Lawrence Radiation Laboratory
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LIVERMORE

UCRL-50977

**COMPARISON OF CALCULATED AND MEASURED RESPONSE
OF A HIGH-RISE BUILDING TO GROUND MOTIONS
PRODUCED BY UNDERGROUND NUCLEAR DETONATIONS**

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MS. date: December 23, 1970

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Abstract

The dynamic response of the Bank of Nevada building in Las Vegas to ground motions resulting from several underground nuclear detonations at the Nevada Test Site is investigated. The events chosen were Handley, Benham, Jorum, and Boxcar.

Both a time-dependent analysis, using the modal superposition method, and a spectral response analysis are presented. A brief outline of both methods is included. The ground motion was measured at the base of the building and at NOAA-NOS seismic stations designated as SE6 and Squires Park. Measured time-history records were used as the exciting motion inputs for the analyses. The peak response of the roof of the building was also measured for each event. In addition, for the Handley Event the time-history response was measured at various levels of the building.

A comparison was made between the calculated and measured peak responses and between time-history responses.

A study was made of the sensitivity of the response calculations to three factors: the mathematical model of the building, the input ground motions, and the value of viscous damping. The results indicate that (1) reasonable modeling assumptions can lead to models whose calculated response is quite different from the measured response; (2) when measured ground motions as near as 800 m from the building are used for the exciting motion, the calculated and measured response values are different; and (3) when a posteriori information of the building (e.g., mode shapes, fundamental periods, and damping) is used, it is possible to develop a model whose calculated response agrees very well with the recorded time-history response of the building.

Introduction

When planning a nuclear event it is essential that good understanding be developed of the response of buildings and other structures to expected ground motions, and of the possibility of damage to such structures. It is conceivable that for future nuclear detonations associated with the Plowshare program the response of surrounding structures and property will be a controlling factor, not only for safety considerations but in the overall planning and feasibility determination of the project.

In an effort to develop this understanding, a study was undertaken to investigate the dynamic response of the Bank of Nevada (B of N) building in Las Vegas to ground motions as the result of several Nevada Test Site (NTS) events. We present here some of the results obtained for the response of the building in its longitudinal direction. The Bank of Nevada building was chosen because measured building response data were available for the Handley Event, enabling us to make a comparison between analytical and measured

time-history response. Peak response data were available for all the events for the B of N building.

The events chosen were Handley, Benham, Jorum, and Boxcar. These events were considered because of the similarities of their yields, depths of burial, and geological configurations. Also, ground motion data existed from at least one common measuring station for these four events.

Most of the studies reported in the literature on the response of high-rise buildings to seismic-type ground motions have been theoretical. To obtain results from such theory in practice is quite difficult for two reasons. First, insufficient data exist regarding such dynamic properties of buildings as natural periods, damping, material properties, etc. Second, there is a lack of measured input ground motion and/or response data for buildings with which to calculate the building's response or to compare with the calculated response. Hence, with the exception of the work of Blume^{1,2} and Hudson,³

theoretical studies usually considered a hypothetical model of some structure, or an actual model in conjunction with an exciting ground motion generated from a past earthquake record.

The degree of confidence that one can associate with any response calculation depends on how well the ground motion is defined and also on the development of a reliable mathematical model of the building. The development of such a model is quite difficult and requires much engineering experience.

In this study, the ground motion measured at the base of the B of N building for the Handley Event is considered as reliable input exciting motion for response calculations. In addition, because the time-history response of the building was also measured for the Handley Event, it was possible to develop a model (L6: see section on building models) of the building such that its calculated response agrees very well with the measured response. Since both a reliable input exciting motion and a reliable model of the B of N building exist, more meaning can be attached to the results of this study.

Current practice in analyzing the response of structures to earthquake-like ground motions is centered about two approaches. One is to use a time-history record of the ground motion with a mathematical model of the structure to determine a time-history response of the model. For a particular model, this analysis will yield what may be termed the "exact" solution. The other approach employs spectral analysis techniques to obtain the peak response of the structure. Should the spectral approach be applied to a structure assumed to be a simple one-degree oscillator, it would also yield the "exact" peak response for an assumed value of viscous damping. However, for a structure such as a high-rise building which cannot usually be represented as a single-degree oscillator, it is necessary to make assumptions regarding modal combination, and the analysis becomes approximate. On the other hand, should the high-rise building vibrate in its fundamental mode (or at least in a combination of only a few of its lowest modes) the effect of these assumptions regarding modal combinations would be minimized. The spectral approach is much simpler to apply but reveals less detailed information regarding the response of the structure than does a time-dependent analysis.

In this study both approaches were employed; a brief outline of each method is presented. Our purpose in making both types of analysis was to compare the accuracy of the spectral method with that of the modal superposition time-history approach.

In order to give some insight into the sensitivity of the response calculations to parametric variations, results are included which were obtained when using (1) various mathematical models of the building, (2) different exciting ground motions and (3) various assumed values of percent critical viscous damping. Since only one building was studied, the resulting range of variation of the different response parameters has little quantitative meaning; however, due to the good agreement between the calculated and measured responses, the range of behavior gives a good qualitative insight for other buildings and other possible exciting motions.

It should be pointed out that, although the same techniques used to analyze the response of structures to seismic motions are also used when designing them, the overall philosophy is often not the same. Because of the many uncertainties involved in design, conservative assumptions are often justified. However, when trying to predict the response of the structure to ascertain the possible onset of damage, such conservative assumptions often lead to unjustified predictions of damage. It is therefore very important that all the response parameters used be as realistic as possible.

