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A RATE-LIMITED MODEL OF
MOLTEN-FUEL/COOLANT INTERACTIONS:
MODEL DEVELOPMENT AND PRELIMINARY CALCULATIONS

by

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NOMENCLATURE

a	$1 - (b/R)$	t	time, sec
A	heat-transfer area per gram of heated coolant, $\text{cm}^2 \text{g}^{-1}$	t_e	pressure-relaxation time constant (see Appendix C), sec
c_0	sonic velocity of unheated coolant, cm sec^{-1}	t_h^*	heating time constant (see Appendix C), sec
C	specific heat of coolant (see Appendix C), $\text{cal g}^{-1} ^\circ\text{K}^{-1}$	T	temperature of heated coolant, $^\circ\text{K}$
C_f	specific heat of fuel, $\text{cal g}^{-1} ^\circ\text{K}^{-1}$	T_f	average temperature of fuel, $^\circ\text{K}$
C_p	specific heat of coolant at constant pressure, $\text{cal g}^{-1} ^\circ\text{K}^{-1}$	T_{fc}	center temperature of fuel particles, $^\circ\text{K}$
C_v	specific heat of coolant at constant volume, $\text{cal g}^{-1} ^\circ\text{K}^{-1}$	U_1, U_2	ejection velocities of unheated coolant, cm/sec
f	friction factor	V	specific volume of heated coolant, $\text{cm}^3 \text{g}^{-1}$
g	gravitational acceleration, cm sec^{-2}	V_g	volume of noncondensable gas per gram of heated coolant, $\text{cm}^3 \text{g}^{-1}$
h	heat-transfer coefficient, $\text{cal cm}^{-2} \text{sec}^{-1} ^\circ\text{K}^{-1}$	V_L	specific volume of saturated coolant liquid, $\text{cm}^3 \text{g}^{-1}$
H	specific enthalpy of heated coolant, cal g^{-1}	V_v	specific volume of saturated coolant vapor, $\text{cm}^3 \text{g}^{-1}$
H_L	specific enthalpy of saturated coolant liquid, cal g^{-1}	w	ratio of fuel mass to sodium mass in the mixing zone
H_v	specific enthalpy of saturated coolant vapor, cal g^{-1}	x	mass fraction of coolant vapor
k_f	thermal conductivity of fuel, $\text{cal cm}^{-1} \text{sec}^{-1} ^\circ\text{K}^{-1}$	z	compressibility factor
L	distance from the fuel-coolant mixing zone to a reflecting surface, cm	$Z, Z_1, Z_2, Z_{in}, Z_{ex}$	axial positions in a single subassembly, cm
L_f	latent heat of fusion of fuel, cal g^{-1}	dQ/dt	heating rate per gram of coolant, $\text{cal sec}^{-1} \text{g}^{-1}$
L_m	length of molten zone, cm	<u>Greek Symbols</u>	
n	adiabatic exponent for a noncondensable gas	α_f	thermal diffusivity of fuel, $\text{cm}^2 \text{sec}^{-1}$
P	pressure in the fuel-coolant mixing zone, atm	α_p	thermal-expansion coefficient of heated coolant, $^\circ\text{K}^{-1}$
P_{in}, P_{ex}	inlet and exit plenum pressures in a single subassembly, atm	β_s	adiabatic compressibility of heated coolant, atm^{-1}
r	radial distance, cm	β_T	isothermal compressibility of heated coolant, atm^{-1}
R	fuel-particle radius, cm	δ	penetration thickness, cm
R_g	gas-law constant, $\text{cm}^3 \text{atm g-mole}^{-1} ^\circ\text{K}^{-1}$	ρ_0	density of unheated coolant, g cm^{-3}
R_h	hydraulic radius of coolant channel, cm	ρ_f	density of fuel, g cm^{-3}
S	flow area per gram of heated coolant, $\text{cm}^2 \text{g}^{-1}$		

Unless otherwise specified, subscript e refers to initial conditions.

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ABSTRACT

A model of molten-fuel/coolant interactions is developed that incorporates various limiting rate processes. Essentially, two competing processes are involved: the heating of coolant by the fuel (which produces pressures) and the expansion of the heated coolant (which reduces pressures) against a constraint given by the surroundings. The coolant heating depends on the complicated processes of fragmentation and mixing as well as on the heat-transfer mechanisms.

Preliminary calculations of pressure generation and voiding histories are made considering a 35-cm-long molten zone in an FFTF subassembly. Two different approximations are used for the heating process, and the results are compared. Both acoustic and inertial formulations are considered for the constraining effects of the unheated coolant external to the fuel-coolant mixing zone.

I. INTRODUCTION

A major safety problem for LMFBR's concerns large pressure pulses and mechanical work that might be produced when molten fuel comes into contact with the sodium coolant. A model of such an interaction between molten fuel and coolant is needed, especially in core-disassembly-accident analyses or in studies of fuel-element failure propagation.

In earlier models, a thermodynamic approach was used to estimate the maximum work available from molten fuel-coolant interactions.^{1,2} However, the thermodynamic approach, which assumes instantaneous heating of the coolant, does not give pressure-time histories that are needed for realistic assessment of structural damage. Pressure-time histories can be obtained only by a transient analysis that takes into account the rate processes involved, such as was done by Padilla.³ Consideration of the rate

processes is also needed to determine to what extent the mechanical work is reduced below the thermodynamic limit.

This report discusses a comprehensive model that incorporates various limiting rate processes. The rate processes considered include the fragmentation and mixing of the molten fuel and coolant in a mixing zone, the heat transfer from the fuel to the coolant, and the expansion of the heated coolant against system constraints. The formulation of the model is described, and preliminary calculations of pressure generation and voiding histories are presented. The calculations are made considering sodium reentry impact with fragmentation and mixing in a 35-cm-long molten zone at the core midplane of an FFTF subassembly.

Realistic calculations with the model depend upon understanding the individual rate processes. Major uncertainties lie in the complicated processes of molten-fuel/coolant mixing and fragmentation, and in the processes of heat transfer. However, these unknown rates have been incorporated into the model as input parameters, and parametric studies of pressure generation in molten-fuel/coolant interactions can be performed for reasonable assumed ranges of these parameters.

II. FORMULATION OF THE MODEL

Consider a volume of molten-fuel/coolant mixture formed in a certain zone of the reactor core. As the coolant is heated rapidly by the fuel, the pressure in the mixing zone increases. The mixing zone then expands against a constraint given by the surroundings. The time history of the pressure and the coolant expansion in the mixing zone is determined by the interaction between two competing processes: the heating of the coolant by the fuel (which produces pressure) and the expansion of the heated coolant (which reduces pressure).

