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232

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A TWO-ZONE POTENTIAL RELAXATION SCHEME FOR HANDLING GRID WIRES

R. F. Smith
S. L. Rompel

June 1, 1973

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LAWRENCE LIVERMORE LABORATORY
University of California, Livermore, California, 94550

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19

Contents

Abstract	1
Introduction	1
Basic Methods	1
One-Wire Test Case	4
Conclusions	5
Acknowledgments	6
References	6
Appendix A. Relaxation in Polar Coordinates	7
Appendix B. Polar Relaxation and Interpolation via Conformal Mapping	10
Appendix C. Program for Relaxation Calculation	12

A TWO-ZONE POTENTIAL RELAXATION SCHEME FOR HANDLING GRID WIRES

Abstract

A relaxation scheme for economically calculating two-dimensional electrostatic potentials in configurations with fine grid wires has been devised and tested. Potentials calculated in this manner agree with potentials calculated with an analytic formula for the case of a one-wire test problem in which the radius of the wire is two orders of magnitude smaller than the outer boundary dimensions.

Introduction

The periodic-focusing direct-conversion unit^{1,2} and the venetian blind converter³ both use grid wires which are quite small compared to other dimensions. There are several reasons why the fields near the grid wires are important. In both devices the electric field at the wires determines the likelihood of sparking and thus determines the maximum voltage which can be used. The exact potential distribution near the wires of the periodic-focusing device has been shown to be unimportant to particle trajectories, because ions are moving rapidly there and small deflections do not prevent them from being captured in the collector cups. However, the venetian blind device depends upon unfocused ballistic trajectories and is quite sensitive to defocusing effects. In that device the secondary electrons must be suppressed by grid wires close to the main collection electrodes. These grid wires are in a geometry which prevents easy estimation of the potentials. Potentials in complicated geometries are usually calculated by a relaxation method. This method is accurate only if the mesh size is comparable to the smallest scale length of the potential. For direct conversion work this length is the radius of the wire, which can be 10^3 times smaller than the other dimensions of interest. Calculations based on uniform subdivisions of such small size would be prohibitively costly in computer time. Instead we combine finely divided regions near the wires with coarse zoning away from the wires.

Basic Methods

We shall consider two-dimensional configurations, such as shown in Figs. 1 and 2. The large-scale potential distributions have rectangular geometry, and the

Fig. 1. Cross-sectional segment of the periodic-focusing collector, showing relative potentials and typical ion orbits. Low-potential grids are shown as dotted circles and high-potential rods as solid circles.

Fig. 2. Detail of two-stage venetian blind converter.

coarse-zoned regions are divided into rectangular cells of dimension Δx by Δy . At each point (i, j) the potential is iterated according to the formula,

$$\phi_{i,j}^N = (1 - \alpha)\phi_{i,j}^{N-1} + \frac{(\Delta x)^2 (\Delta y)^2}{(\Delta x)^2 + (\Delta y)^2} \frac{\alpha}{2} \times \left[\frac{\phi_{i,j-1}^N + \phi_{i,j+1}^{N-1}}{(\Delta y)^2} + \frac{\phi_{i-1,j}^N + \phi_{i+1,j}^{N-1}}{(\Delta x)^2} \right], \quad (1)$$

where N is the present iteration, $N - 1$ is the previous iteration, and α is the so-called over-relaxation parameter.⁴ This parameter is typically about 1.7, and its use greatly speeds the convergence. Too large a value for α can result in instability. The relaxation pattern alternates, going from left to right and top to bottom on odd N and right to left and bottom to top on even N .

As mentioned above, it is impractical to calculate the potentials near grid wires by rectangular relaxation, since the meshes would have to be approximately equal to the wire in size. It is desirable to have a fine scale cylindrical mesh centered on each wire which would be connected in some way with the coarse rectangular mesh.

As shown in Appendix A, if one chooses the n^{th} mesh radius to be $r_n = 2^n r_0$ and $\Delta\theta = 2\pi/p$, where p is an integer, then a relaxation formula for the new value of ϕ at $\theta = m\Delta\theta$, $r = r_n$ in terms of the previous surrounding ϕ values is

$$\phi_n^m = \frac{10}{3} \left[\frac{1}{\left(\frac{p}{\pi}\right)^2 + 6} \right] \phi_{n+1}^m + \frac{8}{3} \left[\frac{1}{\left(\frac{p}{\pi}\right)^2 + 6} \right] \phi_{n-1}^m + \frac{1}{2} \left[\frac{1}{1 + 6\left(\frac{\pi}{p}\right)^2} \right] (\phi_n^{m-1} + \phi_n^{m+1}). \quad (2)$$

An alternative relaxation formula

$$\phi_n^m = \frac{\phi_{n-1}^m + \phi_{n+1}^m}{2 \left[1 + \left(\frac{p}{2\pi}\right)^2 (\ln 2)^2 \right]} + \frac{\phi_n^{m-1} + \phi_n^{m+1}}{2 \left[1 + \left(\frac{2\pi}{p}\right)^2 \left(\frac{1}{\ln 2}\right)^2 \right]}. \quad (3)$$

is derived in Appendix B.

