AEC
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BNWL-149
103-

**PHYSICS RESEARCH QUARTERLY REPORT
APRIL, MAY, JUNE, 1965**

**THE STAFFS OF
REACTOR PHYSICS
EXPERIMENTAL PHYSICS RESEARCH,
AND
CRITICAL MASS PHYSICS**

JULY 15, 1965

Info for 60000 703 JAN 1965
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UNITED STATES ATOMIC ENERGY COMMISSION UNDER CONTRACT AT(45-1)-1830

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BNWL-149
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PHYSICS RESEARCH QUARTERLY REPORT
APRIL, MAY, JUNE, 1965

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July 15, 1965

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Printed in USA. Price \$3.00. Available from the
Clearinghouse for Federal Scientific and Technical Information
National Bureau of Standards
U. S. Department of Commerce
Springfield, Virginia

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LIGHT MODERATOR REACTOR PHYSICS

Summary of Water Moderated Experiments with 1.8 wt% Pu-Al Fuel in the PRCF

J. H. Lauby

Introduction

Critical experiments have been conducted in the Plutonium Recycle Critical Facility (PRCF) with 1.82 wt% Pu-Al fuel. The critical assembly was moderated with light water. Results of these experiments will provide comparisons to measurements made with enriched uranium, mixed oxides of plutonium and uranium, and plutonium containing different isotopic concentrations. They will also furnish tests of analytical methods for calculating reactor parameters.

Results of critical approaches, worth measurements of control elements, reactivity coefficients for various reactor materials, spectrum studies, and reactor kinetics studies are summarized in this report. The results are from a preliminary analysis of work which has been done by U. P. Jenquin, A. L. Pitner, W. P. Stinson, V. O. Uotinen and myself. The details of the work and the final results are being prepared and will be issued subsequently. Further analysis may change the results and reduce the errors.

Previous subcritical experiments⁽¹⁾ using this fuel were performed in the Critical Approach Facility (CAF) for a variety of lattice spacings. The 0.80 in. fuel spacing was chosen for the critical experiments and is near the maximum of the buckling versus H/Pu curve. The H/Pu ratio for the 0.80 in. lattice of 1.8 wt% Pu-Al is 810 and the moderator-to-rod volume ratio is 1.203.

Differences between critical loadings of these and previous experiments with the same fuel lattice are not yet resolved. The total worth of the safety system was \$36 and an antishadowing effect existed between

elements. Teflon when displacing moderator near the center of the core more nearly simulates a void than Styrofoam. The fuel when measured relative to a void had a larger reactivity worth throughout the core than when measured relative to moderator. A comparison of fuel elements with various isotopic concentrations indicates that the reactivity effect of a variation of 1% in the Pu^{240} content is overshadowed by other variations in the fuel composition. The neutron temperatures on the fuel and in the moderator were 340 and 308 °K, respectively.

Experimental Facility

Three templates held the fuel rods vertically aligned in a 0.800 ± 0.001 center-to-center triangular array. The templates were made of 3/4 in. thick Lucite. They were spaced vertically at intervals of 12 1/8 in. above an aluminum support plate. The support plate is 3/4 in. thick with its center line 3/4 in. below the end of the Pu-Al alloy. The fuel was surrounded on all sides by an effectively infinite reflector.

The 1.82 wt% fuel rods were manufactured from fuel made for the Plutonium Recycle Test Reactor (PRTR) by cutting them in half to give two 43.8 in. fuel sections and adding a Zircaloy end cap to each half. The ID of the Zircaloy fuel can is 0.505 ± 0.001 in. with a 0.030 ± 0.003 in. wall thickness. The Pu-Al rod is 0.500 ± 0.001 in. in diameter. The length of the can allows a 0.400 in. gap at the top which is helium filled. End caps and associated tubing beyond the end of the fuel average 12.4 and 21.6 g per rod for the bottom and top caps, respectively. The average plutonium content is 7.12 g per rod. Approximately half the rods had 5.0% and the other half had 6.0% Pu^{240} .

Six control elements were supported above the top of the loading. Three were safety rods spaced 120° apart. Each safety rod comprised four rods spaced as the Pu-Al fuel with their geometrical center at 4.85 in. from the core center. Each of the four safety rod elements consisted of a 1.50 wt% $\text{PuO}_2\text{-UO}_2$ fuel rod 56.904 in. long as the lower part and a 49 in. long polycarbonate upper portion 0.292 in. in diameter covered with

0.030 in. of cadmium. The fuel in the $\text{PuO}_2\text{-UO}_2$ rod is 48 1/2 in. long. Three safety sheets were spaced 120° apart, alternately with the safety rods. The sheets were at the outer edge of the core at a radius of 9.70 in. They were made of cadmium sheets, 7 1/2 x 49 x 0.030 in., with a polycarbonate follower 50 7/8 in. long.

All control elements had a vertical travel of 48 in. In the withdrawn position the lower end of the fuel in the safety rod follower was 11/16 in. above the bottom of the core and the top of the fuel was 5 3/8 in. above the top of the core. In the withdrawn position the lower end of the cadmium section of the safety sheets was 6 1/2 in. above the top of the fuel.

Critical Mass

Two independent measurements were made of the critical loadings. The loading was divided into three annular zones according to the expected radial fuel element worth. Fuel rods with highest Pu^{240} content were loaded into the zone of minimum worth in the first critical approach to minimize the number of fuel rods at critical. The second loading had fuel elements with highest Pu^{240} content in positions of maximum worth to maximize the number of fuel rods at critical.

The first and second loadings resulted, respectively in 486.3 ± 0.1 and 486.7 ± 0.1 fuel rods at critical. There was a 2 in. difference in depth of the top reflector from the first to second loadings but this change was not worth a measurable amount of reactivity with either loading.

Subcritical experiments in the CAF predicted the critical number of rods to be 510.3 ± 0.9 . Notable differences between the critical and subcritical experiments are the presence of polycarbonate safety sheet followers and mixed oxide safety rod followers in the PRCF and Lucite liners 0.650 in. OD with 0.032 in. wall in the CAF experiments. The sheet followers are expected to cause an increase in the critical loading of ~ 4 rods as estimated from the measurement of polycarbonate worth in the center of the core. Replacement of mixed oxide fuel with Pu-Al fuel in positions equivalent to that of safety rods increases the critical number by 0.68 ± 0.07 rods.

From a measurement of Lucite worth, the Lucite liners in the CAF are expected to cause an increase in critical mass of $\lesssim 0.3$ rod/% Lucite in the moderator. Since the liners take up about 8% of the volume of the moderator, the critical mass is expected to be $\lesssim 3$ rods less because of the Lucite than was measured in the CAF. Moderator was allowed to fill the space between the liners and the fuel rods so that reactivity effects because of voids would be minimized. If for some reason voids were trapped in the 0.010 in. thick space between the liners and the fuel rods, the critical mass could be too large. A value for the reactivity worth of such a void between the fuel and a Lucite liner the the CAF was estimated by interpolating from the worth of a measured 0.030 in. thick void in the PRCF. The worth was measured as a function of radius for an isolated void and the results were integrated over the core to obtain an estimate of 25 rods.

All of these corrections resolve the differences between the results from the PRCF and CAF and the critical loading for the unperturbed core is expected to be 482 rods. Further work is required to completely resolve the differences since the space between the fuel rods and the liners was known not to be complete voids. If the number of rods for the critical loading is 482 then calculated multiplications using 510 rods would be high by $\sim 1\% \Delta k/k$.

Evaluation of Safety System Worth

Reactivity worths of the six elements of the safety system were obtained from measurements of flux transients after rapid insertion separately and in combination. The worths were evaluated by comparing transients calculated from kinetics equations assuming step reactivity changes. Each measurement was made with the reactor just critical and the delayed neutron fractions in equilibrium. Only Safety Rod 3 and Safety Sheet 1 were measured separately. Two measurements were made to evaluate the safety rod group and the safety sheet group worths. The largest measured reactivity change was for insertion of all six safety elements as a group. Previous measurements⁽²⁾ indicate the deviations of the individual worths from the average worths for Safety Rod 3 and Safety Sheet 1 to be +1.9% and -1.6%,

respectively. Assuming their individual worths to be equal, the antishadowing effect increases with the worth of elements inserted as shown in Table I.

A portion of Safety Rod 3 was accurately calibrated in 0.24 in. steps by positive period measurements. The insertion of 13.44 in. of rod was worth $54.8 \pm 0.5\%$. Three fuel elements were added symmetrically around the core edge to compensate as required for the loss in excess reactivity as the safety rod was inserted. Reactivity changes as the rod was inserted were positive for the first 2.4 in. and then became negative. The reactivity change for the first 2.4 in. was $< 0.7\%$. This result is reasonable since at a position 2 5/16 in. from the out position, the safety rod follower was centered vertically with the rest of the core.

TABLE I
MEASURED REACTIVITY WORTHS
OF SINGLE AND COMBINED CONTROL ELEMENTS

Control Element	Worth, %	Enhancement, %
Safety Sheet 1	2.13 ± 0.03	
3X (Safety Sheet 1)	6.39	
3 Safety Sheets (together)	7.18 ± 0.05	12
Safety Rod 3	7.39 ± 0.05	
3X (single rod worth)	22.17	
3 Safety Rods (together)	27.32 ± 0.07	23
3X (single sheet and rod worth)	28.56	
3 Safety Sheets and 3 Safety Rods (together)	36.34 ± 1.43	27

Reactivity Coefficients

Voids

The effect of annular voids around fuel elements in the core is shown in Figure 1 as Curve C. For the measurements a fuel rod was centered in a Lucite tube (3/4 in. OD x 5/8 in. ID) to produce a 0.030 in. thick annular void. The void tube extended from the bottom of the fuel rod to the top of the moderator. The reactivity worth of this void was $-3.4 \pm 0.3\%$ at the core center and approached zero at the core edge.

FIGURE 1

Fuel Element and Void Reactivity Worth as a Function of Radius
(The error of a typical measurement is $\leq 0.4\%$.)

The worths of a Lucite tube used to produce annular voids was measured with no fuel inside and the results are shown in Figure 1. The worth of the tube void was equal to the worth of the annular void at the core center. The worth of the tube void increased from $-3.5 \pm 0.3\%$ at the core center to $-10.5 \pm 0.3\%$ at a radius of 70% of the core radius and then decreased again at larger radii.

The reactivity effect of two other types of voids were measured. The void materials had equal geometry and displaced the moderator associated with seven fuel rods for a vertical length of 3 in. The first type of void was molded from Styrofoam with densities measured to be

equivalent to 86 and 36% void when displacing moderator. The second type was machined from Teflon. A comparison of the worths in the radial center of the core shows the Teflon to be equivalent to a 72% void when based on a linear interpolation for the two densities of Styrofoam. These worth measurements were made with the vertical center of the materials 4 7/8 in. above the fuel center. A radial traverse of the reactivity worth was made at this height with the Teflon. The worth in the center was $-6.1 \pm 0.3\%$ and was nearly constant out to 3 in. from the center. The worth became 0 at the core edge and $+2.2 \pm 0.3\%$ at 0.3 in. beyond the edge.

The reactivity effect of an empty section of Lucite tubing (8 in. x 0.588 in. ID) was measured as it was moved vertically through the core in an empty fuel channel. Small corrections for the effects of Lucite were obtained by repeating the measurements with the Lucite filled with water. The curve obtained for the void was approximately a cosine with a maximum value of $+0.9\%$ near the vertical center and -0.5% at 20 in. above and below the position of maximum worth.

Fuel Elements

The worth of 1.8 wt% Pu-Al fuel was measured as a function of radial position when displacing moderator and when added into a tube void. These two worths differed by about 1% at the core center. The worth of fuel, when added into a void at a radius of 86% of the core radius, was 2.0 times the worth when replacing moderator. At the core edge a fuel element was worth $16.5 \pm 0.1\%$ when replacing moderator. Figure 1 shows the results obtained as a function of radius.

Fuel element worths at the center of the core were measured to obtain reactivity differences due to variations in fuel compositions. The worths of fourteen 1.82 wt% and four 2.006 wt% Pu-Al rods were measured in the central fuel position of the core. The rods were divided into five groups of rods which are expected to have nearly the same worth. The average worths of each group, the average of the group averages, and the standard deviations of the averages are summarized in Table II.

