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A STUDY OF THE PERIPHERAL COMPRESSOR

AUTHOR:

P. S. Cates

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Nuclear Division
Oak Ridge Gaseous Diffusion Plant
Oak Ridge, Tennessee

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Author: Phillip Stephen Cates

A B S T R A C T

This investigation was undertaken because of the lack of adequate design information on the peripheral-type compressor, a device having promising potential for a number of compressor applications in the nuclear, chemical, and space industries. The study had three objectives: (1) determine the "state of the art" by conducting a thorough survey of the available literature on peripheral pumps and compressors; (2) analyze the published theories for incompressible flow and propose a hypothesis for performance in compressible flow; and (3) obtain experimental data on gases for correlation with respect to the proposed hypothesis. The theories for incompressible flow operation were summarized and studied with regard to their limitations for application to compressible flow operation. Fundamental laws of continuity, thermodynamics, and motion for compressible flow were applied to control volumes in the peripheral compressor in the form of integral vector relations. From these equations, from experience in compressible flow problems, and from published literature on centrifugal compressor studies, qualitative predictions for peripheral compressor performance at high impeller Mach numbers were made. Functional relationships of dimensionless parameters were derived for correlation of experimental data measured in compressible flow. Data were obtained on an

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A B S T R A C T
(Continued)

2/10 *peripherals*
experimental compressor at speeds from 5000 to 14,000 rpm. in gases with molecular weights of 4 to 400. In addition to the effects of Mach number, the influence of channel, impeller, and nozzle geometries and impeller-to-casing clearances were investigated. Generally, the experimental data confirmed the qualitative predictions for compressible flow operation. The peripheral compressor operated stably at all speeds and flows but exhibited some limitations at high Mach number operation. Methods to improve performance, particularly at high impeller Mach numbers, were developed, and significant advancement of design information was achieved.

A STUDY OF THE PERIPHERAL COMPRESSOR

A Thesis

Presented to

the Graduate Council of
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In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by

Phillip Stephen Cates

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CHAPTER I

DESCRIPTION AND PROJECT SCOPE

The peripheral-type pumping device defined in this thesis has been used for many years in liquid service requiring high delivered head at low flow rates. In recent years, designers and users of compressors have become increasingly interested in the device as a circulator for compressible fluids in applications with similar requirements. Its relatively simple construction and stable operating characteristics make the peripheral compressor attractive for use in the chemical industry, nuclear reactor systems, the petroleum industry, nuclear fuel reprocessing systems, space programs, and as accessory equipment in many other systems. The work discussed here is concerned with the study of the peripheral machine as a compressor. In this thesis, the device will be called a pump when operating in incompressible flow and a compressor when operating in compressible flow. This chapter describes the machine, presents the reasons for the study, and outlines the program followed.

I. DESCRIPTION OF THE PERIPHERAL PUMP AND COMPRESSOR

Many names have been used to identify the peripheral pump or compressor due mainly to several theories proposed by various authors to explain the pumping mechanism. Names that may be found in the literature include drag, regenerative, turbulent, friction, vortex,

turbine, traction, and tangential; however, a more neutral term, peripheral, was chosen until the pumping theory is more firmly established. The general features of the peripheral compressor are shown in Figure 1, which is a schematic of the experimental compressor used in this study. In the usual machine, the fluid enters at the periphery, passes through a channel at the periphery of the impeller (gaining energy from the impeller), and discharges at the periphery. Between the suction and discharge is a stripper or block seal that minimizes leakage flow from the high-pressure outlet region to the low-pressure inlet region.) The clearances between the impeller and the casing at the stripper and at the sides of the impeller are of the order of a few thousandths of an inch in order to keep the leakage losses as low as possible. The conceivable configurations of the impeller and channel are almost infinite; in addition to the usual design of Figure 1, a few other important possibilities are those shown in Figure 2.

Typical shapes of peripheral pump performance curves are illustrated in Figure 3, where delivered head, efficiency, and input power are plotted as a function of capacity. No numbers are given since the values can vary widely and the intent here is to simply point out important features regarding the performance. Of most significance are the shapes of the head and power curves. The delivered head increases with decreasing flow up to and including zero flow. The actual curve may be straight, concave upward, convex upward, or both concave and convex in different parts of the curve; geometric and

FIGURE 1

SCHEMATIC DRAWING OF ORGDP PERIPHERAL COMPRESSOR

a. LONG BLADING

b. HALF-TORUS BLADING

c. HALF-TORUS WITH
CENTER-BODY

d. CONTOURED CHANNEL

e. CHEVRON BLADING

FIGURE 2

VARIOUS IMPELLER AND CHANNEL CONFIGURATIONS

FIGURE 3

TYPICAL SHAPES OF PERIPHERAL PUMP PERFORMANCE CURVES

operating parameters determine the particular shape. The significant result of the negative slope of the head-flow curve is that the machine operates stably at all flows with no surge or stall region. Because of the high delivered heads at low flow, the power curve also has negative slope at all flows. The shape of the efficiency curve is similar to that of other rotary pumps. Peak efficiencies in the order of 50 percent, occurring at about two-thirds of the maximum capacity, are usual. The performance curves for compressible flow operation, although similar, can be somewhat different at high Mach numbers, as will be seen later.

How the peripheral compressor compares with other types will be illustrated in the following discussion. As mentioned above, the peripheral compressor is attractive for applications which require relatively high head or pressure ratio at low flow rates. It is, therefore, characterized as a low-specific-speed compressor, with specific speed defined as

$$N_s = \frac{N \sqrt{Q_s}}{H_s^{3/4}}, \quad (1)$$

where

N = impeller rotational speed, rpm.,

Q_s = suction volume flow, ft.³/sec., and

H_s = isentropic head, ft.-lb_f./lb_m.

This form of specific speed is not dimensionless but is commonly used in American literature. Defining specific speed as above and specific

diameter as

$$D_s = \frac{D H_s^{1/4}}{\sqrt{Q_s}}, \quad (2)$$

where D = impeller diameter, ft., Balje (1)¹ compared the performance of several pump and compressor types in an $N_s - D_s$ diagram, reproduced here as Figure 4. (Balje used the term "drag" instead of peripheral.) This diagram is approximate due to scant data in some instances, but it serves to locate the peripheral compressor performance relative to that of more common types such as the axial compressor and various designs of the centrifugal compressor. Important parameters on Balje's diagram are efficiency, η , and head coefficient, ψ_s . Efficiency is the compressor hydraulic efficiency, which excludes mechanical losses, and the head coefficient is the same used in this thesis, which is

$$\psi_s = \frac{g_c H_s}{U_T^2}, \quad (3)$$

where

ψ_s = isentropic head coefficient,

g_c = gravitational constant, $32.17 \text{ lb}_m \cdot \text{ft.} / \text{lb}_f \cdot \text{sec.}^2$, and

U_T = impeller tip velocity, ft./sec.

Figure 4 shows that for specific speeds below 25 to 30, peripheral compressor efficiencies can exceed those of centrifugal compressors,

¹Numbers in parentheses designate similarly numbered references in the List of References.

FIGURE 4

APPROXIMATE N_s - D_s DIAGRAM FOR SINGLE-DISK PUMPS AND LOW-PRESSURE-RATIO COMPRESSORS [BALJE (1)]

and head coefficients greater than unity are obtained. Generally, however, energy transfer between a fluid and a rotor is such that the compressors which produce the highest heads usually have the lowest maximum efficiency. Samaras and Tyler (2), who made a similar comparison of rotary compressors, expressed this idea as follows:

A complicated and tortuous path of travel involving changes of fluid direction and fluid cross section, and the presence of intense circulation, may be expected to be accompanied by relatively high losses. It is reasonable to suppose, therefore, that a machine of comparatively high loading will be inherently less efficient than a machine producing a smaller head per stage, unless the path of flow through the latter is considerably more complicated. In practice, high stage heads and flow path complication go hand in hand.

As will be seen later, the peripheral compressor does indeed have "intense circulation." Still another comparison of compressors from which the same conclusions may be drawn is given in Reference (3).

In summary, the peripheral compressor is a low-specific-speed unit with usual efficiencies of 30 to 50 percent. It is usually considered for applications in which power requirement is not of prime importance. Positive displacement compressors, such as reciprocating piston, rotary lobe, or diaphragm, are generally inferior due to problems associated with lubrication, sealing, wear, vibration, and/or diaphragm life that result in higher initial and maintenance costs. Since axial flow compressors are beyond the capacity range of the peripheral, the centrifugal is the most common competitor when selection is being made for a particular application. Significant structural advantages of the peripheral compressor over the centrifugal compressor are that no complex flow passages or vaning are required.

No diffuser or scroll is needed so that the overall size is smaller. However, smaller impeller-to-casing clearances are necessary. Performance-wise, while centrifugal compressors are inherently more efficient, this is not always the case at specific speeds of 30 or less. The no-surge characteristic of the peripheral compressor makes stage matching and off-design operation less critical. Since the peripheral compressor delivers heads several times those of the centrifugal compressor, lower rotative speeds, fewer stages, smaller impellers, or a combination of these can be cited as advantages. Single-stage head coefficients greater than 5 have been measured on the peripheral types, compared with about 0.6 to 0.8 for the centrifugal. (The maximum theoretical head coefficient for a radial-bladed centrifugal compressor with zero prerotation is 1.0.)

II. STATEMENT OF PROBLEM

As will be seen in the next chapter, there is considerable information available in the literature on peripheral pumps under conditions of incompressible flow operation. But even with a large amount of data on liquids, there is still a lack of established design criteria for operation in incompressible flow and a lack of a completely adequate theory of the pumping mechanism. The situation is worse in the case of operation in compressible flow, where relatively little work has been published. A few organizations, including the United Kingdom Atomic Energy Authority, Mechanical Technology, Inc., Massachusetts Institute of Technology, Los Alamos Scientific Laboratory,

and Oak Ridge National Laboratory, have done some investigation with gases, but most of their work has been at relatively low Mach numbers. Performance data and analysis where compressibility effects must be considered are much needed in the many areas that affect performance, such as flow passage geometry, impeller geometry, nozzle design, and impeller-to-casing clearances.

With regard to the above considerations, the following three objectives were selected for this study:

1. Determine the "state of the art" by conducting a thorough study of the available literature on peripheral pumps and compressors.
2. Analyze the published theories for incompressible flow and propose a hypothesis for performance in compressible flow.
3. Correlate experimental data on gases with respect to the proposed hypothesis.

III. OUTLINE OF PROGRAM

In addition to acquiring the general information on the peripheral compressor already presented in this chapter to familiarize the reader with the subject, the following program was followed to meet the above objectives. First, all known published literature relating to the peripheral-type pump or compressor was surveyed and existing theories analyzed. These theories on incompressible flow were studied with regard to application to compressible flow, and

areas where they failed to predict performance in compressible flow were investigated. Fundamental laws of continuity, thermodynamics, and motion for compressible flow were applied to control volumes in the peripheral compressor in the form of integral vector equations. Functional relationships using dimensionless parameters were then derived to predict correlation of performance for compressible flow. A parallel to centrifugal compressor analysis was drawn in this development, and qualitative effects of high Mach numbers on losses were predicted. Data were obtained on an experimental compressor at the Oak Ridge Gaseous Diffusion Plant using several gases tested over a wide range of speeds, and those data were correlated in these dimensionless terms. The effects of Mach number and various geometric factors on performance were explored in the experimental program.

CHAPTER II

REVIEW OF THE LITERATURE

Although several theories have been proposed to explain the pumping mechanism of the peripheral pump, they may be grouped into two major classes, regenerative and turbulent. The regenerative theory considers that the fluid contained within the impeller blading at any given instant is thrown out radially, centrifugal fashion, gaining angular momentum in the process. This fluid then flows into the stationary channel, loses some of its angular momentum, and supports the pressure gradient. The fluid then re-enters the impeller at the root of the blading and repeats the process perhaps several times during its path through the pump, hence the name "regenerative." The flow pattern is considered to be orderly, although turbulent, and can be described as helical or circulatory. Experimental observations tend to support the existence of this type of flow pattern.

The turbulent theories may be subdivided into two separate ideas: (1) one recognizes the presence of a circulatory type of flow but assumes the momentum exchange to occur through a violent mixing process in the stationary channel; (2) the other simply considers the impeller as a very rough surface that "drags" the fluid through the passage and promotes momentum exchange through turbulent shear. Prandtl's mixing length theory has been applied to both ideas to aid in the analyses.

As one investigator (4) has noted, most of the published theories are compatible in many ways and all require one or more experimentally determined constants. The following discussion of these theories is divided according to the above classification. Only the major works in each category are described, and comments about other investigations in that category are included if considered pertinent. Unless otherwise noted, the published theories apply only to incompressible flow.

I. REGENERATIVE THEORIES

By far the most extensive work published on this theory is that by a group at the Massachusetts Institute of Technology. Working partly on a grant from the Worthington Corporation, at least six investigators at MIT contributed to the problem. Three seniors (5,6) did much of the experimental work, including flow pattern visualization, which provided information about the pumping mechanism. A rather detailed theoretical investigation by Oelrich (7) was followed by additional refinements and simplification by Wilson et al. (8) and Santalo (9). A somewhat detailed description of their work is given here since the writer believes the regenerative theory comes closest to explaining the behavior of the peripheral compressor.

Experimental observations. Experimental work performed by Lazo and Hopkins (5) and Lutz (6) on a commercial pump with a 5.4-inch diameter impeller formed the basis for the theoretical work that followed. Tests were conducted at seven speeds from 1200 to 4800 rpm.

using air at atmospheric conditions. Internal static pressures were measured with wall taps, and flow patterns were determined with "thread probes." Their studies showed the following:

1. The pressure developed by the pump increased linearly with angular position in the working section of the pump; i.e., $dP/d\theta = \text{constant}$. Wilson and associates (8) later showed five regions of operation--inlet region, acceleration region, linear region, deceleration region, and outlet region.
2. The overall performance curves were of the shape shown in Figure 3, page 5, with a maximum efficiency of about 45 percent.
3. A definite helical-type flow pattern as described on page 13 existed at all flows. The helix became tighter or more compressed as flow was reduced, changing very rapidly near shutoff, and flow reversal in the channel was observed at very low flows.

Hypothesis of operation and mathematical model. Based on the above experimental observations, Oelrich (7) and Wilson et al. (8) formulated a hypothesis of pump operation upon which they constructed their theoretical analysis. They defined a tangential fluid velocity component in the channel, V_θ , and a meridional velocity component, V_m . (Nomenclature used in this thesis is substituted in some instances in order to reduce the number of different symbols.) A tangential or through flow was defined as

$$Q = \int_{A_c} V_\theta dA_c, \quad (4)$$

and a meridional or circulatory flow, Q_m , was associated with V_m . The following quotation from Reference (8) summarizes the hypothesis of the regenerative theory for incompressible flow:

As the circulatory flow passes radially through the rotor, its angular momentum in the direction of rotor motion is increased by virtue of the work of the impeller. To maintain a tangential pressure gradient in the open channel, the angular momentum of the circulatory flow continually decreases after it leaves the rotor. Qualitatively, this indicates that an increase in pressure rise requires an increase in the rate of the circulatory flow, a greater reduction of the tangential velocity along the meridional flow path from the exit of the rotor to its entrance, or both. Reductions in the tangential velocity outside the rotor result in reductions in the capacity Q . The centrifugal field creating the radial pressure gradient inside the rotor is constant at a given speed while the centrifugal field due to the external flow weakens as the tangential velocities decrease; therefore, the circulatory flow rate may be expected to increase as we approach zero through-flow. Both these effects lead to the observed inverse relation between flow and head.

A simplified model was developed as illustrated in Figure 5, and the following assumptions were made:

1. The flow is steady, incompressible, and adiabatic, and internal leakage effects are neglected.
2. There are no end effects and the tangential and meridional flows, Q and Q_m , are independent of circumferential position.
3. The entire pump flow may be represented by the velocities, V_θ and V_m , along a mean streamline.
4. The tangential pressure gradient is independent of radius and is constant throughout the linear region.

a. DIMENSIONS OF CHANNEL AND IMPELLER

b. CONTROL VOLUMES FOR SECTION $d\theta$

FIGURE 5

IMPELLER AND CHANNEL DIMENSIONS AND CONTROL VOLUMES
FOR REGENERATIVE ANALYSIS [WILSON (8)]

5. All the meridional flow, Q_m , leaves the impeller at the tip of the blades.
6. The unit is governed by the same laws of similitude as is other turbomachinery.

An additional assumption made in Oelrich's early analysis was that the process was frictionless. An attempt to include friction in the analytical equation resulted in tremendously complicated expressions beyond solution, so that irreversibilities were introduced in additional terms simply as head losses. The definitions of the following terms complete the hypothetical model description:

$$V_{\theta,2} = \sigma U_2, \quad V_{\theta,1} = \alpha U_1, \quad (5)$$

and

$$dQ_m = V_{m,1} r_1 f d\theta = V_{m,2} r_2 a_1 d\theta, \quad (6)$$

where

U_2 and U_1 = impeller velocities at points 2 and 1,
respectively, in Figure 5, ft./sec.,

σ = a dimensionless "slip factor," assumed constant,

α = a dimensionless blade entrance factor which depends
on flow,

dQ_m = meridional flow in incremental section, $d\theta$,
ft.³/sec., and

r_1, f, r_2, a_1 = dimensions shown in Figure 5, ft.

(Hereafter in this thesis, when terms are not given specific dimensions, it is to be assumed that a consistent set of units is used.)

Theoretical equations. By applying Newton's second law of motion in the form for angular momentum to the open channel control volume in Figure 5(b), Oelrich (7) obtained a relation for the incremental tangential pressure rise in a frictionless pump, which, when modified to include head loss due to tangential shear forces on the channel walls, appeared in Reference (8) as

$$\frac{dP}{\rho} = \frac{dQ_m \omega \left(\sigma r_2^2 - \alpha r_1^2 \right)}{g_c r_g A_c} - dH_t, \quad (7)$$

where

dP = pressure rise in $d\theta$, lb_f/ft^2 ,

ρ = fluid density, lb_m/ft^3 ,

ω = impeller angular velocity, rad./sec. ,

A_c = channel cross-sectional area, ft^2 ,

r_g = radius of centroid of open channel, ft. , and

dH_t = tangential head loss in $d\theta$, $\text{ft.-lb}_f/\text{lb}_m$.

Oelrich also applied angular momentum relations successively to incremental volumes along a simplified mean streamline in the open channel as indicated in Figure 5(a) that resulted in expressions for the tangential velocity, V_θ . Using Equation (4), a cumbersome equation for the channel flow that included a meridional flow term was then derived. Upon simplifications by Wilson et al. and substitution of Equation (7), the following expression was obtained:

$$Q = \frac{1}{2} \frac{r_2}{r_g} Q_{sb} \left(K_1 \sigma + \frac{r_1^2}{r_2^2} K_2 \alpha \right), \quad (8)$$

where

$$Q_{sb} = \text{"solid body" flow} = r_g A_c \omega, \text{ ft.}^3/\text{sec.}, \text{ and}$$

$$K_1, K_2 = \text{dimensionless channel geometric factors.}$$

An expression for the meridional pressure rise in the impeller, $P_2 - P_1$, was derived by writing Bernoulli's steady flow energy equation and the angular momentum expression for the impeller control volume in Figure 5(b). Another expression for the channel pressure drop in the meridional plane, also $P_2 - P_1$, was obtained by applying either Equation (7) and the energy equation (Reference (8)) or the angular momentum relation (Reference (7)) to the channel control volume. Combined and simplified, these two expressions led to

$$g_c H_c = \omega^2 (\sigma r_2^2 - \alpha r_1^2) \left(1 - \frac{Q}{Q_{sb}} \right), \quad (9)$$

where H_c is the head loss in the circulatory or meridional flow, ft.-lb_f/lb_m.

Wilson and associates assumed the meridional flow losses to be comprised of two terms, blade entrance losses and friction losses, and wrote the relation

$$g_c H_c = \frac{(1 - \alpha)^2 U_1^2}{2} + k_c \left(\frac{dQ_m}{r_2 a_1 d\theta} \right)^2, \quad (10)$$

where k_c is a coefficient to be determined experimentally. The first term on the right denotes the losses in the flow entering the blading,

and the second term represents wall friction, turning, and turbulence losses.

Wilson obtained an expression for the overall head delivered by the pump by assuming that all head losses due to tangential flow in the channel and nozzles could be written as $k_t \left(\frac{Q}{D^2} \right)^2$, with k_t determined experimentally, and by integrating Equation (7):

$$g_c H = \frac{\Delta P}{\rho} = \frac{Q_m}{Q_{sb}} (\sigma U_2^2 - \alpha U_1^2) - k_t \left(\frac{Q}{D^2} \right)^2. \quad (11)$$

Defining pump efficiency, η , as the ratio of useful output power to input power, the relation

$$\eta = \frac{\rho Q H}{P_{in}} = \frac{Q/Q_{sb}}{1 + k_t \frac{Q^2/g_c H D^4}{Q_{sb}^2}} \quad (12)$$

was obtained.

