)) |\langle f|v|i \rangle|^2 \quad (\text{III.37})$$

The density matrix approach derives from the idea that the external field is itself a one-body operator. Hence,

$$\begin{aligned} \langle f|v|i \rangle &= \langle f| \sum_{\alpha\beta} v_{\alpha\beta} c_{\alpha}^{\dagger} c_{\beta} |i \rangle \\ &= \sum_{\alpha\beta} v_{\alpha\beta} \langle f| c_{\alpha}^{\dagger} c_{\beta} |i \rangle = \text{Tr}(v\rho^{fi}) \end{aligned} \quad (\text{III.38})$$

Configuration space truncation and Landau renormalization require the introduction of renormalized charges, which are taken to be state independent, e_p^{ren} and e_n^{ren} .⁴⁸⁻⁵⁰

After this renormalization,

$$\langle f|v|i \rangle = \text{Tr}(e^{\text{ren}} v_p^{\text{fi}}) \quad \text{qp} \quad \text{(III.39)}$$

ρ_{qp} is the quasiparticle density matrix. In the case of the electromagnetic field, the V matrix elements may be evaluated without difficulty. Thus, the problem reduced to finding elements of the generalized quasiparticle density matrix. As an example the $0^+ \rightarrow 2^+$ transition in even-even nuclei is quite simple to evaluate because the RPA amplitudes X and Y are easily related to the required quasiparticle density matrix elements.

Suppose as in Sec. III.C.4, $X_{\alpha\beta} = \langle 2|a_{\alpha}^+ a_{\beta}^+ |0 \rangle$ and $Y_{\alpha\beta} = \langle 2|a_{\beta} a_{\alpha} |0 \rangle$ where a_{α}^+ is the creation operator for a quasibogolon - in particular, a_{α}^+ is a transform of the quasiparticle and quasihole operators, c_{α}^+ and c_{α}^- , are, in turn, related to the bare particle and hole operators,

$$a_{\alpha}^+ = U c_{\alpha}^+ - S_{\alpha} V_{\alpha} c_{-\alpha}^- = U c_{\alpha}^+ - V c_{\alpha}^- \quad \text{(III.40a)}$$

$$c_{\alpha}^+ = \sqrt{Z_{\alpha}} C_{\alpha}^+ + \dots \quad \text{(III.40b)}$$

Equation (III.40a) gives

$$c_{\alpha}^+ c_{\beta}^- = -u_{\alpha} v_{\beta} a_{\alpha}^+ a_{\beta}^- - u_{\beta} v_{\alpha} a_{\beta}^+ a_{\alpha}^- \quad \text{(III.41)}$$

+ terms with one creation and one annihilation operator,

so that when antisymmetry and time-reversal invariance of the state,

$|2 \rangle$, are taken into account

$$\langle 2|v|0 \rangle = - \sum_{\alpha > \beta} \sqrt{2} (u_{\alpha} v_{\beta} \pm v_{\alpha} u_{\beta}) \langle \alpha | e_{\tau}^{\text{ren}} v | \beta \rangle (X_{\alpha\beta} \pm Y_{\alpha\beta}) \quad (\text{III.42})$$

$\Omega_{\alpha} > 0$
 $\tau = \text{proton,}$
 neutron

where the $\pm$ signs depend on time-reversal phase of the external field, e.g., (+) for electric and (-) for magnetic transition moment operators.

The density matrix approach (III.39) may also be employed for odd-mass nuclei. In this case, the density matrix elements

$$\rho_{\beta\alpha}^{fi} = \langle f | c_{\alpha}^{\dagger} c_{\beta} | i \rangle$$

may be evaluated according to the technique of Speth.⁴⁸ This method is based on the idea that

$$|i\rangle = P_i \left\{ e^{-iHt} c_Y^{\dagger}(-\infty) |\tilde{0}\rangle, t \rightarrow 0^- \right. \quad (\text{III.43})$$

that is the state $|i\rangle$ is the product of evolution from a simple shell-model excitation and a projection operator P_i which projects onto one eigenstate and normalizes. Hence,

$$\rho_{\beta\alpha}^{fi} = \lim_{\substack{t_{\delta} \rightarrow -\infty \\ t_Y \rightarrow -\infty \\ t_{\alpha} \rightarrow +0 \\ t_{\beta} \rightarrow -0}} \langle \tilde{0} | c_{\delta}(t_{\delta}) c_{\alpha}^{\dagger}(t_{\alpha}) c_{\beta}(t_{\beta}) c_Y^{\dagger}(t_Y) | \tilde{0} \rangle \quad (\text{III.44})$$

The generalized density matrix elements are thus clearly related to the two-nucleon Green's function. Equations of motion for ρ may then be formulated within the framework of the many-body theory.

We shall find it convenient to calculate transition moments of odd-mass nuclei according to the effective field formalism.^{23,51} In an odd-mass nucleus,⁵¹

$$\langle f|v|i \rangle = \langle \tilde{0}|v|\tilde{0} \rangle \delta_{fi} + \sum_{12} \langle f|c_2^+|\tilde{0} \rangle \langle \tilde{0}|c_1|i \rangle \tau_{21}(v; E_i, E_f) \quad (\text{III.45})$$

That is, since the V is a one-body operator, its expectation value between two odd-mass nuclear states is proportional to the single-particle excitation amplitudes of the "before" and "after" states times the sum of all diagrams which involve a transition between two single-particle states. The $\langle \tilde{0}|v|\tilde{0} \rangle$ contribution arises because if we are calculating a nuclear moment, i.e., $i = f$, the diagrammatic contributions to τ measure only the contribution of the odd-nucleon, not the inner-shell particles. One frequently encounters the notation

$$\langle f|\delta v|i \rangle \equiv \langle f|v|i \rangle - \langle \tilde{0}|v|\tilde{0} \rangle \delta_{fi} \quad (\text{III.46})$$

τ is called the vertex function. It is composed of the lowest order contribution from the external field as well as the totality of graphs which connect the incoming and outgoing lines via the nuclear interaction. The diagrammatic formula is shown in Fig. 37. The algebraic expression is simply

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Fig. 37. Graphical contributions to the vertex function. The decomposition of Γ is in the second particle-hole formalism. Lines are FDSM.

$$\begin{aligned}
\tau_{21}(V, E_i, E_f) &= V_{21} - \frac{i}{2\pi} \int d\omega \sum_{34} \Gamma_{1423}(E_i, E_f - E_i + \omega, E_f, \omega) \\
&\quad \cdot G_3(\omega) G_4(E_f - E_i + \omega) V_{43} \\
&= V_{21} - \frac{i}{2\pi} \int d\omega \sum_{34} I_{1423}(E_i, E_f - E_i + \omega, E_f, \omega) \\
&\quad \cdot G_3(\omega) G_4(E_f - E_i + \omega) \tau_{43}(V; E_i, E_f) \quad (\text{III.47})
\end{aligned}$$

where the sum over (3,4) includes (p,h) and (h,p) pairs, the $\frac{1}{2\pi}$ factor accompanies the interaction, and the (-1) factor counts the number of closed loops.

Because the bare particle Green's functions contain high energy poles, it is convenient to carry out the usual Landau renormalization, as well as restricting the configuration space for the summation (3,4). The equations for the renormalized vertex are^{24,49,50}

$$\langle f | \delta v | i \rangle = \sqrt{Z_1 Z_2} \sum_{12} \langle f | c_2^+ | \tilde{0} \rangle \langle \tilde{0} | c_1 | i \rangle \tau_{21}(e^{\text{ren}V}; E_i, E_f) \quad (\text{III.45}')$$

$$\tau_{21}(e^{\text{ren}V}; E_i, E_f) = (eV)_{21}^{\text{ren}} - \frac{i}{2\pi} \int d\omega \sum_{34} F'_{1423}(E_i,$$

$$\begin{aligned}
&E_f - E_i + \omega, E_f, \omega) g_3(\omega) g_4(E_f - E_i + \omega) \tau_{43}(e^{\text{ren}V}; E_i, E_f) \\
&\quad (\text{III.47}')
\end{aligned}$$

The transition between any two nuclear states, i and f, in the odd-mass nucleus may be calculated according to Eq. (III.45') and (III.47').

There are several associated difficulties, however. One is that Eq. (III.47') is an inhomogeneous matrix equation not exactly corresponding to a phonon approximation. Another major difficulty is that I or F' is energy-dependent.

The remaining discussion of electromagnetic transition probabilities will be divided as follows:

- a. Migdal's approximation;
- b. quasiparticle-phonon coupling approximation;
- c. the origin of "higher order" terms;
- d. off-the-energy-shell problems; and
- e. an approximate SCQPC prescription for moments of the external field.

a. Migdal's Approximation. In Eq. (III.45'), we may assume that

$$\begin{aligned} \langle f | c_2^+ | \tilde{0} \rangle &= 1 \\ \langle \tilde{0} | c_1 | i \rangle &= 1 \end{aligned} \quad (\text{III.48})$$

while assuming that g_3 and g_4 contain only one pole. This corresponds to the quasiparticle approximation. If, in addition, F^1 is taken to be energy independent (unretarded), the following system of equations is obtained:²⁴

$$\tau_{21} = (ev)_{21}^{\text{ren}} + \sum_{34} F'_{1423} X_{34} + F'_{1324} Y_{34} \quad (\text{III.49a})$$

and

$$(\Omega - E_3 - E_4)X_{34} - (eV)_{43}^{\text{ren}} = \sum_{56} F'_{3645} X_{56} + F'_{3546} Y_{56} \quad (\text{III.49b})$$

$$(-\Omega - E_3 - E_4)Y_{34} - (eV)_{43}^{\text{ren}} = \sum_{56} F'_{3645} Y_{56} + F'_{3546} X_{56}$$

Since $\Omega = (E_f - E_i)$, Eq. (III.49) may be solved by matrix inversion. The point to be emphasized is that the X's and Y's so obtained are not identical to the X's and Y's corresponding to eigenmodes of the even-mass nucleus. This is not surprising since the (ph) excitation propagating away from the odd-nucleon in Fig. 37 is "off-the-energy shell", i.e., $\Omega = E_f - E_i \neq E_{\text{phonon}}$. Equations (III.48-49), which rely on the quasiparticle approximation, are the usual approximation of Migdal. They may be generalized to quasiparticle Green's functions with more than one pole by obtaining the residues $\langle f|c_2^+|0 \rangle$, $\langle 0|c_1|i \rangle$ via a solution of Dyson's equation and substituting $1/A_{34}(\pm\Omega)$ for $(\pm\Omega - E_3 - E_4)$ in Eq. (III.49b), where

$$A_{34}(\Omega) = \sum_{ij} \frac{z_3^{(i)} z_4^{(j)}}{\Omega - E_3^{(i)} - E_4^{(j)}} \quad (\text{III.50})$$

In all that follows, the one pole approximation will be generalized to Eq. (III.50).

b. The Quasiparticle-Phonon Coupling Approximation. We would like to motivate the following procedure (an approximation to Eq. (III.49b) which is often used for the calculation of transitions between odd-nucleon states:

$$|\psi_i\rangle = \alpha c_1^+ |\tilde{0}\rangle + \beta c_2^+ Q_N^+ |\tilde{0}\rangle$$

$$|\psi_f\rangle = \gamma c_2^+ |\tilde{0}\rangle + \delta c_1^+ Q_N^+ |\tilde{0}\rangle; \quad (\text{III.51})$$

and, finally,

$$\langle \psi_f | V | \psi_i \rangle = (\alpha \cdot \gamma) \langle 2 | V | 1 \rangle + (\beta \gamma + \delta \alpha) \langle \tilde{0} | V | N \rangle^*$$

Often, one of the phonons possesses most of the transition strength, i.e., is very collective. In this case, the quasiparticle transition matrix element is enhanced primarily by coupling to this one collective mode.

Equation (III.47') may be written, assuming the block F' is unretarded,

$$\tau_{21}(V; \Omega = E_i - E_f) = (eV)_{21}^{\text{ren}} + \sum_N \frac{\langle f | c_2^+ Q_N^+ |\tilde{0}\rangle \langle \tilde{0} | V | N \rangle \langle 1 | H_{\text{eff}} | 2N \rangle}{\Omega - E_N}$$

* We note that in an odd-mass deformed nucleus, the phonon transition has a degeneracy factor which differs from the even-even nucleus, viz.

$$\langle \tilde{0} | V | N \rangle_{\text{odd}} = \sqrt{\frac{1 + \delta_{K_N, 0}}{2}} \langle \tilde{0} | V | N \rangle_{\text{even}}.$$

The reason for this is that in the odd mass nucleus, the phonon may couple parallel or antiparallel to the particle $K + \Omega \neq |K - \Omega|$ for $K \neq 0$. In the even-even nucleus $\Omega = 0$.

$$- \sum_N \frac{\langle \tilde{0} | c_{1N}^\dagger | i \rangle \langle \tilde{0} | v | N \rangle \langle 2 | H_{\text{eff}} | 1N \rangle}{\Omega + E_N} \quad (\text{III.52})$$

where

$$\langle \tilde{0} | v | N \rangle = \sum_{56} (x_{56}^N + y_{56}^N) (ev)_{56}^{\text{ren}}$$

and

$$\langle 1 | H_{\text{eff}} | 2N \rangle = \sum_{56} F'_{1625} x_{56}^N + F'_{1526} y_{56}^N.$$

In deriving Eq. (III.52) we have taken advantage of the relation between k and Γ , Eq. (II.31), and the spectral decomposition of k , Eq. (III.32). So far, we have made only the approximation that F' is unretarded.

The utility of Eq. (III.52) is that while we must sum over many phonons, N , the X 's and Y 's are derived in a separate RPA calculation. Then in the case that only for a particular N is $\langle \tilde{0} | v | N \rangle$ significant, the sum may be replaced by the single term.

A further simplification arises if we recall that $\langle f | c_2^\dagger | 0 \rangle = \sqrt{Z_2^{(f)}}$ may be related to the amplitude $\langle f | c_1^\dagger c_N^\dagger | \tilde{0} \rangle$ in the lowest order QPC approximation, viz. Eq. (III.4a)

$$\left. \begin{aligned} \langle f | c_1^\dagger c_N^\dagger | \tilde{0} \rangle &= \frac{\langle 2 | H_{\text{eff}} | 1N \rangle \langle f | c_2^\dagger | 0 \rangle}{E_f - E_i - E_N} \\ \langle i | c_2^\dagger c_N^\dagger | \tilde{0} \rangle &= \frac{\langle 1 | H_{\text{eff}} | 2N \rangle \langle i | c_1^\dagger | 0 \rangle}{E_i - E_f - E_N} \end{aligned} \right\} \quad (\text{III.53})$$

Equation (III.53) combined with Eq. (III.45') yields

$$\begin{aligned}
 \langle f | \delta V | i \rangle &= \sqrt{Z_1^{(i)} Z_2^{(f)}} (eV)_{21}^{\text{ren}} \\
 &+ \langle f | c_1^+ Q_N^+ | \tilde{0} \rangle \langle \tilde{0} | V | N \rangle \\
 &+ \langle i | c_2^+ Q_N^+ | \tilde{0} \rangle \langle \tilde{0} | V | N \rangle \quad (\text{III.54})
 \end{aligned}$$

Equation (III.54) is identical to Eq. (III.51) except for a slight notational change. It is a simplification of Eq. (III.47') in that the inhomogeneous equation for the X's and Y's is not solved, rather they are taken from the RPA calculation of the dominant collective state. This simplification, however, will permit us to improve a different aspect of the approximation in Eq. (III.49),²⁴ namely, the energy dependence of the particle-hole effective interaction.

c. The Origin of "Higher-Order" Diagrams. Let us for the time being neglect the particle-particle channel and Landau and configuration space renormalization, i.e., $F'' = F^{\tilde{F}} = J = 0$ and $F' = F^{\omega} = I$.

The most severe approximation that we have not removed so far is that the particle-hole block has been assumed to be energy independent. Figure 37 for the exact transition amplitude is reminiscent of Fig. 22 for the self-energy. The quantity of interest is exactly equal to a "string of bubbles" graph where each bubble is connected by the energy-dependent I-block.

As in Sec. III.B, we may treat the energy dependence of I in one of two ways:

- i. substitute an exact I with energy dependence into Fig. 37; or,
- ii. use some lower form of the interaction, e.g., $\tilde{I}$, and calculate the "higher order", i.e. energy-dependent, contributions explicitly.

As before, we adopt this approach, and expand the T -block which is a "one-three" vertex.

In Fig. 38, we define an approximate I -block in terms of the short-range (almost energy independent) G -matrix and one- and two-phonon exchange contributions. Figure 39 formulates the vertex function in terms of the "string of bubbles" or phonon and the I -block which has the unique role of initiating the (ph)-bubble string.

Figure 40 then shows the higher-order terms which arise when Fig. 38 is substituted into the I -block of Fig. 39. The description "higher-order" is derived from the decomposition of I into various lower and higher order contributions. If all of the vertices in Fig. 38 could be taken as energy independent, calculating Fig. 39 by substitution of Fig. 38 would provide a more reliable estimation of the vertex function than the usual approximation, where I is unretarded or energy independent.

If G is approximately energy independent, then contributions (ab_1) , (ab_2) , (ac_1) , (ac_2) correspond to the QPC approximation shown in Fig. 41a. Figure 40 (bb_1) , (bb_2) (bc_1) , and (bc_2) cancel the Pauli principle violations implicit in Fig. 41a and shown explicitly in Fig. 41b.

(bb_3) , (bc_3) , (cb_3) , and (cc_3) represent an effective external field operator which connects quasiparticle-phonon configurations. These are reconstructed (for future discussion) in Fig. 41c and 41d. Contributions (cb_1) , (cb_2) , (cc_1) , and (cc_2) represent the coupling of a particle to

Fig. 38. Irreducible particle-hole block decomposed into a short-range contribution plus several long-range phonon exchange graphs.

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Fig. 39. The vertex function in terms of phonons (wiggly line) and the initial I-block. The intermediate lines are FDSM and may be time-ordered so that the external field acts before, during, or after I-block.

(b) $\rightarrow$ (c) Similar to (bb) except --- replaced by ~~~~~

Fig. 40. Short and long range contributions to vertex function when Fig. 38 is substituted into Fig. 39, i.e., initial I-block decomposed. Lines are FDS!! All possible time orderings are explicitly drawn.

where

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(c) → (c) Similar to (c) → (b) except --- replaced by

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+ backwards and other diagrams

(d) → (c) Similar to (d) → (b) except --- replaced by

Fig. 41a. Lowest-order QPC contribution to vertex.

Fig. 41b. EPV's cancelled by (bb_1) , (bb_2) , (bc_1) , and (bc_2) .

Fig. 41c.

Fig. 41d.

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Fig. 41e. Graphs in which higher-order coupling terms of H_{eff} contribute before or after vertex operator.

the two-phonon state followed by a "cross-over" transition. In analogy to even-even nuclei, this process is probably not too important. (db_1) , (db_2) , etc., show a class of graphs in which the self-energy intermediate or particle-phonon configuration iterates several times before decaying via the external field. The structure of these graphs in terms of the H_{eff} defined earlier is shown in Fig. 41e.

d. Off-the-Energy-Shell Contributions to Effective Vertices. The vertex contributions from Fig. 41c and 41d represent a certain difficulty in calculational practice, namely that we must know the vertex function "off-the-energy-shell". Consider Fig. 41c, for example. The intermediate propagators (suppose they have particle labels 3 and 4) will have poles at energies E_k and E_l . The corresponding vertex function is $\tau_{43}(V; E_i, E_f)$. The point is that the $\tau_{43}(V; E_i, E_f)$ will be very much different from $\tau_{43}(V; E_k, E_l)$. The latter would usually be calculated as a matter of routine and would be stored in the computer as one number. The "off-the-energy-shell" vertex requires an entire mesh of numbers which would be tedious to compute and practical to store.

The magnitude and sign of such energy dependent terms fluctuates substantially depending on the value of $\Omega = E_i - E_f$. This is shown in Fig. 42 where we plot (see Eq. (II.114))

(III.55)

$$= \frac{\gamma V}{\Omega - E_N} + \frac{\gamma V}{-\Omega - E_N} = \frac{-2E_N \gamma V}{E_N^2 - \Omega^2}$$

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Fig. 42. Energy dependence of approximate vertex function of Eq. (III.55).

γ = QPC vertex, taken to be unretarded and attractive,

$V = \langle 0 | V | N \rangle$, taken to be positive in Fig. 42,

$\Omega = E_i - E_f \neq E_k - E_l$, and

W_N = phonon energy.

It is clear from the graph that the only time it is safe to approximate $\tau(\Omega)$ as energy independent is when E_N is large and all energy differences of odd-mass nuclear states are approximately zero. This is not the case, for example, in ^{166}Er , where $E_N = 0.8$ MeV for $N = \gamma$ vibration and we are interested in E2 transitions among states at 0.208, 0.668, and 0.752 MeV.

Actually, it turns out that if we use a global propagator formalism, Fig. 41c may be drawn as shown in Fig. 43. The processes (d) and (e), but not (b) and (c) are calculated in one of the contributions to Fig. 41e. In Fig. 44, the specific nature of this claim is detailed.

In the calculations reported later, Fig. 43a,b,c as well as Fig. 41d have not been calculated. The above discussion does not justify that procedure. In fact, Fig. 43b,c may contribute significantly on account of an energy difference being "accidentally" close to a pole in Fig. 42. Figure 41d is proportional to the moment of the excited state (e.g., the $2+$ vibration) which is known⁵² to be of the same order as the E2 transition moment. Birbrair⁵³ has shown that the relatively large magnitude here, compared to the negligible "crossover" transition probability, results from the energy dependence of the vertex function.

e. An Approximate SCQPC Prescription for Moments of the External Field.

The alternative to the exact vertex calculation with the exact energy-dependent I-block is, as we have seen, to separate from the I-block what we

Fig. 43. Explicit contributions (global propagator formalism) to Fig. 41c.

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Fig. 44. Processes included in "higher-order" H_{eff} graphs.

consider to be important energy-dependent terms and evaluate the higher-order terms directly. As in the case of the self-energy, the only I-block we will decompose is the one-three vertex - the remaining ones serve as effective interactions which define the phonon, and this part of the diagram seems to be well approximated by unretarded I-blocks.

We take the "one-three" blocks to be identical to the ones used in the calculation of the self-energy, i.e., the energy independent F' and F'' defined previously. This is consistent with the approximate idea that they are the sum of short-range graphs initiating long-ranged phonons. The diagrams we calculate are shown in Fig. 45. The usual transcriptions from particles to quasiparticles and from quasiparticles to quasibogolons has been assumed.

Figure 45a is calculating by resolving spectrons i and j into quasibogolons α and β , then calculating the transition between them.*

$$\text{Fig. 45a} = \langle i | A_{\alpha}^+ | 0 \rangle \langle f | A_{\beta}^+ | 0 \rangle (u_{\alpha} u_{\beta} - v_{\alpha} v_{\beta}) \langle \beta | (eV)^{\text{ren}} | \alpha \rangle$$

(III.56)

Figure 45b,d,e, etc., summarize the various ways in which the spectron or exact elementary excitation, i , propagates before resolving into the spectron, f , plus a phonon. Figure 45c,f, etc., display the vice versa process.

* Equation (III.56-59) constitute the algorithm for calculating the transition amplitudes quoted in Sec. V. The only additional information necessary is the $N \rightarrow 0$ transition, Eq. (III.42), and the $e^{\text{ren}} = 1.4$ for protons and 0.4 for neutrons.

Fig. 45. Diagrams included in SCQPC calculation of transition amplitudes. Single lines are QB, double lines are EFB (i.e., they are dressed).

Fig. 46. Backwards diagrams neglected.

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$$\text{Fig. 45 b + d + e + h} = \langle i | A_{\beta}^{\dagger} Q_N^+ | \tilde{0} \rangle \langle \tilde{0} | V | N \rangle \quad (\text{III.57})$$

$$\text{Fig. 45 c + f + ...} = \langle f | A_{\alpha}^{\dagger} Q_N^+ | \tilde{0} \rangle \langle \tilde{0} | V | N \rangle \quad (\text{III.58})$$

It should be noted that since H_{eff} includes Fig. 29a, the (iii) and (iv) EPV diagrams in Fig. 41b are cancelled. Since H_{eff} includes Fig. 29b, Fig. 43d and 43e are calculated.

The one further effect of the Pauli Principle is shown in the (i) and (ii) contributions to Fig. 41b. These EPV's are cancelled by Fig. 40 (bb₁) and (bb₂), (same as Fig. 45g).

The (i) and (ii) contributions may be subtracted directly by defining for purposes of insertion into Eq. (III.57,58).

$$\begin{aligned} \langle 0 | V | N \rangle_{i|fN} &= \langle 0 | V | N \rangle_{\text{even-even}} - \\ &\sum_{3,k} \frac{Z_3^{(k)} Z_2^{(f)}}{\omega_N - E_3^{(k)} - E_2^{(f)}} \langle 2 | V | -3 \rangle P^N(23) \end{aligned} \quad (\text{III.59})$$

$$\langle 0 | V | N \rangle_{f|iN} = \text{similarly} \quad .$$

As we have indicated earlier, the neglect of Fig. 41c and 41d is a serious approximation. Also the two-phonon crossovers of Fig. 40 (cb₁), (cb₂), (cc₁), and (cc₂) are neglected. Certain backwards diagrams such as Fig. 45h are included by virtue of the resolvents in Eq. (III.57) and (III.58), while others shown in Fig. 45 are neglected. This is probably not serious, on account of the smallness of the backwards residues. Finally, in all of the diagrams we do calculate, the only phonon (or wiggle) included is the

lowest in energy. As long as this collective mode exhausts a substantial fraction of transition strength allowed by the sum rule, this is a good approximation.

2. One-Nucleon Transfer Reactions

In this subsection, we shall be concerned with nuclear reactions of the type

$$Z_N^A (d,p) Z_N^A + 1_{N+1}$$

$$Z_N^A (d,t) Z_N^A - 1_{N-1}$$

$$Z_N^A (\alpha,t) Z_{+1}^A + 1_N$$

$$Z_N^A (t,\alpha) Z_{-1}^A - 1_N$$

where Z, N are even. In general, these reactions measure the single particle strength of the levels in the residual odd-mass nucleus.

We would like to treat the nuclear reaction in first order perturbation theory and in a convenient formalism. First, however, we indicate graphically the various time-dependent perturbation theory processes which might occur. These are shown in Fig. 47 for the usual (d,p) stripping reaction. In addition, to simple one-neutron stripping, the ingoing or outgoing particles may undergo elastic and inelastic scattering via the nuclear or Coulomb potentials. There is also the exotic possibility of two different nuclear reactions occurring in series, but with the net effect of one-neutron stripping. The totality of such diagrams describes the T-matrix.

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Fig. 47. Perturbation processes for a (d,p) reaction. --- denotes an electromagnetic interaction, wavy a nuclear interaction, $\Rightarrow$ a transfer reaction. Propagators are nuclear eigenstates coupled to plane waves, K .

$$\langle A + 1, 1 | a_p(K') U(\infty, -\infty) a_d^\dagger(K) | A, GS \rangle^*$$

In regard to the latter possibility (process (d) in Fig. 47), we should mention that multiple-transfer diagrams would cease to be of any use in perturbation theory, if the reaction were by a compound nucleus mechanism. A compound nucleus would be a complicated mixture of projectile-core states, requiring an infinite summation similar to Tamm-Dancoff. The usual approximation is to ignore diagrams in which more than one transfer reaction occurs. The physical idea behind this is that at energies just above the Coulomb barrier, (d,p), etc., are surface reactions, the projectile grazing the surface (once only) of the target on its way past. The justification is experimental, viz., the anisotropic distribution of residual particles. Most of the effect of the nuclear and Coulomb potentials can be accounted for via DWBA, which is like Hartree-Fock for nuclear reactions. The idea is to replace all the two-body interactions by an average one-body potential. This may be calculated,⁵⁵ but it is usually taken to be the optical potential.⁵⁶ The plane-wave incoming and outgoing "orbitals" are replaced by distorted waves. For the details of DWBA, the reader may wish to consult references.^{57,58}

At low energies, most of the residual interaction is the long-range Coulomb potential. Process (c) shown in Fig. 47, in which a Coulomb

* We have taken the most straightforward tack in this subsection and presented transfer reactions in terms of the usual Born approximations to the S matrix, which one encounters in most quantum mechanics textbooks, as well as in the literature. It is possible, via a Green's function formalism,⁵⁴ to extract direct transfer cross-sections from a general many-body theory of nuclear reactions.

interaction precedes or follows transfer, can be included in the description of one-nucleon transfer reactions via a coupled channels formulation.⁵⁹ However, this possibility is generally ignored also. It is possible to express then the one-nucleon transfer reaction in terms of a spectroscopic amplitude and a geometric factor:

$$\langle i | T_{d,p}(K, K') | \tilde{0} \rangle \approx \sum_{\alpha} \langle i | c_{\alpha}^{\dagger} | 0 \rangle S_{\alpha}(K, K') \quad (\text{III.60})$$

and similar equations for (p,d), (α,t), etc. In a superfluid nucleus, the interesting (and available) amplitudes are $\langle i | a_{\alpha}^{\dagger} | 0 \rangle$. $a_{\alpha}^{\dagger}$ is related to $c_{\alpha}^{\dagger}$ by Landau renormalization, which introduces a constant factor $\sqrt{Z}$, and the BCS transformation, which introduces a factor u_{α} (stripping) or v_{α} (pickup). If we assume that the i^{th} and j^{th} eigenstates contain single-particle strength from only one orbital, the ratio of integrated cross-sections is simply

$$\frac{\sigma(i)}{\sigma(j)} = \frac{|\langle i | a_{\alpha}^{\dagger} | 0 \rangle|^2}{|\langle j | a_{\alpha}^{\dagger} | 0 \rangle|^2} \quad (\text{III.61})$$

Clearly, the experimentalist and theoretician are compatible at this point. They can both convert their "data" to relative fractions of single-particle strength in various odd-mass eigenstates (bandheads, in deformed nuclei). Thus, the results of Sec. V (tables in Appendix C) are expressed in terms of various percentages of single-particle strength.

IV. DETAILS OF THE CALCULATION

In the previous section, we have proposed a self-consistent theory of quasiparticle-phonon coupling. In Fig. 48, the calculational scheme is depicted in a flowchart. As is usual in a phenomenological theory, the starting point is the choice of theoretical quantities such as the single particle levels and the effective interactions in the (pp) and (ph) channels. Next a calculation of the collective state(s) is performed (RPA). The phonon as well as the quasiparticles (normal) or quasibogolons (superfluid) then constitute the input for a calculation of the self-energy. There is a self-consistency requirement within this step because, as shown in Sec. III, the intermediate propagators should be "dressed" or exact. After the self-energies are consistent, the collective state is recalculated according to the renormalized RPA equations. The phonon and quasiparticle calculations must be iterated to self-consistency because the phonon is renormalized by the fractionation of single particle strength (i.e., the improved self-energy), while the single particle characteristics depend on the renormalized phonon. In effect, this is an "outside do loop". A further self-consistency which should be required of the complete theory is that the pairing field be consistent with the renormalized quasiparticles or bogolons. We have indicated this loop by a dotted line, along with choice of new parameters, because we have only occasionally executed this part of the scheme--in those cases in which we felt it necessary (see later in this section). In the remainder of this section, the details which are particular to each step (or box in Fig. 48) of the calculation are presented.

Fig. 48. Flow chart of self-consistent quasiparticle-phonon coupling (SQPC).

A. Phenomenological Inputs to the Theory-Parameterization

As was indicated in the section on applications of Green's functions, the Hartree-Fock shell model picture is renormalized by Brueckner theory and other high energy processes. The effect is to deplete the shell model strength by approximately 15%. The idea of Fermi liquid theory is to regard the 85% part of the Green's function, which has only one pole, as a quasiparticle or quasihole. The irreducible (pp) and (ph) effective interactions, J and I , are renormalized by the 15% part to become F^E and F^W . Since J and I , as well as F^E and F^W are defined in a rather large model space, configuration space truncation further renormalizes F^E and F^W to F'' and F' . Then, according to the bogolon approximation, the pairing field is calculated from F'' . Quasiparticles and -holes become bogolons and F'' and F' are transformed by various linear combinations of u and v amplitudes into P and Q (see Eq. (II.62)).

1. Quasiparticle Energies

The quasiparticle and -hole energies would be approximately determined by a renormalized BHF calculation, such as in Ref. 60, or a density dependent Hartree-Fock calculation, such as in Ref. 61. This would of course neglect many self-energy contributions which are not included later; however, it would be a very useful starting point. Since the main result of such calculations is to produce an energy dependent and spatially non-local potential well, $U(\epsilon, r)$, we might take the quasiparticle energies and wave functions from a phenomenological potential such as Nilsson⁶² or Woods-Saxon.⁶³ We shall report calculations for nuclei in the rare-earth region, $152 \leq A \leq 178$, so we mention that for deformed nuclei perhaps

the most realistic parameterization for $U(\epsilon, r)$ is that of Vogeler.⁶⁴ However, in the calculations reported here, the Nilsson single particle basis set prescribed by Soloviev⁶⁵ and modified by Faessler⁶⁶ has been used. There are 32 proton orbitals and 36 neutron orbitals in this set. The single particle energies of these orbitals were chosen to fit the experimental energy level data of odd-mass nuclei⁶⁷ and are listed in units of $\hbar\omega$ in Table 1. This set, used both for gamma and octupole states, facilitates comparisons with earlier calculations. The wave functions are obtained by diagonalizing a deformed harmonic oscillator hamiltonian with $\epsilon = 0.29$. The strengths of the spin-orbit ($\ell \cdot s$) and centrifugal (ℓ^2) terms varied for different principal quantum numbers and are given in Table 2.

