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Toroidal Effects on MHD Stability in Tokamaks*

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Toroidal effects on MHD flute, ballooning, and kink modes are examined in an expansion in the inverse aspect ratio a/R , and stability criteria are found.

Magnetohydrodynamic equilibrium and stability considerations dominate the macroscopic behavior of Tokamak discharges, the usual operating regime being close to marginal stability. Therefore, as the aspect ratio R/a is lowered in future Tokamaks, effects associated with higher order terms in a/R will become important. The present calculation examines toroidal effects on flute, kink, and ballooning modes in the MHD limit.

We use the equilibrium formulation of Greene, Johnson and Weimer [1], which utilizes an expansion in $\epsilon \equiv a/R \ll 1$ with β_θ and $q \sim 1$, and extend the preliminary stability analysis of Ware and Haas [2]. To lowest significant order the results agree with those of Shafranov [3]. For a uniform toroidal current distribution inside the plasma column

$$\delta W^{(2)} = \sum_m \frac{2\pi^2 R B_{\theta a}^{(1)2} \xi_{a,m}^{(0)2} (m - nq_a) (m - nq_a - 1 + (a/b)^{2m})}{m (1 - (a/b)^{2m})} \quad (1)$$

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To proceed to higher order in ϵ we expand about the marginal stability criteria predicted by Eq. (1). These conditions are (1) $\xi_a^{(0)} = 0$, (2) $m - nq_a = 0$, and (3) $m - nq_a - 1 + (a/b)^{2m} = 0$. In addition we treat the special limiting cases (4) $a/b \rightarrow 0$, (5) $1 - (a/b)^{2m} \sim \epsilon$ and (6) $1 - (a/b)^{2m} \sim \epsilon^2$.

For cases (1), (2) and (3) we find

$$\delta W^{(3)} = \frac{4\pi^2 B_0^2 a^2}{q_a R} \left[\frac{l_a^{(1)}}{2\pi} \frac{(m - nq_a)}{(1 - (a/b)^{2m})} \xi_a^{(0)2} - \frac{1}{2} \frac{l_a^{(1)}}{2\pi} \xi_a^{(0)2} - \frac{(m - nq_a)}{2a} \int_0^a dr \xi_r^{(0)2} \frac{\partial}{\partial r} \left(r \frac{l_a^{(1)}}{2\pi} \right) \right] \quad (2)$$

and thus obtain the trivial result that either $\xi_a^{(0)} = 0$ or $l_a^{(1)} = 0$ for constant $l_a^{(1)}$. If $l_a^{(1)}$ is not uniform we obtain the usual shift in the criterion for case (3) associated with the current distribution.

Proceeding to next order we find, for $m - nq_a = 0$,

$$\delta W^{(4)} = \frac{\pi^2 B_0^2}{R} \left\{ \int_0^a r dr \left[\left(\frac{l_a^{(1)}}{2\pi} \right)^2 \left(\frac{rd\xi_r^{(0)}}{dr} \right)^2 + (m^2 - 1) \xi_r^{(0)2} \right] - \frac{2}{B_0^2} \left(1 - \frac{1}{q} \right) r p' \xi_r^{(0)2} \right\} + \frac{2(m+1)q^2 R^2}{B_0^4 a^{2m+2}} \left(2 - \left(\frac{b}{a} \right)^{2m+2} \right) \left(\int_0^a r^{m+1} dr p' \xi_r^{(0)2} \right) - \frac{2a^2}{q} \left\{ \frac{l_a^{(2)}}{2\pi} \xi_a^{(0)2} \right\} \quad (3)$$

This is positive definite if $p' < 0$ and both $\xi_a^{(0)}$ and $\xi_a^{(1)}$ vanish, demonstrating the well known flute stabilization due to "Minimum average B" for $q > 1$.

The general results are too lengthy to be quoted here but can be pictured schematically as in Fig. 1.

Fig. 1. Toroidal Modifications to Tokamak Stability

The solid lines in the Figure are the stability criteria predicted by Eq. 1 and the dotted lines indicate the modification introduced by the higher order effects in a/R , arising from cases (1) - (6). We note the asymptotic agreement of the stability criteria.

Two possibly important types of instability modes emerge from this study. First, when $a/b \rightarrow 0$ marginal modes with $\xi_m^{(0)}$ and $\xi_{m+1}^{(0)}$ nonvanishing can coexist in second order and resonate in higher order. Their interaction creates a ballooning type of perturbation. Second, if $\xi_m^{(1)}$ vanishes at a , a trial displacement $\xi_m^{(0)}$ can be chosen which vanishes

almost everywhere except in a narrow region near the surface. This perturbation eliminates all but the last term in Eq. (3) so that $\epsilon_a^{(2)} < 0$ is a necessary condition for stability. We term such a displacement a "peeling" instability and note that it tends to defeat shear stabilization.

The details of these calculations will be given in a later publication.

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