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SCALING PROPERTIES
IN WEAK AND ELECTROMAGNETIC PROCESSES

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ABSTRACT

The scaling properties in high energy weak and electromagnetic processes are reviewed, and their theoretical basis examined.

In this talk, I shall first review the nature of the scaling property that has been recently discovered in various high energy weak and electromagnetic processes, then examine its theoretical basis, and finally comment on its implications for the future of high energy physics.

1. Scaling Hypothesis

The scaling property is the consequence of the scaling hypothesis which was first suggested¹ by Bjorken and others. Here, we wish to state the scaling hypothesis in a form somewhat different from its original formulation, one that is perhaps more directly related to experimental results, and appears to be symmetric with respect to leptons and hadrons. For definiteness, we consider a purely leptonic or semi-leptonic reaction which can be either a second order electromagnetic process or a first order weak interaction process, e.g.,

$$e^+ + e^- \rightarrow \mu^+ + \mu^-$$

$$e^+ + e^- \rightarrow \text{hadrons}$$

$$\nu_\mu + n \rightarrow \mu^- + \text{hadrons, etc.}$$

Furthermore, for the semi-leptonic reaction we shall always sum over all final hadronic channels. Let $d\sigma$ be the appropriate differential cross-section, which can be, in general, written as

$$d\sigma = f(s, q^2, m_\ell, m_N) \times \begin{cases} \alpha^2 \\ G^2 \end{cases}, \quad (1)$$

where the factors α^2 and G^2 are, respectively, the squares of the fine structure constant and of the Fermi constant, depending on whether the process is electromagnetic or weak,

$$s = (\text{center-of-mass energy})^2 ,$$

q^2 represents the various relevant (4-momentum transfer)², m_ℓ denotes the various lepton masses (m_e or m_μ), and m_N denotes the various hadron masses (which can be either the nucleon mass m_N itself, or the ρ and pion masses, etc.).

The scaling hypothesis states that (i) if s and $|q^2|$ are much larger than m_ℓ^2 then it is a good approximation to set $m_\ell = 0$ in the expression for $d\sigma$ and (ii) if s and $|q^2|$ are much larger than m_N^2 , then it is a good approximation to set $m_N = 0$ in the expression for $d\sigma$, provided that all final hadronic channels are summed over. We emphasize that if one does not sum over different final hadron channels, then there would obviously be cases in which one cannot neglect hadron masses. For example, in $e^+e^- \rightarrow \rho^0$, the physical mass and the width of ρ^0 clearly cannot be neglected. One notes further that even in case (i) for the leptons, though not explicitly stated, it is understood that all different final channels of infrared photons are being summed over; otherwise $d\sigma$ would be zero.

According to the scaling hypothesis, for s and $|q^2|$ larger than a few (GeV)², one may set as a good approximation $m_\ell = m_N = 0$; therefore, (1) becomes simply

$$d\sigma = f(s, q^2) \times \begin{cases} \alpha^2 \\ G^2 \end{cases} . \quad (2)$$

Apart from the coupling constant a^2 or G^2 , the differential cross section $d\sigma$ now depends only on s and the various q^2 . These quantities represent (in the natural units $\hbar = c = 1$) the only physical observables with the dimension $(\text{length})^{-2}$. All the consequences of the scaling hypothesis can then be easily derived by a pure and simple dimensional analysis². The scaling hypothesis means simply the absence of any basic physical energy scale, such as m_e and m_N . As we shall see, this enables us to connect various cross sections at a relatively low energy range to those at a much higher energy range.

