AN EXAMINATION OF THE FRACTURE BEHAVIOR OF GRAPHITE THROUGH
THE USE OF SPLITTING TENSILE TESTS*

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(Synopsis)

Comparisons of results from tests conducted on splitting tensile specimens made from AXM and ATJ graphites with analytical predictions indicate that linear-elastic fracture mechanics methods can be used for predicting fracture of graphite under complex stress states. Circular and elliptical disks were tested under lateral, colinear, compressive loads, the forces being applied along a diameter of the circular disks and along the major axis of the elliptical disks. Elliptical disks with major-to-minor axis ratios (α/β) of 1.5 and 2.0 were used. Predictions of failure loads were obtained through the use of finite element analysis methods in combination with linear-elastic fracture mechanics methods.

INTRODUCTION

Methods for predicting fracture of graphite under multiaxial states of stress are required by structural analysts so that reliable design of graphite components can be achieved. These methods must apply to . . . A number of investigators have studied the fracture of graphite under multiaxial states of

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stress⁽¹⁻⁵⁾ They have generally tested thin-walled tubes and have correlated their results with such phenomenological theories as modified maximum strain energy theory, modified maximum stress theory, and the Coulomb-Mohr theory. X

Fracture mechanics methods have been applied to graphite under uniaxial stress states by a number of investigators⁽⁶⁻¹²⁾ These studies as well as a study of the effect of grain size on strength ~~conducted by Knabbs,~~

(13) indicate that fracture mechanics is applicable to graphite. The purpose of the present work was to examine the applicability of fracture mechanics methods

to the prediction of fracture of graphite under biaxial states of stress; *circular and elliptical splitting tensile specimens were used.*

→ The splitting tensile specimens ^{*used were a single set of*} are 1-in. - thick circular or elliptical disks loaded by lateral compressive loads. The circular disks ^{*were*} are 2 in. in diameter and the elliptical disks ^{*had*} have a major axis length of 2 in. The elliptical disks, which ^{*had*} have major-to-minor axis ratios of 1.5 or 2.0, ^{*were*} are loaded at the end of the major axis. Slots ^{*were*} are machined in some of the disks.

nearby
A fairly uniform biaxial stress state exists in the central portion of circular or elliptical disks loaded by lateral compressive loads. The maximum tensile stress occurs normal to the loading direction at the center *and is the maximum principal stress at that point.* of the disk. The minimum principal stress at the center is compressive and

is in the direction of loading. The ratio of the principal stresses is dependent on the shape of the disk. Slots in the central region that are aligned with the loading direction cause premature failures which should be predictable using fracture mechanics methods.

The graphites which were tested and data on these materials required for the analysis are described in the following section. In the next three sections, the method of analysis used to predict the load that should produce fracture of the splitting tensile specimens is explained, the splitting tensile tests are described, and the predictions are compared with the experimental results. The conclusions arising from this study are summarized

We apply the term splitting tensile test to the measurement of the diametral compressive load required to cause a cylindrical or elliptical specimen to split along the loaded diameter.

in the last section.

SUPPORTING DATA

~~The graphites used in this study are described in this section and the data on these materials that are required for the analyses of the splitting tensile specimens are given.~~

Two premium grade commercial graphites, ^{yes} ATJ and AXM, were used in these studies. ATJ graphite is a molded graphite which has a maximum grain size of 0.006 in. It has been produced by Union Carbide Corporation, Carbon Products Division, for a number of years, and its mechanical behavior has been well characterized. It was supplied in 14-in.-diam by 15-in.-long billets.

AXM graphite is an extremely fine grain, isotropic graphite manufactured by POCO Graphite, Inc. This material was supplied in 2-in.-diam by 12-in.-long cylinders.

Uniaxial Tensile Tests

~~Data for analysis of the splitting tensile specimens were obtained from uniaxial tensile tests.~~ The uniaxial tensile behaviors of the two graphites were determined using cylindrical specimens that were loaded through carefully aligned steel cables to insure that bending moments were not applied to the specimens. Electrical resistance foil strain gages were used to measure the axial and transverse strains, and strain-gage-based load cells were used to measure the loads.