2. The (pp) and (ph) Interactions

The interactions employed in a renormalized configuration space (to be specified later) were taken to be point-like in space and time. This is in accord with the suggestion of Migdal.²⁴ In earlier calculations, we have employed both the volume (VDI) and surface (SDI) delta interactions. The SDI is more consistent with the density dependent parameterization of Migdal, in which the particle-hole block is repulsive inside the nuclear volume but attractive outside. Thus we shall assume a spin- and isospin-dependent surface delta interaction.⁶⁸

$$F'' = \xi[SDI]$$

$$F' = \{f + f' t_1 \cdot t_2 + (g + g' t_1 \cdot t_2) \sigma_1 \cdot \sigma_2\} [SDI]$$

$$[SDI] = 2 \left(\frac{28 \text{ MeV}}{A} \right) (R_0 U_0)^{-4} \delta(r_1 - R_0) \delta(r_2 - R_0) \delta(\cos \omega_{12} - 1) \quad (IV.1)$$

Table 1A. The proton single-particle levels $|\alpha\rangle \equiv \Omega\pi[Nn_z\Lambda]$, energies ϵ_α

Z	$\Omega\pi[Nn_z\Lambda]$	$\frac{\epsilon_\alpha}{\hbar\omega_0}$
	1/2 + 440	0.39
	3/2 + 431	0.46
	5/2 + 422	0.67
	1/2 - 301	0.74
	1/2 + 431	0.82
	7/2 + 413	0.98
	1/2 - 550	1.03
	3/2 - 541	1.06
	1/2 + 420	1.08
	3/2 + 422	1.10
60	9/2 + 404	1.20
62	5/2 - 532	1.31
64	5/2 + 413	1.36
66	3/2 + 411	1.42
68	7/2 - 523	1.48
70	1/2 + 411	1.56
72	7/2 + 404	1.66
74	1/2 - 514	1.69
76	5/2 + 402	1.76
	1/2 - 541	1.86
	3/2 + 402	1.90
	1/2 + 400	1.97
	11/2 - 505	2.04
	1/2 + 660	2.11
	3/2 - 532	2.18
	3/2 + 651	2.25
	1/2 - 530	2.32
	5/2 + 642	2.39
	5/2 - 523	2.46

(continued)

Table 1A. (continued)

Z	K[Nn ₂ Λ]	$\frac{\epsilon_{\alpha}}{\hbar\omega_0}$
	3/2 - 521	2.53
	7/2 + 633	2.60
	7/2 - 514	2.70

The single-particle energies, the energy gap and the chemical potential are given in units of $\hbar\omega = 41 A^{-1/3}$ MeV.

Table 1B. The same values as Table 1A but for neutrons in the rare earth region

N	$\Omega\pi(Nn_z A)$	$\frac{\epsilon_\alpha}{\hbar\omega_0}$
	1/2 - 550	0.00
	3/2 - 541	0.06
	5/2 - 532	0.12
	7/2 - 523	0.37
	5/2 + 402	0.39
	1/2 - 541	0.41
	7/2 + 404	0.58
	1/2 + 400	0.60
	5/2 - 514	0.61
	3/2 + 402	0.66
	1/2 - 530	0.82
	11/2 - 505	0.85
	3/2 - 532	0.91
90	1/2 + 660	0.95
92	3/2 + 651	1.00
94	3/2 - 521	1.04
96	5/2 + 642	1.08
98	5/2 - 523	1.11
100	7/2 + 633	1.26
102	1/2 - 521	1.30
104	5/2 - 512	1.36
106	7/2 - 514	1.48
108	9/2 + 624	1.55
110	1/2 - 510	1.64
112	3/2 - 512	1.68
114	1/2 + 651	1.71
	7/2 - 503	1.74
	1/2 + 640	1.75

(continued)

Table 1B. (continued)

N	$K\pi[Nn_z\Lambda]$	$\frac{\epsilon_\alpha}{n\omega_0}$
	9/2 - 505	1.78
	3/2 - 501	1.79
	11/2 + 615	1.83
	1/2 - 770	1.97
	3/2 + 642	2.03
	1/2 - 501	2.09
	5/2 - 503	2.15
	3/2 - 761	2.16

Table 2. Spin-orbit coupling parameter, κ , centrifugal parameter, μ , used in the determination of the single particle wave functions of Table 1

N	μ	κ
3(proton)	0.45	0.05
4(proton)	0.55	0.05
5(proton)	0.55	0.05
6(proton)	0.45	0.05
4(neutron)	0.45	0.05
5(neutron)	0.45	0.05
6(neutron)	0.45	0.05
7(neutron)	0.4	0.05

where ξ , f , f' , g , and g' are dimensionless constants of order unity and A is the nuclear mass. If the approximation is made that all radial wave functions have the same value (namely U_0) at the nuclear surface, then $(R_0 U_0)^{-4} \delta(r_1 - R_0) \delta(r_2 - R_0)$ becomes unity when integrated over the radial coordinates and only the angular integration needs to be performed. In the Migdal formalism, the (ph) matrix element of F' is evaluated in the second (ph) channel and is not antisymmetric. A very fast algorithm for calculating VDI and SDI matrix elements with Nilsson wave functions was developed by the author in the appendix of Ref. 69.

In Table 3, we report the values of f , f' , g , and g' which we have employed in two sets of calculations. The first calculation was an unrenormalized RPA calculation of the $2+$, $0+$, $0-$, and $2-$ phonons, the values of the constants being determined to give the best agreement with experiment. The second calculation is the renormalized RPA and SCQPC which constitutes most of our numerical results. In all cases, the relative magnitude of the four constants was the same. The spin dependence was chosen in accordance with the magnetic moment data²⁶ and the splittings in particle-hole nuclei.²⁷ The overall renormalization for each spin and parity is states in terms of the factor r .

In Table 4, we list the values of ξ which were employed. In a large model space, the particle-particle interaction should be the same for each nucleus as well as for both neutrons and protons. However, this is not true for a truncated configuration space. In fact, the interaction is determined by the requirement that the spurious state should still come at 0.0 MeV.

Table 3. Particle-hole force parameters

$f = -0.9r$		$f' = 0.2r$	$g = g' = 0.5r$
Unrenormalized RPA		r	
2+		0.866	
0+		0.866	
0-		1.33	
2-		1.6	
Renormalized RPA		r	
2+		0.91	
0+		0.91	
0-		1.35	
2-		1.4	

Table 4. Particle-Particle Force Parameters

Nucleus	Proton	Neutron
$^{152}_{\text{Sm}}$	-1.2950	-0.09605
$^{154}_{\text{Sm}}$	-1.3050	-0.9784
$^{154}_{\text{Gd}}$	-1.2730	-0.9708
$^{156}_{\text{Gd}}$	-1.2836	-0.9888
$^{158}_{\text{Gd}}$	-1.2940	-0.9945
$^{160}_{\text{Gd}}$	-1.3040	-1.0450
$^{158}_{\text{Dy}}$	-1.269	-0.9991
$^{160}_{\text{Dy}}$	-1.2796	-1.0057
$^{162}_{\text{Dy}}$	-1.2900	-1.0556
$^{164}_{\text{Dy}}$	-1.3000	-1.0780
$^{164}_{\text{Er}}$	-1.3490	-1.0990
$^{166}_{\text{Er}}$	-1.3570	-1.0886
$^{168}_{\text{Er}}$	-1.3670	-1.0695
$^{170}_{\text{Er}}$	-1.3775	-1.1170
$^{168}_{\text{Yb}}$	-1.3700	-1.1000
$^{170}_{\text{Yb}}$	-1.3820	-1.0800
$^{172}_{\text{Yb}}$	-1.3926	-1.1270
$^{174}_{\text{Yb}}$	-1.4030	-1.1660
$^{176}_{\text{Yb}}$	-1.4165	-1.1960
$^{174}_{\text{Hf}}$	-1.3544	-1.1376
$^{176}_{\text{Hf}}$	-1.3650	-1.1760
$^{178}_{\text{Hf}}$	-1.3750	-1.2080

We have shown that the self-energy may be calculated approximately by restricting the intermediates of the mass operator to phonon-quasi-particle configurations, i.e., to those excitations known to be long range. The interactions which might initiate such nuclear modes are different from the irreducible (pp) and (ph) blocks. They are identical only in the lowest order. In the calculations reported here, however, it is assumed that (ph) phonons are initiated by the (ph) block and (pp) excitations by the (pp) block. In a phenomenological theory, this was done to avoid unreasonable amounts of arbitrary parameter fitting. In other words, we took

$$F''_{13} = F''$$

$$F'_{13} = F' \quad (\text{IV.2})$$

3. The Pairing Field Parameters

The pairing parameters u and v of each level should be determined from the BCS (or renormalized) gap equation using F'' . However, the finite basis truncation of the pairing problem involves a different renormalization than the truncation of the configuration space so that the BCS problem requires the introduction of a different parameter, ξ , in the definition of F'' . Instead, the state-independent Δ 's and λ 's employed by Soloviev⁶⁵ have been adopted. Not only are the empirically fit u 's and v 's of this scheme nearly the same as those determined by using the gap equation and a new phenomenological interaction, but they also permit comparison with earlier schematic model calculations.

In Table 5, we give the Δ , λ , and G for each even and odd proton and neutron number in the rare earth region. The values of Δ and λ for

even nuclear number were taken from Soloviev.⁶⁵ The corresponding constant pairing matrix element was calculated from the gap equation and the single particle levels of Table 2. The odd-nucleon number G parameter is the average of the neighboring even values; the Δ_{odd} and λ_{odd} were then calculated from the gap and number equations.

The Δ and λ given in Table 5 were used to calculate both the phonons and the quasiparticle self-energy. In particular, since particle number is not conserved, calculations are schematically identical in even and odd-mass superfluid systems. The only difference is the interpretation of the computer output.

B. Migdal RPA

The first iteration of the SCQPC calculation is somewhat different in that the quasiparticle strength is not yet split and the phonons can be calculated according to the usual RPA prescription. The numerical method applied for the renormalized RPA problem is, of course, applicable here, and its discussion is deferred until later.

It is important that we should indicate before outlining the procedure for dressing quasiparticle excitations the selection of configurations. For the $2+$, $0+$, $0-$, and $2-$ states[†] in deformed nuclei, configurations were considered in which the sum of the unperturbed quasiparticle energies was less than $41/A^{1/3}$ MeV. That is, if $\Omega_\alpha + \Omega_\beta = K$ and $\pi_\alpha \pi_\beta = \pi_K$ and $E_\alpha^0 + E_\beta^0 \leq 41/A^{1/3}$, then the configuration $|\alpha\beta\rangle$ was allowed.

[†]The $1-$ and $3-$, as well as $1+$ phonons were not included because our calculations did not indicate them to be very collective in character.

Table 5. Pairing Parameters. State Independent Pairing Force Approximation

Protons (units of $\hbar\omega$)

N	G_e	G_o	Δ_e	Δ_o	λ_e	λ_o
60	0.02427		0.124		1.252	
61		0.02422		0.1247		1.2884
62	0.02418		0.127		1.325	
63		0.02415		0.1284		1.3572
64	0.02412		0.129		1.391	
65		0.02407		0.1283		1.423
66	0.02403		0.127		1.458	
67		0.02399		0.1249		1.4907
68	0.02395		0.123		1.528	
69		0.02389		0.1213		1.5631
70	0.02383		0.121		1.601	
71		0.02379		0.122		1.636
72	0.02375		0.123		1.671	
73		0.02372		0.1225		1.7029
74	0.02369		0.121		1.737	
75		0.02367		0.119		1.7711
76	0.02366		0.118		1.808	
77		0.02362		0.1178		1.8425
78	0.02359		0.118		1.877	

Neutrons (units of $\hbar\omega$)

N	G_e	G_o	Δ_e	Δ_o	λ_e	λ_o
88	0.02123				0.917	
89		0.02123		0.1354		0.9446
90	0.02123		0.137		0.968	
91		0.02112		0.136		0.9951
92	0.02112		0.136		1.018	
93		0.02108		0.1339		1.0455

(continued)

Table 5. (continued)

Neutrons (units of $h\nu$)						
N	G_e	G_o	Δ_e	Δ_o	λ_e	λ_o
94	0.02103		0.131		1.068	
95		0.02099		0.1259		1.0975
96	0.02095		0.12		1.123	
97		0.02094		0.1111		1.1516
98	0.02093		0.104		1.195	
99		0.02094		0.1038		1.2373
100	0.02095		0.106		1.273	
101		0.02094		0.1059		1.3068
102	0.02093		0.104		1.341	
103		0.02094		1.1014		1.3777
104	0.02096		0.099		1.419	
105		0.02099		0.109		1.4608
106	0.02102		0.111		1.497	
107		0.02100		0.118		1.5328
108	0.02099		0.124		1.559	
109		0.02093		0.1311		1.5925
110	0.02086		0.135		1.613	
111		0.02077		0.1402		1.6425
112	0.02067		0.142		1.660	
113		0.02073		0.1464		1.6859
114	0.02059		0.146		1.703	
115		0.02055		0.1461		1.7261
116	0.02051		0.145		1.743	

Also, the configuration $|\beta\alpha\rangle$ is identical except for antisymmetry and the associated bogolon density matrices have the symmetry property

$$X_{\alpha\beta} = -X_{\beta\alpha}$$

$$Y_{\alpha\beta} = -Y_{\beta\alpha}$$

It is convenient to calculate matrix sums for only one permutation. This gives rise to the following alterations of the RPA equation:

$$\sum_{\gamma\delta} \left\{ \begin{aligned} & \frac{\xi_{\alpha\beta}^{\pm} \xi_{\gamma\delta}^{\pm}}{2} \langle \alpha\beta | F'' | \gamma\delta \rangle_A \\ & + \eta_{\alpha\beta}^{\pm} \eta_{\gamma\delta}^{\pm} \langle \alpha\bar{\delta} | F' | \bar{\beta}\gamma \rangle \end{aligned} \right\} + \sum_{\gamma\delta} \left\{ \begin{aligned} & \langle \alpha\beta | \pm | \gamma\delta \rangle = \\ & \xi_{\alpha\beta}^{\pm} \xi_{\gamma\delta}^{\pm} \langle \alpha\beta | F'' | \gamma\delta \rangle_A \\ & + \eta_{\alpha\beta}^{\pm} \eta_{\gamma\delta}^{\pm} [\langle \alpha\bar{\delta} | F' | \bar{\beta}\gamma \rangle \\ & \mp \langle \alpha\bar{\gamma} | F' | \bar{\beta}\delta \rangle] \end{aligned} \right. \quad (\text{IV.4})$$

In the case of $K = 0$ configurations, the amplitude

$X_{-\beta, -\alpha}$, $Y_{-\beta, -\alpha}$ must be calculated in addition to $X_{\alpha\beta}$ and $Y_{\alpha\beta}$. Since by time reversal symmetry, they are identical except for a phase, the matrix sums are limited to configurations such that $\alpha < \beta$ and $\Omega_{\alpha} > 0$, i.e., in summing over α and β , the time reverse of α is never considered. This effects further alterations on the RPA matrix elements:

$$\begin{aligned}
 & \sum_{\gamma < \delta} + \sum_{\substack{\gamma < \delta \\ \Omega \gamma > 0}} \\
 & \left. \begin{aligned}
 & \xi_{\alpha\beta}^{\pm} \xi_{\gamma\delta}^{\pm} \langle \alpha\beta | F'' | \gamma\delta \rangle \\
 & + \eta_{\alpha\beta}^{\pm} \eta_{\gamma\delta}^{\pm} \langle \alpha\bar{\delta} | F' | \bar{\beta}\gamma \rangle \\
 & \mp \langle \alpha\bar{\gamma} | F' | \bar{\beta}\delta \rangle
 \end{aligned} \right\} + \left\{ \begin{aligned}
 & \frac{1}{\sqrt{1 + \delta_{\alpha-\beta}}} \frac{1}{\sqrt{1 + \delta_{\gamma-\delta}}} \\
 & \{ \xi_{\alpha\beta}^{\pm} \xi_{\gamma\delta}^{\pm} [\langle \alpha\beta | F'' | \gamma\delta \rangle + \delta_{K,0} \langle \alpha\beta | F'' | \delta\gamma \rangle] \\
 & + \eta_{\alpha\beta}^{\pm} \eta_{\gamma\delta}^{\pm} [\langle \alpha\bar{\delta} | F' | \bar{\beta}\gamma \rangle \mp \langle \alpha\bar{\gamma} | F' | \bar{\beta}\delta \rangle] \\
 & \pm \delta_{K,0} \langle \alpha\gamma | F' | \bar{\beta}\bar{\delta} \rangle - \delta_{K,0} \langle \alpha\delta | F' | \bar{\beta}\bar{\gamma} \rangle \}
 \end{aligned} \right\}
 \end{aligned}
 \tag{IV.5}$$

C. Self-Consistent Quasiparticle-Phonon Coupling

In this part of the calculation, the phonons obtained from RPA, the microscopic two-body matrix elements of the irreducible or effective interactions, the unperturbed bogolons with energy $E_{\alpha}^{(0)}$, and the dressed bogolons with residues and pole energies $Z_{\beta}^{(i)}$ and $E_{\beta}^{(i)}$ are used to determine the residues and pole positions, $Z_{\alpha}^{(i)}$ and $E_{\alpha}^{(i)}$. It is a self-consistent calculation. The details of this procedure, corresponding to the theory of Sec. III.C, can be separated into the following steps.

1. Calculation of the QPC vertices.
2. Construction of lowest order matrices coupling bogolon and phonon-parton pairs. Truncation.
3. Construction of second order effective Hamiltonian between phonon-dressed bogolon pairs.
4. Diagonalization.

5. Truncation of small residues.
6. Recycle to step 2 until results of matrix diagonalization on successive cycles converge.

The procedure must be carried out for all quasiparticles. The protons and neutrons can be done separately. However, even for an odd-proton nucleus, the neutrons are calculated as they renormalize the phonon. It is convenient to calculate the self-energy for each proton (neutron) in one cycle and then repeat using the output of the previous cycle for the "dressed" bogolon input to the next cycle.

1. The QPC Vertices

The QPC vertices are given in Eqs. (II.62), (II.63), and (III.8-11). The multiplicative factor which accompanies $K = 0$ phonons should be noted. It is convenient to combine Eq. (III.34), viz., the RPA equation, and Eq. (III.8) viz., the definition of the QPC vertices, to obtain

$$P(\alpha\beta) = X_{\alpha\beta}/A_{\alpha\beta}(E_N)$$

$$Q(\alpha\beta) = Y_{\alpha\beta}/A_{\alpha\beta}(-E_N) \quad (IV.6)$$

For each pair, $|\alpha\beta\rangle$, such that $\Omega_\alpha + \Omega_\beta = K$, only one set of vertices P, Q, R, S need be calculated. The remainder can be obtained from the symmetry properties, (III.9).

2. Construction of Matrices with Lower Order Matrix Elements

For each bogolon, a matrix of the form shown in Fig. 49 is constructed. In general, there will be more than one bogolon with the same spin and parity and it would not be difficult to calculate all of the same

$$\begin{array}{|c|c|} \hline A = A^+ & B \\ \hline -B = B^+ & -A \\ \hline \end{array}
 \begin{bmatrix} \psi \\ \phi \end{bmatrix} = E \begin{bmatrix} \psi \\ \phi \end{bmatrix}$$

$$A = \begin{array}{|c|c|} \hline E_{\alpha} & \sqrt{Z_j} \langle \beta | H_{\text{eff}} | \alpha \rangle \\ \hline (\text{sym}) & E_{\beta}^{(j)} + E_N + H_{\text{eff}} \\ \hline \end{array}$$

$$B = \begin{array}{|c|c|} \hline 0 & \sqrt{Z_j} \langle \tilde{O} | H_{\text{eff}} | \alpha \beta^{-1} N^{-1} \rangle \\ \hline -\sqrt{Z_j} \langle \tilde{O} | H_{\text{eff}} | \alpha \beta^{-1} N^{-1} \rangle & 0 \\ \hline \end{array}$$

Fig. 49. Matrix problem for quasiparticle Green's functions (odd-mass nuclear energy levels).

$K\pi$ simultaneously. However, they are usually either well separated in energy, different in particle-hole nature, or sufficiently different in the radial and angular coordinate representations that the self-energy does not have off-diagonal elements, i.e., no eigenvector will contain appreciable fractions of both bogolons. We make this assumption throughout. The submatrices A and B are square. The first row and column correspond to the bare bogolon. The others refer to configurations of a phonon and a dressed quasifermion excitation or spectron: That is, $a_{\beta j}^{\dagger} Q_N^+$ where $a_{\beta j}^{\dagger}$ creates the quasifermion eigenstate, j, obtained in the diagonalization for bogolon, β . The matrix elements $\langle \beta N | H_{\text{eff}} | \alpha \rangle$ have been given in Sec. III. C, Eq. (III.21).

Since the configuration space has been limited to a finite set of configurations, $|\alpha\beta\rangle$, then the coupling matrix elements are available only when α_1 couples to β_1 + phonon and the configuration $|\alpha_1 \beta_1\rangle$ is in the space. We have further truncated the dimensions of the SCQPC problem by considering α_1 and β_1 only if they are among the 10 quasiparticles or quasiholes closest to the Fermi energy.

3. The Second Order Effective Interaction Between Phonon-Dressed-Bogolon Pairs

The analytical expressions for these matrix elements have been given also in Sec. III.C.3.c., Eq. (III.22). We remark here that it turns out (numerically and for sound theoretical reasons which will be discussed later) that the higher order diagrams such as those shown in Figs. 16b and 18a,b do not contribute to the renormalization of the phonons. Hence, they do not need to be included until the iterations between RFA and SCQPC have converged. In most of the calculations reported here, the off-diagonal H_{eff}

matrix elements were not included until the last iteration, for this reason. This was primarily for convenience, as these are very tedious to calculate and exhaust a great deal of computer time. The self-consistent intermediates and diagonal second order corrections of the type shown in Fig. 16b, i.e., the Pauli Principle corrections, were included in all iterations.

The energy-dependent denominators were approximated in the following manner:

$$\langle \alpha^i_N | H_{\text{eff}}(\omega) | \beta^j_{N'} \rangle \quad \text{with denominator} \quad E_{\alpha}^{(i)} + E_{\beta}^{(j)} + E_Y(k) - \omega;$$

$$\omega \text{ was taken as } \frac{1}{2} (E_{\alpha}^{(i)} + E_{\beta}^{(j)} + E_N + E_{N'}) \quad . \quad (\text{IV.7})$$

$$\langle \alpha^i_N | H_{\text{eff}}(\omega) | \beta^j_{N'} \rangle \quad \text{with denominator} \quad \omega - E_N - E_{N'} - E_Y^k;$$

This denominator showed a high propensity to accidentally vanish for almost every choice of ω . Though self-consistent diagonalization would avoid the problem, this was not feasible. ω was taken as E_Y^k , i.e., the denominator was taken to be

$$-(E_N + E_{N'}) \quad (\text{IV.8})$$

This assumption would be correct if all quasi bogolons had the same energy, say 1 MeV. In fact, for low energy ones we are interested in calculating, this is not an unreasonable approximation.

4. Diagonalization

The matrix problem shown in Fig. 49 is symmetric, so a number of rapid computational schemes might be used. However, we do not have the option of finding the lowest eigenvalue and disregarding the rest, so the

(rather than a basis state) containing a significant part of the β -quasi-particle strength. There may of course, be more than one such state, u_{β}^i , and the equations are appropriately modified by multiplying off-diagonal matrix elements by residues which are less than unity. Thus we include graphs of the type shown in Fig. 71. The matrix diagonalization problem for the one-quasiparticle Green's function is shown in Fig. 49 (Sec. IV).

4. The even-even problem is still the Bethe-Salpater equation for the variation of the Bogolon density matrix--i.e., no two phonon states. Instead of the problem

$$(\omega - E_{\alpha\beta})X_{\alpha\beta} = P_{\alpha\beta\gamma\delta} X_{\gamma\delta} + Q_{\alpha\beta\gamma\delta} Y_{\gamma\delta}$$

$$(\omega + E_{\alpha\beta})Y_{\alpha\beta} = -P_{\alpha\beta\gamma\delta} Y_{\gamma\delta} - Q_{\alpha\beta\gamma\delta} X_{\gamma\delta}$$

we substitute for $(\omega \pm E_{\alpha\beta})$, $\mp 1/A_{\alpha\beta}(\mp\omega)$ where

$$A_{\alpha\beta}(\omega) = \sum_{ij} \frac{z_{\alpha}^{(i)} z_{\beta}^{(j)}}{\omega - E_{\alpha}^{(i)} - E_{\beta}^{(j)}} \quad (\text{A.25})$$

This matrix diagonalization is energy-dependent and the eigenvectors are not orthogonal. A method for solution of this problem was devised for normal systems by Zawisha and Werner⁷⁰ and the extension to superfluids is carried out in a previous chapter.

Earlier the self-consistent treatment of phonons and quasiparticles was carried out near closed shells by these authors.⁷⁰ We can obtain their equations by simply ignoring all but the zeroth quadrant in the QPC matrix

5. Residues

After having obtained the eigenvectors from the diagonalization outlined above, the forwards and backwards residues are obtained

$$Z_{\alpha}^{(i)} = |\psi_{\alpha}^{(i)}|^2$$

$$Z_{\alpha}^{(i, \text{back})} = |\phi_{\alpha}^{(i)}|^2$$

The dressed bogolon Green's function is then

$$\tilde{g}_{\alpha}(\omega) = \sum_i \frac{Z_{\alpha}^{(i)}}{\omega - E_{\alpha}^{(i)} + i\delta} + \frac{Z_{\alpha}^{(i, \text{back})}}{\omega + E_{\alpha}^{(i)} - i\delta} \quad . \quad (\text{IV.13})$$

Most of the forward residues and all of the backwards residues will be small, and therefore effect no qualitative changes when inserted into another calculation. One then has two choices as to the remaining (large) residues:

- a. renormalize them so that they sum to one;
- b. leave them, as is.

The justification for the first possibility is that, if the residues do not sum to one, the RPA diagonal energies are effectively shifted upward, making the phonon less collective. Also, we have defined the quasiparticle as containing all renormalizations due to small residues of the real particle Green's function, hence the "reduced" residues of the quasiparticle Green's function should sum to 1. The argument in behalf of the second choice is that if a bogolon has a large strength spread among a lot of weak high energy poles, then renormalization (sum = 1) will

weight the low energy poles, without physical justification. The latter objection can be removed and the renormalization procedure used if, whenever a pole has some predetermined strength and it is truncated, the strength is placed in the poles which are close in energy but not truncated.

We choose the following truncation limits:

- a. no residues with $Z < 0.05$;
- b. no backwards residues;
- c. no more than four poles. If the next largest residues have $Z_5 > 0.01$, then the strength will be distributed to the nearest poles according to the following formula:

$$\frac{\Delta Z^{(i)}}{Z_5} = \frac{1/|E_5 - E^{(i)}|}{\sum_{j=1,4} 1/|E_5 - E^{(j)}|} \quad (\text{IV.14})$$

Eq. (IV.14) is merely an ad hoc prescription to minimize the effect of "lost" single particle strength. It turns out to be necessary, i.e., affects numerical results, in only a few cases, notably ^{153}Eu and neighboring isotopes.

D. Renormalized RPA Equations

When the dressed quasiparticles or bogolons have been obtained self-consistently, then the RPA phonon is renormalized, i.e., recalculated, according to equations

$$PX + QY = X/A(\omega)$$

$$PY + QX = Y/A(-\omega)$$

$$A_{\alpha\beta}(\omega) = \sum_{ij} \frac{z_{\alpha}^{(i)} z_{\beta}^{(j)}}{\omega - E_{\alpha}^{(i)} - E_{\beta}^{(j)}} \quad (\text{III.34})$$

The difficulties associated with the diagonalization are two-fold:

- (1) The matrix must be diagonalized repeatedly until self-consistency.
- (2) The eigenvectors are not orthogonal. (In the RPA sense, "orthogonal" means the quasibosons commute.)

We discuss first the method for self-consistent diagonalization. The basic idea was proposed by Zawischa and Werner.⁷⁰ Namely, that as $\omega \rightarrow E_N$, the energy dependent expression may be expanded in a Taylors series about the value of ω on the previous iteration, (i.e., ω_0).

$$\frac{1}{A(\omega)} \cong \frac{1}{A(\omega_0)} + (\omega - \omega_0) \left. \frac{d}{d\omega} \frac{1}{A(\omega)} \right|_{\omega = \omega_0}$$

$$\frac{1}{A(-\omega)} \cong \frac{1}{A(-\omega_0)} + (\omega - \omega_0) \left. \frac{d}{d\omega} \frac{1}{A(-\omega)} \right|_{\omega = \omega_0} \quad (\text{IV.15})$$

Thus we obtain the matrix problem

$$\begin{bmatrix} P - \frac{1}{A(\omega_0)} & Q \\ Q & P - \frac{1}{A(-\omega_0)} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$= (\omega - \omega_0) \begin{bmatrix} \frac{d}{d\omega} \left(\frac{1}{A(\omega)} \right)_{\omega = \omega_0} & 0 \\ 0 & \frac{d}{d\omega} \left(\frac{1}{A(-\omega)} \right)_{\omega = \omega_0} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \quad (\text{IV.16a})$$

It is easily seen that derivative matrix of Eq. (IV.16) is the weighting factor which is needed to normalize the amplitudes X and Y according to the requirement of Eq. (III.35). Thus, we have

$$f([x,y]) \equiv [X \quad Y] \begin{bmatrix} \frac{d}{d\omega} \frac{1}{A(\omega)} & 0 \\ 0 & \frac{d}{d\omega} \frac{1}{A(-\omega)} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = 1 \quad (\text{IV.16b})$$

It is not possible to reduce the dimensions of the matrix problem by a factor of two as in previous RPA calculations,⁷¹ because the diagonal quadrants are not equal. However, a great deal of computer time is saved by taking advantage of the facts that only the diagonal elements are different in the upper left and lower right quadrants and that the metric, or derivative, matrix has no off-diagonal elements. Both matrices are symmetric. In solving (IV.16) for the lowest collective states, we have employed⁷² the method of steepest descents of Hestenes and Karush.⁷² The algorithm was applied first to nuclear physics problems by Kumar and Baranger.⁷³ The advantage of the method is that only one eigenvector is found at a time and the number of computer operations is proportional to the square, rather than the cube, of the matrix dimension. Also, the method converges after only a few iterations when the initial guess is very good--as will generally be the case on all but the first or second outside iterations of the even mass-odd mass self-consistency problem.

We present the equations of the method both as an illustration of the method's simplicity and to have available the appropriate formulae and definitions for later discussion.

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Fig. 50. μ defined as a function of the two-dimensional vector, X . Find the gradient, ξ , in which direction μ increases most rapidly. Minimize μ , in the method of steepest descents, by moving in the opposite direction.

Suppose we seek the lowest eigenvalue and eigenvector, ω_1 and x_1 , such that

$$Hx_1 = \omega_1 Fx_1 \quad (\text{IV.17})$$

where H and F are symmetric matrices. This is equivalent to minimizing the Rayleigh quotient^{72,74}

$$\mu(x) \equiv \frac{h(x)}{f(x)} \quad (\text{IV.18})$$

where

$$h(x) \equiv x H x$$

$$f(x) \equiv x F x, \text{ matrix multiplication understood}$$

$$\mu(x_1) \equiv \omega_1.$$

$\mu(x)$ defines a surface in $(N + 1)$ -space, where N is the dimension of the matrices, H and F . See Fig. 50. The idea is to find the gradient of μ , called ξ , and correct the initial guess for the eigenvector by descending along the μ -surface in the opposite direction of ξ .

$$x' = x - \alpha \xi \quad (\text{IV.19})$$

The gradient is easily shown to be proportional to

$$\xi = Hx - \mu Fx \quad (\text{IV.20})$$

The number α is determined by maximizing the difference

$$\mu(x) - \mu(x - \alpha \xi)$$

The result is

$$\alpha = \frac{c}{b + \sqrt{b^2 + ac}} \quad (\text{IV.21})$$

where

$$a = f(\xi)\eta - h(\xi)\zeta$$

$$b = \frac{1}{2} (h(\xi) f(x) - h(x) f(\xi))$$

$$c = f(x)\eta - h(x)\zeta$$

$$\eta = \xi H x$$

$$\zeta = \xi F x \quad .$$

For reasons of numerical convergence, α is multiplied by a damping factor less than one.

H and F in Eq. (IV.17) correspond to the matrices of renormalized RPA, Eq. (IV.16). The normalization of $[X \ Y]$ in (IV.16b) is just $f(x) = 1$ (IV.24) where $f(x)$ is defined in Eq. (IV.18).

It is also possible to minimize $\mu(x)$ subject to the condition that x be metric orthogonal to a lower eigenvector, viz.

$$x F x_0 = 0 \quad (\text{IV.22})$$

This can be done by projecting out from the initial guess and subsequent gradients the unwanted component.

$$x_{int} = x_{guess} - (x_0^T x_{guess}) F x_0$$

and

$$\xi = t - (x_0^T t) F x_0$$

where

$$t = Hx - \mu Fx \quad . \quad (IV.23)$$

The various self-consistent eigenvectors of Eq. (IV.16) are not orthogonal. Hence, when the second or third lowest eigenvector is sought it is necessary to orthogonalize not to the lowest eigenvector but to the lower solutions of Eq. (IV.16) for the current value of ω . Though the difference is not great, the method of steepest descents will not converge to higher eigenvectors of a matrix unless the trial solutions are orthogonal to the lower "solutions",

$$\text{i.e., } [X_1' \ Y_1'] [F(\omega_2)] \begin{bmatrix} X_2 \\ Y_2 \end{bmatrix} = 0 \quad . \quad (IV.25)$$

A further difficulty is exclusion of the spurious state. Because the equation for the amplitudes of a 0+ phonon is energy dependent, the usual orthogonalization requirement

$$X_s X_1 - Y_s Y_1 \text{ where } X_s = -Y_s = \delta_{\alpha\beta} \frac{u_\alpha v_\alpha}{\sum_\alpha (u_\alpha v_\alpha)^2} \quad (IV.26)$$

does not hold. In fact, if the pairing and phonon problems were solved in a truly self-consistent manner, the occupation probabilities u_α and v_α

would be replaced by more sophisticated amplitudes which would be different for each pole of the bogolon Green's function. Since we have not treated the pairing field self-consistently, it is necessary to select an ansatz to ensure that real physical states do not assume the characteristics (e.g., high (p,t) strength) of the spurious or "neighboring nuclei" state. We shall demand that Eq. (IV.26) be fulfilled. This is correct in the limit of one-pole bogolon Green's functions. Equation (IV.26) may be satisfied by subjecting the variational method of steepest descents to an external condition, viz. the initial guesses and gradients will be "orthogonalized" to the spurious state in the spirit of Eqs. (IV.23)

$$\begin{bmatrix} X \\ Y \end{bmatrix}_{\text{int}} = \begin{bmatrix} X \\ Y \end{bmatrix}_{\text{guess}} - \gamma F_0 \begin{bmatrix} s \\ s \end{bmatrix}$$

$$\begin{bmatrix} \xi_x \\ \xi_y \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix} - \delta F_0 \begin{bmatrix} X_s \\ Y_s \end{bmatrix} \quad (\text{IV.27})$$

where

$$\begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} H \\ F \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} - \mu \begin{bmatrix} F \\ F \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}.$$

H, F are defined in Eq. (IV.16).

$$F_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\gamma = \begin{bmatrix} X_s & Y_s \end{bmatrix} \begin{bmatrix} F_0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}_{\text{guess}}$$

E. Renormalized Gap Equations

In general, phonon and pairing correlations should be treated self-consistently. This means that if the single-particle energies are shifted or split by either process, this should be taken into account in the other. One possible way of approximating this requirement is the renormalized gap equation, discussed in Sec. III.C.5, Eq. (III.36). Computationally, this would mean doing three self-consistent programs instead of two.

