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I-A2412-5

Technical Report

STRUCTURAL INTEGRITY OF SHIPPING CONTAINERS
FOR RADIOACTIVE MATERIALS

PART V: AN ANALYTICAL STUDY OF LONGITUDINAL
VEHICLE-COLLISIONS

by

K. D. Doshi

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

November 1965

Prepared for

DIVISION OF REACTOR DEVELOPMENT
U.S. ATOMIC ENERGY COMMISSION

Contract No. AT(30-1)-2539

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ABSTRACT

A basic approach is outlined for developing analytical procedures to predict vehicle collision behavior. The approach is to idealize the physical system by a non-linear, lumped parameter mathematical model, then to set up the equations of motion of the model and solve these numerically by means of Runge-Kutta propagation techniques for initial value problems. An analysis and digital computer program are provided for numerical evaluation of system response for the longitudinal collision of a tractor semi-trailer carrying a container for irradiated fuel. The analysis given for this specific case presumes foreknowledge of collision input force and of the force-displacement characteristics of deformable elements of the system; in addition, only longitudinal motion of the system was considered. Modifications can be made to the analysis, if future collision test results indicate these are required.

FOREWORD

The analytical study reported herein was carried out during 1964-1965 as part of a research contract between the Atomic Energy Commission and The Franklin Institute. As such, it was not a part of the current Army-AEC joint program on vehicle collisions. The study does represent a necessary initial step in the development of analytical procedures for predicting vehicle collision behavior.

The approach and the analysis, involving formation of a mathematical model of a vehicle-container-tiedown system and solution of the equations of motion of this model by numerical techniques, are quite general and could be applied to vehicle-container configurations other than the particular one studied. Evaluation of the solution that has been obtained for a specific situation awaits results from the joint Army-AEC collision tests. Once these are available, the analytical procedure can be modified and extended, if need be, to provide an acceptable method for predicting vehicle behavior in longitudinal collisions.

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INTRODUCTION

The study of structural response of vehicle-container-tiedown systems* in accident environments is a particularly important problem in context to radioactive materials transportation research. Such a study is essential in bridging the gap existing between our present understanding of container structural integrity and of accident environments encountered by vehicles carrying radioactive material containers. We know little about the dynamic and/or impact loads applied to a container as a result of a transportation accident. Consequently, we are somewhat handicapped in establishing criteria for structural design of containers and tiedown systems.

The Atomic Energy Commission has, in the past year, sponsored analytical and experimental studies in specific aspects of the problem area outlined above. The analytical study, pursued at The Franklin Institute, forms the subject matter of this report. On the other hand, a series of vehicle collision tests has been envisaged under joint sponsorship of the Atomic Energy Commission and Department of the Army. These tests are scheduled for execution sometime in 1965.

The present report considers a vehicle-container-tiedown system and develops an analytical procedure to evaluate its dynamic response under impact loads such as those encountered in a longitudinal collision. Various possibilities of a longitudinal collision are schematically shown in Figure 2**. In the present study, tractor semi-trailer type vehicles (Fig. 1) and irradiated fuel type containers are considered. The approach and the analysis are, however, quite general and would apply to other vehicle-container configurations. For analysis of non-longitudinal vehicle-collisions, a similar approach could be employed.

*A vehicle-container-tiedown system represents a vehicle carrying a container as cargo, with the two connected by a suitable tiedown arrangement.

**Such collisions imply symmetry (about a vertical-longitudinal plane) of the collision and of vehicle system involved in it.

For the purpose of analysis, the physical system is idealized by a non-linear, lumped parameter (mass-spring) system, with masses considered rigid as compared to connections between them. The equations of motion for the idealized system (or, the mathematical model) are set up and solved numerically employing Runge-Kutta propagation techniques for initial value problems. This procedure is incorporated in a digital computer program for numerical evaluation of system response.

Previous studies in the area of vehicle collisions are scarce and restricted generally to specific experimental investigations of automobiles, buses and light trucks and not all of them are available in open literature. Analytical procedure such as presented in this report has, to the author's knowledge, not been considered previously. With such a background and in view of the complex physical setup of a vehicle-container-tiedown system, the present study must be considered a necessary first step toward evolving a more complete analysis of the problem. In this connection, the need for experimental verification of analytical procedures cannot be overemphasized. It is expected that when results from controlled vehicle collision tests are available, the present mathematical model would be improved, and feed-back between analysis and experiment continued until accurate predictions from analysis are obtained.

The report is divided into three sections and an Appendix. In the first section the problem is defined and the approach to its analytical study is described. The concept of a mathematical model of a vehicle-container-tiedown system is discussed and such a model is proposed for a tractor-semi-trailer type vehicle. In the second section, the details pertaining to mathematical analysis of the model are presented. These include equations of motion and propagation techniques for numerical integration of these equations on a digital computer. A computer program (FORTRAN system) for an assumed form of force-displacement characteristics for connecting elements of the model is developed and presented in the Appendix. This program can be enlarged conveniently to incorporate other connection characteristics whenever they become available. Finally, conclusions are presented in section 3.

1. FORMULATION OF THE STUDY

1.1 Problem

Our interest in the present study lies principally in "container" as a unit, and stems from a concern for its structural integrity under various accident environments. We need to define and evaluate conditions of dynamic/impact loading on a container as a result of accidents involving its carrier. The present analysis is confined to one specific type of accident, namely, longitudinal vehicle collisions (Fig. 2). Such collisions imply perfect symmetry (about a longitudinal-vertical plane) of the collision and of the vehicle system involved in it.

The container and its carrier are connected together by the tiedown assembly. Under normal transport conditions, the latter provides sufficient restraint to maintain relative positions of the container and its carrier. This configuration, however, is altered in a longitudinal collision of the carrier (with another object), and depending upon characteristics of the collision and of the physical systems involved therein, the container would experience various forms of dynamic/impact loading. We consider this aspect in a detailed manner in the following.

As a result of the collision (Fig. 2), a vibratory motion develops within the vehicle-container-tiedown system. The tiedown and other connecting elements in the system are stressed dynamically, and the container and other vehicle components accelerate under action of forces transmitted to them through the connecting elements. For example, the container would vibrate as dynamic forces are transmitted to it through the tiedown which connects the container to the vehicle floor.

Several possibilities exist in regard to the overall dynamic response of the physical system. One is that dynamic stresses and deformations in individual connecting elements* do not lead to rupture (or failure) of any one of them. In such an event, the physical system would behave as a "unit", and one should examine its corresponding modes of response in order to evaluate loading on the container, as applied through the tiedown assembly. Another possibility is that one or more of the connecting elements may fail as a result of the collision and,

*The vehicle-container tiedown setup is composed of elements which, for the present study, are grouped into "components" and "connections or connecting elements" according to their response (under collision situations) as discussed on page 7-8.

in turn, lead to disengagement of one or a group of components* from the vehicle-container-tiedown setup. The overall response of the system would depend on the function and location of these elements. For example, in a tractor semi-trailer, rupture of the fifth wheel connection would separate trailer (with container) from tractor, and to evaluate the system response one should examine individual motions (thereafter) of the tractor and the trailer. On the other hand, failure of a connecting element such as an engine mount, would not, in particular, affect the behavior of the system as a 'unit'.

