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A Numerical Method for Transforming
Surface and Space Probe Data

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Abstract

A project was undertaken to find an effective method of relating observed visible light intensity distributions to the actual visible light emission within a confined plasma. The physical system consisted of a cylindrically confined plasma which was bombarded with an electron beam. Visible light was collected by a scanning system from a conical volume of the plasma.

Two programs have been written which calculate the desired light emission from these radiation measurements taken external to the plasma. The first program works with a two dimensional system in which the plasma is contained within a circular surface, and radiation measurements are made with a wedge-shaped surface probe. The second program works with the true three dimensional system in which the plasma is contained within a cylinder, and radiation measurements are made with a conical volume probe.

This work extends that of Nestor and Olson who considered only the planar case with scanning systems of lines and segmental areas.¹

¹ Nestor, O. H. and Olson, H. N., SIAM Review, (July, 1960), 200-207.

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INTRODUCTION:

When studying the physical properties of an inhomogeneous, axially symmetric plasma, the radial distribution of such quantities as radiation intensities must be obtained. However, when observations to determine such properties are made with probes external to the plasma, the data obtained represents an integration of the desired radial distribution over the extent of the plasma. Thus, this external probe data must be transformed in some way to give the desired radial distribution.

This report describes a numerical technique and the corresponding computer programs for determining the radial distribution of light emission within a cylindrically confined plasma. The first program considers only a two dimensional system in which the plasma is contained within a circular surface, and the radiation measurements are made with a wedge-shaped surface probe. The second program considers the true three dimensional system in which the plasma is contained within a cylinder, and the radiation measurements are made with a conical volume probe.

Due to the differences in these two systems and their corresponding computer programs, they will be considered separately in the following report.

Two Dimensional Computer Program

NAME: CIRWEDGE

PROGRAMMER: R. C. Durfee

LANGUAGE: FORTRAN 63

INPUT: Description of Variables Used

1. RADIUS = radius of containing circular surface
2. ALPHA = angle of wedge apex in degrees
3. NOZONE = number of zones into which circular surface is divided
4. HEIGHT = height of wedge apex above the x axis
5. GX = radiation measurements taken by wedge-shaped surface probe
6. SS1= {
 - 1 to specify and read one constant height for all zones
 - 2 to read height at each zone
 - 3 to supply subroutine that calculates height at each zone
7. SS2= {
 - 1 to use test generating routine to calculate GX values
 - 2 to read GX value at each zone
8. SS3= {
 - 1 to printout HEIGHT and GX values used
 - 2 to omit printout of HEIGHT and GX values
9. SS4= {
 - 1 to printout area coefficient matrix calculated
 - 2 to omit printout of area coefficient matrix
10. SS5= {
 - 1 to printout determinant and inverting pivot elements of area coefficient matrix
 - 2 to omit printout of determinant and inverting pivot elements
11. SS6= {
 - 1 to printout inverse matrix of area coefficient matrix
 - 2 to omit printout of inverse matrix

Card Format.

Card 1.	SS1,SS2,SS3,SS4,SS5,SS6	-	FORMAT(6I1)
Card 2.	RADIUS,ALPHA,NOZONE	-	FORMAT(2F10.0,I10)
Card 3.	HEIGHT (If SS1 $\neq$ 3)	-	FORMAT(8E10.0)
Card 4.	GX (If SS2 = 2)	-	FORMAT(8E10.0)

OUTPUT: 1. RADIUS,ALPHA,NOZONE

2. HEIGHT and GX	-	If SS3 = 1
3. Area coefficient matrix	-	If SS4 = 1
4. Determinant and pivot elements	-	If SS5 = 1
5. Inverse of area matrix	-	If SS6 = 1
6. FR (radial distribution of light emitted within the plasma)		

DESCRIPTION:

The numerical method used in solving for the radial distribution of the emitted light within the plasma is based upon several assumptions. The emitted radiation is assumed to be a radial function with negligible attenuation, and this radial function is assumed to be constant across any given zone as will be described below.