TABLE II
REACTIVITY COMPARISON
OF 1.8 wt% AND 2.0 wt% Pu-Al FUEL ELEMENTS

Plutonium, wt%	Rod	Pu ²⁴⁰ , %	Pu-Al wt/rod, g	Average Worth	
				ϵ /rod	m ϵ /g of alloy
The following fuel elements are 44 in. long, 0.5 in. diam, and clad in 0.030 in. Zircaloy.					
1.8	C-33	5.1	390.5	40.4 $\pm$ 0.2	103.4 $\pm$ 0.5
	C-33-H				
	C-8				
	C-8-H				
1.8	D-26	5.6	389.6	38.6 $\pm$ 0.4	99.1 $\pm$ 1.0
	D-26-H				
	B-15				
	B-15-H				
1.8	F-80	5.95	389.4	39.5 $\pm$ 0.6	101.4 $\pm$ 1.5
	F-80-H				
	G-5				
The following fuel elements are 44 in. long, 0.5 in. diam, and clad in 0.035 in. Zircaloy.					
1.8	J-69	6.0	377.7	38.1 $\pm$ 0.6	100.9 $\pm$ 1.6
	T-59				
	I-18				
				Average	101.2 $\pm$ 1.3
The following fuel elements are 36 in. long, 0.5 in. diam, and clad in 0.030 in. Zircaloy.					
2.0	838	16.49	318.25	28.80 $\pm$ 0.05	90.49 $\pm$ 0.16
	851				
	852				
	864				

The 2.0 wt% Pu-Al rods are expected to be the more uniform since they were made especially for critical experiments and not refabricated from existing rods as were the 1.8 wt% Pu-Al rods. This fact is also concluded from the results in Table II. The standard deviation of the average of the worths of the group of 2.0 wt% Pu-Al rods is what would be expected from four measurements of the same rod. Therefore, within the experimental error there is no reactivity differences between the rods of this group. On the other hand, the size of the errors on the averages of the worths of the group of 1.8 wt% Pu-Al rods are 5 to 10 times larger than expected for the same rod. Therefore, the variations of reactivity for these rods are probably due to nonuniformity of the rods. If the difference between the average worth per gram of alloy of 1.8 wt% Pu-Al and that of 2.0 wt% Pu-Al is assumed to be due to the change in concentration which allows the increase of 10.5% Pu²⁴⁰, the change expected in going from 5.1 to 6.0% Pu²⁴⁰ in the 1.8 wt% alloy would be about equal to the nonuniformity observed for the rods. Therefore,

differences in the critical mass are not expected to be observed because of selected positioning of the fuel elements in the core according to their composition. This conclusion is corroborated by the results of the two loadings which were designed to maximize or minimize the number of fuel rods for a critical loading.

Reactor Materials

Rods containing three concentrations of HfO_2 and one of Eu_2O_3 were tested in the central fuel position to obtain information about the material as control rods in water moderated plutonium reactors. The lowest concentrations of HfO_2 and Eu_2O_3 were dispersed in an Al_2O_3 matrix. The other two concentrations of HfO_2 were mixed into aluminum powder and compacted into rods. All of the samples were loaded into Zircaloy tubes 0.372 in. ID with 0.027 in. walls. The two samples mixed in Al_2O_3 were 44 in. long and the two in aluminum powder were 36 in. long. The HfO_2 concentrations were 0.125, 0.54, and 1.08 g/cm³ and the Eu_2O_3 concentration was 0.0369 g/cm³. Reactivity worth of dummy rods with amounts of Al_2O_3 and aluminum equal to that in the poison rods were measured as corrections for the measurements of the poison rods. The worths of Al_2O_3 and aluminum rods were $0.9 \pm 0.2\%$ and $3.9 \pm 0.3\%$, respectively. The reactivity changes due only to neutron absorption in the HfO_2 and Eu_2O_3 are listed in Table III.

TABLE III

MEASURED REACTIVITY WORTHS OF REACTOR MATERIALS

All materials with plate geometry have moderator displacement worth included in the value. The worths of materials with rod geometry have been corrected for reactivity effects of moderator displacement and neutron absorption in Zircaloy or matrix materials.

<u>Material</u>	<u>Sample Weight, g</u>	<u>Position in Core</u>	<u>Sample Geometry</u>	<u>Worth, ¢/g</u>
HfO_2	9.58	Radial Center	44 in. rod	-3.70 $\pm$ 0.02
HfO_2	33.8	Radial Center	36 in. rod	-3.21 $\pm$ 0.04
HfO_2	67.0	Radial Center	36 in. rod	-2.44 $\pm$ 0.02
Eu_2O_3	2.89	Radial Center	44 in. rod	-46.6 $\pm$ 0.5
Copper	1125	Radial Center	44 in. rod	-0.121 $\pm$ 0.001
Aluminum	69.50	Core Center	8 in. rod	-43 $\pm$ 4 $\times 10^{-3}$
Aluminum	151.8	Core Center	Plate	-91 $\pm$ 2 $\times 10^{-3}$
Polycarbonate	90.46	Core Center	Plate	-25 $\pm$ 3 $\times 10^{-3}$
Teflon	122.9	Core Center	Plate	-56 $\pm$ 3 $\times 10^{-3}$
Lucite	67.84	Core Center	Plate	-3.0 $\pm$ 0.5 $\times 10^{-3}$
Polyethylene	69.93	Core Center	Plate	+24 $\pm$ 4 $\times 10^{-3}$

A 1125 g copper rod (44 in. long x 0.425 in. diam) was inserted into fuel positions and the reactivity change measured as a function of radius. This resulted in the worth of the copper rod when replacing moderator. To obtain a correction for the moderator which was removed, the worth of an empty Zircaloy tube (0.372 in. ID with 0.027 in. wall) was measured in the central fuel element position. The worth of the tube plus displaced moderator was $-0.44 \pm 0.09\%$. The copper worth is reported in Table III and is the value relative to air.

Hexagonal templates machined from various reactor materials to enclose 19 fuel rods were tested for reactivity worths. Samples of aluminum, polycarbonate, Teflon, Lucite, and polyethylene were centered radially and vertically in the core for the measurements. These worths are reported in Table III and include the reactivity change because of the displaced moderator.

To obtain the reactivity effect of aluminum when not displacing moderator a short section of aluminum rod (8 x 1/2 in.) was positioned inside of a Lucite void tube. The reactivity change was measured for adding the aluminum rod at the radial and vertical center of the loading. The worth of the aluminum rod is listed in Table III.

A comparison of the Teflon worth to measurements made with aluminum show Teflon to be a reasonably good void simulator in this lattice. The worth of Teflon is $0.124 \pm 0.005 \text{ } \$/\text{cm}^3$ which agrees within the errors with the value $0.130 \pm 0.012 \text{ } \$/\text{cm}^3$ which is obtained after removing the absorption effects from the aluminum plate using the value measured for the rod. The worth of the 86% Styrofoam material 4 7/8 in. above the fuel center at the radial center was $0.135 \pm 0.007 \text{ } \$/\text{cm}^3$, and the Teflon simulated void in the same position was worth $0.118 \pm 0.007 \text{ } \$/\text{cm}^3$. Thus, the Teflon probably is a better void material than Styrofoam.

Moderator Height and Temperature Coefficients

The reactivity coefficients for moderator level changes were determined from positive period measurements. The moderator was lowered from 102.25 to 81.75 in. in 1/4 in. steps (ΔH) and reactivity changes ($\Delta \rho$) determined for each step. The sensitivity ($S = \Delta \rho / \Delta H$) was assumed to apply

to the height at the mid-point of each step. Fuel rods were added at the outer edge of the cylindrical core to compensate for reactivity lost by lowering the moderator. The total measurable worth of the moderator above the top end of the fuel ($93 \frac{3}{4}$ to $99 \frac{3}{4}$ in.) was worth $88.0 \pm 1.0\%$. The reactivity coefficient of moderator level below the top of the core can be described by;

$$S = (54.8 \pm 0.9) + (48.0 \pm 5.0) \times 10^5 (H^{-3} - 1.223 \times 10^{-5}) \quad (1)$$

where S is in $\%$ /in. and H is in inches. H is the effective moderated core height and includes 3.25 ± 0.03 in. for bottom reflector savings. This value was obtained from Reference (1). Equation (1) is valid over the top 8 in. of the fuel core excluding the positions near the top Lucite template. A discontinuity occurs over the thickness of the template because the effective moderator height changes very little as the moderator is lowered in this region. A value of $42 \pm 5 \text{ cm}^2$ was obtained for the migration area from the slope of Equation (1).

Temperature coefficients of reactivity were measured as the reactor temperature was increased step-wise from 19.0 to 35.1°C . The moderator was circulated through an external heater to increase the reactor temperature. The coefficient was taken to be the rate of change of reactivity at the average of the temperature measured. Values of the temperature coefficient are 2.1 ± 0.4 and $1.1 \pm 0.4 \%$ / $^\circ\text{C}$ at 20.1 and 32.5°C , respectively. These values include the reactivity effects because of expansion of the templates.

Spectrum Measurements

Lutetium pins containing 11% Lu_2O_3 in Al_2O_3 were used as activation detectors for measurements of indices T and r which describe the thermal and epi-thermal neutron spectra, respectively. Lutetium pins ($1/4 \times 0.028$ in. diam), both bare and cadmium covered, were distributed vertically and radially in the core. Pins for the vertical traverse were attached to the surface of the central fuel rod. Pins for the radial traverse were positioned, in a Lucite holder, 0.050 in. from each fuel rod. The radial traverse was obtained $16 \frac{3}{4}$ in. above the core center. The bare and cadmium covered pins were activated in separate irradiations.

The results of lutetium measurements are given in Table IV. The neutron temperature was constant vertically from a point 5 in. above the bottom of the fuel to the top of the fuel. Radially it was constant through the core and 5.2 in. beyond the edge. The value of the epi-thermal index r was constant vertically from 5 to 40 in. above the bottom of the fuel. Radially it was constant up to 2 in. from the core edge.

TABLE IV
SPECTRAL INDICES AS MEASURED WITH LUTETIUM
Moderator Temperature at the Time of Irradiation = 299 °K

Position	Spectral Indices	
	$T, ^\circ K$	r
In Water (radially)	308 ± 7	0.0722 ± 0.0002
On Cladding (vertically)	340 ± 4	0.0790 ± 0.0009

A radial flux distribution was measured also with a 0.030 in. diam copper wire. The wire could be positioned closer to the center than the lutetium holder and was 3 3/8 in. above the fuel center along a core diameter. In the same irradiation five copper pins, 1/4 in. long and 0.030 in. in diameter, were placed near the radial center and in the same positions as the lutetium pins. The copper wire was cut, for counting, into lengths equal to a lattice unit and centered on each fuel rod beginning at the center. A comparison of the copper data to that obtained with subcadmium Lu^{176m} activity showed the radial flux distribution to be equivalent at 3 3/8 and 16 3/4 in. above the core center within the accuracy of the measurement.

Copper activity traverses were made 16 3/4 in. above the center to compare the flux shapes of the two critical loadings. Copper pins were loaded into the lucite holder described for the lutetium radial traverse and irradiated. A comparison between this traverse and the copper wire traverse from the first critical loading showed the flux distributions to be identical. The copper cadmium ratio 16 3/4 in. above the core center at the radial center is 12.9 ± 0.3 .

Kinetics

Power fluctuations were recorded on magnetic tape to obtain the ratio β/ℓ of the effective fraction of delayed neutrons to the reactor neutron

lifetime. The fluctuations were sensed by a BF_3 ion chamber placed at the top radial edge of the core. The reactor power was held constant at 43 W during the 30 min recording period. Analysis of the tape for spectral density as a function of frequency results in a value of $33 \pm 3 \text{ sec}^{-1}$ for β/ℓ . If a value of 2.098×10^{-3} is assumed for β , the neutron lifetime ℓ is $(6.4 \pm 0.6) \times 10^{-5} \text{ sec}$.

The same ratio was deduced from the neutron flux transient after a rapid insertion of negative reactivity. The reactor power was monitored with the ion chamber used for the reactor noise recording. A voltage generated by the current from the BF_3 ion chamber was recorded by oscilloscope photograph for the first 2 sec after insertion. Subsequent data was recorded by a digital voltmeter with a typewriter readout. The ratio β/ℓ is obtained by fitting the flux transient data to a series of exponentials. The best fit obtained gives $\beta/\ell = 26 \pm 3 \text{ sec}^{-1}$.

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PuO₂-UO₂ Experiment in the EBWR

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(Abstract of Formal Report BNWL-126)

A plutonium experiment is to be performed in the Experimental Boiling Water Reactor (EBWR) at Argonne National Laboratory and is a joint program between Pacific Northwest Laboratory and Argonne National Laboratory. This program which is to demonstrate the utilization of plutonium in a light-water moderated power reactor is entering Phase 2.^(1, 2) In this phase of the program, the central portion of the core will be loaded with plutonium fuel such that the core reactivity variation with burnup will be caused mainly by burnup of the plutonium zone. Surrounding the plutonium zone are uranium fuel elements which will be used as driver elements. Several rods will be included in the plutonium zone to obtain burnup information from special fuel compositions.

A series of tests will be conducted during the initial loading of the EBWR in order to measure the reactivity worth of fuel rods, safety rods, voids, and boric acid in the moderator.

At various stages of burnup, the series of tests performed at the time of the startup experiments will be repeated. Also, at these stages a series of rods from the plutonium zone will be removed and returned to Pacific Northwest Laboratory for reactivity measurements in the Plutonium Recycle Critical Facility (PRCF) and subsequently analyzed for plutonium concentration, isotopic composition and to obtain other data which will describe the burnup characteristics. The results of calculations of the reactivity changes expected at startup and throughout the burnup of the core are presented in this report. Information obtained during the startup experiments will be used to assess the accuracy of the calculational methods and identify areas of uncertainty in the computations.

The maximum exposure at which the plutonium zone can be made critical at low power is expected to be about 5000 MWd/ton with the moderator hot and 6000 MWd/ton with it cold. However, the plutonium zone is subcritical before reaching 5000 MWd/ton at full power because of large negative reactivity effects due to void and xenon concentrations.

A result from this study is that the plutonium fuel exhibits larger variations of negative reactivity with increased moderator temperature and moderator void changes than does the uranium fuel which surrounds it.