We have chosen to renormalize the gap equation only in certain cases, namely, when backwards coupling simply shifted the bogolon away from the Fermi level. The following equation was used in the constant pairing matrix element approximation

$$\Delta = G \sum_{\alpha} \frac{\Delta}{\sqrt{(\epsilon_{\alpha} - \lambda)^2 + \Delta^2} + \sum_{\text{backward}}} \quad (\text{IV.28})$$

where

$$\sum_{\text{back}} = \sum_{\beta, N} \frac{|\langle \alpha^{-1} \beta N | H_{\text{eff}} | 0 \rangle|^2}{E_{\alpha}' + E_{\beta}' + \omega_N}$$

$$E_{\alpha}' = E_{\alpha}^0 + \sum_{\text{back}}$$

This is essentially a first order treatment; the justification is that the backwards residues are small. The only α and β considered were

protons:	1/2 [411]	neutrons:	1/2 [521]
	3/2 [411]		3/2 [521]
	5/2 [413]		5/2 [523]

The only phonon, N , taken into account was the $2+$ phonon. And the procedure was invoked only in the Dy and Er isotopes.

The reason for this abbreviated scheme is that it turns out that only in the above cases are GS correlations sufficiently strong to reduce the pairing field appreciably. Secondly, the pushing down effect of the forwards core polarization diagram is a more regular and moderate effect. Since it would therefore increase all Δ , in disagreement with experiment, G would have to be decreased. In other words, except for backwards correlations in Dy and Er, the self-consistency requirements of the gap equation can be neglected and the pairing constant, G , renormalized. The value of G used in such a calculation was given in Table 5.

V. RESULTS

A. Even Nuclei

In Sec. III, we have presented a self-consistent treatment of quasiparticles and phonons in a superfluid nucleus. Section IV detailed the mechanisms for carrying out the SCQPC program. In this part of the thesis, the results of our calculations are presented. Those for even-even nuclei are discussed first. One reason for this is that the effects of quasiparticle-phonon coupling and higher order perturbation theory diagrams are less dramatic than for odd-mass nuclei. For this reason, the even nuclei provide a less complicated introduction to the various improvements we have proposed for quasiparticle or quasibogolon propagators, as well as an opportunity to gauge the goodness of several approximations employed.

1. The Effects of Various Approximations and Higher Order Diagrams

a. Truncation. The most time-consuming part of the SCQPC computation are the diagonalizations for the self-energy. The dimensionality is minimized by never coupling phonons to spectrons with less than 5% single particle strength and by not dressing quasibogolons several MeV away from the Fermi surface.

In Table 6, we show the 2+ phonon energies using other values than 5% for the residue cut-off.

Table 6. Energies of 2+ phonon in an SCQPC calculation (only 2+ phonons coupled) with various values of the minimum residue to be included in self-consistent perturbation theory and renormalized RPA.

	$Z_{\min} = 0.1$	0.05	0.025
$^{152}_{\text{Sm}}$	1.42	1.43	1.45
$^{166}_{\text{Er}}$	0.81	0.81	0.82

The most substantial effect of z-truncation is to shift the center of gravity downward for low-energy quasiparticles, hence lowering phonon energies. For $z_{\min} = 0.05$, however, this is not too important, as illustrated in Table 6.

A more severe truncation problem is that of deciding which quasiparticles to "dress". One difficulty here is, first, that fragmentation of high energy single particle states will push the phonon downward in energy, and, secondly, that, unless these states are included, the fragmentation of intermediate energy single particle states will be affected and perhaps even the lowest energy eigenstates of the odd-mass nucleus. We have observed that the phonon is definitely sensitive to quasiparticles within 2 MeV of the Fermi surface. Secondly, we note that to describe odd-mass states with energies (relative to even nucleus) in the range 1-2.5 MeV, one should include coupling to states of approximately 2.5 MeV (which comes to a total of 3.5 MeV if the coupled phonon ≈ 1 MeV). This is due to the size of coupling matrix elements, generally less than 0.5 - 1.0 MeV.

Table 7 shows the results of calculations in which the lowest 5 and 10 quasiparticles on either side of the Fermi surface were included in the odd-mass basis set for QPC diagonalization.

Table 7. Eigenenergies of 2^+ states using various numbers of quasiparticles on each side of Fermi surface

	10	8	5	0 (No QPC)
^{152}Sm	1.43	1.43	1.43	1.56
^{166}Er	0.81	0.81	0.85	0.92

The fifth quasiparticle is generally $\mathcal{O}$ (2 MeV) while the tenth is $\mathcal{O}$ (3.5 MeV). The phonon calculation is sensitive to this difference. Clearly, any irreducible blocks F'' and F' employed will need to be renormalized by the higher energy cases we do not calculate, i.e., we must fit the constant to the truncation. Using ten on either side, we are confident that no couplings of qualitative or significantly quantitative importance are neglected.

b. Higher Order Diagrams. What will be the effect on renormalized RPA of dressing intermediates and second order coupling of phonon-plus-quasiparticle configurations? It is possible to estimate this is an approximate way. In the TDA,

$$A_{12}^{-1}(\omega) X_{12} = P_{1234} X_{34} \quad (V.1)$$

where

$$A_{12}(\omega) = \sum_{i,j} \frac{Z_i Z_j}{\omega - E_1^{(i)} - E_2^{(j)}}$$

Using the expansion,

$$\begin{aligned} \frac{1}{a-x} &= \frac{1}{a} \left(\frac{1}{1 - \frac{x}{a}} \right) = \frac{1}{a} \left(1 + \frac{x}{a} + \left(\frac{x}{a}\right)^2 + \dots \right) \\ &\approx \frac{1}{a} + \frac{(x/a)}{a} + \frac{(x/a)^2}{a}, \quad x < a \end{aligned}$$

We can expand $A_{12}(\omega)$ by moments about the unperturbed quasiparticle or -bogolon energies, $\omega - E_1^{(0)} - E_2^{(0)}$.

$$A_{12}(\omega) \approx \frac{1}{\omega - E_1^{(0)} - E_2^{(0)}} * \left(\sum_m \sigma_m \right)$$

$$\sigma_m = \sum_{i,j} \frac{(E_1^{(i)} - E_1^{(0)} + E_2^{(j)} - E_2^{(0)})^m}{(\omega - E_1^{(0)} - E_2^{(0)})^m} z_1^{(i)} z_2^{(j)} \quad (V.2)$$

The assumptions $\sigma_0 = 1$ and $\sigma_{m>0} = 0$ correspond to the usual RPA. One can predict the effects of renormalized RPA by considering the σ_1 and σ_2 moments. The matrix problem shown in Fig. 49 yields eigenenergies in pairs, $E^{(i)}$ and $-E^{(i)}$, with the residues

$$z^{(i)} = |\langle \psi^{(i)} | \chi_\alpha \rangle|^2 \text{ and } z_{\text{back}}^{(i)} = |\langle \phi^{(i)} | \chi_\alpha \rangle|^2.$$

$$E_\alpha = \langle \chi_\alpha | H_{\text{eff}} | \chi_\alpha \rangle = \sum_{i,j} \langle \chi_\alpha | \psi^{(i)} \rangle \langle \psi^{(i)} | H_{\text{eff}} | \psi^{(j)} \rangle \langle \psi^{(j)} | \chi_\alpha \rangle$$

where i, j sum over $\psi^{(i)}$ and $\phi^{(i)}$ states. Thus

$$E_\alpha = \sum_{E^{(i)} > 0} (z^{(i)} - z_{\text{back}}^{(i)}) E^{(i)} \quad (V.3)$$

Equation (V.3) illustrates two points:

i. If the backwards coupling terms of H_{eff} had been neglected,

$$\sum_{E^{(i)} > 0} z^{(i)} E^{(i)} = E_\alpha \quad (V.3a)$$

ii. When they are not neglected

$$\sum_{E^{(i)} > 0} Z^{(i)} E^{(i)} > E_{\alpha} \quad (V.3b)$$

Hence, the center of gravity of the physical ($E^{(i)} > 0$) solutions of Dyson's equation is pushed up by "backwards" coupling, but is unshifted by the "forwards" diagrams. This means that backwards coupling will also push the solutions of the renormalized RPA toward higher energy.

$$\sigma_2 \propto \lim_{\Delta t \rightarrow 0} \Delta t \left\{ \text{diagram} \right\}, \quad (V.4)$$

according to a derivation by Baranger.⁷⁵ In other words, the RPA is renormalized by the most simple QPC diagram. Processes shown in Figs. 16b, 18a,b do not contribute because the intermediate "does not have time to violate the Pauli principle or get dressed".

The calculations shown in Table 8 verify this important result. ¹⁶⁶Er deviates slightly; however, it is the most sensitive nucleus in the rare earth region to such subtleties. This justifies calculating SCQPC without the off-diagonal higher-order matrix elements. These can then be included on the last iteration in order to get the proper results for the odd-mass nucleus.

2. The Energy Spectra of Even Rare Earth Nuclei

a. Results. The calculated energies for the 2+, 0+, 0-, and 2- phonons are plotted in Figs. 51-54 and compared to the phonon energies determined in an unrenormalized RPA calculation.

It is interesting to contrast the two calculations in light of the σ_1 and σ_2 moments which were introduced. The σ_1 moment arises entirely because of "backwards" coupling; that is, the quasibogolon energies are increased by blocking of favorable ground state correlations. This yields a non-zero first moment because part of the quasibogolon strength is transferred from the $N + 1$ to the $N + 3$ or $N - 1$ nuclei. The σ_2 moment reflects fragmentation of the single particle strength due to mixing with core coupled states.

There are three important trends we wish to note

- i. Backwards correlations dominate the nuclei in the ^{166}Er region (e.g., ^{162}Dy , ^{166}Er , etc.)
- ii. Forwards or normal QPC correlations push the phonons to lower energy (in better agreement with experiment).
- iii. The $2+$ phonon dominates the QPC scheme. $0+$, $0-$, $2-$ phonon couplings are of less or no importance, since they are generally higher in energy and less collective.

The backwards correlations predominate around ^{166}Er because with the low $2+$ energies there is considerable ground state correlation. In particular, the $3/2$ [411], $1/2$ [411], and $5/2$ [413] protons and $3/2$ [521], $1/2$ [521], and $5/2$ [523] neutrons participate substantially in the zero-point motion. RPA Y -amplitudes for configurations involving these orbitals are $\mathcal{O}(0.2)$.

In the lighter rare earths (e.g., Gd), these same quasibogolons have substantial fractionation but the ground state EPV's are not nearly so important. Hence, the phonon energies are decreased. The same is true for the heavier rare earths (e.g., Yb) although the $3/2$ [512], $5/2$ [512], $1/2$ [510], and $7/2$ [514] neutrons are more active in QPC here.

Table 8. Calculations of 2+ phonon by SCQPC (only 2+ coupling) in various approximations to self energy

- Col.(a) includes diagonal elements of Fig. 16b, i.e., exclusion principle violations (EPV), but not dressed intermediates.
- Col.(b) includes dressed intermediate, Fig. 18a, but not Fig. 16b.
- Col.(c) includes both dressing and EPV's, but not off-diagonal terms of Fig. 16b, 18b.
- Col.(d) includes everything

	(a)	(b)	(c)	(d)
^{152}Sm	1.434	1.43	1.43	1.43
^{166}Er	0.820	0.826	0.808	0.849

XBL726-3121

Fig. 53

XBL726- 3120

Fig. 54

In ^{152}Sm and ^{154}Sm , the $0+$ and $0-$ phonons are important in the QPC scheme, primarily because of the "backwards" diagram causing a substantial σ_1 . In fact, the $2+$ vibration is pushed up by including the $0+$ and $0-$. Figures 51-54 also show a few nuclei calculated with only the $2+$ active in QPC. Except around Sm, where the $0+$ and $0-$ backwards diagrams are moderately important, the coupling of the gamma phonon is almost wholly responsible for the shifting of phonon energies from unrenormalized RPA.

b. Renormalization of the Gap. The pushing-up effect of backwards correlations is somewhat modulated by renormalization of the BCS pairing. This renormalization was approximately taken into account for the Dy and Er isotopes according to the prescription of Sec. IV.E. Table 9 shows the decrease in the pairing for several isotopes. It is never more than a 10% lowering.

Table 10 shows several calculations involving the backwards diagrams and renormalized pairing field. It is easily seen that for ^{166}Er , it is necessary to renormalize pairing or take into account blocking. Increasing the constant just makes the EPV's worse. We attempted in the course of this work to include blocking by shifting 1-quasibogolon and 2-quasibogolon energies via a blocking calculation. Nilsson⁷⁶ has argued that blocking should not be included because its effects are approximately cancelled by fluctuation in the number of particles, i.e., $\frac{\partial^2 E}{\partial n^2} \langle N^2 \rangle$. When blocking calculations were done, we found that the two-quasibogolon energies were unrealistically low in energy, e.g. the $5/2$ [512] - $1/2$ [521] state in ^{172}Yb came at approximately 1.0 MeV.

c. Effect of Backwards EPV Diagrams on RPA Amplitudes. Recently, it has been argued by a number of authors that the usual schematic model

Table 9. Calculated Renormalization of the Pairing Field
by "Backwards" Coupling:

Nucleus	$\hbar\omega(\text{vibration})$	Δ^0	Δ'
^{158}Dy protons neutrons	0.95 MeV	0.945	0.9
		1.01	1.00
^{162}Dy protons neutrons	0.9 MeV	0.95	0.87
		0.89	0.83
^{166}Er protons neutrons	0.8 MeV	0.95	0.85
		0.775	0.67

Table 10. $2+$ energies calculated under various conditions:

(a) magnitude of overall particle-hole renormalization, r ;

(b) "old" or renormalized "new" gap parameters; and

(c) type of QPC (none, with or without backwards coupling).

(a)	0.866	0.866	0.866	0.91	0.91
(b)	old gaps	old gaps	old gaps	old gaps	new gaps
(c)	no QPC	QPC	back QPC	back	back
^{152}Sm	1.56	1.48	1.503	1.43	1.43
^{166}Er	0.8	0.61	0.964	0.904	0.8

RPA calculations overestimate ground state correlations. There are two possible reasons:

- i. the quadrupole-quadrupole interaction is too coherent,⁷⁷ and
- ii. exclusion principle violations.⁷⁸

Our own calculations shown in Table 11 tend to support the first explanation but not the second. In particular, the inclusion of all multipoles lowers ground state correlation from that found in schematic calculations.⁶⁶ However, "backwards" coupling does not substantially change the γ -amplitudes of RPA. In our case, this is because the pushing-up effect of backwards diagrams is partially offset by a renormalization of the pairing as well as an increase in the coupling constant.

d. Renormalized RPA for Second States. Although we have not focused our attention on higher-energy incoherent phonons, the renormalized RPA equations do yield a description of these states, and they may be coupled to quasiparticles or -bogolons in Dyson's equation. We find, however, that

- i. While these states are lowered in energy by renormalized RPA,
- ii. the incoherent phonons do not renormalize Dyson's equation or contribute further to the renormalization of RPA.

Table 12 shows the first and second 2^+ states in ^{172}Yb and ^{176}Hf . The second 2^+ state is primarily the $5/2$ [512] - $1/2$ [521] two-neutron configuration. Since both of the quasibogolons in this instance are pushed down by coupling to configurations such as $1/2$ [510] + γ or γ - $3/2$ [521], the second 2^+ states are lowered. Also shown are the first and second excited 0^+ states of ^{176}Hf . As with the 2^+ example, the second state is lowered.

Table 11. RPA Y-amplitudes for ^{166}Er configurations in various calculations.
The phonon energy was 0.8 MeV in all calculations.

	SDI, Faessler 66	Wigner direct all multipoles	Migdal RPA	Renormalized RPA
<u>proton configurations:</u>				
5/2 [413] - 1/2 [411]	0.25	0.16	0.185	0.184
3/2 [411] + 1/2 [411]	0.296	0.176	0.21	0.214
<u>neutron configurations:</u>				
5/2 [523] - 1/2 [521]	0.25	0.193	0.201	0.203
3/2 [521] + 1/2 [521]	0.23	0.212	0.223	0.227

Table 12. First and second phonons calculated in unrenormalized and renormalized RPA. In both calculations, the (ph) constant $r = 0.91$. The renormalized RPA includes coupling of the first 2^+ and 0^+ phonons only.

		unren. RPA (MeV)	ren. RPA (MeV)
^{172}Yb	2_1^+	1.44	1.35
	2_2^+	1.62	1.488
^{176}Hf	2_1^+	1.55	1.44
	2_2^+	1.98	1.88
	0_1^+	1.42	1.32
	0_2^+	1.54	1.47

It turns out, however, that these are incoherent phonons and have small QPC matrix elements. When these experimentally known even-nucleus states are included in the solution for odd-A nuclei, there is no significant fragmentation of single particle states. This is especially noticeable in, for example, the Sm or Er isotopes where renormalization via coupling the lowest 2^+ state lowers the RPA phonon substantially. The second 2^+ state has almost no effect, as shown in Table 13.

e. Renormalized RPA for Pairing Vibrations. Theories for 0^+ states in the rare earth region have been outlined elsewhere.⁷⁹ However, we note here that at neutron numbers $N = 98, 104$, there are gaps in the Nilsson spectrum. This has the effect that nuclei such as ^{166}Er or ^{174}Yb have a 0^+ state similar to ^{208}Pb , i.e., a pairing vibration.²⁹ We note here that it makes only a small contribution to QPC as shown in Table 14. The 2^+ state is altered negligibly. The 0^+ pairing vibration itself is lowered by 70 keV, and this is probably not experimentally significant.

3. Transition Probabilities in Even Rare Earth Nuclei

Figures 55-58 display the $B(E2)$ or $B(E3)$ calculated for the first excited 2^+ , 0^+ , 0^- , 2^- phonons with and without renormalized RPA. As with the energies, the QPC increases the quadrupole correlations, and the $B(E2)$'s are generally enhanced for the 2^+ and 0^+ states. One might expect these correlations to interfere in the octupole mode. The calculations seem to indicate that this effect is not very important, except for the octupole 0^- states at the beginning of the region where the lowered 0^- states do not have a correspondingly enhanced $B(E3)$. The lower $B(E3)$ for 2^- states is a consequence of the increase in 2^- energies which occurred when the W' interaction was decreased to obtain agreement for the ^{166}Er 2^- state.

Table 13. Second 2^+ state as computed in unrenormalized and renormalized RPA. The renormalization includes only coupling of this second 2^+ state.

	unren. RPA	ren. RPA
$^{152}_{\text{Sm}}$	2.007	1.989
$^{166}_{\text{Er}}$	1.809	1.802

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Table 14. States calculated with and without $F''_{13} \approx 0$ in a renormalized RPA calculation ($2+$, $0+$, $0-$, $2-$ coupled).

		$F''_{13} = F''$	$F''_{13} = 0$
^{152}Sm	$2+$	1.43	1.44
	0^+	0.6814	0.663
^{166}Er	$2+$	0.808	0.796
	$0+$	1.118	1.186

B. Odd-A Nuclei

Most of the emphasis in QPC calculations has focused on odd-A nuclei. This is because they are so sensitive to the details of the correlations of the last (odd) proton or neutron. Since the motivation for our work has been to understand microscopically the QPC process in higher as well as lower orders, we should analyze first the dependence of odd-A nuclear spectra on the varying degrees of sophistication in the calculation.

1. The Schematic Effects of Various Approximations and Higher Order Diagrams

a. The Nilsson Levels. Unfortunately, the most important determinant of odd-A nuclear structure in the rare earth region are the not-so-well-determined single particle energy levels. For reasons which we indicate later, the lowest eigenstates are not strongly coupled. Hence, the usual Nilsson-plus-pairing prescription probably yields a fair estimate of the Nilsson energies closest to the Fermi surface.

However, the experimental situation seems to be that these levels can vary by several hundred keV as nucleons are added and the field changes. For this reason, we can expect absolute energies to be limited in accuracy by the uncertainty in the Nilsson levels (~ 200 keV). Except in cases of accidentally degenerate basis states, the wave functions should be fairly reliable, i.e., predictions of enhanced $B(E2)$ or fractionated single particle strength should be consistent with experiment.

b. Quasiparticle-Phonon Coupling. In Figs. 59-61, a summary of QPC schematics is given for ^{153}Eu , ^{167}Er , and ^{173}Yb . Each column corresponds to the spectra calculated within the approximations discussed in the identically lettered subsection; that is, the results calculated with no

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Fig. 59. For explanation of col. (a)-(f), see text, Sec. V.B.1.b.
 Energies are relative to even-mass nucleus, i.e., $E_0 + \lambda$.

XBL726 - 3126

Fig. 60. See caption to Fig. 59.

Fig. 61. See caption to Fig. 59.

backwards or higher order diagrams and "undressed" intermediates are discussed here and displayed in col. (b) of Figs. 59-61. This is the usual microscopic intermediate coupling. There are two systematic trends to which we call attention:

- i. the lowest eigenstates are predominantly single-particle in nature. This is because the coupling matrix element $\langle \alpha | H_{\text{eff}} | \beta N \rangle \propto (U_{\alpha} U_{\beta} - V_{\alpha} V_{\beta})$ for $N =$ particle-hole phonon and α, β quasibogolons.

For the lowest $|\alpha\rangle$, $U_{\alpha} \approx V_{\alpha} \approx 0.7$, thus diminishing the coupling. Also the closest states $|\beta N\rangle$ are ≈ 1 MeV higher in energy. The end results are that the depletion of single-particle strength is approximately 10% or less and that the energy is decreased by ~ 0.1 MeV.

- ii. The primary splittings of single particle strength involve proton orbitals 1/2 [411], 3/2 [411], 5/2 [413] (also 1/2 [400], 3/2 [402], 5/2 [402] in heavier rare earths) and neutron orbitals 1/2 [521], 3/2 [521], 5/2 [523], and 1/2 [510], 3/2 [512], and 5/2 [512]. These quasiparticles, or quasibogolons, are most strongly coupled to the γ -vibration, on account of asymptotically allowed quadrupole matrix elements. In general, the results at this stage are similar to those of Soloviev.⁸⁰ The Q-Q interaction employed in earlier calculations somewhat exaggerates the coupling terms, however, just as in RPA, because it is too coherent.

- c. Pauli Principle Effects on Phonon States. We have indicated earlier that the diagonal matrix elements of Fig. 16b in Sec. III cancel exclusion principle violations in the simultaneous propagation of a quasi-particle or -bogolon and a phonon. The considerable import of these EPV's has not been discussed sufficiently by other authors, so we begin with a

theoretical example. Namely, we calculate the RPA phonon in the usual unrenormalized manner, except any configurations involving a specified orbital are excluded. This provides an estimate of how serious EPV's are. In ^{166}Er , the phonon calculated in the usual manner was 0.8 MeV for $K\pi = 2+$. The rise in energy when certain orbitals were blocked is shown in Table 15, col. (2). The Pauli principle can renormalize the phonon energy by as much as 50%. Next we calculate the diagonal elements of Fig. 16b, viz.,

$$\begin{array}{c} \alpha \\ \uparrow \\ \text{N} \end{array} \begin{array}{c} \text{N} \\ \downarrow \\ \text{B} \\ \uparrow \\ \alpha \end{array} = \sum_{\beta} \frac{|F^N(\alpha\beta)|^2}{2E_{\alpha} + E_{\beta} - E_{\alpha} - E_N} \quad (V.5)$$

These are shown in col. (3) of Table 15. Equation (V.5) is clearly quite accurate for estimating the EPV correction. In general, col. (2) < col. (3). The reason is that col. (2) excludes backwards correlations involving $\bar{\alpha}$, whereas col. (3) correctly does not.

The experimental situation in this regard is somewhat obscure because the same quasibogolons which interfere with phonons also couple very strongly to phonons and the QPC process mixes such configurations with single-particle like states. A disagreement between theory and experiment may be due to EPV's, or bad estimates of Nilsson energies. Certainly, the fact that the high-spin phonon states, which do not mix, are seldom observed is consistent with the theoretical explanation that they are pushed up into a region ~ 1.6 MeV in which they are difficult to observe via the usual nuclear spectroscopy analyses.

Table 15. Effect of Pauli Principle Blocking on ^{166}Er 2+ phonon $E = 0.8$ MeV

blocked quasiparticle	exact diagonalization ($E = 0.8$)	2 nd order correction ($E = 0.8$)
5/2 [413] proton	0.387	0.351
-5/2 [413]	0.00	0.00
3/2 [411]	0.483	0.448
-3/2 [411]	0.059	0.047
7/2 [523]	0.006	0.004
-7/2 [523]	0.009	+0.006
1/2 [411]	0.447	0.448
-1/2 [411]	0.387	0.351
5/2 [402]	0.026	0.04
-5/2 [402]	0.06	0.00
3/2 [651] neutron	0.04	0.026
-3/2 [651]	0.009	0.007
3/2 [521]	0.408	0.356
-3/2 [521]	0.01	0.006
5/2 [642]	0.088	0.063
-5/2 [642]	0.00	0.00
5/2 [523]	0.514	0.45
-5/2 [523]	0.018	0.01
7/2 [633]	0.014	0.00
-7/2 [633]	0.003	0.00
1/2 [521]	0.408	0.356
-1/2 [521]	0.513	0.45

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Fig. 62. ^{169}Tm calculations illustrating pushing-up effect of EPV corrections on phonon states and similar corrections to the collective E2 transitions.

¹⁶⁹Tm provides some experimental evidence for the pushing up effect. The calculated and experimental spectra are shown in Fig. 62. The ground state is the $1/2 [411]$; the gamma vibrations, $1/2 [411] \pm \gamma = 3/2, 5/2$ are expected to occur at 1.9 MeV (the zero of energy is the BCS vacuum, i.e., no quasibogolons). In addition, the $3/2 [411]$ and $5/2 [413]$ states are expected to occur at around 1.5 and 1.7 MeV. It appears that the vibrationally excited states occur at $(0.9 + 1.2 =) 2.1$ MeV or above. This is consistent with our theoretical estimates, in which the gamma vibrations are elevated in energy by ~ 0.4 MeV due to EPV's. Figure 60 showing ¹⁶⁷Er also exhibits the pushing-up effect on neutron negative parity gamma vibrations, in col. (c). The effect is visible, though not so pronounced, in ¹⁵³Eu (Fig. 59) and ¹⁷³Yb (Fig. 61).

d. Backwards Diagram. Column (d) of Figs. 59-61 show very distinctly the influence of the backwards diagrams, such as Fig. 17b, which cancel the EPV which occurs when a quasibogolon, α , propagates simultaneously with a 4- quasibogolon ground state correlation already populating α . The result is an energy increase in all those one quasibogolon states which are frequently involved in even-even ground state correlations. Primarily, these are the orbitals we have noted previously as being active participants in other QPC processes. In general, backwards QPC counteracts the reduction in the odd-even mass difference originating from forwards QPC. It is also responsible for shuffling low-energy states in several nuclei. Unfortunately, it is difficult to compare these results with experiment in the rare earth region, since the odd-even mass difference varies considerably from nucleus to nucleus, as well as the deformed field (ϵ_2, ϵ_4).

single particle and phonon-plus-particle configurations may mix to any degree.

If two (or three, etc.) phonon states of the core are considered, the self-energy contains partially dressed intermediates (the "nested wiggle" diagram in Fig. 68), and one-phonon states are connected to themselves via two phonon intermediates.

ii. Microscopic intermediate coupling

Figure 68 which describes intermediate coupling[†] provides a convenient starting point for more sophisticated theories. One theoretical improvement would be to calculate the same diagrams but with a microscopic phonon and the coupling Hamiltonian derived from the two-body interaction. Such a program was first carried out by Brown, Evans, and Thouless⁸⁸ who calculated the self-energy insertions shown in Fig. 69. The second of the two-self-energy diagrams cancels the Pauli Principle violation shown in Fig. 70, i.e., the contribution of a particle on top of a 2p-2h state in which the same particle is already occupied.

The analytical expressions for the self-energy are

$$\Sigma = \sum_{bN} \frac{|\langle a | H_{\text{eff}} | bN \rangle|^2}{\omega - \epsilon_b - E_n} + \sum_B \frac{|\langle aBN | H_{\text{eff}} | \tilde{0} \rangle|^2}{\omega + \epsilon_B + E_N}$$

$$\langle a | H_{\text{eff}} | bN \rangle = \sum_{cD} \langle aD | V | bc \rangle X_{cD} + \langle ac | V | bD \rangle Y_{cD}$$

$$\langle aBN | H_{\text{eff}} | \tilde{0} \rangle = \sum_{cD} \langle ac | V | BD \rangle X_{cD} + \langle aD | V | Bc \rangle Y_{cD}$$

[†] Note that Fig. 68 and all others in which the phonon is drawn as a wiggly line may be considered to be macroscopic or microscopic.

decreased from 3.3 to 2.5 MeV. This is conceptually equivalent to employing two-phonon as well as one-phonon intermediates in an "undressed" calculation, i.e., Fig. 18a. This additional contribution to the self-energy also pushes the $1/2 [550]$ down. In fact, it has become an "intruder state".

For β -vibrational intruder states of this type there are several necessary features of the calculation:

- i. The coupling matrix element must be large. This is true for very "prolate" or "downsloping" Nilsson levels.
- ii. The quasibogolon must have u or $v \approx 1$. If it is close to Fermi surface, the $u^2 - v^2$ will erase the large coupling matrix element.
- iii. Intermediates must be dressed.

In ^{167}Er , the $1/2 [510]$ mixes with the $5/2 [512] + \gamma$ and the $5/2 [512]$ with the $1/2 [510] + \gamma$ configuration. As a consequence, both states are lowered considerably. Self-consistency reinforces this feature, as shown in Fig. 60, col. (e). In fact, the "reinforcement" consequence of self-consistency is quite general for gamma phonon-quasibogolon coupling in the middle rare earth region.

f. Higher Order Diagrams. We finally wish to consider the off-diagonal matrix elements of the second order processes shown in Figs. 16b, 18b. A glance at Figs. 59-61 indicates how very important these self-energy contributions are.

First, let us review the experimental evidence for such higher order couplings:

- i. Splitting of $K_0 \pm 2$ bands in odd-mass rare earth nuclei.
e.g., ^{165}Ho , ^{167}Er

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Fig. 63. Calculated spectrum of ^{165}Ho . Note especially the splitting of the $K_0 \pm 2$ states.

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Fig. 64. $K_0 \pm 2$ spectrum of ^{167}Er .

(a)

(b)

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Fig. 65. Higher-order contributions to H_{eff} which depress $K_0 - 2$ states in ^{165}Ho and ^{167}Er .

- ii. In general, $E(\text{phonon-plus-quasiparticle}) - E(\text{qp})$ is less than $E(\text{phonon})$. This indicates that additional interactions push down the more complex configuration. In fact, this anomalous depression of phonon states is a common feature in the rare earth nuclei.
- iii. In some cases (e.g., ^{169}Tm) where the phonon and quasiparticle are expected to have significant EPV's, the higher order interaction pushes up the phonon-plus-quasiparticle state.
- iv. In spherical nuclei, the $I = j - 1$ "anomalous" or "intruder states" descend in energy on account of higher order diagrams.⁸¹

With these facts in mind, the $K_0 \pm 2$ splittings in ^{165}Ho (173.2 keV) and ^{167}Er (179.5 keV) provide a simple test of the theory. Experimentally, these states are not substantially mixed with any single particle states. In fact, these are the only two rare earth nuclei where this situation occurs, and both the $K_0 + 2$ and $K_0 - 2$ bands are seen. The two bands should be split only by higher order diagrams and approximately 80 keV of rotational energy. Figures 63-64 show the calculated splittings for these two nuclei. Figure 65 displays the particular diagrams which have large matrix elements. In both cases, the $K_0 - 2$ state contains $\sim 10\%$ 2- octupole vibration. The theoretical $K_0 \pm 2$ splittings are 280 and 50 keV. In the latter case, the inclusion of the rotational energy difference improves agreement with experiment. In ^{165}Ho , it does not. It seems fair to conclude that the diagrams in Fig. 65 contribute substantially to the $K_0 \pm 2$ splittings in the rare earth region, although there may be other competing processes.

Another exceedingly interesting consequence of higher order couplings are the intruder states in the lighter odd-Z rare earths. For

example, Fig. 59 shows that these graphical contributions to the QPC matrix elements lower the $1/2 [500]$ to almost the ground state of ^{153}Eu . Of course, once such a "pushing-down" occurs, it is reinforced by self-consistency. Of interest here is the mechanism for the process, shown in Fig. 66a. The diagonal matrix element of the phonon-plus-dressed quasibogolon configuration is

$$\langle |H_{\text{eff}}| \rangle \approx \frac{|\langle 1/2[550] | H_{\text{eff}} | \langle 1/2[550], 0+ \rangle |^2}{-2E_{\text{Beta Vibration}}} \quad (\text{V.6})$$

This is particularly large (despite several residue factors which are less than one) because the coupling matrix element ≈ 3 (1.4 MeV) while $E_{\beta} \approx 0.7$ MeV. Note that Eq. (V.6) or Fig. 66a describe the H_{eff} contribution which has a "staggered" two-phonon intermediate. In fact, the two-phonon states contribute substantially both on account of the "nested" self-energy contribution, as well as the "staggered" H_{eff} matrix element.