Neither of these graphites exhibited any purely elastic behavior. The stress-strain curves were all non-linear, even at extremely low stress levels, and the strain ratios* decreased as the stress increased. Since the methods

*Strain ratio is the non-linear equivalent of Poisson's ratio, i.e., it is the negative of the ratio of the strain normal to the direction of loading to the strain in the loading direction.

to be used for analyzing the splitting tensile specimens were based on linear elasticity, a linear approximation to the stress-strain curve and a constant value of strain ratio were required. The secant modulus and strain ratio at a selected stress level were chosen for use in the analyses. ~~No~~ *In this way, the non-linear* ~~doubt even better results would have been obtained if the elastic-plastic behavior of the graphite had been taken into account in the analysis.~~ *was accounted for in an approximate fashion so that linear analyses could be used. These values are a better approximation of*

The average strength of three AXM tensile specimens was 7300 psi, and their average strain-to-failure was 0.57%. The secant modulus at a stress of 1000 psi was 1.60×10^6 psi, and the strain ratio at 1000 psi was 0.27. The strength, strain-to-failure, and strain ratio of AXM graphite are all *markedly* ~~remarkably~~ high. *the non-linear behavior than the tangent modulus and initial strain ratio,*

Since ATJ graphite is transversely isotropic, it was necessary to conduct two sets of tensile tests on this material to determine the response to with-grain and across-grain loading. Three across-grain ATJ tensile specimens gave average strength and strain-to-failure values of 2400 psi and 0.41%, respectively. The average secant modulus and average ratio of transverse strain to axial strain at a stress of 500 psi were 0.83×10^6 psi and 0.083, respectively. Four with-grain specimens of ATJ graphite were tested. The average strength was 3500 psi and the average strain-to-failure was 0.36%. The average secant modulus at a stress of 500 psi was 1.39×10^6 psi. The transverse strain varies around the circumference of a with-grain ATJ graphite specimen because of anisotropy. Therefore, the transverse strain was measured in both the with-grain and across-grain direction. The ratio of transverse strain in the across-grain direction to axial strain was 0.14 at 500 psi stress. The ratio of transverse strain in the with-grain direction to axial strain was 0.09 at that same stress.

Fracture Toughness Determinations

(12)
A comprehensive study of the requirements for valid measurements of fracture toughness of graphite ^{has been} made using notched beams tested in four-point flexure. The effects of specimen size, specimen width, notch sharpness, notch depth, and strain energy in the system were examined. Specimen geometries and methods for measuring the fracture toughness of graphite were established. The measure of fracture toughness ^{determined} used was K_{Ic} , which is the stress intensity factor at onset of rapid crack propagation under plane strain conditions.

Values of K_{Ic} were determined for AXM and ATJ graphites ^{were provided through} from that study. The measured value of K_{Ic} for AXM graphite is $1.17 \text{ ksi} \sqrt{\text{in.}}$ with a standard deviation of $0.08 \text{ ksi} \sqrt{\text{in.}}$. Since AXM graphite is isotropic, only one K_{Ic} value is needed. ATJ graphite is transversely isotropic and requires the investigation of three principal combinations of load application and crack propagation directions with respect to grain direction. The measured values of K_{Ic} for ATJ graphite are as follows: for loading in the with-grain direction and crack propagation in the with-grain direction, $0.82 \text{ ksi} \sqrt{\text{in.}}$ with a standard deviation of $0.05 \text{ ksi} \sqrt{\text{in.}}$; for loading in the with-grain direction and crack propagation in the across-grain direction, $0.81 \text{ ksi} \sqrt{\text{in.}}$ with a standard deviation of $0.06 \text{ ksi} \sqrt{\text{in.}}$; for loading in the across-grain direction, $0.69 \text{ ksi} \sqrt{\text{in.}}$ with a standard deviation of $0.04 \text{ ksi} \sqrt{\text{in.}}$.

ANALYSIS

The theory of elasticity solution ⁽¹⁴⁾ for an isotropic circular disk without a slit and loaded in diametral compression gives a calculated axial

compressive stress to transverse tensile stress ratio* of 3 in the central region of the disk. The theory of elasticity solution for an isotropic elliptical disk subjected to opposing compressive forces at the ends of the major axis was developed by Brisbane⁽¹⁵⁾ using the complex variable method. The axial compressive stress to transverse tensile stress ratios in the central regions of elliptical disks are calculated to be 3.8 for $\alpha/\beta = 1.5$ and 6.6 for $\alpha/\beta = 2.0$. The stresses in both the circular and the elliptical disks are reasonably constant within the central regions that extends approximately one-half the distance from the center to the perimeter. From the values quoted, it is seen that a significant range of biaxial stress ratios is covered. Okubo⁽¹⁶⁾ has solved the problem of diametrically compressed circular disks of anisotropic elastic material.