In addition, a strength analysis requires detailed knowledge of the loading and material properties of the individual structural members. The material properties include allowable stresses as well as the elastic constants and effective geometry required for the response analysis. The actual values of these quantities are often not even deterministic. For example, reasonable estimates of Young's modulus for concrete might vary from 10^6 psi to 6×10^6 psi; an allowable stress for concrete in tension might vary from 0 to 300 psi; the effective cross-section properties used for stiffness calculations could vary by a factor of two or three.

Although, generally, neither the response calculation nor the strength calculation is directly proportional to such variations, the results are greatly influenced, particularly the strength analysis results. Again, much engineering experience and judgment are necessary when selecting values for such quantities.

Finally, should the predicted ground motion be of such a magnitude that the overall behavior of the structure ceases to act linearly elastic, then any realistic response predictions become extremely difficult. In fact, only conservative analysis can be made to ensure that no catastrophic failure occurs.

The objectives of this paper can be summarized as follows:

- (1) To demonstrate that the elastic spectral approach and a time-dependent approach

are capable of predicting the dynamic response of structures to seismic motion.

- (2) To indicate the advantages and the disadvantages of the two approaches.
- (3) To indicate the sensitivity of the calculated response to various mathematical models of

the building, different exciting ground motions, and various values of damping.

- (4) To give some insight as to the precision with which the onset of possible damage can be ascertained from the response calculations.

Building Description

The Bank of Nevada is a 10-story building located in downtown Las Vegas, approximately 165 km from

Area 20 of the Nevada Test Site (NTS). A schematic of the building is shown in Fig. 1. Figure 2 shows the

Fig. 1. Schematic of the Bank of Nevada building.

Fig. 2. Maps of NTS and vicinity of Las Vegas.

location of the building relative to NTS; also shown are the National Oceanic and Atmospheric Administration National Ocean Survey (NOAA-NOS) recording stations pertinent to this study (formerly U.S. Coast and Geodetic Survey).

The upper six floors are 69 ft X 138 ft in plan. A parking garage extends the lower four levels to 124 ft X 138 ft. Figures 3 and 4 show framing plans of a typical upper floor and of the third floor. Three steel frames are located in planes B, C, and D of Fig. 1.

Fig. 3. Typical floor plan above the fifth floor of B of N building.

Fig. 5. Rigid frame in plane B (see Fig. 1).

These frames which extend 125 ft above the surrounding grade, consist of steel wide-flange

columns and girders. The building's resistance to seismic and wind forces in the longitudinal direction

rests primarily with these three steel frames. Reinforced concrete slabs (4½ in. thick) at each floor provide a sufficiently rigid horizontal diaphragm to allow us to assume that the three frames displace identically to longitudinal motions. An elevation view of the frame in plane B is shown in Fig. 5. An additional steel frame with reinforced concrete block is located in plane A.

Two reinforced concrete walls (8 in. thick) extend from the roof to the ground level in planes 1 and 6. In addition, walls of reinforced concrete and reinforced concrete block are located along these planes in the lower four levels. In the transverse direction seismic and wind forces are resisted primarily by the two shear walls located in planes 1 and 6.

Building Models

Several mathematical models of the B of N building were developed. All the models are considered to represent the building reasonably well. Their differences reflect the various assumptions made about the structural behavior of the building. These assumptions in turn reflect any known information about past dynamic behavior of the building. For example, if no information were available regarding the response of the building to past ground motions or even to induced motions, then any of the models would be considered reasonable. But if information were available which established, say, the fundamental period of the building, then a good building model should reflect this information.

Before proceeding to a discussion of the different models, a few comments are in order regarding the general dynamic behavior of the building in the longitudinal direction. Above the fifth level the lateral load-carrying capacity rests with three structural-steel frames in planes B, C, and D of Fig. 1. These three frames are assumed to have rigid column-girder connections. That is, full moment transfer is possible between the columns and girders. This assumption might be questionable with regard to the frame in plane C. Below the fifth level the building's longitudinal dynamic behavior is determined not only by these three frames (which extend to ground level), but also by the rigidity introduced by the parking garage. Measurements of building motion during the Handley Event indicate that the fourth and ground levels displaced longitudinally together. This implies that the parking garage introduces a substantial increase in rigidity in these lower levels. Should this be true, lateral shear would be transferred to the soil beneath the building primarily through the concrete-block wall in plane A. Otherwise, the lateral shear would be transferred to the soil through eighteen spread footings (one at the base of each of the frame columns).

Figure 6 illustrates the basic differences among five of the models investigated. The models are designated as L1, L2, L4, L5, and L6 and have fundamental periods of 1.90, 1.58, 1.12, 1.72, and 1.46 sec, respectively. The first three mode shapes are shown for each model. All the models are assumed fixed at ground level. The effect of soil-structure interaction is not included in the study.

For models L1 and L5 the actual stiffness values were assigned to all the columns and girders. These values were calculated from the drawings of the building. The girder stiffness included the structural steel, the 4½-in. reinforced concrete slab at each floor, and any concrete fireproofing. In model L1 a 3-ft portion of the slab was used to derive the girder stiffnesses, whereas in model L5 all of the slab was used. The exterior column stiffness for both models included all of the 8-in. reinforced-concrete walls. The interior column stiffnesses included the structural steel and the concrete fireproofing.

