Simulation and Analysis of Particle Motion in Magnetic
Mirrors with High Frequency Fluctuations

MASTER

R. E. Aamodt*

Department of Astro-Geophysics, University of Colorado
Boulder, Colorado 80302

and

J. A. Byers†

Lawrence Livermore Laboratory, University of California
Livermore, California 94550

July 1972

Abstract

Particle motion in a magnetic mirror containing high frequency short wavelength longitudinal electric fields is followed numerically for many mirror bounce periods. It is found that while the magnetic moment undergoes large rapid changes, these fluctuations seldom imply particle loss from the mirror. This behavior is shown to be completely consistent with nonlinear superadiabatic theory, wherein containment properties of the trap are affected only when particles cross isolated singular surfaces in phase space.

*Research supported in part by the U. S. Atomic Energy Commission under Contract AT(11-1)-2081 with the University of Colorado.

†Work performed under the auspices of the U. S. Atomic Energy Commission.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

24

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Microinstabilities for warm plasmas contained by magnetic mirrors are typically characterized by longitudinal electric fields having frequencies near multiples of the ion cyclotron frequency, and wavelengths, in units of the ion Larmor radius, long along, and short across the external B field. The all important question of particle containment in such fields has progressed analytically by noticing that while the magnetic moment is no longer an adiabatic invariant in such fluctuations, an invariant still exists, almost everywhere along the particle's trajectory.¹ According to this "superadiabatic" theory, it is only when a particle is forced to cross an isolated singular surface (separatrix) in phase space that this invariant is broken and a nonperiodic change induced in the particle's mirror position. Containment of a particle, once contained in this field configuration, is then determined by the accumulative sum of these nonadiabatic transitions along the entire path history of the particle.

An exacting test of this superadiabatic theory in real magnetic fields is both extremely difficult and expensive. However, for a small finite set of chosen initial conditions it is possible to numerically follow particle motions for many mirror cycles, under various field conditions for which the analytic theory predicts different containment characteristics. In this article, we report the results of such a computer study and while it is hazardous to assess the validity of the theory by such a procedure, we show that the various differences in the numerical results are explainable and consistent with the superadiabatic theory.

The field configuration used in the simulation has been chosen to be a divergence and curl free weak magnetic mirror field with components $B_z(X,Z)$, $B_x(X,Z)$, where $B_z \gg B_x$, and an electric potential $\phi(Z,X,t)$ with frequency chosen to be near the minimum of the second harmonic of the cyclotron frequency. In these functionally fixed fields, the exact equations of motion are numerically solved using standard numerical analogs with a corrector that renders the orbit exact if all fields were independent of Z .² With these techniques, particle trajectories with differing initial phases have been calculated for times up to $\frac{10^5}{2\pi}$ cyclotron periods, which for D_2 in a 10KG field corresponds to two milliseconds. For the same set of initial particle phases the field configuration is then changed by independently varying the frequency, amplitude, and wavelengths both perpendicular and parallel to $\underline{B}$ of the electric potential, ϕ . Typical phase space projections for a number of different conditions are presented in Figs. 1 through 4. The time units in all figures are basic cyclotron times, $\Omega_0 t$, however, note the differing overall scales, as indicated by the exponential multiplying factor in the lower left hand corners. Also the displacement along the mirror direction, $Z(t)$, is measured in Larmor radii and in all examples the mirror midpoint is held at $Z = 16$, has a mirror ratio of 1.1, and a mirror maximum at $Z = 30$ and $Z = 2$.

In order that the important features of these trajectories become apparent it is first necessary to review the predictions of the superadiabatic theory.¹ First of all, in the absence of any Z variation of ϕ and $\underline{B}$, the theory predicts that the electric field causes periodic

variation of the magnetic moment, μ , at a frequency $\bar{\omega}$, which for the qualitative purposes of this article can be characterized by the functional form

$$\bar{\omega} = \left[(\omega - \ell\Omega)^2 - \left(\frac{e\phi k^2}{2m\Omega} \right)^2 G_{\ell,k}(H) \right]^{1/2} \quad (1)$$

where ω, k and ϕ are the frequency, wavevector perpendicular to $\underline{B}$, and amplitude of the electric potential; ℓ is the harmonic number which has been assumed close to ω/Ω , and $G_{\ell,k}(H)$ is a complicated function of the time averaged perpendicular Hamiltonian, H , having for $\ell = 2$ an upper bound equal to unity. The essence of the superadiabatic theory is that if the rate of spatial variation induced by mirror fields, $B(Z, X)$ or nonflute behavior ($\frac{\partial}{\partial Z} \phi \neq 0$) of the potential are slow compared to the frequency $\bar{\omega}$, then the quasiperiodic motion implies the existence of an adiabatic invariant, J , which replaces the usual magnetic moment, and neglecting slow drift motion, implies cyclic motion in $Z(t)$. However, in general, due to the Z dependence of $\omega - \ell\Omega$, and ϕ , as a particle traverses the mirror, it may be forced to cross a separatrix where $\bar{\omega} = 0$, and here a typically small nonperiodic transition takes place in J . This change in turn implies a difference in the Z motion, with possible particle loss out the end of the mirror. The problem then is understanding the various ways this nonadiabatic transition can take place, and its implication on confinement.

Two classifications are immediately apparent from (1); one is where $\omega - \ell\Omega \neq 0$ anywhere in the mirror (nonresonant) and the resonant case where $\omega = \ell\Omega$ somewhere along the mirroring orbit.