The five Equations (8) through (12) relate the variables α , Q , H , H_c , Q_m , and η when the parameters k_c , k_t , and σ are determined. The theoretical equations were fitted to the experimental data at three points, $Q = 0$, Q_{max} , and $\frac{1}{2} Q_{max}$, in order to evaluate k_c , k_t , and σ . The theoretical curves agreed very closely with the data; however, apparently the main thing predicted by the equations was the shape of the performance curves since they were fitted to the data at three points. The parameters k_c , k_t , and σ are unknown functions of geometry, with k_c also considered to be a function of flow.

Two plots which are considered important results of the theory developed by the MIT group are shown in Figure 6 (7). The theoretical equations were applied to the commercial pump tested by Lazo, Hopkins, and Lutz in an effort to illustrate various internal performance parameters. The nature of the parameters represented by these calculated curves prevented complete experimental verification. They are shown here mainly to demonstrate qualitatively some of the features of the regenerative theory for incompressible flow. The following observations may be made from these curves:

1. The meridional flow, Q_m , was much larger than the through flow, Q , even at maximum capacity.
2. The blade entrance factor, α , became negative at low flows, indicating a flow reversal in the channel as observed experimentally.
3. The meridional flow, Q_m , reached a maximum and then decreased as capacity was reduced. This was attributed to the fact that the centrifugal field in the impeller was constant while that in the channel decreased then increased again as flow reversal occurred.
4. The ratio Q/Q_{sb} reached a maximum of about 0.77 at maximum flow (zero head). This ratio, or one similar, is frequently used as the flow parameter in dimensionless plots of regenerative pump performance and usually has a value at zero head of 0.7 to 0.8.

a. INTERNAL PERFORMANCE PARAMETERS

b. POWER DISTRIBUTION

FIGURE 6

THEORETICAL PERFORMANCE PARAMETERS CALCULATED
BY REGENERATIVE THEORY [OELRICH (7)]

5. The blade entrance losses represented by far the major source of irreversibilities in the pump, especially at low flows.

Simplified equation. After making additional simplifying assumptions, Santalo (9) derived the following "working equation" from the above theoretical equations for a regenerative pump with incompressible flow:

$$\frac{d\psi}{d\theta} = \frac{2\sqrt{2} d r_1 r_2}{A_c \sqrt{k_c} r_g} \sqrt{1 - \frac{r_1}{r_2}} (C_1 - \lambda) \sqrt{C_2 + \sigma \lambda - 2\lambda^2}, \quad (13)$$

where

$$C_1 = \frac{\sigma}{2} \left(1 + \frac{r_2}{r_1} \right), \quad (13a)$$

$$C_2 = \frac{\sigma \left(1 - \frac{\sigma}{2} \right) - \frac{1}{2} \left(\frac{r_1}{r_2} \right)^2}{1 - \frac{r_1}{r_2}}, \quad (13b)$$

$$\psi = \frac{g_c H}{U_2^2}, \quad (13c)$$

$$\lambda = \frac{Q}{U_2 A_c}, \quad (13d)$$

A_c = channel cross-sectional area of a double-sided pump, and

d = "typical dimension" of the open channel.

With addition of the inlet and outlet losses, the pump equation may be written as

$$\psi = \frac{\Delta\theta_a \cdot 2\sqrt{2} \cdot d \cdot r_1 \cdot r_2}{A_c \sqrt{k_c} \cdot r_g} \sqrt{1 - \frac{r_1}{r_2}} (c_1 - \lambda) \sqrt{c_2 + \sigma\lambda - 2\lambda^2} - k'_t \lambda^2. \quad (14)$$

Here, $\Delta\theta_a$ is the active pumping angle in radians and equals 2π minus (stripper angle + inlet angle + outlet angle). The coefficient k'_t is similar to the previously used k_t . Equation (14) is dimensionless, so the units of the individual terms must be consistent. Santalo indicated that the radius, r_1 , (Figure 5, page 17) could be assumed to equal $\frac{1}{2} (r_o + r_2)$; however, the dimension, d , must be determined experimentally. In Santalo's simplification, the dimension, d , and the meridional flow loss coefficient, k_c , are related in the term $(k_c/2)(dq_m/r_2 d\theta)^2$, which is similar to the meridional flow loss term in Equation (10), page 20. It was not made clear whether d and the impeller blade depth, a_1 , are equivalent.

Other discussions of the regenerative theory. Bartels (10) and Wright (11) presented discussions substantiating the regenerative theory. Bartels reported experimental work that showed the existence of the circulatory-type flow pattern. He used a wax coating on the pumping passages, pulverized coal and ground glass in the water, and a photomicrographic technique to determine the flow pattern. Wright presented a simple theoretical development somewhat similar to that of the MIT group in that the presence of a meridional flow was utilized.

However, some of his assumptions were different than those of the MIT group, so that his derived equation was not of the same form. His equation for the overall performance of a radial-bladed pump when written in dimensionless terms consistent with usage in this thesis may be expressed as

$$\psi = \frac{g_c H}{U_2^2} = \frac{2 \Delta\theta_a a_1 r_g^3 k'_c}{A_c r_2^2} \left(1 - \frac{r_2}{r_g} \lambda\right) - k_t'' \lambda^2, \quad (15)$$

where k'_c and k_t'' are associated with the meridional and through flows, respectively, and must be determined experimentally. The differences in Equations (14) and (15) are probably due primarily to Wright's use of a mean radius instead of actual radii, lack of consideration of a "slip factor" at the blade tip or "entrance factor" at the blade root, and somewhat different assumptions regarding the meridional flow velocity. Wright also made an attempt to analyze the effect of backward-curved blades. By substituting the ratio of the actual fluid angular velocity to the impeller angular velocity (at the impeller tip) for the unity term within the parenthesis in Equation (15), he obtained a reduction in head coefficient that agreed with experimental data.

II. TURBULENT THEORIES--MOMENTUM EXCHANGE FLOW WITH VIOLENT MIXING

The main idea expressed in this group of theories is that the flow passes through the impeller blading passages, but no orderly "whole flow," helical type of pattern exists in the stationary channel.

Instead, the impeller is considered a source of momentum, and the fluid projected from it transports this momentum. This fluid transfers its momentum to the slower moving fluid in the channel through violent turbulent mixing. A Japanese investigator named Yasutoshi Senoo is the foremost proponent of this theory and has published more work on the peripheral pump than has any other single individual. The following discussion deals mainly with his contributions, with those of other followers of this theory mentioned briefly.

Work of Senoo

Senoo published two comprehensive papers (12,13) which report the results of extensive studies made at Kyushu University. The latter paper contains much of the information reported in the former and describes both theoretical developments and experimental results.

Turbulence mechanism and theory. In his discussion of the pumping mechanism, Senoo (13) refuted the regenerative concept and summarized his idea of the mechanism in the following quotation:

Certainly the liquid between the vanes is thrown by a centrifugal force into the pumping passage between the impeller and the casing. This liquid will mix with the liquid in the passage and will transmit the momentum of the impeller to the liquid in the pumping passage. Therefore, the vanes do not serve as the device for increasing pressure but as the device for conveying the momentum of the impeller to liquid. Accordingly the performance of the pump will be decided by the momentum being conveyed to the liquid in the passage.

In his development, Senoo employed a mixing length concept similar to the well-known Prandtl theory. However, a distinct difference in the type of turbulence was emphasized. In the peripheral

pump, the turbulence intensity is much greater than in normal channel flow, and the source of turbulence is the impeller rather than the velocity gradient. However, Senoo stated that the momentum transport mechanism is the same since it results from a motion between two volumes of liquid. In Prandtl's theory of ordinary turbulence (14) there is the relation

$$\tau_t = \frac{\rho}{g_c} l^2 \left| \frac{du}{dy} \right| \frac{du}{dy} = \frac{\mu_t}{g_c} \frac{du}{dy}, \quad (16)$$

where

τ_t = mean turbulent shear stress, $\text{lb}_f./\text{ft.}^2$,

l = mean mixing length, ft.,

u = mean velocity in direction of main flow (x direction),
ft./sec.,

y = direction normal to main flow (perpendicular to wall),
ft.,

ρ = fluid density, $\text{lb}_m./\text{ft.}^3$, and

μ_t = turbulent, or eddy, viscosity, $\text{lb}_m./\text{ft.-sec.}$

Senoo defined the similar expression

$$\tau_t = \frac{\rho}{g_c} l v \frac{du}{dy} = \kappa \frac{\rho}{g_c} y U \frac{du}{dy}, \quad (17)$$

where

v = turbulent velocity in y direction, perpendicular to wall,
ft./sec.,

U = velocity of the moving wall (impeller), ft./sec., and

κ = turbulence intensity coefficient.

Although Senoo recognized that channel curvature could be important in inducing secondary flow, he assumed that its effect could be incorporated in κ and considered a straight passage in his analysis. This passage consisted of two parallel plates a distance, δ , apart, one fixed and the other moving at the velocity, U , inducing turbulence. The right side of Equation (17) was derived by assuming the turbulent velocity, v , to be proportional to $U y/\delta$, and the mixing length, ℓ , to be proportional to δ . According to Senoo, the proportionality factor, κ , is strongly dependent upon the shape and number of impeller vanes.

In order to account for the fact that the mixing length theory is not applicable at the wall where a laminar sublayer exists and molecular viscosity dominates, Senoo transferred the coordinate so that $y = \epsilon + \delta$ at the moving surface. Combining this with the assumption that the turbulent viscosity and the molecular viscosity at the fixed wall are equivalent resulted in the expression

$$\mu_t|_{y=\epsilon} = \mu = \kappa \rho \epsilon U, \quad (18)$$

where μ = molecular viscosity, lb_m./ft.-sec.

Beginning with Equation (17) and assuming that (1) the mean flow is parallel to the walls, (2) the pressure, P , is a function of only x , the coordinate parallel to the walls, and (3) the following relationship exists between forces acting on the liquid,

$$d\tau_t/dy = dP/dx, \quad (19)$$

Senoo developed equations for $\tau_t(y)$ and $u(y)$. Applying boundary conditions of $u = 0$ and $\tau_t = \tau_{t,0}$ at $y = \epsilon$ and $u = U$ at $y = \delta + \epsilon$, he

was able to obtain an equation for the volume flow per unit breadth as

$$q = \int_{\epsilon}^{\epsilon+\delta} u \, dy = U\delta \left(1 + \frac{\epsilon}{\delta} - \frac{1}{\ln(1 + \delta/\epsilon)} \right) - \frac{\delta^2 g_c dP}{\rho \kappa U \, dx} \left(\frac{1}{2} - \frac{\epsilon}{\delta} - \frac{1}{\ln(1 + \delta/\epsilon)} \right). \quad (20)$$

After making further assumptions regarding the shape of the flow passage in order to obtain an equivalent breadth, Senoo obtained an expression for the pump capacity, Q , similar in form to Equation (20). Rewriting that equation to obtain dP/dx , he then integrated to obtain the delivered head, which can be written as

$$H = \int_0^L \frac{1}{\rho} \frac{dP}{dx} \, dx = \frac{\frac{\kappa U^2 L}{g_c \delta} \left[\left(1 + \frac{\epsilon}{\delta} - \frac{1}{\ln(1 + \delta/\epsilon)} \right) - \frac{Q}{UA_c} \right]}{\frac{1}{2} + \frac{\epsilon}{\delta} - \frac{1}{\ln(1 + \delta/\epsilon)}}, \quad (21)$$

where L = total length of pumping passage, ft.

Experimental observations and modifications to equations. Senoo performed many tests on two experimental machines. One pump was used primarily to determine the effects of internal leakage by varying impeller-to-casing clearances from 0.0016 inch to 0.0142 inch, and the other pump was used to test four different channels and nine different impellers with from 0 to 120 vanes. From his testing, Senoo found that inlet and outlet losses and internal leakage should be included

in the head equation. He assumed that the observed head decrease, H' , in the entrance region could be accounted for by subtracting that value from the right hand side of Equation (21). Also, he modified the pumping length by subtracting an ineffective length, L' , from L . The result can be written in dimensionless terms as

$$\psi' = \frac{g_c H}{U^2} = \frac{\frac{\kappa(L-L')}{\delta} \left[\left(1 + \frac{\epsilon}{\delta} - \frac{1}{\ln(1 + \delta/\epsilon)} \right) - \lambda' \right]}{\frac{1}{2} + \frac{\epsilon}{\delta} - \frac{1}{\ln(1 + \delta/\epsilon)}} - \frac{g_c H'}{U^2}, \quad (22)$$

where the primed head coefficient, ψ' , and capacity coefficient, λ' , are based on the average impeller velocity, U .

Senoo is one of only two published investigators who tried to include the effect of internal leakage in the pump equations. His experiments showed that internal leakage across the faces of the impeller and through the stripper section decreased the driving power of the pump, decreased the net capacity, and decreased pump efficiency. Senoo substituted for the capacity, Q , in Equation (21) the expression $Q_0 + \int q \, dx$, where Q_0 represented the net discharge and q a leakage flow which was some function of position along the passage. Assumptions regarding the nature of the leakage flow were made for several cases--small, large, and very large clearances--and the term $\int q \, dx$ was estimated. However, Senoo finally stated that an easier and yet satisfactory method of including the effect of internal leakage was to use an apparent turbulent coefficient that could be estimated as the product of the true coefficient and the ratio of actual shut-off discharge pressure to the ideal shut-off pressure with no leakage.

Senoo stated that the denominator in Equation (22) was only slightly influenced by channel dimensions and was usually about 0.35. With this observation and the apparent turbulence coefficient, his pump equation may be written as

$$\psi' = \frac{\kappa(L-L')}{0.35 \delta} [0.85 - \lambda'] - k_t''' \lambda'^2, \quad (23)$$

where k_t''' is an inlet loss coefficient. In this equation κ , L' , and k_t''' must be determined experimentally. Also, the effective channel depth, δ , must be estimated. A satisfactory estimate for most geometries is

$$\delta = \frac{b^2(2a_2 + 2h) + h^2a}{A_c}. \quad (24)$$

L' and k_t''' are dependent primarily upon the inlet design, while κ is dependent mainly upon impeller vane shape and number and internal clearances. Senoo presented a rather detailed discussion on the important turbulence coefficient, comparing it with ordinary turbulence intensity and pointing out some of the factors which affect its value. From a test with a smooth impeller, Senoo concluded that the ordinary turbulence intensity caused by a velocity gradient was about 1/40 as great as the usual turbulence caused by an impeller with vanes. From tests with several channels and impellers, Senoo concluded that channel shape had much less effect upon κ than did impeller shape. The number of vanes, vane spacing, and curvature at the vane roots were determined as the most influencing factors. At least 32 vanes with as large spacing as possible was recommended. Senoo also reported the influence

of Reynolds number on the turbulence coefficient. With Re defined as $\rho U \delta / \mu$, κ began to decrease as Re became less than 10^4 . Based on many tests by Senoo and others, he reported that the usual value of κ was between 0.02 and 0.03, usually nearer 0.03 for properly designed pumps.

Work of Shimosaka and Yamazaki

Two other Japanese investigators made significant contributions to the study of the peripheral pump. Minoru Shimosaka and Shinzō Yamazaki of the Tokyo Metropolitan University conducted experiments in water that increased the range of pump parameters and developed equations that included impeller and channel dimensions.

Theoretical development. In his theoretical treatment, Shimosaka (15) followed an approach similar to that of Senoo. His assumption regarding the type of pumping action and flow patterns was the same, but he handled his equations a little differently. The following factors were assumed to affect the turbulence (see Figure 5, page 17, for dimension identification): (1) vane axial depth, a_1 , and radial length, a_2 ; (2) distances between channel walls and impeller; (3) number of impeller vanes, Z ; (4) average vane spacing, t_1 , and thickness, t_2 ; (5) impeller peripheral velocity, U_2 ; and (6) the velocity gradient in the channel, du/dy . The channel and impeller dimensions were grouped into a "characteristic channel dimension,"

$$R_m = \frac{A_c}{2a_2 + a}, \quad (25)$$

where A_c is the cross-sectional area of the channel and $(2a_2 + a)$ is

the wetted perimeter of the blading (a is $2a_1$ plus the web thickness between the blades).

With these assumptions and definitions, Shimosaka wrote an equation for the Reynolds, or turbulent, stress as

$$\tau_t = \kappa' \frac{\rho}{g_c} Z \left(\frac{t_1}{a_1} \right)^n \left(\frac{a_1}{a_2} \right) R_m U_2 \frac{du}{dy}, \quad (26)$$

where κ' and n are empirical constants. Equation (26) is similar to Senoo's Equation (17), page 28, but it includes impeller and channel dimensions as factors which affect τ_t . In his development, Shimosaka also neglected channel curvature and assumed a linear pressure rise. Beginning with a force equilibrium equation identical to Equation (19), page 29, he derived equations for $\tau_t(y)$, $u(y)$, Q , and H in similar fashion. However, a basic difference was his assumption of the boundary conditions $u = K'U_2$ and $y = R_m$ at the moving wall. The head equation can be written in dimensionless terms as

$$\psi = \frac{g_c H}{U_2^2} = K'' Z \left(\frac{t_1}{a_1} \right)^n \left(\frac{a_1}{a_2} \right) \left(\frac{D}{R_m} \right) \left(1 - \frac{2}{K'} \lambda \right), \quad (27)$$

where $K'' = 6 \kappa' K' \left(\pi - \frac{1}{2} \theta_s \right)$ and θ_s is the stripper angle, representing the inactive portion of the circumference.

Experimental observations. In a rather extensive experimental program, Shimosaka and Yamazaki (16) tested twenty-six impellers and twelve flow channels using water as the test fluid. With these data, they established values for the constants in their pump equations. The data covered the following ranges in pump variables: (1) A_c , 0.168 to

1.040 in.²; (2) h/b , 0.278 to 1.250; (3) R_m , 0.098 to 0.535 in.; (4) t_1/a_1 , 0.3 to 3.3; (5) Z , 12 to 150; (6) a_1/a_2 , 0.2 to 0.6; (7) D , 3.66 to 5.55 in.; (8) D/R_m , 12.5 to 56.4; (9) side clearance, c_1 , 0.002 to 0.012 in.; and (1) tip clearance, c_2 , 0.002 to 0.028 in. For this range of test variables, K' in Equation (27) was found to be fairly constant at 1.4. Reference (15) indicates that this K' and the one used in the boundary condition for u at the moving wall ($u = K'U_2$) are the same; however, it seems unlikely that the tangential fluid velocity could be 1.4 times the impeller velocity. A value of K' of 1.4 in Equation (27) seems reasonable, however, as λ would be 0.7 at $\psi = 0$, which agrees with results of other investigations.

Shimosaka (15) reported empirical relationships for K'' and n in Equation (27) as follows:

$$K'' = 0.052 \left(\frac{D}{D_m} \right)^{-0.54}, \quad n = 1.60 \text{ for } t_1/a_1 \leq 1.3, \quad (28a)$$

and

$$K'' = 0.24 \left(\frac{D}{R_m} \right)^{-0.93}, \quad n = 0.47 \text{ for } t_1/a_1 > 1.3. \quad (28b)$$

The effect of internal clearances was not incorporated into Equation (27), but the authors reported this effect in terms of the ratios R_m/c_1 and R_m/c_2 . Their tests showed that pump performance deteriorated rapidly if these ratios were decreased below 20. Other conclusions reported were: (1) optimum number of vanes was 24 to 72; (2) optimum a_1/a_2 was 0.45; (3) maximum efficiency of about 50 percent may be expected; and (4) the flow at maximum efficiency was 63 to 76 percent

of the maximum flow at zero head.

Work of Hasinger and Pfleiderer

Two other reports that describe approaches somewhat different than the above but that probably belong in this theory classification are those by Hasinger (17) and Pfleiderer (18). Hasinger's theory could perhaps be classified as regenerative since he considered an "orderly circulatory flow superimposed on the through flow"; however, he also used terms such as "impeller drag," "traction," and "impact diffusion" which indicate some turbulent mixing in the momentum exchange. Based on some visualization studies in which air bubbles employed as tracers were photographed, Hasinger assumed a spiral flow cycle at zero through-flow that consisted of the following phases: (1) acceleration in the blading; (2) impact diffusion in the channel due to the presence of the stripper; (3) re-acceleration in the channel in the reverse direction; and (4) re-entry into the blading. Using momentum, energy, and continuity relationships, he wrote equations for the different processes from which certain conclusions regarding optimum geometry and conditions were drawn. Hasinger was primarily interested in methods to improve the peripheral pump, and a number of his conclusions should be mentioned:

1. Small wheel diameter and high speed, resulting in greater circulatory flow intensity, should be used.
2. Maximum efficiency occurs at a value of Q/UA_c of 0.5 to 0.6 and may reach 50 percent with optimum geometry.
3. A relatively small channel area should be used, with an

axial depth about 60 percent of the blade depth, a radial depth about half the blade depth, and a blade length-to-depth ratio of about 1.5. A blade inlet angle should be set to correspond to a value of Q/UA_c of 0.5 to 0.6, and the blades should be bent slightly forward at the outer diameter.