Models L2 and L6 reflect the possibility of greater stiffness in the first four levels by assigning infinite stiffness to all the columns and girders below the fifth floor. Otherwise, these models are identical to models L1 and L5, respectively.

In model L4 the girders were assigned infinite moments of inertia and the columns infinite areas for axial loads. The flexural and shear properties of the columns were in accordance with the active properties. Model L4, therefore, represents a building deforming laterally in shear. Such a building is commonly referred to in the literature as a "shear building." Although the actual properties of the columns and girders will not allow this building to respond as a "shear building." Although the actual properties of the columns and girders will not allow this building to respond as a "shear building," such a model is included for the sake of comparison.

The total seismic weight of the building (13,080 kips) was calculated according to the actual drawings

Fig. 6. Five of the building models used for the study, designated L1, L2, L4, L5, and L6. The first three mode shapes for each model are shown. The lowest natural period is shown in parentheses; also listed are modal participation factors for the first three modes.

Fig. 7. Geometry, weight distribution, and stiffness properties of columns and girders used in model L6. Numbers in parentheses under floor levels are weights, in kips, of that floor. Underlined numbers are areas, units of ft^2 ; non-underlined numbers are second moments of area, unit of ft^4 .

of the building. The weights of the individual stories shown were assumed concentrated at the roof and floor levels. The same weight distribution was used for all models.

The geometry, weight distribution, and stiffness properties of the columns and girders used for model L6 are shown in Fig. 7. These stiffness properties are the sums of those calculated for frames in planes B, C, and D.

Measured Motion

Two types of measuring instruments were used in this study: strong-motion instruments and velocity meters. The strong-motion instruments included both a Carder displacement meter and a NOAA-NOS accelerometer (which measured and recorded three components of accelerations and displacements). The

velocity meters employed were either NGC-21 or L7 (earlier models were called SCATT). These designations refer to National Geophysical Company and Precision Instrument Company velocity meters, respectively.^{4,5} The strong-motion measurements were used only to obtain peak particle accelerations

and displacements. The measured velocities were recorded on analog tapes which were then processed⁶ by Environmental Research Corporation (ERC) to yield a digital time-history of the velocity, acceleration, and displacement.

tion, and displacement.

NOAA-NOS provided and operated all the instrumentation used for motion measurement in this study (under contract to AEC-NVO).

GROUND MOTION

The ground-motion instrumentation was placed at several NOAA-NOS seismic stations in Las Vegas (see Fig. 2). We felt that ground-motion measurements at Squires Park, at SE-6, and at the base of the B of N building were the most applicable to this study.

Velocity meters were used at all three stations. However, the strong-motion instruments were located only at station SE-6 and at the base of the B of N building. Table 1 summarizes the peak ground motions at SE-6 and Squires Park; the peak measured ground motion at the base of the B of N building is shown in Table 2. The S63° E component shown in these tables coincides with the longitudinal axis of the B of N building, and the N27° E component coincides with the building's transverse direction. (Note: the peak particle ground motions shown are not presented to indicate their usefulness as an index of building response, but rather to show the type of variation of ground motion that can be expected from different events.)

For all the results given in this paper the exciting ground motion took the form of a digital time-history of acceleration. This time-history data for the Jorum Event were derived by ERC. For the Handley, Benham, and Boxcar Events the accelerations were obtained by numerical differentiation (by the authors) of the digital velocity data supplied by ERC.

The original velocity data were recorded every 0.00625 sec. The number of data points ranged from about 23,000 to 45,000 (yielding a record of approximately 143 to 280 sec). In order to avoid having to revise our dynamic computer program to handle such a large number of points, the number of points was reduced to about 2500. This was accomplished in two ways. First, instead of using every point of the record, we selected every tenth point. Second, we used only that portion of record in which the data indicated maximum ground motion.

Figures 8 and 9 show time-history plots of the original data used as ground motion input. The zero times on these figures are arbitrary and are not

Table 1. Peak measured ground motion at SE6 and Squires Park stations.

Event	Direction	Station SE6			Direction	Station Squires Park		
		Displacement (cm)	Velocity (cm/sec)	Acceleration ($10^{-2}g$)		Displacement (cm)	Velocity (cm/sec)	Acceleration ($10^{-2}g$)
Handley	N-S	0.29	0.60	0.62	N27°E	—	1.62	—
	E-W	—	0.51	0.54	S63°E	—	1.62	—
Benham	N-S	0.30	1.09	0.47	N27°E	—	1.38	—
	E-W	0.56	0.86	0.60	S63°E	—	2.18	—
Jorum	N-S	0.31	0.71	0.42	N27°E	—	0.92	—
	E-W	0.24	0.57	0.37	S63°E	—	1.26	—
Boxcar	N-S	0.30	1.20	0.39	N27°E	—	1.06	—
	E-W	0.26	0.72	0.40	S63°E	—	1.35	—

Table 2. Peak measured response of B of N building.