2. Applications

(i) To illustrate the use of the scaling hypothesis, we shall first consider the following two electromagnetic processes, one purely leptonic and the other semi-leptonic:

$$e^+ + e^- \rightarrow \mu^+ + \mu^- \quad (3)$$

and

$$e^+ + e^- \rightarrow \text{hadrons} . \quad (4)$$

It follows from the scaling hypothesis that for

$$s = (\text{center-of-mass energy})^2 \gg m_\mu^2$$

one may set $m_e \approx m_\mu = 0$. The total cross section for the purely leptonic reaction (3) depends then only on a^2 and s . From simple dimensional considerations, one sees that

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) \approx \text{constant} \cdot a^2/s .$$

The constant can be evaluated by using quantum electrodynamics, which is consistent with the scaling hypothesis provided that radiative corrections are neglected; one finds then

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

Similarly, according to the scaling hypothesis, if one sums over all final hadronic channels in the semi-leptonic reaction (4), for $s > \text{a few } (\text{GeV})^2$ one may set $m_N = m_\ell = 0$. A simple dimensional analysis leads to

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = \text{constant} \cdot \alpha^2/s$$

where the constant may be determined by a relatively low energy experiment, which then enables one to predict the cross section in a much higher energy region. The present colliding beam results from Frascati³ are in agreement with the predicted s^{-1} dependence.

(ii) Next, we consider the following weak processes:

$$\nu_e + e^\pm \rightarrow \nu_e + e^\pm \quad (5)$$

and

$$\nu_\mu + N \rightarrow \mu^- + \text{hadrons} . \quad (6)$$

Let q^2 denote the (4-momentum transfer)² between the incident neutrino and the target, and s be the (center-of-mass energy)², as before. For the purely leptonic reaction (5), if s and q^2 are $\gg m_e^2$, on account of the scaling hypothesis, one may set $m_e = 0$ in the expression for $d\sigma$. Similarly, for the semi-leptonic reaction

(6), if s and q^2 are greater than a few $(\text{GeV})^2$, one may set $m_\mu = m_N = 0$, provided all hadron channels are summed over. In either case, the differential cross section is proportional to G^2 and the proportionality factor depends only on q^2 and s . Recalling that the dimension of G is $(\text{length})^2$, one finds from simple dimensional considerations that the differential cross sections of both reactions must be of the form

$$\frac{d\sigma}{dq^2} = G^2 \cdot f\left(\frac{q^2}{s}\right) \quad (7)$$

where f is a dimensionless function depending only on the ratio (q^2/s) , which varies from 0 to 1 since $s = q_{\text{max}}^2$, the maximum value of q^2 . The corresponding total cross sections are of the form

$$\sigma = \text{constant} \cdot G^2 s. \quad (8)$$

According to the usual (Current X Current) theory of the weak interaction, one can readily show that

$$\frac{d\sigma}{dq^2} (\nu_e e^- \rightarrow \nu_e e^-) = \frac{G^2}{\pi}$$

and

$$\frac{d\sigma}{dq^2} (\nu_e e^+ \rightarrow \nu_e e^+) = \frac{G^2}{\pi} \left(1 - \frac{q^2}{s}\right)^2$$

which agree with (7). In Figure 1, we reproduce the results from the CERN neutrino experiment⁴, which gives, after averaging over $N = p$ and n ,

$$\sigma(\nu_\mu + N \rightarrow \mu^- + \text{hadrons}) \cong 0.6 \times 10^{-38} (\text{cm}^2/\text{nucleon}) \times [(E_\nu)_{\text{lab}} \text{ in GeV}]$$

in good agreement with (8).

(iii) As a further example, one may consider the following two electromagnetic processes:

$$e^{\pm} + \mu^{\pm} \rightarrow e^{\pm} + \mu^{\pm} \quad (9)$$

and

$$e^{\pm} + p \rightarrow e^{\pm} + \text{hadrons} \quad (10)$$

in which one sums over all final hadron channels, as is done in the "deep" inelastic experiments of SLAC⁵.