4. Peripheral pump efficiency can be better than centrifugal pump efficiency below a specific speed of 25, primarily due to surface friction effects (compare with Figure 4, page 8).
5. A head coefficient equation based on the assumed optimum conditions was presented as

$$\psi' = \frac{g_c H}{U^2} = 7 \frac{r_g}{\sqrt{A_c}} \left(1 - \frac{Q}{UA_c} \right)^2. \quad (29)$$

Pfleiderer (18) analyzed a pump with a channel entirely at the side of the impeller such as is shown in Figure 2(b), page 4. He assumed a "strong rotational flow" (circulatory, or meridional) superimposed upon a through flow caused by pressure differences at the front and back of the blades. The energy transfer was considered to take place through mixing of these two flows in the channel. The intensity of the meridional flow was assumed to be independent of the through flow and dependent only upon resistance to flow in the meridional path, pump geometry, and impeller speed. Assuming incompressible flow and neglecting channel curvature, channel friction, and inlet and outlet losses, Pfleiderer developed pump equations by applying momentum relationships to a control volume. With nomenclature already introduced, his head equation may be written as

$$(\psi') = \frac{g_c(H)}{U^2} = \frac{Q_m}{U^2 A_c} \left(u_m - \frac{Q}{A_c} \right) \quad (30)$$

Here, (ψ') and (H) are "ideal" values with the above-mentioned losses neglected. Also, u_m is the average tangential velocity of the meridional flow leaving the impeller, and U is the impeller velocity at a radius defined as $r_g + 1/3 (r_2 - r_o)$ (nomenclature in Figure 5(a), page 17, used for geometry of Figure 2(b), page 4).

According to Equation (30) and the assumption that Q_m is independent of Q , the "ideal" head-flow curve is a straight line for the type of pump discussed by Pfleiderer. When a loss term proportional to the flow rate squared was subtracted from the right side of Equation (30), a head-flow curve convex upward was obtained that agreed with experiment. This equation may be written as

$$\psi' = \frac{g_c H}{U^2} = \frac{Q_m}{U A_c} \left(\frac{u_m}{U} - \frac{Q}{U A_c} \right) - k_t^{iv} \left(\frac{Q}{U A_c} \right)^2, \quad (31)$$

where k_t^{iv} is a loss coefficient similar to previous ones. The usual value of u_m/U , which may be likened to a slip factor, was reported as 0.85 and was assumed independent of through flow. Also, the maximum efficiency was usually observed at a $Q/U A_c$ value of approximately 0.5. With these two values substituted in Equation (31), an expression for the dimensionless meridional flow at maximum efficiency can be written as

$$\frac{Q_m}{U A_c} = 2.86 \left[\psi' + k_t^{iv} \left(\frac{Q}{U A_c} \right)^2 \right]. \quad (32)$$

Pfleiderer reported some experimental work by others in which a head coefficient as great as 3.25 and an overall efficiency of 46 percent at the maximum efficiency point was obtained for the type of geometry shown in Figure 2(b), page 4.

III. TURBULENT THEORIES - MOMENTUM TRANSFER THROUGH TURBULENT SHEAR

The theoretical developments in this theory classification do not utilize the meridional flow mechanism, although the existence of such a flow is not always denied. The basic idea is that a very rough surface (the impeller) moves through the fluid, creating shear forces and momentum transfer. This is the simplest of the theories in that one-dimensional flow relationships are used and no attempt is made to separately account for the effects of any meridional flow or any other type of flow into and out of the impeller. However, the theoretical equations still contain coefficients which must be determined experimentally.

Work of Iversen. The paper of H. W. Iversen (19) is probably the one most frequently referenced in discussions of this theory. It contains a simple presentation of the theory with verification by experimental data. His analytical model is shown in Figure 7(a). Writing the force balance indicated in the Figure, combining it with an energy balance on the same control volume, and expressing the power input as the product of the impeller velocity and the shear force on the impeller, Iversen derived the pump equations. In the development,

a. ANALYTICAL MODEL

b. THEORETICAL PERFORMANCE CURVES

FIGURE 7

ANALYTICAL MODEL AND THEORETICAL PERFORMANCE CURVES
FOR TURBULENT SHEAR THEORY [IVERSEN (19)]

he assumed no leakage, no bearing losses, no shaft seal losses, constant shear stress, negligible fluid viscosity effects (inertial effects predominant), and no disk friction losses. He further assumed that the impeller and casing shear stresses, τ_i and τ_w , could be expressed in terms of shear coefficients and relative velocities as follows:

$$\tau_i = \frac{f_i \rho}{2g_c} \left(U - \frac{Q}{A_c} \right)^2, \quad (33a)$$

and

$$\tau_w = \frac{f_w \rho}{2g_c} \left(\frac{Q}{A_c} \right)^2, \quad (33b)$$

where

$$\tau_i = \text{impeller shear stress, lb}_f/\text{ft.}^2,$$

$$\tau_w = \text{casing or wall shear stress, lb}_f/\text{ft.}^2,$$

$$f_i, f_w = \text{dimensionless shear coefficients,}$$

$$U = \text{average effective impeller velocity, ft./sec., and}$$

$$\frac{Q}{A_c} = \text{average through-flow velocity, ft./sec.}$$

With the above assumptions and definitions, the following equations were derived:

$$\psi' = \frac{g_c H}{U^2} = \frac{f_i A_i}{2 A_c} \left[\left(1 - \frac{Q}{UA_c} \right)^2 - \frac{f_w A_w}{f_i A_i} \left(\frac{Q}{UA_c} \right)^2 \right], \quad (34)$$

$$P_{in} = \tau_i A_i U = \frac{f_i A_i \rho U^3}{2g_c} \left(1 - \frac{Q}{UA_c} \right)^2, \quad (35)$$

and

$$\eta = \frac{\rho Q H}{P_{in}} = \frac{Q}{U A_c} \left[1 - \frac{f_w A_w}{f_i A_i} \left(\frac{Q/U A_c}{1 - Q/U A_c} \right)^2 \right]. \quad (36)$$

In these equations, the shear coefficients, f_i and f_w , and the effective impeller velocity, U , must be determined experimentally. The end points of the head-flow curve and the power at zero flow were used to obtain three equations from which these three variables could be evaluated. Therefore, the reported agreement between the equation and the experimental data is really in the shape of the curves. Figure 7(b) shows the theoretical curves for three values of the ratio of casing to impeller friction terms, $f_w A_w / f_i A_i$. In the Figure, H_{so} is the shut-off head (head at zero flow). Iversen assumed that the inlet and outlet losses could be included in the casing shear coefficient, f_w . Based upon tests conducted in water at speeds of 900, 1200, and 1500 rpm., Iversen concluded that the impeller shear coefficient was a function not only of impeller geometry but of channel geometry as well. Also, the effective impeller velocity, U , appeared to be a weighted average which depended upon channel geometry. From his experiments, Iversen reported that the usual model laws associated with other type pumps - also applied to geometrically similar peripheral pumps, such as

$$Q \propto N D^3 \propto U D^2, \quad (37a)$$

$$H \propto N^2 D^2 \propto U^2, \quad (37b)$$

and

$$P_{in} \propto \rho N^3 D^5 \propto \rho U^3 D^2. \quad (37c)$$

Work of Weinig. A report by F. S. Weinig (20) presented a theory similar to that of Iversen but somewhat more comprehensive in that an attempt was made to include the effects of various losses in the equations. Weinig based his development upon what was called "the traction principle," which was slightly different than the friction concept of a very rough impeller in that pressure forces on the blades and fluid exchange between impeller and channel were believed to exist. However, he used the same definitions of shear stresses and shear coefficients as did Iversen, and his first head equation was the same as Equation (34), page 41, except that the head and volume flow were "internal" values. Weinig then accounted for nozzle and leakage losses through the use of appropriate coefficients and derived a pump equation which may be written as

$$\psi' = \frac{g_c H}{U^2} = \frac{f_i A_i}{2A_c} \left\{ \left[1 - \left(\frac{Q}{UA_c} + \sqrt{\frac{\psi' + k_N (Q/UA_c)^2}{k_L}} \right) \right]^2 - \frac{f_w A_w}{f_i A_i} \left[\frac{Q}{UA_c} + \sqrt{\frac{\psi' + k_N (Q/UA_c)^2}{k_L}} \right]^2 - \frac{2A_c k_N}{f_i A_i} \left(\frac{Q}{UA_c} \right)^2 \right\}, \quad (38)$$

where

k_N = a dimensionless nozzle loss coefficient,

k_L = a dimensionless leakage loss coefficient, and

U = an average impeller velocity.

Other terms are defined the same way as in Equation (34), except that here the casing shear coefficient, f_w , does not include nozzle losses as Iversen assumed. In Equation (38), the last term expresses the nozzle losses and the square root term expresses the internal leakage losses.

Probably Weinig's major contribution was his discussion of the various losses and how they affected the head coefficient and efficiency. His presentation may be summarized in Figure 8, which shows qualitatively the effect of slip, disk friction, casing friction, leakage, and nozzle losses on the head coefficient and efficiency. The labeled areas between curves represent the magnitude of the particular loss, which may vary widely depending upon design. Because of the inherent slip loss of the traction principle, the theoretical head coefficient, ψ_{TH} , that would result if all the input power were converted to pressure rise is unattainable. For the same reason, the straight line on the efficiency plot represents the maximum efficiency even if all other losses were eliminated. With proper experimental verification, plots such as these can be useful to the designer in reducing losses.

Other published work. In a discussion of Iversen's paper, W. E. Wilson expressed agreement with Iversen's theory and stated that the concept of circulatory flow and centrifugal action "is both unnecessary and erroneous." In an earlier article of his own on the subject (21), Wilson discussed primarily operation with laminar flow. He developed equations based upon the classical theory of laminar flow between two parallel plates with one moving. Miyagi and Miyadzu (22)

FIGURE 8

QUALITATIVE PERIPHERAL PUMP LOSSES [WEINIG (20)]

presented a similar treatment for laminar flow in which they showed the theoretical maximum efficiency to be $33\frac{1}{2}$ percent. They also presented a theory for turbulent flow in which they used the Prandtl mixing length concept in order to write an expression for the shear stress. In this respect, their development was similar to Senoo's and the final head equation contained two empirical constants. However, unlike Senoo, they did not consider a momentum-exchange flow between the impeller and channel.

Two British investigators, E. Crewdson and E. A. Jackson (23) also believed that the turbulent shear theory best explained the pumping mechanism. Crewdson wrote that the effective radius for computing an average impeller velocity should be taken as $\sqrt[5]{(r_2^5 + r_0^5)/2}$, based primarily upon power considerations (see Figure 5, page 17, for radii identification). Jackson reported some experimental results on impeller blade shapes which indicated that the blades should be rounded at the trailing edges and slanted forward at 45° . This was determined from tests on a stationary model in which resistance to flow was measured.

IV. COMPRESSIBLE FLOW THEORIES

All previous discussions in this chapter of theories proposed for the peripheral pump have considered incompressible flow. There are only two known reports in the open literature which theoretically treat the case of compressible flow, although the United Kingdom Atomic Energy Authority (24,25) and the Oak Ridge National Laboratory (26,27) have published data obtained on units operating in the compressible flow

region. Most of these data, however, were obtained at relatively low Mach numbers, so that compressibility effects were not of major importance. The report by Balje (28) is primarily a discussion of the performance of the drag turbine in compressible flow, but equations for compressor action were also derived. Balje's treatment was like that of Iversen and his equation for compressor head was identical (i.e., the development was based on the turbulent-shear concept and one-dimensional flow). Balje discussed the effects of the channel velocity that varied with circumferential position, but his main concern was the operation of a turbine as a compressor over a portion of the circumference.

Burton (29) presented a more comprehensive treatment of compressor action, but he, too, followed the same approach as did Iversen and Balje. His assumptions were similar to Iversen's in that he considered one-dimensional flow with no radial components, no end effects, and no leakage. Considering density and channel area change with circumferential position, Burton wrote the continuity equation to include flowing mass in both channel and impeller. Writing shear stress equations similar to Iversen's, he then made a force-momentum balance on an incremental control volume. With these expressions plus an energy balance on the control volume and the perfect gas relation, he then had four differential equations for determining internal compressor performance with speed, gas, and geometry known. (Of course, these equations included the unknown casing and impeller shear coefficients as did Iversen's.) Burton transformed his equations into dimensionless

expressions that showed dependence upon the impeller tip Mach number and on the Reynolds number through the shear coefficients. The set of differential equations were programmed for digital computer solution, but satisfactory solutions are not yet available. This writer did not pursue the solutions since some of the important assumptions made do not agree with experimental evidence--especially the assumption of one-dimensional flow. However, the equations may be useful in predicting effects of Mach number and channel area variation and should be solved.

V. ANALYSIS AND COMPARISON OF THEORIES

The preceding pages of this chapter contain condensed presentations of the known published theories that attempt to explain the pumping mechanism of the peripheral pump. The purpose in presenting these discussions was to give the reader a summary of the "state of the art" of the peripheral pump or compressor. The basic differences of opinion concerning the working principles were emphasized, and certain other observations pertinent to each discussion were made. In all cases, the pump equations contained one or more coefficients which must be determined experimentally; therefore, there is no known theory which can completely predict the performance of a new pump geometry, even for incompressible flow. However, each theory seemed to fit experimental data well within its particular range of application after the coefficients were determined. Apparently, then, the concepts of momentum transfer are not as opposed as they appear, and the various

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theories may be compatible in many ways.

Senoo's Analysis

That the theories could be compatible was suggested in a paper by Senoo (4) in which he attempted to resolve the differences in the basic assumptions. Approaching the problem from two viewpoints, the torque-pressure rise relationship and the through flow-pressure rise relationship, Senoo added assumptions of his own in reducing most of the theories to two equations. He neglected casing friction forces and stated that the equation relating impeller torque and internal pressure rise, $T/\theta = A_c r_g (P/\theta)$, was independent of the pumping mechanism. Then, making assumptions regarding the "momentum exchange flow," he obtained two equations for the internal head rise as

$$\left(\psi/\theta \right) \left(\frac{A_c}{D(a_1 + a_2)} \right) = K_3 \left(1 - \frac{Q}{U_2 A_c} \right) \text{ if } V_m \propto U_2, \quad (39a)$$

or

$$\left(\psi/\theta \right) \left(\frac{A_c}{D(a_1 + a_2)} \right) = K_4 \left(1 - \frac{Q}{U_2 A_c} \right)^2 \text{ if } V_m \propto U_2 - Q/A_c, \quad (39b)$$

where

ψ = the dimensionless head coefficient, $g_c H/U_2^2$,

U_2 = impeller peripheral velocity, ft./sec.,

D = impeller diameter, ft.,

A_c = channel area, ft.²,

a_1, a_2 = blading width and height, ft.,

Q = volume flow, ft.³/sec., and

V_m = velocity of momentum-exchange flow, ft./sec.

As noted, the choice of equations depends upon the assumption regarding V_m . Senoo believed that the turbulent theories could also be reduced to these equations since turbulence is a kind of momentum exchange. In the case of turbulent theories, the choice of equation depends on whether it is believed that the turbulence is induced primarily by the impeller (Equation 39a) or is similar to pipe turbulence but of greater intensity (Equation 39b).

Further Analysis

Some additional comments may be made regarding analysis and comparison of the theories. In a comparison of the equations, it should be realized that the definitions of impeller velocity and mean radius vary somewhat. For example, most of the turbulent theories used an average impeller velocity, which was not always defined, while the regenerative theory used the impeller tip velocity. Therefore, some correction in the head coefficient and capacity coefficient is necessary before direct comparison can be made. The assumption is adopted here that an average impeller velocity, U , can be defined as $r_g \omega$, where r_g is the radius to the centroid of the channel and ω is the impeller

angular velocity. With this assumption, $\psi' = \left(\frac{r_2}{r_g} \right)^2 \psi$ and

$\lambda' = \left(\frac{r_2}{r_g} \right) \lambda$, so that now the various equations may be compared in

the same dimensionless terms. The coefficients based on the tip velocity, ψ and λ , were used in a comparison of the fit of several

theoretical equations to data obtained on the compressor shown in Figure 1, page 3. The data used were measured in air at 5000 rpm. at suction conditions of 200°F. and 10 psia. Although air was used, the flow was nearly incompressible since the maximum density change was only 5.8 percent. This unit represents an independent design for comparison of the theories, having the following dimensional data (see Figure 5, page 17):

$$\begin{array}{lll}
 a_1 = 0.500 \text{ in.} & c_1 = 0.007 \text{ in.} & A_c = 1.940 \text{ in.}^2 \\
 a_2 = 1.500 \text{ in.} & c_2 = 0.005 \text{ in.} & Z = 48 \\
 a = 1.200 \text{ in.} & h = 0.200 \text{ in.} & \Delta\theta_a = 5.32 \text{ rad.} \\
 r_o = 3.500 \text{ in.} & b = 0.500 \text{ in.} & t_1 = 0.462 \text{ in.} \\
 r_2 = 5.000 \text{ in.} & r_g = 4.443 \text{ in.} & t_2 = 0.094 \text{ in.}
 \end{array}$$

The data points from the air tests are shown in Figure 9. A smooth curve drawn through these points was extended to determine λ at $\psi = 0$. In order to determine the empirical coefficients, each theoretical equation (except Shimosaka's) was fitted to the data at $\psi = 0$, $\lambda = 0.74$ and $\psi = 4.77$, $\lambda = 0$ and, if necessary to evaluate a third coefficient, at $\psi = 2.04$, $\lambda = 0.5$, which was near the maximum efficiency point. The fit of the theories to the data may be seen in Figure 9. The equations with dimensional data and coefficients evaluated are given below.

MIT group, Equation (14). With the assumption that $d = a_1 = 0.500 \text{ in.}$, this equation becomes

FIGURE 9

COMPARISON OF INCOMPRESSIBLE FLOW THEORIES

$$\psi = \frac{7.183}{\sqrt{k_c}} (1.088\sigma - \lambda) \sqrt{6.667\sigma - 3.33\sigma^2 - 2.41 + \sigma\lambda - 2\lambda^2 - k_t'\lambda^2}, \quad (14a)$$

with $k_c = 2.49$, $\sigma = 1.00$, and $k_t' = 2.16$. Since a value of 1.00 for the slip factor, σ , seems unlikely, a more reasonable value of 0.85 was assumed for comparison. Both curves are shown in Figure 9.

Wright, Equation (15). With dimensional data inserted, this equation becomes

$$\psi = 9.62 k_c' (1 - 1.125\lambda) - k_t'' \lambda^2, \quad (15a)$$

with $k_c' = 0.496$ and $k_t'' = 1.460$.

Senoo, Equation (23). The effective channel dimension, δ , was calculated according to Equation (24), page 32, as $\delta = 0.463$ in. The ineffective channel length, L' , was estimated as r_g times (stripper angle + inlet angle) and assumed constant. Generally, L' will be a function of λ , but data for determining that function were not available. With these assumptions, the equation can be written as

$$\psi = 120.6 \kappa [0.85 - 1.125\lambda] - k_t''' \lambda^2, \quad (23a)$$

with $\kappa = 0.0465$ and $k_t''' = 0.183$.

Shimosaka and Yamazaki, Equation (27). This equation was not fitted to the air data; instead, the empirical Equations (28a) and (28b), page 35, were used. Here, $R_m = 0.462$ in. and $t_1/a_1 = 0.924$, so that $n = 1.60$ and $K'' = 0.00988$. Also, $K' = 1.4$, so that

$$\psi = 3.016 (1 - 1.429\lambda). \quad (27a)$$

Iversen, Equation (34). With impeller surface area, A_i , calculated as 117.81 in.² and channel surface area, A_w , approximated as 186.76 in.², Iversen's equation becomes

$$\psi = 24.00 f_i (1 - 1.125\lambda)^2 - 48.13 f_w \lambda^2, \quad (34a)$$

with $f_i = 0.199$ and $f_w = 0.00508$.

Weinig, Equation (38). Efforts to fit Weinig's equation,

$$\begin{aligned} \psi = 24.00 f_i \left[1 - 1.125 \left(\lambda + \sqrt{\frac{\psi + k_N \lambda^2}{k_L}} \right) \right]^2 \\ - 48.13 f_w \left[\lambda + \sqrt{\frac{\psi + k_N \lambda^2}{k_L}} \right]^2 - k_N \lambda^2, \end{aligned} \quad (38a)$$

to the experimental data at four points in order to evaluate the four unknowns, f_i , f_w , k_L , and k_N , were unsuccessful, even with computer methods. Attempts at simultaneous solutions resulted in negative numbers under the square root signs. It was concluded that the basic shape of the equation was not that of the data. Several values of the empirical coefficients were assumed to determine the general effect of each on the curve shape, and a typical curve is included in Figure 9 for $f_i = 0.470$, $f_w = 0.002$, $k_N = 0.2$, and $k_L = 50$. This curve nearly coincided with Iversen's.