Since theory of elasticity solutions are not available for circular and elliptical disks with slits in them, finite element methods^(17,18) were used in the analysis of the test specimens. Because of symmetry, it was necessary to consider only one quadrant of the disk in each analysis. The quadrant *with smaller elements being used along the loaded axis.* was divided into 234 discrete elements with 243 nodes. *The elements along the loaded axis were made smaller than the other elements.* A load of 1000 lb. and the secant modulus and strain ratio values obtained from the tensile tests were used.

Finite element solutions for the solid disks compared very well with the classical elasticity solutions in the central area of the solid disks. Results for an elliptical disk ($\alpha/\beta = 1.5$) of isotropic AXM graphite are compared in Fig. 1 where the stress distributions along the loaded axis

*For conciseness, stresses and strains normal to the loading direction are referred to as transverse, and stresses and strains in the loading direction are referred to as axial.

of the specimen are plotted. The stress in the direction of the applied load is designated as σ_y , and the stress in the direction normal to the load direction is σ_x . Lack of agreement between the finite element analysis and the classical elasticity results near the points of load application is due to differences in the ways the loads were applied. In the classical elasticity solution it was assumed that a line load was applied, whereas the load was distributed over some area in the finite element analysis. During test of a specimen, the load is actually distributed over some finite area due to deformation in the region of load application. The area increases as the load increases. Comparison of the finite element results ^{for a circular disk} in Fig. 2 shows the differences in the stress distributions along the ^{axis} loaded ^{axis} for concentrated and distributed loads. In the latter case, the load was distributed over an area having a total width of 0.32 in.; for the point load case, the load was applied to only one node, the one on the axis. A distributed load was used in all the calculations to be dismissed here, ^{C.U.} except where noted.

The ratio of compressive stress to tensile stress in the center of the solid disks, as determined by finite element analyses, is given in Table 1 for each type of disk. The stress ratios for the isotropic disks are in agreement with Brisbane's ^e results, but the anisotropy of the ATJ graphite produces somewhat different stress ratios.

The procedure used for determining the relation between stress intensity factor, crack length, and load was developed by Watwood,⁽¹⁹⁾ and by Mowbray.⁽²⁰⁾ It is basically an analog to the compliance method of experimentally determining ~~K_c or G_c~~ ^{of the stress intensity factor at onset of rapid} values. The finite element method is used to compute the strain energy stored in several disks which are identical except for differences in crack lengths. With the strain energy, U , known as a function of crack area, A , Irwin's ⁽²¹⁾ definition of the strain-energy release

propagation, K_c , or the strain-energy release rate at onset of rapid crack propagation, G_c .

crack

rate

$$G = \left(\frac{\partial U}{\partial A} \right) \quad (1)$$

can be used in conjunction with the known relation (4)

$$G = \left\{ (1 - \nu^2)/E \right\} K^2 \quad (2)$$

to determine the relation between the stress-intensity factor, K , and crack length. Here E is the elastic modulus, and ν is the strain ratio. Since the strain energy in an elastic body is proportional to the square of the load, (P) per unit thickness, the relation between the stress intensity factor, crack length, and load is

$$\frac{K}{P} = \sqrt{\frac{E}{(1 - \nu^2)P^2} \frac{\partial U}{\partial A}} \quad (3)$$

The right side of Eq. (3) can be determined from finite-element analyses. The fracture toughness parameter, K_c , which is the stress intensity factor at onset of crack propagation, can then be determined from the load at onset of crack propagation, or vice versa.

Two methods were used to calculate the strain energy in each disk. Since the strain energy stored in an elastic body is equal to the work done by external forces applied to the body, one method used in determining the strain energy was to calculate the work, W , done by the applied loads. The other method was to compute the strain energy in each individual element and to sum these energies to obtain the total strain energy, U , in the body. The strain energy, U , obtained by the second method was slightly less than the work, W . However, when both quantities were plotted as functions of crack length and the curves were differentiated graphically, they gave almost identical values of $\partial U / \partial A$.

The values of K/P for circular and elliptical disks with slits of different lengths were calculated with Eq. (3). The elastic constants and load were the same as those used in the finite element analyses that produced the

curves for determining $\frac{\partial U}{\partial A}$. The calculated values of K/P ^{are listed in Table 2. These values may be used} for the splitting ^{to predict} tensile specimens, if the value of K_{Ic} for the material is known. Alternatively, the measured loads when the specimens break ^{the load necessary to break} might be used in conjunction with the K/P values to determine the value of K_{Ic} .