In the case involving failure of one or more connecting elements, the longitudinal collision might be followed by accidents (secondary effects) involving one or more components of the physical system. To illustrate, consider events following rupture of the fifth wheel connection. The tractor and the trailer (with container) could continue to move separately until each collides individually with other interfering objects, preceded possibly, by a drop into a valley (or a river). The number of possible consequences such as these is very large, and for purposes of a systematic analytical (or experimental) study of the problem it is essential to idealize the physical system and its environments and to consider only the potentially severe happenings. This is accomplished (in the present study) in a two fold manner. First, longitudinal vehicle collisions are idealized to imply a symmetry of collision and of the physical system involved therein. Accordingly, events such as overturning or jack-knifing of the system although common, are excluded as possible primary consequences of the collision. Secondly, consideration of the secondary effects is limited to further longitudinal collisions of the components disengaged from the system, such as trailer (with container) in the previous illustration. Secondary consequences such as overturning are therefore not examined in the present study.

*See footnote on Page 3.

In the light of the above discussion, we can classify the loading conditions for the container in two groups. Depending upon whether the tiedown to the carrier fails or not, the container would either be released from, or would remain tied to, the vehicle (trailer) bed. In the first case, subsequent to its release at a finite velocity, the container could impact upon, or collide with, an external surface or an object. It would then be subject to impact loads. In the second case, on the other hand, it would experience dynamic loads applied to it through the tiedown arrangement. Consequently, it would perform a vibratory motion.

The problem, in a nutshell, can be stated thus: to determine the dynamic response of a vehicle-container-tiedown system in a longitudinal collision (Fig. 2), and in particular, to evaluate the dynamic forces in the tiedown arrangement and the resulting motion of the container.

1.2 Approach

In a broad sense, the approach to the present study should be compatible with the approach to an overall investigation of the vehicle collision problem, which includes the order of studying various types of vehicle collisions and for each collision type, a specific relationship between analysis and experiment. The latter aspect is discussed in a recent report (Ref. 1) by the author, and to a large extent the present analytical study follows the pattern set in the 'analytical-experimental approach' outlined therein. In this connection, the experimental verification and further feed-back between analysis and experiment would come from the joint AEC-DA program.

The present study, with its scope outlined in Section 2.1, is characterized by an almost complete lack of similar efforts in the past. It represents an initial effort toward analytical study of vehicle collisions, and consequently is restricted to consideration of simple and somewhat idealized collision situations. However, its approach to analytical investigation should be quite general and extendable to more complex collision environments. With these remarks, we can discuss the approach to solution of the problem defined in Section 2.1.

The response of a vehicle-container-tiedown setup in a longitudinal collision is a function of two sets of independent variables. The first set consists of the structural system characteristics of the two physical objects involved in the collision, and the second set pertains to initial kinematic conditions, namely, the displacement and velocity distributions within each object at the instant of collision. In evaluating the desired response, one must consider the dynamic structural interaction between the vehicle-container-tiedown system and the object involved in collision with it. Such an object could vary from a simple structure such as a massive retaining wall, to a system as complex as the one under consideration.

The evaluation of the system response as described above, still remains too complicated a problem for an initial study such as the present one. The principal difficulty arises from the need to consider interaction between the physical system under consideration and another object whose structural configuration is variable. To overcome this difficulty and further simplify the analysis, we consider the vehicle-container-tiedown setup by itself and assume that the impact forces resulting from its interaction with another object in collision are known initially. The problem would then reduce to obtaining (analytically) the dynamic response of the physical system under symmetrically applied known impact forces. The latter are presumed known, in preference to other parameters such as imposed displacements, in view of the relative ease and accuracy of making their measurement experimentally.

So far we have considered specific collision situations, examined the resulting patterns of loading on the system, and further reduced the problem to an analysis under application of known impact loads. The first step in such an analysis is to set up a mathematical model of the physical system and thereby idealize the complex vehicle-container structure to a form amenable to mathematical analysis. The mathematical model, by definition, would incorporate the basic behavioral characteristics of the parent system in conjunction with the loading encountered by it. Since the system behavior is only qualitatively understood to begin with, the mathematical model initially formulated would need to be improved in steps with feed-back from appropriate experimentation.

In the next step, structural analysis of the mathematical model under prescribed impact loads is carried out. Equations of motion are set up and appropriate mathematical techniques employed for numerical solution of these equations. The dynamic response of the mathematical model, and hence of the system, can then be obtained by introducing appropriate numerical values.

In the following section, a mathematical model for a representative tractor-semitrailer vehicle carrying a container is developed; this model would be modified and improved as appropriate experimental observations become available.

1.3 Mathematical Model

The mathematical model of the system under consideration would include characteristics determinative of its dynamic response under impact loads encountered in a longitudinal collision. Since these characteristics must be selected from a number of system parameters, we should somehow evaluate relative significance of the latter.

Let us examine intuitively and in a qualitative manner the behavior of a typical tractor-semitrailer (Fig. 1) carrying a container and involved in a longitudinal collision (Fig. 2). Some general observations can be made as follows^{*}, regarding the dynamic response of the vehicle-container-tiedown system.

- a. Under the collision, a vibratory motion develops within the physical system, with its elements performing motion in horizontal and vertical planes.
- b. Motion in the horizontal direction is considered to be unaffected by motion in the vertical direction, and thus can be determined independently of the latter.

^{*}See Ref. 2 for a supplementary discussion.

- c. Under impact loading of the system, certain elements such as the container, front and rear axle (with wheels), engine block, etc. do not deform appreciably. On the other hand, elements such as leaf springs, tiedown, etc. undergo elastic and inelastic deformations.

Elements in the first group are termed as "components" in contrast to "connections or connecting elements" of the second group.

- d. The "components" collectively constitute a large percentage of the total mass (weight) of the physical system, whereas the "connecting elements" provide flexibility or shock absorbing capacity of the system. Fig. 3 shows schematically the weight distribution in a typical vehicle-container-tiedown setup.
- e. Under a given collision situation, the response of the system would depend principally on the total mass and its distribution within the system, and on the dynamic force-displacement characteristics, up to failure, of the individual connecting elements.

These observations, supplemented by a careful examination of structural layout and design of a tractor-semitrailer*, would provide a reasonable basis for developing a mathematical model. In this connection, it would be advantageous to include all of the important behavioral characteristics of the system, although we must consider the efforts necessary to analyze the resulting mathematical model and further, to test and improve it in conjunction with appropriate experimentation. It would be inefficient to proceed with an elaborate mathematical model of a physical system whose behavior is understood at present only in a qualitative way.

*In addition to studying design drawings, one benefits considerably from a visit to assembly plants for tractors and trailers.