Radiation measurements are taken within a conical volume at specified positions, x , within the plasma. The relation between this measured light intensity, $G(x)$, and the emitted light intensity per unit volume, $f(r)$, is

$$G(x) = \int f(r) dV, \quad (1)$$

integrated over the conical volume. Since the determination of $f(r)$ in this spatial system is very difficult, a first approximation using a two dimensional system was solved. In the planar system, the cylinder becomes a circular surface, the cone becomes a wedge, and the integral equation (1) becomes:

$$G(x) = \int_A = F(x) \int f(r) dA = \int_A \int f(r) r dr d\theta . \quad (2)$$

Therefore, the program, CIRWEDGE, has been written to solve the integral equation (2) for $f(r)$. The intensity, $G(x)$, represents the radiation measurements taken by the wedge-shaped surface probe on a circular surface as the wedge sweeps across the surface (see Fig. 1).

Fig. 1 Wedge-Shaped Surface Probe Sweeping Circular Surface

The numerical technique consisted of dividing the circular surface into n zones as shown in Fig. 1 and then finding the shaded area of each zone inside the wedge. By assuming that $f(r)$ is constant across any single zone, the integral equation (2) may be written numerically as:

$$G_i = \sum_{j=1}^n a_{ij} \cdot f(r_j) , \quad (3)$$

where i designates the position, x , of the wedge, j designates the zones of the circle, and a_{ij} designates the area coefficients. The set of n simultaneous equations (3) may then be solved for the radial functions, $f(r_j)$.

The program allows the height, h , of the wedge apex to change as the wedge sweeps across the circular surface. In order to test the program, a known radial function,

$$f(r) = \begin{cases} 1-2r^2, & 0 \leq r \leq 1/2, \\ 2(1-r)^2, & 1/2 \leq r \leq 1, \\ 0, & 1 \leq r, \end{cases}$$

was used to compute G_i 's. These G_i 's were then used as input data to calculate $f(r_j)$'s which could be compared with the known $f(r)$.

RESULTS:

Upon comparing the known $f(r_j)$ with the computed $f(r_j)$, differences on the order of 10^{-5} were obtained, thus indicating that the numerical technique was quite applicable to well-behaved radial functions. Also, the accuracy of the calculated $f(r_j)$'s was increased by letting the height of the wedge apex approach zero and by increasing the number of zones. This improved accuracy was due to the fact that the radial function was not assumed constant over as large a zonal area. Some sample output is given on pages 6-10.

SURFACE WEDGE DATA REDUCTION BY R.C.D.

PARAMETERS

RADIUS = 1.00000000
 WEDGE ANGLE = 3.00000000 (DEGREES)
 X INCREMENT = .10000000
 NO. OF ZONES = 10
 N2 = 11

I	X VALUE	HEIGHT(X)	INTEGRAL - G(X)
1	0	.5000000	2.7263793E-02
2	.1000000	.5000000	2.6484344E-02
3	.2000000	.5000000	2.4208860E-02
4	.3000000	.5000000	2.0627740E-02
5	.4000000	.5000000	1.6072894E-02
6	.5000000	.5000000	1.1092757E-02
7	.6000000	.5000000	6.7067864E-03
8	.7000000	.5000000	3.4291763E-03
9	.8000000	.5000000	1.3047915E-03
10	.9000000	.5000000	2.4396818E-04
11	1.0000000	.5000000	2.4103019E-07