If the volume change of the molten fuel is neglected, the expansion of the mixing zone is due only to the volume change of the heated coolant. (The effect of any noncondensable gas present in the mixing zone will be discussed later.) It is also assumed that the heated coolant is in a state of uniform thermodynamic equilibrium throughout the mixing zone and that the mixing zone does not exchange heat or mass with the surroundings. The change in the volume of the heated coolant, V , is related to the changes in the enthalpy H and pressure P by the first law of thermodynamics:

$$\frac{dH}{dt} = \frac{dQ}{dt} + \frac{V dP}{dt}, \quad (1)$$

and the equation of state of the coolant

$$H = H(P, V), \quad (2)$$

where all quantities refer to a unit mass of the heated coolant, and dQ/dt represents the overall heating rate of the coolant in the mixing zone. The form of dQ/dt depends on the processes of fragmentation and mixing as well as on the heat-transfer mechanisms. An additional relationship between P and V is given by consideration of the dynamics of the constraint furnished by the surroundings:

$$P = f\left(V, \frac{dV}{dt}, \frac{d^2V}{dt^2}\right). \quad (3)$$

Equation 3 represents the constraining effects of the surroundings on the expansion of the mixing zone and may be called the equation of constraint. The surroundings include the unheated coolant, intact fuel pins, and other structural materials. In this report we shall use a one-dimensional model of the constraint; although, it can be extended to a three-dimensional case. With appropriate forms of dQ/dt , Eqs. 1-3 can be solved to give time histories of pressure generation and expansion. The forms of dQ/dt and Eqs. 2 and 3 used in the present analysis are discussed below.

III. EQUATION OF STATE OF THE COOLANT

The actual form of the equation of state used for computation depends on the available experimental data and correlations. For a pure liquid or vapor phase, it is convenient to use a P - V - T rather than a P - V - H relationship. In this case, Eq. 1 is rewritten as

$$C_P \frac{dT}{dt} = \frac{dQ}{dt} + TV\alpha_p \frac{dP}{dt}, \quad (4)$$

where the thermodynamic relation,

$$dH = C_P dT + \left[V - T \left(\frac{\partial V}{\partial T} \right)_P \right] dP,$$

has been used, and α_p is the thermal-expansion coefficient:

$$\alpha_p \equiv \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P.$$

Equation 2 is replaced by

$$V = V(P, T). \quad (5)$$

with

$$\frac{dV}{dt} = -\beta_T V \frac{dP}{dt} + \alpha_P V \frac{dT}{dt}, \quad (6)$$

where β_T is the isothermal compressibility:

$$\beta_T \equiv -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T.$$

For an ideal gas, $\alpha_P = 1/T$ and $\beta_T = 1/P$. For a real gas, a useful form of the equation of state is

$$PV = zR_g T, \quad (7)$$

where z is the compressibility factor.

For a two-phase mixture, Eq. 1 may be used as it stands. For the equation of state, it is convenient to use the following forms of empirical correlations:

$$P = P(T) \text{ (vapor pressure equation),}$$

$$H = xH_v + (1-x)H_l, \quad (8)$$

and

$$\frac{dH}{dt} = x \frac{dH_v}{dt} + (1-x) \frac{dH_l}{dt} + \Delta H_{lv} \frac{dx}{dt}, \quad (9)$$

with

$$H_v = H_v(T) \text{ (enthalpy of saturated vapor);} \quad (10a)$$

$$H_l = H_l(T) \text{ (enthalpy of saturated liquid),} \quad (10b)$$

where x is the mass fraction of vapor and $\Delta H_{lv} = H_v - H_l$ is the latent heat of vaporization. Further,

$$V = xV_v + (1-x)V_l$$

and

$$\frac{dV}{dt} = x \frac{dV_v}{dt} + (1-x) \frac{dV_l}{dt} + \Delta V_{lv} \frac{dx}{dt}, \quad (11)$$

with

$$V_v = V_v(T) \text{ (volume of saturated vapor);} \quad (12a)$$

$$V_l = V_l(T) \text{ (volume of saturated liquid),} \quad (12b)$$

where

$$\Delta V_{lv} = V_v - V_l.$$

IV. HEAT-TRANSFER APPROXIMATIONS

Two different approximations will be used for the heating rate of the coolant in the mixing zone. We consider a mixing zone where fuel particles are uniformly dispersed in the coolant as a result of fragmentation of the molten fuel. This is merely a convenient picture (one can conceive of a dispersion of coolant droplets in the fuel as well). Moreover, the mode of fragmentation and mixing may differ, depending on accident situations, and is not well understood. For the purpose of the present analysis, therefore, we shall regard the fuel particle size as a characteristic measure of the level of fragmentation and mixing.

In both approximations, we assume that the heat transfer is limited only by the thermal-conduction resistance of the fuel. The heat-transfer resistance in the sodium coolant is neglected, so that the temperature of the heated sodium is considered to be uniform throughout the mixing zone. This is a reasonable approximation as long as the fuel particles are in direct contact with the liquid sodium, because the thermal conductivity of liquid sodium is about 30 times higher than that of UO_2 . No account is taken of such thermal resistance as might occur if the fuel particles were blanketed by a film of noncondensable gas or sodium vapor.

A. Quasi-steady-state Heat Transfer

In this approximation, the rate at which one gram of coolant is heated by the fuel, dQ/dt , is given by

$$\frac{dQ}{dt} = hA(T_f - T), \quad (13)$$

where h is the effective heat-transfer coefficient, A is the heat-transfer area per gram of heated coolant, and T_f and T are the average fuel and coolant temperatures. The change in T_f is given by

$$C_f w \frac{dT_f}{dt} = - \frac{dQ}{dt}, \quad (14)$$

where C_f is the specific heat of the fuel and w is the fuel-to-coolant mass ratio.

In this approximation, we assume h and A to be constant. If there is a uniform dispersion in the coolant of fuel particles of radius R , the heat-transfer area A becomes

$$A = 3w/\rho_f R, \quad (15)$$

where ρ_f is the fuel density. The heat-transfer coefficient is approximated by

$$h = k_f/R, \quad (16)$$

where k_f is the fuel thermal conductivity. We then have

$$hA = C_f w \left(\frac{3\alpha_f}{R^2} \right), \quad (17)$$

where α_f is the fuel thermal diffusivity.

