FEB 3 1965

MASTER COPY

GENERAL ATOMIC

DIVISION OF **GENERAL DYNAMICS**

GA-5411

CONF-654-35

BURNUP INDEPENDENT POWER SHAPING BY FUEL ZONING

by

K. R. Van Howe

Contract No. AT(04-3)-167

Project Agreement No. 17

U. S. Atomic Energy Commission

October 28, 1964

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

GENERAL ATOMIC
DIVISION OF
GENERAL DYNAMICS

JOHN JAY HOPKINS LABORATORY FOR PURE AND APPLIED SCIENCE

P.O. BOX 608, SAN DIEGO 12, CALIFORNIA

GA-5411

BURNUP INDEPENDENT POWER SHAPING BY FUEL ZONING

Work Done by:

D. Mangan
M. H. Merrill
F. W. Todt
K. R. Van Howe

Report Written by:

K. R. Van Howe

This report was prepared for
presentation at the American
Nuclear Society Meeting in San
Francisco, California, November,
1964.

Project No. 317.2320
Contract No. AT(04-3)-167
Project Agreement No. 17
U. S. Atomic Energy Commission

October 28, 1964

ABSTRACT

The desirability of shaping the power distribution of a reactor to maximize the power output is well known. One method available for power shaping is fuel zoning. Shain and Green⁽¹⁾ have presented a technique for zoning the fissile material to yield a desired initial power distribution. However, in reactors with significant conversion ratios, it is also necessary to zone the fertile material in order to maintain the initial power shape throughout large burnup periods. A procedure is presented below for determining the fissile and fertile loading distribution which will yield any desired burnup independent power distribution.

The time derivative of the macroscopic fission cross section is:

$$\frac{\partial \Sigma_f^{Fi}(r,t)}{\partial t} = \Sigma_a^{Fe}(r,t) \phi(r,t) \sigma_f^B(r,t) - \Sigma_a^{Fi}(r,t) \phi(r,t) \sigma_f^{Fi}(r,t) \quad (A-1)$$

Where superscript Fe denotes fertile material, Fi denotes fissioning material and B denotes bred fuel. Rearranging terms and defining a quantity $A(r,t)$ as:

$$A(r,t) = \left[1 - \left(\frac{\Sigma_a^{Fe}(r,t)}{\Sigma_a^{Fi}(r,t)} \right) \left(\frac{\sigma_f^B(r,t)}{\sigma_f^{Fi}(r,t)} \right) \right] \sigma_a^{Fi}(r,t) \quad (A-2)$$

one obtains:

$$\frac{\partial \Sigma_f^{Fi}(r,t)}{\partial t} = -\Sigma_f^{Fi}(r,t) A(r,t) \phi(r,t) \quad (A-3)$$

In this derivation, $\sigma_a^{Fi}(r,t)$ and $\sigma_f^{Fi}(r,t)$ are the average microscopic absorption and fission cross sections of fissioning material, e.g.,

$$\sigma_a^{Fi}(r,t) = \frac{\sum_{\text{fissioning material}} \Sigma_a^i(r,t)}{\sum_{\text{fissioning material}} N^i(r,t)} \quad (A-4)$$

A burnup independent power distribution implies that:

$$\frac{\partial \Sigma_f^{Fi}(r,t)}{\partial t} / \Sigma_f^{Fi}(f,t) = - \frac{\partial \phi(r,t)}{\partial t} / \phi(r,t) \quad (A-5)$$

Employing this equation and Eq. (3) and assuming that $A(r,t)$ is

separable in space and time, i.e.,

$$A(r,t) = A(r,o) A(t) \quad (A-6)$$

One obtains the following conditions on the neutron flux and macroscopic fission cross section for the power distribution to be time invariant.

$$\phi(r,t) = \phi(r,o) / [1 - S(r) T(t)] \quad (A-7)$$

And,

$$\Sigma_f^{Fi}(r,t) = \Sigma_f^{Fi}(r,o) [1 - S(r) T(t)] \quad (A-8)$$

Where:

$$S(r) = \phi(r,o) A(r,o) \quad (A-9)$$

$$T(t) = \int_0^t A(t) dt \quad (A-10)$$

If the distribution of fissile material can be made invariant with burnup then the flux distribution should also remain invariant with burnup since the major changes in the diffusion properties of the core are a result of changes in the fissile distribution. The conditions (A-7) and (A-8) for a burnup independent

power distribution can therefore be achieved by making $S(r)$ constant with respect to position.