Another result is that the moderator temperature coefficient of reactivity for the plutonium zone alone becomes less negative with fuel exposure, and finally goes positive near 5000 MWd/ton. At an irradiation which is greater than 7000 MWd/ton for this fuel, the reactivity change due to moderator and fuel heating is positive. However, the reactivity change due to moderator void is calculated to be negative throughout burnup with a magnitude large enough to compensate for the positive temperature effect. Thus, the reactivity effects at full power operation are expected to be negative up to very high exposures (~15,000 MWd/ton). For a pressurized water reactor loaded like the EBWR, the negative reactivity effects of a large void fraction present in the EBWR would be lost, and at exposures greater than 7000 MWd/ton the reactivity response to temperature increases would be positive for the pressurized system.

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HTLTR Driver Fuel Design Analysis

J. J. Regimbal and R. C. Liikala

(Abstract of Informal Report BNWL-CC-142)

This report summarizes results of calculations of fueling requirements for safe and adequate operation of the High Temperature Lattice Test Reactor (HTLTR). General features and operating conditions for the HTLTR concept have been described⁽¹⁾ and results of preliminary calculations of criticality were reported previously.⁽²⁾ Final results of the preliminary study, including an estimate of driver fuel requirements were reported later.⁽³⁾ Results of additional analyses are described in this report.

Driver fuel configurations and flux distributions were computed for reactor models which closely represent the geometry and detail of practical loadings. The accuracy of the computational methods chosen for this work were evaluated using results of experiments conducted with a similar graphite-uranium lattice in the Physical Constants Testing Reactor. The method selected for the HTLTR computations was that which produced the best agreement with the results of that experiment. Explicit account was taken of heterogeneity in the cell and reactor configurations by specifying the detail in THERMOS⁽⁴⁾ and 4K-ANGIE⁽⁵⁾ computations.

The analysis of the HTLTR covered driver fuel enrichments of 5.0, 7.5, and 10.0 wt% $U^{235}O_2$ in UO_2 , conditions of a central test region which contained a void, N-14 gas, and a test lattice poisoned to $k_\infty = 1.0$ and reactor temperatures of 293, 698, 973, and 1273 °K. The use of gadolinium to reduce reactivity changes due to temperature fluctuations of the stack was examined also.

The feasibility of using the 5 wt% elements is of particular interest to maximize the Doppler coefficient yet retain compatibility with the shim rod design. This study shows that the HTLTR can be made critical with this fuel at the least reactive assumed test conditions. The required side load of driver fuel, on an 8 3/8 in. pitch, comprised ~85 elements (2 rows, 5.0 ft long) surrounding the test region. A zone fueled with 120 elements

(20 in. long) at each axial end completes the configuration of driver fuel. Considerations of flux shaping near the ends will probably require changes in the assumed fuel disposition and lengths. Increasing the length of end fuel displaces graphite reflector without increasing the multiplication very much (e. g., increasing the length from 10 to 20 in. is worth only 1.2% $\Delta k/k$). Thus, a different arrangement of fuel between the side and end drivers may be better.

Small fluctuations in the physical temperature of the HTLTR during an experiment will produce extraneous reactivity signals. A reduction of 75% in this sensitivity of reactivity to temperature may be obtained for configurations which employ a blanket of graphite impregnated with 3 to 4 wt% Gd_2O_3 .

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ENGINEERING PHYSICS

Comparison of Theoretical and Experimental
Resonance Absorption in Pu²⁴⁰

C. M. Heeb

Introduction

Experimental values of the Pu²⁴⁰ resonance integral as determined by Physical Constants Testing Reactor (PCTR) reactivity increment measurements are available in the literature.⁽¹⁾ These results can be used to evaluate the applicability of various methods of calculating the Pu²⁴⁰ resonance integral. Specifically, these values can be used to test the numerical methods contained in the codes ZUT and TUZ.⁽²⁾ These codes are based on a suggestion by L. W. Nordheim⁽³⁾ that the general integral slowing down equations in collision density, $F(u) = \Sigma_t(u)\phi(u)$ lend themselves to direct numerical solution for $F(u)$. The resonance integrals can then be calculated from:

$$I = E_1 \int_0^{\infty} \sigma_{a_0}(u) \frac{F(u)}{\sigma_t(u) + \sigma_m} e^{-u} du ,$$

where E_1 is the energy corresponding to the base lethargy and σ_m is the potential scattering cross section of an admixed scatterer per absorber nucleus. The codes, in the present form, contain features which were developed with the U²³⁸ and Th²³² resonances in view and so a test of their applicability to the giant 1 eV Pu²⁴⁰ resonance is of interest. Comparisons of the integral method and the usual NRIA approximation are also of interest since this last approximation is most often used in calculating the absorption at 1 eV in Pu²⁴⁰.

Description of the Experiments

To determine the value of the resonance integral for Pu²⁴⁰, the reactivity difference between pairs of 0.65 in. diam Pu-Al alloy rods differing essentially only in the Pu²⁴⁰ concentration were measured in the PCTR.

An equation of the form

$$\Delta\rho_i = C \left[(N_B^{240} I_B^{240} V_B - N_A^{240} I_A^{240} V_A) \right] \quad (1)$$

was assumed.

$\Delta\rho_i$ = the observed reactivity difference between Rods B and A of the i^{th} pair (ϕ)

I^{240} = the effective resonance integral for the rod (barns)

V = the rod volume (cm^3).

To minimize reactivity difference due to changes in thermal absorption in the rods, a cadmium cover was provided for each rod. The epi-cadmium correction is estimated to be 114 barns. This value, plus the sum of the individual calculated values for each Pu^{240} resonance can be substituted in Equation (1) along with the observed reactivity difference for the i^{th} pair. The experiment thus affords a way of evaluating the methods used to calculate the integrals by applying goodness of fit tests to the data.

Comparison of Experiment and Theory

Using the numerical solution of the slowing down equation available from ZUT for each rod pair, the resonance integrals are shown in Table I below.

TABLE I
CALCULATED ROD RESONANCE INTEGRAL VALUES

Rod	N^{240} , u/barn-cm	I at 1 eV, barns	ΣI above 1 eV, barns	Unresolved*, barns	Epi Cd 1/V, barns	Total barns
1	5.0×10^{-7}	8,259	113	32	114	8,518
2	1.42×10^{-7}	8,210	113	32	114	8,469
3	4.08×10^{-7}	8,074	113	32	114	8,332
4	1.17×10^{-6}	7,712	113	32	114	7,970
5	3.35×10^{-6}	6,865	112	31	114	7,122
6	9.59×10^{-6}	5,312	110	31	114	5,567
7	2.74×10^{-5}	3,453	106	31	114	3,704
8	7.80×10^{-5}	2,045	95	29	114	2,283
8B	2.20×10^{-4}	1,185	76	26	114	1,401

* The TUZ code has been used to estimate the unresolved resonance contribution.

Using the values from Table I, a least squares fit gives the value $\underline{C} = 0.142 \pm 0.0016$ for the constant in the experimental Equation (1). Figure 1 is a plot of this line along with the 95% confidence level bands on the abscissa for a given single observation of the PCTR reactivity increment. The line fit demonstrates a high degree of consistency between the observed experimental results and the method of calculation.

The confidence level band serves to estimate the experimental uncertainty. If $X = I_B N_B V_B - I_A N_A V_A$, then the upper and lower values on the resonance integral are given by

$$I_{\pm} = \frac{1}{N_B V_B} (X_{\pm} + N_A V_A I_A)$$

where $X_{\pm}$ corresponds to the upper and lower band values on X for a given Y.

FIGURE 1
All Points Line
(Reactivity Versus Absorption Difference,
95% CL Band Indicated)

For the purpose of comparing the "calculated" resonance integral values with "experimental" value, the best experimental value is

$$I_{\text{experimental}} = \frac{1}{N_B V_B} (X + N_A V_A I_A),$$

where X is the line value corresponding to observed reactivity for that particular absorber nuclei concentration. The integral value I_A for the other rod in the pair is obtained from the calculated value given from the ZUT code. When all the experimental points are used to determine the exact linear relationship, then the line values become the "true values" in the sense that a large number of repeat measurements would yield the value given by the line if the line obtained represents a good approximation to the true model. The "calculated values" are those obtained through the direct numerical solution for $F(u)$ with subsequent evaluation of the resonance integral.

Table II summarizes the resonance integral values for numerical comparison with the values quoted by Nichols. The wide experimental uncertainty at lower absorber concentrations explains the discrepancy between the integral values given here and the values quoted by Nichols. The method used in arriving at the quoted values was to obtain a value of $\underline{C}$ at the low concentrations (Rods 1, 2, and 3) and then to use this value in a pair wise comparison with increasing absorber concentration from Rod 3. From the uncertainty at low concentration it is apparent that there is a high probability of obtaining an incorrect value of the slope $\underline{C}$ which would then cause an incorrect evaluation of all subsequent absorber integrals. It is of interest to note that the "calculated" values shown in Table II are everywhere within the quoted uncertainty of the "experimental" values even though that uncertainty decreases by almost two orders of magnitude with increasing absorber concentration.

Comparison of the integral method of solution and the NR1A approximation used in the 1 eV resonance yields virtually identical results at the Pu^{240} density range of the experiments.

TABLE II
COMPARISON OF EXPERIMENTAL AND
CALCULATED RESONANCE INTEGRAL VALUES

<u>Rod</u>	<u>N_O n/barn-cm</u>	<u>I_{experimental} barns</u>	<u>I_{calculated}, barns</u>	<u>I_{Nichols} barns</u>
4	3.35×10^{-6}	$7,647 \pm 1,286$	7,122	$6,730 \pm 430$
5	9.59×10^{-6}	$5,826 \pm 456$	5,567	$5,080 \pm 450$
6	2.74×10^{-5}	$3,724 \pm 163$	3,704	$3,300 \pm 370$
7	7.80×10^{-5}	$2,314 \pm 61$	2,283	$1,990 \pm 300$
8	2.20×10^{-4}	$1,389 \pm 26$	1,401	$1,180 \pm 270$

$I_{\text{experimental}} \sim$ the value corresponding to $x = (N_B V_B I_B - N_A V_A I_A)$
 where $x = y/c$ and $N_A V_A I_A$ contains the calculated
 value of the resonance integral.

$I_{\text{calculated}} \sim$ the value calculated with ZUT and TUZ codes.

$I_{\text{Nichols}} \sim$ the original resonance integral values as quoted
 by Nichols.

Conclusion

The numerical procedures embodied in the code ZUT and the NRIA approximation when applied to Pu^{240} resonance absorption yield results which are consistent with the Nichols' experimental results. The values of the Pu^{240} resonance integrals at high Pu^{240} concentration quoted in the original experiment are probably too low for reasons which are clear upon subsequent reanalysis made possible by the availability of more powerful means of computing resonance absorption. It is therefore concluded that both the ZUT code, its progeny NIT and GANIT and the NRIA approximation can be used to calculate Pu^{240} resonance absorption until new experimental evidence to the contrary is produced.

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Physics Analysis of a U-Al Polyethylene System

G. J. Busselman

One goal of the Phoenix fuel program is to develop methods for calculating the nuclear characteristics of plutonium fueled thermal systems. In order to check various calculational schemes, a series of experiments have been selected to cover the variables of interest. The experiments employ Pu-Al circular fuel plates in conjunction with borated polyethylene discs stacked in a cluster of aluminum tubes. The experiment consists of determining the amount of boron necessary to add to the moderator to achieve a k_{∞} of unity for the Pu-Al poly system.

In the first experiment, U^{235} rather than plutonium was used to try out the experimental techniques and calculational methods with a system which has neither the low-lying resonance structure of plutonium nor the contamination problems of a plutonium experiment.

Experiment⁽¹⁾

The fuel consists of 40 mils thick by 1.96 in. diam U-Al discs with 33 wt% uranium, enriched to 93.0 wt% U^{235} . Between each fuel disc there are fifteen 20 mil polyethylene moderator discs (Figure 1). Some of the poly discs contain 0.8 wt% boron. Every disc in the cell has a 0.20 in. diam hole in the center. The just critical condition requires 0.126 atoms of B^{10} per atom of U^{235} . The H/ U^{235} ratio for this system is 217.4. Table I lists the atom densities in the experimentally critical system. Note: These numbers supercede those given in Table III of Reference (1). The experiment consists of placing a 19 tube hexagon of the experimental lattice in the test cavity of the PCTR and determining that k_{∞} of a portion of the hexagon is just critical by a null experiment. That is, there is no effect on the whole reactor reactivity when the test sample is removed. The experimental lattice is some 12 in. in length and only the center 6 in. of the center tube is removed for the null experiment. The remaining discs act as spectrum buffers and assure a flat flux across the test area.

FIGURE 1
Experimental Lattice

Methods

The calculational methods used progressed from the ZODIAC⁽²⁾ code diffusion methods employed for survey type calculation, to a more exacting multigroup transport code.⁽³⁾ The thermal absorption and fission cross sections in all cases are calculated using the SPECTRUM⁽⁴⁾ code with a 20 °C polyethylene kernel and the epithermal cross sections, i. e., above 0.683 eV are taken from the GAM⁽⁵⁾ slowing down code. Thermal scattering cross sections are generated from TEMPEST⁽⁶⁾ using a combination of Wigner and Wigner-Wilkins approximations. The 2200 meters/sec cross section values employed are tabulated in Table II. The epithermal cross section library is based on the work of Liikala.⁽⁷⁾

TABLE I
JUST CRITICAL EXPERIMENTAL COMPOSITIONS
(atoms/barn cm)

	<u>Cell Composition</u>	<u>Fuel Disc Composition</u>	<u>Moderator Region Composition*</u>
Hydrogen	$5.00 \times 10^{-2} \pm 0.04$		7.92×10^{-2}
Carbon	$2.50 \times 10^{-2} \pm 0.02$		3.96×10^{-2}
Aluminum	$4.62 \times 10^{-3} \pm 0.02$	5.48×10^{-2}	
B ¹⁰	$2.90 \times 10^{-5} \pm 0.03$		4.60×10^{-5}
Oxygen	$2.20 \times 10^{-4} \pm 0.02$		3.49×10^{-4}
U ²³⁴	$2.53 \times 10^{-6} \pm 0.04$	3.10×10^{-5}	
U ²³⁵	$2.30 \times 10^{-4} \pm 0.02$	2.73×10^{-3}	
U ²³⁸	$1.45 \times 10^{-5} \pm 0.01$	1.72×10^{-4}	

Note: There is associated with each unit cell some 6.24×10^{-3} atoms/barn cm of aluminum can wall. This material is included in the cross section calculations but is not part of the system which has a reactivity of unity.