SPLITTING TENSILE TESTS

The splitting tensile test specimens were 1-in.-thick disks, either circular with a 2.00 in. diameter or elliptical with a 2.00 in. major axis length ² and a 1.00 or 1.33 in. minor axis length ¹. The disks were subjected to compressive forces applied at the edges and acting along a diameter of the circular specimens or along the major axis ¹ of the elliptical specimens.

Orientation of the AXM specimens with respect to grain direction was not necessary, since AXM graphite is isotropic. However, particular attention to this aspect was necessary for the ATJ graphite specimens. The ATJ disks were cut so that the across-grain direction was in the plane of the disk. The with-grain ATJ specimens were oriented so that the load was applied in the with-grain direction, and the across-grain ATJ specimens were oriented so that the load was applied in the across-grain direction.

Some of the specimens had through-thickness slits in the central portion of the specimen in the direction of load application. The slit was produced by first machining a 0.092-in.-diam hole through the center and normal to the plane of the disk. Following this, a 0.020-in.-thick saw blade was inserted through the hole and used to cut the slit in both directions along the loading line. The blade was shaped such that the slit terminated with a 30 degree included angle ^{and} with a root radius of approximately 0.001 in.

^{The total distance between the tips}
Total length of the slits varied from 0.200 in. to 0.800 in.

The specimens were tested in a commercial compressive subjig and were loaded with a 120,000 lb. capacity BIW universal hydraulic testing machine.

After considerable preliminary experimentation, pieces of 0.035-in-thick cardboard were selected as padding material to prevent crushing of the specimen in the vicinity of the point of load application. One thickness of cardboard was placed between the specimen and the end of the subjig ram at the top and between the specimen and the subjig pedestal on the bottom. This padding material did not prevent premature fracture at the loading points in several instances, however.

All of the solid specimens showed evidence of premature fracture at the loading points, and each of these specimens broke into a number of pieces. A typical fractured solid specimen is shown in Fig. 3. This was a with-grain ATJ elliptical disk with a α/β ratio of 1.5; the failure load was 3600 psi. Considerable evidence of crushing at the loading points can be seen. Other investigators, who have conducted diametral compression tests on solid graphite cylinders were also unable to achieve clean tensile fractures along the loaded diameter. This has consistently resulted in tensile strength values obtained from diametral compression tests of graphite that were in poor agreement with results from uniaxial tensile tests.^(22,23)

A variety of fracture patterns was obtained for the specimens with slits. ^{Typical} Some of the fracture patterns obtained from specimens with 0.400 in.-long slits are shown in Fig 4. The specimen in the upper left failed with a single crack along the loaded axis as would be expected when tensile stress normal to the load direction produces fracture. The specimen at the upper right broke into six pieces. A similar fracture pattern was observed by Rudnick, Hunter, and Holden⁽²⁴⁾ in diametral compression tests of solid circular disks; they referred to it as a "triple-cleft" fracture. They present evidence from photoelastic tests to support their contention that the crack along the loaded axis occurs first and the ^{remaining} other fracturing occurs as a result of the new stress distribution ^{associated with} that occurs in the two half

cylinders. The specimen in the lower left of Fig. 4 failed in one of the variations of the triple-cleft mode of fracture. This specimen broke into four pieces; however, cracks which started near the loading ^{points} can be observed. These cracks indicate that, if the load had not been released, two additional lobes would have been formed. Such cracks were also observed in some specimens that broke into only two pieces. Our observations confirm the conclusion of Rudnick, Hunter, and Holden that fracture of such specimens is caused by the maximum tensile stresses, which act across the loaded diameter.

The fracture pattern exhibited by the specimen in the lower right of Fig. 4 does not include the straight fracture along the loaded axis. Crushing was observed at the loading points at a load of 2250 lb; complete fracture occurred at a load of 2630 lb. The only specimens with slits in which such fractures were observed were the elliptical disks of ATJ graphite with an α/β of 2.0 and a slit length of 0.400 in. All of those specimens failed in this manner, ^{as shown in the lower right of Fig. 4.} ~~It is concluded that failure in all of the other disks with slits was produced by the maximum tensile stress acting across the loaded axis.~~

Two- gage strain-gage rosettes were mounted in the center on one face of each solid specimen so that the axial compressive strain and transverse tensile strain could be measured. The number of specimens tested and average fracture load and strains in the solid disks are recorded in Table 3. The transverse tensile stress and axial compressive stress at fracture were calculated from the measured strains using plane stress elasticity equations and the elastic properties that were obtained from the uniaxial tensile tests. As would be expected from the observed fracture modes, the measured tensile strains and calculated tensile stresses at

were fracture more less than the uniaxial fracture strains and fracture stresses. Because of the sensitivity of the stress distribution near the loading points to the load distribution, it was not possible to determine the stresses in the region of fracture initiation.