Using the criteria that the power density in any region r is equal to the desired value, and that the value of $S(r)$ in any region is equal to the core average value, a computer code, GASP, has been evolved to perform an iterative solution for the required fissile and fertile loading distributions. This code is based on the one-dimensional, four-group diffusion theory calculational scheme of the FEVER⁽²⁾ code. After each iteration the fissile distribution is adjusted to yield the desired power distribution, and the fertile distribution is adjusted to make the value of $S(r)$ in each core region equal to the core average value.

Figure 1 shows the results obtained from a search for an exponential axial power distribution for a case where both the fissile and fertile distribution were adjusted to obtain the desired distribution, and a case where only the fissile distribution was adjusted. The example is for a large graphite moderated and reflected cylindrical reactor using thorium and U-235 as the initial materials. This GASP problem took 3.4 centihours on the IBM 7044. After fuel burnup equivalent to 1.1 fissions per initial fissile atom, the power distribution with fissile and fertile zoning retains the desired shape, while the power distribution with only fissile zoning has experienced a large change.

I. INTRODUCTION

One of the most difficult problems encountered in the design of power reactors is that of isothermalizing the reactor core. If isothermalization of the core can be achieved, then every point within the core can be operated near the maximum temperature and the power density can be maximized with no change in geometry or pumping power. One means available to approach isothermalization is control of the power distribution by core zoning. In the direction transverse to the coolant flow, a flat power distribution is the most desirable. In the direction, parallel to the coolant flow, an exponential power distribution is the most desirable.

A procedure for the determination of the fissile loading distribution required to yield any desired initial power distribution has been described by Shain and Green⁽¹⁾. In reactors with a large fuel burnup the power distribution can change significantly throughout fuel burnup. It is, therefore, not only necessary to produce the desired initial power distribution, but also to maintain this distribution throughout the fuel burnup history. This report contains the derivation of the criteria which must be met to achieve a stable power distribution with any desired shape, and a description of the computer code, GASP, which employs these criteria.

II. STABILITY CRITERION

The following derivation is based on a one neutron energy group model for the sake of simplicity, but is also applicable to multi-group theory. The power density $p(r,t)$ at any space point r and time t is equal to:

$$p(r,t) = \Sigma_f(r,t) \phi(r,t) \quad (1)$$

If the overall power of the reactor is constant with burnup, then the condition for the power distribution to be invariant with burnup is:

$$\frac{\partial p(r,t)}{\partial t} = 0 \quad (2)$$

From Eq. (1) and (2)

$$\frac{\partial p(r,t)}{\partial t} = \Sigma_f(r,t) \frac{\partial \phi(r,t)}{\partial t} + \frac{\partial \Sigma_f(r,t)}{\partial t} \phi(r,t) = 0$$

therefore,

$$\frac{1}{\Sigma_f(r,t)} \left(\frac{\partial \Sigma_f(r,t)}{\partial t} \right) = - \frac{1}{\phi(r,t)} \left(\frac{\partial \phi(r,t)}{\partial t} \right) \quad (3)$$

The rate of change of the macroscopic fission cross section is equal to:

$$\frac{\partial \Sigma_f^{Fi}(r,t)}{\partial t} = \Sigma_a^{Fe}(r,t) \phi(r,t) \sigma_f^B(r,t) - \Sigma_a^{Fi}(r,t) \phi(r,t) \sigma_f^{Fi}(r,t) \quad (4)$$

Superscript Fe denotes fertile material (i.e., Th-232, U-234, U-238, Pu-240), B denotes bred material (i.e., U-233, U-235, Pu-239, Pu-241), and Fi denotes the material which is fissioning at time t (U-233, U-235, Pu-239, Pu-241).

The above expression neglects the time delay produced by the decay chain and any parasitic absorption between an absorption of a neutron in a fertile material and the appearance of the bred material. Neglecting these phenomena is not a serious limitation since the appearance of bred fuel soon comes into equilibrium with the absorption in fertile material and the losses in parasitic absorptions in the decay chain are usually a small effect. In this derivation microscopic cross sections are the average microscopic cross section for an event considering all nuclides which can contribute to the event, e.g.,

$$\sigma_a^{Fi}(r,t) = \frac{\sum_{\text{fissioning material}} \Sigma_a^i(r,t)}{\sum_{\text{fissioning material}} N^i(r,t)} \quad (5)$$

Rearranging terms in Eq. (4), one obtains:

$$\frac{1}{\Sigma_f^{Fi}(r,t)} \left(\frac{\partial \Sigma_f^{Fi}(r,t)}{\partial t} \right) = -A(r,t) \phi(r,t) \quad (6)$$