A somewhat simplified mathematical model of the vehicle-container-tiedown setup (Fig. 3) is presented in (Fig. 4). The physical system is idealized by a lumped parameter, mass-spring system in which mass and spring elements represent the components and connecting elements respectively of the parent system. The mass elements, considered rigid in contrast to flexible springs, are restrained to motion in horizontal directions only. The springs (or connections), in general, are characterized by their non-linear force-displacement relationships. The impact loads $(F[t])^*$ on the system as result of collision are, as explained in Sec. 1.2, assumed known. The mathematical model in Fig. 4 involves the following principal simplifications:

- a. Vertical motion of individual components is omitted, since it is assumed not to affect the horizontal motion.**
- b. The energy absorbed in elastic and/or plastic deformations of individual "components" is assumed negligible as compared to the energy of deformation of individual "connecting elements"
- c. Elements such as the driver's cab are not included in the mathematical model in view of their complex configuration their small energy of deformation relative to the initial kinetic energy of the system.
- d. The force-displacement characteristics of various connecting elements, it is assumed, can be determined analytically and/or experimentally.

Procedures for analysis of the mathematical model are presented in the following section.

* Square bracket represents a functional relationship between dependent (F) and independent (t) variables.

** This appears to be a principal uncertainty with regard to the model; if collision test results show the assumption to be unrealistic, the analysis can be modified to include vertical motion.

2. MATHEMATICAL DETAILS

2.1 Analysis of the Mathematical Model

The analysis is carried out in two principal steps. First, we establish the equations of motion for the idealized system (Fig. 4) and secondly, develop mathematical techniques for numerical solution of these equations. In deriving the latter, the force-time characteristics of impact $F[t]$, are assumed given, as explained in Section 1.2.

2.2 Equations of Motion

First, we define quantities pertinent to analysis of the mathematical model in Fig. 4. In the following definitions, indexes i, j vary from 1, 2 up to 8, the total number of mass elements in the model.

m_i - mass of the i th mass element (also used to designate the element);

$x_i[t]$ - displacement of m_i from its initial, equilibrium position at $t = 0$;

$F_i[t]$ - external impact force resulting from the collision, and applied to i th mass element; for a head-on collision (Fig. 2b,d) only $F_1[t]$ is non-zero and for a rear-end collision (Fig. 2a,c) either $F_6[t]$ or $F_7[t]$ is non-zero depending on whether the trailer frame or axle is hit; for generality of presentation, all $F_i[t]$ are included in the equations of motion;

k_{ij} - designation for the connecting element between masses m_i and m_j ;

$R_{ij}[x_i - x_j]$ - internal force^{*} in the connecting element k_{ij} due to a relative horizontal displacement $(x_i - x_j)$ between m_i and m_j ; it defines the force-displacement characteristic for the connection k_{ij} and in general, is a non-linear relationship between the force R_{ij} and the displacement $(x_i - x_j)$.

*The notation here is abbreviated from $R_{ij} [x_i[t] - x_j[t]]$

To obtain the equations of motion, we should consider the dynamic equilibrium of individual mass elements. At a given time instant 't' during the motion, each mass m_i would be subject to forces $R_{ij} [x_i - x_j]$, $j = 1, 2, 3 \dots 8$, in connections between m_i and other masses, and to external force $F_1[t]$, if non-zero. The time t is measured from the instant of collision.

Let us examine equilibrium of mass m_1 . The forces acting on it are those in connections between m_1 and masses m_2 , m_3 , m_4 , and m_5 , and the impact force $F_1[t]$. These are shown in the sketch below (assuming for convenience that $x_1 > x_2 > x_3 > \dots x_8$).

The second law of motion gives:

$$m_1 \ddot{x}_1 = F_1[t] - R_{12}[x_1 - x_2] - R_{13}[x_1 - x_3] - R_{14}[x_1 - x_4] - R_{15}[x_1 - x_5] \quad (1)$$

where $\ddot{x}_1$ represents the acceleration of m_1 .

Proceeding similarly for mass elements m_2 , $m_3 \dots m_8$, we obtain the following equations:

$$m_2 \ddot{x}_2 = F_2[t] + R_{12}[x_1 - x_2] - R_{24}[x_2 - x_4] \quad (2)$$

$$m_3 \ddot{x}_3 = F_3[t] + R_{13}[x_1 - x_3] \quad (3)$$

$$m_4 \ddot{x}_4 = F_4[t] + R_{14} [x_1 - x_4] + R_{24} [x_2 - x_4] \quad (4)$$

$$m_5 \ddot{x}_5 = F_5[t] + R_{15} [x_1 - x_5] - R_{56} [x_5 - x_6] \quad (5)$$

$$m_6 \ddot{x}_6 = F_6[t] + R_{56} [x_5 - x_6] - R_{67} [x_6 - x_7] - R_{68} [x_6 - x_8] \quad (6)$$

$$m_7 \ddot{x}_7 = F_7[t] + R_{67} [x_6 - x_7] \quad (7)$$

$$m_8 \ddot{x}_8 = F_8[t] + R_{68} [x_6 - x_8] \quad (8)$$

Equations 1-8 constitute the complete set of equations of motion for the mathematical model, and can be expressed in a general form as

$$m_i \ddot{x}_i = F_i[t] + \sum_{j=1}^n \alpha_{ij} R_{ij} [x_i - x_j] \quad (9)$$

where, $i = 1, 2 \dots n$;

n = number of mass elements in the system;

$\alpha_{ij} = -1, 0$ or $+1$ depending on connection between i th and j th mass elements.

The next step is to solve the system of equations (9) and obtain response of the system under prescribed external forces and initial ($t = 0$) conditions. The latter include the initial displacements $x_i = x_i^0$, and the initial velocities $\dot{x}_i = \dot{x}_i^0$ of individual mass elements of the system.

* In a longitudinal collision $F_i[t]$ is zero for all mass elements except the ones impacted.

2.3 Solution of the Equations of Motion

Equation (9) represents a system of second order, nonlinear, ordinary differential equations and as far as we know no techniques are available for an analytical solution, i.e. a closed form or a series solution, of such equations at large. In their most general form the functions R_{ij} , the force-displacement relationships, would involve nonlinearities of different kinds and therefore, pursuit of an analytical solution would not be rewarding for the present problem.

Other possibilities of solution include analog methods and numerical digital techniques. The analog methods require setting up physical systems whose equations of state have the same form as Eqs. (9) and the desired solution is obtained by specific measurements of analog system response. Such a method proves economical once the analog setup is established. However, it lacks the flexibility essential for the problem under study. Any changes in system characteristics could become expensive in time and money. Furthermore, the use of specific analog systems as tools of research cannot be as widespread as digital computers, for example. On the other hand, numerical methods employing digital computers for computation provide the desired flexibility and ease of handling, and are ideally suited for the solution of the equations of motion.

The system of equations (9) defines an initial value problem with time as independent variable and for its numerical solution, propagation techniques (Ref. 3) are especially useful. In these methods, the independent variable (t) is separated into discrete steps of finite size, and derivatives ($\ddot{x}_i$) are approximated by functional values of the dependent variables (x_i) at the discrete time (t) values. Beginning with the initial values of the dependent variables the successive values are obtained by utilizing the equations of motion. Various procedures are available for approximating the derivatives $\ddot{x}_i$ in terms of the discrete x_i values, and for starting and continuing the numerical solution from a given set of initial values. Each such procedure is characterized by ease of com-

putation and by an error level associated with approximate expression of derivatives $\ddot{x}_i$. For a detailed discussion of these procedures in general, one may consult (Ref. 4).