Strain gages were mounted at the tips of the saw cuts of the slotted specimens to aid in determining the onset of crack initiation. These strain gages were oriented normal to the direction of loading, ^{with the edge of the} ~~and the first grid~~ ~~of the strain gage was~~ ^{and a portion of the gage being} only 1/32 in. from the tip of the saw cut. The electrical continuity of the gage was destroyed when the crack propagated through one of the grid wires. This event always occurred when the load was a maximum. In some cases, an accelerated increase in strain was observed at 90% or more of maximum load, which probably was a result of a small amount of initial slow crack growth. ~~In some cases, no accelerated increase in strain was observed.~~ Because the load at the instant of loss of electrical continuity of the gage was well defined, this load, which is the maximum load, was taken as that corresponding to crack initiation.

COMPARISON OF MEASURED AND CALCULATED RESULTS

The average fracture load for each set of specimens with machined slots is listed in Table 4.

291 The loads necessary to fracture disks with slits were predicted using the values ^{of} K_{Ic} given earlier and the K/P values tabulated in Table 2. These predictions are also given in Table 4. For AXM graphite, all of the predictions are within 10% of the average measured fracture load, while for ATJ graphite, three out of the twelve sets of specimens tested failed at loads that were much less than those predicted. Two of the sets were elliptical disks with an α/β of 2.0 and a 0.400-in. slit length, both with-grain and across-grain,

which were observed to crush at the loading points prior to fracture.

This crushing changed the stress distribution in the disk so that the predictions were no longer valid. The predicted fracture loads for these disks were higher than the breaking loads for similar solid disks. The other set of specimens that failed at a load much lower than the predicted load was

The ^{third} set of with-grain elliptical disks that had an α/β of 1.5 and a 0.400 in.-long slit. The ^{failures at loads well below those predicted loads} discrepancy cannot be explained as well for this set, but

It is suspected that some crushing at the loading points occurred in these specimens. Overall, the predictions are in good agreement with the experimental results, except as noted. Since elastic analyses were used to determine the K/P values, the better agreement obtained for the AXM graphite may be associated with the fact that the stress-strain curve for this material is more nearly linear than those for ATJ graphite.

CONCLUSIONS

The good agreement between the predicted fracture loads and the measured fracture loads for the specimens with slits indicates that fracture mechanics methods are applicable for predicting failure of graphite under biaxial states of stress where one stress is tensile and the other is compressive. Other combined stress states must be examined before general applicability to graphite is established, but fracture mechanics methods look promising.

The results of the study also show that the finite element method can be used effectively in failure predictions. Because of the versatility of this method in handling varied and complex geometries, this demonstrated utility is of importance.

In those cases where the specimens failed in the expected

manner, that is, by splitting along the loaded axis, the predictions agreed very well with the experimental results. The agreement was poor when the specimens did not fail in the expected manner.

7/6/91 The final observation from this study is that it is extremely important that the assumed failure mode be correct if accurate fracture predictions are to be achieved.

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APPENDIX

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Table 1. Calculated Ratio of Axial Compressive Stress to Transverse Tensile Stress at Center of Solid Splitting Tensile Specimens

Graphite Orientation.	α/β	$\frac{\sigma_y}{\sigma_x}$ <i>subscript y</i> <i>Subscript x</i>
AXM (isotropic)	1.0	3.0
	1.5	3.8
	2.0	6.6
ATJ with-grain <i>all cracks</i>	1.0	2.8
	1.5	4.0
	2.0	6.9
<i>crack</i> ATJ across-grain <i>loading</i>	1.0	3.6
	1.5	4.0
	2.0	6.2

Table 2. Calculated Ratio of Stress Intensity Factor to Load for Splitting Tensile Specimens with Slots