In this problem, there are three independent invariant variables: s , q^2 and $p \cdot q$ where s is the center-of-mass-energy squared, q is the virtual photon momentum, and p denotes either the 4-momentum of the initial proton [in (10)] or that of the initial muon [in (9)]. It is customary to introduce a dimensionless variable, called the scaling variable

$$\omega \equiv -2p \cdot q / q^2 .$$

According to the scaling hypothesis, for s and q^2 greater than a few $(\text{GeV})^2$, one may set $m_e = m_N = 0$; therefore, the differential cross section depends only on q^2 , $s = q_{\text{max}}^2$ and ω . From simple dimensional considerations, one deduces that for the deep inelastic ep scattering

$$\frac{d^2\sigma}{dq^2 d\omega} = \frac{\alpha^2}{(q^2)^2} F\left(\frac{q^2}{s}, \omega\right) . \quad (11)$$

For the purely lepton reaction (9), because $e\mu$ scattering is an elastic process, one has $p^2 = (p+q)^2 = -m_{\mu}^2$. Consequently, the scaling variable ω

equals 1 and the corresponding function F is, therefore, proportional to $\delta(u-1)$.

One may write, instead of (11),

$$\frac{d\sigma}{dq^2} (e^\pm \mu^\pm \rightarrow e^\pm \mu^\pm) = \frac{\alpha^2}{(q^2)^2} f\left(\frac{q^2}{s}\right) . \quad (12)$$

The $\frac{q^2}{s}$ dependence in both (11) and (12) can be explicitly evaluated by using quantum electrodynamics, since it involves only lepton variables. One finds (for $m_e = m_\mu = 0$)

$$\frac{d\sigma}{dq^2} (e^\pm \mu^\pm \rightarrow e^\pm \mu^\pm) = \frac{4\pi \alpha^2}{(q^2)^2} \left[1 - \frac{q^2}{s} + \frac{1}{2} \left(\frac{q^2}{s} \right)^2 \right] \quad (13)$$

and

$$\frac{d^2\sigma}{dq^2 du} (e^\pm p \rightarrow e^\pm + \text{hadrons}) = \frac{4\pi \alpha^2}{(q^2)^2} \left[\left(\frac{1}{u} - \frac{q^2}{s} \right) (vW_2) + \left(\frac{q^2}{s} \right)^2 W_1 \right] \quad (14)$$

where W_1 and vW_2 are called structure functions; both are dimensionless and depend on u only. As shown in Figure 2, the validity of the scaling hypothesis has been verified by the recent SLAC data⁵.

3. Theoretical Difficulties

While the statement of the scaling property is simple, it turns out to be rather difficult to find a solid theoretical basis of such a property for the hadrons. This difficulty is connected with the so-called "mass singularities" of local field theories⁶. In the case of quantum electrodynamics, such mass singularities are well known. They are connected with the high degeneracy between all states consisting of any number of pairs and photons moving along the same direction and with the same total momentum. The approximation $m_q = 0$ can only be made in the first Born term; it leads to logarithmic divergences in higher order radiative corrections. Experimentally, the mass singularity is supported by the fact that all zero-mass particles, such as the neutrino, the photon and the graviton, are found to be neutral. Fortunately, in quantum electrodynamics, the coupling constant is small, and therefore the zero mass approximation is a good one provided the energy ν is not extraordinarily high, so that $\alpha \ln \frac{\nu}{m_q}$ is $\ll 1$. For the strong interaction of hadrons, the problem becomes more serious because of the large coupling constant associated with strong interactions. Let me briefly review various previous theoretical attempts and their difficulties:

(i) Parton model

While the original parton idea of Feynman⁷ has important heuristic values, it nevertheless puts a special emphasis on the infinite-momentum frame of reference. It is suggested that in the infinite-momentum frame, the electromagnetic property of the assumed pointlike constituents of the physical nucleon can be treated as that of an assembly of independent free particles. The "infinite momentum frame", by itself,

is clearly not a Lorentz invariant concept. Furthermore, one can easily show⁷ that, in general, the direction of the infinite momentum cannot be arbitrary. It must be limited to a certain restrictive set of directions, depending on the virtual photon momentum; otherwise, the mass of each of the pointlike constituents has to be lighter than that of the physical proton, and that would be too unphysical. Naturally, this leads to questions of whether the parton model, especially of a spin $\frac{1}{2}$ particle such as the physical proton, can be derived from a relativistically invariant theory.