Where $A(r,t)$ is defined as:

$$A(r,t) = \left[1 - \left(\frac{\Sigma_a^{Fe}(r,t)}{\Sigma_a^{Fi}(r,t)} \right) \left(\frac{\sigma_f^B(r,t)}{\sigma_a^{Fi}(r,t)} \right) \right] \sigma_a^{Fi}(r,t) \quad (7)$$

Employing Eq. (3) and (6), one obtains the following relationships for the time behavior of $\phi(r,t)$ and $\Sigma_f^{Fi}(r,t)$ which must be satisfied to achieve a burnup invariant power distribution.

$$\phi(r,t) = \frac{\phi(r,0)}{\left[1 - \phi(r,0) \int_0^t A(r,t) dt \right]} \quad (8)$$

$$\Sigma_f^{Fi}(r,t) = \Sigma_f^{Fi}(r,0) \left[1 - \phi(r,0) \int_0^t A(r,t) dt \right] \quad (9)$$

As burnup proceeds, the major changes in the diffusion properties of the core come about because of changes in the macroscopic absorption and production cross sections of fissile material. If

the spatial distribution of these properties (Eq. 9) can be made invariant with burnup, then the flux distributions should be invariant with burnup and the conditions of Eq. (8) and (9) met.

Assuming the quantity $A(r,t)$ is separable in space and time,

$$A(r,t) = A(r,0) A(t) \quad (10)$$

one obtains the following for Eq. (8) and (9)

$$\phi(r,t) = \frac{\phi(r,0)}{1 - S(r) T(t)} \quad (11)$$

$$\Sigma_f^{Fi}(r,t) = \Sigma_f^{Fi}(r,0) [1 - S(r) T(t)] \quad (12)$$

where,

$$S(r) \equiv \phi(r,0) A(r,0) \quad (13)$$

and,

$$T(t) \equiv \int_0^t A(t) dt \quad (14)$$

If now the quantity $S(r)$ in Eq. (11) and (12) is made independent of position, the flux and power distributions will be space invariant with fuel burnup. The criterion for the power distribution to be stable with fuel burnup is therefore:

$$\left[1 - \left(\frac{\Sigma_a^{Fe}(r,0)}{\Sigma_a^{Fi}(r,0)} \right) \left(\frac{\sigma_f^B(r,0)}{\sigma_f^{Fi}(r,0)} \right) \right] \sigma_a^{Fi}(r,0) \phi(r,0) \neq f(r) \quad (15)$$

This criterion can be satisfied by making the value of $S(r)$ in any core region equal to the core average value, i.e.,

$$S(r) = \bar{S} \quad (16)$$

This approach to stabilizing the power distribution was first suggested by R. Shanstrom of General Atomic.

III. METHOD OF SOLUTION

With the two criteria:

- a) The initial power in any core region r , $p(r,0)$, must equal the desired power in that region $p^*(r)$ and,
- b) The quantity $S(r)$ in every region r must equal the core average value of $S(r)$, i.e., $\bar{S}$,

it is fairly simple to set up an iterative scheme for the solution of the fissile and fertile fuel loading distributions which will yield any arbitrarily specified power distribution, and maintain this power distribution throughout the fuel burnup.

A computer code, GASP, has been evolved to perform this iterative solution using the basic four-group diffusion theory calculational scheme of the FEVER⁽²⁾ code. The desired region-wise power distribution $p^*(r)$ is used as a first guess of the source distribution $F(i)_r$ for all space points i in region r . A single sweep through the diffusion equations is performed with an assumed distribution of both fertile material, $N^{Fe}(r)$, and fissile material, $N^{Fi}(r)$. The resultant neutron flux distribution is then employed to obtain the fertile and fissile material distributions which are required to satisfy the criteria (a) and (b) above for the desired

stable power distribution.

The fertile atom density distribution for iteration $k+1$ required to meet the stability criterion of Eq. (16) is:

$$N_{k+1}^{Fe}(r) = N_k^{Fe}(r) \left[1 + \frac{\left[(S_k(r) - \bar{S}_k) N_k^{Fi}(r) \frac{\sigma_{f_k}^{Fi}(r)}{\sigma_{f_k}^B(r)} \right]}{\Sigma_{a_k}^{Fe}(r) \phi_k(r)} \right] \quad (17)$$