The second formula [Eq. (3)] seems to have better limiting behavior than the first, although no flaw in the derivation of Eq. (2) is apparent to the authors.

In order to implement the overlapping of the two relaxation zones, one might proceed as follows:

1. Specify the potentials on the rectangular mesh boundary and also on the grid wires; for example, points 1, 2, 3 on Fig. 3.
2. Relax the potential on the rectangular mesh, ignoring the wires.
3. From the ϕ values on the rectangular mesh, interpolate to get the potentials on the outer radius of the polar zone at each wire, such as points 4 and 5.
4. Holding ϕ fixed at points 4, 5, etc., do the polar relaxation. This is where the grid potential is felt.
5. Now from the polar ϕ values, interpolate as in Appendix B to find ϕ on the rectangular mesh points which lie within the polar region and are nearest the wire, such as point 6.
6. Holding ϕ fixed at points such as 6, return to step 2 and repeat the process.

Fig. 3. Overlapping of rectangular and variable-radii polar relaxation zones. The grid wire is at point 1.

Once the potentials are computed for the square and polar meshes, for each (x,y) location, one must ask whether there is a grid wire nearby, and if so, which polar table must be used to compute ϕ .

For the rectangular mesh, Δx and Δy need not be the same. Since trouble is anticipated if the wire is too close to

a rectangular mesh point, the wires should be located roughly in the center of a mesh rectangle. A plausible choice for the radius of the polar region is somewhat larger than two rectangular mesh spacings.

One-Wire Test Case

The above two-zone relaxation scheme was tested in the case of a planar grid triode. Spangenberg⁵ gives an approximate but accurate expression for the potential in a configuration of evenly spaced grid wires between two parallel plates. Figure 4

Fig. 4. Geometry of one-wire test problem. The boundary conditions are $\phi = 0$ and $\phi = 2$.

shows the 13×36 square mesh used in the test problem. The fixed boundary potentials are zero on the left edge, 2 on the edge $x = 36$, and -1 at the wire of radius 0.1 located at (11.5, 6.5). The potentials on the top and bottom boundaries are specified by the analytic expression for the potential, which is

$$\phi(x,y) = K_1 \ln \left\{ e^{\frac{4\pi d_1}{a}(x-d_1)} + 1 - 2 \cos \left[\frac{2\pi d_2}{a} \left(y - \frac{a}{2} \right) \right] e^{\frac{2\pi d_2}{a}(x-d_1)} \right\} + K_2 x,$$

where $d_1 = 11.5$, $d_2 = 24.5$, $a = 13$, $K_1 = 0.1203$, and $K_2 = -0.02357$ for the present geometry and potentials.

The potentials obtained from the analytic formula are compared with the numbers obtained from the relaxation scheme in Table 1. Only the numbers on the line $x = 11$ which passes close to the wire are shown. There are two columns of relaxed potentials. Sixteen angular divisions were used. The numbers listed under ϕ_{equal} are obtained with equally spaced radii of 0.1, 0.404, 0.707, 1.011, 1.314, 1.618, 1.921, and 2.225. The numbers under ϕ_{varied} were obtained with the relaxation formula [Eq. (2)], using

Table 1. Comparison of analytic and relaxed potentials at $x = 11$.

y	Φ_{analytic}	Φ_{equal}	Φ_{varied}
0	-0.119817	-0.119817	-0.119817
1	-0.126810	-0.130345	-0.127022
2	-0.148615	-0.155823	-0.148968
3	-0.188162	-0.199562	-0.188693
4	-0.252294	-0.268799	-0.252986
5	-0.357746	-0.379900	-0.357338
6	-0.546669	-0.574233	-0.541065
7	-0.546669	-0.574235	-0.541066
8	-0.357746	-0.379901	-0.357338
9	-0.252294	-0.268801	-0.252986
10	-0.188162	-0.199565	-0.188693
11	-0.148615	-0.155826	-0.148968
12	-0.126810	-0.130347	-0.127022
13	-0.119817	-0.119817	-0.119017

mesh radii of 0.1, 0.2, 0.4, 0.8, 1.6, and 3.2. The numbers obtained with variable mesh spacing agree quite well with the semi-exact analytic values. The error seems to be about 1% and is on the correct side of the "true" answer, i.e., the relaxed values underestimate the effect of the wire. The relaxed numbers are nearly symmetric with respect to the line $y = 6-1/2$, as they must be.

Conclusions

The two-zone relaxation method for handling grid wires in two-dimensional electrostatic potential problems appears to work quite well, and a progressive increase of mesh radius at greater distances from the wires is a considerable improvement over a constant mesh radius. It is intuitively clear that the polar zones should not overlap each other or a boundary. We speculate that over-relaxing is much more important in the coarse rectangular mesh than in the polar meshes, because the former involve many more mesh points.