AREA COEFFICIENTS											
EQ.	COL.	1	2	3	4	5	6	7	8	9	10
1	1	5.2204-03	5.2429-03	5.2379-03	5.2366-03	5.2361-03	5.7596-03	6.8068-03	7.8540-03	8.9012-03	9.9484-03
	1		2	3	4	5	6	7	8	9	10
2	1	8.7852-04	8.1789-03	5.7451-03	5.4703-03	5.3723-03	5.7649-03	6.8068-03	7.8540-03	8.9012-03	9.9484-03
	1		2	3	4	5	6	7	8	9	10
3	1	0	1.2600-03	1.0428-02	6.4344-03	5.8570-03	5.8509-03	6.8068-03	7.8540-03	8.9012-03	9.9484-03
	1		2	3	4	5	6	7	8	9	10
4	1	0	0	1.5544-03	1.2290-02	7.0950-03	6.2836-03	6.8068-03	7.8540-03	8.9012-03	9.9484-03
	1		2	3	4	5	6	7	8	9	10
5	1	0	0	0	1.8048-03	1.3897-02	7.7119-03	6.9528-03	7.8540-03	8.9012-03	9.9484-03
	1		2	3	4	5	6	7	8	9	10
6	1	0	0	0	0	2.0277-03	1.5335-02	8.2884-03	7.8593-03	8.9012-03	9.9484-03
	1		2	3	4	5	6	7	8	9	10
7	1	0	0	0	0	0	2.2314-03	1.6647-02	8.8746-03	8.9012-03	9.9484-03
	1		2	3	4	5	6	7	8	9	10
8	1	0	0	0	0	0	0	2.4206-03	1.7862-02	9.5681-03	9.9484-03
	1		2	3	4	5	6	7	8	9	10
9	1	0	0	0	0	0	0	0	2.5986-03	1.8998-02	1.0352-02
	1		2	3	4	5	6	7	8	9	10
10	1	0	0	0	0	0	0	0	0	2.7676-03	2.0066-02
	1		2	3	4	5	6	7	8	9	10
11	1	0	0	0	0	0	0	0	0	0	2.9292-03

NO. PIVOT ELEMENTS

1	5.22040327e-03
2	7.2966413e-03
3	9.5979647e-03
4	1.1376609e-02
5	1.2902003e-02
6	1.4259209e-02
7	1.5498496e-02
8	1.6642171e-02
9	1.7691686e-02
10	1.8658905e-02

DETERMINANTS 6.51518322e-20

INVERSE AREA COEFFICIENTS

EQ.	COL.	1	2	3	4	5	6	7	8	9	10
1	1	2.1377+02	-1.3203+02	-3.2487+01	-1.4221+01	-7.4257+00	7.7313+00	6.6567+00	4.9427+00	3.3608+00	2.3713+00
		1	2	3	4	5	6	7	8	9	10
2	1	-2.4956+01	1.4850+02	-6.6335+01	-1.9409+01	-9.0263+00	7.6119+00	7.1321+00	5.2377+00	3.5663+00	-2.5155+00
		1	2	3	4	5	6	7	8	9	10
3	1	3.2606+00	-1.9372+01	1.1218+02	-4.9343+01	-1.4665+01	7.9635+00	8.1496+00	5.9537+00	4.0564+00	-2.8615+00
		1	2	3	4	5	6	7	8	9	10
4	1	-4.4426+01	2.6396+00	-1.5236+01	9.4385+01	-4.0882+01	-1.1763+01	-9.2500+00	7.2829+00	-4.9160+00	-3.4709+00
		1	2	3	4	5	6	7	8	9	10
5	1	0.2032+02	-3.6861+01	2.1346+00	-1.3181+01	8.3083+01	-3.5479+01	-1.1007+01	9.5334+00	6.3704+00	-4.5020+00
		1	2	3	4	5	6	7	8	9	10
6	1	-8.8059+03	5.2327+02	-3.0332+01	1.8711+00	-1.1794+01	7.5044+01	3.1403+01	-1.1644+01	8.9059+00	-6.2199+00
		1	2	3	4	5	6	7	8	9	10
7	1	1.2661+03	-7.5236+03	4.3559+02	-2.6903+01	1.6958+00	-1.0790+01	6.8952+01	2.8362+01	-1.2321+01	-9.1429+00
		1	2	3	4	5	6	7	8	9	10
8	1	-1.8407+04	1.0938+03	-6.3341+03	3.9112+02	-2.4654+01	1.5687+00	-1.0024+01	6.4183+01	-2.6225+01	-1.3992+01
		1	2	3	4	5	6	7	8	9	10
9	1	2.7224+05	-1.6177+04	9.3681+04	-5.7846+03	3.6462+02	-2.3200+01	1.4826+00	9.4926+00	6.0793+01	-2.7290+01
		1	2	3	4	5	6	7	8	9	10
10	1	-3.7545+06	2.2310+05	-1.2920+04	7.9776+04	-5.0286+03	3.1996+02	-2.0447+01	1.3091+00	-8.3840+00	5.3594+01
		1	2	3	4	5	6	7	8	9	10
11	1	0	0	0	0	0	0	0	0	0	2.9292+03