In summary, it can be seen that only the MIT equation fitted this particular set of data--and quite well, although, of course, the equation was fitted at three points. Shimosaka's empirical equation predicted considerably poorer performance than was measured on this

machine. Iversen's and Weinig's equations did not predict the shape of the data at all, and Senoo's equation had too little curvature. Other conclusions which may be stated concerning general performance and geometric characteristics of the peripheral pump resulting from the literature study on incompressible flow are as follows:

1. Pressure rise in the effective pumping passage was linear.
2. Channel velocity was constant under given conditions.
3. Maximum efficiency observed was 40 to 50 percent.
4. Maximum efficiency occurred at $\lambda = 0.45$ to 0.6 .
5. Maximum λ at $\psi = 0$ was about 0.7 .
6. Head coefficients of 6 at shutoff and 3 at maximum efficiency were obtained.
7. Optimum geometries varied somewhat, but they may be summarized as follows: number of vanes, 24 to 72; a_2/a_1 , 1.5 to 3; h/b , 0.4; and R_m/c_1 and R_m/c_2 , greater than 20.

Whether any of these conclusions can be applied to compressible flow operation will be discussed in the following chapters.

CHAPTER III

PREDICTED PERFORMANCE IN COMPRESSIBLE FLOW

The previous chapter dealt mainly with existing theories for peripheral pump performance in incompressible flow. The discussions of these theories were intended to present the "state of the art"--to point out significant features of the theories and to briefly compare them. This chapter will discuss some of the limitations of the incompressible flow theories with regard to peripheral compressor performance in compressible flow. The major portion of this chapter will be devoted to the development of thermodynamic and aerodynamic relations and a discussion of a hypothesis for compressible flow operation, with special consideration of the influence of high Mach numbers.

I. LIMITATIONS OF INCOMPRESSIBLE FLOW THEORIES

One important limitation of the existing theories already mentioned is that there are two or more coefficients which must be determined experimentally. To date, there is not enough information to predict these coefficients for a new pump design. This fact, coupled with the differences among the basic pumping mechanisms assumed in the theories, poses a problem even for incompressible flow.

There are a number of assumptions made for the incompressible flow case which are not compatible with compressible flow. One of these, of course, is that of constant fluid density. Introduction of

variable density renders invalid the assumption that certain variables are independent of angular position. For example, the average through-flow velocity in the channel, $\bar{V}_\theta$ or $\bar{u}$, can no longer be assumed constant at a given inlet volume or mass flow (for constant channel area, A_c). Therefore, the volume flow rate through the compressor channel does not remain independent of angular position--even if leakage is ignored. Other variables perhaps not so obvious are probably also affected by variable fluid density. In the regenerative concept, these likely include the circulatory or meridional flow intensity, Q_m , and velocity, V_m . Consequently, one would suspect that $dP/d\theta$, or dP/dx , is not constant in a peripheral compressor. Reasons for such a suspicion will be given later.

Not only would the above variables be affected by compressibility, but the various experimental coefficients such as k_c , k_t , κ , f_i , f_w , and σ would also likely become functions of angular position unless they are only weakly dependent upon through-flow velocity. Of course, average values may be taken as in the incompressible flow case, but this could result in error if the functions are not nearly linear or known.

An additional effect that is not usually important in incompressible flow but must be considered in compressible flow operation is the fluid temperature rise. This is particularly important at low flows, where the efficiency becomes quite low. High temperature rise at low flows can have significant effect upon the head-flow curve.

II. THERMODYNAMIC--AERODYNAMIC CONSIDERATIONS

An exact theoretical treatment of turbulent, compressible flow in a peripheral compressor would be extremely difficult and probably impossible with present day knowledge. Such an analysis of even the much better known centrifugal compressor has not been successfully made to the writer's knowledge. Theoretical treatments of flow through centrifugal compressors have generally consisted of relaxation solutions to frictionless flow equations, whereas actual design is based primarily on experimentally obtained information. Therefore, since the flow in the peripheral compressor is even more complex, a complete theoretical treatment was considered beyond the scope of this thesis. Although a one-dimensional approach such as reported in References (28) and (29) could have been followed, such an analysis was not considered to adequately describe the physical flow patterns known to exist in the compressor. Therefore, a semi-empirical approach was taken wherein the compressor characteristics were assumed to follow certain functional relationships derived from dimensional analysis. Then, the effects of compressibility and geometric factors on performance were predicted in a qualitative manner.

Fundamental concepts and definitions. Five basic physical "laws" may be used to describe the motion and thermodynamics of the gas passing through a peripheral compressor: (1) the equation of continuity, which is a statement of the conservation of mass; (2) the

first law of thermodynamics, which is a statement of the conservation of energy; (3) Newton's second law of motion, one form of which equates the impeller torque with the net change in angular momentum of the gas; (4) the second law of thermodynamics, which is used to define an ideal compression process; and (5) the equation of state. The equations representing the mathematical statements of these laws can be found in some form in most discussions of turbomachinery and fluid dynamics in the literature. The continuity, angular momentum, and energy equations are presented here as integral vector relations--forms not frequently presented in the literature. Brief derivations of these relations, based on some compressor technology lectures presented by D. W. Burton at the Oak Ridge Gaseous Diffusion Plant in 1963 (30), are given in Appendix B.

For a control volume, v , entirely filled with fluid and enclosed by a continuous surface, S , the continuity equation may be written as

$$\int_v \frac{\partial \rho}{\partial t} dv + \int_S \rho \vec{n} \cdot \vec{V} dS = 0, \quad (40)$$

where

ρ = fluid density,

t = time,

$\vec{V}$ = fluid velocity vector, and

$\vec{n}$ = unit normal vector directed outward from the surface, S .

This equation states simply that the net flux of material into the volume equals the rate of increase of material within the volume. Applied to the control volume in a peripheral compressor shown in

Figure 10, including leakage, this equation becomes for steady flow

$$\int_{A_c} \rho \vec{n} \cdot \vec{V} dA_c + \int_{A_b} \rho \vec{n} \cdot \vec{V} dA_b + \int_{A_L} \rho \vec{n} \cdot \vec{V} dA_L = 0, \quad (41)$$

where A_c , A_b , and A_L are channel, blade, and leakage areas, respectively.

In the right-handed cylindrical coordinate system (R , θ , Z) with the Z -axis along the shaft centerline, the R -axis in the radial direction of the impeller, and the θ -axis in the direction of rotation, $\vec{n} \cdot \vec{V} = V_\theta$ over surfaces A_{c,θ_2} and A_{b,θ_2} and $\vec{n} \cdot \vec{V} = -V_R$ over surface A_L . Therefore, Equation (41) can be written as

$$\int_{A_{c,\theta_2}} \rho V_\theta dA_c - \int_{A_{c,\theta_1}} \rho V_\theta dA_c + \int_{A_{b,\theta_2}} \rho V_\theta dA_b - \int_{A_{b,\theta_1}} \rho V_\theta dA_b - \int_{\theta_1}^{\theta_2} \rho V_R (2c_1 r_o) d\theta = 0. \quad (42)$$

An important observation to be made from Equation (42) is that for compressible flow in a peripheral compressor with constant channel area, the average channel velocity, $\bar{V}_{\theta,c}$, (or volume flow $Q_c = \bar{V}_{\theta,c} A_c$) is not generally constant but is influenced by three factors: (1) leakage through the clearance, c_1 ; (2) variable density with circumferential position; and (3) variable mass in the impeller blading with circumferential position. The significance of this fact and its consideration in design will be discussed later.

The first law of thermodynamics for gas flow through a control volume can be written as

FIGURE 10

CONTROL VOLUME IN PERIPHERAL COMPRESSOR

$$\begin{aligned}
Q + W &= \Delta E_{\text{FLOW}} + \Delta E_{\text{STORAGE}} = [\Delta U + \Delta FE + \Delta KE + \Delta PE]_{\text{FLOW}} \\
&\quad + [\Delta U + \Delta KE + \Delta PE]_{\text{STORAGE}} \\
&= \left[m_{\text{out}} \left(u + \frac{V^2}{2g_c} + \frac{P}{\rho} + \frac{gZ}{g_c} \right)_{\text{out}} - m_{\text{in}} \left(u + \frac{V^2}{2g_c} + \frac{P}{\rho} + \frac{gZ}{g_c} \right)_{\text{in}} \right]_{\text{FLOW}} \\
&\quad + \left[m_{\text{final}} \left(u + \frac{V^2}{2g_c} + \frac{gZ}{g_c} \right)_{\text{final}} - m_{\text{initial}} \left(u + \frac{V^2}{2g_c} + \frac{gZ}{g_c} \right)_{\text{initial}} \right]_{\text{STORAGE}},
\end{aligned} \tag{43}$$

where

Q = heat added to the gas, ft.-lb_f.,

W = work done on the gas, ft.-lb_f.,

m = mass, lb_m.,

u = internal energy of the gas per unit mass, ft.-lb_f./lb_m.,

$\frac{V^2}{2g_c}$ = kinetic energy of the gas per unit mass, ft.-lb_f./lb_m.,

P = pressure of the gas, lb_f./ft.²,

ρ = gas density, lb_m./ft.³, and

$\frac{gZ}{g_c}$ = potential energy of the gas per unit mass, ft.-lb_f./lb_m.

Here, u , V , P , ρ , and Z are average values at the specified location or time. With the assumptions of steady flow and negligible difference in potential energy and the definitions of enthalpy, h , and total or stagnation enthalpy, h_t , as

$$h = u + P/\rho, \tag{44}$$

and

$$h_t = h + \frac{V^2}{2g_c} = u + \frac{P}{\rho} + \frac{V^2}{2g_c}, \quad (45)$$

the energy equation may be written as

$$Q + W = m (h_{t,2} - h_{t,1}). \quad (46)$$

A steady flow integral vector relation for a control volume, v , with the surface, S , can be derived (Appendix B) as

$$\int_S \rho h_t \vec{V} \cdot \vec{n} dS = - \int_S \vec{q} \cdot \vec{n} dS + \int_v \rho \vec{F} \cdot \vec{V} dv + \int_S \vec{\tau} \cdot \vec{V} dS, \quad (47)$$

where

$\vec{q}$ = the heat flux vector,

$\vec{n}$ = unit normal vector outward from the control surface, S ,

$\vec{F}$ = body force vector acting on the fluid (such as gravity), and

$\vec{\tau}$ = viscous stress vector.

Equation (47) states that the net flux of total enthalpy through the control surface is equal to the heat addition plus the work done on the volume by body forces and viscous surface tractions. Applied to the control volume of Figure 10, page 61, denoting the rate of work done on the gas per unit mass as $\dot{E}_{gas}$, the time-averaged energy equation becomes

$$\begin{aligned} \dot{E}_{gas}|_v = & \int_{A_c, \theta_2} \overline{\rho h_t V_\theta} dA_c - \int_{A_c, \theta_1} \overline{\rho h_t V_\theta} dA_c + \int_{A_b, \theta_2} \overline{\rho h_t V_0} dA_b \\ & - \int_{A_b, \theta_1} \overline{\rho h_t V_\theta} dA_b + \int_{S_{cas}} \overline{\vec{q} \cdot \vec{n}} dS_{cas} + \int_{S_{imp}} \overline{\vec{q} \cdot \vec{n}} dS_{imp} - \int_{\theta_1}^{\theta_2} \overline{\rho h_t V_R (2c_1 r_o)} d\theta, \end{aligned} \quad (48)$$

where heat flow through the control surface is assumed to take place only at the casing walls and at the impeller surfaces. Usually, flow through an uncooled compressor is assumed adiabatic. However, this may not be true in the peripheral compressor under certain conditions of low flows, high gas temperatures, relatively larger casing area for heat transfer, and long flow path. For the compressor as a whole, the energy equation in terms of energy per unit mass may be written as

$$E_{\text{gas}} + Q = \bar{h}_{t,d} - \bar{h}_{t,s}, \quad (49)$$

where the total enthalpies at the discharge and suction of the compressor are mass-averaged values and Q is heat added per unit mass.

The integral relation equating the flux of angular momentum with the moment of forces about the Z-axis for a control volume, v , with the surface, S , in the absolute system may be derived (Appendix B) as

$$\int_S \overline{\rho R V_{\theta} \vec{n} \cdot \vec{V}} dS = \int_v \overline{\rho R F_{\theta}} dv - \int_S \overline{\vec{n} \cdot \vec{e}_{\theta}} R P dS + \int_S \overline{\vec{n} \cdot \vec{R}} \tau_{\theta} dS, \quad (50)$$

for time-averaged flow. A similar equation in the relative flow system is

$$\int_S \overline{\rho R V_{\theta} \vec{n} \cdot \vec{W}} dS = \int_v \overline{\rho R F_{\theta}} dv - \int_S \overline{\vec{n} \cdot \vec{e}_{\theta}} R P dS + \int_S \overline{\vec{n} \cdot \vec{R}} \tau_{\theta} dS. \quad (51)$$

In these equations, F_{θ} is the θ -directed body force, which will be assumed negligible, $\vec{W}$ is the velocity vector in the relative system,

$$\begin{aligned} \vec{W} &= \vec{e}_R W_R + \vec{e}_{\theta} W_{\theta} + \vec{e}_Z W_Z \\ &= \vec{V} - \vec{\omega} \times \vec{r} \\ &= \vec{e}_R V_R + \vec{e}_{\theta} V_{\theta} + \vec{e}_Z V_Z - \vec{e}_{\theta} R \omega, \end{aligned} \quad (52)$$

and $\vec{\tau}_\theta$ is the viscous stress vector in the θ -direction,

$$\vec{\tau}_\theta = \vec{e}_R \tau_{R\theta} + \vec{e}_\theta \tau_{\theta\theta} + \vec{e}_Z \tau_{Z\theta}. \quad (53)$$

Equations (50) and (51) may be applied to the peripheral compressor by writing them for the control volumes and surfaces of Figure 11. For the absolute system consisting of the volume, v^* , with the surface, S^* , Equation (50) with the body force, F_θ , neglected becomes

$$\int_{S^*} \overline{\rho R V_\theta \vec{n} \cdot \vec{V}} dS = \int_{S^*} \overline{\vec{n} \cdot R \vec{\tau}_\theta} dS - \int_{S^*} \overline{\vec{n} \cdot \vec{e}_\theta} R P dS, \quad (50a)$$

or

$$\int_{S_1, S_2, S_4^*} \overline{\rho R V_\theta \vec{n} \cdot \vec{V}} dS = \int_{S_1, S_2, S_3, S_4^*} \overline{\vec{n} \cdot R \vec{\tau}_\theta} dS - \int_{S_3} \overline{\vec{n} \cdot \vec{e}_\theta} R P dS, \quad (50b)$$

since there is no flow through the casing surface, S_3 , and $\vec{n} \cdot \vec{e}_\theta = 0$ on an axisymmetric surface (the compressor inlet and outlet surfaces, S_1 and S_2 , are considered as portions of a cylindrical surface). For the relative flow system consisting of the control volume, v^{**} , with the control surface, S^{**} , Equation (51) with the body force, F_θ , neglected becomes, using Equation (52)

$$\begin{aligned} \int_{S_4^{**}} \overline{\rho R V_\theta \vec{n} \cdot \vec{W}} dS &= \int_{S^{**}} \overline{\rho R V_\theta \vec{n} \cdot \vec{V}} dS - \int_{S^{**}} \overline{\rho R^2 V_\theta \omega \vec{n} \cdot \vec{e}_\theta} dS \\ &= \int_{S^{**}} \overline{\vec{n} \cdot R \vec{\tau}_\theta} dS - \int_{S^{**}} \overline{\vec{n} \cdot \vec{e}_\theta} R P dS, \end{aligned} \quad (51a)$$

or

$$\int_{S_4^{**}} \overline{\rho R V_\theta \vec{n} \cdot \vec{V}} dS = \int_{S_4^{**}, S_5} \overline{\vec{n} \cdot R \vec{\tau}_\theta} dS - \int_{S_5} \overline{\vec{n} \cdot \vec{e}_\theta} R P dS, \quad (51b)$$

- S_1 = INLET SURFACE
- S_2 = OUTLET SURFACE
- S_3 = CASING SURFACE, INCLUDING STRIPPER
- S_4^* = SURFACE SURROUNDING IMPELLER BOUNDING ABSOLUTE VOLUME v^*
- S_4^{**} = SURFACE SURROUNDING IMPELLER BOUNDING RELATIVE VOLUME v^{**}
- S_5 = IMPELLER SURFACE
- v^* = ABSOLUTE VOLUME BOUNDED BY S_1 , S_2 , S_3 , AND S_4^*
- v^{**} = RELATIVE VOLUME BOUNDED BY S_4^{**} AND S_5

FIGURE 11

VOLUMES AND SURFACES FOR MOMENTUM EQUATIONS

since there is no flow through the impeller surface, S_5 , and $\vec{n} \cdot \vec{e}_\theta = 0$ on the surface, S_4^{**} . Since the unit vector, $\vec{n}$, normal to the surfaces, S_4^* and S_4^{**} , are in the opposite directions, Equations (50b) and (51b) may be combined as

$$\begin{aligned} \int_{S_5} \overline{\vec{n} \cdot \vec{R} \vec{\tau}_\theta} dS - \int_{S_5} \overline{\vec{n} \cdot \vec{e}_\theta} RP dS &= \int_{S_1} \overline{\rho R V_\theta \vec{n} \cdot \vec{V}} dS + \int_{S_2} \overline{\rho R V_\theta \vec{n} \cdot \vec{V}} dS \\ &- \int_{S_1} \overline{\vec{n} \cdot \vec{R} \vec{\tau}_\theta} dS - \int_{S_2} \overline{\vec{n} \cdot \vec{R} \vec{\tau}_\theta} dS - \int_{S_3} \overline{\vec{n} \cdot \vec{R} \vec{\tau}_\theta} dS + \int_{S_3} \overline{\vec{n} \cdot \vec{e}_\theta} RP dS. \end{aligned} \quad (54)$$

The left side of Equation (54) represents the impeller torque, while the first two terms on the right side represent the net change of angular momentum of the gas as it enters and leaves the compressor. The third and fourth terms on the right side of Equation (54) are surface tractions at the inlet and outlet surfaces and can usually be neglected, and the fifth and six terms represent casing torques due to viscous friction and pressure forces, respectively. For constant channel dimensions, the last term would have non-zero value only over the stripper surfaces projected onto a meridional plane (i.e., where $\vec{n} \cdot \vec{e}_\theta \neq 0$).

The efficiency reported in the literature for a turbomachine is frequently not well defined, perhaps because there are several efficiencies which may be used as a measure of the "idealness" of the machine. Precise definition of the ideal process is therefore important, and the second law of thermodynamics may be used indirectly to define such a process. A commonly used ideal process is one at

constant entropy, or reversible and adiabatic. Defining E_{ideal} as the energy per unit mass required to give the desired output by means of an ideal process and E_{gas} as the rotor energy input to the gas per unit mass, the isentropic efficiency, η_s , is

$$\eta_s = E_{ideal,s} / E_{gas}, \quad (55)$$

and

$$E_{ideal,s} = h'_{t,d} - \bar{h}_{t,s}, \quad (56)$$

where $h'_{t,d}$ is the total enthalpy at the fictitious discharge total temperature, $T'_{t,d}$, for the isentropic process ending at the actual discharge total pressure, $\bar{P}_{t,d}$. Another $E_{ideal,s}$ could be defined where the entropy at discharge temperature is the reference condition and a fictitious suction temperature is defined, but this definition will not be used in this thesis. Two forms of the isentropic efficiency may be defined depending on whether or not the actual process is adiabatic:

$$\eta_s = \frac{h'_{t,d} - \bar{h}_{t,s}}{\bar{h}_{t,d} - \bar{h}_{t,s}} \quad (57)$$

for the adiabatic process, and

$$\eta_{aero} = \frac{h'_{t,d} - \bar{h}_{t,s}}{(\bar{h}_{t,d} - \bar{h}_{t,s}) - Q} \quad (58)$$

for the process with heat transfer, with η_{aero} called the aerodynamic efficiency (Stanitz (31)).