This approximation accounts in a rough way for a finite rate of fragmentation and mixing of the molten fuel and coolant. The quasi-steady-state approximation implies that, by the time the fragmentation of the fuel is completed, there already exists a significant temperature gradient in the fuel particles. The temperature gradient is assumed to occur over the particle radius.

B. Transient Conduction

Consider a uniform dispersion of fuel particles of radius R suddenly formed in the coolant by instantaneous fragmentation of the molten fuel. We assume that the resulting transient heat transfer from the fuel into the coolant is controlled by the thermal-conduction resistance of the fuel particles.

To solve the problem of transient conduction, we employ the integral method. An energy balance over the fuel particle gives

$$\frac{d}{dt} \int_0^R r^2 T_f dr = \alpha_f R^2 \left[\frac{\partial T_f}{\partial r} \right]_{r=R}. \quad (18)$$

The fuel temperature T_f is a function of the radial distance from the particle center, r , as well as of time. We shall use a linear temperature profile for T_f . When the penetration thickness δ is smaller than R , we assume

$$\frac{T_f - T}{T_{f0} - T} = \begin{cases} \frac{R}{\delta} \left(1 - \frac{r}{R}\right) & \text{for } R - \delta \leq r \leq R \\ 1.0 & \text{for } 0 \leq r \leq R - \delta \end{cases} \quad (19)$$

When $\delta \geq R$, we assume

$$\frac{T_f - T}{T_{fc} - T} = 1 - \frac{r}{R} \quad \text{for } 0 \leq r \leq R, \quad (20)$$

where T_{fc} is the center temperature of the particle.* The linear profile assumes that T_{fc} is constant at the initial fuel temperature, T_{f0} , until δ becomes equal to R and then changes with time. Note that the coolant temperature T is not constant but is a function of time. The change in T is given by the energy equation (Eq. 1) for the heated coolant.

With the temperature profile of Eq. 19 for $\delta \leq R$, Eq. 18 becomes

$$\frac{dT}{dt} \left[1 - \frac{1}{4}(1 + a + a^2 + a^3) \right] + \frac{1}{4}(T_{f0} - T)(1 + 2a + 3a^2) \frac{da}{dt} = \frac{3\alpha_f}{R^2} \frac{T_{f0} - T}{a - 1}, \quad (21)$$

where $a = 1 - (\delta/R)$. Equation 21 gives the change in δ with time. The heating rate dQ/dt is given by

$$\frac{dQ}{dt} = C_{fw} \left(\frac{3\alpha_f}{R^2} \right) \frac{T_{f0} - T}{1 - a} \quad \text{for } \delta \leq R. \quad (22)$$

With the temperature profile of Eq. 20 for $\delta \geq R$, Eq. 18 becomes

$$\frac{d}{dt} \left(\frac{3T + T_{fc}}{4} \right) = \frac{3\alpha_f}{R^2} (T - T_{fc}), \quad (23)$$

which gives the change in T_{fc} with time. The heating rate, dQ/dt , is given by

$$\frac{dQ}{dt} = C_{fw} \left(\frac{3\alpha_f}{R^2} \right) (T_{fc} - T) \quad \text{for } \delta \geq R. \quad (24)$$

*For the case of constant coolant temperature, the linear profile appears to approximate the exact solution better than a quadratic or cubic profile; see Appendix I of Ref. 8.

For this approximation, dQ/dt is infinite at zero time, so numerical solutions must start from some time greater than zero. Appendix A gives approximate analytic solutions to be used for small times.

V. ONE-DIMENSIONAL MODEL OF CONSTRAINT

We consider a molten zone of length L_m in a single subassembly and assume that the molten fuel is mixed with all the coolant in the flow channel of length L_m . The subassembly can is assumed to be rigid, so that the mixing zone can expand only in the axial direction. We shall consider two formulations of the constraining effects of the coolant columns above and below the mixing zone. The formulations represent a special case of Eq. 3.

A. Acoustic Constraint of Infinite Extent

Assume that the coolant columns above and below the fuel-coolant mixing zone are of infinite extent and compressible, and do not exchange heat or mass with the mixing zone. The acoustic approximation for the compressible flow, both upward and downward, gives

$$P(t) - P_0 = \rho_0 c_0 \frac{dZ}{dt} \quad (25)$$

where $P(t)$ is the pressure in the mixing zone, P_0 is the initial pressure throughout, ρ_0 and c_0 are the density and sonic velocity of the unheated sodium in the loading column, and $Z(t)$ is the position of the interface between the mixing zone and the loading column measured relative to the stationary median plane (see Fig. 1). The above acoustic constraint is valid only for an initial period of $2L/c_0$, where L is the distance from the mixing zone boundary to the nearest free reflecting surface. The acoustic formulation is accurate during the initial shock phase of the molten-fuel/coolant interaction. It can be extended to longer times by taking into account reflected waves and center-of-mass motion.

$$P(t) - P_0 = \rho_0 c_0 \frac{dZ}{dt} + \frac{\rho_0 c_0}{S} \frac{dV_{tot}}{dt}$$

$$V_{tot} = V + V_1 + V_2$$

Fig. 1

One-dimensional Acoustic Constraint of Infinite Extent. ANL
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The flow velocity dZ/dt is related to the change in the total mixing volume V_{tot} by

$$\frac{dV_{tot}}{dt} = S \frac{dZ}{dt} \quad (26)$$

where S is the flow area (both upward and downward) per gram of heated coolant. Neglecting the volume change of the fuel, we have

$$\frac{dV_{\text{tot}}}{dt} = \frac{dV}{dt} + \frac{dV_g}{dt} \quad (27)$$

where V_g is the volume of any noncondensable gas present per gram of heated coolant. Assuming that this is an ideal gas and undergoes an adiabatic change, we have

$$PV_g^n = P_0 V_{g0}^n = \text{constant}, \quad (28)$$

where n is the adiabatic exponent and V_{g0} is the initial gas volume. Substitution of Eqs. 26-28 into Eq. 25 gives

$$\frac{dV}{dt} = \frac{S}{\rho_0 c_0} (P - P_0) + \frac{P_0^{1/n} V_{g0}}{nP[1+(1/n)]} \frac{dP}{dt} \quad (29)$$

The second term in the right-hand side represents the compliance of any noncondensable gas that may be initially present in the mixing zone.