An additional test of the method would be the calculation of potentials near the electron suppression grids of a venetian blind collector. These potentials have been measured on resistance paper by Phil Maslin.

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Appendix A Relaxation in Polar Coordinates

This is an annotation of work done by Don Moore (December 1971). In Polar coordinates, Laplace's equation is

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2}. \quad (\text{A-1})$$

If the notation of Fig. A-1 is used, the difference equation is

Fig. A-1. Notation used in the development of Eq. (2).

$$\begin{aligned} & \frac{\phi_{n+1}^m - 2\phi_n^m + \phi_{n-1}^m}{(\Delta r)^2} + \frac{1}{r_n} \frac{\phi_{n+1}^m - \phi_{n-1}^m}{2\Delta r} \\ & + \frac{1}{r_n^2} \frac{\phi_n^{m+1} - \phi_n^m + \phi_n^{m-1}}{(\Delta \theta)^2} = 0. \end{aligned} \quad (\text{A-2})$$

This is true when the radial increments are equal. When they are unequal, we will resort to a Taylor expansion in r

to obtain $\partial^2 \phi / \partial r^2$ and $\partial \phi / \partial r$. The expansions are

$$\phi(r + \Delta r_+, \theta) = \phi(r, \theta) + \frac{\partial \phi}{\partial r} \Delta r_+ + \frac{\partial^2 \phi}{\partial r^2} \frac{(\Delta r_+)^2}{2} + \dots \quad (\text{A-3})$$

and

$$\phi(r - \Delta r_-, \theta) = \phi(r, \theta) - \frac{\partial \phi}{\partial r} \Delta r_- + \frac{\partial^2 \phi}{\partial r^2} \frac{(\Delta r_-)^2}{2} - \dots \quad (\text{A-4})$$

By eliminating $\partial \phi / \partial r$ we find

$$\frac{\partial^2 \phi_n^m}{\partial r^2} = \frac{2\phi_{n+1}^m}{\Delta r_+ (\Delta r_- + \Delta r_+)} - \frac{2\phi_n^m}{\Delta r_+ \Delta r_-} + \frac{2\phi_{n-1}^m}{\Delta r_- (\Delta r_- + \Delta r_+)}. \quad (\text{A-5})$$

Similarly one can eliminate $\partial^2 \phi / \partial r^2$ from Eqs. (A-3) and (A-4) to obtain

$$\frac{\partial \phi_n^m}{\partial r} = \frac{\Delta r_- \phi_{n+1}^m}{\Delta r_+ (\Delta r_- + \Delta r_+)} + \frac{(\Delta r_- - \Delta r_+) \phi_n^m}{\Delta r_+ \Delta r_-} - \frac{\Delta r_+ \phi_{n-1}^m}{\Delta r_- (\Delta r_- + \Delta r_+)}. \quad (\text{A-6})$$

Suppose the n^{th} grid has radius r_n , where

$$r_n = r_0 + \Delta r \frac{K^n - 1}{K - 1}. \quad (\text{A-7})$$

Thus $\Delta r_+ = K^n \Delta r$, $\Delta r_- = K^{n-1} \Delta r$, where K is a constant. Substituting into Eqs. (A-5) and (A-6), we find

$$\frac{\partial \phi_n^m}{\partial r} = \frac{\phi_{n+1}^m}{K^n(K+1)\Delta r} + \frac{(K-1)\phi_n^m}{K^n \Delta r} - \frac{K^2 \phi_{n-1}^m}{K^n(K+1)\Delta r} \quad (\text{A-8})$$

and

$$\frac{\partial^2 \phi_n^m}{\partial r^2} = \frac{2K\phi_{n+1}^m}{K^{2n}(K+1)(\Delta r)^2} - \frac{2K\phi_n^m}{K^{2n}(\Delta r)^2} + \frac{2K^2\phi_{n-1}^m}{K^{2n}(K+1)(\Delta r)^2}. \quad (\text{A-9})$$

Substituting into Eq. (A-1), with $\Delta\theta = 2\pi/p$, where p is an integer

$$\begin{aligned} & \left[\frac{2Kr_n^2}{K^{2n}(K+1)(\Delta r)^2} + \frac{r_n}{K^n(K+1)\Delta r} \right] \phi_{n+1}^m + \left[\frac{(K-1)r_n}{K^n \Delta r} - \frac{2Kr_n^2}{K^{2n}\Delta r^2} \right] \phi_n^m \\ & + \left[\frac{2K^2r_n^2}{K^{2n}(K+1)(\Delta r)^2} - \frac{K^2r_n}{K^n(K+1)\Delta r} \right] \phi_{n-1}^m + \frac{p^2}{4\pi^2} [\phi_n^{m+1} - 2\phi_n^m + \phi_n^{m-1}] = 0 \end{aligned} \quad (\text{A-10})$$