Another process that is commonly used in compressor technology is the polytropic-adiabatic, or simply polytropic, process. Many

writers consider the polytropic analysis better than the isentropic analysis for describing compressor performance. In terms of total conditions, the ideal polytropic process energy output may be written as

$$E_{\text{ideal,p}} = \int_{\bar{P}_{t,s}}^{\bar{P}_{t,d}} \frac{dP_t}{\rho}, \quad (59)$$

with the polytropic process path equation defined as

$$P_t / \rho_t^n = \text{constant}, \quad (60)$$

where n is the commonly called polytropic exponent. The polytropic efficiency is therefore

$$\eta_p = E_{\text{ideal,p}} / E_{\text{gas}} = \int_{\bar{P}_{t,s}}^{\bar{P}_{t,d}} \frac{dP_t}{\rho} / (\bar{h}_{t,d} - \bar{h}_{t,s}). \quad (61)$$

The polytropic efficiency is sometimes called the isentropic efficiency of an infinitesimally small stage since it can be derived by considering the differential state equation for a pure substance (in terms of total properties),

$$dh_t = \frac{dP_t}{\rho_t} + T_t ds_t, \quad (62)$$

for an infinitesimal isentropic compression. The polytropic efficiency attempts to account for the temperature rise due to frictional dissipation into heat and is considered a better representation of true output/input efficiency. The path Equation (60) is an assumed path of a reversible process between the end state points (the actual path is generally unknown), and the reversible heat input necessary to complete

the energy balance for this process is equivalent to the actual frictional dissipation. If the availability concept is used, it can be shown that part of the energy represented by the increase in discharge temperature is available for doing work (Stepanoff 32). Other excellent discussions of the polytropic analysis are given in References (30, 31, 33, 34, 35, 36). A comparison of peripheral compressor data analyzed by the two methods is included in the next chapter. The thermodynamics of the compression process is presented in Figure 12 on an enthalpy-entropy diagram, where both total and static property representations are shown (the exponent n is not necessarily constant in the general case).

The final fundamental "law" describing peripheral compressor performance is the equation of state, which may be written as

$$P = Z \rho \frac{R_O}{MW} T \quad (63)$$

where

R_O = the universal gas constant, 1545 ft.-lb_f./lb_m.-mole °R.,

MW = the gas molecular weight, lb_m./lb_m.-mole,

P = absolute pressure, lb_f./ft.²,

T = absolute temperature, °R.,

ρ = density, lb_m./ft.³, and

Z = a compressibility factor.

The compressibility factor, Z , is discussed in most texts on thermodynamics. Equation (63) may be applied to real gases if Z is known as

FIGURE 12

COMPRESSION PROCESSES ON ENTHALPY-ENTROPY DIAGRAM

a function of pressure and temperature. The excellent paper by Schultz (36) presents real gas analyses as applied to compressor processes and discusses two other compressibility functions which evolve from the analyses. Schultz shows that errors of several percent in work output and efficiency can exist under certain conditions if perfect gas relationships are used instead of the real gas equations, so that this possibility should be remembered in any analysis of compressor performance. However, since the objective of this thesis is to present a general analysis of the peripheral compressor and since data for performance correlation were obtained at low pressures with $Z \approx 1$, this refinement will not be used further. Perfect gas relationships will be used with the specific heat capacities considered as known functions of temperature only or as average values. With the "total" equation of state of a perfect gas,

$$P_t = \rho_t \frac{R_o}{MW} T_t = \rho_t R T_t, \quad (63a)$$

the expression for the total enthalpy of a perfect gas,

$$h_t = c_p T_t = c_p T + \frac{V^2}{2g_c}, \quad (45a)$$

the relation between temperatures and pressures for an isentropic process of a perfect gas (derived from $P_t/\rho_t^k = \text{constant}$),

$$\frac{T'_{t,d}}{T_{t,s}} = \left(\frac{\bar{P}_{t,d}}{\bar{P}_{t,s}} \right)^{\frac{k-1}{k}}, \quad (64)$$

and Equations (56) and (57), page 68, expressions for isentropic work

or head, H_s , and efficiency, η_s , may be written as

$$\begin{aligned}
 H_s &= E_{\text{ideal},s} = c_p \left(T'_{t,d} - \bar{T}_{t,s} \right) \\
 &= c_p \bar{T}_{t,s} \left[\left(\frac{\bar{P}_{t,d}}{\bar{P}_{t,s}} \right)^{\frac{k-1}{k}} - 1 \right] = \frac{kR}{k-1} \bar{T}_{t,s} \left[\left(\frac{\bar{P}_{t,d}}{\bar{P}_{t,s}} \right)^{\frac{k-1}{k}} - 1 \right] \quad (65)
 \end{aligned}$$

and

$$\begin{aligned}
 \eta_s &= E_{\text{ideal},s} / E_{\text{gas}} = c_p \left(T'_{t,d} - \bar{T}_{t,s} \right) / c_p \left(\bar{T}_{t,d} - \bar{T}_{t,s} \right) \\
 &= \left[\left(\frac{\bar{P}_{t,d}}{\bar{P}_{t,s}} \right)^{\frac{k-1}{k}} - 1 \right] / \left(\frac{\bar{T}_{t,d}}{\bar{T}_{t,s}} - 1 \right). \quad (66)
 \end{aligned}$$

Similar expressions for polytropic work or head, H_p , and efficiency, η_p , may be derived for a perfect gas if the exponent, n , is assumed constant. From Equations (60) and (63a), pages 69 and 72, respectively, the relationship between pressures and temperatures becomes

$$\frac{\bar{T}_{t,d}}{\bar{T}_{t,s}} = \left(\frac{\bar{P}_{t,d}}{\bar{P}_{t,s}} \right)^{\frac{n-1}{n}}. \quad (67)$$

With Equations (59) and (61), page 69, and Equation (67), the expressions for H_p and η_p may be written as

$$H_p = E_{\text{ideal},p} = \frac{nR}{n-1} \bar{T}_{t,s} \left[\left(\frac{\bar{P}_{t,d}}{\bar{P}_{t,s}} \right)^{\frac{n-1}{n}} - 1 \right], \quad (68)$$

and

$$\begin{aligned}
\eta_p &= \frac{nR}{n-1} \bar{T}_{t,s} \left[\left(\frac{\bar{P}_{t,d}}{\bar{P}_{t,s}} \right)^{\frac{n-1}{n}} - 1 \right] / \frac{kR}{k-1} \bar{T}_{t,s} \left(\frac{\bar{T}_{t,d}}{\bar{T}_{t,s}} - 1 \right) \\
&= \left(\frac{n}{n-1} \right) \left(\frac{k-1}{k} \right). \quad (69)
\end{aligned}$$

In this section, the line over certain quantities was used to indicate time-averaged or mass-averaged values. This notation will be dropped in further discussion, with it understood that measured variables, or parameters based on these variables, are time- or mass-averaged values.

Functional relationships of dimensionless terms. In the previous section basic concepts were applied to the peripheral compressor in a similar way that they have been applied to analysis of other turbomachinery, although presented in a little different manner than usually found in the literature. Because of the difficult if not impossible task of obtaining analytical solutions to the fundamental equations of motion and energy when applied to the internal flow in the peripheral compressor, a method similar to one used to describe centrifugal compressor performance was adopted. This method utilizes the concept of dimensional analysis to derive dimensionless parameters which may be written in functional form and used to present compressor characteristics on so-called performance maps. Under appropriate conditions of similarity (ideally, geometric, kinematic, dynamic, and thermal), the performance of other compressors may be predicted from these performance maps. It was assumed that the peripheral compressor obeys

the laws of similitude that apply to centrifugal compressors, and a set of dimensionless parameters was chosen to correlate experimental data. The dimensional analysis method will not be discussed in detail here since it is described frequently in the literature, with References (30, 31, 33, 34, 35, 37, 38, 39, 40, 41, 42) having good discussions as applied to compressors.

The variables which determine or affect peripheral compressor performance may be divided into dependent and independent types. The dependent variables are a measure of useful output such as total pressure ratio, $P_{t,d}/P_{t,s}$, isentropic head, H_s , or polytropic head, H_p , and a measure of energy necessary to obtain that output, E_{gas} . Independent variables include: (1) gas properties such as the gas constant, R (or molecular weight, MW), specific-heat capacity at constant pressure, c_p , viscosity, μ , and thermal conductivity, κ ; (2) inlet total temperature, $T_{t,s}$, and total density, $\rho_{t,s}$; (3) impeller rotational speed, N ; (4) mass flow rate, w ; and (5) geometric variables such as impeller dimensions, casing dimensions, clearances, and nozzle location and dimensions. The geometric variables for a peripheral compressor are large in number and to completely account for all of them would be extremely difficult; however, it is likely that only a few have significant effect upon performance. For geometrically similar machines, only a characteristic dimension such as impeller tip diameter, D , is necessary in the analysis.

One set of non-dimensional parameters derived from the above considerations, in addition to the total pressure ratio and isentropic efficiency, consists of the isentropic head coefficient,

$$\psi_s = \frac{g_c H_s}{U_T^2}, \quad (70a)$$

a flow term called specific mass flow,

$$\phi_t = \frac{w}{P_{t,s} D^2} \sqrt{\frac{RT_{t,s}}{g_c k}} \quad (70b)$$

a speed term called impeller tip Mach number,

$$M_{t,T} = \frac{U_T}{\sqrt{k g_c RT_{t,s}}}, \quad (70c)$$

an impeller tip or "machine" Reynolds number for viscous effects,

$$Re_{t,T} = \frac{\rho_{t,s} U_T D}{\mu}, \quad (70d)$$

the ratio of specific heat capacities for compressibility effects,

$$k = \frac{c_p}{c_v}, \quad (70e)$$

a Prandtl number for heat transfer effects,

$$Pr = \frac{c_p \mu}{k}, \quad (70f)$$

and a number of geometric factors,

$$g_1 = \frac{R_m}{D}, \quad g_2 = \frac{R_m}{c_1}, \quad g_3 = \frac{a_1}{a_2}, \quad g_4 = \frac{h}{b}, \text{ etc.}, \quad (70g)$$

where R_m is the characteristic channel dimension identified by Equation (25), page 33, and the other dimensions are identified on Figure 5, page 17. In the above parameters, the impeller tip speed,

U_T , equals πND . Other parameters which are applicable to peripheral compressor analysis are the polytropic efficiency, the polytropic head coefficient,

$$\psi_p = \frac{g_c H_p}{U_T^2}, \quad (70h)$$

a capacity coefficient based on channel area,

$$\lambda = \frac{w}{\rho_{t,s} A_c U_T} = \frac{Q_s}{A_c U_T} \quad (70i)$$

and an input power coefficient,

$$\gamma_p = \frac{w E_{gas}}{\rho_{t,s} N^3 D^5}. \quad (70j)$$

The specific speed, N_s , and specific diameter, D_s , defined by Equations (1), page 6, and (2), page 7, may also be useful although they are not dimensionless.

With the above dimensionless parameters, the following functional relationships were assumed for the peripheral compressor:

$$\frac{P_{t,d}}{P_{t,s}} = f_1 \left(\phi_t, M_{t,T}, Re_{t,T}, k, Pr, g_1, g_2, g_3, \text{etc.} \right), \quad (71a)$$

$$\eta_s \text{ or } \eta_p = f_2 \left(\phi_t, M_{t,T}, Re_{t,T}, k, Pr, g_1, g_2, g_3, \text{etc.} \right), \quad (71b)$$

and

$$\gamma_p = f_3 \left(\phi_t, M_{t,T}, Re_{t,T}, k, Pr, g_1, g_2, g_3, \text{etc.} \right), \quad (71c)$$

or

$$\psi_s \text{ or } \psi_p = f_4 \left(\lambda, M_{t,T}, Re_{t,T}, k, Pr, g_1, g_2, g_3, \text{etc.} \right), \quad (71d)$$

$$\eta_s \text{ or } \eta_p = f_5 \left(\lambda, M_{t,T}, Re_{t,T}, k, Pr, g_1, g_2, g_3, \text{etc.} \right), \quad (71e)$$

and

$$\gamma_p = f_6 \left(\lambda, M_{t,T}, Re_{t,T}, k, Pr, g_1, g_2, g_3, \text{etc.} \right). \quad (71f)$$

Some of the parameters in Equations (71a) through (71f) may be neglected as a first approximation. In these expressions, the geometric factors are not necessary if only geometrically similar machines are being considered. Also, the Prandtl number seldom has significant influence because flow through compressors usually approaches adiabatic. (However, it may be more important in peripheral compressors than in other types because of the low flows and longer path through the machine.) The exact influence of the Reynolds number is not fully understood, but in many compressor applications its value is high enough (greater than 10^6) so that a small change from a test or design value does not appear to have appreciable effect on performance (42). Finally, the ratio of specific heats is an index of compressibility, i.e., an indication of density change (and velocity change) as the gas passes through the compressor. Therefore, for kinematic similarity and equal velocity triangles in two machines, the k value must be the same. However, this is not possible (or desirable) in many cases, so that some adjustment may be necessary. An excellent discussion of errors and limitations involved in comparing performance with gases of different k is given by Davis (39). An attempt to account for this effect was made by including k in the flow term, ϕ_t , in a way similar to that followed with some success by Sheets (41).

III. HYPOTHESIS OF COMPRESSIBLE FLOW OPERATION

The purpose of this section is to propose a hypothesis of peripheral compressor characteristics in compressible flow, with emphasis on the influence of high Mach numbers. This is done in a qualitative manner for reasons already discussed. Assumptions are made which are based partly on application of the fundamental concepts presented in the previous section, partly on observations of others in incompressible flow, and partly on experience with compressible flow.

Pumping Mechanism

The assumption is made that the flow pattern is a combination of the regenerative flow and the turbulent momentum-exchange flow--that is, the gas follows a circulatory or meridional path in a somewhat orderly pattern but with a high degree of turbulence. It is believed that many of the results obtained from Oelrich's regenerative analysis (see Figure 6, page 23, for example) can be applied at least qualitatively to the peripheral compressor with modifications for compressible flow. These are discussed below, with the effects of compressibility on performance emphasized.

Circulation intensity and tangential pressure rise. Whereas linear pressure rise with circumferential position was predicted and observed with incompressible flow, consideration of the regenerative concept along with Equation (42), page 60, leads to the conclusion of a non-linear pressure rise in compressible flow. For a given net mass flow and constant channel area, the average channel velocity, $\bar{V}_{\theta, c}$,

decreases with circumferential position due to density increase in both the channel and impeller. Therefore, for the same reasons stated on page 16, the pressure rise should increase more rapidly as the discharge is approached; i.e., $\frac{dP}{d\theta}$ is not a constant but rather a function of θ (actually, an unknown function of flow and geometry). A corollary, or perhaps prerequisite, of this statement is that the meridional flow intensity is not constant for a given net flow but increases with angular position because of the decreased centrifugal field in the channel. These effects would be expected to be more pronounced the greater the Mach number and compressibility (greater density change).

Blade inlet losses. Since blade entrance losses were by far the greatest of losses, except at high flow, according to the regenerative analysis (Figure 6, page 23), a discussion of them with particular reference to high Mach number effects is warranted. If the blading losses are really that significant, any increase in them would lower the output head or pressure ratio. The effects of Mach number on blade incidence losses reported by References (30, 33, 43) lead to the conclusion that performance at high impeller tip Mach number, $M_{t,T}$, may be poor. For subsonic velocities, according to Balje (43), impulse-type losses due to the difference between blade angle and flow angle are proportional to the Prandtl factor, $1/\sqrt{1-M_{rel,1}^2}$, where $M_{rel,1}$ is the actual relative Mach number at the blade entrance. Obviously, incidence losses are not infinite at $M_{rel,1} = 1$, but this factor illustrates the increase in those losses with increasing Mach number. The relative Mach

number, $M_{rel,l}$, is not generally the same as the impeller tip Mach number, $M_{t,T}$, but the latter can be used as a measure of this effect, particularly since the radii at blade entrance and blade tip do not usually differ greatly.

Carryover flow and temperature effect. What may be an important influence is the gas that is carried through the stripper within the impeller blading. This carryover flow can be as great as the net flow, even at maximum net flow, and is obviously greater at low flow. The expected effect of the carryover of hot gas into the inlet region would be to reduce pressure ratio and the head calculated using the temperature measured in the inlet pipe. The mixing of this hot gas with the cooler inlet flow results in a higher effective inlet temperature, but this higher temperature is not the one used in the head Equations (65) or (68), page 73. Therefore, a droop in the head-flow curve at reduced flow (where efficiency is lower and temperature rise greater) would be expected. Similarly, since pressure ratio can be expressed as

$$\frac{P_{t,d}}{P_{t,s}} = \left[1 + \eta_s \frac{E_{gas}}{c_p T_{t,s}} \right]^{\frac{k}{k-1}} \quad (72)$$

for the adiabatic case by rewriting Equation (66), page 73, the pressure ratio will decrease with increasing inlet temperature if the efficiency and energy input remain constant. Therefore, a droop in the pressure-ratio curve at low flow would also be expected. Another effect to add to those already mentioned is that of expansion losses of the hot gas as it leaves the stripper section and the mixing losses of this gas with

the inlet flow. These losses would be expected to increase with pressure ratio and impeller tip Mach number.

Influence of Impeller Geometry

The influence of the impeller design variables is difficult to predict. The discussion of blade inlet losses suggests that the blading should have an angle other than 90° with the plane of the impeller. Figure 13 shows inlet velocity diagrams at various circumferential positions that one would expect at a given compressible flow in a constant area channel. Unfortunately, the meridional velocity, $V_{m,1}$ and the channel tangential velocity, $V_{\theta,1}$, are not known and can be only qualitatively indicated. However, based on assumptions stated above regarding density change and change in meridional flow intensity with circumferential position, velocity diagrams can be roughly estimated. In the inlet region, expansion of higher pressure gas coming through the stripper probably causes a higher channel velocity than would normally exist from the inlet flow only. Also, the resulting mixing and random motions would preclude an orderly circulatory flow pattern in this region, although some meridional velocity may exist. Further around the passage, the circulatory flow is assumed to develop, which, combined with some pressure rise, should result in greater $V_{m,1}$ and lower $V_{\theta,1}$. As pressure increases and density increases, $V_{\theta,1}$ would further decrease, and $V_{m,1}$ would further increase due to weakening of the centrifugal field in the channel. Different sets of velocity diagrams would of course exist at different flow rates.

FIGURE 13

QUALITATIVE VELOCITY DIAGRAMS AT ENTRANCE TO
BLADING FOR A GIVEN FLOW

If the velocity diagrams in Figure 13 are even roughly correct, it is easy to imagine how high blade entrance losses could occur in a straight-bladed impeller ($\beta_b = 90^\circ$). These diagrams indicate that some forward leaning of the blading would reduce these losses. The optimum angle would likely be a function not only of impeller dimensions but of channel dimensions and especially flow rate.

Impeller blading outlet angle (at impeller tip) may be considered in a similar manner as is done in centrifugal compressor analysis. That is, more blades give better guidance to the gas but cause higher frictional losses, so that some compromise must be made. No information is available concerning the optimum number of blades except the small amount in the incompressible flow literature which indicated that 24 to 72 blades are recommended. The slip factor, $\sigma = V_{\theta,2}/U_2$, which may be thought of as a measure of the impeller's ability to transfer energy to the gas, probably depends not only on the number of blades but on channel dimensions as well. Reduction of the radial depth between the impeller and the channel would force more of the gas to leave the impeller at the side at a smaller radius and with less energy. Backward curved impeller blades would probably increase efficiency but decrease energy transfer, as occurs in centrifugal compressors with backward-curved blading. Conversely, forward-bent blading would be expected to produce higher heads, but with greater losses. As the impeller tip Mach number approaches one, the slip factor (and the ability of the impeller to transfer energy to the gas) would be expected to decrease, especially for nearly straight or forward-bent blades.

The axial and radial depths of the impeller blading, a_1 and a_2 (see Figure 5, page 17), probably have optimum dimensions for a given channel geometry. It is believed that a_1 should be no less than about one-third of a_2 to prevent restriction of the circulatory flow pattern. From the previous discussion of carry-over flow through the stripper, a small blade slot volume would be indicated in order to reduce the detrimental effects of the carry-over flow. However, the smaller blading volume could mean less energy transfer to the gas per unit time (less gas in the impeller), so that one effect must be weighed against the other.

Influence of Channel Geometry

It should be apparent from the previous discussions that the effects of the various parts of the peripheral compressor cannot be separated since one influences the other. A varying channel cross-section area, decreasing in direction of flow, is indicated for compressible flow operation due to density change. The desired rate of area change would depend upon the particular condition of operation, but consideration of blade entrance velocity diagrams indicates that the area should vary in a manner to give constant β_1 (see Figure 13, page 83) with the circumferential position so that a given blading angle can be more advantageously employed.

Under the assumption made in this section regarding the type of flow pattern, a contoured channel such as shown in Figure 2(d), page 4, should be aerodynamically better than the conventional rectangular shape. However, economics may prohibit use of such a channel in

practice unless benefits to be gained outweigh the additional fabrication costs. If the rectangular channel is used, the ratio h/b undoubtedly is an important factor, but perhaps more important is the "characteristic channel dimension," R_m , proposed by Shimosaka (15) which includes the channel area and impeller blade dimensions (see Equation (25), page 33).