Among various available techniques for numerical solution of Eq. (9), the Runge-Kutta method (Ref. 5) is perhaps best suited. It prescribes a uniform procedure for evaluating the unknowns x_i for all values of time t . The method has a high order of accuracy at all time steps including the first one. In view of rapid rise and decay of forcing functions $F_i[t]$, it is extremely important that the first time step should provide a high degree of accuracy. Finally, the method is flexible in its usage; the size of discrete time step can be changed in the course of computation without great difficulty.

Since (Ref. 5) deals extensively with the Runge-Kutta method, we would briefly discuss only its details pertaining to numerical solution of Eq. (9).

2.4 Runge-Kutta Method

We are dealing with a set of equations

$$m_i \ddot{x}_i = F_i[t] + \sum_{j=1}^n \alpha_{ij} R_{ij}[x_i - x_j] \quad (9)$$

with $i = 1, 2, 3, \dots, n$

Equivalently, one can write,

$$\dot{x}_i = X_i[x_1, x_2, x_3, \dots, x_n; t] \quad (10)$$

The initial conditions are,

$$\left. \begin{array}{l} x_i[0] = x_i^0 \\ \dot{x}_i[0] = \dot{x}_i^0 \end{array} \right\} \quad i = 1, 2, 3, \dots, n \quad (11)$$

Designating the discrete time increment as ' Δt ', the procedure for evaluating the dependent variable x_i^{s+1} at $(s+1)$ th time step can be described as follows with reference to Eq. (10).

$$x_i^{s+1} = x_i^s + v_i^s + \frac{(\Delta t)^2}{6} (w_{i1} + 2w_{i2})$$

and

$$v_i^{s+1} = v_i^s + \frac{(\Delta t)^2}{6} (w_{i1} + 4w_{i2} + w_{i3}) \quad (12)$$

where

$$v_i^s = \Delta t (\dot{x}_i^s)$$

$$\left. \begin{aligned} w_{i1} &= X_i [x_j^s, j = 1, 2, 3 \dots n; t^s] \\ w_{i2} &= X_i \left[\left(x_j^s + \frac{v_j^s}{2} + \frac{w_{j1}(\Delta t)^2}{8} \right), j = 1, 2, 3 \dots n; t^s + \frac{\Delta t}{2} \right] \\ w_{i3} &= X_i \left[\left(x_j^s + v_j^s + \frac{w_{j2}(\Delta t)^2}{2} \right), j = 1, 2, 3 \dots n; t^s + \Delta t \right] \end{aligned} \right\} (13)$$

and

$$t^s = \text{time at } (s)\text{th step}$$

$$= (s)\Delta t, \text{ if all time steps are uniform in size } \Delta t.$$

Starting with the initial values x_i^0 and $\dot{x}_i^0$, the displacements and velocities at successive time intervals are obtained by repeated application of Equations (12) and (13). In this connection, the functions X_i in Equation (13) are defined by Equation (9).

The procedure described above is ideally suited for a digital computer usage in view of the repetitive nature of computations. Various aspects of programming are discussed in the following section.

2.5 Numerical Solution

The computer program for obtaining numerical solution of Equation (9) can be set up in three segments to perform the following operations:

- a. input - output;
- b. Runge-Kutta scheme, Equation (12) for advancing the solution in discrete time steps; and
- c. evaluation of functions X_i , Equation (13), at every time step.

Programming for 'a' and 'b' above is straightforward and does not need any explanation.

The step 'c' by virtue of Equations (9) and (10) reduces to evaluation of functions $R_{ij}[x_i - x_j]$, $j = 1, 2, 3 \dots n$, for a given set of displacements x_m , $m = 1, 2, 3 \dots n$. R_{ij} defines the force-displacement characteristics of connecting element k_{ij} (Fig. 4) of the physical system, and is in general a nonlinear relationship between force R_{ij} and displacement $(x_i - x_j)$. The program would require one subroutine for every distinct functional relationship R_{ij} .

A computer program incorporating the basic numerical scheme (operations 'a' and 'b') and a typical force-displacement relationship is developed and presented in the Appendix. It is written for use on the Honeywell 1400 digital computer, but can be adapted after slight modification to FORTRAN (II) systems. The functional relationship (R_{ij}) incorporated in the program is shown graphically in Fig. 5; it is characterized by a bilinear, hysteretic, inelastic spring typical of certain tiedown connections. Additional relationships whenever they become available, can be incorporated in the program in the form of subroutines.

3. CONCLUSIONS

The presentation in this report describes the initial effort in an analytical study of the vehicle collision problem. The mathematical model and its analysis as outlined provide a means for evaluating dynamic response of a vehicle-container-tiedown setup under impact loads such as those encountered in longitudinal collisions.

In formulating the approach to the present study, we have assumed that the impact loads are known 'a priori', thus implying that the analytical efforts would be supplemented by a series of vehicle collision tests. Such an arrangement would provide feedback between analysis and experiment, essential for a rational development of analytical procedures. In this connection, certain aspects of the present analytical study must be described quantitatively before one can obtain numerical results to compare with experimental observations.

One group of input parameters for analysis pertains to force-displacement relationships (R_{ij}) of various connecting elements in the physical system. These relationships significantly affect the system response and could not be assumed arbitrarily. Furthermore, each connection must be individually considered and full-scale testing of some of them may be the only reliable way to evaluate their force-displacement characteristics. Although the effort involved in doing so would not be extensive, the information must be obtained prior to experimental verification of analytical procedures presented in this report. Once these characteristics are known, the computer program (Appendix) can be modified if necessary, and the system response can be evaluated for a prescribed impact loading.

The present study could be extended, as shown necessary by appropriate vehicle collision tests, to develop a more representative mathematical model of the physical system under longitudinal collision environments.

Modifications could include vertical motion of individual vehicle components and the structural interaction between the physical system and another object in a collision. Consideration of non-longitudinal vehicle collisions (angle, overturn) would then be a logical next step to complete the development of acceptable analytical procedures for predicting the behavior of vehicles during any collision.