Event	Direction	Displacement (cm)	Velocity (cm/sec)	Acceleration ($10^{-2}g$)	Location of instrument
Handley	S63°E	0.90	3.87	1.80	Roof
	S63°E	—	2.62	—	8th floor
	S63°E	—	1.53	—	5th floor
	S63°E	—	1.43	—	4th floor
	S63°E	0.497	1.24/1.55 ^a	0.50	Ground
	N77°E	0.579	1.10	0.45	Ground
Beakham	S63°E	0.85	—	1.50	Roof
Jorum	S63°E	0.83	—	1.20	Roof
Boxcar	S63°E	0.92	—	1.40	Roof

^aTwo velocity meters at this location.

related to each other or to travel time for the first arrivals of the stress wave. The figures show the appreciable variation in ground motion from station to station and for the different events. The events

were of similar yield and were all located in Area 20 of NTS. Variations of this sort are common and indicate the difficulty in obtaining reliable input ground motion.

BUILDING MOTION

Although in many cases more than one component of the motion was recorded at the different stations throughout the building, only that component which coincides with the building's longitudinal axis (S63°E) is included.

A strong-motion instrument was located on the roof of the B of N building for all four events and, as mentioned earlier, also at ground level for the Handley Event. Velocity meters were located on the roof, eighth, fifth, and fourth floors, and at ground level for the Handley Event. All these instruments were located approximately along a vertical centerline, the intersection of planes 4 and C of Fig. 1. The

peak measured values recorded from these instruments are shown in Table 2. An inspection of the velocity records shows that these peak velocity values occur at approximately the same time.

From these (analog tape) velocity records, acceleration and displacement time-history digital records were provided by ERC. Unfortunately, the acceleration records appeared questionable, and we felt that more work was needed to check their authenticity. Therefore, only absolute displacement time-history records of the building's longitudinal motion are included. Plots of these displacement records are shown for the various levels in Figs. 10 through 13.

Fig. 8. Ground acceleration vs time for the Handley Event (S63°E component), measured at (a) SE6, (b) Squires Park, (c) base of the building. Derived from measured velocity record.

Fig. 9. Ground acceleration vs time at Squires Park (S63°E component) for events (a) Benham, (b) Jorum, and (c) Boxcar. Derived from measured velocity records.

Fig. 10. Displacement vs time, roof, Handley Event. Derived from measured velocity record.

Fig. 11. Displacement vs time, eighth floor, Handley Event. Derived from measured velocity record.

Fig. 12. Displacement vs time, fifth floor, Handley Event. Derived from measured velocity record.

Fig. 13. Displacement vs time, ground level, Handley Event. Derived from measured velocity record.

Time-Dependent Analysis

The time-history response of the building to the different recorded ground motions was obtained by using the modal superposition method on a lumped-mass representation of the building. A lumped-mass model is obtained by lumping the building's inertial properties at the floor levels and replacing the structural members with massless beam-like elements having equivalent stiffness characteristics. Sixty lumped masses were used, each having three degrees of freedom: two translational and one rotational. The masses were located at the intersection of the columns and girders. For more details refer to the section on Building Models.

As stated earlier, this study is concerned with the response of the B of N building only in the longitudinal direction. Transverse building motions are assumed uncoupled from longitudinal motions, and therefore the transverse component of the ground motion is not included. Furthermore, the vertical component of the ground motion introduces negligible longitudinal building motion. Hence, only the longitudinal component of the ground motion is used as the exciting input. The building is assumed fixed at ground level and only a viscous-type damping is employed.

Although the procedures used to calculate the time-dependent response of the building are well documented throughout the literature,⁷⁻⁹ a brief discussion is given in order to make this report self-contained.

Once the lumped-mass model of the building has been established, the equations of motion can be easily derived, given arbitrary ground motion. First, consider a system with 1 single degree of freedom as shown in Fig. 14. When the excitation of the mass is due only to base motion, it is clear that the equation of motion can be written as

$$m\ddot{x} + c(\dot{x} - \dot{g}) + k(x - g) = 0, \quad (1)$$

or

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{g}, \quad (2)$$

where

x = absolute displacement,
 $u = x - g$, the relative displacement,
 g = base motion,

Fig. 14. System with one degree of freedom. K — linear spring constant, m — mass of system, c — constant, relating damping force to velocity of mass, $x(t)$ — absolute displacement of mass, $g(t)$ — absolute displacement of floor (or ground).

and the dots indicate derivatives with respect to time. The extension to the system with many degrees of freedom yields the following equation of motion:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -\ddot{g}(t)[M], \quad (3)$$

where

$[M]$ and $\{M\}$ = diagonal and column mass matrices, respectively,
 $[C]$ = viscous damping matrix,
 $[K]$ = stiffness matrix,
 $\{u\}$ = relative displacement matrix (includes rotations),

and

$\ddot{g}(t)$ = input ground acceleration.

It can be seen by inspecting Eqs. (1) and (2) that it is advantageous to use relative displacements and to express the input ground motion in terms of acceleration.

Instead of directly integrating all the equations that comprise the matrix system (3) (in the more general case we considered this comprised 180 equations) we simplify the problem by transforming from physical coordinates to a new coordinate system in which the equations become uncoupled into the modal equations of the structure. (For many structures only the lower modes are of any importance in the response of the structure, so only the first few modes need to be considered.)

The key to the transformation are the eigenvalues and eigenvectors obtained from the solution of the equation

$$([K] - \omega_i^2 [M]) \{\phi_i\} = \{0\}, \quad (4)$$

where ω_i is the eigenvalue, or undamped natural frequency, of the i th mode and $\{\phi_i\}$ is the eigenvector of the i th mode. Physically, $\{\phi_i\}$ is the modal deformation of the j th mass in the i th mode. The undamped natural frequencies ω_i are important in the establishment of a reliable model.