Influence of Nozzle Geometry

Nozzle location and geometry can be quite important, particularly at high flows. The spacing of the nozzles should provide as long a pumping path through the compressor as feasible, yet the stripper must be circumferentially long enough to prevent high leakage--at least as long as the span including two adjacent impeller vanes. To promote the assumed circulatory flow pattern, the nozzles should be oriented so that turning and mixing are minimized. From experience with compressible flow through other types of compressor nozzles and piping turns, it is known that the dimensionless coefficient usually used to represent the pressure losses is a function of Mach number and increases at high Mach numbers. Therefore, the inlet and outlet losses in a peripheral compressor would be expected to increase with Mach number. Although the average flow Mach number in the nozzle pipes may be relatively low and not necessarily directly associated with the impeller tip Mach number, locally high flow Mach numbers which could occur at high impeller Mach numbers would cause an increase in the nozzle losses. Therefore, it is assumed that the impeller Mach number can be used as a rough indication of compressibility effects in the nozzles, with

nozzle losses increasing as the impeller Mach number approaches one.

CHAPTER IV

OBSERVED PERFORMANCE IN COMPRESSIBLE FLOW

An experimental program was conducted at the Oak Ridge Gaseous Diffusion Plant to determine the performance of the peripheral compressor operating under compressible flow conditions and to test the hypotheses advanced in the previous chapter. The test loop, experimental apparatus, measurement methods, and data reduction techniques are discussed in Appendix C. The experimental compressor shown schematically in Figure 1, page 3, was tested at speeds from 5000 rpm. to 14,000 rpm. on gases whose molecular weights ranged from 4 to 400. These gases and some of their more important properties are listed in Table I, and the important dimensions of compressor designs discussed in this chapter are listed in Table II. With the selection of gases and the facility speed range capability, the impeller tip Mach number could be varied from less than 0.1 to over 2.0. Some of the data presented here have been published previously by the writer (44, 45); however, most of the data have not been published before.

I. GENERAL CHARACTERISTICS WITH SEVERAL GASES

The first objective of the experimental program was to determine the characteristics of the peripheral compressor with a wide variety of gases, speeds, and Mach numbers. The design designated ORGDP-1-0 in Table II was used for this purpose. (In the design designations, the

TABLE I
TEST GASES WITH PROPERTIES AT 200°F.

Gas Name	Code or Symbol	Molecular Weight, MW	Specific Heat, c_p , $\frac{\text{Btu.}}{\text{lb}_m \cdot ^\circ\text{R.}}$	Specific Heat Ratio, $k = c_p / c_v$	Viscosity, μ , $\frac{\text{lb}_m}{\text{ft.} \cdot \text{sec.}}$
Helium	He	4.0	1.242	1.668	1.54×10^{-5}
Air	Air	29.0	0.242	1.398	1.45×10^{-5}
Monochlorodi- fluoromethane	CHClF_2 (R-22)	85.5	0.168	1.159	1.02×10^{-5}
Sulfur Hexafluoride	SF_6	146.6	0.181	1.082	1.24×10^{-5}
Perfluoro-dimethyl- cyclohexane	C_8F_{16}	400.0	0.203*	1.025*	0.79×10^{-5}

*At 77°F.

TABLE II

ORGDP EXPERIMENTAL PERIPHERAL COMPRESSOR DIMENSIONS

Design	h,in.	b,in.	a,in.	a ₁ ,in.	a ₂ ,in.	r _r ,in.	Z	c ₁ ,in.	c ₂ ,in.	A _c ,in. ²
ORGDP-1-0	0.200	0.500	1.200	0.500	1.500	1.250	48	0.012	0.005	1.940
ORGDP-3-2	0.500	0.875	1.200	0.500	1.500	1.250	48	0.028	0.015	4.100
ORGDP-4-2	0.500	0.875	1.200	0.500	1.500	1.250	48	0.028	0.015	4.100
ORGDP-4-3	0.500	0.875	1.200	0.500	1.500	1.250	48	0.030	0.020	4.100
ORGDP-4-4	0.500	0.875	1.200	0.500	1.500	1.250	48	0.015	0.015	4.100
ORGDP-5-3	0.200	0.500	1.200	0.500	1.500	1.250	48	0.030	0.020	1.940
ORGDP-5-4	0.200	0.500	1.200	0.500	1.500	1.250	48	0.015	0.015	1.940
ORGDP-9-4	0.350	0.688	1.200	0.500	1.500	1.250	48	0.015	0.015	2.966
ORGDP-13-4	(Variable)		1.200	0.500	1.500	1.250	48	0.015	0.015	2.818
ORGDP-15-4	0.500	0.875	1.200	0.375	1.500	1.250	48	0.015	0.015	4.100
ORGDP-16-4	0.200	0.500	1.200	0.375	1.500	1.250	48	0.015	0.015	1.940
ORGDP-17-4	0.500	0.875	1.200	0.250	1.500	1.250	48	0.015	0.015	4.100
ORGDP-18-4	0.200	0.500	1.200	0.250	1.500	1.250	48	0.015	0.015	1.940
ORGDP-22-4	0.100	0.375	1.200	0.500	1.500	1.250	48	0.015	0.015	1.320
ORGDP-24-4	0.100	0.500	1.200	0.500	1.500	1.250	48	0.015	0.015	1.720
ORGDP-25-4	0.100	0.688	1.200	0.500	1.500	1.250	48	0.015	0.015	2.322
ORGDP-26-4	0.200	0.375	1.200	0.500	1.500	1.250	48	0.015	0.015	1.515
ORGDP-27-4	0.200	0.688	1.200	0.500	1.500	1.250	48	0.015	0.015	2.579
ORGDP-28-4	0.350	0.375	1.200	0.500	1.500	1.250	48	0.015	0.015	1.808
ORGDP-29-4	0.350	0.500	1.200	0.500	1.500	1.250	48	0.015	0.015	2.270
ORGDP-56-4	0.200	0.500	1.200	0.500	1.500	1.250	48	0.015	0.015	1.940
ORGDP-59-4	0.200	0.500	1.400	0.500	1.500	---	36	0.015	0.015	1.980
ORGDP-60-4	0.200	0.500	1.400	0.500	1.500	---	36	0.015	0.015	1.980
ORGDP-61-4	0.200	0.500	1.400	0.500	1.500	---	36	0.015	0.015	1.980
ORGDP-62-4	0.200	0.500	1.400	0.500	1.500	---	36	0.015	0.015	1.980
ORGDP-63-4	0.200	0.500	1.400	0.500	1.500	---	36	0.015	0.015	1.980

Note: For each impeller, D = 10.000 in. and t₂ = 0.080 to 0.090 in. Refer to Figure 5, page 17, and nomenclature in Appendix A for identification of symbols.

first number denotes a geometric configuration, and the second number refers to a certain set of clearances.) All five gases listed in Table I were tested in this design. Most of the data were obtained at 2 psia. suction pressure and at 90°F. and 200°F. suction temperatures, although the lighter gases were also tested at 6 psia. and 10 psia. in order to achieve more accurate pressure and temperature-rise measurements. Test speeds were 5000, 7500, 9000, 11,000, and 14,000 rpm.; also, some shut-off data were obtained at intermediate speeds.

Head Coefficients and Efficiencies

Figure 14 shows some of the data obtained on the ORGDP-1-0 design presented as isentropic head coefficient versus capacity coefficient, with impeller tip Mach number as the speed parameter. Also shown are approximate isentropic efficiency contours and a line of average maximum efficiency. Smooth curves drawn through the data points were extrapolated to zero flow when shut-off data were not available and to zero head. All the curves showed the droop at low flows which was discussed in the previous chapter as a probable temperature effect resulting from low efficiency at reduced flow. The lower Mach number curves (some of which are not shown for better clarity) fell in a small band in the upper portion of the performance map. Generally, incompressible flow data reported in the literature would collapse onto a single curve when plotted in this manner. However, the compressible flow data exhibited a marked decrease in the level of the $\psi_s - \lambda$ curve as Mach number increased above about 0.3 to 0.4. This result is better seen in Figure 15, where the head coefficient at three values of the flow term

FIGURE 14

$\psi_s - \lambda$ CURVES FOR ORGDP-1-0 COMPRESSOR WITH
 APPROXIMATE η_s CONTOURS

FIGURE 15

ψ_s AS FUNCTION OF $M_{t,T}$ FOR ORGDP-1-0 COMPRESSOR

is plotted as a function of Mach number. A combination of the high Mach number effects discussed in Chapter III--blade entrance losses, stripper expansion and mixing losses, and nozzle losses, which were predicted to cause poorer performance at high Mach numbers--would seem a logical explanation of the loss of head and efficiency at the greater Mach numbers.

Isentropic efficiencies obtained at several impeller Mach numbers are shown in Figure 16. From this Figure and Figure 14, it can be seen that efficiency generally decreased with increasing Mach number and that peak efficiency occurred at 56 to 62 percent of maximum flow (at zero head), although maximum λ values ranged from 0.25 to 0.72. In the incompressible flow data reported in the literature, maximum efficiency generally occurred at 60 to 80 percent of the fairly consistent 0.7 value for maximum λ . Maximum efficiencies greater than 50 percent were obtained on the ORGDP-1-0 design, although maximum efficiency as low as 10 percent was measured at tip Mach numbers greater than 1.0. Most of the peak efficiencies were near 40 percent for impeller tip Mach numbers less than 0.5.

A comparison of head coefficients and efficiencies calculated by the isentropic and the polytropic analyses are presented in Figure 17. Low Mach number (nearly incompressible) and high Mach number (highly compressible) cases were selected for the comparison. Helium and SF_6 at 9000 rpm. represent the test data near the lowest and the highest pressure ratios, respectively, that were obtained on the ORGDP-1-0 design. The differences between the polytropic and isentropic methods

FIGURE 16

ISENTROPIC EFFICIENCIES OF ORGDP-1-0 COMPRESSOR

FIGURE 17

COMPARISON OF ISENTROPIC AND POLYTROPIC ANALYSES

of analysis are greater at the higher pressure ratios, as would be expected due to the divergence of constant pressure lines on the $h-s$ diagram. Although the differences between η_s and η_p or between ψ_s and ψ_p can be fairly large, percentagewise, the absolute differences are not great in the range of pressure ratios considered here; therefore, the isentropic analysis was considered adequate for the purposes of this thesis. (However, as mentioned previously, the departure from adiabatic compression could be significant under certain conditions and should be considered in a more refined analysis.)

Pressure Ratios

Often a more useful presentation of compressor performance is that of total pressure ratio as a function of specific mass flow, ϕ_t , and impeller tip Mach number, $M_{t,T}$. The data obtained on the ORGDP-1-0 peripheral compressor were correlated in this manner. Figure 18 shows a partial performance map of this type, where the Mach number was again treated as a speed parameter. Only about half of the performance curves obtained on this design are shown in order to improve clarity, but the other data correlated in similar manner. Plotting performance in terms of these variables is advantageous when test conditions vary in that the compressor characteristics can be referenced to some standard conditions. The impeller Mach number, rather than speed or type of gas, positions the curve on this plot when Reynolds number effects can be assumed negligible and if the influence of different values of k can be accounted for by including k in ϕ_t . The data obtained on the ORGDP-1-0 design showed that pressure ratios increased with Mach number up to $M_{t,T} \approx 0.77$.

FIGURE 18

TOTAL PRESSURE RATIO AS FUNCTION OF SPECIFIC
MASS FLOW FOR ORGDP-1-0 COMPRESSOR

Above that value, pressure ratio decreased with increasing Mach number, indicating a maximum obtainable pressure ratio.

A relationship between pressure ratio, isentropic head coefficient, and impeller tip Mach number can be derived by combining their defining equations as

$$\frac{P_{t,d}}{P_{t,s}} = \left[1 + (k-1)\psi_s M_{t,T}^2 \right]^{\frac{k}{k-1}} \quad (73)$$

By a binomial series expansion, Equation (73) can be written as

$$\frac{P_{t,d}}{P_{t,s}} = 1 + k \psi_s M_{t,T}^2 + \frac{k}{2} \psi_s^2 M_{t,T}^4 + \dots, \quad (73a)$$

which converges if $(k-1)^2 \psi_s^2 M_{t,T}^4 < 1$, which is generally true for rotary compressors. For region of constant ψ_s , small variation in k , and low $M_{t,T}$, Equation (73a) may be approximated as

$$\frac{P_{t,d}}{P_{t,s}} - 1 = A M_{t,T}^B \quad (73b)$$

Plotting pressure ratio as a function of impeller Mach number for several values of ϕ_t resulted in Figure 19. This Figure clearly shows that pressure ratio reached a maximum at each flow and that the data from the different gases correlated very well on this type of plot. As may be seen, Equation (73b) applied to only a portion of each curve--at Mach numbers below 0.4 to 0.6--and that the constants, A and B, were different for each flow. Since ψ_s itself was found to be a function of $M_{t,T}$ at the higher Mach numbers, Equation (73b) was not expected to hold at the higher Mach numbers.

FIGURE 19

TOTAL PRESSURE RATIO AS FUNCTION OF IMPELLER TIP
MACH NUMBER FOR ORGDP-1-0 COMPRESSOR

Internal Measurements

In an attempt to determine flow patterns and pressures inside the compressor and to gain some insight regarding the poor performance at high Mach number, static pressures and flow angles were measured at several points as described in Appendix C.

Pressure rise in channel. In nearly all tests, static pressures were measured at seven points at a 4.500-inch radius. Results typical of those measured at low and high impeller Mach numbers are presented in Figure 20, where the channel pressure has been normalized by dividing by the suction static pressure. Air at 11,000 rpm. ($M_{t,T} = 0.383$) was selected as typical of the low Mach number data and SF₆ at 11,000 rpm. ($M_{t,T} = 1.04$) as representative of the high Mach number results. The pressure rise at low Mach number was similar to that reported in the literature for incompressible flow, except the rise in gases was not as linear. The slight depression of channel pressure near the inlet shown in Figure 20 for the air was usually observed in incompressible flow also. The large reduction in static pressure at high Mach number, which was as much as 40 percent at large flows, is considered due to (1) nozzle, mixing, and blade entrance losses at high flow and (2) expansion, mixing, and blade entrance losses at low flow. Local points of near choking conditions are believed to exist at impeller Mach numbers near 1.0. Although no internal velocity measurements were made to confirm this belief, consideration of the nonuniform velocity profiles in turning such as exists at the inlet leads to that conclusion.. The statements made in Chapter III regarding losses at high impeller

FIGURE 20

CHANNEL PRESSURE RISE IN ORGDP-1-0 COMPRESSOR

Mach number appear to be verified by the test results. Also, the non-linearity of the pressure rise, especially at conditions of greater compressibility, was predicted from density considerations.

Flow angles in channel. Limited measurements of internal flow angles were made primarily to establish whether or not a helical-type flow pattern exists in the peripheral compressor. Data were obtained in air at speeds of 5000, 7500, 11,000, and 14,000 rpm., covering an impeller Mach number range of 0.19 to 0.54. For ease of measurement and to reduce the effect of the presence of the probe in the gas stream, the probing was done in a larger channel (ORGDP-3-2). The probing was accomplished with an 0.125-inch diameter cylindrical yaw probe traversing in an axial direction at a 4.500-inch radius. Four angular positions were probed: 37° , 124° , 236° , and 323° , measured in the direction of rotation from the centerline of the stripper. The flow angle data substantiated the assumption of the circulatory flow pattern by showing a radially inward component of fluid velocity in the channel. Flow angles as great as 60° were measured. (Purely tangential flow would be 0° and purely radially inward flow would be 90° .) The following observations were made from these probing tests:

1. The flow in the channel had a radially inward component at the three stations nearest the discharge for all flows and at the station near the inlet for flows less than $\lambda = 0.3$. At flows near $\lambda = 0.3$ and greater, the gas velocity directions near the inlet had no definite pattern. At all flows, however, the circulatory pattern was established by the time

the gas reached the second station (at 124°).

2. The flow angles increased with increasing axial distance from the impeller.
3. The flow angles increased with increasing circumferential distance from the inlet, as expected from density change considerations.
4. The flow angles increased as flow was reduced. Flows near shutoff were not explored, so that flow reversal (angle greater than 90°) was not actually measured. However, the data indicate that the same kind of results reported by Oelrich (7) would have been obtained, with the exceptions due to compressibility.

In the above observations, an increase in the flow angle was assumed to indicate an increase in meridional flow intensity. The type of pressure rise distributions measured tend to substantiate this assumption. Although angle measurements were not made at impeller Mach numbers near 1.0, there is no reason to believe that similar flow patterns did not exist there, too. In fact, the existence of helical flow can be used to at least partially explain the low maximum λ values measured at high Mach numbers. Obviously, if the flow was not tangential, its actual Mach number was greater than that represented by the average through-flow velocity. For example, if the flow angle was 45° , the actual velocity was over 40 percent greater than $V_{\theta,c}$. This effect, plus that of expansion of the higher-pressure carryover gas to greater volume, would necessitate lower $\lambda_{\max}$ values.

Specific Speeds and Specific Diameters

Since specific speed, N_s , and specific diameter, D_s , defined by Equations (1), page 6, and (2), page 7, are frequently used by designers, some mention of the experimental values is warranted. Of course, a large range of values were obtained during testing of a given design at a given speed, but the values at maximum efficiency are those of interest. Typical values of specific speed and specific diameter obtained on the ORGDP-1-0 design are presented in Table III. As may be seen, N_s generally was in the range of 12 to 30, and D_s ranged from 5.3 to 6.4. These numbers are approximately in the region of those presented by Balje (Figure 4, page 8). Design changes reported later in this chapter resulted in specific speeds up to 50 and specific diameters as low as 3 at maximum efficiency.

TABLE III
TYPICAL VALUES OF SPECIFIC SPEED AND SPECIFIC DIAMETER
OBTAINED ON THE ORGDP-1-0 COMPRESSOR

Test Gas	Max η_s , %	N_s	D_s
He	40	15	5.3
He	41	13	5.6
Air	40	12	5.8
Air	52	12	6.0
CHClF ₂	23	22	5.6
CHClF ₂	37	17	5.8
SF ₆	19	22	6.0
SF ₆	36	17	5.8
C ₈ F ₁₆	10	30	6.4

II. INFLUENCE OF CHANNEL AND NOZZLE GEOMETRY

The next phase of the experimental program was to investigate the effects of channel and nozzle geometry on the compressor performance.

Nozzle geometry. The original nozzle designs, consisting of circular pipes entering the cylindrical wall as shown in Figure 1, page 3, were believed to be causing excessive losses at high flows and Mach numbers, as indicated by the pressure distribution. Therefore, the modifications shown in Figure 21 were made in an effort to reduce these losses. Both the inlet and outlet nozzles were enlarged; additionally, the stripper was cut back on the inlet side and a splitter was installed in the suction nozzle. These latter changes were intended to help develop the circulatory flow pattern.

Figure 22 shows the effects of these modifications typical of low and high impeller Mach numbers. The results obtained on a small and a large channel (see Table II, page 90) are shown to illustrate the interrelationship between channel and nozzle geometry. The air data for the ORGDP-1-0 design were obtained at 200°F. suction temperature while all the other data were measured at 90°F.; however, the small difference in $M_{t,T}$ resulting from this temperature difference was not enough to significantly mask the nozzle effects. Also, the slight difference in clearances between the ORGDP-1-0 and ORGDP-5-4 designs should have little effect. Upon comparing the data before and after modification, the most significant change to be seen is the high Mach number

FIGURE 21

NOZZLE MODIFICATIONS

FIGURE 22

EFFECTS OF NOZZLE GEOMETRY ON PRESSURE RATIO AND FLOW

performance with the large channel. Simple modification of the nozzles resulted in almost doubling of the maximum flow and significant increase of the pressure ratio at all flows. Similar, but not as remarkable, results were observed with the smaller channel. In air and other low Mach number cases, some improvement at high flow resulted from the nozzle modifications, but low flow performance was impaired--particularly with the small channel. Apparently, the nozzle losses were not as important at low Mach numbers; in the case of the small channel, the maximum flows were small enough to cause overall performance to be poorer because the benefits from the enlarged nozzles did not offset the loss due to decreased pumping passage length.

Sample internal pressure distributions for SF_6 and the large channel are shown in Figure 23. These measurements, made at approximately the same flow rates, show the benefits of both the inlet nozzle and the outlet nozzle modifications. The depression of pressure in the inlet region was greatly reduced, and some gain at the outlet was observed also.