Now let $r_0 = \Delta r$, so that $r_n = \Delta r \frac{K^n + K - 2}{K - 1}$, $n = 0, 1, 2, \dots$. One obtains

$$\phi_n^m = a_n \phi_{n+1}^m + b_n \phi_{n-1}^m + c_n \phi_n^{m+1} + d_n \phi_n^{m-1}, \quad (\text{A-11})$$

where

$$a_n = \frac{c_+(n)}{c_0(n)}, \quad b_n = \frac{c_-(n)}{c_0(n)}, \quad c_n = d_n = \frac{c(n)}{c_0(n)},$$

and

$$c_+(n) = \frac{3K^{n+1} - K^n + 2K^2 - 4K}{K^{2n+2} - K^{2n}}, \quad (\text{A-12a})$$

$$c_-(n) = \frac{3K^{n+2} - K^{n+3} + 2K^3 - 4K^2}{K^{2n+2} - K^{2n}}, \quad (A-12b)$$

$$c(n) = \frac{(K-1)p^2}{4\pi^2(K^n + K-2)}, \quad (A-12c)$$

and

$$c_0(n) = \frac{2p^2 K^{2n}(K-1)^2 + 8\pi^2 K(K^n + K-2)^2 - 4\pi^2 K^n(K-1)^2(K^n + K-2)}{4\pi^2(K^n + K-2)K^{2n}(K-1)} \quad (A-12d)$$

In the special case $r_n = 2^n r_0$, $\Delta\theta = 2\pi/p$, we have Eq. (2),

$$\phi_n^m = \frac{10}{3} \left[\frac{1}{\left(\frac{p}{\pi}\right)^2 + 6} \right] \phi_{n+1}^m + \frac{8}{3} \left[\frac{1}{\left(\frac{p}{\pi}\right)^2 + 6} \right] \phi_{n-1}^m + \frac{1}{2} \left[\frac{1}{1 + 6\left(\frac{\pi}{p}\right)^2} \right] (\phi_n^{m+1} + \phi_n^{m-1}).$$

In circular symmetry, where $\phi_n^{m+1} = \phi_n^m$, this reduces to

$$\phi_n^m = \frac{5}{9} \phi_{n+1}^m + \frac{4}{9} \phi_{n-1}^m. \quad (A-13)$$

This is independent of p , as it should be, but does not seem to reproduce a logarithmic variation in potential between two concentric cylinders.

Appendix B

Polar Relaxation and Interpolation via Conformal Mapping

Suppose we use conformal mapping to transform a pie-wedge into a rectangle (Fig. B-1).

Fig. B-1. Notation used in the development of Eq. (3).

We wish to expand the potentials at B' and A' in Taylor series and eliminate $\partial\phi/\partial u$ in analogy with Appendix A. This time we have transformed away the first derivative from the Laplacian. Dropping the primes for convenience,

$$\phi_B = \phi_P + \frac{\partial\phi}{\partial u} h_2 + \frac{1}{2} \left(\frac{\partial^2\phi}{\partial u^2} h_2^2 \right) + \dots \quad (\text{B-1})$$

and

$$\phi_A = \phi_P - \frac{\partial\phi}{\partial u} h_1 + \frac{1}{2} \left(\frac{\partial^2\phi}{\partial u^2} h_1^2 \right) + \dots \quad (\text{B-2})$$

Eliminating $\partial\phi/\partial u$ we find

$$\frac{\partial^2\phi_P}{\partial u^2} = \frac{2(h_1\phi_B + h_2\phi_A)}{h_1h_2(h_1 + h_2)} - \frac{2}{h_1h_2} \phi_P \quad (\text{B-3})$$

Similarly

$$\frac{\partial^2 \phi_D}{\partial v^2} = \frac{1}{h_3^2} (\phi_C + \phi_D) - \frac{2}{h_3^2} \phi_P. \quad (B-4)$$

Substituting into $\partial^2 \phi / \partial u^2 + \partial^2 \phi / \partial v^2 = 0$, we obtain

$$\phi_P = \frac{h_1 h_2 (\phi_C + \phi_D) + \frac{2h_3^2}{h_1 + h_2} (h_1 \phi_B + h_2 \phi_A)}{2(h_1 h_2 + h_3^2)}. \quad (B-5)$$

This can be checked in various limits. If the mesh is square, $h_1 = h_2 = h_3 = h_4$ and one obtains the usual result,

$$\phi_P = \frac{1}{4} (\phi_A + \phi_B + \phi_C + \phi_D).$$

If the problem is one-dimensional or circularly symmetric, $\phi_C = \phi_D = \phi_P$ and Eq. (B-5) reduces to

$$\phi_P = \frac{h_1 \phi_B + h_2 \phi_A}{h_1 + h_2} = \frac{\ell n \frac{r_2}{r_1} \phi_B + \ell n \frac{r_3}{r_2} \phi_A}{\ell n(r_3/r_1)}. \quad (B-6)$$

This is, of course, the exact answer for the potential between two concentric cylinders when r_2 is considered to be variable, and it is independent of h_3 . The limit $\epsilon \rightarrow 0$ of $r_3/r_2 = 1 + \epsilon = r_2/r_1$ and $\Delta\theta = \epsilon$ is also correct in this second treatment.