B. Inertial Constraint in a Single Subassembly

Assume that the coolant columns above and below the fuel-coolant mixing zone are incompressible and do not exchange heat or mass with the mixing zone. The heated coolant in the mixing zone expands against the inertia of the unheated coolant columns.

$$\frac{dU_2}{dt} = -g + \frac{P - P_{\text{ex}}}{\rho_0 (Z_{\text{ex}} - Z_2)} - \frac{1}{2} U_2^2 \frac{f}{R_h}$$

$$\frac{dU_1}{dt} = g + \frac{P - P_{\text{in}}}{\rho_0 Z_1} - \frac{1}{2} U_1^2 \frac{f}{R_h}$$

Fig. 2. Inertial Constraint in a Single Subassembly.
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A macroscopic momentum balance gives the equation of motion of the loading coolant columns (see Fig. 2):

$$\frac{dU_2}{dt} = -g + \frac{P - P_{\text{ex}}}{\rho_0 (Z_{\text{ex}} - Z_2)} - (1/2) U_2^2 \frac{f}{R_h} \quad (30)$$

$$\frac{dU_1}{dt} = g + \frac{P - P_{\text{in}}}{\rho_0 Z_1} - (1/2) U_1^2 \frac{f}{R_h} \quad (31)$$

where P_{ex} is the exit plenum pressure, P_{in} is the inlet plenum pressure, Z_{ex} is the position of the exit plenum (measured from the inlet plenum), Z_2 is the position of the interface between the mixing zone and the upper coolant column, Z_1 is the position of the interface between the mixing zone and the lower coolant column, U_2 is the ejection velocity of the upper coolant column, U_1 is the ejection velocity of the lower coolant column, g is the acceleration due to gravity, f is the friction factor, and R_h the hydraulic radius of the coolant channel.

The ejection velocities U_1 and U_2 are related to the change in V_{tot} by

$$\frac{dV_{tot}}{dt} = \frac{S}{2}(U_1 + U_2) = \frac{S}{2}\left(-\frac{dZ_1}{dt} + \frac{dZ_2}{dt}\right). \quad (32)$$

Neglecting the volume change of the fuel, we have

$$\frac{dV}{dt} = \frac{S}{2}(U_1 + U_2) + \frac{P_0^{1/n} V g_0}{n P^{1+(1/n)}} \frac{dP}{dt}. \quad (33)$$

VI. PRELIMINARY CALCULATIONS; RESULTS AND DISCUSSION

Calculations have been made considering a molten-fuel/coolant interaction that might occur following sodium reentry into a 35-cm-long molten zone at the core midplane of an FFTF subassembly. These calculations represent an initial examination of the effects of various limiting rate processes.

The calculations assumed a uniform dispersion of fuel particles of radius 117μ in the liquid sodium and used a fuel-to-sodium mass ratio of 8.3. The initial temperatures used for the sodium and fuel were 1100 and 2900°K, respectively. The fuel-particle size used is the median of the particle-size distribution observed in laboratory experiments.⁴ The flow area per gram of the heated sodium, S , was 0.077 cm^2 , corresponding to the 35-cm-long molten zone considered.

The calculations used the thermodynamic properties of sodium given by Golden and Tokar.⁵ Although the sodium properties are given only for temperatures up to 1644°K, they were extrapolated and used for sodium temperatures as high as 2500°K (see Appendix B). The coupled Eqs. 1-3 were solved numerically using an Argonne National Laboratory subroutine based on the Bulirsch-Stoer method.⁶ Table I summarizes the forms of Eqs. 1-3 used in the calculations.

A. Acoustic Constraint of Infinite Extent

Table II summarizes the parameters and initial conditions used and the peak pressures calculated. Figure 3 shows details of the pressure-time histories for quasi-steady-state heat transfer. The case of transient conduction is presented in Fig. 4. The corresponding temperature histories are given in Figs. 5 and 6. Figure 7 is a pressure-volume plot of the results. Solid curves represent the case when there was no noncondensable gas present in the mixing zone. Broken curves indicate the cushioning effect of the presence of a noncondensable gas in the mixing zone. The initial volume of the gas was 0.27 cc per gram of the heated sodium or 20% of the initial sodium volume. From these figures, we make the following observations.

TABLE 1. Summary of Pertinent Equations

Heat Transfer	Quasi-steady State		$\frac{dQ}{dt} = C_f w \left(\frac{3\alpha_f}{R^2} \right) (T_f - T), \quad C_f w \frac{dT_f}{dt} = - \frac{dQ}{dt}$
	Transient Conduction	$\delta \leq R$	$\frac{dQ}{dt} = C_f w \left(\frac{3\alpha_f}{R^2} \right) \frac{T_{f_0} - T}{1 - a}$ $\frac{dT}{dt} \left[1 - \frac{1}{4} (1 + a + a^2 + a^3) \right] + \frac{1}{4} (T_{f_0} - T) (1 + 2a + 3a^2) \frac{da}{dt} = \frac{3\alpha_f}{R^2} \frac{T_{f_0} - T}{a - 1}$
		$\delta \geq R$	$\frac{dQ}{dt} = C_f w \left(\frac{3\alpha_f}{R^2} \right) (T_{fc} - T)$ $\frac{d}{dt} \left(\frac{3T + T_{fc}}{4} \right) = \frac{3\alpha_f}{R^2} (T - T_{fc})$
Energy Conservation for Coolant	Liquid Phase		$C_p \frac{dT}{dt} = \frac{dQ}{dt} + TV\alpha_p \frac{dP}{dt}$
	Two-phase		$\frac{dH}{dt} = \frac{dQ}{dt} + V \frac{dP}{dt}$
Equation of State	Liquid Phase		$\frac{dV}{dt} = -\beta_T V \frac{dP}{dt} + \alpha_p V \frac{dT}{dt}$
	Two-phase		$P = P(T) \text{ (Vapor-pressure Equation)}$ $\frac{dH}{dt} = \left[x \frac{dH_v}{dT} + (1-x) \frac{dH_l}{dT} \right] \frac{dT}{dt} + (H_v - H_l) \frac{dx}{dt}$ $\frac{dV}{dt} = \left[x \frac{dV_v}{dT} + (1-x) \frac{dV_l}{dT} \right] \frac{dT}{dt} + (V_v - V_l) \frac{dx}{dt}$
Constraint	Acoustic		$\frac{dV}{dt} = \frac{S}{\rho_0 c_0} (P - P_0) + \frac{P_0^{1/n} V_0}{n P^{1+(1/n)}} \frac{dP}{dt}$
	Inertial		$\frac{dV}{dt} = \frac{S}{2} (U_1 + U_2) + \frac{P_0^{1/n} V_0}{n P^{1+(1/n)}} \frac{dP}{dt}$ $\frac{dU_2}{dt} = -g + \frac{P - P_{ex}}{\rho_0 (Z_{ex} - Z_2)} - \frac{1}{2} U_2^2 \frac{f}{R_h}, \quad U_2 = \frac{dZ_2}{dt}$ $\frac{dU_1}{dt} = g + \frac{P - P_{in}}{\rho_0 Z_1} - \frac{1}{2} U_1^2 \frac{f}{R_h}, \quad U_1 = - \frac{dZ_1}{dt}$