Channel geometry. A series of experiments was run in which the dimensions of the channel were systematically varied by employing removable inserts. The modified nozzles described above were used in these tests, which were made on air at 11,000 rpm. with $M_{t,T} = 0.42$. Suction pressure was 8 psia. and suction temperature was 90°F. in the tests. Figure 24 shows the data plotted as total pressure ratio versus specific mass flow for the eleven channels tested. Generally, each increase in channel area, A_c , whether made by increasing channel radial

FIGURE 23

EFFECTS OF NOZZLE GEOMETRY ON CHANNEL PRESSURE RISE

FIGURE 24

EFFECTS OF CHANNEL GEOMETRY ON PRESSURE RATIO

depth, h , or axial depth, b , shifted the performance curve to the right and upward. A notable exception was the case of the largest channel, which showed poor performance at low flows. Included for comparison are the data obtained on a channel which was designed to decrease losses in the meridional flow (ORGDP-13-4). As expected, significant improvement was achieved by this design. These curves illustrate an important advantage of the peripheral compressor. With a given impeller, speed, and casing, the performance can be greatly shifted by changing inserts in the channel. Therefore, should flow requirements shift in a given application, it is quite likely that a simple channel insert modification will permit use of the same basic machine.

Perhaps a more useful presentation of the channel performance data is the $\psi_s - \lambda$ plot shown in Figure 25, since the channel area is included in λ . This plot shows that a larger axial depth generally produced higher heads with the impeller used in these tests. The ratio of radial to axial depth, h/b , did not appear to be a controlling factor. Senoo (4) reported incompressible flow tests which showed that h/b of 0.4 was an optimum value, although he observed that the influence of h/b was not uniform. A tabulation of the characteristic channel dimension, R_m , defined by Equation (25), page 33, is included in Figure 25 and indicates that R_m may be a better criterion of optimum geometry. Although more data with different impeller dimensions are necessary to firmly establish this hypothesis, these data indicate optimum performance at $R_m \approx 0.6$ to 0.7 .

FIGURE 25

 ψ_s - λ CURVES FOR SEVERAL CHANNELS

Figure 25 also clearly shows the improvement with the contoured channel. Head coefficients 11 to 16 percent greater than those of the best rectangular channels were achieved by this design. Included in Figure 25 are maximum isentropic efficiencies and the λ -values at which they occurred. The maximum efficiencies were measured at $\lambda = 0.50 \pm 0.05$, and R_m again seemed to be indicative of performance, with maximum η_s occurring at $R_m = 0.5$ to 0.7 .

III. INFLUENCE OF IMPELLER GEOMETRY

The first changes made to impeller geometry were variations in axial blading depth, a_1 . With all other dimensions held the same, three impellers were machined with depths of 0.500 inch, 0.375 inch, and 0.250 inch. The results of tests on these three impellers with a small and a large channel are shown in Figure 26 for air at 11,000 rpm. and in Figure 27 for SF_6 at 11,000 rpm. In air, the 0.500-inch deep blading was significantly better than the others with the larger channel, but only slightly better than the 0.375-inch deep blading with the smaller channel. It is believed that the shallow-bladed impeller limited the meridional flow and hence the ability of the impeller to exchange momentum with the gas. In SF_6 , a different result was observed; the 0.250-inch deep blading was best with the smaller channel while the 0.375-inch blading was best with the larger channel. A likely explanation is that the smaller expansion and mixing losses through the stripper that would exist with the shallower blading offset the momentum-exchange effect discussed above.

FIGURE 26

AIR TEST RESULTS ON IMPELLERS WITH DIFFERENT AXIAL DEPTHS

FIGURE 27

SF_6 TEST RESULTS ON IMPELLERS WITH DIFFERENT AXIAL DEPTHS

Another series of impeller tests was made in order to determine the effect of leaning the blades forward with respect to the plane of the impeller. To facilitate machining at various angles, the thickness of the impeller was reduced to 0.400 inch at and below the blades, and the blades protruded paddle-wheel fashion with an axial depth of 0.500 inch. Also, the number of blades was reduced from 48 to 36 per side for the same reason. In addition to the conventional 90° blading, impellers with 75°, 60°, 45°, and 30° blading were tested. These impellers were tested in air and SF₆ at 7000 and 11,000 rpm. Typical results are the 11,000 rpm. air data presented in Figure 28, which show that the 45° blading was the best of those tested. Pressure ratios at approximately 50 percent maximum flow are shown as functions of the blade angle in Figure 29 for each test condition. The 45° blading was consistently best, and each forward-bent blading design was better than the conventional 90° blading at the higher Mach numbers, as predicted in Chapter III.

Another point to be made regarding the data in Figure 28 is that concerning blade shape. The dashed curve shown represents the performance of the original impeller used in most of the previously discussed tests. These particular impeller tests were made in a new casing with slightly different nozzles than those shown in Figure 21, page 107 (the newer nozzles were a little wider and did not consume as much of the circumference). Otherwise, the ORGDP-56-4 design was the same as the ORGDP-5-4 design represented in previous curves. The performance of this original impeller was significantly better than its 90°,

FIGURE 28

INFLUENCE OF IMPELLER BLADING ANGLE

FIGURE 29

PRESSURE RATIO AS FUNCTION OF IMPELLER BLADE ANGLE

paddle-wheel type, counterpart (ORGDP-59-4). Although the latter impeller had 36 instead of 48 blades per side, this blade number difference is not believed significant as negligible difference in performance was seen between a 48-bladed, 90° paddle-wheel impeller and the 36-bladed impeller. Therefore, the better performance of the original impeller was apparently due to the curved surface at the root of the blading. This was expected from consideration of the circulatory flow patterns since the curved surface would help guide the flow through the blading.

IV. INFLUENCE OF IMPELLER-TO-CASING CLEARANCES

One of the reported disadvantages of the peripheral pump or compressor is the need for small clearances between the impeller and casing in the stripper and at the sides of the impeller. This was kept in mind in designing the original experimental compressor, which had 0.007-inch side clearances and 0.005-inch tip clearance in the stripper. However, at speeds above 11,000 rpm., it was necessary to increase the side clearances to 0.012-inch due to shaft thermal expansion. Repeat runs in air at the lower speeds with the larger clearances showed negligible differences in performance. Additional tests at larger clearances (up to $c_1 = 0.030$ -inch and $c_2 = 0.015$ -inch) with a large channel, reported in Reference (45), showed little detrimental effects from the increased clearances. It was assumed then that the high carry-over flow through the stripper within the impeller blading masked the clearance effects.

Additional data on two sets of clearances were obtained in air and SF_6 at 7000 rpm. and 11,000 rpm. using a small and a large channel. The small channel data are presented in Figure 30 and the large channel data in Figure 31. With the small channel, performance was generally better with the lesser clearance, although little difference was seen in SF_6 at 11,000 rpm. ($M_{t,T} = 1.04$). With the larger channel, there was little difference in performance at all four test conditions; in fact, in SF_6 at 11,000 rpm., better performance was observed at the larger clearance. This latter result confirmed the earlier test results reported in Reference (45). Shimosaka's criterion that the ratio of the characteristic channel dimension, R_m , to side clearance, c_1 , should be greater than 20 (see page 35) appears to apply to these and the earlier results. Of course, more data are necessary to firmly establish this ratio as a true criterion of clearance effects.

There is no known explanation for the one case of better performance with the larger clearance; however, a possible cause is the high degree of dependence of the SF_6 performance at 11,000 rpm. upon channel area. Although the clearance increase represents less than 3 percent increase in the channel area (referenced to the small channel), comparison of Figures 30 and 31 reveals that the 111 percent area enlargement resulted in more than 100 percent increase in flows and pressure ratios in SF_6 at 11,000 rpm. For example, at 2:1 pressure ratio, the 111 percent channel enlargement resulted in 149 percent flow increase; therefore, the additional 6 percent flow increase resulting from the clearance increase could be at least partially due to the small area increase (since $3 \times \frac{149}{111} \approx 4$).

FIGURE 30

CLEARANCE EFFECTS WITH SMALL CHANNEL

FIGURE 31

CLEARANCE EFFECTS WITH LARGE CHANNEL

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

In Chapter II, the theories proposed for incompressible flow operation in the peripheral pump were presented, discussed, and compared. Since the essential features of these theories were summarized with the discussion of each and conclusions were presented at the end of that chapter, little additional comment is necessary here. It was concluded that each of the incompressible theories requires empirical coefficients and that insufficient liquid data exist to completely predict performance or to firmly establish whether the regenerative theory with circulatory flow, the turbulent theory with momentum-exchange flow, or the turbulent shear theory best describes the pumping mechanism. A summary of experimental observations in incompressible flow was presented on page 55.

In Chapter III, it was shown that none of the incompressible flow theories can be used to predict performance with compressible flow because of certain assumptions made in their derivations. However, the regenerative concept of circulatory flow was assumed the best explanation of compressible flow operation, with recognition of the existence of a high degree of turbulence. Five fundamental laws of thermodynamics, fluid motion, and mass and energy conservation were applied to the peripheral compressor. Although solutions to the resulting equations were not obtained, certain qualitative conclusions regarding

compressor performance and expressions relating performance parameters were derived from them. As is frequently done in turbo-compressor analysis, functional relationships derived from dimensional analysis were assumed for the peripheral compressor. Also, predictions were made in a qualitative manner regarding peripheral compressor performance, particularly at high impeller Mach numbers.

Test data obtained on an experimental compressor operated on five gases at several speeds up to 14,000 rpm. and data measured to evaluate the influences of channel, nozzle, impeller, and clearance geometries were presented in Chapter IV in plots of dimensionless parameters. Generally, the data confirmed the predictions made for compressible flow operation. From the analyses of the data, the following conclusions may be drawn:

1. The peripheral compressor operated satisfactorily without surging or unstable operation with gases having a range of molecular weights of 4 to 400. However, performance at high impeller Mach numbers (near and above 1.0) became poorer, probably because of increased losses due to blade entrance, nozzle, stripper expansion, and mixing. A maximum pressure ratio was observed with each design tested regardless of increase in impeller speed or Mach number. It is believed that locally high flow Mach numbers and possibly choking in the channel are the reasons for this performance deterioration.

2. Correlation of the data on the different gases in terms of the dimensionless parameters was confirmed. Either total pressure ratio, $P_{t,d}/P_{t,s}$, and specific mass flow, ϕ_t , or isentropic head coefficient, ψ_s , and capacity coefficient, λ , correlations with impeller tip Mach number, $M_{t,T}$, as a speed parameter were good. Generally, pressure ratio or head coefficient data at a given Mach number collapsed fairly well onto a single curve regardless of type of gas; the small discrepancies are believed due to Reynolds number, specific heat ratio, and possibly Prandtl number effects.
3. Isentropic efficiencies as great as 50 percent were measured, although generally they were in the 30 to 40 percent range, with maximum values as low as 10 percent at impeller tip Mach numbers above 1.0. No definite correlation of efficiency with Mach number was found, although it generally decreased with increasing Mach number. Maximum efficiencies occurred at approximately 60 percent of maximum flow with the original nozzles and near 70 percent of maximum flow with the enlarged nozzles.
4. Internal pressure and flow angle measurements confirmed the predicted non-linear channel pressure rise in compressible flow and the helical flow patterns. Also, large depression of pressure in the inlet region was observed at high Mach numbers.

5. Nozzle modifications in the form of enlargement greatly improved high Mach number performance and significantly reduced the pressure losses in the inlet. However, low Mach number performance at low flows was impaired, especially with small channels.
6. Channel test results showed no definite optimum ratio of radial depth, h , to axial depth, b . The characteristic channel dimension, R_m , suggested by Shimosaka (15) appeared to be an indicator of performance, with best performance observed at $R_m \approx 0.6$ to 0.7 . The best channel tested had a contoured shape designed to reduce losses in the circulatory flow.
7. Limited test data on the influence of impeller blading axial depth, a_1 , indicated that a large depth was better at low Mach numbers while a small depth was as good or better at high Mach numbers.
8. Impeller blade-angle test results confirmed the prediction that forward-inclined blades would improve performance by reducing blade entrance losses in the circulatory flow. Any forward-inclination of the blading improved performance, and an angle of 45° was best with the channel tested.
9. Impeller-to-casing clearances were shown to be important with small channels, but less so with large channels. Shimosaka's criterion of R_m/c_1 and R_m/c_2 greater than 20 for insignificant clearance effect agreed with the limited test data.

This study was undertaken to establish the "state-of-the-art" of the peripheral pump and compressor and to investigate the performance of the peripheral compressor on compressible fluids in a general way. More data are needed to substantiate some of the tentative conclusions drawn and to establish adequate design information. The following suggestions and recommendations are made regarding future exploration and development of the peripheral compressor performance (some of these studies would make good projects for master's thesis work):

1. Extensive internal probing of several geometries tested over a wide range of conditions is needed to better establish the flow mechanisms in compressible fluids and to provide better background for theoretical treatments. Flow visualization techniques would be useful in such a study.
2. Much more data on impeller and channel geometries are necessary to establish optimum geometric parameters. The design variables h , b , a_1 , a_2 , and R_m should be included in such a study.
3. Data are needed on geometrically similar machines to test the similitude hypothesis. Although the data from this study correlated well in the dimensionless parameters used, the scaling effect was not investigated.
4. The influence of Reynolds number on performance should be studied. The investigation of the effects of channel and impeller surface roughness would be appropriate in such a study.

5. Additional clearance data are needed for a wide variety of designs. Of particular interest would be whether R_m/c_1 and R_m/c_2 are really criteria of clearance effects. Clearances are quite important from the standpoint of machining tolerances (and cost) and thermal expansion.
6. The effects of heat transfer should be explored. It is believed that the departure from adiabatic compression can be great under certain conditions.

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LIST OF REFERENCES

1. Balje, O. E., "A Study on Design Criteria and Matching of Turbomachines: Part B - Compressor and Pump Performance and Matching of Turbo-components," American Society of Mechanical Engineers Paper No. 60-WA-231, 1960.
2. Samaras, D. G., and Tyler, R. A., "The Relative Merits of Rotary Compressors," Journal of the Aeronautical Sciences, Vol. 15, No. 10, 1948, pp. 625-631.
3. Yeapple, F. D., "Rotors Ride on Gas," Product Engineering, Vol. 34, No. 21, 1963, pp. 116-123.
4. Senoo, Y., "A Comparison of Regenerative-Pump Theories Supported by New Performance Data," Transactions of the American Society of Mechanical Engineers, Vol. 78, 1956, pp. 1091-1102.
5. Hopkins, T. J., and Lazo, L. R., "Theoretical and Experimental Analysis of a Regenerative Turbine Pump, the Sta-Rite TH-7," Senior Thesis, Massachusetts Institute of Technology, January, 1953.
6. Lutz, G. F., "Experimental Investigation of the Pressure Distribution in a Regenerative Turbine Pump, the Sta-Rite TH-7," Senior Thesis, Massachusetts Institute of Technology, May, 1953.
7. Oelrich, J. A., "Development of an Analysis of a Regenerative Pump," Thesis for the Degree of Mechanical Engineer, Massachusetts Institute of Technology, August, 1953.
8. Wilson, W. A., Santalo, M. A., and Oelrich, J. A., "A Theory of the Fluid-Dynamic Mechanism of Regenerative Pumps," Transactions of the American Society of Mechanical Engineers, Vol. 77, 1955, pp. 1303-1316.
9. Santalo, M. A., Discussion of Reference (4), Transactions of the American Society of Mechanical Engineers, Vol. 78, 1956, pp. 1097-1099.
10. Bartels, J., Discussion of Reference (19), Transactions of the American Society of Mechanical Engineers, Vol. 77, 1955, p. 23.
11. Wright, A. M., Discussion of Reference (19), Transactions of the American Society of Mechanical Engineers, Vol. 77, 1955, pp. 25-27.

12. Senoo, Y., "Theoretical Research on Friction Pump," Reports of the Research Institute for Fluid Engineering, Japan, Vol. 5, No. 1, 1948, pp. 23-38.
13. Senoo, Y., "Researches on Peripheral Pump," Reports of Research Institute for Applied Mechanics, Japan, Vol. 3, No. 10, 1954, pp. 53-113.
14. Knudsen, J. G., and Katz, D. L., Fluid Dynamics and Heat Transfer, McGraw-Hill Book Company, Inc., New York, 1958, pp. 113-114.
15. Shimosaka, M., "Research on the Characteristics of Regenerative Pump (2nd Report, Theoretical Research on the Performance)," Bulletin of the Japan Society of Mechanical Engineers, Vol. 3, No. 10, 1960, pp. 191-199.
16. Shimosaka, M., and Yamazaki, S., "Research on the Characteristics of Regenerative Pump (1st Report, Influences of Flow Channel and Impeller)," Bulletin of the Japan Society of Mechanical Engineers, Vol. 3, No. 10, 1960, pp. 185-190.
17. Hasinger, S., "A Study of the Peripheral Pump," Wright Air Development Center Technical Report 57-333, May, 1957.
18. Pfleiderer, C., "Side-Channel Pumps" in Die Kreiselpumpen fur Flussigkeiten und Gase Wasserpumpen, Ventilatoren, Turbogebblase Turbokompressoren, Springer-Verlag, Berlin, 1955, pp. 571-584.
19. Iversen, H. W., "Performance of the Periphery Pump," Transactions of the American Society of Mechanical Engineers, Vol. 77, 1955, pp. 19-28.
20. Weinig, F. S., "Analysis of Traction Pumps," Wright Air Development Center Technical Report 54-554, June, 1955.
21. Wilson, W. E., "Analysis of Turbine Pumps," Product Engineering, Vol. 18, No. 10, 1947, pp. 163-166.
22. Miyagi, O., and Miyadzu, A., "Theory of the Westco-type Rotary Pump," Technology Reports of the Tohoku Imperial University, Japan, Vol. 13, No. 1, 1939, pp. 264-288.
23. Crewdson, E., "Water-ring Self-priming Pumps," with an addendum by E. A. Jackson, Proceedings of the Institution of Mechanical Engineers, Vol. 170, No. 13, 1956, pp. 407-425.
24. Wright, J. R., and Kirtley, W. F., "Design of a Universal Compressor for Gas Circulation in In-Pile Loops and Similar Applications," Paper No. 48 presented at the Symposium on the Design of Experimental Equipment for Use in Nuclear Energy Research, U.K.A.E.A. Research Establishment, Harwell, Berkshire, April 11-14, 1961.

25. Burgoyne, T., "Flow Tests on Peripheral-Type Compressor," U.K.A.E.A. Research and Development Branch (Windscale) Technical Note 145, June, 1954.
26. Namba, I. K., "Development of Regenerative Compressor for Helium Circulation," U.S.A.E.C. Report ORNL-TM-218, Oak Ridge National Laboratory, July 20, 1962.
27. Grindell, A. G., "Development of Small Compressors for Gas-Cooled In-Pile Loops," Shafts Seals for Compressors and Turbines for Gas-Cooled Reactor Application, U.S.A.E.C. Report TID-7604, December, 1959, pp. 191-199.
28. Balje, O. E., "Drag-Turbine Performance," Transactions of the American Society of Mechanical Engineers, Vol. 79, 1957, pp. 1291-1304.
29. Burton, D. W., "Review of Regenerative Compressor Theory," Rotating Machinery for Gas-Cooled Reactor Application, U.S.A.E.C. Report TID-7631, June, 1962, pp. 228-242.
30. Burton, D. W., Unpublished Information (Unclassified), Oak Ridge Gaseous Diffusion Plant, Union Carbide Corporation, Nuclear Division, Oak Ridge, Tennessee.
31. Stanitz, J. D., "Centrifugal Compressors," Gas Turbine and Free Piston Engine Lectures, University of Michigan, 1958.
32. Stepanoff, A. J., Turboblowers, John Wiley & Sons, Inc., New York, 1955, pp. 8-11, 18-25, 76-132.
33. Ferguson, T. B., The Centrifugal Compressor Stage, Butterworths, London, 1963.
34. Shepherd, D. G., Principles of Turbomachinery, The Macmillan Co., New York, 1956, pp. 10-47, 125-140.
35. O'Neill, P. P., and Wickli, H. E., "Predicting Process Gas Performance of Centrifugal Compressors from Air Test Data," Transactions of the American Society of Mechanical Engineers, Vol. 84, Series B, No. 1, 1962, pp. 248-264.
36. Schultz, J. M., "The Polytropic Analysis of Centrifugal Compressors," Transactions of the American Society of Mechanical Engineers, Vol. 84, Series A, No. 1, 1962, pp. 69-82.
37. Wiesner, F. J., Jr., "Practical Stage Performance Correlations for Centrifugal Compressors," American Society of Mechanical Engineers Paper No. 60-HYD-17, 1960.