One observes the same sort of enhancement quite frequently with the γ -vibrations, coupled to the orbits indicated previously (e.g., $5/2 [512]$). Phonon states are often pushed downward by diagonal terms such as shown in Fig. 66b. More often, however, (because of phase relations) pushing down is caused by mixing (or off-diagonal matrix elements) of the type shown in Fig. 55.

We conclude this section on the schematic effects of various approximations and higher order diagrams by remarking on their importance. We have found QPC to be enhanced by self-consistency and by higher order coupling of phonon-plus-quasibogolon configurations. Exclusion principle violations have been shown to have a definite effect on calculated nuclear

(a)

(b)

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Fig. 66. "Staggered" two-phonon contributions to H_{eff} which (a) further depress intruder state in ^{153}Eu , etc. and (b) lower phonon coupled states in odd-neutron rare earth isotopes.

spectra. Certain phonon configurations are pushed to much higher energies when EPV corrections are made. Backwards diagrams have been shown to have a measurable effect not only on the pairing field and the odd-even mass difference, but also on the relative ordering of levels.

2. Level Schemes for Selected Odd-Mass Nuclei

With the previous section as background, we turn now to a brief presentation of several odd-A rare earth nuclei. Although the exact details of such spectra are often dependent on such factors as Nilsson levels and unspecified higher order diagrams, we think that the results in each nucleus present a fair description of the available experimental picture.⁸²

Tables 29 through 42 in Appendix C list the lowest eigenstates in odd-Z and odd-N nuclei for $Z = 63, 65, 67, 69, 71$, and $N = 91, 93, 95, 97, 99, 101, \text{ and } 103$.

a. Odd-Z, Even-N Nuclei. The Eu isotopes, as well as the lighter Tb isotopes, exhibit what were earlier called intruder states. These occur when a very prolate orbital (such as $1/2 [550]$) is far above or below the Fermi energy, and the beta vibration is low in energy. It is interesting that the intruder goes away as we go from ^{157}Tb to ^{161}Tb . This is because the beta vibration becomes less coherent. In the lighter rare earths, one observes substantial gamma vibration coupling among the $1/2 [411]$, $3/2 [411]$, and $5/2 [413]$ quasibogolons. These instances are labeled (AAA) in the tables. Among the odd-proton nuclei, coupling among the $1/2 [400]$, $3/2 [402]$, $5/2 [402]$, $7/2 [404]$, and $9/2 [404]$ levels (labeled (BBB) in the tables) is also important. Below and including Ho, the

chief effect is that orbitals such as $1/2 [400]$ become intruders, often lowered in energy by as much as 1 MeV due to the effects of self-consistency and higher order matrix elements in addition to the usual QPC. In Tm and Lu, collective transitions among these states dominate the low energy level scheme. Exclusion principle violations have a substantial effect on the (AAA)-phonon levels around ^{166}Er , often raising their energy by several hundred keV.

b. Even-Z, Odd-N Nuclei. We preface our remarks here by noting that N-mixing ($1/2 [660]$ with $1/2 [400]$ and $3/2 [651]$ with $3/2 [401]$ and coriolis coupling will make substantial alterations in our energy level schemes. This is especially true for the Gd isotopes. As for the odd-proton nuclei, there are two sets of levels which influence markedly the QPC predictions. In the lighter nuclei, the $1/2 [521]$, $3/2 [521]$, and $5/2 [523]$ quasibogolons (labeled (CCC) in the tables) are often coupled to the $2+$ gamma vibration and the resultant configurations highly mixed. The set (DDD) $1/2 [510]$, $3/2 [512]$, $5/2 [512]$, and $7/2 [514]$ are similar in effect to the (BBB) set in odd-Z nuclei, namely, their observation in the Er nuclei results primarily from the intruder-like enhancements already discussed. Also, EPV's are again of the order of several hundred keV around ^{166}Er for the (CCC)-phonons.

3. Transition Probabilities

As is well known,^{80,82} the main consequences of QPC are the fragmentation of single particle strength and the enhancement of gamma ray transition probabilities over the single-particle estimate. We have presented in the previous subsection the results for the first phenomenon.

In this subsection, we tabulate for several nuclei transition moments (with and without QPC enhancement) and $B(E2)$'s and $B(E3)$'s for transitions connecting the most interesting quasiparticle-phonon coupled levels discussed previously. Although these transitions are not exhaustive of the level energy data given in the Appendix and the previous section, we indicate several important trends in the region.

a. Interference of Collective and Single-Particle Transition Moments. According to Eq. (III.56-58), the transition amplitude connecting two odd-mass nuclear levels is the sum of three terms: the first is a single particle transition amplitude; the second and third are collective in origin. An examination of Tables 16-26 clearly demonstrates the constructive and destructive interference of these terms. It should first be noted that the single particle contributions are not very much less than the collective ones. This is especially true of odd-proton nuclei in which the effective charge is several times larger than in odd-neutron nuclei. Secondly, one can see that states lowest in energy are coherent, whereas the higher states are connected to other nuclear levels by $B(E2)$'s or $B(E3)$'s which are smaller on account of incoherence. These observations apply to all of the nuclei studied. We may consider ¹⁷³Lu as an example. In this instance, the 7/2 [404] ground state may be coulomb excited to one of several 3/2+ states. The lowest in energy is at 1.13 MeV and contains 83% 3/2 [402] and only 13% of the gamma vibration built on the ground state. The transition amplitude connecting the 3/2 [402] Nilsson orbital component to 7/2 [404] is -0.41 (units are s.p.u.^{1/2}). The 13% gamma component corresponds to a component in the wave function with amplitude 0.36. When this is multiplied by the

$$\Delta E(\text{fig. 73a}) = \frac{|\langle \{j, 3^-\}_{j_p} | H_{\text{coup}} | j_p \rangle|^2}{\epsilon_j + \hbar\omega - \epsilon_{j_p}} \quad (\text{A.11a})$$

$$\Delta E(\text{fig. 73b}) = \sum_{\substack{j_1 > \text{Fermi} \\ \text{level}}} \frac{|\langle j | H_{\text{coup}} | \{j_1, 3^-\}_{j_1} \rangle|^2}{\epsilon_j - \hbar\omega - \epsilon_{j_1}} \quad (\text{A.11b})$$

$$\Delta E(\text{fig. 73c}) = \sum_{\substack{j_1 < \text{Fermi} \\ \text{level}}} \frac{|\langle \{jj_1\} 3^- | H_{\text{coup}} | 3^- \rangle|^2}{\epsilon_j + \epsilon_{j_1} - \hbar\omega} \quad (\text{A.11c})$$

$$\Delta E(\text{fig. 73d}) = \frac{|\langle \{j, 3^-\}_{j_h} | H_{\text{coup}} | j_h \rangle|^2}{\epsilon_j + \epsilon_{j_h} + \hbar\omega} \quad (\text{A.11d})$$

However, perturbation theory is feasible only when these splittings are small.

The macroscopic model may be extended to include 3 valence particles plus core excitations.⁹⁶ In this case, the particles with the core via the usual H_{coup} . The particle-particle interaction is taken to be the pairing force.

From the point of view of matrix diagonalizations, this is the logical extension of the intermediate coupling calculations of ²⁰⁹Pb to ²¹¹Pb. The calculations carried out include an application to the Ag isotopes⁹⁷ where intruder states have been observed.

ii. Microscopic Particle-Phonon Coupling with higher order effects included

Another important class of higher order contributions to QPC is that which results from the microscopic nature of the collective mode. Figure 73c and also Fig. 76, are examples of the collective mode decoupling

c. Exclusion Principle Violations. When vibrations are treated microscopically the enhancement of the γ -transition probabilities is a consequence of the coherency of many particle-hole pairs which participate in the phonon. If one or more pairs are not allowed to contribute, because of Pauli principle considerations, then we might expect the collective enhancement to be less. This is especially so in the rare earth region where there are only about five dominant (ph) contributions to the phonon. Actually, the EPV's have two consequences within the calculation. One is to raise the phonon energy, thus making the mixing of phonons into low energy states more difficult by the usual second-order perturbation theory denominator argument. Another feature of the exclusion principle is that the phonon transition itself is reduced in accordance with Eq. (III.59), which corresponds to the graphical contribution depicted in Fig. 45g. This phenomenon is noticeable for odd-proton nuclei, but not too considerable for odd-neutron nuclei, since the effective charge of neutrons is less. As examples, we consider ^{169}Tm and ^{167}Er . Both of these nuclei have vibrations which are dominated by only a few components. In ^{169}Tm , the ground state is $1/2 [411]$. The $3/2$ and $5/2$ states at 1.935 and 1.504 MeV are predominantly gamma vibration. The unhindered $M(E2)$ for the gamma transition is $1.34 \text{ s.p.u.}^{1/2}$. This value is lowered by 0.24 and 0.37 when the $1/2 [411]$, $K = -1/2$, and $K = 1/2$ levels are respectively blocked. In ^{167}Er , the $1/2 [521]$ is low in energy and the $3/2$ and $5/2$ gamma vibrations, if unmixed and unhindered, would have an $M(E2)$ of $1.58 \text{ s.p.u.}^{1/2}$. Blocking for $K = -1/2$, $+1/2$ yields in these instances decreases in this

amplitude of only 0.088 and 0.076. This EPV effect is substantial not only for nuclei in the ^{166}Er region where the $3/2 [411] + 1/2 [411]$ and $5/2 [413] - 1/2 [411]$ proton and $3/2 [521] + 1/2 [521]$ and $5/2 [523] - 1/2 [521]$ neutron configurations are dominant in the gamma vibration, but also around ^{152}Sm where the $1/2 [660] - 1/2 [660]$ and $3/2 [651] - 3/2 [651]$ contributions to the beta vibration are quite large.

d. Effects of Dressed Intermediate or SCPT on Transition

Probabilities. An interesting consequence of SCPT is that one can obtain enhanced transitions between states, neither one of which is predominantly single particle in character. These are essentially equivalent to one-phonon states. ^{167}Er provides a simple example. The $7/2^-$ state at 0.982 MeV (calculated) is 78% $7/2 [514]$ and 17% gamma vibration coupled to the $3/2^-$ state at 1.054 MeV. This $3/2^-$ state is 20% $3/2 [512]$, 35% gamma vibration plus the $7/2^-$ state at 0.982 MeV, and 38% gamma vibration plus the $1/2^-$ state at 0.697 MeV. Another $7/2^-$ state is predicted at 1.93 MeV with 16% $7/2 [514]$ and 82% gamma phonon coupled to the $3/2^-$ state. Thus the first $7/2^-$ is predominantly single particle while the second is predominantly $7/2 [514] - \gamma + \gamma$, i.e., two-phonon in nature. The $3/2^-$ is a mixture of a one-phonon contribution, $7/2 [514] - \gamma$ and a two-phonon contribution, $\gamma + \gamma - 5/2 [512] = \gamma - 1/2$. As a consequence, the following $B(E2)$'s are predicted:

$$\begin{array}{lll} 7/2 - (1.93) \rightarrow 3/2 - (1.054) & B(E2) = 1.42 & \text{s.p.u.}, \\ 7/2 - (0.982) \rightarrow 3/2 - (1.054) & B(E2) = 2.78 & \text{s.p.u.}, \\ 1/2 - (0.697) \rightarrow 3/2 - (1.054) & B(E2) = 1.85 & \text{s.p.u.}, \\ 1/2 - (0.697) \rightarrow 5/2 - (0.268) & B(E2) = 1.93 & \text{s.p.u.} \end{array}$$

e. The Intruder States in Eu and Tb. We have predicted, on the basis of self-consistent intermediates in the self-energy and second order QPC matrix elements, intruder states in Eu and Tb. These are the $1/2$ [550] and $3/2$ [541] which mix with themselves plus a beta vibration and the $1/2$ [420] which mixes with the $1/2^-$ intruder coupled to the octupole (0^-) vibration. We quote in Tables 16 and 17 the $B(E2)$ and $B(E3)$ transition probabilities between these levels as they are fairly enhanced and may help experimentalists to identify these previously unseen levels.

In this section, we have seen that the pairing field is renormalized by backwards diagrams and that phonon-plus-quasiparticle states can be pushed so high in energy by Pauli principle corrections that they are not observed experimentally. In addition, self-consistency and higher order diagrams have been shown to enhance the "pushing down" effect of the usual QPC on predominantly single particle states. Transition probabilities are subject to these factors also. Thus, our calculations show quite clearly the dramatic effects of exclusion principle corrections and higher order terms in the QPC.

Table 16. $M(E2)$ matrix elements between intruder states in ^{153}Eu .
Numbers in parens are without QPC enhancement.

$3/2 [541] @ 1.02$	
$1/2 [550]$	
@ 0.115	(0.256), 0.751
$B(E2)_{\text{s.p.u.}}$	
$1/2 [550]$	0.564

Table 17. $M(E3)$ matrix elements between intruder states in ^{153}Eu .
Numbers in parens are without QPC enhancement.

$1/2 [420] @ 0.747$	
$1/2 [550]$	
@ 0.115	(0.16), 1.26
$B(E3)_{\text{s.p.u.}}$	
$1/2 [550]$	1.587

Table 18. $M(E2)$ matrix elements in ^{161}Tb . Numbers in parens are without QPC enhancement

	1/2 [4_{11}] @ 0.750	1/2 [4_{11}] @ 0.441
3/2 [4_{11}] @ 0.0	(0.245), -0.2	(0.156), 1.49
5/2 [4_{13}] @ 0.186	(-0.09), -0.86	(-0.061), 0.63
$B(E2)_{\text{s.p.u.}}$		
3/2 [4_{11}]	0.04	2.25
5/2 [4_{13}]	0.74	0.4

Table 19. $M(E2)$ matrix elements in ^{169}Tm . Numbers in parens are without QPC enhancement.

	$3/2^+ @ 1.935$	$3/2^- [411] @ 0.481$	$5/2^+ @ 1.504$	$5/2^- [413] @ 0.831$
$1/2^- [411]$				
@ 0.0	(0.), 1.06	(-0.3), -0.58	(-0.05), 0.84	(0.305), 0.678
$1/2^+ @ 1.3$	-	(0.), 0.57	-	(0.), 0.466
$B(E2)_{\text{s.p.u.}}$				
$1/2^+ @ 0.$	1.124	0.3364	0.706	0.46
$1/2^+ @ 1.3$	-	0.33	-	0.22

Table 20. $M(E2)$ matrix elements in ^{173}Lu . Numbers in parens are without QPC enhancement.

	3/2 [402] @ 1.128	3/2 [402] @ 1.534
7/2 [404]		
@ 0.0	(-0.41), -0.923	(-0.1665), 0.815
$B(E2)$ s.p.u.		
7/2 [404]	0.852	0.664

Table 21. $M(E2)$ matrix elements in ^{173}Lu . Numbers in parens are without QPC enhancement.

	$5/2 [413] @ 1.892$	$5/2 [413] @ 1.242$	$3/2 [411] @ 1.77$	$3/2 [411] @ 0.837$
$1/2 [411]$				
@ 0.1	(0.175), (-0.6)	(0.417), 0.88	(-0.15), 0.78	(-0.5), -0.886
$B(E2)_{\text{s.p.u.}}$				
$1/2 [411]$	0.36	0.774	0.61	0.785

Table 22. $M(E2)$ matrix elements in ^{155}Gd . Numbers in parens are without QFC enhancement.

	5/2 [523] @ 0.237	3/2 [521] @ 0.016
1/2 [521] @ 0.948	(-0.09), -0.8	(0.086), 1.042
$B(E2)$ s.p.u.		
1/2 [521]	0.64	1.09

Table 23. $M(E2)$ matrix elements in ^{167}Er . Numbers in parens are without QPC enhancement.

	$3/2^- [521]$ @ 1.064	$5/2^- [523]$ @ 0.656	$3/2^- = 1/2^- \oplus \gamma$ @ 1.642	$5/2^- = 1/2^- \oplus \gamma$ @ 1.5
$1/2^- [521]$ @ 0.331	(0.051), -0.53	(0.03), 0.26	(0.), 1.31	(0.), 1.45
$1/2^- @ 1.24$	(0.), 0.869	(0.), 1.243	0.	0.
$B(E2)_{\text{s.p.u.}}$				
$1/2 @ 0.33$	0.281	0.068	1.71	2.1
$1/2 @ 1.24$	0.755	1.55	0.	0.

Table 24. $M(E2)$ matrix elements in ^{167}Er . Numbers in parens are without QPC enhancement.

	1/2 [510] @ 0.697	7/2 [514] @ 1.927	7/2 [514] @ 0.982
3/2- [512] @ 1.054	(-0.001), 1.36	(-0.0414), 1.19	(-0.083), -1.67
5/2 [512] @ 0.268	(-0.031), 1.39	-	-
$B(E2)_{\text{s.p.u.}}$			
3/2- [512]	1.85	1.42	2.78
5/2- [512]	1.932	-	-

Table 25. $M(E2)$ matrix elements in ^{173}Yb . Numbers in parens are without QPC enhancement.

	3/2 [521] @ 1.967	3/2 [521] @ 1.348	5/2 [523] @ 1.718	5/2 [523] @ 1.12
1/2 [521] @ 0.122	(-0.13), 0.44	(-0.08), -1.11	(0.09), -0.78	(0.1), 0.914
$B(E2)_{s.p.u.}$				
1/2 [521]	0.193	1.23	0.608	0.835

Table 26. $M(E2)$ matrix elements in ^{173}Yb . Numbers in parens are without QPC enhancement.

	$1/2 [510] @ 1.4$	$1/2 [510] @ 1.2$	$7/2 [514] @ 0.282$
$3/2 [512]$			
@ 1.11	(0.), 0.3	(0.), 0.526	(-0.122), -1.072
$5/2 [512]$			
@ 0.0	(-0.047), 0.97	(-0.1), -0.73	-
$B(E2)_{s.p.u.}$			
$3/2 [512]$	0.09	0.277	1.15
$5/2 [512]$	0.94	0.533	-

VI. CONCLUSION

The work which we have reported in previous sections and the appendix has as its aim the understanding of correlations between the quasifermion and quasiboson elementary excitations in the atomic nucleus.

Two sections have been devoted to introduction and summary of the current status of the problem. In Sec. II, the Green's function formalism was introduced and the graphical techniques of perturbation theory were outlined. This provided a theoretical framework within which an approach to the many-body problem was presented. The bare NN -interaction was replaced by the reaction matrix, and the full space of one-particle states was replaced by a model space. Proceeding on the assumption that only two-nucleon correlations were important in the nucleus, the interactions between two particles (pp) and between a particle and a hole (ph) were described in terms of irreducible blocks. The one-particle self-energy, already separated into the Brueckner-Hartree-Fock and a residual term, was further divided into those terms which substantially fractionate the single particle strength and the remaining "regular" contribution. The quasiparticle Green's function was written down as containing only the contributions from the "analytic" part of the self-energy. The regular part was renormalized into the (pp) and (ph) blocks. The prescriptions for configuration space truncation and obtaining the lowest vibrational modes via RPA were presented. The extension of the above techniques to superfluids was described in terms of bogolons. An alternative "anomalous" Green's function formalism was indicated. Although some of these steps are, in fact, only carried out via a

phenomenological parameterization, their totality justifies a picture of the nucleus in terms of "quasi" particles, phonons, and effective interactions.

In Appendix B, we described the various efforts to determine nuclear states from quasiparticles, phonons, and an effective interaction. Most of these theories could be described quite simply in terms of Green's functions. The bootstrap and boson expansion techniques, however, were indicated as alternatives to the diagrammatic method outlined in Sec. III.

Section III contains the theory which we have worked out to describe quasiparticle-phonon coupling. Using Feynman diagrams as a pictorial guide, it was shown how to correct for several kinds of Pauli exclusion principle violations: namely, in the simultaneous propagation of a quasiparticle and a phonon and in the simultaneous propagation of a quasiparticle and a 2p-2h ground state correlation. In addition, nested and staggered two-phonon contributions to the self-energy were drawn and it was shown that these could be calculated by considering self-consistent intermediate propagators, in the first case, and by defining an effective second order coupling term between one-phonon-plus-quasiparticle configurations, in the second case. We have neglected several third order coupling terms which have been calculated by other authors. We felt it would be interesting if one could "derive" QPC without the usual phenomenological motivation. Indeed, the dynamical equation for the self-energy may be separated into the Hartree-Fock term plus a generalized core-polarization term, in which the phonon is initiated by the G matrix and terminated by the irreducible (ph)-block, I. Higher order contributions to the self-energy are implicit in the energy dependence (or complicated contributions inside) of I.

A generalized quasifermion Green's function was defined to calculate the time evolution of a one-nucleon excitation. Although the quasifermions included only the simple zero-order quasibogolons and the fermion-like pair composed of one phonon and one spectron, or odd-mass eigenstate, it was shown that the poles of this Green's function corresponded to the inclusion of all the contributions to the self-energy discussed earlier. The solution of the dynamical equation for the quasifermion Green's function was obtained via a matrix diagonalization which mixed forwards and backwards quasibogolons and forwards and backwards phonon-plus-spectron pairs.

The theory was applied in Sec. V to the calculation of even- and odd-mass rare earth isotopes. The significant results for the even nuclei were that the higher order (two-phonon and EPV) contributions to the self-energy did not renormalize the phonon. This was expected from the theory. Also, the backwards correlations made a substantial contribution around ^{166}Er , both renormalizing the pairing field and bringing the trend of the $2+$ energies into better agreement with experiment. In the odd-mass rare earth isotopes, we found exclusion principle violations to have substantial effects on the energy level spectra--in all cases pushing the energies of the eigenstates upwards. In many nuclei, this seems to explain the absence of certain phonon states. The two-phonon contributions implicit in the self-consistency and second order coupling terms were also responsible for major energy level shifts, e.g., the consistent pushing-down of the $1/2 [510]$ and $3/2 [512]$ levels in odd-neutron isotopes. For the same reasons, an intruder $1/2 [550]$ state was computed at low excitation energy in the Eu and Tb isotopes. It was

also found that the second order coupling terms were responsible (at least in part) for the $K_0 \pm 2$ splittings experimentally observed in ^{165}Ho and ^{167}Er . In the section on transition probabilities, it was interesting to explore the consequences of QPC on fractionation of the one-nucleon stripping and pick-up strength. Typically for low-energy states, the single particle $B(E2)$ strength and the collective $B(E2)$ are in phase. Nevertheless, the incoherent states can be observed experimentally, and the single particle and collective terms should interfere destructively, thereby substantially reducing the expected $B(E2)$. It was also found that Pauli principle corrections can reduce the transition probabilities of a phonon state by as much as 40%.

Certainly, our calculations have verified the prominent role of the Pauli principle on nuclear spectra in the QPC scheme, as well as the importance of high order contributions to the self-energy.

Our efforts in this area have stimulated our interest in a number of further theoretical investigations. A more exact description of the energy dependence of the (ph)-block, I , should be sought. It would be worthwhile to study the variation of this energy dependence throughout the periodic table. We suspect that it would be quite substantial for "vibrational" nuclei. Having derived or at least postulated a reasonable energy dependence, a prescription is then badly needed for solving the Dyson and Bethe-Salpater equations with an energy dependent I -block.

Another fundamental study to be undertaken is the self-consistent treatment of the "normal" and "anomalous" self-energies--in other words, the interference of pairing and phonon correlations. Such a thorough re-examination of pairing should probably yield energy dependent u and v

factors as well as an algorithm for properly orthogonalizing to the spurious state. In the calculation of external fields, we have neglected some higher order diagrams which, though off-the-energy-shell, are most assuredly of some consequence. These should also be examined further.

There are also several interesting applications of the SCQPC method outlined here to other regions of the periodic table. The renormalized RPA (or TDA) equation should describe accurately the phonon coupled states in the odd-odd nuclei. Also, we suspect that the $I = j - 2$ states which are often quite as low in energy as the anomalous $I = j - 1$ intruder states in spherical nuclei can be explained in terms of the nested and staggered two-phonon contributions to the self-energy. Not only are the vector coupling coefficients favorable, but the low spin case will be lowered more by configuration interaction, i.e., the second order coupling term.

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APPENDIX

A. Notation

Table 27 presents a summary of the notation which is employed in equations, that is, letterings and indices for creation and annihilation operators, Green's functions, interactions, etc. This listing is not exhaustive of the notation employed. It does, however, serve to distinguish the more subtle aspects (e.g., the difference between g and G , or between F' , F'' , F^E , and F^W).

Table 28 should be used in conjunction with the various Feynman diagram figures. As we will make use of "dressed" and "undressed" propagators for particles, quasiparticles, and bogolons, it will not be possible to distinguish these, as is customary, by single and double lines. Instead, if there would be any ambiguity we include in the figure caption an explanation of the propagator lines. This will be in the form of the abbreviations in Table 28.

Table 27. Notation

C_{α}^{+}	creation operator for a "bare" shell-model particle
c_{α}^{+}	creation operator for a quasiparticle
Λ_{α}^{+}	creation operator for a "bare" or shell-model bogolon
a_{α}^{+}	creation operator for a quasibogolon, i.e., an elementary excitation of the fermion system after the BCS transformation and Landau renormalization
G, K	one- and two-nucleon Green's functions for <u>particles</u> (fully dressed)
G^0, K^0	zeroth order or "undressed" one- and two-nucleon <u>particle</u> Green's functions
g, K	one- and two-nucleon Green's functions for quasiparticles
$\tilde{G}, \tilde{K}$	one- and two-nucleon Green's functions for <u>bogolons</u>
$\tilde{g}, \tilde{k}$	one- and two-nucleon Green's functions for <u>quasibogolons</u>
$\alpha, \beta, \gamma, \text{etc.}$ $1, 2, \text{etc.}$	Greek letters and arabic numerals used interchangeably as single particle labels. The Greek letters will appear often in equations where other indices, such as i, j, N, N' , appear (esp. the SCQPC equations). The arabic numerals will appear when more than three or four Greek letters would be needed (esp. the Bethe-Salpater equations).
i, j, k	denote poles of the single-particle Green's functions, i.e., eigenmodes of odd nucleons
N, N', S	denotes poles of the two-nucleon Green's functions, i.e., phonons
a_{β}^{+i}	creation operator for a <u>spectron</u> , that is the i^{th} eigenmode containing an appreciable fraction of the single particle (β) strength
$Z_{\beta}^{(i)}$	residue of quasiparticle or bogolon, β , at i^{th} pole of Green's function
Q_N^{+}	creation operator for a phonon
$N-1, N, N+1$	denote number of nucleons. N will generally refer to a phonon index, unless $N-1, N+1$, etc. appear in same context.

(continued)

Table 27. (continued)

$b_{\alpha}^{+}, b_{\beta N}^{+}$	creation operator for a <u>quasifermion</u>
$\mathcal{L}$	"dressed" quasifermion Green's function
$\mathcal{L}^0$	"undressed" quasifermion Green's function (i.e., before H_{eff} diagonalized)
V	two-body interaction
G	Brueckner reaction matrix, not to be confused with G, the particle Green's function
J	irreducible block in particle-particle channel, contains $n + 2$ particle-n hole intermediates both inside and outside model space ($n \geq 1$)
I, I_1, I_2	irreducible block in particle-hole (ph) channel. The subscripts, 1 or 2, refer to the first and second (ph) channels, respectively.
$\Gamma^{\xi}, \Gamma^{\omega}$	Landau renormalized J and I
F^{ξ}, F^{ω}	Landau renormalized J and I multiplied by fractions of single particle strength which is not included in regular part of Green's function, G, i.e. $\langle \alpha\beta F^{\omega} \gamma\delta \rangle = \sqrt{Z_{\alpha} Z_{\beta} Z_{\gamma} Z_{\delta}} \langle \alpha\beta \Gamma^{\omega} \gamma\delta \rangle .$
F'', F'	F^{ξ} and F^{ω} after configuration space truncation
P, Q, R, S	"irreducible" interactions which scatter two quasibogolons into two quasibogolons (P), four quasibogolons into vacuum (Q), one quasibogolon into three (R), and <u>vice versa</u> (S).
$P^N(\alpha\beta)$, etc.	quasiparticle phonon vertex
T	one-body kinetic energy operator
U	one-body shell model or Hartree-Fock potential
V	one-body external field
H_{eff}	the approximate vertices between quasifermions

Table 28. Abbreviations for propagator lines in Feynman diagrams

SM	corresponds to shell model propagator, G^0 .
BHF	G^0 , where "shell model" potential is Brueckner-Hartree-Fock
FDSM	corresponds to G , fully dressed Green's function of particle
LQ	Landau quasiparticle, one-pole with energy renormalized by Brueckner and other high energy processes in mass operator, corresponds to g^0 .
FDLQ	fully dressed (including single-particle strength fractionation) Landau quasiparticle, corresponds to g .
SMB	shell model bogolon, $\tilde{G}^0$, the BCS transform of shell model particles and holes.
FDB	$\tilde{G}$, fully dressed bogolon
LQB	quasibogolon, $\tilde{g}^0$, the BCS transform of quasiparticles and -holes.
FDQB	fully dressed quasibogolon, $\tilde{g}$.
QF	quasifermion, corresponds to χ^0 .
EEB	<u>spectrons</u> , corresponds to elementary excitations of interacting bogolon space, i.e., excitations corresponding to some pole of FDQB.

B. Review of QPC Literature

The most successful and widely accepted model of low energy nuclear structure can be ascribed to Bohr and Mottelson⁷ and is widely referred to as the unified model. This picture of nuclear motions unifies the concepts of macroscopic or collective nuclear modes and the shell model.

Except for rotational motion which we do not consider here, there are two kinds of elementary nuclear excitation: vibration and single particle motion. These modes are approximately independent. Due to pairing, there are seldom low-energy shell-model (or two quasiparticle) excitations in even-even nuclei. The lowest states are usually the 2⁺ quadrupole or 3⁻ octupole vibrational modes. In odd-mass nuclei, the low-energy nuclear levels are obtained by promoting the odd nucleon to different shells. The exact description of the energy levels and transition probabilities in a particular nucleus requires that we consider the coupling of these two modes, as well as the interaction with high energy modes (such as the incoherent 2⁺ states or the giant dipole resonance). If we assume the high energy modes can be neglected (because, for example, their coupling by second order perturbation theory involves large energy denominators), then we seek to improve the description of low-energy nuclear phenomena via quasiparticle-phonon coupling (QPC).

Within the past 5 or 10 years, a number of theories or models have been developed to improve the description of both the even-even nuclei--collective states--and the odd-mass nuclei--quasiparticles. A review article is certainly needed. It would seem that newcomers to this field are faced with a proliferation of approaches to one or more aspects of

the quasiparticle-phonon problem. Usually, very little attention is paid to comparisons of the various methods. In this section, we summarize a number of phonon and/or quasiparticle theories which we have studied, within the Green's function formalism. The comparisons are mostly mathematical or theoretical in nature. No attempt will be made to judge the theory or model on the basis of agreement with experimental data.

In order to maintain some semblance of order in the presentation, we divide the QPC[†] models and theories into the following classifications:

1. Descriptions of the Odd-Mass Nuclei
 - a. Weak and intermediate coupling, Low order diagrams
 - i. Macroscopic or phenomenological
 - ii. Microscopic
 - b. Higher order diagrams
 - i. Macroscopic
 - ii. Microscopic
2. Descriptions of Even-Mass Nuclei
 - a. Renormalization of RPA by the Pauli Principle
 - b. Two-Phonon Admixtures
 - i. Macroscopic
 - ii. Microscopic
3. Self-Consistent Descriptions of Even- and Odd-Mass Nuclei
 - a. Diagrammatic theories
 - b. Spectral decomposition theories
 - c. Boson expansions

[†]In this section only, we use the word quasiparticle to include all kinds of single particle excitations: shell model, bogolon, etc. For simplicity, the lines in the diagrams may be taken as SM.

1. Descriptions of Odd Mass Nuclei

a. Weak and intermediate coupling

i. Macroscopic and phenomenological models

A quite general particle-core coupling Hamiltonian was first written down by Lawson and Uretsky⁸³ and by de Shalit,⁸⁴ viz.

$$H_{\text{coup}} = \sum_k \gamma_k T^{(k)}(\text{core}) \cdot t^{(k)}(\text{particle}) \quad (\text{A.1})$$

where $T^{(k)}$ is an irreducible tensor which acts in the coordinate space specifying the core motion and $t^{(k)}$ similarly for particle motion. γ_k is a phenomenologically determined constant. In first order perturbation theory, the state $|J_c J\rangle$ would be shifted in energy by each multipole in the sum (A.1)

$$\begin{aligned} \Delta E_k(J) &= \gamma_k \langle J_c J | T^{(k)} \cdot t^{(k)} | J_c J \rangle \\ &= (-1)^{J_c + j + J} \gamma_k \begin{Bmatrix} J_c & j & J \\ j & J_c & k \end{Bmatrix} \langle J_c || T^{(k)} || J_c \rangle \langle j || t^{(k)} || j \rangle. \quad (\text{A.2}) \end{aligned}$$

It can be shown from the usual Racah algebra that the "center of gravity" for each multiplet coincides with the unperturbed position

$$\sum_J (2J + 1) \Delta E_k(J) = 0 \quad \text{for } k \neq 0 \quad (\text{A.3})$$

The $k = 0$ contribution, γ_0 , can be included in the definition of the single particle energies, i.e., only $k \neq 0$ terms correspond to physical coupling of single particle and core modes.

The most obvious application of Eq. (A.2) is to a multiplet of states composed of a shell model particle, say $p_{1/2}$, coupled to a core state, say the $2+$ state. (In the example we have chosen, there would be two states in the multiplet; a $3/2^-$ and a $5/2^-$ state.)