RESULTS - F(R)=(1-2R)**2 OR 2(1-R)**2

I	R VALLE	CALC. F(R)	TRUE F(R)	DIFFERENCE
1	.050000	9.80095953E-01	9.95000000E-01	1.49000467E-02
2	.150000	9.45451284E-01	9.55000000E-01	9.54871569E-03
3	.250000	8.68635108E-01	8.75000000E-01	6.36489212E-03
4	.350000	7.52854102E-01	7.55000000E-01	2.14589793E-03
5	.450000	5.97618827E-01	5.95000000E-01	2.61882655E-03
6	.550000	4.11605896E-01	4.05000000E-01	6.60589645E-03
7	.650000	2.49791451E-01	2.45000000E-01	4.79045066E-03
8	.750000	1.29344566E-01	1.25000000E-01	4.34456607E-03
9	.850000	4.79684814E-02	4.50000000E-02	2.96848140E-03
10	.950000	5.34155865E-03	5.00000000E-03	3.41558652E-04

Three Dimensional Computer Program

NAME: CYLCONEPROGRAMMER: R. C. DurfeeLANGUAGE: FORTRAN 63INPUT: Description of Variables Used

1. RADIUS = radius of containing cylinder
2. ALPHA = angle of conical probe's apex in degrees
3. NOZONE = number of zones into which cylinder is divided
4. HEIGHT = height of cone apex above the x axis
5. GX = radiation measurements taken by conical volume probe
6. $SS1 = \begin{cases} 1 & \text{to specify and read one constant height for all zones} \\ 2 & \text{to read height at each zone} \\ 3 & \text{to supply subroutine that calculates height at each zone} \end{cases}$
7. $SS2 = \begin{cases} 1 & \text{to printout HEIGHT and GX values used} \\ 2 & \text{to omit printout of HEIGHT and GX values} \end{cases}$
8. $SS3 = \begin{cases} 1 & \text{to printout volume coefficient matrix calculated} \\ 2 & \text{to omit printout of volume coefficient matrix} \end{cases}$
9. $SS4 = \begin{cases} 1 & \text{to printout determinant and inverting pivot elements of volume coefficient matrix} \\ 2 & \text{to omit printout of determinant and inverting pivot elements} \end{cases}$
10. $SS5 = \begin{cases} 1 & \text{to printout inverse matrix of volume coefficient matrix} \\ 2 & \text{to omit printout of inverse matrix} \end{cases}$
11. $SS6 = \begin{cases} 1 & \text{to compute total cone probe volume two different ways for comparison purposes} \\ 2 & \text{to omit calculation of total volume of conical probe} \end{cases}$

Card Format

Card 1.	SS1,SS2,SS3,SS4,SS5,SS6	-	FORMAT(6I1)
Card 2.	RADIUS,ALPHA,NOZONE	-	FORMAT(2E10.0,I10)
Card 3.	HEIGHT (If SS1 $\neq$ 3)	-	FORMAT(8E10.0)
Card 4.	GX	-	FORMAT(8E10.0)

OUTPUT:

1.	RADIUS,ALPHA,NOZONE		
2.	HEIGHT and GX	-	If SS2 = 1
3.	Volume coefficient matrix	-	If SS3 = 1
4.	True conical volume and summed conical volume	-	If SS6 = 1
5.	Determinant and pivot elements	-	If SS4 = 1
6.	Inverse of volume matrix	-	If SS5 = 1
7.	FR (radial distribution of light emitted within the plasma)		

DESCRIPTION:

The numerical method used in solving for the radial distribution of the emitted light within the plasma is based upon several assumptions. The emitted radiation is assumed to be a radial function with negligible attenuation, and this radial function is assumed to be constant across any given zone as will be described below.

Radiation measurements are taken within a conical volume at specified positions, x , within the plasma. The relation between this measured light intensity, $G(x)$, and the emitted light intensity per unit volume, $f(r)$ is

$$G(x) = \int f(r) \, dV, \quad (5)$$

integrated over the conical volume.

Therefore, the program, CYLCONE, has been written to solve the integral equation (5) for $f(r)$. The intensity, $G(x)$, represents the radiation measurements from a conical volume of the plasma taken by an external probe as the probe sweeps across the cylinder (see Fig. 2).