* Contains both borated and unborated poly discs.

TABLE II
CROSS SECTIONS
(2200 meters/sec)

<u>As Used</u>				<u>BNL Values</u>				<u>Reference</u>
<u>σ_a</u>	<u>σ_f</u>	<u>ν</u>	<u>η</u>	<u>σ_a</u>	<u>σ_f</u>	<u>ν</u>	<u>η</u>	
0.3302				0.332				8
0.0038				0.0034				8
0.2283				0.235				8
3837.0				3837.0				8
94.49				105.0				9
676.77	577.99	2.43	2.075	678.2	577.1	2.442	2.078	10
2.71				2.71				11

Results

The results are summarized in Table III. The first two entries indicate that the diffusion theory thermal flux depression in the fuel region is insufficient. The diffusion depression factor is 0.9739 where the double P_3 method gives 0.8963 using the same cross sections. The second heterogeneous entry in Table III is derived from a homogeneous calculation (i. e., no spatial flux variation) with the thermal cross sections reduced for those materials belonging in the fuel plate. No adjustments were made to the other energy groups. This is our "standard survey approach" for plate cores, and agrees quite well with the experiment.

TABLE III
METHODS COMPARISON

	k_∞	
	<u>Homogeneous</u>	<u>Heterogeneous</u>
ZODIAC, Diffusion Theory	1.0888	1.0587
ZODIAC, Homogeneous with Thermal Group Self Shielding Factors from DP_3		1.0108
Sn(double S_3), Transport Theory, 6 Angles	1.0889	1.0359
Sn(double S_5)		1.0224
Sn(double S_7)		1.0173
Sn(double S_{13})		1.0120
Sn(double S_{16})		1.0110
Sn(double S_{31})		1.0093
Sn(double S_{16}) Selected Angles		1.0130
Sn(double S_{34}) Selected Angles		1.0121

The transport theory analysis made use of Program S-XIII⁽³⁾ and the same set of cross sections as used in the diffusion analysis. The number of double angles was varied from 3 to 31. Two attempts were made at selecting the angular areas of most importance in order to achieve a high degree of accuracy with a lower number of angles. The results are shown in Table III and indicate that selection of the "correct angular set is not straightforward.

Uncertainties

The accuracy of the experiment is estimated to be $\pm 0.04\%$ or ± 0.0000025 in k (reactivity) for the entire PCTR. The accuracy to which

the test cell is just critical is estimated as ± 0.001 . No correction is made for streaming as the experimenters feel it is negligible. The flux dip in the borated discs was neglected. This omission is justified by the experimental result which shows that rearranging the borated discs so that there were half as many, twice as thick per cell, resulted in no measurable change in reactivity.

The composition uncertainties listed in Table I carry with them an uncertainty in reactivity. Table IV lists these uncertainties based on perturbation coefficients from the Sn code. These values do not reflect microscopic cross section shifts caused by changing the absorption to slowing down ratio for the cell. Separate calculations using ZODIAC (TEMPEST and diffusion theory) rather than SPECTRUM and transport theory yield substantially the same uncertainty estimates. These latter calculations do include changes in the microscopic cross sections due to spectral shifts.

TABLE IV

PERTURBATIONS

(Double S_{13} case, no changes in the microscopic cross sections)

	Δk for <u>+1% Concentration</u>	<u>Table I Uncertainty</u>
Hydrogen	-0.001032	± 0.00082
Carbon	-0.00014	± 0.00011
Aluminum	-0.00016	± 0.00007
B ¹⁰	-0.00435	± 0.00448
U ²³⁴	-0.00014	± 0.00022
U ²³⁵	-0.00435	± 0.00378
U ²³⁸	-0.00015	± 0.00010
Fuel Region	+0.00426	
Moderator Region	-0.00528	
Sum of the Δk 's		± 0.00958
RMS of the Δk 's		± 0.00593
Diffusion Theory, Including Shifts in Micro Cross Sections		
All Uranium	+0.00468	± 0.00407
B ¹⁰	-0.00433	± 0.00446

Conclusions

The "best" calculated number is 1.0093 employing a 31 double angle, 4 energy group, Sn method. The uncertainty in the experimental result due

to uncertainties in the composition is about ± 0.006 . In addition, the uncertainty due to experimental technique is of the order of ± 0.001 . On this basis it would appear that the calculations and the experimental results are in good agreement.

It would also appear that for this particular system the use of diffusion theory coupled with an improved spatial thermal flux calculation gives adequate results ($k_{\infty} = 1.0108$).

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Preliminary Analysis of Some Pu-Al
Polyethylene Experiments in the PCTR

G. J. Busselman

A series of experiments⁽¹⁾ were scheduled for the PCTR which would yield critical composition and plate thicknesses for some Pu-Al borated polyethylene systems. One requirement of the experiments is that the "test cell" be buffered with like material to achieve spectral and flux flattening across the test cell. These plutonium systems consist of unclad Pu-Al discs which are sandwiched between polyethylene discs and then placed in aluminum tubes. This operation must take place in a glove box and is a rather lengthy job as the whole system contains some 34,000 discs. Obviously one would like to minimize the number of times that the discs be rearranged.

The calculations summarized here were made primarily to provide this preexperiment information. The exact compositions were not known prior to the experiment, so some discrepancy in reactivity was expected.

The plutonium compositions used in the calculations are given in Table I.

TABLE I
PLUTONIUM COMPOSITIONS

Type	Pu ²³⁹ at. %	Pu ²⁴⁰ at. %	Pu ²⁴¹ at. %	Pu ²⁴² at. %
I	90.48	8.19	1.25	0.08
II	77.80	18.89	3.01	0.30
III	67.42	26.86	4.76	0.96

The results of the calculations are summarized in Figure 1. The first experiment of the series was the 20 mils, H/Pu of 200, 27% Pu²⁴⁰ case. The calculated B¹⁰/Pu ratio of this system was normalized to the experimental value and the set of normalized calculations was then used as a basis for loading the buffer zones of the other experiments. The absolute errors in B¹⁰/Pu—which range from 10 to 20% corresponding to about 2.5 to 5% in k_∞—have not yet been fully resolved.

FIGURE 1
 Characteristics of Critical Pu-Al
 Borated Polyethylene Systems

The calculations made use of the SPECTRUM code⁽²⁾ with a polyethylene kernel⁽³⁾ to obtain thermal cross sections and the GAM slowing down code to generate epithermal group constants. These cross sections were then used in the S-XIII⁽⁴⁾ transport theory code to carry out a 4 energy group, 13 double angle calculation.

The choice of multigroup energy boundaries was based on the following considerations:

- The fission spectrum is contained entirely in the first group (10 MeV $\rightarrow$ 1.17×10^4 eV).

- The Pu²⁴⁰ giant resonance lies mainly within the third group (2.38 – 0.638 eV).
- All other resonance effects are left to be accounted for in the second group (1.17×10^4 – 2.38 eV).
- The fourth group (0.683 – 0.0 eV) covers the thermal effects. The microscopic cross sections used are based on the work of Liikala⁽⁵⁾ with the exception of the Pu²³⁹.
- Plutonium-239 is normalized to the Sher⁽⁶⁾ 2200 meters/sec values.

Various methods were compared for obtaining the Pu²⁴⁰ absorption cross section near the 1 eV resonance. For most calculations the NRIA approximation was found to be adequate.

As shown in Figure 1, the discrepancy between calculated and measured B^{10}/Pu is greater for the higher H/Pu systems. The analysis of these very thermal systems should benefit from a more exact representation of the thermalization process. For this purpose the space dependent thermalization code THERMOS is being used. Preliminary results indicate that the discrepancy in k_{∞} can be reduced to less than 1%.

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HIGH TEMPERATURE REACTOR PHYSICS

Spherical and Slab Geometry Solutions to a Two-Region Boundary Value Problem Giving Temperature Increase During an Exponential Power Rise

R. J. Hoch, R. M. Humes, and W. W. Brown

In calculations for the High Temperature Lattice Test Reactor a solution to the following boundary value problem in heat conduction was needed.

Heat is generated in spherical pellets of radius (a) which are embedded in graphite. With each pellet associate a concentric sphere of graphite of radius (b). An exponential power excursion with inverse period (ω) occurs and the temperature rise $\theta(r, t)$ is to be found.

Differential Equation

$$k_f \Delta^2 \theta_f(r, t) + S_0 e^{\omega t} = c_f \rho_f \frac{\partial \theta_f(r, t)}{\partial t}, \quad 0 < r \leq a$$

$$k_g \Delta^2 \theta_g(r, t) = c_g \rho_g \frac{\partial \theta_g(r, t)}{\partial t}, \quad a \leq r \leq b$$

Boundary and Initial Conditions

$$\theta_f(a, t) = \theta_g(a, t)$$

$$k_f \frac{\partial \theta_f(a, t)}{\partial r} = k_g \frac{\partial \theta_g(a, t)}{\partial r}$$

$$\frac{\partial \theta_f(0, t)}{\partial r} = 0$$

$$\frac{\partial \theta_g(b, t)}{\partial r} = 0$$

$$\lim_{t \rightarrow -\infty} \theta(r, t) = 0, \quad 0 < r \leq b$$

Definition of Symbols

$\theta_f(r, t)$ temperature rise of fuel pellet °C.

$\theta_g(r, t)$ temperature rise in graphite region associated with the pellet °C.

k_f conductivity of fuel material, $\frac{\text{cal}}{\text{cm}^\circ\text{C sec}}$

k_g conductivity of graphite, $\frac{\text{cal}}{\text{cm}^\circ\text{C sec}}$

c_f heat capacity of fuel material, $\frac{\text{cal}}{\text{g}^\circ\text{C}}$

c_g heat capacity of graphite, $\frac{\text{cal}}{\text{g}^\circ\text{C}}$

ρ_f density of fuel material $\frac{\text{g}}{\text{cm}^3}$

ρ_g density of graphite, $\frac{\text{g}}{\text{cm}^3}$

S_o rate of energy generation per unit volume of fuel material at time zero, $\frac{\text{cal}}{\text{cm}^3 \text{ sec}}$

$\frac{1}{\omega}$ time required for one e-fold increase in energy generation rate, (ω has dimensions sec^{-1})

a radius of fuel pellet, cm.

b radius of the spherical volume of graphite that is associated with a fuel pellet, cm.

Solution

Separate variables into time dependent and spatial parts and apply boundary and initial conditions.

$$\theta_f(r, t) = e^{\omega t} \left\{ B \left[\sum_{n=0}^{\infty} \frac{f^n r^{2n}}{(2n+1)!} \right] + \frac{S_o}{c_f \rho_f \omega} \right\}, \quad 0 \leq r \leq a$$

$$\theta_g(r, t) = e^{\omega t} \left\{ (GB + H) \left[\sum_{n=0}^{\infty} \frac{g^{n+1} r^{2n+1}}{(2n+2)!} \right] + F \sum_{n=0}^{\infty} \frac{g^n r^{2n}}{(2n+1)!} \right\}, \quad a \leq r \leq b$$

where

$$\begin{aligned}
 g &= \omega \frac{c_g \rho_g}{k_g} \\
 f &= \omega \frac{c_f \rho_f}{k_f} \\
 F &= - \frac{\sum_{n=0}^{\infty} g^{n+1} b^{2n} \frac{(2n+1)}{(2n+2)!}}{\sum_{n=0}^{\infty} g^n b^{2n-1} \frac{2n}{(2n+1)!}} \\
 G &= \frac{\sum_{n=0}^{\infty} f^n a^{2n} \frac{1}{(2n+1)!}}{\sum_{n=0}^{\infty} g^{n+1} a^{2n+1} \frac{1}{(2n+2)!} + F \sum_{n=0}^{\infty} g^n a^{2n} \frac{1}{(2n+1)!}} \\
 H &= \left[\frac{S_o}{c_f c_f \omega} \right] \frac{1}{\sum_{n=0}^{\infty} g^{n+1} a^{2n+1} \frac{1}{(2n+2)!} + F \sum_{n=0}^{\infty} g^n a^{2n} \frac{1}{(2n+1)!}} \\
 B &= \frac{k_g H \left[\sum_{n=0}^{\infty} g^{n+1} a^{2n} \frac{(2n+1)}{(2n+2)!} + F \sum_{n=0}^{\infty} g^n a^{2n-1} \frac{2n}{(2n+1)!} \right]}{k_f \sum_{n=0}^{\infty} f^n a^{2n-1} \frac{2n}{(2n+1)!} - k_g G \left[\sum_{n=0}^{\infty} g^{n+1} a^{2n} \frac{(2n+1)}{(2n+2)!} + F \sum_{n=0}^{\infty} g^n a^{2n-1} \frac{2n}{(2n+1)!} \right]}
 \end{aligned}$$