TABLE II. Calculational Results for Acoustic Constraint

Heat Transfer	Quasi-steady State		Transient Conduction	
Parameters	w (fuel-to-sodium mass ratio) = 8.3 R (fuel-particle radius) = 117 μ S (flow area) = 0.077 cm ² /g Na, which corresponds to a 35-cm-long molten zone in an FFTF subassembly.			
Initial Conditions	$T_{\text{Na}} = 1100^{\circ}\text{K}$, $T_{\text{fuel}} = 2900^{\circ}\text{K}$,		$P_0 = 1 \text{ atm}$,	$V_0 = 1.35 \text{ cc/g}$.
Noncondensable Gas	$V_{g0} = 0$	$V_{g0} = 20\% \text{ of } V_0$	$V_{g0} = 0$	$V_{g0} = 20\% \text{ of } V_0$
Peak Pressure during Liquid-phase Heating	$t = 0.35 \text{ msec}$ $P = 472 \text{ atm}$ $T_{\text{Na}} = 1306^{\circ}\text{K}$	$t = 1.79 \text{ msec}$ $P = 312 \text{ atm}$ $T_{\text{Na}} = 1792^{\circ}\text{K}$	$t = 0.056 \text{ msec}$ $P = 3286 \text{ atm}$ $T_{\text{Na}} = 1751^{\circ}\text{K}$	$t = 0.231 \text{ msec}$ $P = 1221 \text{ atm}$ $T_{\text{Na}} = 1995^{\circ}\text{K}$
Vaporization Starts	$t = 5.28 \text{ msec}$ $P = 127 \text{ atm}$ $T_{\text{Na}} = 2193^{\circ}\text{K}$	$t = 5.25 \text{ msec}$ $P = 127 \text{ atm}$ $T_{\text{Na}} = 2195^{\circ}\text{K}$	$t = 2.84 \text{ msec}$ $P = 163 \text{ atm}$ $T_{\text{Na}} = 2304^{\circ}\text{K}$	$t = 2.39 \text{ msec}$ $P = 174 \text{ atm}$ $T_{\text{Na}} = 2335^{\circ}\text{K}$
Peak Pressure during Two-phase Heating	$t = 14.9 \text{ msec}$ $P = 169 \text{ atm}$ $T_{\text{Na}} = 2321^{\circ}\text{K}$	$t = 14.8 \text{ msec}$ $P = 169 \text{ atm}$ $T_{\text{Na}} = 2321^{\circ}\text{K}$	$t = 5.0 \text{ msec}$ $P = 169 \text{ atm}$ $T_{\text{Na}} = 2322^{\circ}\text{K}$	$t = 4.6 \text{ msec}$ $P = 182 \text{ atm}$ $T_{\text{Na}} = 2358^{\circ}\text{K}$

Fig. 4

Pressure-Time Histories for Acoustic Constraint; Transient Conduction. (Broken curves indicate the case in which the mixing zone contains a noncondensable gas. The initial volume of the gas is 20% of the sodium volume.) ANL Neg. No. 900-482 Rev. 1.

Fig. 3

Pressure-Time Histories for Acoustic Constraint; Quasi-steady-state Heat Transfer. (Broken curves indicate the case in which the mixing zone contains a noncondensable gas. The initial volume of the gas is 20% of the sodium volume.) ANL Neg. No. 900-483 Rev. 1.

Fig. 5. Temperature-Time Histories for Acoustic Constraint; Quasi-steady-state Heat Transfer with No Noncondensable Gas Present

Fig. 6. Temperature-Time Histories for Acoustic Constraint; Transient Conduction with No Noncondensable Gas Present

Fig. 7. Effect of Heating Rates on Pressure Generation; Acoustic Constraint

depends upon the values assumed for the properties of sodium under conditions for which measurements are not available.

Only after the mixing zone has expanded for several milliseconds does the pressure relax sufficiently to allow the heated sodium to vaporize.

2. The transient-conduction approximation gives a peak pressure higher by an order of magnitude than does the quasi-steady-state heat-transfer approximation, because of the higher initial heating rate. The

1. The peak pressures occur during the liquid-phase heating and expansion when boiling is suppressed. The high initial heat flux into the sodium produces a large pressure with rapid thermal expansion of the liquid sodium in the mixing zone. The initial rate of pressure rise is determined by the sodium thermal pressure coefficient and the initial rate of temperature increase of the liquid sodium in the mixing zone. (The thermal pressure coefficient is the quotient of the thermal-expansion coefficient α_p and the isothermal compressibility β_T .) The dependence of the peak pressure upon various parameters is more complicated and is discussed in Appendix C for the different heat transfer approximations used here. In any case, the peak pressure

very high initial heating rate for the transient-conduction approximation results from the assumption of instantaneous fragmentation and dispersion of the fuel in the sodium. A finite rate of fragmentation and mixing would reduce the heat flux into the sodium and therefore the resulting peak pressure.

3. The presence of the 20 vol % noncondensable gas in the mixing zone reduced the peak pressure and also delayed its time of occurrence. For the quasi-steady-state heat-transfer approximation, the peak pressure was reduced from 472 to 312 atm, and the time at which the peak pressure occurred changed from 0.35 to 1.79 msec. For the transient-conduction approximation, the peak pressure was reduced from 3286 to 1221 atm, and the corresponding time changed from 0.056 to 0.231 msec. With the noncondensable gas, the initial pressure rise was slow, so the heated sodium started to vaporize early in the process. When the gas was sufficiently pressurized, the pressure increased rapidly and exceeded the sodium vapor pressure. The sodium vapor then recondensed completely, and the peak pressure occurred during this second phase of liquid expansion, as shown in Figs. 3 and 4.