The fissile atom density distribution for iteration $k+1$ required to meet the criterion for the desired power distribution is obtained from:

$$N_{k+1}^{Fi}(r) = \frac{N_k^{Fi}(r) p^*(r)}{p_k(r)} \quad (18)$$

The source distribution, $F_{k+1}(i)$, for iteration $k+1$ at mesh point i is obtain from:

$$F_{k+1}(i)_r = \nu \Sigma_{f_k}(i) \phi_k(i) \left(p^*(r) / p_k(r) \right) \quad (19)$$

for all i in r

If the input concentration of Th-232 in a region is non-zero, then this nuclide is adjusted according to Eq. (17). If Th-232 is

zero but U-238 is non-zero, then U-238 is adjusted according to Eq. (17). All fuel materials present other than the primary fertile material (Th-232 or U-238) are adjusted according to Eq. (18). After each iteration the nuclide distributions are renormalized to maintain the same total loadings of all materials that were present in the initial guess. The iterative scheme is continued until the error in both the power distribution and the stability criterion are within specified limits, ϵ_p and ϵ_s respectively:

$$\epsilon_p = \text{Max} \left| \frac{p^*(r) - p(r)}{p^*(r)} \right| \quad (20)$$

$$\epsilon_s = \text{Max} \left| \frac{\bar{S} - S(r)}{\bar{S}} \right| \quad (21)$$

This iterative solution appears to converge very rapidly (<50 iterations) for thorium-uranium systems where $S(r)$ is a positive value. This is true even for the extreme case of producing a stable exponential axial power distribution with a 1.6 peak-to-average power. Some difficulties have been encountered in the convergence of the iterative solution for cases where $S(r)$ is zero or negative. These difficulties will be discussed later in the section on "General Considerations."

IV. GENERAL CONSIDERATIONS

The procedure developed above assumes that the variation in the flux distribution with burnup is primarily dependent upon the variation in the fissile distribution. By making the fissile distribution invariant with time, the flux distribution for most cases will remain invariant with time. The power distribution will therefore remain invariant with burnup. If a burnable poison or other high cross section non-fissile material is present in significant quantities, it will burn out at a rapid rate and may cause a shift in the flux distribution irrespective of the constancy of the fissile distribution. Care must therefore be taken in the initial zoning of such a high cross section material. If this high cross section material is loaded in a manner which maintains the same k_{∞} distribution as the correctly zoned core without this material, the flux distribution will be unaltered both with the initial loading of this material, and after it has burned out. The condition that the change of flux distribution with burnup is primarily controlled by the change in the fissile distribution is then met.

Some difficulties have been encountered in obtaining convergence in the iterative scheme presented above for cases where the quantity $\bar{S}$ is near zero or negative (that is, the macroscopic fission cross section is constant or increases with burnup) and a severe change from the normal mode power distribution is desired. The general tendency is to get into oscillations in the fertile material distribution between iterations. If the quantity $\bar{S}$ is near zero or slightly negative in all regions the macroscopic fission cross section distribution is already stabilized and the zoning of the fertile material not required. By inspection of Eq. (11) and (12), it can be seen that fertile zoning is not required if the absolute value of the product of $S(r)$ and the total burnup time is much less than unity. For most practical reactors with a negative value of $S(r)$ the above product is much less than unity, and a desired stable power distribution can be achieved zoning only the initial fissile inventory. The problem of convergence with negative values of $S(r)$ although not severe for most practical reactors, is being investigated. It is hoped that some type of deceleration in the iteration scheme will solve this problem.

V. SAMPLE PROBLEM

For the purpose of a sample problem, a cylindrical graphite-moderated and reflected reactor with a 242.84 cm core radius and a 472.44 cm core length is employed. The initial fissile and fertile materials are U-235 and Th-232. The desired axial power shape is an exponential, $P(z) = 1.59 e^{-.002117z}$. Five axial core zoning regions were used. An increase in the number of core zoning regions will, of course, improve the approximation to the true exponential. The initial power distribution and the distribution after an exposure equivalent to 1.1 fissions per initial fissile atom which result from the zoning predicted by the GASP code are given in Figure 1. The initial and final distributions obtained from a uniform thorium distribution and a zoned uranium distribution are also presented in this figure. The power distribution using the zoning from the GASP code experiences very little change with exposure while a considerable shift in the power is experienced when only the fissile material is zoned to obtain the desired initial power distribution.

Figure 1

REFERENCES

- 1) Shain, R. B., and Green, W. B., "Fuel Distribution for an Arbitrarily Specified Power Distribution and Multiplication Factor," Transactions of the American Nuclear Society, Volume 4, Number 2, Page 256, November, 1961.
- 2) Todt, F., "FEVER, A One-Dimensional Few-Group Depletion Program for Reactor Analysis," General Atomic Report, GA-2749, November 28, 1962.