The analogous problem in slab geometry was also of interest. Using the variable x to represent slab thicknesses and assuming the slabs have infinite extent in y and z , we replace Δ^2 by $\frac{\partial^2}{\partial x^2}$ and $\frac{\partial}{\partial r}$ by $\frac{\partial}{\partial x}$ in the stated problem. Following the approach outlined by W. A. Horning and H. C. Corben,⁽¹⁾ the following solution is obtained.

$$\theta_f(x, t) = \frac{S_o e^{\omega t}}{\rho_f c_f \omega} \left\{ 1 - \frac{\cosh \gamma_f x}{\cosh \gamma_f a + \frac{k_f}{k_g} \frac{\gamma_f}{\gamma_g} (\sinh \gamma_f a) [\coth(b-a) \gamma_g]} \right\}, \quad 0 < x \leq a$$

$$\theta_g(x, t) = \frac{S_o e^{\omega t}}{\rho_f c_f \omega} \left\{ \frac{\frac{k_f}{k_g} \frac{\gamma_f}{\gamma_g} \frac{\sinh \gamma_f a}{\sinh(b-a) \gamma_g} \cosh(b-x) \gamma_g}{\cosh \gamma_f a + \frac{k_f}{k_g} \frac{\gamma_f}{\gamma_g} (\sinh \gamma_f a) [\coth(b-a) \gamma_g]} \right\}, \quad a \leq x \leq b$$

where

$$\gamma_g = \sqrt{\frac{\omega \rho_g c_g}{k_g}} \quad \text{and} \quad \gamma_f = \sqrt{\frac{\omega \rho_f c_f}{k_f}}$$

Physical Interpretation

Solutions in closed form, utilizing separation of variables, are possible in the above cases because of the initial condition $\lim_{t \rightarrow -\infty} \theta(r, t) = 0$. Using instead the condition $\theta(r, 0) = 0$, which may seem more natural, prohibits the separation of variables and apparently complicates solving the problem. The physical interpretation of the situation described by the minus infinity condition used here may in fact be just as realistic as requiring a perfectly flat temperature distribution at time zero. Since $\theta(r, t)$ gives only the increase in temperature over ambient temperature, one can make the initial distribution as flat as he pleases simply by calling the "initial instant" some time prior to time zero where $\theta(r, t)$ is very small. The gross temperature gradient due to heat leakage out of the matrix as whole is neglected in this model.

Evaluation of Solutions

Calculation of the infinite series in the spherical solution is not as formidable a task as it may appear for many cases of interest. For example, for UO_2 pellets in graphite with $a \sim 4 \times 10^{-3}$, $b \sim 10^{-2}$ and $\omega < 10$, convergence to four significant figures requires only two nontrivial terms in the series. It is helpful to realize that regardless of geometry the result at r or x equal zero must be between $\theta_{\max}$ and $\theta_{\min}$.

where

$$\theta_{\max} = S_o e^{\omega t} / \rho_f c_{\min_f} \omega \text{ (fuel dissipates no heat)}$$

$$\theta_{\min} = S_o e^{\omega t} / \rho_f c_{f+g} \omega \text{ (fuel and graphite share heat instantly)}$$

and where $c_{\min_f}$ is a low estimate of the mean heat capacity of the fuel and

c_{f+g} is a high estimate of $\left(c_f + \frac{\text{mass of graph}}{\text{mass of fuel}} c_g \right)$.

REFERENCE

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Neutron Pulsing Possibilities in HTLTR

R. M. Humes

A pulsed source of neutrons has become an increasingly useful tool in reactor studies. One of these uses, that of measuring the sub-criticality of a given assembly, has apparently been tested only on hydrogenously moderated reactors. The presence of a pulsed source of reactor startup on HTLTR made it desirable to test this method on a graphite moderated assembly.

From the work of Garelis and Russell⁽¹⁾ it may be seen that the shutdown reactivity in dollars is given by

$$\$ = \frac{\alpha - \frac{k_1 \beta}{\ell}}{k_1 \beta / \ell} \quad (1)$$

where α is the prompt neutron decay constant and the term $\frac{k_1 \beta}{\ell}$ is obtained from the observed decay curve by evaluating

$$\int_0^\infty N_p e^{-\frac{k_1 \beta}{\ell} t} dt - \int_0^\infty N_p dt = \frac{N_d}{R} \quad (2)$$

where N_p is the prompt neutron counting rate, N_d is the (constant) delayed neutron counting rate, and R is the repetition rate of the pulsed source.

In the derivation of Equation (2) it has been assumed that $\lambda < R < \alpha$ where λ (which is equal to 4.5 sec^{-1}) is the decay constant of the shortest lived delayed neutron precursor. In actuality, since the two shortest lived groups are relatively unimportant, the first inequality is apparently well satisfied by $R = 10$. A difficulty in graphite would appear to arise in satisfying the second inequality, since $\alpha = \frac{1}{\ell} (1 - k + k\beta) \approx 10^{+3} (1 - k + k\beta) \approx 6 (1 + \$)$.

The experiment was performed on PCTR with a $\text{UO}_2\text{-PuO}_2$ lattice in the central test cavity. The pulsed neutron source was mounted approximately at the center of the north face of the reactor with its power supply

in the immediate vicinity of the source. The liquid scintillator neutron detector and its power supply were positioned in the opening of the moveable east face of the reactor. The signals from the detector were conducted to a time channel analyzer in the control room. The pulser which supplied both neutron pulse initiation and analyser sweep initiation was located in the analyser console.

A typical neutron decay curve includes background due to the presence of the reactor startup source which was necessarily partially inserted to avoid period scrams from the integrated effect of the pulsing.

Due to the improper operation of the first few channels of the TCA, causing the initial rise and approach to equilibrium to be lost, the data was analyzed by noting that in the first approximation the prompt neutron decay may be represented by

$$N_p = N_o e^{-\alpha t} . \quad (3)$$

Inserting Equation (3) into Equation (2), we find that

$$\frac{k_1 \beta}{\ell} \doteq \frac{N_d \alpha^2}{N_o R + N_d \alpha} , \quad (4)$$

or using Equation (1):

$$\beta \doteq \frac{N_o R}{N_d \alpha} . \quad (5)$$

Since the deviation of the initial portion of the curve from the behavior represented by Equation (3) will not cause a large change in the integrals of Equation (2), the values represented by (4) and (5) will very closely approximate the true values.

The data was analyzed with the assistance of the IBM 7090 program GLEX in fitting the experimental data to the form

$$N = (N_d + N_b) + N_p e^{-\alpha t} , \quad (6)$$

where N_b is the background caused by the presence of the startup source and Pu-Al fuel, and was evaluated by taking the average of a number of analyzer runs in the absence of the pulsed source.

Pulsing rates of 1, 5, and 10 sec^{-1} were investigated at each of the three reactor configurations:

- Safety discs withdrawn, 6 control rods closed, and 2 control rods open
- Safety discs withdrawn and all 8 control rods closed
- Safety discs inserted and all 8 control rods closed

The subcriticality estimates for each case, together with the values estimated from control rod calibrations are shown in Table I.

TABLE I
SHUTDOWN ESTIMATES BY PULSED SOURCE METHOD

Reactor Condition	Pulse Rate	α	$\frac{k_1 \beta}{\ell}$	Shutdown in \$		Rod Calibration Value	Bkg	N_d
6 Rods Closed	1	36.09 ± 1.49	21.75 ± 4.10	$0.66 \pm$	0.10	\$2.75	1596 ± 5	48 ± 7
	5	38.29 ± 4.79	21.45 ± 6.31	$0.79 \pm$	0.20	\$2.75	1620 ± 5	82 ± 16
	10	36.79 ± 4.19	10.49 ± 3.67	$2.51 \pm$	0.78	\$2.75	1823 ± 7	123 ± 35
8 Rods Closed	1	37.75 ± 0.60	4.92 ± 2.76	$6.67 \pm$	3.74	\$3.75	985 ± 2	9 ± 5
	5	38.97 ± 2.66	9.74 ± 2.53	$3.00 \pm$	0.72	\$3.75	2963 ± 5	40 ± 9
	10	39.32 ± 3.15	0.71 ± 3.17	$54.74 \pm$	246.33	\$3.75	1800 ± 5	4 ± 18
Safety Discs Inserted	1	84.24 ± 2.51	- -	- -		\$7.75	359 ± 2	-4 ± 4
	10	76.78 ± 2.58	4.99 ± 2.52	$14.40 \pm$	7.23	\$7.75	616 ± 3	14 ± 7

It should be noted that essentially all of the uncertainty in $k_1 \beta / \ell$ and \$ arises due to the uncertainty in N_d . This occurs because, with the reactor source partially inserted, the delayed neutron contribution from pulsing with a relatively weak ($\sim 10^5$ n/burst) source is found as a small difference between two large numbers. This may be taken to give some indication of difficulties which may be expected with plutonium fueled lattices in the test cavity. In the HTLTR case, however, the pulsed source strength at the reactor should be 10^4 - 10^5 higher and background due to the presence of Pu-Al should not be higher than experienced here and indeed in most cases should be less.

Thus, from the 6 rods closed case which is the only one having reasonable statistics, it would appear that there will be available to HTLTR a technique to measure subcritical reactivities which should be easily programable on PMACS. The range in which this technique will be useful will be dependent on the natural background caused by the loading of the reactor but should be fairly wide.

An additional use for the pulsed neutron source data, which was not checked in this case due to the poor statistics, may be the measurement of the delayed neutron fraction, β , and the prompt neutron lifetime, ℓ . This may be seen by noting that:

$$\ell = \frac{k_1 \beta}{\alpha} (1 + \$) \text{ and } \ell' = \frac{k_1' \beta'}{\alpha'} (1 + \$') \text{ and } k = 1 + \beta \$.$$

Assuming that measurements are made such that $\ell = \ell'$, $\beta = \beta'$ and equating we obtain:

$$\beta = \frac{\alpha' (1 + \$) - \alpha (1 + \$')}{\alpha' \$ (1 + \$) - \alpha \$' (1 + \$')} ,$$

$$\ell = \frac{\beta (1 + \$) (1 + \beta \$)}{\alpha} ,$$

which may be obtained from the evaluation of $\$$ noted above.

REFERENCE

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HEAVY MODERATOR REACTOR PHYSICS

Pu-Al - Polyethylene Experiments in the PCTR*

D. G. Albertson

A series of experiments with hydrogen-moderated plutonium systems has been performed in the Physical Constants Test Reactor PCTR.⁽¹⁾ The boron concentration required to reduce the infinite multiplication factor k_{∞} to unity was measured for various Pu^{240} concentrations, fuel plate thicknesses, and moderator-to-fuel ratios. These experiments yield basic reactor physics data which provide a check on the accuracy of calculational methods and parameters for these systems. The principal components for these experiments were Pu-Al (20 wt% plutonium, containing two different concentrations of Pu^{240} : 27 and 19%), pure polyethylene, and borated polyethylene (0.90 wt% natural boron). Table I summarizes the experimental configurations and results.

TABLE I

SUMMARY OF EXPERIMENTAL RESULTS

<u>H/Pu, atom/atom</u>	<u>Fuel Thickness, in.</u>	<u>Pu^{240} Concentration, % by Weight of Plutonium</u>	<u>B^{10}/Pu atom/atom</u>	<u>Cadmium Ratio, 0.02 in. diam Gold</u>
203	0.020	27	0.152 ± 0.003	1.61
202	0.060	27	0.135 ± 0.003	- -
200	0.100	27	0.114 ± 0.002	1.88
960	0.020	27	0.089 ± 0.002	5.17
214	0.020	19	0.197 ± 0.004	1.61

The materials used in the experiment were in the form of disks, 1.960 in. in diameter and 0.020 in. thick. These disks were loaded into aluminum cans (12.0 in. inside length, 2.0 in. ID and 1/16 in. wall thickness) in uniform arrangements. The cans were stacked in a close packed (touching) hexagonal array with a small air filled interstitial gap. The central 22 cans of a 61 can hexagonal lattice were loaded in this manner. The outer 24 cans contained U-Al and borated polyethylene acting as a flux

* This summary has been submitted for presentation at the November, 1965 American Nuclear Society Meeting in Washington, D. C.

modifying region between the PCTR and the experimental lattice. The remaining 15 cans were empty.

The experimental method was essentially a reactivity comparison of a section of the experimental lattice to a void. Since the void has, by definition, unit k_{∞} , this comparison^(1, 2) allows determination of the critical boron concentration for the lattice being studied. A requirement of this method is that the neutron flux be spatially constant throughout that part of the lattice to be compared to a void. In this way an infinite lattice is simulated. Figure 1 shows the neutron flux as measured by axial traverses with both bare and cadmium covered BF_3 chambers. A sensitivity measurement showed that a 14% error in the BF_3 cadmium ratio would induce an error in the critical boron concentration of about 2%, which corresponds to an error in k_{∞} of about 0.5%. An analysis of the experiment is described in Reference (3).

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2. D. G. Albertson. "U-Al - Polyethylene Experiment in the PCTR," Physics Research Quarterly Report - July, August, September, 1964, HW-84369. General Electric Company, Richland, Washington. October 15, 1964.
3. G. J. Busselman. Unpublished Data, Battelle Northwest, Richland, Washington, November, 1965.