(ii) Perturbation theory

In the literature, there have been several attempts to try to derive the scaling property from the usual relativistic local field theory. So far, the only success has been limited to either the trivial case of free particles (free except for their electromagnetic interaction), or the unphysical case of a super-renormalizable ϕ^3 -type theory⁸ in which all particles must be of zero spin. For the physically interesting case of spin $\frac{1}{2}$ charged particles with some non-electromagnetic interaction, straightforward calculation in perturbative expansions leads to logarithmic deviations from scaling behavior⁹.

(iii) Perturbation theory with cut-offs

Efforts have been made to introduce a transverse momentum cut-off¹⁰ to the perturbation theory. However, the transverse momentum cut-off in the field theoretical derivation of scaling leads to a formalism and a scattering amplitude that are current-conserving only in the infinite momentum frame and in the scaling region. Therefore, it is difficult to see how one may derive such an ad hoc cut-off procedure in a bona fide relativistic and gauge invariant field theory.

(iv) Light cone commutator

A straightforward application of field equations for interacting spin $\frac{1}{2}$ particles leads to a current commutator that is more singular than that for free fields¹¹; therefore, it does not seem to yield the desired scaling properties. Efforts have been made to introduce the so-called "formal manipulation" of current operators¹². However, at present the theoretical foundation of such rules appears to be quite uncertain. In particular, if one identifies such a "formal manipulation" with the usual Feynman regulator with a negative metric, then it is possible to show that while one may regularize the light-cone commutator, it is not possible to regularize the deep inelastic cross section unless such negative-metric particles are indeed produced asymptotically; this would violate unitarity and is certainly unrelated to the SLAC experiments.

Remarks:

We note that in both ep and vp type experiments the magnitudes of the deep inelastic cross sections are comparable to those of the corresponding purely leptonic ones. This strongly suggests that we may characterize the electromagnetic and weak interactions of hadrons, like those of the leptons, by some local interactions. On the other hand, as mentioned before, in a local field theory there is this difficulty of the "mass-singularity". Remembering that for leptons, while the "mass-singularity" becomes important in the infinite energy range, the approximation $m_l = 0$ remains a good one over quite an extensive intermediate energy range

$$m_l < \nu < m_l' \exp(a^{-1}) .$$

For the hadrons, the theoretical difficulties discussed above are all associated with

the mathematical infinite energy limit; only in such a limit are the deep inelastic scaling phenomena determined by the current commutator on the light cone. Comparison with the similar situation for leptons at least raises the question that perhaps for hadrons one should also regard the scaling property to be valid not necessarily in the infinite energy limit, but only in an intermediate energy range, one that includes all presently available machine energies. This view seems all the more reasonable, since at infinite energy and infinite 4-momentum transfer very likely the higher order electromagnetic and weak effects would become comparable in magnitude to the so-called strong interaction effects. In such a case, the observed deep inelastic cross sections are no longer simply related to the appropriate light-cone commutator. Experimentally, the physical "scale" for scaling is known to be only $\sim 0(m_N)$ which is also of the same order of magnitude as the physical scale for the elastic electromagnetic form factors of the nucleon. Thus, to explain the observed scaling phenomena it is certainly not necessary to require the relevant current commutator to have the desired behavior as an operator equation valid on the mathematical light cone. All that is needed is to construct theories in which the zero-mass approximation is a good one for the matrix element of the current commutator after it has been averaged over the physical nucleon state and at values of s and $|q^2|$ that are large but need not be infinite.

As we shall see, it is indeed possible to develop such a bona fide local field theory, provided one regards the physical nucleon as a composite, not represented by a single elementary local field.

4. Bound-State Model

Next, I wish to discuss some recent theoretical progress¹³ made in this direction; this work was done in collaboration with S. D. Drell. Our basic view is to regard the physical proton p as a bound state of some local fields. To illustrate this bound-state concept, let me first discuss a simple model¹⁴.