In higher orders of perturbation theory, the off-diagonal matrix elements

$$\langle J_c \| T^{(k)} \| J'_c \rangle \langle j \| t^{(k)} \| j' \rangle \quad (A.4)$$

contribute to the energies in the multiplet.

Some very recent quasiparticle-core coupling calculations may be analyzed in terms of de Shalit's Hamiltonian.⁸⁵ In these calculations, only the $J_c = 0+, 2+$ core states are considered and only the $k = 1, 2$ terms in the tensor expansion are retained:

$$H_{\text{coup}} = -\xi J^{(1)} \cdot j^{(1)} - \eta Q^{(2)} \cdot q^{(2)} \quad (A.5)$$

The matrix elements of $j^{(1)}$ and $q^{(2)}$ are easily derived between shell model states. Also, it is clear that $\langle J_c | j^{(1)} | J'_c \rangle = \delta_{cc, J'_c}$. The $J \cdot j$ term is analogous to the coriolis term in strong coupling and gives rise to a quasi-rotational spectrum (Lande interval rule). In spherical nuclei, such a term is not necessary⁸⁶ to explain experimental⁸⁶ results if the quadrupole term is diagonalized in a large model space, i.e. J_c corresponding to one-, two-, and three-phonon core states. The $Q \cdot q$ term physically corresponds to coupling of the particle to quadrupole oscillations of the core.

The choice for $\langle J_c | Q | J_c \rangle$ depends on how we define the collective or core coordinates. Sometimes the core matrix elements are taken as parameters. Alternatively, Q may be taken as the mass quadrupole operator $\sum_i q_i^{(2)}$ or the charge quadrupole operator $\sum_p e_p^{\text{eff}} q_p^{(2)} + \sum_n e_n^{\text{eff}} q_n^{(2)}$ where e_p^{eff} and e_n^{eff} are the effective proton and neutron charges. In the latter case, $\langle 2 || Q || 0 \rangle$ is proportional to the square root of the experimentally observed $B(E2: 2 \rightarrow 0)$ and $\langle 2 || Q || 2 \rangle$ to the experimentally observed static quadrupole moment of the 2^+ state. In spherical nuclei, $\langle 0 || Q || 0 \rangle = 0$.

In most calculations which begin with the de-Shalit coupling term, the interaction is diagonalized among the basis states $|J_c j J\rangle$. This is intermediate coupling.

In the weak coupling limit, i.e., off-diagonal matrix elements much less than diagonal energy differences, the first- (Eq. (A.2)) and second- (Eq. (A.5)) order perturbation theory terms should be sufficient.

$$\Delta E_{J_c j J} = \sum_{J_c, j'} \frac{|\langle J_c j J | H_{\text{coup}} | J_c, j' J \rangle|^2}{E_{J_c j J}^{(0)} - E_{J_c, j' J}^{(0)}} \quad (\text{A.6})$$

In the original papers,^{83,84} "weak" meant first-order perturbation theory. In the remaining discussion, we shall include second order terms as well under this heading. The physical meaning of weak coupling is, nevertheless, clear.

The Bohr-Mottelson^{7,87} particle-vibration coupling is mathematically a special case of the de-Shalit interaction

$$H_{\text{coup}} = - \sum_{\lambda=2,3} \sum_{\mu=-\lambda}^{\lambda} K_{\lambda}(r) Y_{\lambda\mu}^*(\theta, \phi) \alpha_{\lambda\mu} \quad (\text{A.7})$$

However, its physical origin is more definite

$$K_{\lambda}(r) = V_c r \frac{df(r)}{dr}; f(r) = \frac{1}{1 + \exp\left(\frac{r-R}{d}\right)}$$

V_c , R , d being the depth, radius, and diffuseness of the Woods-Saxon potential.

$Y_{\lambda\mu}^*$ and $\alpha_{\lambda\mu}$ refer to the particle coordinates and the core deformation respectively. Hence Eq. (A.7) represents the correlation of the particle motion with the oscillating core. The sum over λ is restricted to 2, 3 because the only low-energy shape fluctuations are known to be the quadrupole and octupole modes.

It should also be noted that α_{λ} connects core states which differ by one phonon, hence the Bohr-Mottelson H_{coup} does not have diagonal terms.

Having written down explicitly the formulation of these weak and intermediate coupling theories, we can construct their graphical equivalents. In fact, the equation could have been derived using Green's functions.

Weak coupling corresponds to the approximation

$$G = G_0 + G_0 \Sigma G \approx G_0 + G_0 (\Sigma_{\text{QPC}}) G_0 \quad (\text{A.8})$$

shown in Fig. 67. It is only applicable when G is not much different from G_0 , that is, when there is no appreciable splitting of the single particle strength.

In the intermediate coupling,⁸⁷ the self-energy approximation is included to all orders. See Fig. 68. This allows that $G_0 \neq G$, or that G may have several poles with significant residues. In other words,

$$\begin{array}{c} \uparrow \\ \Sigma \end{array} \approx \begin{array}{c} \uparrow \\ \text{wavy line} \end{array}, \quad \text{wavy line} = \langle 0_j J | H_{\text{coup}} | J_c j' J \rangle$$

XBL 726-1031

Fig. 67. The one nucleon Green's function in the weak coupling limit.

$\uparrow$ = SM; $\uparrow$ = FDSM; Σ = phonon.

XBL 726-1032

Fig. 68. The one-nucleon Green's function in the intermediate coupling approximation.

Fig. 69. The self-energy insertions of Brown, Evans, and Thouless.
Lines are S.F.

= microscopic particle-phonon coupling vertex, Eq. (A.9).

XBL 726-1033

Fig. 70. The second diagram in Fig. 69 cancels this Pauli principle violation.

single particle and phonon-plus-particle configurations may mix to any degree.

If two (or three, etc.) phonon states of the core are considered, the self-energy contains partially dressed intermediates (the "nested wiggle" diagram in Fig. 68), and one-phonon states are connected to themselves via two phonon intermediates.

ii. Microscopic intermediate coupling

Figure 68 which describes intermediate coupling[†] provides a convenient starting point for more sophisticated theories. One theoretical improvement would be to calculate the same diagrams but with a microscopic phonon and the coupling Hamiltonian derived from the two-body interaction. Such a program was first carried out by Brown, Evans, and Thouless⁸⁸ who calculated the self-energy insertions shown in Fig. 69. The second of the two-self-energy diagrams cancels the Pauli Principle violation shown in Fig. 70, i.e., the contribution of a particle on top of a 2p-2h state in which the same particle is already occupied.

The analytical expressions for the self-energy are

$$\Sigma = \sum_{bN} \frac{|\langle a | H_{\text{eff}} | bN \rangle|^2}{\omega - \epsilon_b - E_n} + \sum_B \frac{|\langle aBN | H_{\text{eff}} | \tilde{0} \rangle|^2}{\omega + \epsilon_B + E_N}$$

$$\langle a | H_{\text{eff}} | bN \rangle = \sum_{cD} \langle aD | V | bc \rangle X_{cD} + \langle ac | V | bD \rangle Y_{cD}$$

$$\langle aBN | H_{\text{eff}} | \tilde{0} \rangle = \sum_{cD} \langle ac | V | BD \rangle X_{cD} + \langle aD | V | Bc \rangle Y_{cD}$$

[†] Note that Fig. 68 and all others in which the phonon is drawn as a wiggly line may be considered to be macroscopic or microscopic.

X and Y are the usual RPA amplitudes. ϵ_a = particle energy = $h_a - \lambda$.
 ϵ_B = hole energy = $\lambda - h_B$, the h 's denoting simple shell model energies with the zero such that λ lies somewhere between the filled and unfilled shells. $\langle a d | V | b c \rangle$ denotes a two-body matrix element of an effective interaction. The insertion of $\sum$ to all orders in the propagator is accomplished by diagonalizing the matrix:

ϵ_a	$\langle a H_{\text{eff}} bN \rangle$	...	0	$\langle aBN H_{\text{eff}} \tilde{0} \rangle$	...
$\langle a H_{\text{eff}} bN \rangle$	$\epsilon_b + E_N$	0	$\langle aBN H_{\text{eff}} \tilde{0} \rangle$	0	0
$\vdots$	0	$\vdots$	$\vdots$	0	0
$\vdots$		$\vdots$	$\vdots$		
0	$\langle aBN H_{\text{eff}} \tilde{0} \rangle$	...	$-\epsilon_A$	$\langle A H_{\text{eff}} BN \rangle$	...
$\langle aBN H_{\text{eff}} \tilde{0} \rangle$	0	0	$\langle A H_{\text{eff}} BN \rangle$	$-\epsilon_B - E_N$	0
$\vdots$	0	0	$\vdots$	0	...
$\vdots$			$\vdots$		

which couples together $N+1$ and $N-1$ nuclei.

From a practical point of view, the most important sophistication of Brown et al.'s microscopic intermediate coupling is the extension to superfluids--because so many nuclei are not "normal" (ph) systems. This program of quasiparticle-phonon coupling (QPC) was carried out for spherical nuclei by Kisslinger and Sorenson⁸⁹ and in deformed nuclei by

Soloviev.^{80,90} Their derivations are based on quasiboson commutation algebra and the pairing-plus-quadrupole model interaction rather than diagrams. The backwards correlations (i.e., Pauli Principle violating terms) were neglected. Also, the vertex only includes coupling of the particle to the quadrupole field or the (ph) part of the superconducting phonon. The pairing (or pp) fluctuation is ignored. The actual numerical problem is not very difficult at all. It is just a diagonalization of the upper left quadrant of the matrix on the previous page except that $\langle \alpha | H_{\text{eff}} | \beta N \rangle$ is corrected for the percentage of the time α and β are particle (hole) and N is (ph) in nature, i.e.,

$$\langle \alpha | H_{\text{eff}} | \beta N \rangle = - \chi \sum_{\gamma \delta} \langle \alpha | Q | \beta \rangle \langle \gamma | Q | \bar{\delta} \rangle (U_{\alpha} U_{\beta} - V_{\alpha} V_{\beta}) (U_{\gamma} V_{\delta} + U_{\delta} V_{\gamma}) \cdot (X + Y)_{\gamma \delta} \quad (\text{A.10})$$

after allowing for the separable matrix elements.

All of the microscopic intermediate coupling theories suffer from several other difficulties. We can mention three of these here: Intermediate propagators are not dressed, i.e., no two, three, etc., phonon intermediates. The RPA phonon is solved assuming no fragmentation of single-particle strengths. Higher order diagrams--including other forms of exclusion principle violation--are not considered. These points correspond to neglect of the perturbation processes shown in Fig. 71.

b. Higher Order diagrams

The diagrams such as those shown in Fig. 71 can be calculated in a variety of ways and several methods have been proposed. We examine first the theories in which the phonons are macroscopic "surfbons" and the coupling interaction is of the Bohr-Mottelson variety. The fully

(a) intermediate propagators
dressed

(b) RPA phonons renormalized by
splitting of single particle strength

Higher order diagrams
included.

(c)

(d)

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Fig. 71. Processes in microscopic intermediate coupling.
Lines are SM or SMB.

microscopic theories are in some cases conceptually identical, the only difference being that the phonon is microscopic.

i. Macroscopic Particle-Phonon coupling with higher order effects included

The graphical contributions to the self-energy shown in Fig. 71a,d, are part of a subclass of graphs in which the self-energy contains at any time only one particle propagator line but one or more phonons. It is easy to see that graphs containing more than one phonon will be important whenever the energy of the phonon is low, or when the single particle strength is substantially fractionated, i.e., intermediates should be "dressed". In fact, these graphs are included in most "intermediate coupling" calculations, such as those in Refs. 91.

These diagrams may be calculated by diagonalizing the H_{coup} in Eq. (A.7) in a space $|N(R)J_c, j\rangle$, that is, N phonons, each with angular momentum R , coupled to J_c and a single particle with angular momentum j , J_c and j coupled to a total angular momentum J . Of course, there is some limit placed on the number of phonons, N , in order to obtain a manageable numerical problem.

Parr⁹² has performed a perturbation calculation for ^{201}Tl in which diagrams such as those shown in Fig. 71 are calculated to sixth order in the interaction H_{coup} . In this case, the fragmentation of single particle strength is slight so it is not surprising that the results are almost identical to exact diagonalization.⁹³

Parr⁹⁴ and Hamamoto⁹⁵ have proposed that phonon-particle configurations such as those in the ^{209}Bi sextuplet (see Fig. 72) may be shifted by higher order diagrams. In particular, the diagrams shown in Fig. 72 should

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Fig. 72. The ^{209}Bi sextuplet. Seven states between 2.49 and 2.74 MeV correspond to the coupling of the core excitation ($J_c = 3^-$) to the $j = 9/2$ - proton.

Fig. 73. Diagrams of Hamamoto which give rise to splitting of ^{209}Bi multiplet. Lines are SM.

Fig. 74. Mixing of j (single particle) states by off-diagonal terms in the self-energy.

Fig. 75. This violation of the Pauli Principle is cancelled by Fig. 73c.

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contribute. Calculations have actually been carried out by Hamamoto by solving a coupled set of differential equations[†]

$$(E - N_1 \hbar \omega - H_0) R_{j_1 N_1 j}(r) = \sum_{N_2 j_2} \langle j_1 N_1 j | H_{\text{coup}} | j_2 N_2 j \rangle R_{j_2 N_2 j}(r) \quad (\text{A.11})$$

Equation (A.11) is identical to an intermediate coupling calculation except that the radial wave function of the particle is permitted to vary also. In fact, this has the effect of mixing different shell model orbits as shown in Fig. 74. Hamamoto states that by calculating Fig. 73a in this manner, Fig. 73d is also included. Similarly, small admixtures of two phonon one particle states calculated in Fig. 73b automatically include Fig. 73c. In other words, diagonalizing the self-energy among shell model states both below and above the Fermi surface is equivalent to calculating certain backwards diagrams. An alternative procedure suggested by Hamamoto is to calculate all four contributions of Fig. 73 explicitly, thus yielding a second order perturbation theory of the multiplet structure in ²⁰⁹Bi. The analytic expressions for these four contributions are:^{††}

[†] N_1 refers to the number of phonons.

^{††} The ϵ_{j_1} (when j_1 is beneath the Fermi level) are $\lambda - h_{j_1}$, as is our convention. If Eqs. (A.11b) and (A.11c) are written in terms of the shell model energies h_j , the denominators are identical. This illustrates the validity of Hamamoto's proposal for mixing radial wave functions, thereby treating A.11b,c (or Fig. 73b,c) simultaneously.

$$\Delta E(\text{fig. 73a}) = \frac{|\langle \{j, 3^-\}_{j_p} | H_{\text{coup}} | j_p \rangle|^2}{\epsilon_j + \hbar\omega - \epsilon_{j_p}} \quad (\text{A.11a})$$

$$\Delta E(\text{fig. 73b}) = \sum_{\substack{j_1 > \text{Fermi} \\ \text{level}}} \frac{|\langle j | H_{\text{coup}} | \{j_1, 3^-\}_{j_1} \rangle|^2}{\epsilon_j - \hbar\omega - \epsilon_{j_1}} \quad (\text{A.11b})$$

$$\Delta E(\text{fig. 73c}) = \sum_{\substack{j_1 < \text{Fermi} \\ \text{level}}} \frac{|\langle \{jj_1\} 3^- | H_{\text{coup}} | 3^- \rangle|^2}{\epsilon_j + \epsilon_{j_1} - \hbar\omega} \quad (\text{A.11c})$$

$$\Delta E(\text{fig. 73d}) = \frac{|\langle \{j, 3^-\}_{j_h} | H_{\text{coup}} | j_h \rangle|^2}{\epsilon_j + \epsilon_{j_h} + \hbar\omega} \quad (\text{A.11d})$$

However, perturbation theory is feasible only when these splittings are small.

The macroscopic model may be extended to include 3 valence particles plus core excitations.⁹⁶ In this case, the particles with the core via the usual H_{coup} . The particle-particle interaction is taken to be the pairing force.

From the point of view of matrix diagonalizations, this is the logical extension of the intermediate coupling calculations of ²⁰⁹Pb to ²¹¹Pb. The calculations carried out include an application to the Ag isotopes⁹⁷ where intruder states have been observed.

ii. Microscopic Particle-Phonon Coupling with higher order effects included

Another important class of higher order contributions to QPC is that which results from the microscopic nature of the collective mode. Figure 73c and also Fig. 76, are examples of the collective mode decoupling

for a finite time into a particle and a hole followed by a different recoupling or a simple two-nucleon interaction.

Hafele⁹⁸ attempted to calculate the effects of certain microscopic subtleties on the structure of the ^{209}Bi multiplet. Indeed, the structure or ordering of the multiplet is dominated by these corrections. Hafele calculated the energies of the multiplet states of ^{209}Pb and ^{209}Bi using several "heuristic" formulae

$$\Delta E_{\text{pauli}} = (2J_{\text{core}} + 1) \sum_{qh} D_{qh}^2 x_{qh}^2 \sum_{j_{pq}=\text{odd}} (2j_{pq} + 1) \left\{ \begin{matrix} J & j_h & j_{pq} \\ j_q & j_q & j_{\text{core}} \end{matrix} \right\} \quad (\text{A.12a})$$

$$\Delta E_{\text{pairing}} = -G \frac{2j_{\text{core}} + 1}{2J + 1} \sum_{j_h} x_{qh}^2 \delta(j, j_h) \quad (\text{A.12b})$$

The former equation adds to the configuration energy those "illegal" (odd j_{pq}) contributions which come from not antisymmetrizing the quasi-boson and fermion, D_{qh} is a separable particle-hole matrix element, x_{qh} is a TDA amplitude. (Fig. 75a.) The latter formula simply calculates the pairing interaction between a particle and a particle-hole phonon after recoupling to pair of particles and a hole. (Fig. 76.)

Quite recently, Bes and Broglia⁴⁴ have calculated the odd-mass nuclei such as ^{209}Pb and ^{209}Bi , by diagonalizing an effective Hamiltonian among (ph) phonon-plus-particle configurations and (pp) or pairing phonon-plus-hole configurations. The effective Hamiltonian matrix elements are shown in Fig. 78.

The physical processes which are included are

- the usual mixing of particles to phonon-particle states, Fig. 78c,e,i;
- pairing vibrations in addition to shape vibrations, Fig. 78a-d, i-j;
- backwards correlations, i.e., occupation of certain 2p-2h states by GS correlations, Fig. 78d,h,j;
- two phonon intermediates, Fig. 78b,f;
- recoupling graphs, Fig. 78a,g;
- Pauli Principle violations, Fig. 78.

The particle-phonon coupling vertices are calculated microscopically.

For example, the vertex connecting a pairing vibration-plus-hole configuration to a particle is shown in Fig. 77a. The pairing phonon may be resolved into its microscopic components, as shown in Fig. 77b, and then the residual interaction, V , couples the ingoing and outgoing propagators. The analytical expression is

$$\langle j_{pp}, j_h | H_{\text{eff}}^{\text{QPC}} | j_p \rangle = \sum_{p_1 p_2} X_{p_1 p_2}^J \langle p_1 p_2 | V | \text{ph} \rangle + \sum_{h_1 h_2} Y_{h_1 h_2}^J \langle h_1 h_2 | V | \text{ph} \rangle \quad (\text{A.13})$$

where X and Y are the usual RPA amplitudes. The effective Hamiltonian between phonon-plus-particle configurations contains two coupling matrix elements in the numerator and an energy difference in the denominator.

$$H_{\text{eff}}(\omega) = [\text{sign}] \frac{|H_{\text{eff}}^{\text{QPC}}|^2}{\omega - E_{\text{int}}} \quad (\text{A.14})$$

The numerator matrix elements are almost always assumed to be energy independent, however, the denominator does introduce an energy dependence,

Fig. 76. The microscopic pairing correction suggested by Hafele. Lines are SM. $\overline{H}$ interaction is a constant pairing ($J = 0$) matrix element.

Fig. 77(a). Vertex coupling a particle to a configuration composed of a pairing vibration, $\overline{H}$, and a hole....

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Fig. 77(b). ...decomposed into its microscopic components. Lines are SM.

$\overline{H}$ is an effective (pp) interaction.

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Fig. 78. Second order graphs contributing to the effective interaction of mes and Broglia. (e-h) are microscopic analogues of Fig. 73. (a-d, i-j) include coupling of pairing vibrations. $\uparrow, \downarrow$ = SM particles and holes.

$\sim$ = (ph), ^{208}Pb , vibration. $\parallel$ = (pp), ^{210}Pb , vibration.

viz, $\omega - E$ (intermediates). E (intermediates) is a summation of all the eigenenergies for the intermediate phonon and particle lines. ω is the energy variable associated with the propagator for a particle-plus-vibration excitation. A considerable difficulty is that the poles of this propagator (which are found by diagonalizing H_{eff} and correspond to eigenvalues) will occur in the variable ω . In other words, the diagonalization must be done self consistently because H_{eff} will be different for different eigenvalues of the Hamiltonian matrix. Two possible approximation schemes are proposed: $\omega = E$ of the main unperturbed component of the wave function (valid when this component $\approx 90\%$ of the wave function). This procedure yields wave functions which are not completely orthogonal $\omega = (E_{\text{in}} + E_{\text{out}})/2$. ω is the average of the two diagonal energies when an off-diagonal matrix element is being calculated between configurations. Eigenvectors are orthogonal. (The procedure is valid only when $(\omega - E_{\text{int}}) \gg [H_{\text{eff}}^{\text{QPC}}]$.) Either approximation works well in the lead region. The important simplification here is that the matrices are small because the configurations which are mixed contain phonons and one particle rather than three shell model excitations. Physically, the approach is satisfying because it couples what we believe to be elementary modes of nuclear excitation. The method's applicability to other regions of the periodic table is limited only by the neglect of self-energy diagrams in the propagators. The equations have been written down only for the normal (nonsuperfluid) system and a pairing-plus-quadrupole Hamiltonian. Thus, the corresponding term which is calculated in Ref. 99, is double counted.

Arita and Horie⁹⁹ have proposed the possibility of mixing particle-vibration coupled states in ²⁰⁹Bi and ²⁰⁷Pb via a system of equations

which approximate the mechanisms shown in Fig. 79, in addition to the usual one particle coupling (c.f., Fig. 77a). These are easily seen to be third order graphs which in general contain, in the numerator, two microscopic H_{eff} vertices and one two-body interaction, and, in the denominator, two energy differences. For example, Fig. 79a may be written

$$\sum_{php'h'} \frac{\langle J | H_{\text{eff}} | ph \rangle \langle ph' | V | hp' \rangle \langle p'h' | H_{\text{eff}} | J \rangle}{(\omega - (\epsilon_p + \epsilon_h + \epsilon_q))(\omega - (\epsilon_{p'} + \epsilon_{h'} + \epsilon_q))} \quad (\text{A.15})$$

If we set the energy variable

$$\omega = \epsilon_q + E_J \quad (\text{A.15a})$$

and note that from RPA that

$$\langle ph | H_{\text{eff}} | J \rangle = X_{ph}^J (E_J - \epsilon_p - \epsilon_h) \quad (\text{A.15b})$$

then Eq. (A.15) simplifies to

$$\sum_{php'h'} X_{ph}^J X_{p'h'}^J \langle ph' | V | hp' \rangle \quad . \quad (\text{A.16})$$

This is the approximation of Arita and Horie.⁹⁹

We should also point out that, because

$$\text{Diagram 1} = \sum_{p'h'} p' \text{Diagram 2} \quad (\text{A.17})$$

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Fig. 79. Third-order graphical contributions to effective interaction between particle-core coupled configurations.

is an effective interaction, e.g., Q-Q.

which is just the microscopic definition of a phonon, Fig. 79a is included in the diagonal energy of the phonon-particle coupled state. Also Fig. 79c is included in Fig. 78, while Fig. 79e,g are included in Fig. 78a. There are the remaining third order graphs (Fig. 79b,d,f) which we might predict to be unimportant since they involve two-body matrix elements which are "disconnected" from the coherent propagation of a phonon. However, the analogous 4th order graphs (to Fig. 79b,d,f) are included by diagonalizing the H_{eff} of Bes, et al.^{hh} The diagrammatical proof is shown in Fig. 80 and derives from the fact that when a matrix is diagonalized, graphs such as Fig. 80 are calculated. The 4th order graphs we are talking about contribute to Fig. 80a. See Fig. 80b.

The point we wish to make is that a "complete" set of contributions to H_{eff} between particle-vibration coupled states includes graphs calculated in both second and third order, while the use of phonons as elementary excitations eliminates the necessity for including many graphs with order 3, 4, 5 etc.

All of the methods mentioned so far begin from the idea that the low-lying state of the odd-mass nucleus is some combination of a particle state and a particle + phonon state. Why not a simple shell model calculation in a basis of 1p and 2p-1h states? See Fig. 81. Indeed, this is a superior calculation for several reasons. First, if the QPC model is valid, it will fall out of the diagonalization anyway--an excellent check on our most basic assumption. Secondly, the incoherent phonons, which are higher in energy, are implicitly included. Thirdly, diagrams of the type shown

Fig. 80. Fourth and fifth order graphs are automatically included in a matrix diagonalization.

Fig. 81. A three quasiparticle diagonalization includes the usual phonon-plus-particle contributions, (a), along with incoherent contributions such as (b). For convenience, effective interaction $\equiv \text{X}$. Lines are SM or SMB.

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Fig. 82. "Backwards" diagrams included in ETDA and ERPA-type 3-quasiparticle diagonalizations.

in Figs. 78-79 are taken into account. One major disadvantage is that the matrices we diagonalize are very large. And taking into account the backwards (ETDA and ERPA) diagrams shown in Fig. 82 is very expensive. The first diagram does, however, include exclusion principle corrections due to ground state correlations. The second admits backwards amplitudes into the phonon. In many regions of the periodic table both effects are important. Also, the intermediate propagators are not easily dressed by their self-energy parts. (In fact, we know of no calculation to do so.)

In normal nuclei, Arita and Horie⁹⁹ have diagonalized a residual interaction among all 2p-1h states for ²⁰⁹Bi. The calculation is a straightforward shell-model diagonalization. No backwards diagrams are taken into account; the constituent particles and holes are of the shell model variety. (They are not "dressed".)

In superfluid systems we might mention the treatments of Bes,¹⁰⁰ of Kuo¹⁰¹ and quite recently a theory of Kuriyama¹⁰² and associates. These methods differ in a number of ways. The first calculation¹⁰⁰ is in a deformed nucleus and rather than coupling all possible 3 quasiparticle pairs, they consider diagonalizing the pairing-plus-quadrupole Hamiltonian in a basis of quasiparticles, a_Y^+ , coupled to the allowed bosons of RPA, $A_{\alpha\beta}^+$ and $A_{\alpha\beta}$ where $\alpha, \beta \neq \gamma$. Hence, basis = $\{|a_0^+\rangle, |a_Y^+ A_{\alpha\beta}^+\rangle, |a_Y^+ A_{\alpha\beta}\rangle\}$. This has the advantage of taking the backwards part of the RPA phonon into account, while consideration is limited to 2+ phonons. No effort is made to exclude small admixtures of the (number nonconservation) spurious state-plus-bogolon which is a linear combination of $|\alpha_Y A_{\alpha-\gamma}^+\rangle$ -type configurations. In the lowest state--which does turn out to be approximately the same as

the 2^+ phonon-plus-bogolon state--this spurious admixture is negligible. Bes' method permits one to evaluate the importance of Pauli violations by simply including the $|\alpha \alpha \gamma\rangle$ -type configurations on an "experimental" basis. He finds that the energy spectra can be fit either with or without cancelling the violations in QPC by changing the coupling constant, although the amount it is changed must vary from nucleus to nucleus--some phonons being more collective than others in the rare earth region. Kuo's ETDA¹⁰¹ is carried out for spherical nuclei and a proper normalization procedure for states of the type $\{|j_1 j_1 j_1\rangle J\rangle$ is prescribed. All allowed singles and triples, $|a_\delta^+\rangle$, $|a_\alpha^+ a_\beta^+ a_\gamma^+\rangle$, $|a_{\gamma-\beta-\alpha}^+ a_{-\alpha}^+\rangle$, and their conjugates, are initially considered. Coupling matrix elements are derived from the equations of motion. In order to obtain a symmetric diagonalization problem (ETDA rather than ERPA), $|a_{\gamma-\beta-\alpha}^+ a_{-\alpha}^+\rangle$ is immediately excluded from the basis set. This has the effect of ignoring diagrams like the ERPA diagram in Fig. 82b, and we might suspect that wherever ground state correlations are important this is an error, whose magnitude is comparable to the relative (to RPA) error of TDA. The method includes diagrams like Fig. 82a (ETDA).

Recently, a quasispin formalism has been developed by Kuriyama¹⁰² to explain odd-mass nuclei. Arguing that diagrams like those of Fig. 83a are smaller than the one in Fig. 83b in the superfluid by a factor $(u^2 - v^2)/(2uv)$, coupling of singles to triples is neglected. Using the quasi-spin formalism, they are able to exclude the spurious (number fluctuation) states--although at the expense of also excluding pair vibrations. This yields a matrix problem

$$\begin{bmatrix} D_3 & -A & 0 & 0 \\ A & -D_1 & B_1 & B_3 \\ B_3 & B_1 & D_1 & -A \\ 0 & 0 & A & -D_3 \end{bmatrix} \begin{bmatrix} \psi_{3/2} \\ \psi_{-1/2} \\ \psi_{+1/2} \\ \psi_{-3/2} \end{bmatrix} = \omega \begin{bmatrix} \psi_{3/2} \\ \psi_{-1/2} \\ \psi_{+1/2} \\ \psi_{-3/2} \end{bmatrix} \begin{matrix} \langle \psi | : a_{\alpha}^+ a_{\beta}^+ a_{\gamma}^+ : | 0 \rangle \equiv \psi_{3/2} \\ \langle \psi | : a_{-\beta} a_{-\alpha} a_{\gamma}^+ : | 0 \rangle \equiv \psi_{-1/2} \end{matrix} \quad (A.18a)$$

In contrast to Kuo, ERPA terms (rather than ETDA) are kept and one obtains the Hermitian metric problem:

$$\begin{bmatrix} D_3 & -A \\ -A & D_1 \end{bmatrix} \begin{bmatrix} \psi_{3/2} \\ \psi_{-1/2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \psi_{3/2} \\ \psi_{-1/2} \end{bmatrix} \quad (A.18b)$$

The justification for ignoring B_1 and B_3 is that in the limit of the degenerate model, they include the same $(u^2 - v^2)$ factors as previously neglected. Away from closed shells, neglect of diagrams of the type in Fig. 83a is known to be substantial and we consider this to be a most important shortcoming of the theory.

It seems, on the other hand, that they might be taken into account by simply including the singles in an expanded matrix diagonalization which would still be metric Hermitian. The matrix elements are well known and the quasispin projection would not affect the singles.

It should be noted of the microscopic theories we have discussed, that in the expansion for the self-energy, "dressed" intermediates are not calculated. This is a most important shortcoming which is remedied only in the "self-consistent" theories.

2. Microscopic Descriptions of Even Nuclei

In even-even nuclei, the lowest order microscopic phonon is obtained in the Tamm-Dancoff approximation (Fig. 84a). If $|0\rangle$ is a vacuum of all shell-model (or Hartree-Fock) excitations, then in a normal system

$$|JM\rangle = \sum_{ph} X_{ph} A_{ph}^+ |0\rangle$$

$$A_{ph}^+ = \sum_{\substack{M_p M_h \\ p h}} \langle j_p m_p j_h m_h | JM \rangle C_{M_p}^+ C_{M_h}^+ \quad . \quad (A.19a)$$

In a superfluid,

$$|JM\rangle = \sum_{\alpha\beta} X_{\alpha\beta} A_{\alpha\beta}^+ |BCS\rangle$$

$$A_{\alpha\beta}^+ = \sum_{M_\alpha M_\beta} \langle j_\alpha m_\alpha j_\beta m_\beta | JM \rangle a_{M_\alpha}^+ a_{M_\beta}^+ \quad (A.19b)$$

At this level of approximation, there are no conflicts with the exclusion principle. There are also no ground state (GS) correlations, no dressing of quasiparticles, and no two-phonon admixtures. In the RPA (Fig. 84b), GS correlations are taken into account by admitting "backwards" amplitudes.

$$|JM\rangle = \sum_{ph} (X_{ph} A_{ph}^+ - Y_{ph} A_{ph}) |\psi_0\rangle$$

$$|\psi_0\rangle = |\text{phonon vacuum}\rangle = (1 + \xi A^+ A^+ + \xi A^+ A^+ A^+ A^+ + \dots) |0\rangle \quad (A.20)$$

Fig. 83. Diagram (a) is included in the microscopic theory of Kuriyama, et al. (b) and (c) are not.

Fig. 84(a). TDA diagram. Lines are π particles (pp) or particle and hole (ph).

$\times$ $\equiv$ effective two-nucleon interaction.

Fig. 84(b). RPA diagram.

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Fig. 85. Pauli principle violations of RPA.

Fig. 86. Exchange diagrams which cancel Fig. 85(a,b)....

...are "included" in irreducible interaction.

Fig. 87. Exchange diagram which corrects (raises) RPA phonon for EPV's with correlated ground state.

XBL 726-1042

Fig. 86. First order perturbations of quasiparticles in Neergard and Vogel's approximation to renormalized RPA. Lines are SM or SMB.

Not only are backwards diagrams included in the string of bubbles describing the phonon, but unlinked clusters are included in the GS expansion.

a. Pauli Principle considerations

Unfortunately, the price we pay for such elegance is the neglect of the Pauli Principle. And as GS correlations increase, so also must our concern for this neglect. Actually there are two forms of exclusion principle violations. First the coincident bubbles in a string should not contain the same quasiparticles. Figure 85a,b show the problem. Figure 85a is not so severe because we do not have to worry about $J \neq 0$ phonons having so many backwards pairs which constitute exclusion violations (EPV) of the frontwards part--this is for combinatorial reasons mostly. Violations of this sort are cancelled by the diagrams shown in Fig. 86.

Migdal's RPA equations (where the interactions are irreducible blocks) take care of this rather cleverly. In terms of irreducible blocks Fig. 86a,b are included in Fig. 86c,d.