FIGURE 1
 BF₃ Axial Traverse (taken adjacent to central can)
 27% Pu²⁴⁰, H/Pu = 200 (atom/atom) 0.100 in. Fuel Thickness

CRITICAL MASS PHYSICS

Foil Activation Studies in Two Plutonium Critical Systems

D. R. Skeen

INTRODUCTION

Experiments were performed in two separate critical assemblies to provide neutron energy dependent data for calculational comparisons. In these experiments, six sets each of gold and copper pin-type detectors were shielded with solutions containing various concentrations of boron and irradiated in two separate critical assemblies. Calculations were then made with the IBM 7090 codes, THERMOS ⁽¹⁾ and DTF ⁽²⁾, to calculate the relative activities of the foils, and comparisons were made with the experimental reaction rates.

EXPERIMENT

The experimental detector assembly consisted of a mixture of sodium tetraborate $[\text{Na}_2\text{B}_4\text{O}_7 \cdot 10 \text{H}_2\text{O}]$ mixed in a glycerin solution $[\text{C}_3\text{H}_5(\text{OH})_3]$ which surround the detectors (gold and copper) in a tube, as shown in Figure 1. The pertinent dimensions of the detectors and the detector assembly are listed below according to region number.

FIGURE 1

A Diagram of the Experimental Detector Tube Apparatus

Region 1

This region contained the detectors which were made from gold and copper wire. These detectors were in the shape of pins, 0.250 ± 0.003 in. long and 0.050 ± 0.001 in. in diameter. The detectors were placed end to end in the assembly to form a long cylinder which could then be separated for counting.

Regions 2 and 4

These two regions were made of aluminum tubing with a 0.028 in. wall thickness. The inner tube (Region 2) had a 0.125 in. OD and the outer tube a 0.500 in. OD. The inside tube was held in place by nylon spacers at intervals of about 6 in. This permitted a region of constant separation between the tube where the boron shielding was placed.

Region 3

This region contained the boron shielding (sodium tetraborate-glycerin) solutions with the concentrations as given in Table I.

TABLE I
BORON SHIELDING SOLUTION CONCENTRATIONS

<u>Solution</u>	<u>Sodium Tetraborate*</u> <u>$\text{Na}_2\text{B}_4\text{O}_7 \cdot 10 \text{H}_2\text{O}$</u> <u>(g/cc)</u>	<u>Glycerin</u> <u>$\text{C}_3\text{H}_5(\text{OH})_3$</u> <u>(g/cc)</u>
1	0.0	1.260
2	0.0554	1.208
3	0.1152	1.152
4	0.2207	1.070
5	0.3774	0.943
6	0.6082	0.785

* Contains natural boron

The gold and copper detectors, which were shielded with these solutions, were irradiated in the two critical systems described below:

Irradiations in a Fully Reflected Sphere

These experiments involved a fully reflected stainless steel sphere (Figure 2) containing a plutonium nitrate solution. The experimental detector assembly was placed along a diameter of the sphere in a 0.50 in. ID stainless steel reentrant tube. The monitor detectors for these irradiations were placed about 3 in. away from the core in the water reflector. The parameters of this experiment are listed in Figure 2. The detectors were then gamma counted with a NaI scintillation crystal and the counting rates in the critical core (relative to the monitor) for each boron shield were fitted, by a least squares process, to the function

$$A(r) = \frac{A_0 \sin Br}{Br}$$

where $A(r)$ is the activity of each detector at position r , B is a constant and A_0 is the activity at the sphere center. This permitted the average activity for the detector-cover combination to be calculated from an average over the spherical core. The tables of these values may be seen in Table II for the gold and copper detectors.

DIMENSIONS

SS Sphere ID = 15.21 in.
 Sphere Wall Thickness = 0.048 in.
 SS Reentrant Tube ID = 0.510 in.
 Reentrant Tube Thickness = 0.035 in.

CORE CONCENTRATIONS

Pu (4.6% Pu²⁴⁰) = 25.93 g/liter
 H⁺ = 0.521 g/liter
 NO₃ = 59.97 g/liter
 H₂O = 976.58 g/liter
 H/Pu Ratio = 1000

FIGURE 2

A Diagram of the Fully Reflected Spherical Critical Assembly

TABLE II
CENTER COUNT RATES (LEAST SQUARES AVERAGED)
FOR THE PLUTONIUM NITRATE SOLUTION CORE

Solution Number	<u>Gold</u>		<u>Copper</u>	
	$A_o \pm \Delta A_o$	% Error	$A_o \pm \Delta A_o$	% Error
1	3.638 ± 0.044	1.21	6.818 ± 0.202	2.96
2	2.733 ± 0.033	1.20	4.533 ± 0.107	2.36
3	2.401 ± 0.035	1.46	3.500 ± 0.093	2.65
4	1.810 ± 0.027	1.49	2.408 ± 0.044	1.84
5	1.365 ± 0.024	1.79	1.752 ± 0.042	2.39
6	1.000 ± 0.013	1.29	1.000 ± 0.031	3.06

The errors quoted in Table II are from the standard deviation errors in the least squares fit and the standard deviation errors in the counting rates of the monitor foils.

Irradiation in an End Reflected Rectangular Parallelopiped

In this set of irradiations, the detector assembly was placed along a center line of the end reflected rectangular parallelopiped through one side of the Lucite reflector as shown in Figure 3. The core of this critical assembly was made up of plutonium oxide mixed in polystyrene. The parameters for this system are also given in Figure 3.

DIMENSIONS

Core Size = 12.21 x 12.21 x 8.65 in.
 Reflector Thickness = 6.0 in.

CORE CONCENTRATIONS

Core Material = $\text{PuO}_2(\text{CH})_n$
 $\text{Pu}(2.2\% \text{ Pu}^{240}) = 1.120 \text{ g/cc}$
 $\text{CH} = 0.913 \text{ g/cc}$

REFLECTOR CONCENTRATIONS

$\text{C}_5\text{H}_8\text{O}_2 = 1.20 \text{ g/cc}$

FIGURE 3

A Diagram of the End Reflected Rectangular Critical Assembly

The monitor detectors for these irradiations were placed in the Lucite reflector approximately at the peak of the thermal flux (~ 2 in. away from the core) on the side opposite the foil assembly.

The detector data was processed in the same manner as those from the spherical vessel, except that the least squares fit was made to the function:

$$A(r) = A_0 \cos Br.$$

Values of A_0 are given in Table III.

TABLE III
CENTER LINE COUNT RATES (LEAST SQUARES AVERAGED)
FOR THE PLUTONIUM-POLYSTYRENE CORE

Solution Number	<u>Gold</u>		<u>Copper</u>	
	$A_0 \pm \Delta A_0$	% Error	$A_0 \pm \Delta A_0$	% Error
1	1.361 ± 0.026	1.93	1.575 ± 0.031	1.96
2	1.304 ± 0.017	1.33	1.421 ± 0.032	2.29
3	1.261 ± 0.020	1.60	1.355 ± 0.023	1.71
4	1.160 ± 0.014	1.22	1.167 ± 0.019	1.62
5	1.081 ± 0.013	1.23	1.102 ± 0.018	1.63
6	1.000 ± 0.008	0.80	1.000 ± 0.013	1.39

The plots of all experimental reaction rates are shown in Figure 4.

CALCULATION

The problem is to calculate the average flux within the small region of detector material, due to the flux depression and spectrum shifting within the covering material which shields the reactors. For this study the IBM 7090 transport theory programs, DTF and THERMOS, were chosen for the epithermal and thermal (< 0.683 eV) neutron flux ranges, respectively.

The calculation was set up as a five region problem in cylindrical geometry. The first four regions were those of the detectors, experimental detector apparatus, boron solutions, and the stainless steel reentrant tube

FIGURE 4
Experimental Reaction Rates
as a Function of Cover Absorption Concentration

(in the case of the solution experiment). The fifth and outermost region was the fissionable medium. The general problem was run as an infinite cell case, so that when the flux, as a function of position and energy, became constant (moving from the inside of the cell outward) this would correspond to the unperturbed flux of the system. The outer radius of Region 5 was set at 12.6 cm, and from an examination of Figure 5, it is evident that this was amply large enough to include all flux depression caused by the insertion of the detector tube.

From these computer calculations, values of $\bar{\phi}/\phi_0$ (the ratio of the average flux in the detector to the unperturbed flux) were calculated for each of 18 energy groups (including one thermal group as defined by Thermos). The total reaction rates for each of the detector-shield combinations were then calculated from the unperturbed flux as calculated by the DTF code. Comparisons of the calculated and experimental reaction rates are shown in Table IV.

FIGURE 5

Thermos Values of $\varphi_i / \varphi_{\text{outside}}$ as a Function of Radius

From an examination of Table 4, it is seen that the calculated reaction rates for the gold detectors are not in good agreement with experiment. The observed differences indicate that the calculated reaction rates do not fall off as fast (with increasing boron concentration) as the experimental reaction rates. This error could come from too large an epithermal contribution to the calculated reaction rates. Consider the ratio of reaction rates

$$\frac{R_1}{R_2} = \frac{\sigma_{\text{th}} \varphi_1^{\text{th}} + \sigma_{\text{epi}} \varphi_1^{\text{epi}}}{\sigma_{\text{th}} \varphi_2^{\text{th}} + \sigma_{\text{epi}} \varphi_2^{\text{epi}}}$$

where the subscript 2 corresponds to a case of more boron shielding than the subscript 1. Since boron is principally a $1/V$ absorber, an increase in the boron concentration from solution 1 to solution 2 would result in a larger change in φ^{th} than in φ^{epi} . If the value of σ_{epi} used in the reaction rate

TABLE IV

A COMPARISON OF EXPERIMENTAL AND CALCULATED REACTION RATES

Boron Solution	Spherical Experiment			Boron Solution	Rectangular Experiment		
	Experimental Count Rates	Calculated Count Rates	Deviation %		Experimental Count Rates	Calculated Count Rates	Deviation %
	Gold Detectors				Gold Detectors		
1	3.638	3.402	- 6.49	1	1.361	1.291	- 5.14
2	2.733	2.722	- 0.40	2	1.304	1.251	- 4.06
3	2.401	2.265	- 5.66	3	1.261	1.217	- 3.50
4	1.810	1.788	- 1.22	4	1.160	1.174	+ 1.21
5	1.365	1.398	+ 2.42	5	1.081	1.121	+ 3.70
6	1.000	1.096	+ 9.60	6	1.000	1.065	+ 6.50
	Copper Detectors				Copper Detectors		
1	6.818	6.363	- 6.67	1	1.575	1.561	- 0.89
2	4.533	4.636	+ 2.27	2	1.421	1.442	+ 1.48
3	3.500	3.552	+ 1.49	3	1.355	1.322	- 2.44
4	2.408	2.468	+ 2.49	4	1.167	1.205	+ 3.26
5	1.752	1.646	- 6.05	5	1.102	1.093	- 0.81
6	1.000	1.053	+ 5.30	6	1.000	0.991	- 0.90

calculation were too large, then the calculated change in reaction rate (upon applying additional boron shielding) would be smaller than the true change.

In the DTF calculation the value used for 5 eV gold resonance absorption was calculated from BNL-325⁽³⁾ resonance parameters averaged over a $1/E$ flux spectrum (which would be the resonance cross section for a dilute material). Since the 0.05 in. gold rod is far from dilute, the calculated self-shielding within the gold would tend to be too small and, hence, the effective resonance cross section too large.

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MONTE CARLO Calculated Values
for the Critical Mass of Pu²³⁸ as a Function of Hydrogen
to Fuel Atomic Ratios and Pu²³⁹ Enrichment

L. L. Carter

The use of Pu²³⁸ as a heat source has resulted in the production of significant quantities of this isotope. The critical mass of Pu²³⁸ in various metal, oxide, and solution mixtures becomes important from a nuclear safety standpoint when handling or storing this isotope.

While Pu²³⁸ is an even-even isotope and has a low thermal fission cross section, recent cross section measurements by Butler, et al., ⁽¹⁾ have shown it to have a larger fission cross section at one MeV than the corresponding fission cross section of Pu²³⁹. Previous calculations on bare metal spheres have indicated a Pu²³⁸ critical mass somewhat less than that of Pu²³⁹. ⁽²⁾ The purpose of this paper is to repeat the previous calculations with a new cross section set. In addition, the critical mass of PuO₂ will be calculated, the change in the critical mass due to enriching the Pu²³⁸ with 20% Pu²³⁹ will be studied, and the change in the critical mass due to the addition of small quantities of water will be evaluated.

The MONTE CARLO calculations were made with the SMC computer code for three region concentric spherical geometry. The energy loss of the neutron due to inelastic scattering events was determined from the evaporation model with the scattering assumed to be isotropic in the center of mass system. Elastic scattering events below 0.02 MeV were assumed to be isotropic in the center of mass system. Above 0.02 MeV, the elastic scattering cross section as a function of angle was defined by the first six terms of a Legendre series.

$$\sigma_s(E, \mu) = \frac{\sigma_s(E)}{2} \sum_{\ell=0}^6 a_{\ell} P_{\ell}(\mu)$$

Average energy group cross sections were computed for 1440 energy groups eliminating the need for time consuming cross section interpolations. The Pu²³⁹ cross sections were obtained from the RBU basic library. ⁽³⁾ The

Pu^{238} cross sections were obtained from a recent update of the RBU library which used the most recent Pu^{238} cross section measurements available. (1, 4, 5) The same anisotropic scattering coefficients were used for both Pu^{238} and Pu^{239} . (6) The anisotropic scattering coefficients for oxygen were obtained from a recent compilation by Joanou. (7) The identical set of Pu^{239} cross sections was used in a previous calculation on a Pu^{239} -metal reactor. (2) The computed critical mass was about 9% larger than the measured value, indicating that the Pu^{239} cross sections are relatively reliable for fast reactor calculations.