Fig. 2 Conical Volume Probe Sweeping Cylinder

The numerical technique consisted of dividing the cylinder into n zones as shown in Fig. 2 and then finding the volume of each zone inside the cone. By assuming that $f(r)$ is constant across any single zone, the integral equation (5) may be written numerically as:

$$G_i = \sum_{j=1}^n v_{ij} \cdot f(r_j), \quad (6)$$

where i designates the position, x , of the cone, j designates the zones of the cylinder, and v_{ij} designates the volume coefficients. The set of n simultaneous equations (6) may then be solved for the radial functions, $f(r_j)$.

The program allows the height, h , of the cone's apex to change as the cone sweeps across the cylinder. To insure that the volume coefficients, v_{ij} , were correct, the individual zonal volumes were summed up for each position of the probe and compared with the known volume of the conical probe. Since the volume coefficients were found to be correct, it was not necessary to calculate $G(x)$ values from a known $f(r)$ for testing purposes as was done in the planar system. This would have been extremely difficult if not impossible.

RESULTS:

Upon comparing the summed zonal volumes with the known conical volume, differences on the order of 10^{-11} were obtained, thus indicating that the zonal volume coefficients were correct. By taking some $G(x)$ values calculated in the planar program, CIRWEDGE, we were able to calculate $f(r_j)$'s which had the same general appearance as did the known $f(r_j)$'s in CIRWEDGE. If the $G(x)$ values had been calculated for the spatial system, then the $f(r_j)$'s would have been completely correct. Some sample output is given on pages 15-19.

CYCLICAL SPACE PROBE DATA TRANSFORMATION BY RCD

PARAMETERS

RADIUS = 1.00000000
 WEDGE ANGLE = 3.00000000 (DEGREES)
 X INCREMENT = .05000000
 NO. OF ZONES = 20
 NZ = 21

I	X VALUE	HEIGHT(X)	INTEGRAL = G(X)
1	.0000000	.8000000	1.5271000--02
2	.0500000	.8000000	1.5141000--02
3	.1000000	.8000000	1.4753000--02
4	.1500000	.8000000	1.4120000--02
5	.2000000	.8000000	1.3261000--02
6	.2500000	.8000000	1.2204000--02
7	.3000000	.8000000	1.0985000--02
8	.3500000	.8000000	9.6454000--03
9	.4000000	.8000000	8.2370000--03
10	.4500000	.8000000	6.8187000--03
11	.5000000	.8000000	5.4571000--03
12	.5500000	.8000000	4.2175000--03
13	.6000000	.8000000	3.1303000--03
14	.6500000	.8000000	2.2099000--03
15	.7000000	.8000000	1.4629000--03
16	.7500000	.8000000	8.8738000--04
17	.8000000	.8000000	4.7595000--04
18	.8500000	.8000000	2.0939000--04
19	.9000000	.8000000	6.4965000--05
20	.9500000	.8000000	8.5076000--06
21	1.0000000	.8000000	7.0155000--11