4. The peak pressure that occurs during the liquid-phase expansion increases with increasing length of molten zone. Detailed discussion is given in Appendix C.

5. The work done can best be visualized from the pressure-volume plot, Fig. 7. The transient-conduction approximation gives a larger amount of work than does the quasi-steady-state heat-transfer approximation. This is so because transient conduction approximation gives a higher heating rate, resulting in a higher pressure.

B. Inertial Constraint in a Single Subassembly

The calculations assumed that the coolant columns above and below the fuel-coolant mixing zone were initially stagnant. The parameters used were: $P_{ex} = 1.456$ atm; $P_{in} = 7.485$ atm; $Z_{ex} = 420$ cm; Z_2 (at $t = 0$) =

215 cm; Z_1 (at $t = 0$) = 180 cm; $f/R_h = 0.0542$ cm⁻¹. These values are appropriate to a 35-cm-long molten zone in an FFTF subassembly.

Fig. 8. Pressure-Time Histories for Inertial Constraint

Figure 8 shows pressure-time histories for both approximations of quasi-steady-state heat transfer and transient conduction. Corresponding temperature histories are given in Fig. 9. The inertial constraint overestimates the initial pressure peak as discussed in the next section.

Fig. 9
Temperature-Time Histories for
Inertial Constraint

Figure 10 gives voiding histories. The lower channel becomes completely voided in 25 msec for the quasi-steady-state heat-transfer approximation, and in 18 msec for the transient-conduction approximation.

Fig. 10. Voiding Histories in an FFTF Subassembly.
ANL Neg. No. 900-481.

C. Acoustic and Inertial Constraints

Pressure-time histories were taken from Fig. 3 (acoustic constraint) and Fig. 8 (inertial constraint) for the quasi-steady-state heat-transfer approximation. They are compared in Fig. 11.

The acoustic constraint is accurate up to the acoustic unloading time $2L/c_0$ when the shock wave is terminated. At this time, the unloading rarefaction wave will depressurize the heated sodium, resulting in flashing of the sodium. The inertial constraint, on the other hand, gives too high a peak shock pressure. However, it does give approximately the correct shock impulse, so it can be used for all times in calculations of work done and total impulse. A practical approximation appears to be use of the

acoustic constraint up to the unloading time and use of the inertial constraint for continuation to longer times. At this transition the ejection velocity of unheated sodium column is given by use of momentum conservation. The transition is indicated by a dotted line in Fig. 11.

Fig. 11

Pressure-Time Histories for an Acoustic Constraint of Infinite Extent and for an Inertial Constraint; Quasi-steady-state Heat Transfer. ANL Neg. No. 900-477.

D. Effect of Change of Fuel Volume

In the formulation of the model, we neglected the change in fuel volume. To examine this, consider a UO_2 -sodium mixture which consists of one gram of sodium and w grams of UO_2 . The total volume of the mixture is assumed constant; i.e.,

$$V_{\text{tot}} = V_f + V = \text{constant}$$

or

$$dV_{\text{tot}} = dV_f + dV = 0. \quad (34)$$

Suppose the UO_2 is solid, or it does not solidify if molten. Assume the UO_2 -sodium mixture is surrounded by an adiabatic wall. The temperature changes of UO_2 and sodium are related by

$$C_{pw} dT_f = -C dT. \quad (35)$$

The volume changes of UO_2 and sodium are, respectively,

$$dV_f = (-\beta_f dP + \alpha_f dT_f) V_f, \quad (36)$$

and

$$dV = (-\beta dP + \alpha dT) V, \quad (37)$$

where subscript f refers to the fuel. For convenience, we dropped subscripts P and T from α_P and β_T . Substitution of Eqs. 35-37 into Eq. 34 and rearrangement gives

$$\Delta P = \underbrace{\left(\frac{\alpha}{\beta} \Delta T\right)}_{\text{The value of } \Delta P \text{ when the fuel volume change is neglected}} \underbrace{\frac{1 - \frac{\rho C \alpha_f}{\rho_f C_f \alpha}}{1 + \frac{w \rho_f \beta_f}{\rho_f \beta}}}_{\text{correction factor}} \quad (38)$$

The value of
 ΔP when the
 fuel volume
 change is
 neglected

where V_f/V has been replaced by $w\rho/\rho_f$. When $w = 10$ g, the correction factor is about 0.9 for molten UO_2 and somewhat higher for solid UO_2 , so that the error in neglecting this effect is about 10%.

Now consider solidification of molten UO_2 at the melting point. Suppose w grams of molten UO_2 completely solidify without temperature change. The increase in sodium temperature is given by

$$wL_f = C(\Delta T), \quad (39)$$

where L_f is the latent heat of fusion. The change in sodium volume is

$$\Delta V = V \left(-\beta \Delta P + \alpha \frac{wL_f}{C} \right), \quad (40)$$

The change in UO_2 volume is

$$\Delta V_f = w \left[\frac{1}{(\rho_f)_{\text{solid}}} - \frac{1}{(\rho_f)_{\text{liquid}}} \right] = -0.011w. \quad (41)$$

Putting Eqs. 40 and 41 into Eq. 34, we find

$$\Delta P = \underbrace{\frac{\alpha}{\beta} \frac{wL_f}{C}}_{\text{The value of } \Delta P \text{ when the fuel volume change is neglected}} \underbrace{\left(1 - \frac{0.011C}{\alpha L_f V} \right)}_{\text{correction factor}}. \quad (42)$$

The value of
 ΔP when the
 fuel volume
 change is
 neglected

The correction factor is about 0.9. Thus, neglecting the volume change of fuel causes only 10% error in ΔP . Therefore, these corrections are not needed within the present state of the art in modeling fuel-coolant interactions.

APPENDIX A

Transient-conduction Solutions for Small Times

For the transient-conduction approximation, the heating rate dQ/dt is infinite at zero time, and numerical solutions must start from some time greater than zero. This difficulty is avoided by using the following approximate solutions for small times. The solutions have been derived considering the heated coolant to be initially a single-phase liquid.