If we specialize to a variable mesh grid wire problem where $r_n = r_0 2^n$ and $\Delta\theta = 2\pi/p$, then $h_1 = \ell n 2 = h_2$, and $h_3 = \Delta\theta$. We obtain Eq. (3)

$$\phi_n^m = \frac{\phi_{n-1}^m + \phi_{n+1}^m}{2 \left[1 + \left(\frac{p}{2\pi} \right)^2 (\ell n 2)^2 \right]} + \frac{\phi_n^{m-1} + \phi_n^{m+1}}{2 \left[1 + \left(\frac{2\pi}{p} \right)^2 \left(\frac{1}{\ell n 2} \right)^2 \right]},$$

where the notation of Appendix A, $(\ell n 2)^2 = 0.4805$, is used.

The coefficients are different from those of Appendix A, and the latter do not seem to reduce to the correct answers in the limits of 1D or circular symmetry, yet the fault is not apparent to the authors.

The above transformation is also useful in interpolating from the known potentials at points A, B, C, D to that at point Q on Fig. B-1.

Appendix C

Program for Relaxation Calculation

*ID 164MMR RELAXER RCMPEL BCX P350C0001

```
*FORTRAN
PROGRAM RELAXER(RELAXINPUT, TAPE2=RELAXINPUT, LRELAX, TAPE5=LRELAX,
PTAPE100, TAPE3=LRELAX)
OPTIMIZE
CODE ANALYSIS
COMMON ALPHA, ANGLE(32)
C, CONVT(1300)
D, ORN(16), DRN2( 6), DTHE2, DX, DX2, DY, DY2
I, IMAX, IMX, IMY(16), IX
J, JMAX, JMX, JMY(16), JX
N, NAME(2), NCMAX, NDR, NDR1, NDRM, NT, NT1, NT2, NW
P, PHI1, PHI2, PHI , PHI4, PI
R, R(16)
S, SQRAD, SQPTS(4), SQTHE(4)
T, THETA, TWDXDY, TWDXDI
W, WIREX(16), WIREY(16), WIRPOT(16), WIRAD(16)
X, X, XJ(37)
COMMON Y, YI(37)
COMMON/STORAGE/AREA, PX(18), PY(18)
PARAMETER IID=48, JD=48, NUD=16, N2D=32, NTD=32, ND=16, IDJD=2304)
DIMENSION PHI( ID, JD), PHIAPP( ID, JD), PHIPC( ID, JD)
DIMENSION POLPHI( NDD, N2D), POLX( ND, NDD, NTD), POLY( ND, NCC, NTD)
REAL IOX, IOY
INTEGER DIRECT, PHI01( ID, JD)
DATA INTO, INT1, NC/0, 1, 1/
LCM PASTC
COMMON/PASTC/PHIP( IDJD)
INTFAC = 3
NW = 0
PI = 3.1415926535898

C*****
C NT MUST BE A MULTIPLE OF 4
CALL ASSIGN(2, 0, 10HRELAXINPUT, 0)
CALL ASSIGN(61, 3)
CALL CREATE(6HRELAX, 40000, RS)
CALL CRTID(2HRS, 1, 1)
CALL REGST

C*****
C READ TITLE CARD
C**CARD 1**
READ(2, TWO2) NAME
WRITE(5, TWO2) NAME

C*****
C READ NCMAX, NUMBER OF POLAR RADII AND THETA DIVISICNS
C**CARD 2**
READ(2, TWO3) NCMAX, NDR, NT
P = NT
WRITE(5, FI21)
THETA = (2.*PI)/P
SINT2 = SIN(.5*THETA)
WRITE(5, FI24) NCMAX, NDR, NT, THETA, SINT2
NDR = NDR + 1
NT1 = NT + 1
NDR1 = NDR + 1
N2T = NT + NT
NT2 = NT + 2
N2TM = N2T - 1
NCRM = NDR - 1

C*****
```



```

C      READ RECTANGULAR MESH SIZE AND BOUNDARY POTENTIALS
C**CARD 3**
      READ(2,TWO0) JMAX,IMAX,PHI1,PHI2,PHI3,PHI4
      WRITE(5,FIV1)
      WRITE(5,TWO0) JMAX,IMAX,PHI1,PHI2,PHI3,PHI4
      JMX = JMAX + 1
      IMX = IMAX + 1
      DD = JMX
C*****
C**CARD 4**
      READ(2,TWO1) X,Y,ALPHA
      DX = X/DD
      DD = IMX
      WRITE(5,FIV2)
      DY = Y/DD
      WRITE(5,TWO1) DX,X,DY,Y,ALPHA
      IX = IMAX + 2
      JX = JMAX + 2
      IX1 = IX + 1
      JX1 = JX + 1
      WRITE(5,FIV3)
C*****
C      READ WIRE POTENTIALS AND LOCATIONS
REOM0 CONTINUE
      NW = NW + 1
C*****