Of course, the irreducible blocks are never calculated--only fit to experiment. And there is further the question of whether in such cases the irreducible contribution is well approximated as a short-range (δ force) interaction.

Another way in which RPA violates the Pauli exclusion Principle is that one cannot build (ph)-bubbles on top of GS correlations when the p or h is already there in the correlated GS. One should take this into account by calculating diagrams such as Fig. 87. Neglect of these diagrams is of crucial importance in transitional regions where RPA tends to become unstable and is certainly of quantitative importance wherever there are GS correlations.

The most general attempts to remedy this situation have been those of Rowe.¹⁰³ The renormalized quasiboson is defined

$$Q^+ = \sum_{ph} \frac{X_{ph} A_{ph}^+ - Y_{ph} A_{ph}}{\sqrt{\langle \psi_0 | [A_{ph}, A_{ph}^+] | \psi_0 \rangle}} \quad (\text{A.21a})$$

The renormalized RPA problem is obtained by replacing all single particle energies by the centroid of one nucleon stripping strength, ϵ_p^+ , while replacing single hole energies by the centroid of the pick-up strength, ϵ_h^- . All two-body matrix elements are multiplied by the ground state density matrices such as in the denominator of Eq. (A.21):

$$\langle ph' | V | hp' \rangle \rightarrow [\langle \psi_0 | [A_{ph}, A_{ph}^+] | \psi_0 \rangle \langle \psi_0 | [A_{p'h}, A_{p'h}^+] | \psi_0 \rangle]^{1/2} \langle ph' | V | hp' \rangle \quad (\text{A.21b})$$

The renormalized RPA^{103,104} calculates these densities from the RPA correlated ground state in a self-consistent manner. The so-called exact particle-hole method¹⁰⁵ determines from a preliminary diagonalization of 0p-0h and 2p-2h configurations.

The methods of Ikeda et al.¹⁰⁶ and Hara¹⁰⁷ neglect the renormalization of single particle energies and the ground state correlation factor is taken as $1 - \rho_\alpha - \rho_\beta$ ($1 - \rho_p - \rho_h$ in a normal system) where

$$\rho_\alpha = \langle \psi_0 | a_\alpha^+ a_\alpha | \psi_0 \rangle \approx \sum \frac{|\langle \alpha \beta N | H_{eff} | \psi_0 \rangle|^2}{E_\alpha + E_\beta + E_N} \quad (\text{A.22a})$$

a_α^+ is the creation operator for a bogolon for the general case of a superfluid. The equations are derived only for the pairing-plus-quadrupole force.

The renormalized RPA problem of Rowe is treated by Neergard and Vogel¹⁰⁸ using Green's functions and the pairing-plus-quadrupole schematic interaction. Self-energy diagrams of the form shown in Fig. 88 are incorporated in zeroth and first order in the single particle propagator. However, in order to make the numerical problem tractable in the usual analytic way, the frontwards diagram is ignored. The end result is that the polarization contribution is neglected and the correction is dependent only on the matrix element $\langle \alpha\beta N | H_{\text{eff}} | 0 \rangle$. The theory is an improvement over that of Refs. 106, 107 because the effect of blocking ground state correlations is included in the diagonal energies—where in the worst cases the effect is numerically more substantial. In particular, as in the above approximations, ρ_α is given by Eq. (A.22a),

$$E_\alpha^+ = E_\alpha + \sum_{\beta N} \frac{|\langle \alpha\beta N | H_{\text{eff}} | 0 \rangle|^2}{E_\alpha + E_\beta + E_N}$$

$$E_\alpha^- = \frac{1}{\rho_\alpha} \sum_{\beta N} \frac{|\langle \alpha\beta N | H_{\text{eff}} | 0 \rangle|^2 (E_\beta + E_N)}{(E_\alpha + E_\beta + E_N)^2} \quad (\text{A.22b})$$

The poles of the Bethe-Salpater equation are given by

$$1/2\chi = \sum_{\alpha\beta} |\langle \alpha | Q | \beta \rangle|^2 \left[\frac{(\eta_{\alpha\beta}^+)^2 (1 - \rho_\alpha - \rho_\beta) (E_\alpha^+ + E_\beta^+)}{(E_\alpha^+ + E_\beta^+)^2 - E^2} \right. \\ \left. - (\xi_{\alpha\beta}^+)^2 \left(\frac{\rho_\alpha (E_\alpha^- - E_\beta^+)}{(E_\alpha^- + E_\beta^+)^2 - E^2} + \frac{\rho_\beta (E_\beta^- + E_\alpha^+)}{(E_\alpha^+ + E_\beta^-)^2 - E^2} \right) \right] \quad (\text{A.23})$$

For a (ph)-vibration, the second factor is usually very small because not only are ρ_α and $E_\alpha^- + E_\beta^- / (E_\alpha^- + E_\beta^-)^2 - E^2$ small, but also $\xi_{\alpha\beta}^+$ will be small since $\eta_{\alpha\beta}^+$ must be large to obtain much (ph) GS correlation. Also, note that $\rho_\alpha = \mathcal{O}(5\%$ at most) whereas $(E_\alpha^+ + E_\beta^+ - E_\alpha - E_\beta) / (E_\alpha + E_\beta - E)$ is considerably larger, as much as 33% in the most extreme cases.

b. Two-Phonon Admixtures

In subsec. 2a theories have been discussed which improve the microscopic phonon description by including the backwards core polarization diagram. Some rather different attempts include the forwards core polarization. The second phonon itself has generally been taken as macroscopic, but recently has been computed microscopically.

i. Intermediate Particle-Phonon Coupling (PPC) Calculations

It is straightforward to diagonalize a two-particle residual interaction and a PPC term in an intermediate coupling basis $|j_1 j_2 N(R); J \rangle$. The effect of the residual interaction is to mix configurations $|j_1 j_2 N(R); J \rangle$ and $|j_3 j_4 N(R); J \rangle$ while the PPC term changes the number of surface phonons by ± 1 , thus including diagrams such as shown in Fig. 89. It is seen that Fig. 89a is the usual PPC self-energy contribution to the single particle propagator, while Fig. 89b represents the phonon exchange contribution to the effective interaction. There are a number of such calculations which might serve as examples: $^{42}\text{Ca}^\dagger$, ^{206}Pb , ^{210}Pb , $^{210}\text{Po}^\dagger$

[†]Ref. 109.

^{††}Ref. 110.

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Fig. 89. Two-particle-plus-phonon diagrams calculated in intermediate PPC calculations. (a) depicts a self-energy contribution to a single-particle propagator. (b) displays a phonon-exchange coupling of the two particles. Lines are SM.

ii. Microscopic

The most common microscopic two-phonon calculation is by the boson expansion method, whose discussion is deferred until the next section on self-consistent theories. The most simple-minded (though large-dimensioned) is simply to diagonalize the residual two-body interaction in a $2p-2h$, $3p-1h$, or $4p-0n$ configuration space, thus obtaining core excitation admixtures into pairing vibrations ($3p-1h$) or simply two-(ph)-phonon admixtures into the lowest vibrational mode ($2p-2h$). An example is the ^{180}O calculation of Lo Iudice.¹¹¹ Actually, the emphasis of that calculation is on the inclusion of diagrams like Fig. 89b rather than Fig. 89a. Both contributions are contained in such a diagonalization.

The calculation of Broglia, et al.¹¹² combines a microscopic hamiltonian with the simplicity of a small basis set of elementary excitations. Using the multipole interactions in the (pp) and (ph) channels, the one phonon states in ^{208}Pb , ^{206}Pb , ^{210}Pb are solved for and then coupled to two phonon ^{208}Pb states. The residual matrix elements between these configurations are formed from microscopic diagrams up to sixth order. We shall not draw these here, as they are similar in spirit to Fig. 78 and listed completely in Ref. 112.

3. Self-Consistent Descriptions of Even- and Odd-Mass Nuclei

In evaluating the various theories which correct the odd-mass nucleus for various orders of QPC and which self-consistently renormalize the even-mass collective state, we begin with those theories which come directly from Feynman diagram techniques as they provide the most convenient framework for analysis.

Fig. 90. Diagrams included in SCQPC diagonalization. $\uparrow$ are dressed intermediate p. operators. Lines are quasiparticles in Landau renormalized normal systems, or quasibogolons in a superfluid nucleus.

Fig. 91(a). Effective interaction between "forward" phonon-plus-quasiparticle couples.

Fig. 91(b). Effective interaction between "forward" and "backward" couples.

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a. Diagram theories

As indicated in an earlier section, we may evaluate the odd-mass nucleus in the following manner, once we have obtained an RPA or even-mass phonon and the usual QPC vertices:

1. Phonons and quasiparticles (or spectrons in the self-consistent version) are coupled to a quasifermion, $b_{\beta N}$. The truncated set of such elementary excitations and the usual quasiparticle fermion operators a_{α}^{+} is approximately linearly independent. We solve an integral equation for the poles and residues of the generalized GF

$$\mathcal{L} = \langle \tilde{O} | T \{ b_1(t_1) b_2^{+}(t_2) \} | \tilde{O} \rangle \quad (A.24)$$

where b_1 is either $b_{\beta N}$ or $b_{\alpha} = a_{\alpha}$. The resultant equations are similar in the normal system to the coupled equations of RPA for ^{210}Pb and ^{206}Pb . That is, we would solve for ^{209}Pb and ^{207}Pb at the same time. For superfluid systems, similar to even-even RPA, we obtain a non-physical set of solutions whose energies are the negative of the eigenenergies in the physical set. We thus include all diagrams containing forwards and backwards quasifermions. See Fig. 90.

2. In the work done so far, the "interaction" between phonon-quasiparticle type fermions is a sum of the second order graphs and it is taken to be energy independent. See Fig. 91a. The backwards vertex between such configurations is taken as zero, as in Fig. 91b.

3. Self-consistent perturbation theory of Dyson's equation is achieved by taking the phonon-quasiparticle-quasifermions to be $a_{\beta}^{+} Q_N$ instead of $a_{\beta}^{+} Q_N$, where a_{β}^{+} is the creation spectron for an eigenstate

(rather than a basis state) containing a significant part of the β -quasi-particle strength. There may of course, be more than one such state, u_{β}^i , and the equations are appropriately modified by multiplying off-diagonal matrix elements by residues which are less than unity. Thus we include graphs of the type shown in Fig. 71. The matrix diagonalization problem for the one-quasiparticle Green's function is shown in Fig. 49 (Sec. IV).

4. The even-even problem is still the Bethe-Salpater equation for the variation of the Bogolon density matrix--i.e., no two phonon states. Instead of the problem

$$(\omega - E_{\alpha\beta})X_{\alpha\beta} = P_{\alpha\beta\gamma\delta} X_{\gamma\delta} + Q_{\alpha\beta\gamma\delta} Y_{\gamma\delta}$$

$$(\omega + E_{\alpha\beta})Y_{\alpha\beta} = -P_{\alpha\beta\gamma\delta} Y_{\gamma\delta} - Q_{\alpha\beta\gamma\delta} X_{\gamma\delta}$$

we substitute for $(\omega \pm E_{\alpha\beta})$, $\mp 1/A_{\alpha\beta}(\mp\omega)$ where

$$A_{\alpha\beta}(\omega) = \sum_{ij} \frac{z_{\alpha}^{(i)} z_{\beta}^{(j)}}{\omega - E_{\alpha}^{(i)} - E_{\beta}^{(j)}} \quad (\text{A.25})$$

This matrix diagonalization is energy-dependent and the eigenvectors are not orthogonal. A method for solution of this problem was devised for normal systems by Zawisha and Werner⁷⁰ and the extension to superfluids is carried out in a previous chapter.

Earlier the self-consistent treatment of phonons and quasiparticles was carried out near closed shells by these authors.⁷⁰ We can obtain their equations by simply ignoring all but the zeroth quadrant in the QPC matrix

problem above and additionally setting all off-diagonal elements not in the first row or column equal to zero. The authors calculate $\langle \alpha | H_{\text{eff}} | \beta N \rangle$ using Γ^{ω} as in this work although no justification is presented. Particles and holes are treated separately, i.e., no backwards diagrams. Truncation, SCPT, and the even-even problem are all treated in the above fashion. We should note that these authors solve the odd-mass problem by solving a dispersion relation for the zeros of Dyson's equation and calculating the residues by differentiation. Such a procedure is so unwieldy as to prevent the higher order and backwards diagrams from being taken into account. The technique of analytically expressing the Green's function in terms of a matrix diagonalization is a necessary element of the extended theory.

Since the static quadrupole moments of 2+ states in spherical nuclei are currently receiving so much attention, we mention the very simple yet accurate prescription of Birbrair.⁵³ The graphical contributions to the 2+ moment are shown in Fig. 92. Clearly, the second "core-polarizing" graph is related to the self-energy dressing on quasiparticles which make up the phonon in the renormalized RPA. A particularly satisfactory feature of Birbrair's calculation is that, while the RPA phonon is taken to be harmonic, the energy-dependent effective field vertex is large for the 2+ moment ($\Delta E = 0$), but small for the 2+ $\rightarrow$ 0+ electromagnetic transition ($\Delta E = 2E_n$). This is the experimental situation also.

b. Spectral Decomposition Theories

Goswami¹¹³ and co-workers, as well as Klein¹¹⁴ with his collaborators, have authored on-going theoretical efforts to describe even- and odd-mass nuclear states self-consistently. These works should be

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Fig. 92. External field calculation of Birbrair for 2+ moments.

} $\equiv$ harmonic RPA phonon. The times of the three vertices may be ordered arbitrarily. Lines are IQB.

regarded as an alternative to diagrammatic analyses of self-energies so we will not attempt to formulate them from that point of view. The algebraic framework of both the bootstrap theory (Goswami) and the generalized Hartree-Fock approximation or GHFA (Klein) is the equations-of-motion method coupled to spectral decomposition.

In bootstrap, the commutator of quasiparticles (or bogolons, in the jargon of our work) and the pairing-plus-quadrupole hamiltonian is derived in the usual equations-of-motion manner and the matrix elements of triples between the vacuum or even-even correlated ground state are evaluated by interposing a complete set of even-even states. The power of the method is to truncate this set to those which are collective, namely the ground and first excited states.

$$\begin{aligned}
 (E_{\alpha} - E_J) \langle \psi_{\alpha} | a_{\mu}^{\dagger} | \psi_J \rangle &= \langle \psi_{\alpha} | [H, a_{\mu}^{\dagger}] | \psi_J \rangle = E_{\mu} \langle \psi_{\alpha} | a_{\mu}^{\dagger} | \psi_J \rangle \\
 &+ \frac{1}{4} \sum_{\xi \beta \gamma \delta} \langle \alpha \beta | V | \gamma \delta \rangle_A \langle \psi_{\alpha} | [N(C_{\xi}^{\dagger} C_{\beta}^{\dagger} C_{\delta} C_{\gamma}), a_{\mu}^{\dagger}] | \psi_J \rangle \quad (A.26)
 \end{aligned}$$

The latter normal ordered term ($C^{\dagger}$ creates particles, $a^{\dagger}$ bogolons) gives rise to quintuplets (which are ignored) and triplets. The triplet

$$\langle \psi_{\alpha} | a_{\beta}^{\dagger} a_{\delta} a_{\gamma} | \psi_J \rangle = \sum_{J'} \langle \psi_{\alpha} | a_{\beta}^{\dagger} | \psi_{J'} \rangle \langle \psi_{J'} | a_{\delta} a_{\gamma} | \psi_J \rangle \quad (A.27)$$

The amplitude $\langle \psi_J | a_{\delta} a_{\gamma} | \psi_J \rangle$ is obtained by bootstrapping, i.e., interposing a complete set of odd-mass nuclear states. One finally obtains a matrix diagonalization problem of the form shown in Fig. 93. In particular, the

$$\begin{bmatrix}
 E_{\alpha} & \langle \alpha | H | \beta N \rangle & 0 & \langle \alpha | H | \beta^{-1} N \rangle \\
 \langle \alpha | H | \beta N \rangle & E_{\beta} + E_N + H_{22}^{\text{for}} & \langle \alpha | H | \beta^{-1} N \rangle & H_{22}^{\text{back}} \\
 0 & \langle \alpha | H | \beta^{-1} N \rangle & -E_{\alpha} & -\langle \alpha | H | \beta N \rangle \\
 \langle \alpha | H | \beta^{-1} N \rangle & H_{22}^{\text{back}} & -\langle \alpha | H | \beta N \rangle & -E_{\beta} + E_N - H_{22}^{\text{for}}
 \end{bmatrix}$$

Fig. 93. The matrix diagonalization problem of Goswami. The expressions for the matrix elements are given in Ref. 113.

matrix couples the following four types of amplitudes: $a_{\alpha}^{+}|0\rangle$, $a_{\beta}^{+}Q_N^{+}|0\rangle$, $a_{-\alpha}|0\rangle$, $a_{-\beta}Q_N^{+}|0\rangle$, where a_{α}^{+} creates a BCS or bogolon quasiparticle and Q_N^{+} a phonon.

The theory of Klein and Dang, which has been presented in several installments in the literature, the most recent of which¹¹⁵ generalizes for a non-separable interaction the techniques developed earlier, is called GHFA. The central distinction between the GHFA and bootstrap is that the GHFA works in the particle representation and does the pairing correlations simultaneous to treatment of ground state phonon correlations. This is accomplished by defining the forwards and backwards amplitudes as generalized u's and v's and calculating a state dependent pairing correlation $\Delta(J, J')$, which serves to couple particles and holes. An almost identical approach has recently been worked out independently by Malov.^{116†} In this respect, this is an improvement over both bootstrap and SCQPC of Immele and Struble because $\Delta(2, 2) \neq \Delta(0, 0)$ and the effect is to weaken the total amount of GS correlations due to the quadrupole force.

Except for this substantial difference in the treatment of the pairing field, the two approaches have the common feature that matrix elements in the odd mass nuclear problem are proportional to the $2^{+} \rightarrow 0^{+}$ transition moment (Q_{20}) or the static 2^{+} moment (Q_{22}). These, in turn, must be determined from the even-even problem. Instead of solving the even-even problem except using RPA at the first iteration, Q_{20} and similar matrix elements are derived by interposing a complete set of states of the odd nucleus

[†]Malov's theory¹¹⁶ purports to derive from Green's functions. The actual equations are derived via the equations of motion technique of Klein. The theory should not be regarded as equivalent to the "graphical" Green's function approaches outlined earlier.

$$Q_{20} = \sum_{\omega\beta\gamma} Q_{\alpha\beta} \langle 2|a_{\alpha}^{\dagger}|\gamma\rangle \langle \gamma|a_{\beta}^{\dagger}|0\rangle \quad (\text{A.28})$$

The even-even energy, ω , is then determined from the relation:

$\omega Q_{20}^+ = \langle 2|[H, Q^+]|0\rangle$. The authors assert that Q_{22} may be obtained for the excited state in a similar manner—and one of the stated aims of these approaches is to calculate the $2+$ moment.

The most crucial input to the bootstrapping procedure is the so-called (Goswami) "backwards" amplitude. The physical (or graphical) interpretation of the "backwards" amplitudes is not clear. Nalcioglu¹¹⁷ has recently interpreted these amplitudes as arising because of the Pauli Principle violations shown in Fig. 75. It would be interesting if this connection could be made more definite, perhaps by deriving from the bootstrap diagonalization the known (from this thesis) correction to the phonon-plus-quasiparticle diagonal energy arising on account of the Pauli principle, namely

$$\Delta E(BN) = \sum_{\gamma} \frac{|\langle \beta\gamma|H|N\rangle|}{E_{\beta} + E_{\gamma} - E_N} \quad (\text{A.29})$$

Also it is clear that $\langle \psi_{\text{odd}}|a_{\alpha}^{\dagger}|\psi_0\rangle$ may have considerable overlap with $\langle \psi_{\text{odd}}|a_{-\beta}|\psi_2\rangle \approx \sum_{\alpha\beta} \langle \psi_{\text{odd}}|a_{\alpha}^{\dagger}|\psi_0\rangle$. Indeed it is this amplitude $\langle \psi_{\text{odd}}|a_{-\beta}|\psi_2\rangle$, which is needed to evaluate quadrupole moments by the spectral decomposition technique. However, its inclusion necessitates extensive use of orthogonalization and normalization difficulties.¹¹⁹

The definition of the backwards amplitudes is the central difference between the spectral decomposition and graphical SCQPC theories. In the latter theory the backwards amplitude $a_{-\beta Q-N}|\tilde{0}\rangle$ corrects for Pauli principle violations of the sort shown in Fig. 70, namely that the configuration $a_{\alpha}^{+}|\tilde{0}\rangle$ contains EPV's on account of ground state correlations. This difference manifests itself algebraically in the diagonal matrix elements in the lower right quadrant:

$$\text{bootstrap: } -E_{\beta} + E_N$$

$$\text{SCQPC: } -E_{\beta} - E_N$$

Curiously, the off-diagonal matrix elements connecting the frontwards and backwards amplitudes are the same.

In the bootstrap and GHFA, this situation has two undesirable consequences. One is that the eigenvalues will not occur in $(+E, -E)$ -type pairs. It is therefore difficult to separate the "physical" odd-mass eigenstates. Secondly, the diagonal element, $-E_{\beta} + E_N$, is much closer to E_{α} (it may even be positive), hence, the "pushing up effect" will be exaggerated. This explains the unrealistically large pushing effect in Ref. 118. (This may be reduced when proper orthogonalization procedures are developed.¹¹⁹)

It seems useful at this point to simply enumerate the differences between the spectral decomposition approaches and SCQPC.

i. Spectral theories do not properly account for backwards correlations of the type shown in Fig. 69. This is associated with the diagonal elements $-E_{\beta} + E_N$.

ii. Spectral theories do not treat the intermediate quasiparticles self-consistently, i.e., the a_{β}^{+} of $a_{\beta}^{+}Q_N^{+}$. A consequence of this is that in predicting the energy difference between, for example, the $g_{9/2}$ level and the $\{g_{9/2}, 2^{+}\}_{7/2}$ levels in Tc isotopes the $g_{9/2}$ which is coupled to the 2^{+} should be "dressed" by the backwards diagrams and hence higher in energy.

iii. The matrix elements connecting configurations of the type βN and $\beta' N'$ are different. This distinction can best be shown graphically as in Fig. 79b,d, where the bootstrap H_{eff} between phonon-quasiparticle configurations is shown. This should be compared to the SCQPC H_{eff} in Fig. 88a.

iv. In bootstrap, the even-even nucleus is calculated according to a spectral method, whereas in SCQPC it is recalculated by a renormalized RPA type matrix diagonalization. Though it is difficult to compare, these two approaches probably yield similar results--namely the lowering of the phonon energy, E_N , due to dispersion of the single particle strength.

v. In GHFA, pairing and phonon correlations are treated simultaneously. In SCQPC, one may iterate back and forth to find renormalized pairing fields. SCQPC and bootstrap do not allow for a reduction in the pairing field in the excited states, i.e., $\langle 2|\Delta|2 \rangle \neq \langle 0|\Delta|0 \rangle$.

c. Boson Expansions

Another alternative microscopic theory for both even- and odd-mass nuclei is formulated in terms of the expansion algebra of ideal bosons and fermions. The essential ingredient is a non-unitary transformation of the usual Slater determinant configuration space into an ideal boson-ideal

Fig. 94. Diagrams included in boson expansion or HRP formalisms.
Lines are S.I or S.MB.

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Fig. 95. Two-phonon admixtures which contribute to 2^+ quadrupole moment in boson expansion formalism.

fermion space. The ideal space offers several unique advantages. First, ideal bosons and fermions commute (or anticommute) exactly. One's basis space is relatively simple because the Pauli Principle is incorporated into the transcribed Hamiltonian. Secondly, one has a straightforward prescription for describing two-phonon states (i.e., the product of two ideal bosons plus a few corrections arising from the boson expansion algebra). The complimentary disadvantage of the boson expansion algebra is that the transformed Hamiltonian which operates in ideal space is exceedingly complicated. A number of articles have been written, in which the extensive mathematical details of this many-body technique are presented.¹²⁰⁻¹²⁶ We shall limit our consideration here to a comparison with other self-consistent theories of even- and odd-mass nuclei.

The description of even-nuclei incorporates several nice features.^{120,121,124} The most important are that by including two-phonon (HRPA) contributions to the usual one-phonon state, one calculates explicitly

- i. the contribution of core polarization to the quasiparticle self-energy (Fig. 94a),
- ii. core polarization and two-phonon contributions to the effective interaction (Fig. 94b,c),
- iii. static $2+$ moments, from two-boson admixtures into the lowest state (Fig. 95), and
- iv. exclusion principle violations.

Each one of these features, however, derives in a manner different from the previously discussed diagrammatic theories.

i. The use of dressed propagators in the Bethe-Salpater equation is formulated in terms of multiple poles in the Green's function. Poles with small residues are neglected. Nested (two- or three-) phonon contributions to the "dressing" (Fig. 71a) are included because of self-consistency of the Green's functions. "Dressing" in boson formulations is incorporated via two and three boson admixtures (i.e., expansion of the configuration space). Because of computational difficulties, it is almost impossible to include nested phonon contributions of higher order.

ii. Two-phonon and core polarization contributions to the effective interaction are implicit in the energy dependence of the irreducible block in the diagram-based theory. Although the prescription for obtaining the block from the bare N-N interaction is known in principle, it is usually approximated by a zero-range (in time and space) interaction. In the boson methods, two-phonon and core polarization processes are calculated explicitly. This is certainly an advantage over the SCQPC theory because such processes are quantitatively significant and the irreducible block is not point-like. On the other hand, to avoid double-counting, the "bare" interaction (e.g., G) should be employed as the Hamiltonian in boson expansion techniques because coupling terms such as the quadrupole-quadrupole force are already "renormalized" by higher order processes.

iii. The calculation of static and transition moments in a diagrammatic theory derives from an effective vertex formalism superimposed on a relatively simple description of the excited state. The boson expansion technique concentrates the sophisticated algebra in the description of

the excited state and computes moments from the expectation value (in ideal space) of the bare external field.

iv. Since ideal bosons commute, the difficulties discussed earlier with regard to the Pauli Principle are transcribed into the "ideal" hamiltonian. While some EPV's are calculated explicitly in the diagram theory (e.g., the backwards or ground state correlations diagram), others are ignored completely (e.g., Fig. 86). We should mention that the backwards diagram problem (Fig. 69b) is neatly avoided because the ground state is an ideal boson-ideal fermion vacuum, not just a quasiboson vacuum.

The description of odd-mass nuclei via ideal bosons and fermions has been pursued by Yamamura¹²² and by Simard and Banville.¹²⁷ The theory of one ideal boson and one ideal fermion produces numerical results which are identical to those of Kuo.¹⁰¹ As in Kuo, the EPV diagrams are treated properly. Some higher order diagrams (e.g., nested phonon self-energy dressings) are ignored. If the five quasiparticle transcription of the hamiltonian were carried out, some of these contributions would be included. To the extent that one diagonalizes bigger and more complicated "ideal" hamiltonians, one can always duplicate the results of self-consistent expressions.

The most unsatisfactory aspect of the boson transcription is that brute force prevails. In transforming the cumbersome Pauli Principle considerations from basis states to the hamiltonian, matrix operations (not to mention extensive algebraic transformation from real to ideal) of 3,4,5-quasiparticle basis spaces are still required. We do not wish to

minimize the importance of the Pauli Principle in finite systems, such as nuclei. However, it seems much more physical to describe the nucleus in terms of elementary excitations like phonons and quasiparticles and add corrections as they are appropriate.

c. Tables of Odd-A Eigenstates

In this section of the Appendix, we list the odd-proton and odd-neutron eigenstates for fourteen rare earth nuclei. The corresponding discussion is contained in Sec. V.B.2.

The tables are self-explanatory with two exceptions:

1. The third column lists for each eigenstate the unperturbed quasibogolon which may be fractionated into the eigenstate. The energy is an unperturbed one. The other components are phonon-plus-spectron, i.e., the third column giving an energy corresponding to an eigenstate sharing a percentage of the designated quasibogolon. The zero of energy was taken to be the experimental ground state. The spin and parity of the state are those of the unperturbed quasibogolon component.
2. The "comments" column often contains letter abbreviations AAA, BBB, CCC, DDD, EPV. AAA denotes that the odd-proton state is composed dominantly of the $3/2$ [411], $1/2$ [411], or $5/2$ [413] quasibogolons coupled to the even-even ground state or gamma vibration. BBB similarly for $1/2$ [400], $3/2$ [402], $5/2$ [402], $7/2$ [404] proton. CCC similarly for the $1/2$ [521], $3/2$ [521], $5/2$ [523] neutrons, and DDD similarly for the $1/2$ [510], $3/2$ [512], $5/2$ [512], $7/2$ [514] neutrons. EPV means that this predominantly phonon (usually gamma vibration

coupled to odd nucleon) state is pushed up to higher energies by Pauli exclusion principle considerations. These notations have been included because they denote what are frequently interesting experimental states.

Table 29.

EU153.
 PHONON ENERGIES (2+,0+,0-,2-).. 1.461, .698, 1.144, 1.566
 ODD PROTON EIGENSTATES.

ENERGY	PER CENT	QUASIBOGOLON OR SPECTRON (MEV)	PHONON	

-.029	87	2.5 [532] (.074)	--	
	5	2.5 [532] (-.029)	0+	
	3	.5 [420] (.747)	2-	
0.	95	2.5 [413] (.010)	--	AAA
	3	2.5 [413] (0.)	0+	
.015	24	.5 [550] (1.720)	--	INTRUDER
	60	.5 [550] (.015)	0+	
	7	.5 [550] (1.319)	0+	
	3	.5 [550] (2.607)	0+	
	2	.5 [550] (3.391)	0+	
.066	93	1.5 [411] (.121)	--	AAA
	1	.5 [411] (.844)	2+	
	2	3.5 [523] (.508)	2-	
.508	94	3.5 [523] (.387)	--	
	2	1.5 [411] (.066)	2-	
.595	88	4.5 [404] (.581)	--	BBB
	9	4.5 [404] (.595)	0+	
.720	2	2.5 [413] (.010)	--	
	97	2.5 [413] (0.)	0+	
.723	5	2.5 [532] (.074)	--	
	94	2.5 [532] (-.029)	0+	
.747	45	.5 [420] (1.367)	--	INTRUDER + 0-
	4	1.5 [422] (.955)	2+	
	34	.5 [550] (.015)	0-	
	11	2.5 [532] (-.029)	2-	
.794	0	1.5 [411] (.121)	--	
	99	1.5 [411] (.066)	0+	
.844	84	.5 [411] (.865)	--	
	8	1.5 [411] (.066)	2+	
	3	.5 [411] (.844)	0+	
.955	80	1.5 [422] (1.229)	--	
	9	.5 [420] (.747)	2+	
	2	.5 [420] (1.134)	2+	
	2	1.5 [422] (.955)	0+	
	3	1.5 [541] (1.020)	0-	
1.020	51	1.5 [541] (1.507)	--	INTRUDER
	13	.5 [550] (.015)	2+	
	13	1.5 [541] (1.020)	0+	
	4	1.5 [422] (.955)	0-	

	12	1.5 [411] (.066)	0-	
1.134	12	.5 [420] (1.367)	--	
	2	1.5 [422] (.955)	2+	
	30	.5 [420] (.747)	0+	
	2	.5 [420] (1.134)	0+	
	36	.5 [550] (.015)	0-	
	16	2.5 [532] (-.029)	2-	
1.183	0	2.5 [413] (.010)	--	
	99	2.5 [532] (-.029)	0-	
1.200	0	2.5 [532] (.074)	--	
	99	2.5 [413] (0.)	0-	
1.203	5	1.5 [541] (1.507)	--	
	5	.5 [550] (.015)	2+	
	6	3.5 [523] (.508)	2+	
	3	1.5 [541] (1.020)	0+	
	80	1.5 [411] (.066)	0-	
1.211	0	3.5 [523] (.387)	--	
	97	3.5 [523] (.508)	0+	
	3	1.5 [411] (.066)	2-	
1.251	3	.5 [411] (.865)	--	AAA
	59	2.5 [413] (0.)	2+	
	29	1.5 [411] (.066)	2+	
	7	.5 [411] (.844)	0+	
1.319	19	.5 [550] (1.720)	--	
	5	2.5 [532] (-.029)	2+	
	31	.5 [550] (.015)	0+	
	20	.5 [550] (1.319)	0+	
	13	.5 [420] (.747)	0-	
	3	2.5 [413] (0.)	2-	
1.328	9	4.5 [404] (.581)	--	
	90	4.5 [404] (.595)	0+	
1.408	1	4.5 [514] (1.760)	--	
	87	2.5 [532] (-.029)	2+	
	11	2.5 [413] (0.)	2-	
1.408	1	.5 [550] (1.720)	--	
	90	2.5 [532] (-.029)	2+	
	2	.5 [550] (.015)	0+	
	2	.5 [550] (1.319)	0+	
	4	.5 [411] (.844)	0-	
1.421	0	2.5 [532] (.074)	--	
	3	.5 [550] (.015)	2+	
	97	2.5 [532] (.723)	0+	
1.475	0	2.5 [532] (.074)	--	
	96	.5 [550] (.015)	2+	
	3	2.5 [532] (.723)	0+	
1.476	92	3.5 [404] (1.546)	--	
	6	3.5 [404] (1.476)	0+	
1.514	2	.5 [550] (1.720)	--	

	3	.5 [550] (.015)	0+	
	5	.5 [550] (1.319)	0+	
	6	.5 [420] (.747)	0-	
	82	2.5 [413] (0.)	2-	
1.517	6	.5 [420] (1.367)	--	
	2	1.5 [422] (.955)	2+	
	59	.5 [420] (.747)	0+	
	26	.5 [550] (.015)	0-	
	3	.5 [550] (1.319)	0-	
1.559	11	1.5 [541] (1.507)	--	
	81	.5 [550] (.015)	2+	
	4	1.5 [541] (1.020)	0+	
1.590	1	.5 [411] (.865)	--	
	3	2.5 [413] (0.)	2+	
	6	1.5 [411] (.066)	2+	
	89	.5 [411] (.844)	0+	
1.590	0	2.5 [413] (.010)	--	
	100	.5 [550] (.015)	2-	
1.595	0	1.5 [411] (.121)	--	
	99	.5 [550] (.015)	2-	
1.611	4	4.5 [514] (1.760)	--	
	9	2.5 [532] (-.029)	2+	
	86	2.5 [413] (0.)	2-	
1.655	1	1.5 [422] (1.229)	--	
	95	1.5 [422] (.955)	0+	
	3	1.5 [541] (1.020)	0-	
1.695	2	2.5 [523] (.387)	--	
	3	3.5 [523] (.508)	0+	
	94	1.5 [411] (.066)	2-	
1.714	6	3.5 [413] (2.080)	--	AAA EPV
	92	1.5 [411] (.066)	2+	
1.742	0	4.5 [404] (.581)	--	AAA EPV
	97	2.5 [413] (0.)	2+	
	3	2.5 [532] (-.029)	2-	
1.767	0	.5 [550] (1.720)	--	
	2	.5 [411] (.844)	0-	
	96	1.5 [411] (.066)	2-	
1.769	71	4.5 [514] (1.760)	--	
	4	2.5 [532] (-.029)	2+	
	5	4.5 [514] (1.769)	0+	
	16	4.5 [404] (.595)	0-	
	2	2.5 [413] (0.)	2-	
1.871	6	.5 [411] (.865)	--	AAA
	36	2.5 [413] (0.)	2+	
	55	1.5 [411] (.066)	2+	

Table 30.