VOLUME COEFFICIENTS

EQ. CAL.	1	2	3	4	5	6	7	8	9	10	11
1	1.3489-04	1.4063-04	1.4172-04	1.4469-04	1.4890-04	1.5423-04	1.6066-04	1.6818-04	1.7578-04	1.8547-04	1.9723-04
12	2.0907-04	2.2199-04	2.3598-04	2.5105-04	2.6721-04	2.8444-04	3.0274-04	3.2212-04	3.4258-04		
	1	2	3	4	5	6	7	8	9	10	11
2	1.2358-05	1.8492-04	1.5536-04	1.5067-04	1.5224-04	1.5633-04	1.6207-04	1.6916-04	1.7745-04	1.8591-04	1.9759-04
12	2.4933-04	2.2216-04	2.3610-04	2.5112-04	2.6723-04	2.8442-04	3.0270-04	3.2206-04	3.4249-04		
	1	2	3	4	5	6	7	8	9	10	11
3	1.7819-05	2.2939-04	1.7595-04	1.6430-04	1.6342-04	1.6667-04	1.7230-04	1.7971-04	1.8953-04	1.9872-04	
12	2.1012-04	2.2271-04	2.3644-04	2.5131-04	2.6729-04	2.8438-04	3.0257-04	3.2186-04	3.4223-04		
	1	2	3	4	5	6	7	8	9	10	11
4	1.9799-05	2.6748-04	1.9618-04	1.7899-04	1.7589-04	1.7833-04	1.8382-04	1.9143-04	2.0077-04		
12	2.1156-04	2.2359-04	2.3707-04	2.5166-04	2.6742-04	2.8433-04	3.0237-04	3.2153-04	3.418-04		
	1	2	3	4	5	6	7	8	9	10	11
5	1.1454-04	3.0122-04	2.1558-04	1.9369-04	1.8893-04	1.9175-04	1.9513-04	2.0404-04			
12	2.1343-04	2.2523-04	2.3807-04	2.5224-04	2.6767-04	2.8431-04	3.0213-04	3.2111-04	3.4122-04		
	1	2	3	4	5	6	7	8	9	10	11
6	1.2920-04	3.3132-04	2.5422-04	2.0848-04	2.0231-04	2.0365-04	2.0907-04				
12	2.1226-04	2.2755-04	2.3958-04	2.5314-04	2.6802-04	2.8435-04	3.0187-04	3.2060-04	3.405-04		
	1	2	3	4	5	6	7	8	9	10	11
7	1.4256-04	3.6019-04	2.5220-04	2.2221-04	2.1581-04	2.1689-04					
12	2.2245-04	2.3101-04	2.4183-04	2.5451-04	2.6879-04	2.8454-04	3.0165-04	3.2004-04	3.3568-04		
	1	2	3	4	5	6	7	8	9	10	11
8	1.5495-04	3.8668-04	2.6565-04	2.3786-04	2.2962-04						
12	2.3045-04	2.3620-04	2.4519-04	2.5659-04	2.6994-04	2.8499-04	3.0154-04	3.1950-04	3.3875-04		
	1	2	3	4	5	6	7	8	9	10	11

I	X VALUE	TRUE CONE VOLUME	SUMMED CONE VOLUME	DIFFERENCE
1	0	4.19350525e-03	4.19350521e-03	3.97903932e-11
2	.0500000	4.19350525e-03	4.19350547e-03	-1.22554411e-09
3	.1000000	4.19350525e-03	4.19350575e-03	-5.09066476e-10
4	.1500000	4.19350525e-03	4.19350549e-03	7.60223884e-10
5	.2000000	4.19350525e-03	4.19350521e-03	3.53566065e-11
6	.2500000	4.19350525e-03	4.19350585e-03	-6.02653927e-10
7	.3000000	4.19350525e-03	4.19350476e-03	4.89421836e-10
8	.3500000	4.19350525e-03	4.19350522e-03	2.54658516e-11
9	.4000000	4.19350525e-03	4.19350481e-03	4.38490133e-10
10	.4500000	4.19350525e-03	4.19350670e-03	-1.45644208e-09
11	.5000000	4.19350525e-03	4.19350622e-03	-9.69862413e-10
12	.5500000	4.19350525e-03	4.19350492e-03	3.21666427e-10
13	.6000000	4.19350525e-03	4.19350520e-03	4.80895324e-11
14	.6500000	4.19350525e-03	4.19350520e-03	4.69526640e-11
15	.7000000	4.19350525e-03	4.19350541e-03	-1.66210157e-10
16	.7500000	4.19350525e-03	4.19350503e-03	2.01566763e-10
17	.8000000	4.19350525e-03	4.19350521e-03	3.88808995e-11
18	.8500000	4.19350525e-03	4.19350571e-03	-4.62480272e-10
19	.9000000	4.19350525e-03	4.19350455e-03	6.95763447e-10
20	.9500000	4.19350525e-03	4.19350511e-03	1.33468347e-10
21	1.0000000	4.19350525e-03	4.19350525e-03	0