References

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I-A2412-5

Reviewed by:

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Project Engineer

Approved by:

N.R. Droulard
Technical Director

FIGURE ILLUSTRATIONS

Fig. 1 - A TYPICAL CARRIER

Fig. 2 - LONGITUDINAL VEHICLE COLLISIONS

TRACTOR

1. FRAME	1000 LB
2. ENGINE AND TRANSMISSION	3250 LB
3. DRIVER'S CAB	1300 LB
4. FRONT AXLE	1600 LB
5. REAR AXLE	4000 LB
OTHER	<u>2450 LB</u>
TOTAL TRACTOR WEIGHT	13,600 LB

6. FIFTH WHEEL	500 LB
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TRAILER

7. FRAME	5300 LB
8. REAR AXLE	<u>4300 LB</u>
TOTAL TRAILER WEIGHT	9600 LB

9. CONTAINER	30,000 LB
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<i>TOTAL WEIGHT</i>	53,700 LB
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Fig. 3 - WEIGHT DISTRIBUTION IN A TYPICAL
VEHICLE-CONTAINER-TIEDOWN SYSTEM

m₅ - FIFTH WHEEL

m₆ - TRAILER FRAME

m₇-TRAILER REAR AXLE

m₈ - CONTAINER

1_{ij} - CONNECTIONS IN VERTICAL DIRECTION

Fig. 4 - MATHEMATICAL MODEL

Fig. 5 - TYPICAL FORCE-DISPLACEMENT RELATIONSHIP, R_{ij}

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APPENDIX

APPENDIX

This Appendix contains a listing of the computer program (Honeywell 1400) developed to solve the equations of motion. The force-displacement characteristics R_{ij} , included in the program are of the type shown in Fig. 5. The program consists of the following segments.

1. "VEHICL" - this is the main program. It reads the input, calls the subroutines in appropriate order, and if desired, punches out parameter values as input to future continuation of the problem. Sense switches 1 and 2 are used.
2. "SUBSET" - this subroutine sets the indexes "K" and "L" for use in "SUBFJK", depending upon distribution and arrangement of the connecting elements (K_{ij}) in the given mathematical model. This subroutine should, therefore, be modified whenever additional connecting elements are introduced in the latter. The subroutine, as presented, is based on a nine mass system, with " m_5 " representing mass of the cab assembly.
3. "SUBFJK" - this subroutine computes the force "FJK(I)" in a given connection due to a relative displacement " $X(K) - X(L)$ ". The relationship between force and displacement is assumed of the type in Fig. 5.
4. "SBPRNT" - this subroutine prints the output at every time step.
5. "SUBACC" - this subroutine computes accelerations ($\ddot{x}_i$) from the equations of motion. It is set up for a nine mass system and for a head-on collision situation, i.e. for $F_1[t] \neq 0$.
6. "TRANS" - this subroutine computes displacements " $X(I)$ " and velocities " $V(I)$ " of individual mass elements of the system, employing Runge-Kutta scheme, Equations (12) and (13).

SYMBOLS USED IN THE PROGRAM

$A(I)$ - acceleration $\ddot{x}_i$, Eq. (10)

ADDV, ADDX - vectors to store the velocity and displacement values in course of computing "X" and "V"

DT - time step Δt uniform over first 'N' steps

DT1 - time step Δt uniform after first 'N' steps

DTSQ2 - $(\Delta t)^2/2$

DUM, DUM1, DUMMY - variables in the program

EN(I), EP(I) - force-displacement moduli Fig. 5, of individual connecting elements

F(I) - input force ($F_1[t]$) values at time intervals Δt

FJK(I) - force in the (i)th connecting element at time t^s

FJK1(I) - force FJK(I) at time preceeding (s)th time step when the force is positive

FJK1N(I) - same as FJK1(I) except being negative

FORCE - FCI at a given time instant

G - gravitational constant "g"

I, ICOUNT - counters

INDEX(I), INDEXN(I) - counters used in conjunction with the subroutine SUBFJK, to keep an account of the force history in course of time; assumes value of 1, 2 or 3.

J, JCOUNT, JJ, JTEST,
K, KTEST, L - counters

M - number of mass elements in the system

MTEST - counter

N, N1 - number of time steps for which the solution is desired; N corresponds to time interval DT and N1 to DT1

NCOUNT(I) - counter
 NN - number of connecting elements in the system
 NTEST - counter
 PN, PP - force-displacement moduli, Fig. 5
 RELX(I) - $(x_i - x_j)$
 SM(I) - m_i
 SN(I), SP(I) - slack in the connection, Fig. 5
 USN(I), USP(I) - ultimate strengths of connecting elements,
 Fig. 5
 V(I) - velocity of (i)th mass elements
 X(I) - displacement of (i)th mass element.
 X1(I) - X(I) at preceeding time step
 YPN(I), YPP(I) - yield point values for connecting elements,
 Fig. 5

TITLE	VEHICL		M 100
	COMMON	A, ADDV, ADDX, DT, DT1, DTSG2, DUM, DUM1, DUMMY, EN, EP,	M 101
1		F, FJK, FJK1, FJKIN, FORCE, G, I, ICOUNT, INDEX, INDEXN,	M 102
2		J, JCOUNT, JJ, JTEST, K, KTEST, L, M, MTEST, N, NCOUNT,	M 103
3		NN, NTEST, N1, PN, PP, RELX, SM, SN, SP, USN, USP, V, X,	M 104
4		X1, YPN, YPP	M 105
	DIMENSION	A(20), ADDV(20), ADDX(20), EN(25), EP(25), F(501),	M 106
1		FJK(25), FJK1(25), FJKIN(25), INDEX(25), INDEXN(25),	M 107
2		NCOUNT(25), PN(25), PP(25), RELX(25), SM(20), SN(25),	M 108
3		SP(25), USN(25), USP(25), V(20), X(20), X1(20),	M 109
4		YPN(25), YPP(25)	M 110
	RFAD 209, JJ		M 111 1
199	RFAD 214, DUMMY		M 112 2
	READ 214, DUMMY		M 112.1 3
	READ 210, M, NN, N, N1, DT, DT1, G		M 113 4
	DO 200 I = 1,M		M 114 5
	READ 211, SM(I), X(I), V(I)		M 115 6
	V(I) = V(I)*DT		M 116 7
	X1(I) = 0.0		M 117 8
200	SM(I) = SM(I)/G		M 118 9
	DO 201 I = 1,NN		M 119 10
201	RFAD 212, EP(I), EN(I), PP(I), PN(I)		M 120 11
	DO 202 I = 1,NN		M 121 12
202	RFAD 212, YPP(I), YPN(I), USP(I), USN(I)		M 122 13
	DO 203 I = 1,NN		M 123 14
	FJK1(I) = 0.0		M 124 15
	FJKIN(I) = 0.0		M 125 16
203	RFAD 213, SP(I), SN(I), INDEX(I), INDEXN(I), NCOUNT(I)		M 126 17
	J = 2*(N+N1) + 1		M 127 18
	DO 204 I = 1,J		M 128 19

204	READ 214, F(I)	M 129	20
209	FORMAT (I5)	M 130	21
210	FORMAT (4I5,3F15.5)	M 131	22
211	FORMAT (3F15.5)	M 132	23
212	FORMAT (4F15.5)	M 133	24
213	FORMAT (2F15.5,3I5)	M 134	25
214	FORMAT (F15.5)	M 135	26
	J = 0	M 136	27
	KTEST = 1	M 137	28
	CALL SBPRNT	M 138	29
	DTSQ2 = DT*DT/2.0	M 139	30
	IF (N1) 215,215,216	M 139.1	31
215	N = N + 1	M 139.2	32
	GO TO 217	M 139.3	33
216	N1 = N1 + 1	M 139.4	34
217	JCOUNT = 1	M 139.5	35
	ICOUNT = N	M 139.6	36
218	DO 230 J = JCOUNT,ICOUNT	M 140	37
219	JTEST = 1	M 141	38
	KTEST = 1	M 142	39
	MTEST = 2*J - 1	M 143	40
220	FORCE = F(MTEST)	M 144	41
C	THIS INSTRUCTION IS INTRODUCED FOR USE WHEN F VECTOR IS DEFINED	M 144.1	
C	ANALYTICALLY AND F(MTEST) HAS TO BE CALCULATED	M 144.2	
	DO 221 I = 1,NN	M 145	42
	CALL SUBSET	M 146	43
	CALL SUBFJK	M 147	44
221	CONTINUE	M 148	45
	CALL SBPRNT	M 149	46
	IF (N1) 222,222,223	M 150	47