TB157.

PHONON ENERGIES (2+, 0+, 0-, 2-). 1.155, .760, 1.323, 1.452
ODD PROTON EIGENSTATES.

ENERGY	PER CENT	QUASIBOSON OR SPECTRON (MEV)	PHONON	
0.	93	1.5 [411] (.029)	--	AAA
	1	3.5 [413] (.2729)	2+	
	1	.5 [411] (.484)	2+	
.148	94	2.5 [413] (.140)	--	AAA
	3	2.5 [413] (.148)	0+	
.200	97	3.5 [523] (.121)	--	
.255	93	2.5 [532] (.353)	--	
	2	2.5 [532] (.255)	0+	
	2	.5 [420] (1.202)	2-	
.484	84	.5 [411] (.480)	--	AAA
	4	2.5 [413] (.148)	2+	
	7	1.5 [411] (0.)	2+	
	2	.5 [411] (.484)	0+	
.623	26	.5 [550] (2.195)	--	INTRUDER
	60	.5 [550] (.623)	0+	
	7	.5 [550] (1.969)	0+	
	3	.5 [550] (3.161)	0+	
.762	1	1.5 [411] (.029)	--	
	99	1.5 [411] (0.)	0+	
.826	11	.5 [411] (.480)	--	AAA
	37	2.5 [413] (.148)	2+	
	51	1.5 [411] (0.)	2+	
.920	3	2.5 [413] (.140)	--	
	96	2.5 [413] (.148)	0+	
.965	0	3.5 [523] (.121)	--	
	98	3.5 [523] (.200)	0+	
	1	1.5 [411] (0.)	2-	
.992	23	.5 [541] (2.515)	--	INTRUDER
	9	.5 [550] (.623)	0+	
	7	.5 [541] (2.461)	0+	
	51	.5 [541] (.992)	0+	
1.025	1	1.5 [541] (1.980)	--	2+, 0- VIB. MIX
	60	3.5 [523] (.200)	2+	
	25	1.5 [411] (0.)	0-	
	13	.5 [411] (.484)	2-	
1.031	2	2.5 [532] (.353)	--	
	98	2.5 [532] (.255)	0+	
1.043	96	3.5 [404] (1.102)	--	BBB

	2	3.5 [404] (1.043)	0+	
1.049	93	4.5 [404] (1.009)	--	888
	3	4.5 [404] (1.049)	0+	
1.202	43	.5 [420] (1.837)	--	
	6	1.5 [422] (1.375)	2+	
	18	.5 [411] (.484)	0+	
	5	.5 [550] (.623)	0-	
	26	2.5 [532] (.255)	2-	
1.225	9	.5 [420] (1.837)	--	
	2	1.5 [422] (1.375)	2+	
	80	.5 [411] (.484)	0+	
	6	2.5 [532] (.255)	2-	
1.310	86	4.5 [514] (1.305)	--	
	7	2.5 [532] (.255)	2+	
	4	4.5 [514] (1.310)	0+	
1.351	0	5.5 [505] (3.843)	--	
	100	3.5 [523] (.200)	2+	
1.369	0	.5 [550] (2.195)	--	
	91	2.5 [532] (.255)	2+	
	1	.5 [550] (.623)	0+	
	7	.5 [411] (.484)	0-	
1.370	6	1.5 [541] (1.980)	--	
	17	3.5 [523] (.200)	2+	
	54	1.5 [411] (0.)	0-	
	10	.5 [411] (.484)	2-	
1.375	77	1.5 [422] (1.695)	--	
	13	.5 [420] (1.202)	2+	
	3	.5 [420] (1.225)	2+	
	1	.5 [420] (2.109)	2+	
	2	1.5 [422] (1.375)	0+	
1.421	7	4.5 [514] (1.305)	--	
	93	2.5 [532] (.255)	2+	
1.493	0	3.5 [523] (.121)	--	
	98	1.5 [411] (0.)	2-	
1.502	0	2.5 [532] (.353)	--	
	98	2.5 [413] (.148)	0-	
1.512	8	3.5 [413] (2.559)	--	AAA EPV
	88	1.5 [411] (0.)	2+	
1.501	0	.5 [420] (1.837)	--	
	99	.5 [411] (.826)	0+	
1.594	33	1.5 [541] (1.980)	--	
	47	.5 [550] (.623)	2+	
	3	1.5 [541] (1.594)	0+	
	7	1.5 [541] (1.940)	0+	
	2	1.5 [422] (1.375)	0-	
	6	1.5 [411] (0.)	0-	
1.601	0	.5 [550] (2.195)	--	

	99	2.5 [413] (.148)	2-	
1.603	0	4.5 [514] (1.305)	--	
	100	2.5 [413] (.148)	2-	
1.623	1	2.5 [413] (.140)	--	
	98	2.5 [532] (.255)	0-	
1.661	86	2.5 [402] (1.794)	--	888
	5	4.5 [404] (1.049)	2+	
	2	2.5 [402] (1.661)	0+	
	3	4.5 [514] (1.310)	2-	
1.667	2	3.5 [413] (2.559)	--	
	5	1.5 [411] (0.)	2+	
	92	3.5 [523] (.200)	0-	
1.701	0	.5 [411] (.480)	--	
	95	2.5 [532] (.255)	2-	
1.702	1	1.5 [411] (.029)	--	
	98	3.5 [523] (.200)	2-	
1.707	0	4.5 [404] (1.009)	--	
	3	2.5 [413] (.148)	2+	
	96	2.5 [532] (.255)	2-	
1.726	1	4.5 [404] (1.009)	--	AAA EPV
	95	2.5 [413] (.148)	2+	
1.745	0	.5 [411] (.480)	--	AAA EPV
	57	2.5 [413] (.148)	2+	
	40	1.5 [411] (0.)	2+	
1.757	1	.5 [541] (2.515)	--	
	8	2.5 [532] (.255)	2+	
	10	.5 [541] (.992)	0+	
	80	.5 [411] (.484)	0-	
1.781	0	2.5 [532] (.353)	--	
	98	.5 [550] (.623)	2+	
1.903	0	1.5 [411] (.029)	--	AAA EPV
	44	.5 [411] (.484)	2+	
	55	.5 [411] (.826)	2+	
1.911	0	2.5 [413] (.140)	--	AAA EPV
	38	.5 [411] (.484)	2+	
	61	.5 [411] (.826)	2+	
2.034	17	1.5 [402] (2.808)	--	888
	81	3.5 [404] (1.043)	2+	

Table 31.

TB161.

PHONON ENERGIES (2+,0+,0-,2-).. .919, 1.094, 1.342, 1.249
ODD PROTON EIGENSTATES.

ENERGY	PER CENT	QUASIBOGOLON OR SPECTRON (MEV)	PHONON	

0.	90	1.5 [411] (.016)	--	AAA
.186	97	3.5 [523] (.116)	--	
.286	94	2.5 [413] (.137)	--	AAA
.280	95	2.5 [532] (.364)	--	
.441	26	.5 [411] (.497)	--	AAA
	23	2.5 [413] (.186)	2+	
	48	1.5 [411] (0.)	2+	
.750	66	.5 [411] (.497)	--	AAA
	17	2.5 [413] (.186)	2+	
	12	1.5 [411] (0.)	2+	
.834	0	1.5 [541] (2.019)	--	
	75	3.5 [523] (.186)	2+	
	9	1.5 [411] (0.)	0-	
	8	.5 [411] (.750)	2-	
	8	.5 [411] (.441)	2-	
1.075	93	4.5 [404] (1.040)	--	BBB
	3	2.5 [413] (.186)	2+	
	2	2.5 [532] (.280)	2-	
1.080	97	3.5 [404] (1.134)	--	BBB
1.095	1	1.5 [411] (.016)	--	
	98	1.5 [411] (0.)	0+	
	1	3.5 [523] (.186)	2-	
1.099	0	5.5 [505] (3.879)	--	
	100	3.5 [523] (.186)	2+	
1.140	45	.5 [420] (1.876)	--	
	14	1.5 [422] (1.279)	2+	
	37	2.5 [532] (.280)	2-	
1.147	4	.5 [550] (2.235)	--	
	93	2.5 [532] (.280)	2+	
1.184	5	4.5 [514] (1.340)	--	
	95	2.5 [532] (.280)	2+	
1.218	0	3.5 [523] (.116)	--	
	47	3.5 [523] (.186)	0+	
	53	1.5 [411] (0.)	2-	
1.258	7	3.5 [413] (2.599)	--	AAA EPV
	91	1.5 [411] (0.)	2+	

1.279	65	1.5 [422] (1.733)	--	
	25	.5 [420] (1.140)	2+	
	2	.5 [420] (2.582)	2+	
	3	.5 [420] (1.727)	2+	
1.282	1	2.5 [413] (.137)	-	
	97	2.5 [413] (.186)	0+	
1.349	0	3.5 [523] (.116)	--	
	53	3.5 [523] (.186)	0+	
	46	1.5 [411] (0.)	2-	
1.364	92	4.5 [514] (1.340)	--	
	5	2.5 [532] (.280)	2+	
1.369	4	1.5 [541] (2.019)	--	
	7	3.5 [523] (.186)	2+	
	85	1.5 [411] (0.)	0-	
	2	.5 [411] (.441)	2-	
1.418	0	2.5 [532] (.364)	--	
	99	2.5 [532] (.280)	0+	
1.427	0	4.5 [514] (1.340)	--	
	100	2.5 [413] (.186)	2-	
1.430	0	1.5 [411] (.016)	--	AAA EPV
	15	.5 [411] (.750)	2+	
	84	.5 [411] (.441)	2+	
1.433	0	2.5 [413] (.137)	--	AAA EPV
	16	.5 [411] (.750)	2+	
	84	.5 [411] (.441)	2+	
1.436	0	.5 [550] (2.235)	--	
	100	2.5 [413] (.186)	2-	
1.498	1	4.5 [404] (1.040)	--	
	3	2.5 [413] (.186)	2+	
	96	2.5 [532] (.280)	2-	
1.547	0	2.5 [532] (.364)	--	
	96	2.5 [413] (.186)	0-	
	2	.5 [411] (.441)	2-	
1.562	3	4.5 [404] (1.040)	--	AAA EPV
	94	2.5 [413] (.186)	2+	
	2	2.5 [532] (.280)	2-	
1.623	1	.5 [411] (.497)	--	AAA
	57	2.5 [413] (.186)	2+	
	37	1.5 [411] (0.)	2+	

Table 32.

HO165.

PHONON ENERGIES (2+,0+,0-,2-).. .774, 1.187, 1.317, 1.251
ODD PROTON EIGENSTATES.

ENERGY	PER CENT	QUASIBOSON OR SPECTRON (MEV)	PHONON	
0.	95	3.5 [523] (-.015)	--	
	1	1.5 [411] (.166)	2-	
.166	90	1.5 [411] (.136)	--	AAA
	1	3.5 [413] (3.375)	2+	
	1	3.5 [413] (1.256)	2+	
.318	81	.5 [411] (.130)	--	AAA
	7	2.5 [413] (.523)	2+	
	4	1.5 [411] (.166)	2+	
.461	9	.5 [411] (.130)	--	AAA
	25	2.5 [413] (.523)	2+	
	64	1.5 [411] (.166)	2+	
.479	0	1.5 [541] (2.481)	--	
	78	3.5 [523] (0.)	2+	
	6	1.5 [411] (.166)	0-	
	14	.5 [411] (.318)	2-	
.523	94	2.5 [413] (.435)	--	AAA
	2	2.5 [532] (.676)	0-	
.601	95	3.5 [404] (.668)	--	BBB
	1	1.5 [402] (2.238)	2+	
	1	1.5 [402] (1.169)	2+	
.676	94	2.5 [532] (.740)	--	
	2	2.5 [413] (.523)	0-	
.765	0	5.5 [505] (3.346)	--	
	100	3.5 [523] (0.)	2+	
1.016	95	4.5 [514] (.860)	--	
1.169	15	1.5 [402] (2.327)	--	BBB
	81	3.5 [404] (.601)	2+	
	2	.5 [400] (1.676)	2+	
	2	3.5 [523] (0.)	2-	
1.185	0	3.5 [523] (-.015)	--	
	100	3.5 [523] (0.)	0+	
1.201	84	2.5 [402] (1.333)	--	BBB
	2	.5 [411] (.318)	2+	
	2	.5 [411] (.461)	2+	
	2	.5 [400] (2.710)	2+	
	3	.5 [400] (1.676)	2+	
	4	4.5 [514] (1.016)	2-	
1.214	0	1.5 [411] (.136)	--	AAA

	18	.5 [411] (.318)	2+	
	81	.5 [411] (.461)	2+	
1.216	0	2.5 [413] (.435)	--	AAA EPV
	15	.5 [411] (.318)	2+	
	85	.5 [411] (.461)	2+	
1.256	11	3.5 [413] (3.064)	--	AAA
	40	1.5 [411] (.166)	2+	2+,0- VIB. MIX
	48	3.5 [523] (0.)	0-	
1.274	0	1.5 [411] (.136)	--	
	2	.5 [411] (.318)	2+	
	3	.5 [411] (.461)	2+	
	95	3.5 [523] (0.)	2-	
1.351	0	1.5 [411] (.136)	--	
	9	3.5 [404] (.601)	2+	
	91	1.5 [411] (.166)	0+	
1.357	6	1.5 [541] (2.481)	--	
	2	3.5 [523] (0.)	2+	
	77	1.5 [411] (.166)	0-	
	9	.5 [411] (.318)	2-	
	4	3.5 [404] (.601)	2-	
1.359	15	.5 [541] (2.039)	--	
	34	2.5 [532] (.676)	2+	
	47	.5 [411] (.318)	0-	
	2	.5 [411] (.461)	0-	
1.438	0	4.5 [514] (.860)	--	
	99	2.5 [532] (.676)	2+	
1.451	1	3.5 [413] (3.064)	--	AAA
	52	1.5 [411] (.166)	2+	2+,0- VIB. MIX
	47	3.5 [523] (0.)	0-	
1.481	2	3.5 [523] (-.015)	--	
	97	1.5 [411] (.166)	2-	
1.495	12	.5 [541] (2.039)	--	
	65	2.5 [532] (.676)	2+	
	22	.5 [411] (.318)	0-	
1.504	0	.5 [411] (.130)	--	
	100	.5 [411] (.318)	0+	
1.541	34	.5 [420] (2.337)	--	
	17	1.5 [422] (1.607)	2+	
	12	2.5 [413] (.523)	2+	
	3	1.5 [411] (.166)	2+	
	6	.5 [411] (.318)	0+	
	23	2.5 [532] (.676)	2-	
1.558	0	2.5 [532] (.740)	--	
	4	4.5 [514] (1.016)	2+	
	95	.5 [411] (.318)	2-	
1.592	62	4.5 [404] (1.482)	--	BBB
	27	2.5 [413] (.523)	2+	
	8	2.5 [402] (1.201)	2+	

1.607	57	1.5 [422] (2.193)	--	
	1	3.5 [413] (1.256)	2+	
	25	.5 [420] (1.541)	2+	
	3	.5 [420] (2.843)	2+	
	3	.5 [420] (2.058)	2+	
	6	.5 [420] (1.607)	2+	
	2	.5 [411] (.318)	2+	
1.642	0	1.5 [411] (.135)	--	AAA EPV
	8	3.5 [413] (1.256)	2+	
	71	.5 [411] (.318)	2+	
	14	.5 [411] (.461)	2+	
	2	3.5 [523] (0. .1)	2-	
1.646	0	.5 [411] (.130)	--	
	100	.5 [411] (.461)	0+	
1.676	19	.5 [400] (2.834)	--	BBB
	49	2.5 [402] (1.201)	2+	
	3	1.5 [402] (2.238)	2+	
	2	1.5 [402] (2.596)	2+	
	25	1.5 [402] (1.169)	2+	
1.799	0	2.5 [413] (.435)	--	AAA EPV
	79	.5 [411] (.318)	2+	
	14	.5 [411] (.461)	2+	
	4	2.5 [413] (.523)	0+	
1.875	15	4.5 [404] (1.482)	--	AAA, BBB
	70	2.5 [413] (.523)	2+	
	11	2.5 [402] (1.201)	2+	
	3	2.5 [532] (.676)	2-	
1.842	1	.5 [411] (.130)	--	AAA EPV
	63	2.5 [413] (.523)	2+	
	26	1.5 [411] (.166)	2+	
	2	2.5 [532] (.676)	2-	

Table 33.

TM167.

PHONON ENERGIES (2+,0+,0-,2-).. .920, 1.235, 1.766, 1.782
ODD PROTON EIGENSTATES.

ENERGY	PER CENT	QUASIBUGOLON OR SPECTRON (MEV)	PHONON	

0.	93	.5 [411] (-.048)	--	AAA
	2	2.5 [413] (.830)	2+	
	2	1.5 [411] (.489)	2+	
.131	98	3.5 [523] (.152)	--	
	2	1.5 [411] (.489)	2-	
.163	97	3.5 [404] (.215)	--	BBB
	1	1.5 [402] (1.758)	2+	
.395	98	4.5 [514] (.370)	--	
.489	93	1.5 [411] (.462)	--	AAA
	2	.5 [411] (0.)	2+	
	2	3.5 [523] (.131)	2-	
.704	93	2.5 [402] (.793)	--	BBB
	1	.5 [400] (2.269)	2+	
	2	.5 [400] (1.389)	2+	
	1	4.5 [514] (.395)	2-	
.830	90	2.5 [413] (.833)	--	AAA
	8	.5 [411] (0.)	2+	
.888	0	1.5 [541] (2.938)	--	
	85	3.5 [523] (.131)	2+	
	15	.5 [411] (0.)	2-	
.916	17	1.5 [402] (1.746)	--	BBB
	80	3.5 [404] (.163)	2+	
	1	.5 [400] (1.389)	2+	
1.049	0	5.5 [505] (2.748)	--	
	100	3.5 [523] (.131)	2+	
1.115	97	2.5 [532] (1.165)	--	
	1	.5 [420] (2.244)	2-	
1.146	0	.5 [411] (-.048)	--	AAA
	28	2.5 [413] (.830)	2+	
	55	1.5 [411] (.489)	2+	
	16	.5 [411] (0.)	0+	
1.254	0	.5 [411] (-.048)	--	
	6	2.5 [413] (.830)	2+	
	10	1.5 [411] (.489)	2+	
	84	.5 [411] (0.)	0+	
1.308	2	1.5 [411] (.462)	--	AAA EPV
	96	.5 [411] (0.)	2+	

1.308	0	2.5 [532] (1.165)	--	
	98	4.5 [514] (.395)	2+	
1.368	0	3.5 [523] (.152)	--	
	100	3.5 [523] (.131)	0+	
1.371	84	.5 [541] (1.466)	--	
	5	.5 [541] (1.371)	0+	
	10	.5 [411] (0.)	0-	
1.389	25	.5 [400] (2.244)	--	BBB
	63	2.5 [402] (.704)	2+	
	4	1.5 [402] (1.758)	2+	
	8	1.5 [402] (.916)	2+	
1.406	0	3.5 [404] (.215)	--	
	100	3.5 [404] (.163)	0+	
1.449	7	2.5 [413] (.833)	--	AAA EPV
	91	.5 [411] (0.)	2+	
1.522	7	4.5 [404] (1.931)	--	BBB
	91	2.5 [402] (.704)	2+	
1.634	1	4.5 [514] (.370)	--	
	99	4.5 [514] (.395)	0+	
1.723	0	1.5 [402] (1.746)	--	
	98	1.5 [411] (.489)	0+	
	2	3.5 [523] (.131)	2-	
1.758	68	1.5 [402] (1.746)	--	
	19	3.5 [404] (.163)	2+	
	2	.5 [400] (2.269)	2+	
	9	.5 [400] (1.389)	2+	
1.831	1	.5 [400] (2.244)	--	BBB
	14	2.5 [402] (.704)	2+	
	85	1.5 [402] (.916)	2+	
1.836	1	3.5 [404] (.215)	--	BBB
	99	1.5 [402] (.916)	2+	
1.866	0	3.5 [404] (.215)	--	AAA EPV
	99	1.5 [411] (.489)	2+	
	1	3.5 [523] (.131)	0-	
1.971	55	4.5 [404] (1.931)	--	AAA, BBB
	37	2.5 [413] (.830)	2+	
	7	2.5 [402] (.704)	2+	

Table 34.

Tm¹⁵⁹.
 PHONON ENERGIES (2+,0+,0-,2-).. 1.009, 1.235, 2.048, 1.960
 ODD PROTON EIGENSTATES.

ENERGY	PER CENT	QUASIBUGOLON OR SPECTRON (MEV)	PHONON	
<hr/>				
0.	93	.5 [411] (-.023)	--	AAA
	2	2.5 [413] (.831)	2+	
	1	1.5 [411] (.481)	2+	
.155	98	3.5 [523] (.168)	--	
.190	98	3.5 [404] (.229)	--	BBB
	1	1.5 [402] (1.774)	2+	
.395	99	4.5 [514] (.379)	--	
.481	94	1.5 [411] (.469)	--	AAA
	2	1.5 [411] (.481)	0+	
	1	3.5 [523] (.155)	2-	
.719	95	2.5 [402] (.793)	--	BBB
	1	.5 [400] (2.325)	2+	
	2	.5 [400] (1.509)	2+	
.831	92	2.5 [413] (.832)	--	AAA
	5	.5 [411] (0.)	2+	
1.008	0	1.5 [541] (2.915)	--	
	86	3.5 [523] (.155)	2+	
	14	.5 [411] (0.)	2-	
1.037	20	1.5 [402] (1.733)	--	BBB
	78	3.5 [404] (.190)	2+	
1.130	97	2.5 [532] (.159)	--	
	1	2.5 [532] (1.130)	0+	
1.162	0	5.5 [505] (2.727)	--	
	100	3.5 [523] (.155)	2+	
1.194	0	.5 [411] (-.023)	--	AAA
	15	2.5 [413] (.831)	2+	
	31	1.5 [411] (.481)	2+	
	53	.5 [411] (0.)	0+	
1.298	1	.5 [411] (-.023)	--	AAA
	18	2.5 [413] (.831)	2+	
	35	1.5 [411] (.481)	2+	
	46	.5 [411] (0.)	0+	
1.376	1	1.5 [411] (.469)	--	AAA EPV
	97	.5 [411] (0.)	2+	
1.401	0	2.5 [532] (1.159)	--	
	99	4.5 [514] (.395)	2+	

1.436	95	.5 [541] (1.456)	--	
	1	.5 [541] (1.436)	0+	
	3	.5 [411] (0.)	0-	
1.450	0	3.5 [523] (.168)	--	
	99	3.5 [523] (.155)	0+	
1.464	0	3.5 [404] (.229)	--	
	100	3.5 [404] (.190)	0+	
1.504	5	2.5 [413] (.832)	--	AAA EPV
	94	.5 [411] (0.)	2+	
1.509	28	.5 [400] (2.226)	--	888
	63	2.5 [402] (.719)	2+	
	3	1.5 [402] (1.774)	2+	
	5	1.5 [402] (1.037)	2+	
1.625	16	4.5 [404] (1.916)	--	888
	83	2.5 [402] (.719)	2+	
1.649	0	4.5 [514] (.379)	--	
	100	4.5 [514] (.395)	0+	
1.698	2	1.5 [411] (.469)	--	
	91	1.5 [411] (.481)	0+	
	7	3.5 [523] (.155)	2-	
1.774	70	1.5 [402] (1.733)	--	888
	21	3.5 [404] (.190)	2+	
	2	.5 [400] (2.325)	2+	
	5	.5 [400] (1.509)	2+	
	2	1.5 [411] (.481)	0+	
1.935	0	3.5 [404] (.229)	--	AAA EPV
	100	1.5 [411] (.481)	2+	
1.956	0	2.5 [532] (1.159)	--	
	2	.5 [541] (1.436)	2+	
	97	.5 [411] (0.)	2-	
1.965	73	4.5 [404] (1.916)	--	888
	8	2.5 [413] (.831)	2+	
	17	2.5 [402] (.719)	2+	
	1	4.5 [404] (1.965)	0+	
1.977	0	2.5 [402] (.793)	--	
	99	2.5 [402] (.719)	0+	
2.012	5	.5 [400] (2.226)	--	888
	17	2.5 [402] (.719)	2+	
	77	1.5 [402] (1.037)	2+	

Table 35.

LUL13.

PHONON ENERGIES (2+,0+,0-,2-).. 1.485, 1.085, 2.128, 1.762
ODD PROTON EIGENSTATES.

ENERGY	PER CENT	QUASIBOGOLON OR SPECTRON (MEV)	PHONON	
0.	98 1	3.5 [404] (.033) 1.5 [402] (1.128)	-- 2+	BBB
.077	96 1 2	.5 [411] (.176) 2.5 [413] (1.242) 1.5 [411] (.837)	-- 2+ 2+	AAA
.098	99	4.5 [514] (.100)	--	
.341	97 1	2.5 [402] (.398) .5 [400] (1.453)	-- 2+	BBB
.556	98 1	3.5 [523] (.576) 1.5 [411] (.837)	-- 2-	
.837	90 6 3	1.5 [411] (.944) .5 [411] (.097) 3.5 [523] (.556)	-- 2+ 2-	AAA
1.000	99	.5 [541] (.995)	--	
1.092	0 100	3.5 [404] (.033) 3.5 [404] (0.)	-- 0+	
1.128	83 13 3	1.5 [402] (1.258) 3.5 [404] (0.) .5 [400] (1.453)	-- 2+ 2+	BBB
1.185	0 100	4.5 [514] (.100) 4.5 [514] (.098)	-- 0+	
1.189	1 99	.5 [411] (.176) .5 [411] (.097)	-- 0+	
1.242	84 15	2.5 [413] (1.339) .5 [411] (.097)	-- 2+	AAA
1.432	0 99	2.5 [402] (.398) 2.5 [402] (.341)	-- 0+	
1.453	65 30 4	.5 [400] (1.735) 2.5 [402] (.341) 1.5 [402] (1.128)	-- 2+ 2+	BBB
1.534	13 86	1.5 [402] (1.258) 3.5 [404] (0.)	-- 2+	BBB
1.567	6 91 2	2.5 [532] (1.680) 4.5 [514] (.098) .5 [411] (.097)	-- 2+ 2-	

1.633	82	2.5 [532] (1.680)	--	
	8	4.5 [514] (.098)	2+	
	1	2.5 [532] (1.633)	0+	
	8	.5 [411] (.097)	2-	
1.667	0	3.5 [523] (.576)	--	
	100	3.5 [523] (.556)	0+	
1.704	0	1.5 [402] (1.258)	--	
	97	.5 [411] (.097)	2+	
	3	1.5 [411] (.837)	0+	
1.762	0	1.5 [532] (3.221)	--	
	100	3.5 [404] (0.)	2-	
1.763	0	1.5 [541] (3.451)	--	
	100	3.5 [404] (0.)	2-	
1.765	0	5.5 [505] (2.224)	--	
	100	3.5 [404] (0.)	2-	
1.770	6	1.5 [411] (.944)	--	AAA EPV
	93	.5 [411] (.097)	2+	
1.790	0	2.5 [402] (.398)	--	
	100	.5 [411] (.097)	2+	
1.805	0	1.5 [532] (3.221)	--	
	18	3.5 [523] (.556)	2+	
	82	.5 [411] (.097)	2-	
1.845	2	4.5 [404] (2.450)	--	B88
	38	2.5 [402] (.341)	2+	
1.858	0	.5 [411] (.176)	--	
	100	2.5 [402] (.341)	2+	
1.873	0	2.5 [402] (.398)	--	
	99	4.5 [514] (.098)	2-	
1.877	0	2.5 [532] (1.680)	--	
	80	.5 [411] (.097)	2-	
1.892	15	2.5 [413] (1.339)	--	AAA EPV
	85	.5 [411] (.097)	2+	
1.924	0	1.5 [402] (1.258)	--	
	3	.5 [411] (.097)	2+	
	97	1.5 [411] (.837)	0+	
2.021	10	5.5 [505] (2.224)	--	
	90	3.5 [523] (.556)	2+	
2.022	25	.5 [400] (1.735)	--	B88
	69	2.5 [402] (.341)	2+	
	5	1.5 [402] (1.128)	2+	

Table 36.

G0155.

PHONON ENERGIES (2+,0+,0-,2-).. 1.303, .699, 1.221, 1.543
ODD NEUTRON EIGENSTATES.

ENERGY	PER CENT	QUASIBOGOLON OR SPECTRON (MEV)	PHONON	

0.	90	1.5 [651] (-.017)	--	
	1	.5 [660] (.140)	2+	
	5	1.5 [651] (0.)	0+	
.016	91	1.5 [521] (.038)	--	CCC
	2	.5 [521] (.948)	2+	
	2	3.5 [633] (1.100)	2-	
.124	94	1.5 [532] (.169)	--	
	4	.5 [530] (.586)	2+	
.140	92	.5 [660] (.038)	--	
	2	.5 [660] (.140)	0+	
.203	91	2.5 [642] (.168)	--	
	4	2.5 [642] (.203)	0+	
.237	94	2.5 [523] (.304)	--	CCC
	2	.5 [521] (.946)	2+	
	2	2.5 [642] (.203)	0-	
.459	98	5.5 [505] (.463)	--	
	2	5.5 [505] (.459)	0+	
.586	79	.3 [530] (.637)	--	
	10	1.5 [532] (.114)	2+	
	2	.5 [530] (.586)	0+	
	5	.5 [660] (.140)	0-	
.689	6	1.5 [651] (-.017)	--	
	9	.5 [660] (.140)	2+	
	79	1.5 [651] (0.)	0+	
	4	1.5 [521] (.016)	0-	
.754	1	1.5 [532] (.169)	--	
	83	1.5 [532] (.114)	0+	
	8	1.5 [521] (.016)	0+	
	3	1.5 [651] (0.)	0-	
	5	.5 [660] (.140)	2-	
.920	4	2.5 [642] (.168)	--	
	4	.5 [660] (.140)	2+	
	91	2.5 [642] (.203)	0+	
.948	38	.5 [521] (1.493)	--	CCC
	32	1.5 [521] (.016)	2+	
	15	2.5 [523] (.237)	2+	
	5	.5 [530] (.586)	0+	
	2	.5 [521] (.948)	0+	
	5	.5 [660] (.140)	0-	

.982	0	1.5 [521] (.038)	--	BETA VIB., EPV
	5	1.5 [532] (.114)	0+	
	91	1.5 [521] (.016)	0+	
1.051	10	.5 [521] (1.493)	--	
	3	1.5 [532] (.114)	2+	
	11	2.5 [523] (.237)	2+	
	43	.5 [530] (.586)	0+	
	30	.5 [660] (.140)	0-	
	2	1.5 [651] (0.)	2-	
1.073	0	2.5 [523] (.304)	--	
	98	2.5 [523] (.237)	0+	
1.100	73	3.5 [633] (1.217)	--	
	25	1.5 [521] (.016)	2-	
1.225	2	5.5 [505] (.463)	--	
	98	5.5 [505] (.459)	0+	
1.231	6	1.5 [402] (1.705)	--	
	15	.5 [660] (.140)	2+	
	57	1.5 [532] (.114)	0-	
	20	1.5 [521] (.016)	0-	
1.272	0	.5 [660] (.038)	--	
	89	1.5 [651] (0.)	2+	
	7	2.5 [642] (.203)	2+	
1.273	2	.5 [521] (1.493)	--	CCC
	2	1.5 [532] (.114)	2+	
	34	1.5 [521] (.016)	2+	
	29	2.5 [523] (.237)	2+	
	19	.5 [530] (.586)	0+	
	9	.5 [521] (.948)	0+	
1.354	6	2.5 [512] (1.917)	--	
	80	2.5 [642] (.203)	0-	
	13	.5 [660] (.140)	2-	
1.371	0	1.5 [651] (-.047)	--	
	4	.5 [660] (.140)	2+	
	1	1.5 [651] (0.)	0+	
	91	1.5 [651] (.689)	0+	
	3	1.5 [521] (.016)	0-	
1.383	0	3.5 [514] (2.789)	--	CCC
	99	1.5 [521] (.016)	2+	
1.399	0	3.5 [633] (1.217)	--	
	96	1.5 [651] (0.)	2+	
	3	1.5 [532] (.114)	2-	
1.420	0	.5 [660] (.038)	--	
	5	1.5 [651] (0.)	2+	
	61	2.5 [642] (.203)	2+	
	1	.5 [660] (.140)	0+	
	20	1.5 [532] (.114)	2-	
	3	1.5 [521] (.016)	2-	
1.457	0	3.5 [514] (2.789)	--	
	65	1.5 [532] (.114)	2+	

	1	1.5 [521] (.016)	2+
	34	1.5 [651] (0.)	2-
1.470	0	1.5 [532] (.169)	--
	1	.5 [530] (.586)	2+
	2	1.5 [532] (.114)	0+
	6	1.5 [521] (.016)	0+
	86	1.5 [651] (0.)	0-
1.510	40	1.5 [402] (1.705)	--
	3	.5 [660] (.140)	2+
	2	1.5 [651] (0.)	0+
	3	1.5 [651] (.689)	0+
	5	1.5 [532] (.114)	0-
	42	1.5 [521] (.016)	0-
1.518	7	.5 [530] (.637)	--
	67	1.5 [532] (.114)	2+
	4	1.5 [521] (.016)	2+
	2	.5 [521] (.948)	0+
	3	.5 [660] (.140)	0-
	10	1.5 [651] (0.)	2-
	6	2.5 [642] (.203)	2-
1.523	0	4.5 [624] (3.305)	--
	100	2.5 [642] (.203)	2+
1.524	1	2.5 [642] (.168)	--
	2	2.5 [642] (.203)	0+
	97	2.5 [523] (.237)	0-
1.567	0	4.5 [514] (2.062)	--
	100	2.5 [523] (.237)	2+
1.575	0	2.5 [642] (.168)	--
	95	.5 [660] (.140)	2+
	4	2.5 [642] (.203)	0+
	1	.5 [530] (.586)	2-
1.577	38	1.5 [402] (1.705)	--
	5	.5 [660] (.140)	2+
	5	1.5 [651] (0.)	0+
	4	1.5 [651] (.689)	0+
	17	1.5 [532] (.114)	0-
	25	1.5 [521] (.016)	0-
1.664	3	1.5 [402] (1.705)	--
	62	.5 [660] (.140)	2+
	9	1.5 [651] (0.)	0+
	5	1.5 [651] (.689)	0+
	20	1.5 [532] (.114)	0-
1.795	1	.5 [400] (2.134)	--
	5	2.5 [642] (.203)	2+
	56	.5 [660] (.140)	0+
	29	1.5 [532] (.114)	2-
1.890	5	.5 [400] (2.134)	--
	23	.5 [660] (.140)	0+
	23	.5 [530] (.586)	0-
	31	2.5 [523] (.237)	2-

CCC

BETA VIIb., EPV

BETA VIIb., EPV

Table 37.