The isotope densities used in these calculations are summarized in Table I. These densities were obtained by linear volume fraction weighting the base densities:

Plutonium Metal	19.6 g/cc
Plutonium Oxide	11.345 g/cc
Water	1.000 g/cc

A summary of the results of the calculations is given in Table II. The MONT CARLO calculated k_{eff} values listed in column four correspond to the spherical radii listed in column two. The perturbation in these radii necessary to yield a critical system was estimated from one group buckling equation:

$$R_c = (R + \lambda) \sqrt{\frac{k_\infty - k_{\text{eff}}}{(k_\infty - 1) k_{\text{eff}}}} - \lambda \quad (1)$$

where

k_{eff} = Multiplication factor as calculated by MONTE CARLO for sphere of radius R.

k_∞ = Infinite multiplication factor calculated by MONTE CARLO.

λ = Extrapolation distance (assumed to be 3 cm).

R_c = Critical sphere radius.

It should be noted that the critical radius obtained from Equation (1) is relatively insensitive to the extrapolation distance, since the MONTE CARLO k_{eff} calculations are near critical.

TABLE I
ISOTOPE DENSITIES USED IN THE MONTE CARLO
CALCULATIONS SUMMARIZED IN TABLE II

(Densities are in atoms/b-cm)

Description	Pu ²³⁸	Pu ²³⁹	H	O
Plutonium Metal (100% Pu ²³⁸)	0.049596	0.00000	0.00000	0.00000
Plutonium Metal (80% Pu ²³⁸ , 20% Pu ²³⁹)	0.039677	0.009919	0.00000	0.00000
Plutonium-Water Mixture, H/Pu = 5 (100% Pu ²³⁸)	0.010535	0.00000	0.052673	0.026337
Plutonium-Water Mixture, H/Pu = 5 (80% Pu ²³⁸ , 20% Pu ²³⁹)	0.008428	0.002107	0.052673	0.026337
Plutonium-Water Mixture, H/Pu = 50 (100% Pu ²³⁸)	0.001303	0.00000	0.065124	0.032562
Plutonium-Water Mixture, H/Pu = 50 (80% Pu ²³⁸ , 20% Pu ²³⁹)	0.001042	0.0002605	0.065124	0.032562
Plutonium-Water Mixture, H/Pu = 200 (80% Pu ²³⁸ , 20% Pu ²³⁹)	0.0002657	0.00006643	0.066432	0.033216
PuO ₂ (100% Pu ²³⁸)	0.025304	0.00000	0.00000	0.050608
PuO ₂ (80% Pu ²³⁸ , 20% Pu ²³⁹)	0.020243	0.005061	0.00000	0.050608
PuO ₂ -Water Mixture, H/Pu = 1 (100% Pu ²³⁸)	0.018358	0.00000	0.018358	0.045895
PuO ₂ -Water Mixture (80% Pu ²³⁸ , 20% Pu ²³⁹ , H/Pu = 1)	0.014686	0.003672	0.018358	0.045895

TABLE II
MONTE CARLO CALCULATIONS ON PLUTONIUM SYSTEMS
WITH HIGH Pu²³⁸ CONTENT
(Bare Spheres)

Description	Radius, cm	Mass Pu, kg	MONTE CARLO Calculated, k _{eff}	MONTE CARLO Estimated Critical Radius, cm	MONTE CARLO Estimated Critical Pu Mass, kg
Plutonium Metal (100% Pu ²³⁸)	4.100	5.658	0.9993 ± 0.0103	4.104	5.68 ± 0.24
Plutonium Metal (80% Pu ²³⁸ , 20% Pu ²³⁹)	4.300	6.527	1.0089 ± 0.0141	4.250	6.30 ± 0.33
Plutonium-Water Mixture, H/Pu = 5 (100% Pu ²³⁸)	∞	∞	0.8920 ± 0.028	--	--
Plutonium-Water Mixture, H/Pu = 5 (80% Pu ²³⁸ , 20% Pu ²³⁹)	16.00	68.89	1.0133 ± 0.036	15.67	64.7 ± 12.4
Plutonium-Water Mixture, H/Pu = 50 (100% Pu ²³⁸)	∞	∞	0.209 ± 0.046	--	--
Plutonium-Water Mixture, H/Pu = 50 (80% Pu ²³⁸ , 20% Pu ²³⁹)	∞	∞	1.002 ± 0.047	--	--
Plutonium-Water Mixture, H/Pu = 200 (80% Pu ²³⁸ , 20% Pu ²³⁹)	∞	∞	0.779 ± 0.044	--	--
PuO ₂ (100% Pu ²³⁸)	7.35	16.63	1.027 ± 0.025	7.136	15.2 ± 1.2
PuO ₂ (80% Pu ²³⁸ , 20% Pu ²³⁹)	7.80	19.88	1.028 ± 0.013	7.570	18.2 ± 0.8
PuO ₂ -Water, H/Pu = 1 (100% Pu ²³⁸)	14.00	83.38	1.169 ± 0.023	11.57	47.1 ± 7.1
PuO ₂ -Water, H/Pu = 1 (80% Pu ²³⁸ , 20% Pu ²³⁹)	11.20	42.70	1.072 ± 0.015	10.36	33.8 ± 1.9

The critical mass of plutonium listed in column 6 of Table II represents the total plutonium critical mass ($\text{Pu}^{238} + \text{Pu}^{239}$). This critical mass rises sharply as the H/Pu atomic ratio increases due to the effective Pu^{238} fission cutoff at about 0.1 MeV. The critical mass of plutonium oxide containing hydrogen at an H/Pu atomic ratio of unity was evaluated, since some moisture is generally present in the oxide. Even this small amount of hydrogen increases the critical mass by about a factor of two.

Two calculations were also made for spherical cores reflected with 10 cm of stainless steel. The reflected critical mass of Pu^{238} metal (19.6 g/cc) was calculated to be 3.53 ± 0.25 kg, or about 62% of the unreflected critical mass. The reflected critical mass of $\text{Pu}^{238}\text{O}_2$ (11.345 g/cc) was calculated to be 7.6 ± 1.0 kg plutonium or about 50% of the unreflected critical mass.

In summary, the spherical critical mass of Pu^{238} in a metal or oxide mixture has been found to be somewhat less than the corresponding critical mass of Pu^{239} . The unreflected critical mass of Pu^{238} metal (19.6 g/cc) was calculated to be 5.68 ± 0.24 kg, while the unreflected critical mass of $\text{Pu}^{238}\text{O}_2$ (11.345 g/cc) was calculated to be 15.2 ± 1.2 kg. The error limits given represent 90% confidence limits for the statistical fluctuations in the MONTE CARLO analysis. These error limits do not include other errors which might be present, such as the cross section set that was used.

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Resonance Absorber Admixed in the Moderator
of Heterogeneous Arrays

C. R. Richey

Experimental data to establish criticality control specifications for enriched uranium rods undergoing dissolution are extremely limited, (1) hence accurate theoretical techniques are needed. A principal difficulty in treating the problem theoretically is that the resonance absorbing U^{238} is admixed in the aqueous solution in which the rods are immersed. This paper describes an extension of the Adler-Nordheim method (2) for treating resonance capture by an absorbing lump to include resonance capture in the moderator.

THEORETICAL FORMULATION

Resonance Capture by an Absorbing Lump

Following Nordheim, (3) the average collision density in an absorbing lump imbedded in an absorbing moderator is

$$F_O(u) = (1 - P_O) \left[\frac{1}{\alpha_O} \int_{u - \Delta_O}^u F_O(u') \frac{\sigma_{S_O}}{\sigma_{t_O} + \sigma_m} du' + \frac{1}{\alpha_m} \int_{u - \Delta_m}^u F_O(u') \frac{\sigma_m}{\sigma_{t_O} + \sigma_m} du' \right] \quad (1)$$

$$+ P_1 \left[\frac{1}{\alpha_1} \int_{u - \Delta_1}^u F_1(u') \frac{\sigma_{S_1}}{\sigma_{t_1}} du' + \frac{1}{\alpha_O} \int_{u - \Delta_O}^u F_1(u') \frac{C \sigma_{S_O}}{\sigma_{t_1}} du' \right]$$

$$\text{with } \Delta_i = \log \frac{1}{1 - \alpha_i} \quad (2)$$

$$\text{and } u = \log \frac{E_1}{E} \quad (3)$$

where E_1 is an energy above which the asymptotic form of the collision density holds,

$$F_{as} = \sigma_t \frac{e^u}{E_1}. \quad (4)$$

The cross sections in Equation (1) are in reference to one fuel absorber nucleus; i. e.,

$$\sigma_m = \frac{N_m}{N_o} (\sigma_m)_{\text{actual}}, \quad \sigma_{s_1} = \frac{N_1 V_1}{N_o V_o} (\sigma_{s_1})_{\text{actual}},$$

$$\sigma_t = \sigma_{t_o} + \frac{N_m}{N_o} (\sigma_m)_{\text{actual}} = \sigma_{t_o} + \sigma_m,$$

$$\sigma_{t_1} = \frac{N_o' V_1 \sigma_{t_o} + N_1 V_1 (\sigma_{s_1})_{\text{actual}}}{N_o V_o} = C \sigma_{t_o} + \sigma_{s_1}$$

where $C = \frac{N_o' V_1}{N_o V_o}$ with N_o and N_o' being the atom densities of the absorber

in the lumped volume (V_o) and the moderator volume (V_1), respectively. The other notation is that of Nordheim. ⁽³⁾

If the resonance is narrow compared to the average energy loss in collision with the moderator or admixture in the absorbing lump, Equation (1) can be simplified by inserting the asymptotic form, Equation (5), for the corresponding integral terms. From the reciprocity theorem

$$P_1 = \frac{\sigma_{t_o} + \sigma_m}{\sigma_{t_1}} P_o \quad (5)$$

and the integral equation becomes

$$F_o(u) = (1 - P_o) \left[\frac{1}{\alpha_o} \int_{u - \Delta_o}^u F_o(u') \frac{\sigma_{t_o}}{\sigma_{t_o} + \sigma_m} du' + \sigma_m \frac{e^u}{E_1} \right] + \left(\frac{\sigma_{t_o} + \sigma_m}{\sigma_{t_1}} \right) P_o \left[\sigma_{s_1} \frac{e^u}{E_1} + \frac{1}{\alpha_o} \int_{u - \Delta_o}^u F_1(u') \frac{C \sigma_{s_o}}{\sigma_{t_1}} du' \right]. \quad (6)$$

In the narrow resonance approximation (NR), the absorption is assumed to be large in an interval small compared to $\alpha_o E$, the maximum energy loss in an absorber collision. Applying the (NR) approximation, the quantities inside the integral in Equation (7) are replaced by their asymptotic values, Equation (4), and $\sigma_{s_o} = \sigma_{t_o} = \sigma_{p_o}$. On rearranging

$$F_o(u) = (1 - P_o) \sigma_{p_o} \frac{e^u}{E_1} + \left[\sigma_m + P_o \sigma_{t_o} \right] \frac{e^u}{E_1} - P_o (\sigma_{t_o} + \sigma_m) \frac{C (\sigma_{t_o} - \sigma_{p_o})}{\sigma_{t_1}} \frac{e^u}{E_1} \quad (7)$$

The resonance integral is defined as ⁽³⁾

$$I = E_1 \int F_o(u) \frac{\sigma_{a_o}}{\sigma_{t_o} + \sigma_m} e^{-u} du \quad (8)$$

and on substitution of Equation (7) becomes in the (NR) approximation

$$I_{NR}^A = \int \frac{\sigma_{a_o} [\sigma_{p_o} + \sigma_m + P_o (\sigma_{t_o} - \sigma_{p_o})]}{\sigma_{t_o} + \sigma_m} du - \int \sigma_{a_o} \frac{C \sigma_{r_o}}{\sigma_{t_1}} P_o du \quad (9)$$

where $\sigma_{r_o} = \sigma_{t_o} - \sigma_{p_o}$. The first term on the right hand side of

Equation (9) is the usual expression for the resonance integral in the (NR) approximation; the second term on the right hand side in effect modifies the surface absorption of the lump for resonance shielding by the absorbing moderator.