The model that we shall discuss consists of only three fields: a spin $\frac{1}{2}$ charged field $\psi(x)$, a pseudoscalar neutral meson field $\pi(x)$ and a scalar neutral gluon field $\phi(x)$. [The model can, of course, be easily extended to include charged meson fields.] The interaction Lagrangian density is assumed to be

$$\mathcal{L}_\pi + \mathcal{L}_\phi \quad (15)$$

where

$$\mathcal{L}_\pi = \kappa_0 \pi^2 \bar{\psi} \psi,$$

$$\mathcal{L}_\phi = f_0 \bar{\psi} \gamma_4 \psi \phi,$$

f_0 and κ_0 are unrenormalized coupling constants. Since $\mathcal{L}_\pi$ is a superrenormalizable interaction and $\mathcal{L}_\phi$ is a renormalizable one, the usual renormalization process can be easily carried out. The physical proton p is assumed to be a bound system $(\psi\pi)$ in the $s_{\frac{1}{2}}$ state. Therefore, the field ψ is of opposite parity from p . In order for p to be the lowest baryon state, the renormalized coupling constants f and κ must satisfy

$$\frac{f\kappa}{4\pi} \sim 0(m_\pi) \quad (16)$$

To describe the bound state, one can easily set up the Bethe-Salpeter equation. In the ladder approximation (which will be removed in later discussions), the wave function $\Phi(k)$ is determined by

$$\Phi(k) = \text{diagram} = \text{diagram}$$

where k is the relative momentum between ϕ and π , and $V(q)$ is the covariant potential generated by the gluon field ϕ . Since $V(q) \sim O(\frac{1}{q^2})$ at large momentum transfer q , one can easily establish that

$$\Phi(k) \sim O(\frac{1}{k^2}) \quad \text{as } k^2 \rightarrow \infty. \quad (17)$$

In the simple model in which only ϕ is charged, the standard minimal electromagnetic interaction is

$$i \phi^\dagger \gamma_4 \gamma_\lambda \phi A_\lambda$$

where A_λ denotes the electromagnetic field. In the "parton" language¹, one would then say that the charged constituent (or parton) ϕ has a "point-like" electromagnetic structure. The evaluation of the electromagnetic form factors of the physical nucleon can be readily carried out. By using the diagram

one finds the familiar result¹⁵ that both $F_1(q^2)$ and $F_2(q^2)$, being proportional to the square of the wave function, decrease as q^{-4} (apart from $\ln q^2$ factors) at large q^2 .

In this theory, the field ψ represents the interpolating field of the continuum $(p\pi)$. Because of parity conservation, $\psi \neq p$; ψ is consequently not an interpolating field for any single stable particle. Therefore, in any collision process ψ cannot appear in the asymptotic states. The following are two typical diagrams for

$$e + p \rightarrow e + p + \dots,$$

where the final hadron state " $\dots$ " can consist of any number of mesons, π or ϕ :

(i)

(ii)

Because of the convergence property of the wave function $\bar{\psi}(k)$, one derives the desired scaling property.

The ladder approximation is not necessary, since the inclusion of any crossed diagrams such as

leads only to additional convergent integrals. The asymptotic behavior of $\Phi(k) \sim \frac{1}{k^2}$ is therefore unaltered. All the above conclusions on the elastic form factors and the deep-inelastic scaling property remain valid if we include (at least iteratively) all crossed diagrams in the description of the bound state p . The only diagrams that may lead to non-scaling results are diagrams in which hard mesons (i. e., mesons with large transverse momentum) are emitted with a large probability, such as

To limit the probability of such hard meson emission we insist that

$$e \equiv \frac{f^2}{4\pi} \text{ should be small,} \quad (17)$$

say $\lesssim 0(10^{-1})$. This condition is certainly compatible with the coupling constant constraint (16) for the bound-state description. The details of the model are given in Ref. 14. The following is a summary of the main features of this model:

1. Scaling holds (including all diagrams) for deep inelastic ep and en reactions, provided one neglects high order electromagnetic corrections and provided the laboratory virtual photon energy ν , in units of GeV, is larger than $O(1)$ but less than $O(e^{1/\epsilon})$. Since ϵ is a free (though non-zero) parameter, one may approach the light cone as nearly as possible by taking the limit $\epsilon \rightarrow 0+$.