C**CARDS **
      READ(2,TWO1) WIRPOT(NW),WIRAD(NW),WIREX(NW),WIREY(NW)
      IF (WIRAD(NW).EQ.0.) GO TO NOMOR
      WRITE(5,TWO1) WIRPOT(NW),WIRAD(NW),WIREX(NW),WIREY(NW)
      GO TO REOM0
NOMOR CONTINUE
      NW = NW - 1
C*****
C      GENERATE EQUATIONS CONSTANTS
      DX2 = DX*DX
      DY2 = DY*DY
      DTHE2 = THETA*THETA
      TWOXDY = 2.*(DX2 + DY2)
      C3 = P*ALOG(2.)/(2.*PI)
      C1 = 2.*(1. + C3**2)
      C2 = 2.*(1. + (1./C3)**2)
      TWOQDD = 2.*SQRT(DX2 + DY2)
      TWOXD1 = 1./TWOXDY
      WRITE(5,FIV8)
      WRITE(5,TWO1) DX2,DY2,DTHE2,TWOXDY,C1,C2,C3,TWOQDD,TWOXD1
C*****
C      GENERATE X AND Y VALUES
      EJE = 0
      DO GENX J = 1,JX
        XJ(J) = EJE*DX
        EJE = J
      GENX CONTINUE
      WRITE(5,FII2)
      WRITE(5,FIVE) (XJ(J),J=1,JX)
      EYE = 0
      DO GENY I = 1,IX
        YI(I) = EYE*DY
        EYE = I
      GENY CONTINUE

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      YI = YI(1)
      WRITE(5,F111)
      WRITE(5,FIVE) (YI(I),I=1,IX)
C*****
C      COMPUTE POINTS FOR INTERPOLATION FOR CARTESIAN FROM POLAR GRID
      SQRAD = .25*TWSSQD
      WRITE(5,F115)
      SQTHE1 = ASIN(.5*DY/SQRAD)
      SQTHE(1) = SQTHE1
      SQTHE(2) = PI - SQTHE1
      SQTHE(3) = PI + SQTHE1
      SQTHE(4) = 2.*PI - SQTHE1
      WRITE(5,FIVE) SQRAD,(SQTHE(L),L=1,4)
C*****
C      COMPUTE PHI TO CHECK
      ARITHREG XJJ
      XJJ = XJ(1)
      DO APPRX J = 1,JX
        X1 = EXP(.9666*(XJJ - 11.5)) + 1.
        X2 = EXP(.4833*(XJJ - 11.5))
        X3 = .02357*XJJ
        XJJ = XJ(J+1)
        ARITHREG YII
        YII = YI(1)
        DO APPRX I = 1,IX
          PHIAPP(I,J) = .1203*ALOG(X1 - 2.*COS(.4833*(YII - 6.5))*X2)
          - X3
          YII = YI(I+1)
        APPRX CONTINUE
        RELEASE XJJ,YII
        XI = XJ(1)
        WRITE(5,FIV7)
        CALL PRINT2(PHIAPP,JX,IX,INT1,INT0,XJ,YI,10,JO)
C*****
C      COMPUTE ANGLE VALUES FOR POLAR RELAXATION
      ANG = 0
      DO ANGLK K = 1,NT2
        ANGLE(K) = ANG*THETA
        ANG = K
      ANGLK CONTINUE
      WRITE(5,F114)
      WRITE(5,FIVE) (ANGLE(L),L=1,NT2)
      XMI = YMI = .11328
      XMA = 1.
      YMA = (Y/X)*XMA + YMI
      CALL MAPS(X1,X,Y1,Y XMI,XMA,YMI,YMA)
C*****
C      SET BOUNDARY VALUES AND PLOT GRID LINES
      YII = YI(2)
      DO BCJ I = 2,IMX
        PHI(I,1) = PHI4
        PHI(I,JX) = PHI2
        CALL LINEP(X1,YII,X,YII)
        YII = YI(I+1)
      BCJ CONTINUE
      XJJ = XJ(2)
      DO BCJ J = 2,JMX
        PHI(1,J) = PHI1
        PHI(1,J) = .1203*ALOG(EXP(.9666*(XJJ - 11.5)) + 1.
          + 2.*EXP(.4833*(XJJ - 11.5))) - .02357*XJJ

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      PHI(IX,J) = PHI(1,J)
      CALL LINEP(XJJ,YI,XJJ,Y)
      XJJ = XJ(J+1)
HCJ  CONTINUE
C****
C      INITIALIZE PHI FOR N-1
      WRITELCM(PHI,PHIP,1DJ0)
      WRITE(5,F123)
      CALL PRINT2(PHI,JX,IX,INT1,INTC,XJ,YI,1D,JD)
C****
C      COMPUTE POLAR VALUES FOR X, Y, R, LOG(R) AND PLOT R ON GRID