222	IF (J=N) 224,230,230	M 151	48
223	IF (J=N=N1) 224,230,230	M 152	49
224	CALL SUBACC	M 153	50
	CALL TRANS	M 154	51
	MTEST = MTEST + 1	M 155	52
	KTEST = KTEST + 1	M 156	53
	GO TO (220,220,225,230), KTEST	M 157	54
225	JTEST = -1	M 158	55
	GO TO 220	M 159	56
230	CONTINUE	M 160	57
	<hr/> GO TO (233,231,231,231), KTEST <hr/>	M 161	58
231	DTSQ2 = DT1*DT1/2.0	M 162	59
	DO 232 I = 1,M	M 163	60
232	V(I) = V(I)*DT1/DT	M 164	61
	DT = DT1	M 165	62
	JCOUNT = N+1	M 166	63
	ICOUNT = N+N1	M 167	64
	GO TO 218	M 168	65
C	BEGIN SECOND LOOP	M 169	
233	IF (SENSE SWITCH 1) 240,234	M 170	66
234	DO 235 I = 1,M	M 171	67
	SM(I) = SM(I)*G	M 172	68
	V(I) = V(I)/DT	M 173	69
235	PUNCH 211, SM(I), X(I), V(I)	M 174	70
	DO 236 I = 1,NN	M 175	71
236	PUNCH 212, FP(I), EN(I), PP(I), PN(I)	M 176	72
	DO 237 I = 1,NN	M 177	73
237	PUNCH 212, YPP(I), YPN(I), USP(I), USN(I)	M 178	74
	DO 238 I = 1,NN	M 179	75
238	PUNCH 213, SP(I), SN(I), INDEX(I), INDEXN(I), NCOUNT(I)	M 180	76

IF (SENSE SWITCH 2) 239,240	M 180.1	77
239 PAUSE	M 180.2	78
240 JJ = JJ = 1	M 181	79
IF (JJ) 241,241,199	M 182	80
241 STOP	M 183	91
END	M 184	92

AUTOMATH SYSTEM

*** SOURCE PROGRAM LISTING ***

SUBROUTINE SUBSET		E 100	
<hr/>			
COMMON		A, ADDV, ADDX, DT, DT1, DTSC2, DUM, DUM1, DUMMY, EN, EP, M 101	
1	F, FJK, FJK1, FJK1N, FORCE, G, I, ICOUNT, INDEX, INDEXN, M 102		
2	J, JCOUNT, JJ, JTEST, K, KTEST, L, M, MTEST, N, NCOUNT, M 103		
3	NN, NTEST, N1, PN, PP, RELX, SM, SN, SP, USN, USP, V, X, M 104		
4	X1, YPN, YPP M 105		
DIMENSION		A(20), ADDV(20), ADDX(20), EN(25), EP(25), F(501), M 106	
1	FJK(25), FJK1(25), FJK1N(25), INDEX(25), INDEXN(25), M 107		
2	NCOUNT(25), PN(25), PP(25), RELX(25), SM(20), SN(25), M 108		
3	SP(25), USN(25), USP(25), V(20), X(20), X1(20), M 109		
4	YPN(25), YPP(25) M 110		
<hr/>			
	GO TO (10,10,10,10,10,11,12,13,14), I	E 111	1
10	K=1	E 112	2
	L=I+1	E 113	3
	GO TO 20	E 114	4
11	K=2	E 115	5
	L=4	E 116	6
	GO TO 20	E 117	7
12	K=6	E 118	8
	L=7	F 119	9
	GO TO 20	E 120	10
13	K=7	E 121	11
	L=8	E 122	12
	GO TO 20	E 123	13
14	K=7	E 124	14
	L=9	E 125	15
<hr/>			
20	RETURN	E 126	16
	END	E 127	17

AUTOMATH SYSTEM

*** SOURCE PROGRAM LISTING ***

SUBROUTINE SUBFJK

F 100

COMMON A, ADDV, ADDX, DT, DT1, DTSQ2, DUM, DUM1, DUMMY, EN, EP, M 101
 1 F, FJK, FJK1, FJK1N, FORCE, G, I, ICOUNT, INDEX, INDEXN, M 102
 2 J, JCOUNT, JJ, JTEST, K, KTEST, L, M, MTEST, N, NCOUNT, M 103
 3 NN, NTEST, N1, PN, PP, RELX, SM, SN, SP, USN, USP, V, X, M 104
 4 X1, YPN, YPP M 105

DIMENSION A(20), ADDV(20), ADDX(20), EN(25), EP(25), F(501), M 106
 1 FJK(25), FJK1(25), FJK1N(25), INDEX(25), INDEXN(25), M 107
 2 NCOUNT(25), PN(25), PP(25), RELX(25), SM(20), SN(25), M 108
 3 SP(25), USN(25), USP(25), V(20), X(20), X1(20), M 109
 4 YPN(25), YPP(25) M 110

IF (NCOUNT(I)) 99,25,25

F 126

1

25 DUM = X(K) - X(L)

F 127

2

DUM1 = X1(K) - X1(L)

F 128

3

IF (JTEST) 75,28,28

F 129

4

28 RELX(I) = DUM

F 130

5

IF (DUM) 29,30,29

F 131

6

29 IF (DUM1/DUM) 31,39,39

F 132

7

30 FJK(I) = 0.0

F 133

8

31 IF (DUM1) 36,39,32

F 134

9

32 NTEST = 2

F 135

10

K = INDEX(I)

F 136

11

GO TO (40,34,33), K

F 136.1

12

33 YPP(I) = FJK1(I)

F 137

13

SP(I) = DUM1 - FJK1(I)/EP(I)

F 138

14

34 INDEX(I) = 1

F 139

15

GO TO 40

F 140

16

36 NTEST = 3

F 141

17

L = INDEXN(I)