2. Because of (17), as $\epsilon \rightarrow 0+$, there are only soft meson emissions; the transverse momentum distribution of the mesons is $\sim k_{\perp}^{-6} d^2 k_{\perp}$ at large transverse momentum $k_{\perp}$.

3. As the scaling variable $x \equiv \omega^{-1} \rightarrow 1$, both W_1 and νW_2 approach $(1-x)^3$. This is rather encouraging, since it is in good agreement with the present experimental result, unlike most spin $\frac{1}{2}$ parton models¹⁶ with an ad hoc transverse momentum cut-off which lead naturally to a linear $(1-x)$ dependence as $x \rightarrow 1$.

4. As mentioned before, the elastic form factors are $\sim O(q^{-4})$ at large q^2 . In this model, all masses are of the order of m_{π} or m_p ; this then "explains" why the physical scale for deep inelastic scaling, as well as that for elastic form factors, are both of the order of 1 GeV.

The idea that p is the bound state of $(\psi\pi)$ and ψ is the interpolating field of the $(p\pi)$ continuum differs from the bootstrap idea in a fundamental way. Here, ψ is a local field; its interactions with the photon and with intermediate bosons (if they exist) must be of local character. If one wishes, one may also view this bound-state description as a Lorentz-invariant, gauge-invariant formulation of the parton model, in which the usual "point-like" assumption for the electromagnetic vertex of the constituent emerges simply as the standard minimal electromagnetic interaction in a local field theory.

5. Remarks

Assuming that the bound-state concept is correct, the theoretical basis of scaling can therefore be understood at least qualitatively. While many of the details remain to be worked out, for most of the experimental applications, as we have discussed, all that is necessary is simply to use dimensional analysis, and that, after a little while, could become quite dull. Fortunately, there are good reasons to believe that the scaling hypothesis may not be an exact law of nature in the extremely high-energy limit. Besides the problem of mass singularities, mentioned earlier, we shall show that there must exist a new basic high energy scale, hitherto undiscovered. Since scaling means the absence of a basic physical energy scale, the presence of such a new basic energy scale therefore means the breakdown of scaling. It is well-known that at a center-of-mass energy higher than 300 GeV, the present Fermi theory of weak interactions would violate its unitarity limit. A new scale may also set in due to strong interactions, e.g., if real quarks do exist. For the Fermi theory, a natural possibility is to regard the basic scale to be given by the Fermi constant itself,

$$G^{-1/2} \sim 300 \text{ GeV} .$$

On the other hand, if one assumes the weak interaction is governed by the same dimensionless constant α as that in the electromagnetic interaction, then the relevant scale can be much lower,

$$(\alpha/G)^{1/2} \sim 30 \text{ GeV} .$$

[A more careful consideration may lead^{17, 18} to 37.3 GeV, or higher values¹⁹.]

Throughout the study of microscopic physics, new frontiers are opened whenever a new basic energy scale is reached. We have the case of atomic and molecular physics with the electron-volt energy scale, nuclear physics with the MeV energy scale and the present strong-interaction physics with the GeV energy scale. In each case, at the energy scale of interest, we encounter a vastly rich structure of multiple energy levels and detailed dynamics; yet, when viewed against a much larger energy scale, this superstructure simply dissolves into the continuum. The recent discovery of the scaling property strongly indicates that we are now again in a transition region: The familiar GeV scale is no longer significant, but the still higher new high-energy scale is, as yet, unreachd. While scaling is important, the future discovery of its violation should be of even greater significance.

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Figure Captions

- Figure 1. Experimental data from CERN on total neutrino cross sections as a function of laboratory neutrino energy (Ref. 4).
- Figure 2. Experimental data from SLAC on structure functions in deep inelastic $e p$ scattering (Ref. 5).

FIGURE 1

FIGURE 2