New modal coordinates, η , are introduced by the transformation

$$\{u\} = \{\phi\} \{\eta\}, \quad (5)$$

where $\{\phi\}$ is a matrix whose columns are the eigenvectors of Eq. (4).

Introducing the new coordinates into Eq. (3) and premultiplying both sides by $\{\phi\}^T$, we obtain

$$\begin{aligned} \{\phi\}^T [M] \{\phi\} \{\ddot{\eta}\} + \{\phi\}^T [C] \{\phi\} \{\dot{\eta}\} \\ + \{\phi\}^T [K] \{\phi\} \{\eta\} = -\ddot{x}(t) \{\phi\}^T \{M\} \end{aligned} \quad (6)$$

It is well known from matrix theory that the eigenvectors of Eq. (4) are orthogonal relative to the weighting factor $[M]$ so that

$$\{\phi\}^T [M] \{\phi\} = \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & \\ 0 & & R_N \end{bmatrix} = [R], \quad (7)$$

where N = total number of lumped masses

$$\text{and } R_i = \sum_{j=1}^N m_j \phi_{ji}^2.$$

Some assumption must be made relative to the damping in the system. To uncouple the modes requires that $[C] = \alpha \cdot [K]$, where α is a scaling factor. If this is done and Eqs. (4) and (7) are used, then Eq. (6) can be written as

$$\begin{aligned} \{\ddot{\eta}\} + \alpha \begin{bmatrix} \omega_1^2 & & \\ & \omega_2^2 & \\ & & \omega_N^2 \end{bmatrix} \{\eta\} \\ + \begin{bmatrix} \omega_1^2 & & \\ & \omega_2^2 & \\ & & \omega_N^2 \end{bmatrix} \{\eta\} = -\ddot{x}(t) [R]^{-1} \{\phi\}^T \{M\}. \end{aligned} \quad (8)$$

The damping indicated in Eq. (8) is not of the usual form. The more standard notation is

$$\begin{aligned} \{\ddot{\eta}\} + \begin{bmatrix} 2\beta_1 \omega_1 & & \\ & 2\beta_2 \omega_2 & \\ & & 2\beta_N \omega_N \end{bmatrix} \{\dot{\eta}\} \\ + \begin{bmatrix} \omega_1^2 & & \\ & \omega_2^2 & \\ & & \omega_N^2 \end{bmatrix} \{\eta\} = -\ddot{x}(t) [R]^{-1} \{\phi\}^T \{M\}, \end{aligned} \quad (9)$$

where β_i is the ratio of actual damping to critical viscous damping in the i th mode.

We see that Eqs. (9) are uncoupled into the principal modes of the system. The equations of motion in the i th mode can now be written as

$$\ddot{\eta}_i + 2\beta_i \omega_i \dot{\eta}_i + \omega_i^2 \eta_i = -\gamma_i \ddot{x}(t), \quad (10)$$

where

$$\gamma_i = [R]^{-1} [\phi]^T \{M\} = \frac{\sum_{j=1}^N m_j \phi_{ji}}{\sum_{j=1}^N m_j \phi_{ji} \phi_{ji}}$$

The quantity γ is often referred to as the "modal participation factor."

The time-history response of the system in any one mode can be obtained by a method to be discussed later. The time-history response of the j th coordinate for N modes is

$$u_j = \sum_{i=1}^N \phi_{ji} \eta_i \quad (11)$$

All the calculations required to obtain the time-history response were performed by a computer program called TRILIN,¹⁰ which was developed in the Structural Dynamics Group of the Nuclear Test Engineering Division.

Although all the results included used input data in the form of a digital time-history of acceleration, in several cases velocity input was used. To accommodate this change, the computer program was altered so that by using a special difference scheme of integration to solve Eq. (10), a velocity input motion could be used. (This is the procedure used by ERC.⁶) For the cases presented here, no appreciable difference was observed in the calculated response, whether acceleration or velocity was used as input.

Spectral Analysis

RESPONSE SPECTRA

Response spectra were developed for several reasons. First, we needed the spectra to calculate the spectral response of the various building models. Second, we wished to obtain a more quantitative feeling for the variation in measured ground motion by comparing the different spectra. Third, the spectra would illustrate the effect of viscous damping on the dynamic response calculations. And finally, we wanted to compare the spectra we developed with those developed by ERC. This comparison would signal any errors introduced by our reducing the number of original data points from the measured motion records (see section on ground motion).

The name "response spectrum" is used with various meanings in the literature on the dynamic response of structures. In addition, other names, such as "pseudo-relative-velocity spectrum" (PSRV), used by ERC, have been applied to essentially the same concept. Therefore, we include a brief discussion to define these names.

"Response spectrum," as used in this paper, is defined as the maximum response of a single-degree-of-freedom system to a prescribed exciting motion of its support (or ground); this response is plotted against the undamped natural period for various frac-

tional values of critical viscous damping. The response may be a measure of displacement (S_D), velocity (S_V), or acceleration (S_A). This definition is consistent with that used in most structural engineering studies involving earthquake-like exciting motion of the support.