GD157.

PHONON ENERGIES (2+,0+,0-,2-).. 1.238, .765, 1.224, 1.515
ODD NEUTRON EIGENSTATES.

ENERGY	PER CENT	QUASIBOGOLON OR SPECTRON (MEV)	PHONON	

-.030	88	2.5 [642] (-.008)	--	
	6	2.5 [642] (-.030)	0+	
	1	.5 [530] (.767)	2-	
0.	90	1.5 [521] (-.040)	--	CCC
	2	.5 [521] (.835)	2+	
	2	1.5 [521] (0.)	0+	
	2	3.5 [633] (.891)	2-	
.047	94	1.5 [651] (.016)	--	
	2	.5 [660] (.178)	2+	
.048	95	2.5 [523] (.070)	--	CCC
	2	.5 [521] (.835)	2+	
.178	89	.5 [660] (.191)	--	
	4	.5 [660] (.178)	0+	
.298	93	1.5 [532] (.289)	--	
	5	.5 [530] (.767)	2+	
.731	98	5.5 [505] (.742)	--	
	2	5.5 [505] (.731)	0+	
.736	7	2.5 [642] (-.008)	--	
	3	.5 [660] (.178)	2+	
	88	2.5 [642] (-.030)	0+	
	2	2.5 [523] (.048)	0-	
.767	72	.5 [530] (.934)	--	
	15	1.5 [532] (.298)	2+	
	8	.5 [660] (.178)	0-	
.835	55	.5 [521] (1.126)	--	CCC
	42	2.5 [523] (.048)	2+	
.870	1	1.5 [651] (.016)	--	
	11	.5 [660] (.178)	2+	
	77	1.5 [651] (.047)	0+	
	8	1.5 [521] (0.)	0-	
.891	92	3.5 [633] (.862)	--	
	6	1.5 [521] (0.)	2-	
.915	1	1.5 [521] (-.040)	--	
	14	1.5 [532] (.298)	0+	
	75	1.5 [521] (0.)	0+	
	8	1.5 [651] (.047)	0-	
.974	0	2.5 [523] (.070)	--	
	97	2.5 [523] (.048)	0+	

1.034	9	.5 [521] (1.126)	--	CCC
	50	1.5 [521] (0.)	2+	
	11	2.5 [523] (.048)	2+	
	11	.5 [521] (.835)	0+	
	6	.5 [660] (.178)	0-	
1.057	1	1.5 [532] (.389)	--	
	86	1.5 [532] (.298)	0+	
	10	1.5 [521] (0.)	0+	
	3	.5 [660] (.178)	2-	
1.167	0	.5 [660] (.191)	--	
	21	1.5 [651] (.047)	2+	
	75	2.5 [642] (-.030)	2+	
	3	.5 [521] (.835)	0-	
1.180	23	2.5 [512] (1.538)	--	
	72	2.5 [642] (-.030)	0-	
	3	.5 [660] (.178)	2-	
1.225	0	4.5 [624] (2.907)	--	
	100	2.5 [642] (-.030)	2+	
1.291	7	.5 [660] (.191)	--	
	72	1.5 [651] (.047)	2+	
	23	2.5 [642] (-.030)	2+	
1.331	0	3.5 [633] (.862)	--	
	95	1.5 [651] (.047)	2+	
	2	3.5 [633] (.891)	0+	
1.359	2	.5 [530] (.934)	--	CCC
	4	1.5 [521] (0.)	2+	
	25	2.5 [523] (.048)	2+	
	25	.5 [530] (.767)	0+	
	34	.5 [660] (.178)	0-	
	9	1.5 [651] (.047)	2-	
1.372	0	2.5 [642] (-.008)	--	
	3	.5 [660] (.178)	2+	
	3	2.5 [642] (-.030)	0+	
	4	2.5 [642] (.736)	0+	
	89	2.5 [523] (.048)	0-	
1.402	0	4.5 [514] (2.403)	--	CCC EPV
	99	2.5 [523] (.048)	2+	
1.410	2	1.5 [651] (.016)	--	
	22	.5 [660] (.178)	2+	
	29	1.5 [532] (.298)	0-	
	46	1.5 [521] (0.)	0-	
1.413	0	3.5 [514] (2.396)	--	CCC EPV
	100	1.5 [521] (0.)	2+	
1.452	5	3.5 [633] (.862)	--	
	4	1.5 [651] (.047)	2+	
	38	3.5 [633] (.891)	0+	
	52	1.5 [521] (0.)	2-	
1.466	65	2.5 [512] (1.538)	--	

	22	2.5 [642] (-.030)	0-
1.482	0	2.5 [642] (-.008)	--
	36	.5 [660] (.178)	2+
	55	2.5 [642] (.736)	0+
	8	2.5 [523] (.048)	0-
1.500	1	.5 [530] (.934)	--
	22	1.5 [651] (.047)	2-
	71	2.5 [642] (-.030)	2-
1.501	1	3.5 [514] (2.396)	--
	46	1.5 [532] (.298)	2+
	4	3.5 [633] (.891)	0-
	50	1.5 [651] (.047)	2-
1.520	2	5.5 [505] (.742)	--
	98	5.5 [505] (.731)	0+
1.522	1	1.5 [402] (2.042)	--
	16	.5 [660] (.178)	2+
	17	1.5 [651] (.047)	0+
	15	1.5 [532] (.298)	0-
	51	1.5 [521] (0.)	0-
1.528	0	.5 [660] (.191)	--
	5	.5 [530] (.767)	0-
	2	1.5 [521] (0.)	2-
	91	2.5 [523] (.048)	2-
1.542	0	2.5 [642] (-.008)	--
	57	.5 [660] (.178)	2+
	2	2.5 [642] (-.030)	0+
	41	2.5 [642] (.736)	0+
1.549	0	1.5 [521] (-.040)	--
	2	1.5 [532] (.298)	0+
	7	1.5 [521] (0.)	0+
	89	1.5 [651] (.047)	0-
1.559	0	3.5 [514] (2.396)	--
	54	1.5 [532] (.298)	2+
	4	3.5 [633] (.891)	0-
	42	1.5 [651] (.047)	2-
1.560	1	4.5 [624] (2.907)	--
	99	2.5 [523] (.048)	2-
1.581	1	.5 [530] (.934)	--
	38	1.5 [532] (.298)	2+
	2	1.5 [521] (0.)	2+
	11	.5 [660] (.178)	0-
	41	1.5 [651] (.047)	2-
	7	2.5 [642] (-.030)	2-
1.879	19	.5 [521] (1.126)	--
	3	1.5 [521] (0.)	2+
	29	2.5 [523] (.048)	2+
	15	.5 [521] (.835)	0+
	30	.5 [521] (1.034)	0+

CCC

Table 38.

DY161.

PHONON ENERGIES (2+,0+,0-,2-).. .917, .957, 1.369, 1.275
ODD NEUTRON EIGENSTATES.

ENERGY	PER CENT	QUASIBOGOLON OR SPECTRON (MEV)	PHONON	
0.	91	2.5 [642] (.056)	--	
	1	.5 [660] (.461)	2+	
	5	2.5 [642] (0.)	0+	
.055	93	2.5 [523] (.051)	--	CCC
	2	.5 [521] (1.062)	2+	
	1	2.5 [523] (.055)	0+	
.240	89	1.5 [521] (.146)	--	CCC
	3	1.5 [521] (.240)	0+	
.294	94	1.5 [651] (.310)	--	
	3	.5 [660] (.461)	2+	
.461	86	.5 [660] (.580)	--	
	6	1.5 [651] (.294)	2+	
	2	2.5 [642] (0.)	2+	
	3	.5 [660] (.461)	0+	
.635	18	.5 [521] (.923)	--	CCC
	16	1.5 [521] (.240)	2+	
	61	2.5 [523] (.055)	2+	
	1	.5 [521] (1.062)	0+	
	1	.5 [521] (.635)	0+	
.659	88	1.5 [532] (.826)	--	
	8	.5 [530] (1.017)	2+	
.742	97	3.5 [633] (.670)	--	
.908	1	.5 [660] (.580)	--	
	3	1.5 [651] (.294)	2+	
	95	2.5 [642] (0.)	2+	
.926	0	4.5 [624] (2.681)	--	
	100	2.5 [642] (0.)	2+	
.935	6	2.5 [642] (.056)	--	
	4	.5 [660] (.461)	2+	
	86	2.5 [642] (0.)	0+	
	4	2.5 [523] (.055)	0-	
1.017	55	.5 [530] (1.429)	--	
	30	1.5 [532] (.659)	2+	
	2	.5 [660] (.461)	0-	
	3	1.5 [651] (.294)	2-	
	9	2.5 [642] (0.)	2-	
1.062	57	.5 [521] (.923)	--	CCC
	34	1.5 [521] (.240)	2+	
	2	.5 [521] (1.062)	0+	

	2	.5 [521] (.635)	0+	
1.138	85	2.5 [512] (1.326)	--	DDD
	3	.5 [510] (.645)	2+	
	3	.5 [510] (1.865)	2+	
	3	2.5 [642] (0.)	0-	
1.176	2	2.5 [523] (.051)	--	
	97	2.5 [523] (.055)	0+	
1.188	0	4.5 [514] (2.937)	--	CCC EPV
	100	2.5 [523] (.055)	2+	
1.206	0	3.5 [633] (.670)	--	
	94	1.5 [651] (.294)	2+	
	2	3.5 [633] (.742)	0+	
	4	1.5 [521] (.240)	2-	
1.222	99	5.5 [505] (1.223)	--	
1.232	2	1.5 [521] (.146)	--	
	86	1.5 [521] (.240)	0+	
	10	1.5 [651] (.294)	0-	
1.233	5	.5 [660] (.580)	--	
	70	1.5 [651] (.294)	2+	
	2	2.5 [642] (0.)	2+	
	21	2.5 [523] (.055)	2-	
1.292	0	4.5 [514] (2.937)	--	
	100	2.5 [642] (0.)	2-	
1.301	2	.5 [530] (1.429)	--	
	6	1.5 [532] (.659)	2+	
	9	1.5 [521] (.240)	2+	
	4	2.5 [523] (.055)	2+	
	79	2.5 [642] (0.)	2-	
1.328	0	4.5 [624] (2.681)	--	
	99	2.5 [523] (.055)	2-	
1.337	2	1.5 [651] (.310)	--	
	60	.5 [660] (.461)	2+	
	31	1.5 [651] (.294)	0+	
	6	1.5 [521] (.240)	0-	
1.352	2	.5 [660] (.580)	--	
	18	1.5 [651] (.294)	2+	
	78	2.5 [523] (.055)	2-	
1.425	0	2.5 [642] (.056)	--	
	91	.5 [660] (.461)	2+	
	6	2.5 [642] (0.)	0+	
	2	2.5 [523] (.055)	0-	
1.430	0	3.5 [514] (2.174)	--	CCC EPV
	2	1.5 [532] (.659)	2+	
	97	1.5 [521] (.240)	2+	
1.483	0	3.5 [514] (2.174)	--	
	98	1.5 [532] (.659)	2+	

1.503	3	2.5 [512] (1.326)	--	
	95	2.5 [642] (0.)	0-	
1.545	5	.5 [521] (.923)	--	
	8	1.5 [532] (.659)	2+	
	5	1.5 [521] (.240)	2+	
	7	2.5 [523] (.055)	2+	
	73	1.5 [651] (.294)	2-	
1.553	0	3.5 [514] (2.174)	--	
	4	3.5 [633] (.742)	0-	
	94	1.5 [651] (.294)	2-	
1.602	1	.5 [521] (.923)	--	
	24	1.5 [532] (.659)	2+	
	12	1.5 [521] (.240)	2+	
	2	2.5 [523] (.055)	2+	
	47	.5 [521] (.635)	0+	
	11	1.5 [651] (.294)	2-	
1.610	0	1.5 [521] (.146)	--	
	5	.5 [521] (1.062)	2+	
	90	.5 [521] (.635)	2+	
	3	1.5 [532] (.659)	0+	
1.612	1	1.5 [532] (.826)	--	
	1	.5 [521] (.635)	2+	
	77	1.5 [532] (.659)	0+	
	3	1.5 [651] (.294)	0-	
	16	.5 [660] (.461)	2-	
1.614	0	2.5 [523] (.051)	--	CCC
	7	.5 [521] (1.062)	2+	
	91	.5 [521] (.635)	2+	
	1	2.5 [642] (0.)	0-	
1.812	4	.5 [510] (3.341)	--	DDD
	19	2.5 [512] (1.138)	2+	
	63	.5 [660] (.461)	0-	
1.865	12	.5 [510] (3.341)	--	DDD
	63	2.5 [512] (1.138)	2+	
	22	.5 [660] (.461)	0-	
1.967	76	3.5 [514] (2.174)	--	DDD
	3	1.5 [512] (4.001)	2+	
	4	1.5 [512] (2.469)	2+	
	1	1.5 [512] (2.481)	2+	
	12	3.5 [633] (.742)	0-	
	2	1.5 [651] (.294)	2-	

Table 39.

ERI 65.
 PHONON ENERGIES (2+,0+,0-,2-).. .821, 1.274, 1.532, 1.486
 ODD NEUTRON EIGENSTATES.

ENERGY	PER CENT	QUASIBOGOLON OR SPECTRON (MEV)	PHONON	
0.	92 3	2.5 [523] (.053) 2.5 [523] (0.)	-- 0+	CCC
.042	94 2	2.5 [642] (.065) 2.5 [642] (.042)	-- 0+	
.240	97	3.5 [633] (.189)	--	
.390	91 3	1.5 [521] (.273) 1.5 [521] (.390)	-- 0+	CCC
.477	94 4	1.5 [651] (.514) .5 [660] (.656)	-- 2+	
.517	38 17 41	.5 [521] (.420) 1.5 [521] (.390) 2.5 [523] (0.)	-- 2+ 2+	CCC
.608	91 3 3	2.5 [512] (.806) .5 [510] (3.066) .5 [510] (1.182)	-- 2+ 2+	DDD
.656	67 8 22	.5 [660] (.841) 1.5 [651] (.477) 2.5 [642] (.042)	-- 2+ 2+	
.769	55 12 28	.5 [521] (.420) 1.5 [521] (.390) 2.5 [523] (0.)	-- 2+ 2+	CCC
.879	0 100	4.5 [624] (2.146) 2.5 [642] (.042)	-- 2+	
.893	16 7 75	.5 [660] (.841) 1.5 [651] (.477) 2.5 [642] (.042)	-- 2+ 2+	
.942	87 9	1.5 [532] (1.115) .5 [530] (1.256)	-- 2+	
1.023	0 96	1.5 [651] (.514) 3.5 [633] (.240)	-- 2+	
1.170	0 39 61	4.5 [505] (3.831) 2.5 [523] (0.) 2.5 [512] (.608)	-- 2+ 2+	CCC, DDD
1.181	0 61 39	4.5 [505] (3.831) 2.5 [523] (0.) 2.5 [512] (.608)	-- 2+ 2+	CCC, DDD

1.182	13	.5 [510] (2.802)	--	000
	86	2.5 [512] (.608)	2+	
1.256	49	.5 [530] (1.751)	--	
	38	1.5 [532] (.942)	2+	
	3	1.5 [521] (.390)	2+	
	7	2.5 [642] (.042)	2-	
1.281	0	3.5 [633] (.189)	--	
	97	1.5 [651] (.477)	2+	
	1	3.5 [633] (.240)	0+	
1.298	3	2.5 [523] (-.053)	--	
	10	.5 [521] (.517)	2+	
	79	2.5 [523] (0.)	0+	
	7	2.5 [642] (.042)	0-	
1.353	10	.5 [660] (.841)	--	
	65	1.5 [651] (.477)	2+	
	23	2.5 [523] (0.)	2-	
1.399	3	2.5 [642] (.065)	--	
	27	.5 [660] (.656)	2+	
	55	2.5 [642] (.042)	0+	
	15	2.5 [523] (0.)	0-	
1.412	0	1.5 [521] (.273)	--	CCC
	16	.5 [521] (.769)	2+	
	76	.5 [521] (.517)	2+	
	5	1.5 [521] (.390)	0+	
1.421	84	3.5 [514] (1.642)	--	000
	3	1.5 [512] (3.331)	2+	
	1	1.5 [512] (3.743)	2+	
	6	1.5 [512] (1.868)	2+	
	4	3.5 [633] (.240)	0-	
1.428	0	2.5 [523] (-.053)	--	CCC
	26	.5 [521] (.769)	2+	
	67	.5 [521] (.517)	2+	
	6	2.5 [523] (0.)	0+	
1.491	1	4.5 [624] (2.146)	--	
	99	2.5 [523] (0.)	2-	
1.495	0	2.5 [642] (.065)	--	
	67	.5 [660] (.656)	2+	
	33	2.5 [642] (.042)	0+	
1.504	3	1.5 [651] (.514)	--	
	94	.5 [660] (.656)	2+	
1.513	0	.5 [521] (.420)	--	
	9	1.5 [521] (.390)	2+	
	3	2.5 [523] (0.)	2+	
	36	2.5 [642] (.042)	2-	
1.522	0	4.5 [505] (3.831)	--	
	100	2.5 [642] (.042)	2-	
1.523	0	3.5 [514] (1.642)	--	CCC EPV
	99	1.5 [521] (.390)	2+	

1.527	4	.5 [660] (.841)	--
	18	1.5 [651] (.477)	2+
	77	2.5 [523] (0.)	2-
1.539	2	1.5 [521] (.273)	--
	18	.5 [521] (.769)	2+
	54	1.5 [521] (.390)	0+
	4	1.5 [651] (.477)	0-
	20	3.5 [633] (.240)	2-
1.559	99	5.5 [505] (1.537)	--
1.632	0	.5 [521] (.420)	--
	56	1.5 [521] (.390)	2+
	20	2.5 [523] (0.)	2+
	2	.5 [521] (.517)	0+
	2	1.5 [651] (.477)	2-
	13	2.5 [642] (.042)	2-
1.640	0	3.5 [633] (.189)	--
	2	1.5 [651] (.477)	2+
	94	3.5 [633] (.240)	0+
	3	1.5 [521] (.390)	2-
1.642	0	3.5 [514] (1.642)	--
	98	1.5 [532] (.942)	2+
1.721	1	2.5 [523] (-.053)	--
	7	2.5 [523] (0.)	0+
	11	2.5 [512] (.608)	0+
	81	2.5 [642] (.042)	0-
1.829	0	.5 [521] (.420)	--
	7	1.5 [532] (.942)	2+
	1	2.5 [523] (0.)	2+
	3	.5 [521] (.769)	0+
	82	.5 [521] (.517)	0+
	6	1.5 [651] (.477)	2-
1.849	0	1.5 [521] (.273)	--
	21	1.5 [521] (.390)	0+
	4	1.5 [651] (.477)	0-
	73	3.5 [633] (.240)	2-
1.868	23	1.5 [512] (3.095)	--
	2	.5 [521] (.769)	2+
	72	3.5 [514] (1.421)	2+

CCC EPV

000

Table 40.

ER167.
 PHONON ENERGIES (2+,0+,0-,2-)... .782, 1.278, 1.750, 1.481
 ODD NEUTRON EIGENSTATES.

ENERGY	PER CENT	QUASIBOGOLON OR SPECTRON (MEV)	PHONON	
0.	94 4	3.5 [633] (.038) 3.5 [633] (0.)	-- 0+	
.268	87 3 7	2.5 [512] (.480) .5 [510] (2.610) .5 [510] (.697)	-- 2+ 2+	DDD
.331	89 3	.5 [521] (.164) .5 [521] (.331)	-- 0+	CCC
.656	95	2.5 [523] (.508)	--	CCC
.697	17 73 6	.5 [510] (2.420) 2.5 [512] (.268) 1.5 [512] (1.054)	-- 2+ 2+	DDD
.727	96	2.5 [642] (.697)	--	
.734	1 95 4	1.5 [651] (1.237) 3.5 [633] (0.) .5 [521] (.331)	-- 2+ 2-	
.839	0 100	4.5 [505] (3.443) 2.5 [512] (.268)	-- 2+	DDD
.982	78 17	3.5 [514] (1.275) 1.5 [512] (1.054)	-- 2+	DDD
1.054	20 35 2 4 38	1.5 [512] (2.711) 3.5 [514] (.982) 3.5 [514] (1.927) .5 [510] (2.610) .5 [510] (.697)	-- 2+ 2+ 2+ 2+	DDD
1.064	87 5 3	1.5 [521] (.962) .5 [521] (.331) 3.5 [633] (0.)	-- 2+ 2-	CCC
1.191	92 3	1.5 [651] (1.237) .5 [660] (1.353)	-- 2+	
1.237	0 30 62 6	.5 [521] (.164) 1.5 [521] (1.064) 2.5 [523] (.656) .5 [521] (.331)	-- 2+ 2+ 0+	CCC
1.309	4 96	3.5 [633] (.038) 3.5 [633] (0.)	-- 0+	
1.353	50 8	.5 [660] (1.589) 1.5 [651] (1.191)	-- 2+	

	42	2.5 [642] (.727)	2+	
1.451	48	4.5 [624] (1.770)	--	
	51	2.5 [512] (.268)	2-	
1.496	6	2.5 [512] (.480)	--	000
	93	.5 [510] (.697)	2+	
1.499	0	2.5 [523] (.508)	--	CCC EPV
	93	.5 [521] (.331)	2+	
	2	2.5 [523] (.656)	0+	
1.508	6	1.5 [521] (.962)	--	
	13	.5 [521] (.331)	2+	
	80	3.5 [633] (0.)	2-	
1.525	0	5.5 [505] (2.308)	--	
	99	3.5 [514] (.982)	2+	
1.539	0	4.5 [624] (1.770)	--	
	99	2.5 [642] (.727)	2+	
1.590	28	.5 [660] (1.589)	--	
	13	1.5 [651] (1.191)	2+	
	55	2.5 [642] (.727)	2+	
1.596	0	5.5 [505] (2.308)	--	
	100	3.5 [633] (0.)	2-	
1.642	1	1.5 [521] (.962)	--	CCC EPV
	77	.5 [521] (.331)	2+	
	3	1.5 [521] (1.064)	0+	
	15	3.5 [633] (0.)	2-	
1.650	4	1.5 [512] (2.711)	--	000
	33	3.5 [514] (.982)	2+	
	59	.5 [510] (.697)	2+	
1.690	83	1.5 [532] (1.875)	--	
	2	.5 [530] (2.977)	2+	
	8	.5 [530] (1.963)	2+	
	2	.5 [510] (.697)	2+	
1.691	2	.5 [521] (.164)	--	
	7	2.5 [523] (.656)	2+	
	90	.5 [521] (.331)	0+	
1.711	5	.5 [510] (2.420)	--	000
	14	2.5 [512] (.268)	2+	
	2	2.5 [512] (1.496)	2+	
	75	1.5 [512] (1.054)	2+	
	2	.5 [510] (.697)	0+	
1.766	0	3.5 [514] (1.275)	--	
	1	1.5 [512] (1.054)	2+	
	98	3.5 [633] (0.)	0-	
1.816	0	2.5 [512] (.480)	--	
	98	2.5 [512] (.268)	0+	
1.823	0	2.5 [642] (.697)	--	
	99	.5 [521] (.331)	2-	

1.845	0	4.5 {505} (3.443)	--	
	100	2.5 {523} (.656)	2+	
1.865	0	1.5 {651} (1.237)	--	
	4	3.5 {633} (0.)	2+	
	95	.5 {521} (.331)	2-	
1.927	16	3.5 {514} (1.275)	--	000
	82	1.5 {512} (1.054)	2+	
1.938	0	2.5 {642} (.697)	--	
	60	2.5 {642} (.727)	0+	
	39	2.5 {512} (.268)	0-	
1.952	0	3.5 {633} (.038)	--	
	99	1.5 {651} (1.191)	2+	
1.963	37	.5 {530} (2.526)	--	
	30	1.5 {532} (1.690)	2+	
	11	1.5 {521} (1.064)	2+	
	2	2.5 {523} (.656)	2+	
	10	.5 {510} (.697)	0+	
	8	2.5 {642} (.727)	2-	

Table 41.

ER169.

PHONON ENERGIES (2+,0+,0-,2-)... .955, 1.070, 2.051, 1.658
ODD NEUTRON EIGENSTATES.

ENERGY	PER CENT	QUASIBOGOLON OR SPECTRON (MEV)	PHONON	
0.	39	.5 [521] (.047)	--	CCC
	2	2.5 [523] (.812)	2+	
	4	.5 [521] (0.)	0+	
.023	91	2.5 [512] (.139)	--	DDD
	1	.5 [510] (2.233)	2+	
	2	.5 [510] (1.602)	2+	
	2	.5 [510] (.737)	2+	
	1	2.5 [512] (.023)	0+	
.090	97	3.5 [633] (.119)	--	
	1	3.5 [633] (.090)	0+	
.600	90	3.5 [514] (.765)	--	DDD
	2	1.5 [512] (2.307)	2+	
	7	1.5 [512] (.998)	2+	
.737	20	.5 [510] (1.852)	--	DDD
	74	2.5 [512] (.023)	2+	
	4	1.5 [512] (.998)	2+	
.812	64	2.5 [523] (.917)	--	CCC
	34	.5 [521] (0.)	2+	
.947	0	4.5 [505] (2.855)	--	
	100	2.5 [512] (.023)	2+	
.997	2	1.5 [651] (1.666)	--	
	93	3.5 [633] (.090)	2+	
	5	.5 [521] (0.)	2-	
.998	31	1.5 [512] (2.136)	--	DDD
	52	3.5 [514] (.600)	2+	
	2	.5 [510] (2.233)	2+	
	3	.5 [510] (1.602)	2+	
	10	.5 [510] (.737)	2+	
1.033	33	1.5 [521] (1.388)	--	CCC
	62	.5 [521] (0.)	2+	
	3	3.5 [633] (.090)	2-	
1.043	0	5.5 [615] (3.218)	--	
	100	3.5 [633] (.090)	2+	
1.097	3	.5 [521] (.047)	--	
	94	.5 [521] (0.)	0+	
1.113	80	4.5 [624] (1.227)	--	
	2	4.5 [624] (1.113)	0+	
	17	2.5 [512] (.023)	2-	

1.135	97	2.5 [642] (1.116)	--	
1.295	35	2.5 [523] (.917)	--	CCC
	62	.5 [521] (0.)	2+	
	2	2.5 [523] (.812)	0+	
1.296	1	3.5 [633] (.119)	--	
	99	3.5 [633] (.090)	0+	
1.378	1	2.5 [512] (.139)	--	
	98	2.5 [512] (.023)	0+	
1.426	0	5.5 [505] (2.737)	--	
	100	3.5 [514] (.600)	2+	
1.543	42	1.5 [521] (1.388)	--	CCC
	34	.5 [521] (0.)	2+	
	22	3.5 [633] (.090)	2-	
1.602	30	.5 [510] (1.852)	--	DDD
	17	2.5 [512] (.023)	2+	
	2	1.5 [512] (2.307)	2+	
	39	1.5 [512] (.998)	2+	
	10	.5 [510] (.737)	0+	
1.696	2	2.5 [512] (.139)	--	DDD
	97	.5 [510] (.737)	2+	
1.726	0	.5 [521] (.047)	--	CCC
	2	1.5 [521] (1.543)	2+	
	21	1.5 [521] (1.033)	2+	
	74	2.5 [523] (.812)	2+	
	2	.5 [521] (0.)	0+	
1.756	0	1.5 [512] (2.136)	--	DDD
	15	3.5 [514] (.600)	2+	
	84	.5 [510] (.737)	2+	
1.835	22	1.5 [521] (1.388)	--	
	2	.5 [521] (0.)	2+	
	5	1.5 [521] (1.033)	0+	
	70	3.5 [633] (.090)	2-	
2.233	37	.5 [510] (1.852)	--	
	2	2.5 [523] (1.295)	2+	
	4	2.5 [512] (.023)	2+	
	4	1.5 [512] (2.307)	2+	
	50	1.5 [512] (.998)	2+	
	1	1.5 [512] (2.723)	2+	

Table 42.

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PHONON ENERGIES (2+,0+,0-,2-).. 1.339, 1.163, 2.079, 1.766
ODD NEUTRON EIGENSTATES.

ENERGY	PER CENT	QUASIBOGOLON OR SPECTRON (MEV)	PHONON	

0.	92	2.5 [512] (.037)	--	DDD
	4	.5 [510] (1.191)	2+	
	3	2.5 [512] (0.)	0+	
	1	4.5 [624] (.763)	2-	
.122	95	.5 [521] (.217)	--	CCC
	1	2.5 [523] (1.120)	2+	
	1	.5 [521] (.122)	0+	
.282	96	3.5 [514] (.344)	--	DDD
	2	1.5 [512] (1.109)	2+	
	1	3.5 [514] (.282)	0+	
.403	98	3.5 [633] (.420)	--	
.763	96	4.5 [624] (.756)	--	
	3	4.5 [624] (.763)	0+	
1.109	51	1.5 [512] (1.631)	--	DDD
	43	3.5 [514] (.282)	2+	
	5	.5 [510] (1.191)	2+	
1.120	55	2.5 [523] (1.382)	--	CCC
	45	.5 [521] (.122)	2+	
1.191	74	.5 [510] (1.354)	--	DDD
	17	2.5 [512] (0.)	2+	
	5	1.5 [512] (1.109)	2+	
	2	1.5 [512] (2.000)	2+	
1.254	3	2.5 [512] (.037)	--	
	97	2.5 [512] (0.)	0+	
1.348	23	1.5 [521] (1.870)	--	CCC
	76	.5 [521] (.122)	2+	
1.397	15	.5 [510] (1.354)	--	DDD
	80	2.5 [512] (0.)	2+	
	3	1.5 [512] (1.109)	2+	
	1	.5 [510] (1.191)	0+	
1.469	1	.5 [521] (.217)	--	
	98	.5 [521] (.122)	0+	
1.515	0	4.5 [505] (2.338)	--	DDD EPV
	100	2.5 [512] (0.)	2+	
1.529	1	3.5 [514] (.344)	--	
	99	3.5 [514] (.282)	0+	
1.577	93	2.5 [642] (1.590)	--	

	1	2.5 [512] (0.)	0-	
	5	.5 [521] (.122)	2-	
1.600	8	1.5 [651] (2.153)	--	
	70	3.5 [633] (.403)	2+	
	21	.5 [521] (.122)	2-	
1.711	1	4.5 [624] (.756)	--	
	27	4.5 [624] (.763)	0+	
	71	2.5 [512] (0.)	2-	
1.718	44	2.5 [523] (1.382)	--	CCC
	53	.5 [521] (.122)	2+	
	2	2.5 [523] (1.120)	0+	
1.719	0	3.5 [633] (.420)	--	
	100	3.5 [633] (.403)	0+	
1.738	0	5.5 [615] (2.696)	--	
	100	3.5 [633] (.403)	2+	
1.845	95	.5 [651] (1.842)	--	
	1	.5 [521] (.122)	0-	
1.872	9	1.5 [651] (2.153)	--	
	29	3.5 [633] (.403)	2+	
	62	.5 [521] (.122)	2-	
1.899	4	2.5 [642] (1.590)	--	
	4	4.5 [624] (.763)	2+	
	1	2.5 [512] (0.)	0-	
	90	.5 [521] (.122)	2-	
1.949	94	3.5 [503] (2.054)	--	
	4	1.5 [501] (2.317)	2+	
	1	5.5 [615] (2.670)	2-	
1.967	59	1.5 [521] (1.870)	--	CCC
	22	.5 [521] (.122)	2+	
	19	3.5 [633] (.403)	2-	
1.990	38	.5 [640] (2.125)	--	
	1	.5 [521] (.122)	0-	
	57	2.5 [512] (0.)	2-	
2.000	39	1.5 [512] (1.631)	--	DDD
	53	3.5 [514] (.282)	2+	
	3	.5 [510] (1.191)	2+	
	3	1.5 [512] (1.109)	0+	

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