INVERSE VOLUME COEFFICIENTS

EQ. CAL.	1	2	3	4	5	6	7	8	9	10	11
1	1.0249+04-7.3050+03-1.547-03-5.3823+02-3.0739+02-2.0173+02-1.4418+02-1.0921+02-8.6111+01-7.0017+01-5.7997+01										
12	-4.2070+04-4.1482+03-3.8152+01-3.0236+01-2.7668+01-2.1313+01-2.2548+01-1.2412+01-2.2132+01										
	1	2	3	4	5	6	7	8	9	10	11
2	-3.9452+04-1.0166+04-4.452-03-7.0706+02-3.6920+02-2.2582+02-1.5572+02-1.1548+02-8.9837+01-7.2394+01-5.9594+01										
12	-5.0194+04-4.2299+03-3.8771+01-3.0702+01-2.8049+01-2.1590+01-2.2917+01-1.2557+01-2.2575+01										
	1	2	3	4	5	6	7	8	9	10	11
3	1.9152+03-4.6867+03-7.7922-03-3.2359+03-5.4620+02-2.9599+02-1.8563+02-1.3079+02-9.8811+01-7.7947+01-6.3199+01										
12	-9.2703+01-4.4102+01-3.8126+01-3.1718+01-2.8877+01-2.2189+01-2.3495+01-1.2869+01-2.2889+01										
	1	2	3	4	5	6	7	8	9	10	11
4	1.0759+02-2.3106+03-3.6416+03-6.6261+03-2.7482+03-4.5775+02-2.5297+02-1.6074+02-1.1452+02-8.7264+01-6.9211+01										
12	-5.6751+01-4.6984+01-4.0261+01-3.3302+01-3.0155+01-2.3111+01-2.4377+01-1.3345+01-2.3571+01										
	1	2	3	4	5	6	7	8	9	10	11
5	4.3882+02-1.1820+03-1.5652+03-3.3397+03-5.8722+03-2.4339+03-4.0305+02-2.2494+02-1.4390+02-1.0321+02-7.8902+01										
12	-6.3111+01-5.1332+01-4.3419+01-3.5612+01-4.1199+01-2.4428+01-2.5524+01-1.4016+01-2.4765+01										
	1	2	3	4	5	6	7	8	9	10	11
6	-2.4054+02-6.1968+02-1.0303+03-1.7771+03-3.0785+03-5.3364+03-2.2209+03-3.6310+02-2.0487+02-1.3173+02-9.4685+01										
12	-7.2833+01-5.7750+01-4.7937+01-3.8846+01-3.4521+01-2.6218+01-2.7297+01-1.4915+01-2.5212+01										
	1	2	3	4	5	6	7	8	9	10	11
7	1.2842+02-3.3053+02-5.5003+02-9.4872+02-1.6435+03-2.8489+03-4.9306+03-2.0577+03-3.3406+02-1.8981+02-1.2219+02										
12	-8.5337+01-6.7388+01-5.4435+01-4.3363+01-3.7961+01-2.8622+01-2.9500+01-1.6095+01-2.8081+01										
	1	2	3	4	5	6	7	8	9	10	11
8	-6.9557+01-1.7919+02-2.2793+02-5.1387+02-8.9022+02-1.5431+03-2.6707+03-4.6101+03-1.9297+03-3.1153+02-1.7789+02										
12	-1.1500+02-9.2537+01-6.4025+01-4.9757+01-4.2663+01-3.1845+01-3.2382+01-1.7632+01-3.0457+01										
	1	2	3	4	5	6	7	8	9	10	11

I	R VALUE	CALC. F(R)
1	.0250000	1.00953887.e+01
2	.0750000	2.07918936.e+00
3	.1250000	2.70938832.e+00
4	.1750000	2.62946454.e+00
5	.2250000	2.01993966.e+00
6	.2750000	3.20014007.e+00
7	.3250000	2.47824009.e+00
8	.3750000	2.45795039.e+00
9	.4250000	1.76545155.e+00
10	.4750000	1.02458429.e+00
11	.5250000	6.00292447.e-01
12	.5750000	3.36055186.e-01
13	.6250000	2.01596681.e-01
14	.6750000	1.07575552.e-01
15	.7250000	1.13793075.e-01
16	.7750000	2.07958246.e-01
17	.8250000	6.23482926.e-01
18	.8750000	1.62533291.e+01
19	.9250000	3.03015916.e+02
20	.9750000	-2.20593695.e+33

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