The "response spectrum" as defined can be developed as follows. Consider the system with one degree of freedom shown in Fig. 14. The equilibrium equation for this system can be written as

$$m\ddot{x} + c\dot{x} + kx = c\dot{g} + kg \quad (12)$$

or

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{g}, \quad (13)$$

where $u = x - g$. Either of the equations may be used to determine the response of the mass to ground (or support) motion. The choice will depend on the form of the ground motion. Equation (12) would be used if the time-history description of the ground motion were in terms of displacement and velocity. In this study the response spectra were calculated with a previously developed

computer code called SPONC,¹⁰ which requires input of ground motion in the form of acceleration and, therefore, Eq. (13) was used.

For convenience the damping in the system is expressed as a fraction of critical viscous damping, β , which in terms of the parameters of Fig. 14 is $(2\beta = c/(km))^{1/2}$. The spring-mass characteristics are described by the undamped circular frequency of oscillation $\omega = (k/m)^{1/2}$ or by the undamped period $T = 2\pi (m/k)^{1/2}$. Thus, Eq. (13) can be rewritten as

$$\ddot{u} + 2\omega\beta\dot{u} + \omega^2 u = -\ddot{g} \quad (14)$$

The exact solution of the relative displacement of the mass is obtained by solving Eq. (14). It is

$$u = \frac{-1}{\omega(1-\beta^2)^{1/2}} \int_0^t \ddot{g}(\tau) e^{-\beta\omega(t-\tau)} \sin[\omega(1-\beta^2)^{1/2}(t-\tau)] d\tau \quad (15)$$

Differentiating Eq. (15), we find the relative velocity of the mass to be

$$\dot{u} = -\int_0^t \ddot{g}(\tau) e^{-\beta\omega(t-\tau)} \cos[\omega(1-\beta^2)^{1/2}(t-\tau)] d\tau + \frac{\beta}{(1-\beta^2)^{1/2}} \int_0^t \ddot{g}(\tau) e^{-\beta\omega(t-\tau)} \sin[\omega(1-\beta^2)^{1/2}(t-\tau)] d\tau \quad (16)$$

and a further differentiation yields the absolute acceleration of the mass as

$$\ddot{x} = \frac{\omega(1-\beta^2)}{(1-\beta^2)^{1/2}} \int_0^t \ddot{g}(\tau) e^{-\omega\beta(t-\tau)} \sin[\omega(1-\beta^2)^{1/2}(t-\tau)] d\tau + 2\omega\beta \int_0^t \ddot{g}(\tau) e^{-\omega\beta(t-\tau)} \cos[\omega(1-\beta^2)^{1/2}(t-\tau)] d\tau \quad (17)$$

By definition, the relative displacement, S_D , the relative velocity, S_v , and the absolute acceleration, S_a , are

$$\begin{aligned} S_D &= u^{max} \\ S_v &= \dot{u}^{max} \\ S_a &= \ddot{x}^{max} \end{aligned} \quad (18)$$

Thus, for a prescribed ground acceleration $\ddot{g}(t)$, the relative displacement, relative velocity, and absolute acceleration spectra can be calculated from Eq. (18) by assuming various values of the natural period for some specified percentage value of critical damping.

For the response of structures to earthquake-like excitation, the relative velocity spectrum and the absolute acceleration spectrum are often replaced by "pseudo relative velocity" (PSRV) and "pseudo absolute acceleration" (PSAA) spectra, defined as

$$\text{PSRV} (\approx S_v) \equiv \omega S_D$$

and

$$\text{PSAA} (\approx S_a) \equiv \frac{1}{\omega^2} S_D \quad (19)$$

These spectra can be derived from Eqs. (16) and (17) by first assuming that the damping is small (say, β less than 10%), in which case the quantity $(1-\beta^2)^{1/2}$ may be replaced by one, then by dropping the second term of these equations since they are of the same order of magnitude as β , and finally by replacing the cosine function in Eq. (16) by a sine function.

To evaluate the accuracy of these approximations a comparison of the PSRV and S_v spectra are shown in Fig. 15 for assumed values of 2, 5, and 10% for viscous damping. It is seen that over the period range of 0.4-2.5 sec, and for damping values less than 10%, the approximations introduce no significant error. Therefore, for the present study PSRV and PSAA spectra will be considered as equivalent to S_v and S_a spectra.

Figure 16 shows the response spectra developed from ground motions measured at Squires Park for each of the events considered in this study for an assumed 5% critical viscous damping. In Fig. 17 the response spectra are again plotted along with the spectra developed by ERC. The good agreement indicates that no appreciable error was introduced by the reduction in the number of data points from the ERC ground-motion records.

The response spectra for Handley, generated from ground motions measured at SE6, Squires Park, and at the base of the B of N building, are shown in Fig. 18.

Variation in both the ground motion and the response can be observed in these figures. The effect of the assumed value of viscous damping can be seen both in Fig. 15 and in Fig. 19 where the response

spectrum for Handley was generated from ground motions measured at the base of the B of N building for assumed values at critical viscous damping of 0, 5, and 10%.

Fig. 15. Comparison of Handley S_v and PSRV spectra for values of viscous damping of 2, 5, and 10%. Input ground motions measured at Squires Park (S63°E component).

Fig. 16. Response spectra from each of the four events, critical viscous damping assumed 5%. Input ground motions measured at Squires Park (S63°E component).

SPECTRAL RESPONSE

Spectral response analysis was performed on all five of the models, using the mode shapes and frequencies calculated in the modal analysis.