      IDX = 1./DX
      ICY = 1./DY
      GO RACKY N = 1,NW
      WIREXN = WIREX(N)
      WIREYN = WIREY(N)
      WIRADN = WIRAD(N)
      CALL POINT(WIREXN,WIREYN,1)
      DRNN = .5*(SQRAD - WIRADN)
      I = WIREYN*IDY + 1.
      J = WIREXN*IDX + 1.
      IWY(N) = I
      JWY(N) = J
      PHI01(1,J) = 1
      PHI01(1,J+1) = 1
      PHI01(1+1,J) = 1
      PHI01(i+1,J+1) = 1
      DRN(N) = CRNN
      DRN2(N) = DRNN*DRNN
      TWON = 1
      DO RPLUT NR = 1,NDR
      RAD = TWON*WIRADN
      TWON = 2**NR
      R(NR) = RAD
      ANG = ANGLE(1)
      DO POLXY K = 1,NT2
      RSA = RAD*SIN(ANG) + WIREYN
      RCA = RAD*COS(ANG) + WIREXN
      ANG = ANGLE(K+1)
      POLX(N,NR,K) = RCA
      POLY(N,NR,K) = RSA
      PX(K) = RCA
      PY(K) = RSA
POLXY  CONTINUE
      CALL TRACE(PX,PY,NT1)
RPLUT  CONTINUE
      WRITE(5,F1V4)
      WRITE(5,F1V6) (R(L),L=1,NDR)
RADXY  CONTINUE
      WRITE(5,F1V5)
      CALL PRINT3(POLX,NW,NT2,NDR,INT1,INTC,ANGLE,R,ND,NDD,NTD)
      WRITE(5,F113)
      CALL PRINT3(POLY,NW,NT2,NDR,INT1,INTC,ANGLE,R,ND,NDD,NTD)
      DIRECT = - 1
      JCOUNT = 2
      JONE = JMX
      ICDUNT = 2
      IONE = IMX
BEGIN  CONTINUE
      DIRECT = - DIRECT

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J = JCOUNT
JCOUNT = JONE
JONE = J
I = ICOUNT
ICOUNT = IONE
IONE = I

C*****
C DO RECTANGULAR RELAXATION
C RELAX IN X DIRECTION
DO HOLDI I = 2,IMX
  IR = IX1 - I
  DO RELAX J = JONE,JCOUNT,DIRECT
    IF (PHIC1(I,J).GT.0) GO TO RX1
    PHI(I,J) = ((DX2*(PHI(I+1,J) + PHI(I-1,J)) + DY2*(PHI(I,J+1)
    + PHI(I,J-1)))*TDXDI - PHI(I,J))*ALPHA + PHI(I,J)
RX1 CONTINUE
    IF (PHIC1(IR,J).GT.0) GO TO RELAX
    PHI(IR,J) = ((DX2*(PHI(IR+1,J) + PHI(IR-1,J)) + DY2*(PHI(IR,J+1)
    + PHI(IR,J-1)))*TDXDI - PHI(IR,J))*ALPHA + PHI(IR,J)
RELAX CONTINUE
HOLDI CONTINUE
C*****
C RELAX IN Y DIRECTION
DO HOLDJ J = 2,JMX
  JR = JX1 - J
  DO RELAX I = IONE,ICOUNT,DIRECT
    IF (PHIC1(I,J).GT.0) GO TO RY1
    PHI(I,J) = ((DX2*(PHI(I,J+1) + PHI(I,J-1)) + DY2*(PHI(I+1,J)
    + PHI(I-1,J)))*TDXDI - PHI(I,J))*ALPHA + PHI(I,J)
RY1 CONTINUE
    IF (PHIC1(I,JR).GT.0) GO TO RELAY
    PHI(I,JR) = ((DX2*(PHI(I,JR+1) + PHI(I,JR-1)) + DY2*(PHI(I+1,JR)
    + PHI(I-1,JR)))*TDXDI - PHI(I,JR))*ALPHA + PHI(I,JR)
RELAY CONTINUE
HOLDJ CONTINUE
C*****
C INTERPOLATE TO GET VALUES ON POLAR GRID
DO POLRX N = 1,NW
  WIRPON = WIRPOT(N)
  WIREXN = WIREX(N)
  WIREYN = WIREY(N)
  CALL POINT(WIREXN,WIREYN,I)
  DO POLBC L = 1,NT
    POLPHI(1,L) = WIRPON
    CALL LAG2(PHI,YI,XJ,INTFAC,POLY(N,NDR,L),POLX(N,NDR,L),
    CPOLPHI(NDR,L),ID,JD)
    POLPHI(1,L+NT) = POLPHI(1,L)
    POLPHI(NDR,L+NT) = POLPHI(NDR,L)
POLBC CONTINUE
C*****
C RELAX USING R.R.S. METHOD
L1 = 2
L2 = N2TM
DO I R I = 1,2
  DO POLAX NR = 2,NDRM
    NM = NDR1 - NR
    DO POLA M = L1,L2
      POLPHI(NR,M) = (POLPHI(NR-1,M) + POLPHI(NR+1,M))/C1
      + (POLPHI(NR,M-1) + POLPHI(NR,M+1))/C2
      POLPHI(NM,M) = (POLPHI(NM-1,M) + POLPHI(NM+1,M))/C1