For the acoustic constraint,

$$T - T_0 = (T_{f_0} - T_0) \left[1 - \exp \left(- \frac{C_{fw}}{B_2} \frac{3\sqrt{\alpha_f t}}{R} \right) \right]; \quad (A.1)$$

$$P - P_0 = \frac{1}{1 + B_1} \frac{\alpha_P}{\beta_T} (T - T_0); \quad (A.2)$$

$$V - V_0 = \frac{1}{1 + B_1} \left(\frac{S}{\rho_0 c_0} \right) \int_0^t (P - P_0) dt + \left(\frac{B_1}{1 + B_1} \right) \alpha_P V_0 (T - T_0); \quad (A.3)$$

$$a = 1 - \frac{2\sqrt{\alpha_f t}}{R}; \quad (A.4)$$

where

$$B_1 = \frac{V g_0}{\beta_T V_0 n P_0} \quad \text{and} \quad B_2 = C_v + \frac{T_0}{41.29} \frac{\alpha_P^2}{\beta_T} \frac{B_1}{1 + B_1} V_0.$$

The subscript 0 denotes initial values, and all property values are evaluated at the initial sodium temperatures. The specific heat at constant volume is given by $C_v = C_p \beta_s / \beta_T$, where β_s is the adiabatic compressibility of the heated sodium. These solutions are valid if

$$t \ll \frac{R^2}{\alpha_f} \quad \text{and} \quad t \ll \frac{\beta_T V_0 \rho_0 c_0}{S}.$$

For the inertial constraint with no noncondensable gas present in the mixing zone,

$$T - T_0 = (T_{f_0} - T_0) \left[1 - \exp \left(- \frac{C_{fw}}{C_v} \frac{3\sqrt{\alpha_f t}}{R} \right) \right]; \quad (A.5)$$

$$P - P_0 = \left(\frac{\alpha_P}{\beta_T} \right) (T - T_0); \quad (A.6)$$

$$V - V_0 = \frac{S}{2} \int_0^t (U_1 + U_2) dt; \quad (A.7)$$

$$a = 1 - \frac{2\sqrt{\alpha_f t}}{R}; \quad (A.8)$$

$$U_2 = \int_0^t \left(-g + \frac{P - P_{ex}}{\rho_0(Z_{ex} - Z_2)} \right) dt; \quad (A.9)$$

$$Z_2(t) - Z_2(t=0) = \int_0^t U_2 dt; \quad (A.10)$$

$$U_1 = \int_0^t \left(g + \frac{P - P_{in}}{\rho_0 Z_1} \right) dt; \quad (A.11)$$

$$Z_1(t) - Z_1(t=0) = - \int_0^t U_1 dt. \quad (A.12)$$

The above solutions are valid if

$$t \ll \frac{R^2}{\alpha_f} \quad \text{and} \quad t \ll \sqrt{\frac{\beta V_0 \rho_0 L}{S}},$$

where L is the loading column length, either the upper or the lower column length, whichever is shorter.

Similar approximate solutions may be derived when the heated coolant is initially a two-phase mixture.

APPENDIX B

Thermodynamic Properties of Sodium Used in Calculations

The thermodynamic properties of sodium used were taken from ANL-7323,⁵ and the following forms were employed.

For the liquid region (T in °K),

$$C_p(\text{cal/g } ^\circ\text{K}) = 0.389352 + 1.99078 \times 10^{-4}T + 11.0542 \times 10^{-8}T^2; \quad (\text{B.1})$$

$$\alpha_p(^{\circ}\text{K}^{-1}) = 0.21968 \times 10^{-3} + 0.81226 \times 10^{-7}T + 0.97135 \times 10^{-11}T^2 \\ + 0.68998 \times 10^{-15}T^3; \quad (\text{B.2})$$

$$\beta_s = 0.69651 \times 10^{-5} + 0.26741 \times 10^{-7}T - 0.14891 \times 10^{-10}T^2 \\ + 0.82082 \times 10^{-14}T^3; \quad (\text{B.3})$$

$$\beta_T = \beta_s + \frac{VT\alpha_p^2}{C_p}, \quad (\text{B.4})$$

where β_s is the adiabatic compressibility.

For the two-phase region (T in °K),

$$P(\text{atm}) = T^{-0.61344} \exp\left(15.3838 - \frac{12767.8}{T}\right) \quad (\text{B.5})$$

(Vapor-pressure equation);

$$H_L(\text{cal/g}) = -0.92130 \times 10^3 + 0.19434 \times 10T - 0.10510 \times 10^{-2}T^2 \\ + 0.22418 \times 10^{-6}T^3; \quad (\text{B.6})$$

$$H_V(\text{cal/g}) = 0.18605 \times 10^4 - 0.14691 \times 10T + 0.10381 \times 10^{-2}T^2 \\ - 0.23238 \times 10^{-6}T^3; \quad (\text{B.7})$$

$$V_L(\text{cc/g}) = V_{L_0} \left[\exp \int_{T_{L_0}}^T \alpha_s dT \right], \quad (\text{B.8})$$

where

α_s = thermal-expansion coefficient of saturated liquid

$\cong \alpha_p$ given above,

and V_{L_0} and T_{L_0} represent reference liquid specific volume and temperature, respectively. Also,

$$V_g(\text{cc/g}) = zR_gT/P, \quad (\text{B.9})$$

where

$$\begin{aligned} R_g &= \text{gas constant per 1 g of sodium} = 3.565 \text{ cc-atm/}^\circ\text{K-g;} \\ z &= \text{compressibility factor} \\ &= -0.97258 + 0.42417 \times 10^{-2}T \\ &\quad - 0.28996 \times 10^{-5}T^2 + 0.55424 \times 10^{-9}T^3. \end{aligned} \quad (\text{B.10})$$

The sodium properties given above are tabulated in ANL-7323⁵ for sodium temperatures up to 1644°K. For higher temperatures, no reliable data are available. Therefore, we used Eqs. B.1-B.10 for sodium temperatures as high as 2500°K, although there is no physical basis for doing so. At the critical temperature of 2573°K, the vapor-pressure equation used, Eq. B.5, gives a somewhat lower critical pressure (271.7 atm) than the value of Dillon *et al.* (350 atm).⁷

We decided to calculate the values of β_T from β_S by the thermodynamic relation, Eq. B.4, rather than directly using the values of β_T tabulated in ANL-7323. The tabulated values do not appear consistent with Eq. B.4. The calculated values were found to be 20-30% larger than those tabulated.

Although the enthalpy data were taken from ANL-7323,⁵ we changed the reference temperature. The enthalpy of saturated liquid sodium at 25°C was taken to be zero.