From Eqs. (10) and (11) the maximum relative displacement, the maximum relative velocity, and the absolute maximum acceleration of the j th mass in the

Fig. 17. Comparison of response spectra plotted in Fig. 16 with spectra computed by ERC.⁶

i th mode of a many-degree-of-freedom system can be expressed as

$$\begin{aligned} u_j^{max} &= \phi_j u_1^{max} \\ \dot{u}_j^{max} &= \phi_j \dot{u}_1^{max} \\ \ddot{u}_j^{max} &= \phi_j (\ddot{u}_1 + \gamma_j \ddot{z})^{max} \end{aligned} \quad (20)$$

By comparing Eqs. (10) and (14) and also using Eqs. (11) and (12), we can express u_1^{max} , $\dot{u}_1^{max}$, and $(\ddot{u}_1 + \gamma_j \ddot{z})^{max}$ as $\gamma_i S_{D,i}$, $\gamma_i S_{V,i}$, and $\gamma_i S_{A,i}$,

respectively. Replacing the corresponding terms in Eq. (20) by their spectral identities results in

$$\begin{aligned} u_j^{max} &= \gamma_i \phi_j S_{D,i} \\ \dot{u}_j^{max} &= \gamma_i \phi_j S_{V,i} \\ \ddot{u}_j^{max} &= \gamma_i \phi_j S_{A,i} \end{aligned} \quad (21)$$

The maximum inertial forces at mass point j in mode " i " is seen to be

$$F_j^{max} = \gamma_i \phi_j m_j S_{A,i} \quad (22)$$

Fig. 18. Response spectra for Handley Event, generated from ground motions measured at SE6, Squires Park, and at the base of the B of N building, 5% damping (S63°E component).

The maximum story shear at the level between M and $M-1$ in mode "T" can be expressed as

$$V_{TM} = \tau_1 S_{a,j} \sum_{j=1}^M m_j \phi_{jM} \quad (23)$$

where M_s is the number of levels. The shear at the base of the system, usually referred to as "base shear," is obtained by setting $M = 1$ in Eq. (23).

The moments in mode "T" can be obtained by the summation of the forces multiplied by the height of

Fig. 19. Response spectra for Handley Event, generated from ground motions measured at the base of the B of N building, for values of damping of 0, 5, and 10% (S63°E component).

each force above the point of interest. For example, the moment at the base of the system is

$$M_{b,i} = \sum_{j=1}^n p_{ij}^{max} h_j$$

where h_j = height of mass j above the base. It is improbable at any instant of time that the spectral values (for example, $S_{D,1}$, $S_{D,2}$, and $S_{D,3}$) will occur simultaneously. Thus the spectral response procedure does not permit the determination through modal combinations, of maximum displacements, velocities, acceleration, forces, shears, or moments. Approximations such as root-mean-square or absolute sum are usually made.

Results

Table 3. Calculated maximum roof response using time-dependent analysis, longitudinal direction.

Event	Station	Model	Damping (%)	Calculated max. roof acceleration ($10^{-2} g$)	Calculated max. roof displacement (cm)
Handley	Sq. Pk.	L6	2	3.7	1.55 ^a
	Sq. Pk.	L6	5	2.3	1.25 ^a
	Sq. Pk.	L6	10	1.5	1.22 ^a
	SE6	L6	5	1.5	0.98 ^a
	B of N	L6	5	1.4	0.96
	B of N	L5	5	2.0	1.40
	B of N	L4	5	2.2	1.62
	B of N	L2	5	1.6	1.22
	B of N	L1	5	2.5	1.90
Benham	Sq. Pk.	L6	5	2.3	1.30 ^a
Jorum	Sq. Pk.	L6	5	1.6	1.10 ^a
Buxton	Sq. Pk.	L5	5	1.4	0.85 ^a

^aIncludes an estimated 0.10 cm for ground displacement.

Tables 3 and 4 summarize the calculated peak roof responses, including both the spectral and the time-dependent results. The spectral values were calculated by using the plots in Figs. 15, 16, and 19 and Eq. (21). Both the root-mean-square approximation and the sum of the absolute modal maxima are shown. Only the first three modes participated effectively in the calculations. As expected, because the building is vibrating primarily in its fundamental mode, the agreement between the spectral and time-dependent peak values is very good.

The measured peak roof responses are shown in Table 3 for all the events considered. A comparison between the measured and calculated values indicates the importance of having a reliable exciting motion and a reliable model; it also shows the variation in the calculated response.

A time-history response was calculated for each of the models developed using the modal superposition method; all the different measured ground motions were used as exciting motion input. Only part of the results are included. Figures 20 through 24 show the calculated time-history response of the absolute displacement of the roof, eighth and fifth floors for each of the five models. The exciting ground motion was the measured motion at the building's base for the

Table 4. Calculated maximum roof response using spectral analysis, longitudinal direction. Handley Event, Squires Park Station (S83°E), 5% damping.