F 142

18

GO TO (40,38,37), L

F 142.1

19

37	YPN(I) = FJKIN(I)	F 143	20
	SN(I) = DUM1 - FJKIN(I)/EN(I)	F 144	21
38	INDEXN(I) = 1	F 145	22
	GO TO 40	F 146	23
39	NTEST = 1	F 147	24
40	IF (DUM) 60,97,41	F 148	25
41	IF (DUM-SP(I)) 42,42,50	F 149	26
42	GO TO (43,40,45), NTEST	F 150	27
43	K = INDEX(I)	F 151	28
	GO TO (46,45,44), K	F 151.1	29
44	YPP(I) = FJK1(I)	F 152	30
	SP(I) = DUM1 - FJK1(I)/EP(I)	F 153	31
45	INDEX(I) = 1	F 154	32
46	FJK(I) = 0.0	F 155	33
	GO TO 95	F 156	34
50	IF (DUM-DUM1) 55,51,52	F 157	35
51	FJK(I) = FJK1(I)	F 158	36
	GO TO 95	F 159	37
52	DUMMY = DUM - SP(I) - YPP(I)/EP(I)	F 160	38
	IF (DUMMY) 53,53,54	F 161	39
53	FJK(I) = EP(I)*(DUM-SP(I))	F 162	40
	INDEX(I) = 2	F 163	41
	GO TO 95	F 164	42
54	FJK(I) = YPP(I) + PP(I)*DUMMY	F 165	43
	INDEX(I) = 3	F 166	44
	GO TO 95	F 167	45
55	K = INDEX(I)	F 168	46
	GO TO (40,53,56), K	F 168.1	47
56	YPP(I) = FJK1(I)	F 169	48
	SP(I) = DUM1 - FJK1(I)/EP(I)	F 170	49

<hr/> IF (SP(I)=DUM) 53,45,45 <hr/>		F 170.1	50
60	IF (SN(I)=DUM) 61,61,66 <hr/>	F 171	51
61	GO TO (62,64,40), NTEST	F 172	52
62	L = INDEXN(I)	F 173	53
	GO TO (65,64,63), L	F 173.1	54
63	YPN(I) = FJKIN(I)	F 174	55
	SN(I) = DUM1 - FJKIN(I)/EN(I)	F 175	56
64	INDEXN(I) = 1	F 176	57
65	FJK(I) = 0.0	F 177	58
	GO TO 96 <hr/>	F 178	59
66	IF (DUM=DUM1) 68,67,71	F 179	60
67	FJK(I) = FJKIN(I)	F 180	61
	GO TO 96	F 181	62
68	DUMMY = DUM - SN(I) - YPN(I)/EN(I)	F 182	63
	IF (DUMMY) 70,69,69	F 183	64
69	FJK(I) = EN(I)*(DUM-SN(I))	F 184	65
	INDEXN(I) = 2	F 185	66
	GO TO 96	F 186	67
70	FJK(I) = YPN(I) + PN(I)*DUMMY	F 187	68
	INDEXN(I) = 3	F 188	69
	GO TO 96	F 189	70
71	L = INDEXN(I)	F 190	71
	GO TO (40,69,72), L	F 190.1	72
72	YPN(I) = FJKIN(I)	F 191	73
	SN(I) = DUM1 - FJKIN(I)/EN(I)	F 192	74
	IF (SN(I)=DUM) 64,64,69 <hr/>	F 193	75
75	IF (DUM) 84,92,76 <hr/>	F 194	76
76	IF (DUM=SP(I)) 92,92,77	F 195	77
77	IF (DUM=DUM1) 82,78,79	F 196	78
78	FJK(I) = FJK1(I)	F 197	79

GO TO 93	F 198	80
79 DUMMY = DUM - SP(I) - YPP(I)/EP(I)	F 199	81
IF (DUMMY) 80,80,81	F 200	82
80 FJK(I) = EP(I)*(DUM-SP(I))	F 201	83
GO TO 93	F 202	84
81 FJK(I) = YPP(I) + PP(I)*DUMMY	F 203	85
GO TO 93	F 204	86
82 IF (DUM1 = DUM - FJK1(I)/EP(I)) 83,92,92	F 205	87
83 FJK(I) = FJK1(I) - (DUM1-DUM)*EP(I)	F 206	88
GO TO 93	F 207	89
84 IF (SN(I)=DUM) 92,92,85	F 208	90
85 IF (DUM=DUM1) 87,86,90	F 209	91
86 FJK(I) = FJK1N(I)	F 210	92
GO TO 93	F 211	93
87 DUMMY = DUM - SN(I) - YPN(I)/EN(I)	F 212	94
IF (DUMMY) 89,88,88	F 213	95
88 FJK(I) = EN(I)*(DUM-SN(I))	F 214	96
GO TO 93	F 215	97
89 FJK(I) = YPN(I) + PN(I)*DUMMY	F 216	98
GO TO 93	F 217	99
90 IF (DUM1 = DUM - FJK1N(I)/EN(I)) 92,92,91	F 218	100
91 FJK(I) = FJK1N(I) - (DUM1-DUM)*EN(I)	F 219	101
GO TO 93	F 220	102
92 FJK(I) = 0.0	F 221	103
93 GO TO 99	F 222	104
95 FJK1(I) = FJK(I)	F 223	105
IF (FJK(I)=USP(I)) 99,99,98	F 224	106
96 FJK1N(I) = FJK(I)	F 225	107
IF (FJK(I)=USN(I)) 98,99,99	F 226	108
97 FJK1(I) = 0.0	F 227	109

FJK1N(I) = 0.0	F 228	110
GO TO 99	F 229	111
98 NCOUNT(I) = -1	F 230	112
FJK(I) = 0.0	F 231	113
99 RETURN	F 232	114
END	F 233	115

AUTOMATH SYSTEM

* * * SOURCE PROGRAM LISTING * * *

SUBROUTINE SBPRNT		S 100	
COMMON	A, ADDV, ADDX, DT, DT1, DTSG2, DUM, DUM1, DUMMY, EN, EP, M 101		
1	F, FJK, FJK1, FJKIN, FORCE, G, I, ICOUNT, INDEX, INDEXN, M 102		
2	J, JCOUNT, JJ, JTEST, K, KTEST, L, M, MTEST, N, NCOUNT, M 103		
3	NN, NTEST, N1, PN, PP, RELX, SM, SN, SP, USN, USP, V, X, M 104		
4	X1, YPN, YPP M 105		
DIMENSION A(20), ADDV(20), ADDX(20), EN(25), EP(25), F(501), M 106			
1	FJK(25), FJK1(25), FJKIN(25), INDEX(25), INDEXN(25), M 107		
2	NCOUNT(25), PN(25), PP(25), RELX(25), SM(20), SN(25), M 108		
3	SP(25), USN(25), USP(25), V(20), X(20), X1(20), M 109		
4	YPN(25), YPP(25) M 110		
GO TO (305,340,340,340), KTEST		S 110	1
305	IF (J=1) 306,331,331	S 111	2
306	PRINT 326	S 112	3
	PRINT 315	S 113	4
	PRINT 316, DT, DT1, G, N, N1, M, NN	S 114	5
	PRINT 317	S 115	6
	DO 307 I=1,NN	S 116	7
307	PRINT 318, EP(I), EN(I), PP(I), PN(I)	S 117	8
	PRINT 319	S 118	9
	DO 308 I=1,NN	S 119	10
308	PRINT 318, YPP(I), YPN(I), USP(I), USN(I)	S 120	11
	PRINT 320	S 121	12
	DO 309 I=1,NN	S 122	13
309	PRINT 321, SP(I), SN(I), INDEX(I), INDEXN(I), NCOUNT(I)	S 123	14
	PRINT 322	S 124	15
	DO 310 I=1,M	S 125	16
	DUMMY = V(I)/DT	S 126	17
	DUM = SM(I)*G	S 126.1	18
310	PRINT 323, X(I), DUMMY, DUM	S 127	19