For the energy dependence of the cross sections, the usual Doppler-broadened, Breit-Wigner line shape is taken with interference scattering being ignored, ⁽²⁾ i.e.,

$$\sigma_a = \sigma_o \psi \Gamma_Y / \Gamma,$$

$$\sigma_{rs} = \sigma_o \psi \Gamma_Y / \Gamma,$$

$$\sigma_o = 4\pi g \lambda^2 \Gamma_n / \Gamma$$

where the line shape function ψ is given by

$$\psi(\xi, x) = \frac{\xi}{\sqrt{4\pi}} \int_{-\infty}^{+\infty} dy \frac{\exp\left[-\frac{\xi}{4}(x-y)^2\right]}{1+y^2}$$

with

$$x = \frac{2(E-E_i)}{\Gamma} \text{ and } \xi = \left[\frac{(A+1)\Gamma^2}{E_i KT} \right]^{1/2}.$$

The resonance integral for a specific resonance at E_i is therefore written in the notation of Adler-Nordheim. ⁽²⁾

$$RI_{NR}^A = \frac{\sigma_{p_o} + \sigma_m}{E_i} \Gamma_Y J(\xi, \beta) + \frac{\sigma_o \Gamma}{E_i} L[t, \xi, \beta, \beta_m] - L(t, \xi, \beta_m, \beta) \quad (10)$$

where

$$J(\xi, \beta) = \int_0^\infty dx \frac{\psi}{\psi + \beta}, \quad (11)$$

$$L(t, \xi, \beta_m, \beta) = \int_0^\infty dx P_o \left[t \left(\frac{\psi}{\beta} + 1 \right) \right] \frac{\psi^2}{\psi + \beta_m}, \quad (12)$$

$$t = N_o (\sigma_{p_o} + \sigma_m) \bar{r}, \quad \beta = \frac{\sigma_{p_o} + \sigma_m}{\sigma_o}, \text{ and } \beta_m = \frac{\sigma_{p_o} + C^{-1} \sigma_{s1}}{\sigma_o}.$$

For resonances whose absorption is large over an energy decrement greater than $\alpha_0 E$, a scattering collision with an absorber atom will not remove the neutron from the resonance region. In the Narrow Resonance-Infinite Mass Approximation (NRIA) wide resonances are treated as narrow resonances with the scattering parameters within the resonance region equated to zero: i. e.,

$$\sigma_{p_0} = 0.0$$

and

$$\sigma_0 \rightarrow \sigma_0 \frac{\Gamma_Y}{\Gamma}.$$

The resonance integral becomes in the (NRIA) approximation

$$RI_{NRIA}^A = \frac{\sigma_m \Gamma}{E_i} \left[J(\xi, \beta') + \frac{1}{3} L(t', \xi, \beta', \beta') - \frac{1}{3_m} L(t', \xi, \beta_m', \beta') \right] \quad (13)$$

where

$$\beta' = \frac{\sigma_m \Gamma}{\sigma_0 \Gamma_Y}$$

$$\beta_m' = \frac{C^{-1} \sigma_{s_1}}{\sigma_0} \frac{\Gamma}{\Gamma_Y}$$

$$t' = N_0 \sigma_m \bar{r}.$$

Resonance Absorber Admixed in the Moderator

The average collision density in the moderator is

$$\begin{aligned} F_1(u) = (1 - P_1) & \left[\frac{1}{\alpha_1} \int_{u-\Delta_1}^u F_1(u') \frac{\sigma'_{s_1}}{\sigma_{t_0} + \sigma'_{s_1}} du' + \frac{1}{\alpha_0} \int_{u-\Delta_0}^u F_1(u') \frac{\sigma_{s_0}}{\sigma_{t_0} + \sigma'_{s_1}} du' \right] \\ & + P_0 \left[\frac{1}{\alpha_0} \int_{u-\Delta_0}^u F_0(u') \frac{C^{-1} \sigma_{s_0}}{C^{-1} \sigma_{t_0} + \sigma'_m} du' + \frac{1}{\alpha_m} \int_{u-\Delta_m}^u F_0(u') \frac{\sigma'_m}{C^{-1} \sigma_{t_0} + \sigma'_m} du' \right] \end{aligned}$$

where all cross sections are now in reference to one absorber nucleus in the moderator; i. e.,

$$\sigma'_m = \frac{N_m V_o}{N'_o V_1} (\sigma_{sm})_{\text{actual}} \quad \sigma'_{s_1} = \frac{N_1}{N'_o} (\sigma_{s_1})_{\text{actual}}$$

$$\sigma'_{t_o} = (\sigma_{t_o})_{\text{actual}} \quad \sigma'_{p_o} = (\sigma_{p_o})_{\text{actual}}$$

and $C^{-1} = \frac{N_o V_o}{N'_o V_1}$. Proceeding as in the previous section, the resonance

integral in the (NR) approximation is

$$RI_{NR}^M = \int \sigma_{a_o} \frac{\sigma'_{s_1} + \sigma_{p_o}}{\sigma'_{s_1} + \sigma_{t_o}} du - (C^{-1} \sigma'_{s_1} - \sigma'_m) \int \frac{\sigma_{a_o} \sigma_{r_o}}{(\sigma'_{s_1} + \sigma_{t_o})^2} P_o du. \quad (15)$$

The first term on the right hand side of Equation (13) is the usual expression for a homogeneous mixture.⁽³⁾ Two effects due to the lumped absorber are evident in the second term of Equation (13); 1) capture in the lump absorber that would have otherwise been captured in the moderator absorber and 2) capture in the moderator absorber of neutrons transferred into resonance by the admixture in the absorbing lump.

Substitution of the Doppler-broadened, Briet-Wigner line shapes gives the resonance integral for a narrow resonance at E_i ,

$$RI_{NR}^M = \frac{\sigma_{p_o} + \sigma_m}{E_i} \Gamma_\gamma J(\xi, \beta_m) - \frac{C^{-1} \sigma_{s_1} - \sigma_m}{E_i} \Gamma_\gamma K(t, \xi, \beta_m, \beta) \quad (16)$$

where

$$K(t, \xi, \beta_m, \beta) = \int_0^\infty dx \frac{\psi^2}{(\psi + \beta_m)^2} P_o \left[t \left(\frac{\psi}{\beta} + 1 \right) \right] \quad (17)$$

$$\beta_m = \frac{\sigma_{p_o} + \sigma'_m s_1}{\sigma_o}$$

$$\beta = \frac{\sigma_{p_o} + C \sigma'_m}{\sigma_o}$$

$$t = N_o (\sigma_{p_o} + C \sigma'_m) \bar{r}.$$

The corresponding resonance integral for a wide resonance at E_i is

$$RI_{NRIA}^M = \frac{\sigma_m}{E_i} \Gamma_\gamma J(\xi, \beta'_m) - \frac{(C^{-1} \sigma'_m s_1 - \sigma'_m)}{E_i} \Gamma_\gamma K(t', \xi, \beta'_m, \beta')$$

where

$$\beta'_m = \frac{\sigma'_m s_1}{\sigma_o} \frac{\Gamma}{\Gamma_\gamma}$$

$$\beta' = \frac{C \sigma'_m}{\sigma_o} \frac{\Gamma}{\Gamma_\gamma}$$

$$t' = N_o C \sigma'_m \bar{r}.$$

Unresolved Resonances

Following Adler, et al, the statistical distribution of neutron widths given by Porter and Thomas is taken into account for unresolved resonances;⁽²⁾

$$P(y) dy = \frac{1}{\sqrt{2\pi}} \frac{e^{-y/2}}{\sqrt{y}} dy$$

where $y = \Gamma_n^0 / \langle \Gamma_n^0 \rangle$ and $\langle \Gamma_n^0 \rangle$ is the average reduced neutron widths. For

unresolved resonances, the (NR) approximation holds and the resonance integral for the lumped absorber, Equation (10), becomes

$$\begin{aligned}
 RI_{\text{unres}}^A = & \frac{\Gamma_Y}{D} \left\{ (\sigma_{p_o} + \sigma_m) \left[\int_{E_c}^{\infty} \frac{dE}{E} \frac{1}{\sqrt{2\pi}} \int_0^{\infty} dy \frac{e^{-y/2}}{\sqrt{y}} J(\xi, \beta) \right. \right. \\
 & + \left. \int_{E_c}^{\infty} \frac{dE}{E} \frac{1}{\sqrt{2\pi}} \int_0^{\infty} dy \frac{e^{-y/2}}{\sqrt{y}} \frac{1}{\beta} L(t, \xi, \beta, \beta) \right] \\
 & - (\sigma_{s_1} + \sigma_{p_o}) \int_{E_c}^{\infty} \frac{dE}{E} \frac{1}{\sqrt{2\pi}} \int_0^{\infty} dy \frac{e^{-y/2}}{\sqrt{y}} \frac{1}{\beta_m} L(t, \xi, \beta_m, \beta) \left. \right\}
 \end{aligned}$$

where D is the average level spacing, E_c the cut-off energy above which the resonances are considered as unresolved and

$$\begin{aligned}
 \xi &= [(A+1)/kT]^{1/2} \left(\langle \Gamma_n^o \rangle_y + \frac{\Gamma_Y}{E} \right) \\
 \beta_m &= \left(\frac{\sigma_{p_o} + C^{-1} \sigma_{s_1}}{2.6 \times 10^6} \right) \left(E + \frac{\Gamma_Y \sqrt{E}}{y \langle \Gamma_n^o \rangle} \right) \\
 \beta &= \left(\frac{\sigma_{p_o} + \sigma_m}{2.6 \times 10^6} \right) \left(\frac{\Gamma_Y \sqrt{E}}{y \langle \Gamma_n^o \rangle} \right)
 \end{aligned}$$

Choosing ξ as the integration variable, the averages for J, L/β_m and L/β are

$$\bar{J} = \frac{1}{\sqrt{2\pi}A} \int_B^{\infty} \frac{d\xi}{\sqrt{\xi-B}} J(\xi, \beta) \exp \left[-(\xi-B)/2A \right]$$

$$\left(\frac{\bar{L}}{\bar{\beta}_m}\right) = \frac{1}{\sqrt{2\pi A'}} \int_B^{\infty} \frac{d\xi}{\xi G_1} \sqrt{\xi - B} L(t, \xi, \beta_m, \beta) \exp \left[-(\xi - B)/2A' \right]$$

$$\left(\frac{\bar{L}}{\bar{\beta}}\right) = \frac{1}{\sqrt{2\pi A'}} \int_B^{\infty} \frac{d\xi}{\xi G_2} \sqrt{\xi - B} L(t, \xi, \beta, \beta) \exp \left[-(\xi - B)/2A' \right]$$

where

$$\begin{aligned} \xi &= A' y + B, & \beta_m &= G_1 \frac{\xi}{\xi - B}, & \beta &= G_2 \frac{\xi}{\xi - B} \\ A' &= \left(\frac{A+1}{kT} \right)^{1/2} < \Gamma_n^0 > & B &= \left(\frac{A+1}{kT} \right)^{1/2} \frac{\Gamma_Y}{\sqrt{E}} \\ G_1 &= \frac{\sigma_{s_1} + \sigma_{p_0}}{2.6 \times 10^6} & G_2 &= \frac{\sigma_{p_0} + \sigma_m}{2.6 \times 10^6} E \end{aligned}$$

The resonance integral for the lumped absorber for unresolved resonance can, therefore, be written

$$RI_{unres}^M = \frac{\Gamma_Y}{D} \left\{ (\sigma_{p_0} + \sigma_m) \left[\int_0^{\infty} \bar{J} \, dn + \int_0^{\infty} \left(\frac{\bar{L}}{\bar{\beta}} \right) \, dn \right] - (\sigma_{s_1} + \sigma_{p_0}) \int_0^{\infty} \frac{\bar{L}}{\bar{\beta}_m} \, dn \right\}$$

where $n = \log \frac{E}{E_c}$. The corresponding unresolved resonance integral for the moderator absorber is

$$RI_{unres}^M = \frac{\Gamma_Y}{D} \left[(\sigma_{p_0} + \sigma'_{s_1}) \int_0^{\infty} \bar{J} \, dn - (C^{-1} \sigma'_{s_1} - \sigma_m) \int_0^{\infty} \frac{\bar{K}}{\bar{\beta}_m} \, dn \right]$$

where

$$\left(\frac{\bar{K}}{\bar{\beta}_m}\right) = \frac{1}{\sqrt{2\pi A'}} \int_B^{\infty} \frac{d\xi}{\xi G_1} \sqrt{\xi - B} K(t, \xi, \beta_m, \beta) \exp \left[-(\xi - B)/2A' \right],$$

$$G_1 = \frac{\sigma_{s_1} + \sigma_{p_o}}{2.6 \times 10^6} E, \quad G_2 = \frac{\sigma'_m + \sigma_{p_o}}{2.6 \times 10^6} E.$$

Total Resonance Integral

The combined resonance integral for the unit cell consisting of a lumped absorber imbedded in a moderator admixed with the resonance absorber expressed in the terms of the resonance integrals for the lump and moderator is

$$RI_T = \frac{N_o V_o \sum_i (RI)_i^A + N'_o V_1 \sum_i (RI)_i^M}{N_o V_o + N'_o V_1}$$

where the summation includes contributions from resolved and unresolved resonances.

Comparison of Experiment and Theory

The change in the material buckling of a heterogeneous array on replacing the water moderator by an uranyl nitrate solution has been measured by Bennett and Block.⁽¹⁾ These measurements were conducted in a rhomboid lattice exponential assembly containing 0.925 in. diameter uranium rods at a lattice spacing of 1.5 in. Both the uranium rods and the uranyl nitrate solution (431 g U/l) were enriched to 1.007 wt% U²³⁵.

The above method for calculating the resonance integral with resonance absorber in the moderator is utilized in a modified version of the GAMTEC-II code.⁽⁴⁾ A comparison of the bucklings calculated by the modified GAMTEC-II code and those measured by Bennett and Block⁽¹⁾ is given in Table I.

TABLE I
COMPARISON OF EXPERIMENTAL AND CALCULATED BUCKLINGS

<u>Method of Determination</u>	B_m^2, m^{-2}		
	<u>Water Moderation</u>	<u>Uranyl Nitrate Solution Moderator</u>	<u>$\Delta B_m^2, m^{-2}$</u>
Experiment	33.84	12.52	21.32
Calculated	36.98	15.64	21.34

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