Event	Model	Calculated maximum roof acceleration, $10^{-2} g$		Calculated maximum roof displacement, cm	
		$\sum$	rms	$\sum$	rms
Handley	L1	0.042	0.028	2.68	2.53
	L2	.027	.020	1.20	1.15
	L4	.034	.026	0.89	0.88
	L5	.034	.022	1.56	1.47
	L6	.031	.023	1.21	1.16
Benham	L1	.041	.028	2.73	2.60
	L2	.036	.045	2.29	2.20
	L4	.039	.032	1.10	1.06
	L5	.033	.038	2.70	2.57
	L6	.035	.026	1.40	1.35
Jorum	L1	.029	.019	1.59	1.49
	L2	.026	.019	1.19	1.15
	L4	.025	.020	0.67	0.66
	L5	.028	.019	1.34	1.26
	L6	.028	.021	1.10	1.06
Buxton	L1	.042	.028	2.68	2.53
	L2	.027	.020	1.20	1.15
	L4	.034	.026	0.89	0.88
	L5	.034	.022	1.56	1.47
	L6	0.025	0.018	0.93	0.89

^aIncludes an estimated 0.10 cm for ground displacement.

Handley Event. Five percent of critical viscous damping was assumed. Superimposed on each of the calculated plots is the measured time-history response. Since the exciting motion used in these

Fig. 20. Model L1: calculated and measured response of (a) the roof, (b) eighth and (c) fifth floors, Handley Event, 5% damping.

Fig. 21. Model L2: calculated and measured response of (a) the roof, (b) eighth and (c) fifth floors, Handley Event, 5% damping.

Fig. 22. Model L4: calculated and measured response of (a) the roof, (b) eighth and (c) fifth floors, Handley Event, 5% damping.

Fig. 23. Model L5: calculated and measured response of (a) the roof, (b) eighth and (c) fifth floors, Handley Event, 5% damping.

Fig. 24. Model L6: calculated and measured response of (a) the roof, (b) eighth and (c) fifth floors, Handley Event, 5% damping.

figures is considered to be reliable, the variations in the calculated response spectra are assumed to result from the use of the different models.

We see from these figures that the calculated response of model L6 agrees much more favorably with the measured response than do the other models. This was expected for the following reason. All the models were developed before any measured time-history information from the Handley Event was available. However, we did have peak response data for the roof, eighth, fifth, fourth, and ground levels, and an estimate (1.37 sec) of the fundamental period of the building (see Table 1). Therefore, because the fundamental period of model L6 (1.46 sec) most closely agrees with the building's measured fundamental period and because model L6 better reflects the increased stiffness provided by the parking garage in the lower levels (also indicated by the peak measured response), this model was considered to best represent the building. (Note that in the absence of such information about the building's response before the Handley Event, all decisions regarding its response have to be based on a consideration of all models.)

The time-history relative displacement response at the roof is shown in Figs. 25 and 26 for model L6 for different exciting ground motions. Figure 25 shows the calculated response for several events using the motion at Squires Park as the exciting input. Note that even though these events were similar in yield, geographical location, and depth of burial, considerable variation in the calculated response is evident. Figure 26 considers only the Handley Event; the exciting ground motions were measured at the base of the building, at Squires Park (800 m away), and at SE6 (7 km away). Again considerable variation in the calculated response can be seen even though both the recording stations, Squires Park and SE6, are quite close to the B of N building.

The variation of the calculated response for various assumed values of percent viscous damping can be seen in Table 3. A damping value of 5% seems to give the best agreement with measured data for base-of-building exciting input; however, if Squires Park ground motion is used as input, a damping value of 10% seems more reasonable. Blume¹ has attempted to deduce the damping value for a structure by using very approximate models with Squires Park ground motion input. Our results indicate that this can easily lead to erroneous values of damping. Thus it appears that the use of any model or exciting motion which cannot be considered reliable could lead to an erroneous inference regarding the damping value for a structure.

Fig. 25. Calculated roof response model L6, using ground motion at Squires Park as input. Event was (a) Benham, (b) Jorum, (c) Boxcar.

Fig. 26. Calculated roof response, model L6, for Handley Event. Input was ground motion measured at (a) SE6, (b) Squires Park, (c) base of the building.

Summary and Conclusions

This study has shown that:

- (1) Both the elastic spectral approach and the time-dependent approach can reasonably predict the response of high-rise buildings to seismic motion when both the exciting motion and the model are reliable. Although the application of the time-dependent analysis requires more time and effort than the PSRV method, it reveals much more detailed information regarding the response of the structure. In addition, should the structure be vibrating in more than one of its fundamental modes, the PSRV method will yield conservative estimates of the structure's response.
- (2) The calculated response of the different possible models of the B of N building varied over a wide range for the same exciting motion, and also for different exciting

motions when the same model was used. The effect of damping was also shown to be important.

There are several important points to note:

- (1) All the models were reasonable models of the same building.
- (2) All the ground motions were generated by nuclear explosions of very similar yield and geographic location and yet the character of the time histories and response spectra varied greatly from event to event and locally from station to station.

The results of this study indicate that modeling is extremely important and that unless empirical data exist for comparison, a range of models should be constructed. No less important is the precision with which the ground motion can be predicted for any meaningful response calculation.

Acknowledgments

The authors wish to express their appreciation to V. N. Karpenko, head of Nuclear Test Engineering Division for initially suggesting and supporting this study and to A. B. Miller for many helpful discussions.

In addition, the authors wish to thank John A. Blume and Associates of San Francisco, in particular S. Freeman and K. Honda, for their helpful comments regarding the various building models and also for providing us with the engineering drawings of the building and with ground motion data.

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Printed in USA. Available from the National Technical
Information Center, National Bureau of Standards,
U. S. Department of Commerce, Springfield, Virginia 22151
Price: Printed Copy \$3.00; Microfiche \$0.65.