PRINT 324						S 128	20
JCOUNT = 2*(N+NI) + 1						S 128.1	21
DO 311 I=1,JCOUNT						S 129	22
311 PRINT 325, F(I)						S 130	23
GO TO 340						S 131	24
<hr/>							
314 FORMAT(103H	X	V	RELX	S 131.1	25		
1 SP SN FJK/)				S 131.2			
315 FORMAT (15H INPUT VALUES///)				S 132	26		
316 FORMAT (7H DT= E12.5,8H DT1= E12.5,6H G= E12.5,7H N= I3.5				S 133	27		
18H NI= I3.7H M= I2.8H NN= I2///)				S 133.1			
317 FORMAT (67H EP EN PP				S 134	28		
1 PN/)				S 135			
318 FORMAT (4F18.5/)				S 136	29		
319 FORMAT(//67H YPP YPN USP				S 137	30		
1 USN/)				S 138			
320 FORMAT (//77H SP SN INDEX				S 139	31		
1 INDEXN NCOUNT/)				S 140			
321 FORMAT (2F18.5,7H I1.14H I1.14H IS				S 141	32		
12/)				S 141.1			
322 FORMAT(//49H X V SM/)				S 142	33		
323 FORMAT (3F18.5/)				S 143	34		
324 FORMAT(//14H F(I)/)				S 144	35		
325 FORMAT (F18.5/)				S 145	36		
326 FORMAT (1H1)				S 146	37		
327 FORMAT (4H J=I3///)				S 147	38		
328 FORMAT (6F18.5/)				S 148	39		
329 FORMAT (2F18.5/)				S 149	40		
330 FORMAT (24H 4F18.5/)				S 150	41		
<hr/>							
331 PRINT 326				S 151	42		
I = J-1				S 152	43		
PRINT 327, I				S 153	44		
PRINT 314				S 153.1	45		
IF (M-MN) 333,333,332				S 154	46		

332	K = M	S 155	47
	L = NN	S 156	48
	GO TO 334	S 157	49
333	K = NN	S 158	50
	L = M	S 159	51
334	DO 339 I=1,K	S 160	52
	DUMMY = V(I)/DT	S 161	53
	IF (I=L) 335,335,336	S 162	54
335	PRINT 328, X(I), DUMMY, RELX(I), SP(I), SN(I), FJK(I)	S 163	55
	GO TO 339	S 164	56
336	IF (M=NN) 338,339,337	S 165	57
337	PRINT 329, X(I), DUMMY	S 166	58
	GO TO 339	S 167	59
338	PRINT 330, RELX(I), SP(I), SN(I), FJK(I)	S 168	60
339	CONTINUE	S 169	61
340	RETURN	S 170	62
	END	S 171	63

AUTOMATH SYSTEM

* * * S O U R C E P R O G R A M L I S T I N G * * *

SUBROUTINE SUBACC

A 100

COMMON A, ADDV, ADDX, DT, DT1, DTSC2, DUM, DUM1, DUMMY, EN, EP, M 101
 1 F, FJK, FJK1, FJKIN, FORCE, G, I, ICOUNT, INDEX, INDEXN, M 102
 2 J, JCOUNT, JJ, JTEST, K, KTEST, L, M, MTEST, N, NCOUNT, M 103
 3 NN, NTEST, N1, PN, PP, RELX, SM, SN, SP, USN, USP, V, X, M 104
 4 X1, YPN, YPP M 105

DIMENSION A(20), ADDV(20), ADDX(20), EN(25), EP(25), F(501), M 106
 1 FJK(25), FJK1(25), FJKIN(25), INDEX(25), INDEXN(25), M 107
 2 NCOUNT(25), PN(25), PP(25), RELX(25), SM(20), SN(25), M 108
 3 SP(25), USN(25), USP(25), V(20), X(20), X1(20), M 109
 4 YPN(25), YPP(25) M 110

A(1) = (FORCE - FJK(1) - FJK(2) - FJK(3) - FJK(4) - FJK(5))/SM(1) A 110 1

A(2) = (FJK(1) - FJK(6))/SM(2) A 111 2

A(3) = (FJK(2))/SM(3) A 112 3

A(4) = (FJK(3) + FJK(6))/SM(4) A 113 4

A(5) = (FJK(4))/SM(5) A 114 5

A(6) = (FJK(5) - FJK(7))/SM(6) A 115 6

A(7) = (FJK(7) - FJK(8) - FJK(9))/SM(7) A 116 7

A(8) = (FJK(8))/SM(8) A 117 8

A(9) = (FJK(9))/SM(9) A 118 9

RETURN A 119 10

END A 120 11

AUTOMATH SYSTEM

*** SOURCE PROGRAM LISTING ***

SUBROUTINE TRANS		T 100	
COMMON	A, ADDV, ADDX, DT, DT1, DTSQ2, DUM, DUM1, DUMMY, FN, EP, M 101		
1	F, FJK, FJK1, FJKIN, FORCE, G, I, ICOUNT, INDEX, INDEXN, M 102		
2	J, JCOUNT, JJ, JTEST, K, KTEST, L, M, MTEST, N, NCOUNT, M 103		
3	NN, NTEST, N1, PN, PP, RELX, SM, SN, SP, USN, USP, V, X, M 104		
4	X1, YPN, YPP M 105		
DIMENSION A(20), ADDV(20), ADDX(20), EN(25), EP(25), F(501), M 106			
1	FJK(25), FJK1(25), FJKIN(25), INDEX(25), INDEXN(25), M 107		
2	NCOUNT(25), PN(25), PP(25), RELX(25), SM(20), SN(25), M 108		
3	SP(25), USN(25), USP(25), V(20), X(20), X1(20), M 109		
4	YPN(25), YPP(25) M 110		
DO 113 I = 1,M		T 110	1
DUMMY = A(I)*DTSQ2		T 111	2
GO TO (110,111,112,112), KTEST		T 112	3
110	ADDV(I) = V(I) + DUMMY/3.0	T 113	4
	ADDX(I) = ADDV(I) + X(I)	T 114	5
	X1(I) = X(I)	T 115	6
	X(I) = X(I) + DUMMY/4.0 + V(I)/2.0	T 116	7
GO TO 113		T 117	8
111	DUM1 = X1(I) + DUMMY + V(I)	T 118	9
	X1(I) = X(I)	T 119	10
	X(I) = DUM1	T 119.1	11
	ADDV(I) = ADDV(I) + 4.0*DUMMY/3.0	T 120	12
	ADDX(I) = ADDX(I) + 2.0*DUMMY/3.0	T 121	13
GO TO 113		T 122	14
112	X(I) = ADDX(I)	T 123	15
	V(I) = ADDV(I) + DUMMY/3.0	T 124	16
113	CONTINUE	T 125	17
RETURN		T 126	18
END		T 127	19

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