In the first, nonresonant case, we see in order that $\bar{\omega} = 0$, φ must exceed a critical value, φ_c , given (for $\ell = 2$) by

$$\frac{e\varphi_c}{m\Omega^2/k^2} = \frac{2(\omega - \ell\Omega)_{\min}}{\Omega} \quad (2)$$

For potential fluctuations above this level, particles still may not cross a separatrix, so that all that can be said in general is for $\varphi < \varphi_c$, particles once contained should be contained for long times, and for $\varphi > \varphi_c$ the motion may begin to scatter.

For such nonresonant cases Fig. 1 illustrates typical computer results when $\varphi < \varphi_c$ where φ has been chosen flute-like (independent of Z). A finer time resolve also indicates the magnetic moment variation changing along Z according to Eqn. (1). The particular case illustrated corresponds to $\frac{\delta\omega}{\Omega}_{\min} = .05$, and a potential amplitude $e\varphi = \frac{1}{2}e\varphi_c = 10\%$ of the initial kinetic energy, indeed a huge fraction of the kinetic energy and also the reason why the magnetic momentum fluctuations are of order 200%. For the same $\frac{\delta\omega}{\Omega}_{\min}$ but smaller φ , the size of the fluctuations in μ decrease, however those particles which are contained on the first half cycle of the mirror transit, remain periodic in $Z(t)$ as in Fig. 4, except for a difference in the mirroring period.

For the nonresonant case when $\varphi_{\max} = \varphi_c$, Fig. 2 is representative, showing very weak scatterings in the mirroring position. For Fig. 2, φ is nonflute and extremely small in the mirror where the particle is turning around, in which case the adiabatic invariant J becomes approximately μ . Therefore, the nonadiabatic changes in J are in

fact approximately illustrated in Fig. 2(a) by the differing values of μ where Z is a maximum or minimum. It is probably correct that after extremely many mirror cycles this particle would leak out of the mirror.

Other runs with different initial particle phase conditions in the nonresonant case when $\varphi > \varphi_c$ indicated again only weak scatter in mirroring which is probably indicative of the fact that these particles were not in orbits which were being forced to cross separatrices. Also, in the nonresonant case when the Z variation of φ was increased to twice that of Fig. 2, then variations in J appeared to be approximately doubled, and this is consistent with the nonadiabatic transition theory³ where changes in J across singular surfaces are proportional to $\frac{d\varphi}{dt}$ at the time of separatrix crossing.

When φ was increased to much larger than φ_c , and held flute-like the runs looked typically as those in Fig. 3 which itself is a resonant case where $\omega - \ell\Omega = 0$ near the mirror midpoint. In this strong field limit it appears that the linear frequency, $\omega - \ell\Omega$, is almost everywhere irrelevant; and in this limit the adiabatic invariant becomes only a function of Z through the $\varphi(Z)$ amplitude variation.¹ As φ was always held constant in these large φ limits, J becomes an exact constant of the motion; and particles are perfectly contained. However, in the vicinity of singular points in phase space the coefficients of φ in the equations of motion identically vanish, and then near these points the linear orbit correction, $\omega - \ell\Omega$, is important. Therefore, in all strong φ cases, according to superadiabatic theory, and as evidenced in all computer runs, only very

infrequent scatterings should occur; containment being almost as good as when $\varphi < \varphi_c$ in the nonresonant, $\omega - \ell\Omega \neq 0$, cases. Another way of understanding this particular phenomena is that the strong nonlinearities rapidly detune any phase coherent resonance, and thereby limit secular type excursions. Such a nonlinear saturation of particle losses may be the explanation of reported experimental improvement in containment in the presence of external rf sources on a mirror machine.⁴

For low and intermediate values of φ resonant modes act on the particles quite differently than in the nonresonant case. Where $\omega - \ell\Omega = 0$, the system is strongly nonlinear, and near the ends of the system where the nonlinear terms in (1) are small compared to $\omega - \ell\Omega$, the system is essentially linear. For such an implied change in phase space topology the particle is almost always forced to cross singular surfaces during one mirror transit and thereby scatter appreciably in a number of transit times. Fig. 4 illustrates this well as the field configuration, and initial particle phases are identical to those of Fig. 3, except that in Fig. 4 the potential φ is fifty times smaller.

In summary, what these many computer runs have shown is that the containment properties of weak mirrors seem to be completely consistent with those calculated with the superadiabatic theory at least for times of order $\frac{10^5}{2\pi}$ cyclotron periods. Furthermore, the overall results are that containment is a strong function of orbit phase, and if almost everywhere along the trajectory the conditions $\Omega_0 \gg \bar{\omega} \gg \omega_{\text{mirror}}$ are satisfied, then moderate value of fluctuating potentials bounce lead to the most rapid particle losses.

References

1. R. E. Aamodt, Phys. Rev. Letters, 27, 135 (1971).
2. C. K. Birdsall, and D. Fuss, (to be published).
3. R. E. Aamodt, and E. F. Jaeger, (to be published).
4. D. G. Swanson, R. W. Clark, P. Korn, S. Robertson, C. B. Wharton, Phys. Rev. Letters, 28, 1015 (1972).

Figure Captions

Fig. 1 Magnetic moment, and particle motion along mirror field when electric field is nonresonant, flute, and subcritical amplitude

Fig. 2 Magnetic moment, and particle motion along mirror field when electric field is nonresonant, non-flute and just critical amplitude

Fig. 3 Magnetic moment, and particle motion along mirror field when electric field is resonant, flute, and extremely large amplitude

Fig. 4 Magnetic moment, and particle motion along mirror field when electric field is resonant, flute, and moderate amplitude

NO VS TIME

FIG. 1(a)

FIG. 1 (b)

MU VS TIME

FIG 2 (a)

Z VS TIME

FIG 2 (b)

TIME VS TIME

FIG 3(a)

Z VS TIME

FIG 3(b)

MU VS TIME

FIG 4(a)

FIG 4(b)