Graphite Orientation	α/β	Slot Length (in.)	K/P (in. ^{-3/2})
<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin-right: 10px;"> AXM (isotropic) ATJ with-grain across-grain </div> <div style="margin-left: 20px;">→</div> </div>	1.0	0.200	0.223
		0.400	0.288
		0.600	0.387
		0.800	0.472
	1.5	0.400	0.276
		0.600	0.368
		0.800	0.492
	2.0	0.400	0.230
		0.600	0.319
		0.800	0.431
	1.0	0.400	0.239
		0.800	0.364
	1.5	0.400	0.239 0.213
		0.800	0.411
	2.0	0.400	0.204
		0.800	0.399
	1.0	0.400	0.290 0.339
		0.800	0.580
	1.5	0.400	0.290
		0.800	0.528
	2.0	0.400	0.234
		0.800	0.434

Table 3. Average Data from Solid Splitting Tensile Specimens

Graphite Orientation	α/β	Number of Specimens	Average Fracture Load (lb)	Average Transverse Strain At Fracture (%)	Average α_{in} Axial Stress At Fracture (%)	Transverse Tensile Stress At Fracture (psi)	Axial Compressive Stress At Fracture (psi)
AXM (isotropics)	1.0	4	10,790	0.467	-0.694	4830	-9800
	1.5	4	7,620	0.241	-0.461	2010	-6830
	2.0	4	5,440	0.156	-0.368	980	-5620
ATJ with-grain	1.0	3	4,400	0.261	-0.456	1660	-6100
	1.5	4	3,740	0.175	-0.381	1020	-5140
	2.0	4	2,590	0.089	-0.274	430	-3740
cc. across-grain	1.0	4	5,290	0.290	-0.734	3220	-5820
	1.5	4	3,770	0.147	-0.508	1470	-4090
	2.0	4	2,900	0.084	-0.445	660	-3630

Table 4. Comparison of Predicted and Measured Fracture Loads for Splitting Tensile Specimens with Slots

Graphite Orientation	α/β	Slit Length (in.)	Number of Specimens	Predicted Fracture Load (lb.)	Average Fracture Load (lb.)
<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin-right: 10px;"> AXM (isotropic) </div> <div style="border-left: 1px solid black; border-right: 1px solid black; height: 100px; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0;"></div> <div style="position: absolute; top: 50%; left: 50%; transform: translate(-50%, -50%);">→</div> </div> </div>	1.0	0.200	3	5,250	5,170
		0.400	3	4,060	4,090
		0.600	3	3,020	3,180
		0.800	3	2,480	2,590
	1.5	0.400	3	4,240	4,590
		0.600	3	3,180	3,170
		0.800	3	2,380	2,460
	2.0	0.400	2	5,090	5,310
		0.600	2	3,670	4,010
		0.800	1	2,710	2,840
<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin-right: 10px;"> ATJ with-grain </div> <div style="border-left: 1px solid black; border-right: 1px solid black; height: 100px; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0;"></div> <div style="position: absolute; top: 50%; left: 50%; transform: translate(-50%, -50%);">→</div> </div> </div>	1.0	0.400	3	2,890	2,620
		0.800	3	1,900	1,800
	1.5	0.400	3	3,240	2,480
		0.800	3	1,680	1,590 ^a
	2.0	0.400	3	4,050	2,440 ^a
		0.800	3	1,730	1,640
<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin-right: 10px;"> across-grain </div> <div style="border-left: 1px solid black; border-right: 1px solid black; height: 100px; position: relative;"> <div style="position: absolute; top: 0; left: 0; right: 0; bottom: 0;"></div> <div style="position: absolute; top: 50%; left: 50%; transform: translate(-50%, -50%);">→</div> </div> </div>	1.0	0.400	3	2,420	2,440
		0.800	3	1,420	1,610
	1.5	0.400	3	2,860	2,880
		0.800	3	1,550	1,600 ^a
	2.0	0.400	3	3,500	2,540 ^a
		0.800	3	1,890	1,730

^aCrushing was observed at the loading points prior to the maximum load.

FIG. 1. Stress Distribution Along Loaded Axis of
AXM Graphite Elliptical Disk with $\alpha/\beta = 1.5$
Under 1000-lb. Lateral Compressive Load

Fig. 2. Stress Distribution Along Loaded Diameter of
AXM Graphite Circular Disk Under 1000-lb.
Diametral Compressive Load.

Fig. 3. Solid With Grain ATJ Splitting
Tensile Specimen with $\alpha/\beta = 1.5$ After Test.

Fig. 4... Broken Splitting Turtle Specimens
Which Contained 0.400-in. Long Slits