APPENDIX C

Pressure Pulses during Liquid-phase Heating and Expansion

We examine analytically the pressure pulse developed during the liquid-phase heating and expansion. To do this, we rewrite Eq. 4 as

$$C \frac{dT}{dt} = \frac{dQ}{dt}, \quad (C.1)$$

where C is the coolant specific heat of some sort. (Strictly, C is neither C_p nor C_v .) Substitution of Eqs. 29 and C.1 into Eq. 6 gives

$$\frac{dP}{dt} + \frac{S}{\beta_T V_0 \rho_0 c_0} (P - P_0) = \frac{\alpha_P}{\beta_T} \frac{dT}{dt} = \frac{\alpha_P}{\beta_T C} \frac{dQ}{dt}, \quad (C.2)$$

where the volume of noncondensable gas, V_{g0} , has been assumed to be zero.

The formal solution of Eq. C.2 is

$$\Delta P = \exp\left(-\frac{t}{t_e}\right) \int_0^t \frac{\alpha_P}{\beta_T} \frac{1}{C} \frac{dQ}{dt} \exp\left(\frac{t}{t_e}\right) dt, \quad (C.3)$$

where $t_e = \beta_T V_0 \rho_0 c_0 / S$ is essentially a pressure relaxation time constant and $\Delta P = P - P_0$. In Eq. C.3 we have assumed constant property values and $V \cong V_0$. We shall consider the following three cases, in all of which $dQ/dt \equiv 0$ and $P \equiv P_0$ for $t < 0$.

1. $dQ/dt = q = \text{Constant}$

Equation C.3 becomes

$$\Delta P = \frac{\alpha_P}{\beta_T} \frac{q}{C} t_e \left[1 - \exp\left(-\frac{t}{t_e}\right) \right], \quad (C.4)$$

and

$$\Delta P_{\max} = \frac{\alpha_P}{\beta_T} \frac{q}{C} t_e = \frac{\alpha_P V_0 \rho_0 c_0}{CS} q \quad (C.5)$$

or

$$\Delta P_{\max} = \frac{\alpha_P \rho_0 c_0}{2C} L_m q, \quad (C.6)$$

since $L_m = 2V_0/S$. Thus the peak pressure is directly proportional to the initial length of molten zone, L_m . This peak pressure is really the asymptotic value of the pressure, under acoustic constraint, for a constant heat flux q .

2. Quasi-steady-state Heat Transfer

In this case,

$$C \frac{dT}{dt} = \frac{dQ}{dt} = hA(T_f - T) = -C_f w \frac{dT_f}{dt}. \quad (C.7)$$

Equation C.7 is solved to give

$$\frac{dQ}{dt} = \frac{C}{t_h^*} (T_{eq} - T_0) \exp \left[-\left(\frac{t}{t_h^*} \right) \right], \quad (C.8)$$

where

$$t_h^* = \frac{1}{hA} \left(\frac{1}{C} + \frac{1}{C_f w} \right)^{-1}$$

is a heating time constant and

$$T_{eq} = \frac{CT_0 + C_f w T_{f0}}{C + C_f w}$$

is the final equilibrium temperature. With the use of Eq. C.8, Eq. C.3 becomes

$$\Delta P = \left(\frac{\alpha_P}{\beta_T} \right) \frac{(T_{eq} - T_0)}{t_h^*} \left(\frac{1}{t_e} - \frac{1}{t_h^*} \right)^{-1} \left[\exp \left(-\frac{t}{t_h^*} \right) - \exp \left(-\frac{t}{t_e} \right) \right], \quad (C.9)$$

and we have

$$\Delta P_{max} = \frac{\alpha_P}{\beta_T} (T_{eq} - T_0) \left(\frac{t_e}{t_h^*} \right)^{t_h^*/(t_h^* - t_e)} \quad (t_e \neq t_h^*). \quad (C.10)$$

We note that

$$\Delta P_{max} \rightarrow \frac{\alpha_P}{\beta_T} (T_{eq} - T_0) \text{ as } \frac{t_h^*}{t_e} \rightarrow 0$$

and

$$\Delta P_{max} \rightarrow 0 \text{ as } \frac{t_h^*}{t_e} \rightarrow \infty.$$

The peak pressure increases with increasing initial molten-zone length, as t_e is proportional to the molten-zone length.

3. Transient Conduction

When $\delta \ll R$, Eq. 22 becomes

$$C \frac{dT}{dt} = \frac{dQ}{dt} = \frac{3}{2} \frac{C_f w}{R} \sqrt{\frac{\alpha_f}{t}} (T_{f_0} - T). \quad (C.11)$$

Equation C.11 is solved to give

$$\frac{dQ}{dt} = \frac{3}{2} \frac{C_f w}{R} \sqrt{\frac{\alpha_f}{t}} (T_{f_0} - T_0) \exp\left(-\frac{3C_f w}{C} \frac{\sqrt{\alpha_f t}}{R}\right). \quad (C.12)$$

With the use of Eq. C.12, Eq. C.3 becomes

$$\begin{aligned} \Delta P = & \exp\left(-\frac{t}{t_e}\right) \int_0^t \frac{\alpha_P}{\beta_T} \frac{3}{2} \frac{C_f w}{CR} \sqrt{\frac{\alpha_f}{t}} (T_{f_0} - T_0) \\ & \times \exp\left(-\frac{3C_f w}{C} \frac{\sqrt{\alpha_f t}}{R} + \frac{t}{t_e}\right) dt. \end{aligned} \quad (C.13)$$

The use of Eq. C.12 is equivalent to assuming an infinite source of heat in the fuel. We find $\Delta P_{\max}$ by putting $dP/dt = 0$ in Eq. C.2:

$$\Delta P_{\max} = t_e \frac{\alpha_P}{\beta_T} \frac{1}{C} \frac{dQ}{dt} \text{ at } t = t_{\max}.$$

The value of $t_{\max}$ is found by putting $\frac{d(\Delta P)}{dt} = 0$ in Eq. C.13 and solving for t . To first-order approximation, $t_{\max} \cong t_e$. Hence,

$$\Delta P_{\max} \cong \frac{\alpha_P}{\beta_T} (T_{f_0} - T_0) \frac{3C_f w}{2C} \frac{\sqrt{\alpha_f t_e}}{R} \exp\left(-\frac{3C_f w}{C} \frac{\sqrt{\alpha_f t_e}}{R}\right). \quad (C.14)$$

Thus, the peak pressure is approximately proportional to the square root of t_e or the initial molten-zone length if $\sqrt{\alpha_f t_e} \ll R$ (which is true in most cases of interest).

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