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                                + (POLPHI(NM,P-1) + POLPHI(NM,P+1))/C2
POLA      CONTINUE
          POLPHI(NR,1) = POLPHI(NR,NT1)
          POLPHI(NR,N2T) = POLPHI(NR,NT)
          POLPHI(NM,1) = POLPHI(NM,NT1)
          POLPHI(NM,N2T) = POLPHI(NM,NT)
POLAX CONTINUE
          L1 = L2
          L2 = 2
ITER CONTINUE
C*****
C      INTERPOLATE TO FIX ALUES FOR RECTANGULAR GRID
      DO REINT L = 1,4
          CALL LAG2(POLPHI,R,ANGLE,INTFAC,SORAD,SCTHE(L),SQPTS(L),NDD,
N2D)
REINT CONTINUE
          I = IMY(N)
          J = JMX(N)
C*****
C      STORE POLAR VALUES ON RECTANGULAR GRID
          PHI(I,J+1) = SQPTS(1)
          PHI(I,J) = SQPTS(2)
          PHI(I+1,J) = SQPTS(3)
          PHI(I+1,J+1) = SQPTS(4)
POLRX CONTINUE
C*****
C      CHECK CONVERGENCE
C      SUM THE SQUARE OF THE DIFFERENCES OF CYCLE N AND N-1
C      READ PHI OF N-1
      READLCM(PHIPC,PHIP,IDJD)
      ARITHREG PHIR,PHIPR
      SUM = 0.
      DO VERG I = 1,IX
          PHIR = PHI(I,1)
          PHIPR = PHIPC(I,1)
          DO VERG J = 1,JX
              SUM = SUM + PHIPR*PHIPR - PHIR*PHIR
              PHIR = PHI(I,J+1)
              PHIPR = PHIPC(I,J+1)
VERG CONTINUE
      CONVT(NC) = SUM
      RELEASE PHIR,PHIPR
      SUM = ABS(SLM)
      IF (SUM.LE..00005) GO TO PRPTS
C*****
C      SAVE PHI FOR N-1
      WRITELCM(PHI,PHIP,IDJD)
      IF (NC.GT.1.AND.MOD(NC,10).NE.0) GO TO UPNC
PRPTS CONTINUE
      WRITE(5,F122) NC
      WRITE(5,FIVE) (CONVT(N),N=1,NC,10)
      WRITE(5,F116)
      WRITE(5,FIVE) (SQPTS(L),L=1,4)
      WRITE(5,FIV6)
      CALL PRINT2(POLPHI,N2T,NDR,INT1,INT0,ANGLE,R,NDD,N2D)
      IF (SUM.LE..00005) GO TO ENDED
UPNC CONTINUE
      NC = NC + 1
      IF (NC.LE.NCMAX) GO TO BEGIN
ENDED CONTINUE

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C*****
C PRINT FINAL VALUES
  WRITE(5,F11C)
  CALL PRINT2(PHI,JX,IX,INT1,INT0,XJ,YI,ID,JD)
  CALL FRAME
  CALL UOOTOF
  CALL GIVHSP(3HP35,6HLRELAX,ERR)
  CALL PLOT5
  CALL QUIT
TWO0 FORMAT(2I5,4F10.5)
TWO1 FORMAT(5F10.5)
TWO2 FORMAT(2A10)
TWO3 FORMAT(5I5)
FIVE FORMAT(10(2XF10.3))
FIV1 FORMAT(/10H JMAX IMAX3X4HPHI16X4HPHI26X4HPHI36X4HPHI4)
FIV2 FORMAT(/4X2+DX8X1HX9X2HDY8X1HY9X5HALPHA)
FIV3 FORMAT(/5HWIRES/43H POTENTIAL RADIUS X LCCATION Y LCCATION)
FIV4 FORMAT(/1HR)
FIV5 FORMAT(/18HX VALUES FOR POLAR)
FIV6 FORMAT(/16HPOLAR PHI VALUES)
FIV7 FORMAT(/17HPHI APPROXIMATION)
FIV8 FORMAT(/9HCONSTANTS)
FI10 FORMAT(/3HPHI)
FI11 FORMAT(/1HY)
FI12 FORMAT(/1HX)
FI13 FORMAT(/18HY VALUES FOR POLAR)
FI14 FORMAT(/5HTHETA)
FI15 FORMAT(/24HSQUARE RADIUS AND THETA)
FI16 FORMAT(/16HNEW POINT VALUES)
FI21 FORMAT(/51HNCMAX NUMBER OF RADII AND THETA DIVISIONS AND THETA)
FI22 FORMAT(/33HRESULTS OF CONVERGENCE TEST AFTER15,7H CYCLES)
FI23 FORMAT(/13HPHI INITIALLY)
FI24 FORMAT(3I5,(F10.5))
END
*DATA RELAXINPUT
TEST CASE 4
  500 5 16
  35 12+0. +2. +0. +0.
+36. +13. +1.5
-1. +.1 +11.5 +6.5
+0.

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