38. Macaluso, C. A., "The Correlation of Turbo-compressor Performance with Various Gases," American Society of Mechanical Engineers Paper No. 60-HYD-15, 1960.
39. Davis, H., "Equivalent Performance Parameters for Turboblenders and Compressors," Transactions of the American Society of Mechanical Engineers, Vol. 80, 1958, pp. 108-116.
40. Del Mar, B. E., "Presentation of Centrifugal-Compressor Performance in Terms of Nondimensional Relationships," Transactions of the American Society of Mechanical Engineers, Vol. 67, 1945, pp. 483-490.
41. Sheets, H. E., "Nondimensional Compressor Performance for a Range of Mach Numbers and Molecular Weights," Transactions of the American Society of Mechanical Engineers, Vol. 74, 1952, pp. 93-102.
42. Davis, H., Kottas, H., and Moody, A. M. G., "The Influence of Reynolds Number on the Performance of Turbomachinery," Transactions of the American Society of Mechanical Engineers, Vol. 73, 1951, pp. 499-509.
43. Balje, O. E., "A Contribution to the Problem of Designing Radial Turbomachines," Transactions of the American Society of Mechanical Engineers, Vol. 74, 1952, pp. 451-472.
44. Cates, P. S., "Performance Characteristics of a Peripheral Compressor," Rotating Machinery for Gas-Cooled Reactor Application, U.S.A.E.C. Report TID-7690, May, 1964.
45. Cates, P. S., "Peripheral-Compressor Performance on Gases with Molecular Weights of 4 to 400," American Society of Mechanical Engineers Paper No. 64-WA/FE-25, 1964.
46. Vavra, M. H., Aero-Thermodynamics and Flow in Turbomachines, John Wiley & Sons, Inc., New York, 1960, pp. 1-155.
47. The American Society of Mechanical Engineers Power Test Code 19, Part 5, Chapter 4, 1949..

APPENDIXES

APPENDIX A

NOMENCLATURE

- a = Total axial width of impeller
- a_1 = Axial depth of blading
- a_2 = Radial length of blading
- A_2 = Orifice area
- A_b = Cross-sectional area of blades
- A_c = Cross-sectional area of channel
- A_i = Impeller surface area (cylindrical surface)
- A_L = Leakage flow area
- A_w = Casing or wall surface area
- A, B = Constants in Equation (73b), page 99
- B = Ratio of casing to impeller friction terms, Figure 7,
page 40
- b = Axial depth of channel
- c_1, c_2 = Impeller-to-casing side and tip clearances, respectively
- C_1, C_2 = Constants in Equation (13), page 24
- c_p = Specific heat capacity at constant pressure
- c_v = Specific heat capacity at constant volume
- d = Santalo's "typical dimension" of the open channel
- D = Impeller diameter

- D_1 = Inside diameter of pipe upstream of orifice
 D_2 = Inside diameter of orifice
 D_s = Specific diameter
 $\vec{e}_R, \vec{e}_\theta, \vec{e}_Z$ = Unit vectors in cylindrical coordinate system
 e = Total energy per unit mass
 E = Energy transferred
 E_{gas} = Energy input to gas
 E_{ideal} = Ideal energy output
 f = Dimension in Figure 5, page 17
 f_i = Impeller shear coefficient
 f_w = Casing shear coefficient
 $\vec{F}$ = Body force vector per unit mass
 FE = Flow energy
 g = Acceleration of gravity, geometric factor
 g_c = Gravitational constant, $32.17 \text{ lb}_m \cdot \text{ft.} / \text{lb}_f \cdot \text{sec.}^2$
 h = Radial depth of channel, enthalpy per unit mass
 H = Compressor or pump delivered head
 H_c, H_t = Circulatory and tangential head losses, respectively
 H' = Senoo's head decrease in entrance region
 k = Ratio of specific heat capacities, c_p/c_v
 $\left. \begin{matrix} k_c, k'_c \\ k_t, k'_t, k''_t \end{matrix} \right\}$ = Loss coefficients in regenerative theories

- k_t''', k_t^{iv} = Loss coefficients in turbulent theories
 k_L = Leakage loss coefficient in Weinig's equations
 k_N = Nozzle loss coefficient in Weinig's equations
 K = Orifice coefficient
 K_1, K_2 = Geometric factors in Equation (8), page 20
 K_3, K_4 = Coefficients in Equations (39a) and (39b), page 49
 K', K'' = Coefficients in Shimosaka's equations
 KE = Kinetic energy
 ℓ = Mixing length
 L = Length of pumping channel
 L' = Ineffective length of pumping channel
 m = Mass
 M = Mach number
 MW = Molecular weight
 n = Exponent in Equation (26), page 34, polytropic exponent
 $\vec{n}$ = Unit vector normal to control surface, S
 $\vec{n} \cdot \vec{\tau}_\theta$ = Component of viscous stress vector, $\vec{\tau}$, in θ -direction
 N = Impeller rotational speed
 N_S = Specific speed
 P = Pressure, power
 PE = Potential energy
 Pr = Prandtl number
 $\vec{q}$ = Heat flux vector
 q = Volume flow per unit breadth

Q = Volume flow, heat added to gas

Q_m = Meridional volume flow

Q_{sb} = Solid body volume flow, $r_g \omega A_c$

$\vec{r}$ = Position vector

r_g = Radius to centroid of channel

$\left. \begin{matrix} r_o, r_1, r_2 \\ r_3, r_r \end{matrix} \right\}$ = Radii in Figure 5, page 17

R = Radius in cylindrical coordinate system, specific gas constant

R_m = Shimosaka's characteristic channel dimension,
 $A_c / (2a_2 + a)$

R_o = Universal gas constant, 1545 ft.-lb_f./lb_m.-mole - °R.

Re = Reynolds number

s = Entropy

S = Control surface

t = Time

t_1, t_2 = Blade spacing and thickness, respectively

T = Temperature

u = Internal energy per unit mass, fluid velocity in x-direction

U = Internal energy, impeller velocity

v = Control volume, fluid velocity in y-direction

V = Absolute fluid velocity

$\vec{V}$ = Velocity vector in absolute system

- w = Mass flow rate
 W = Relative velocity, work done on gas
 $\vec{W}$ = Velocity vector in relative system
 x = Direction of impeller movement in two-dimensional, rectangular coordinate system
 y = Direction perpendicular to impeller
 Y = Orifice expansion parameter
 Z = Number of impeller blades per side, axial distance in compressor, compressibility factor, height above datum
 α = Blade entrance factor
 β = Fluid angle, orifice diameter ratio
 β_b = Blade angle relative to plane of impeller
 γ_p = Input power coefficient
 δ = Distance from impeller to fixed wall
 ϵ = Senoo's y -coordinate at fixed wall
 η = Efficiency
 θ = Angular position in compressor, coordinate in cylindrical system
 $\Delta\theta_a$ = Active pumping angle
 θ_s = Stripper angle
 κ = Turbulence intensity coefficient, thermal conductivity
 κ' = Turbulence coefficient in Shimosaka's equations
 λ = Capacity coefficient based on impeller tip velocity

λ' = Capacity coefficient based on average impeller velocity

μ = Molecular viscosity

μ_t = Turbulent, or eddy, viscosity

$\vec{\pi}$ = Total stress vector at a point on the surface, S

ρ = Fluid density

σ = Slip factor

τ = Shear stress

τ_t = Turbulent shear stress

$\vec{\tau}$ = Viscous stress vector at a point on the surface, S

ϕ_t = Specific mass flow

ψ = Head coefficient based on impeller tip velocity

ψ' = Head coefficient based on average impeller velocity

ω = Impeller angular velocity

$\vec{\omega}$ = Angular velocity vector

Subscripts

1 = Blade root or entrance, control volume inlet

2 = Blade tip or exit, control volume exit

aero = Aerodynamic

c = Circulatory, channel

cas = Casing

d = Compressor discharge

i = Impeller

imp = Impeller

m = Meridional

max = Maximum
p = Polytropic
rel = Relative
R, θ ,Z = Cylindrical coordinates
s = Compressor suction, isentropic
so = Shutoff
t = Total, tangential, turbulent
T = Blade tip
x,y,z = Cartesian coordinates
w = Casing wall

APPENDIX B

DERIVATION OF INTEGRAL VECTOR RELATIONS

The continuity, angular momentum, and energy equations presented in Chapter III in integral vector form can be derived from application of the basic laws of fluid motion and thermodynamics to a control volume, v , with the surface, S . Similar derivations can be found in Vavra (46), but the forms presented below were condensed from Burton (30).

Continuity equation. With $\vec{n}$ the unit normal vector directed outward from the surface, S , the amount of fluid leaving through the incremental area, dS , can be represented by

$$\rho \vec{n} \cdot \vec{V} dS ,$$

so that the net flux of fluid into the volume, v , is

$$- \int_S \rho \vec{n} \cdot \vec{V} dS .$$

This net flux of fluid into the volume is equal to the rate of increase of material within the volume; therefore,

$$- \int_S \rho \vec{n} \cdot \vec{V} dS = \int_v \frac{\partial \rho}{\partial t} dv , \quad (74)$$

which is the same as Equation (40), page 59.

Energy equation. The derivation of this vector equation begins with the statement of the conservation of energy by the first law of

thermodynamics. Letting the total energy content of the fluid be represented by $E = \rho e$ (see Equation (43), page 62), the rate of change of the fluid total energy in the control volume can be written as

$$\int_V \frac{\partial \rho e}{\partial t} dv + \int_S \rho e \vec{V} \cdot \vec{n} dS ,$$

where the first term represents storage energy and the second term, flow energy. This rate of energy increase is equal to the rate of heat addition to the control volume and the rate of work done on the volume by surface and body forces. Heat addition rate through the control surface is

$$- \int_S \vec{q} \cdot \vec{n} dS ,$$

where $\vec{q}$ is the heat flux vector. The body-force work rate, where $\vec{F}$ is the body force vector per unit mass (such as gravity), can be represented by

$$\int_V \rho \vec{F} \cdot \vec{V} dv .$$

Finally, the rate of work done on the fluid by surface stresses can be written as

$$\int_S \vec{\pi} \cdot \vec{V} dS ,$$

where $\vec{\pi}$ is the stress vector at each point on the surface, S . Since an excellent detailed discussion of the stress conditions of a fluid particle is contained in Vavra (46), only brief definitions will be presented here. The vector, $\vec{\pi}$, can be written as

$$\vec{\pi} = \vec{e}_R (\vec{\pi}_R \cdot \vec{n}) + \vec{e}_\theta (\vec{\pi}_\theta \cdot \vec{n}) + \vec{e}_Z (\vec{\pi}_Z \cdot \vec{n}) \quad (75)$$

where

$$\begin{aligned} \vec{\pi}_R &= \vec{e}_R \pi_{RR} + \vec{e}_\theta \pi_{R\theta} + \vec{e}_Z \pi_{RZ} , \\ \vec{\pi}_\theta &= \vec{e}_R \pi_{\theta R} + \vec{e}_\theta \pi_{\theta\theta} + \vec{e}_Z \pi_{\theta Z} , \\ \vec{\pi}_Z &= \vec{e}_R \pi_{ZR} + \vec{e}_\theta \pi_{Z\theta} + \vec{e}_Z \pi_{ZZ} , \end{aligned} \quad (76)$$

and $\vec{e}_R$, $\vec{e}_\theta$, and $\vec{e}_Z$ are unit vectors in the cylindrical coordinate system. The pressure stress and the viscous stress can be separated from the total stress by

$$\vec{\pi} = \vec{\tau} - \vec{n} P , \quad (77)$$

where $\vec{\tau}$ is defined by relations similar to Equations (75) and (76), with the viscous stress, τ , substituted for π . The expression for $\vec{\tau}_\theta$ in Equation (53), page 65, is obtained after assuming that $\tau_{\theta R} = \tau_{R\theta}$ and $\tau_{\theta Z} = \tau_{Z\theta}$. Following usual convention, the subscripts to the shear stress components are interpreted as follows: $\tau_{\theta R}$ is the shear force per unit area in the direction of the R-axis acting on the face perpendicular to the θ -axis. Also, $\vec{\tau}_\theta$ is the viscous force vector per unit area acting on the face perpendicular to the θ -axis. Similarly, $\vec{\pi}_\theta$ is the total surface force vector per unit area acting on the face perpendicular to the θ -axis and includes the pressure stress in the component normal to that face.

Therefore, the energy equation can be written as

$$\begin{aligned}
\int_v \frac{\partial \rho e}{\partial t} dv + \int_S \rho e \vec{V} \cdot \vec{n} dS &= - \int_S \vec{q} \cdot \vec{n} dS + \int_v \rho \vec{F} \cdot \vec{V} dv \\
&+ \int_S \vec{\tau} \cdot \vec{V} dS - \int_S P \vec{n} \cdot \vec{V} dS .
\end{aligned} \tag{78}$$

With the assumption of steady flow, the first term in Equation (78) drops out, and if potential energy is neglected, the total energy per unit mass becomes

$$\begin{aligned}
e &= u + \frac{1}{2g_c} |\vec{V}|^2 \\
&= h - P/\rho + \frac{1}{2g_c} |\vec{V}|^2 \\
&= h_t - P/\rho .
\end{aligned} \tag{79}$$

The assumption of steady flow and the substitution of Equation (79) into Equation (78) result in Equation (47), page 63.

Flux of angular momentum. Newton's second law of motion applied to a control volume, v , with the surface, S , states that the net rate of change in momentum of the fluid within the volume equals the sum of the external forces acting on the volume. The rate of change in momentum can be expressed as

$$\frac{D}{Dt} \int_v \rho \vec{V} dv ,$$

where $\frac{D}{Dt}$ is the total, or material, derivative,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \nabla . \tag{80}$$

The external forces acting on the volume, v , consist of body forces (such as gravity),

$$\int_V \rho \vec{F} dv ,$$

and surface forces,

$$\int_S \vec{\pi} dS = \int_S \vec{\tau} dS - \int_S \vec{n} P dS . \quad (81)$$

Therefore, the rate of change of linear momentum in the absolute system is

$$\frac{D}{Dt} \int_V \rho \vec{V} dv = \int_V \rho \vec{F} dv + \int_S \vec{\tau} dS - \int_S \vec{n} P dS . \quad (82)$$

The rate of change of angular momentum is equal to the sum of the external torques acting on the volume, v . A relation for this equality can be obtained by taking the vector cross-product between the position vector, $\vec{r}$, and Equation (82), where

$$\vec{r} = R \vec{e}_R + Z \vec{e}_Z \quad (83)$$

in cylindrical coordinates. (In this application, $\vec{r}$ has its origin on the compressor axis.) This vector cross-product is

$$\begin{aligned} \frac{D}{Dt} \int_V \rho (\vec{r} \times \vec{V}) dv &= \int_V \rho (\vec{r} \times \vec{F}) dv + \int_S (\vec{r} \times \vec{\tau}) dS \\ &\quad - \int_S (\vec{r} \times \vec{n}) P dS . \end{aligned} \quad (84)$$

With the vector equality,

$$\frac{D}{Dt} \int_V \rho \vec{F} dv = \int_V \rho \frac{D\vec{F}}{Dt} dv , \quad (85)$$

where $\vec{F}$ is some arbitrary vector function of position, $\vec{F}(\vec{r}, t)$, the first term in Equation (84) becomes

$$\frac{D}{Dt} \int_V \rho (\vec{r} \times \vec{V}) dv = \int_V \rho \frac{D(\vec{r} \times \vec{V})}{Dt} dv . \quad (86)$$

In compressors, the torque about the compressor axis is of interest; therefore, only the $\vec{e}_Z$ component of Equation (84) need be considered. With Equations (80) and (86), the $\vec{e}_Z$ component of Equation (84) is

$$\begin{aligned} \int_V \rho \frac{\partial RV_\theta}{\partial t} dv + \int_V \rho \vec{V} \cdot \nabla RV_\theta dv &= \int_V \rho R F_\theta dv \\ &+ \int_S \vec{n} \cdot R \vec{\tau}_\theta dS - \int_S \vec{n} \cdot \vec{e}_\theta R P dS . \end{aligned} \quad (87)$$

With the assumption of steady or time-averaged flow and the use of a vector identity, the continuity equation, and the divergence theorem, Equation (87) becomes

$$\begin{aligned} \int_S \overline{\rho R V_\theta \vec{n} \cdot \vec{V}} dS &= \int_V \overline{\rho R F_\theta} dv + \int_S \overline{\vec{n} \cdot R \vec{\tau}_\theta} dS \\ &- \int_S \overline{\vec{n} \cdot \vec{e}_\theta R P} dS , \end{aligned} \quad (88)$$

which is Equation (50), page 64. A similar procedure is followed in deriving Equation (51), page 64, which applies to the relative system.

APPENDIX C

DATA ACQUISITION AND REDUCTION

The following pages present a description of the experimental apparatus, data acquisition methods, and computation techniques. An TRM 7090 computer program was written to perform all calculations from the raw input data. Perfect gas equations were used in calculating the various parameters according to the relationships presented in Chapter III. The properties of mixtures were calculated based on application of the Gibbs-Dalton Law.

General description of facility. The experimental facility was located in the K-1401 building at the Oak Ridge Gaseous Diffusion Plant. A schematic of the test loop and auxiliary equipment is shown in Figure 32. The primary process gas loop consisted of the experimental compressor, a gas-to-water heat exchanger, a 3-inch, air-operated control valve, five orifice lines with manual block valves, gas feed and withdrawal systems, a surge volume, and the associated instrumentation described below. A lube system consisting of a 1/3 hp. motor and pump, an oil-to-water heat exchanger, oil filters, and pressure and temperature safety switches was used to supply oil to the compressor and motor bearings. A mass spectrometer consisting of a Beckman Instruments Company line-recorder rack and General Electric Company control panels was used to monitor concentration of foreign gases in the test gas. (In the early testing, samples were drawn from the loop in bottles for analysis.) The cooling water system shown in

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FIGURE 32
SCHEMATIC DRAWING OF TEST LOOP

Figure 32 removed the heat of compression and controlled the compressor suction temperature. For the tests at elevated temperature, the loop was enclosed in an electrically heated housing in which the temperature was controlled at 200°F. For charging the loop with the process gas, the loop was evacuated and the gas was fed in from bottles or cylinders. Effort was made to keep the process gas purity above 98 percent, but not always with success due to the small loop volume. A buffered shaft seal on the compressor was satisfactory in reducing inleakage from that source, and a surge volume helped reduce the effect of unavoidable inleakage.

Drive system. The original experimental compressor was that shown schematically in Figure 1, page 3, plus the nozzle modifications that were made later. A modified Worthington compressor shaft and bearing housing assembly with tilting-pad journal bearings was used to support the peripheral compressor casing and impeller. An Allis-Chalmers Size 12 compressor frame with modified shaft and bearing system was used with later experimental designs. The compressor was driven by a Westinghouse variable-frequency motor rated at 50 hp. at 240 cps. Capability of both 2-pole and 4-pole operation permitted the speed range of 5000 to 14,000 rpm. with the available 150 to 240 cps. supply. A Burke Electric Company 75 kva. motor-generator set driven by an Allis-Chalmers dc. motor-generator set supplied the electrical power.

Pressures. Process gas static pressures were measured at the orifices, at the compressor inlet and discharge, and at seven points in

the compressor channel through 1/8-inch diameter holes drilled flush with the inner surfaces. Orifice fore-pressures and compressor suction pressure were measured on the absolute scale, while differential measurements were made for the orifice pressure drop and the compressor pressure rise. Taylor Instrument Company absolute and differential pressure transmitters sensed the gas pressures and provided pneumatic signals to Bourdon-type pressure gages. The transmitters and gages were calibrated together in place, and the resulting calibration equations were included in the computer program. Total pressures were computed from the static pressures and calculated Mach numbers.

Temperatures. Gas total temperatures were sensed by chromel-alumel thermocouples made into probes with half-cylinder shells around the junction. The probes were furnace-calibrated and the resulting calibration equations were included in the computer program. Although the recovery factor of these probes was considered high based on the design, a factor appreciably less than 100 percent would not cause significant error since gas velocities at points of measurement were not great. Gas temperatures were measured by a Leeds and Northrup Company No. 8662 Portable Precision Potentiometer, while other temperatures (oil, casing, water, etc.) were recorded on an L and N Type G Speedomax Recorder. Gas static temperatures were calculated from mass flow, total temperature, static pressure, and area by solution of simultaneous equations. With static and total temperatures known, Mach numbers and then total pressures could be calculated. Gas properties in the compressor were based on an average of the compressor suction and discharge temperatures.

Flows. The orifices used to measure mass flows were of standard ASME design with vena-contracta pressure taps. The orifices were calibrated in water by the Y-12 plant over the Reynolds number range expected. Calibration equations in the form of the orifice coefficient as a function of Reynolds number were included in the computer program. For a pipe diameter, D_1 , and orifice diameter, D_2 , the equation for mass flow is, from Reference (47),

$$w = 0.668 A_2 KY \sqrt{\rho_1 \Delta P}, \quad (89)$$

in which

$$Y = 1 - (0.45 - 0.31\beta^4) \Delta P/P_1 k, \quad (89a)$$

where

w = mass flow, $\text{lb}_m./\text{sec.}$,

A_2 = orifice area = $\frac{\pi}{4} D_2^2$, in.^2 ,

K = orifice coefficient = $f(\text{Re}_2)$,

Y = expansion parameter,

ρ_1 = density upstream of orifice, $\text{lb}_m./\text{ft.}^3$,

P_1 = orifice forepressure, $\text{lb}_f./\text{in.}^2$,

ΔP = orifice pressure drop, $\text{lb}_f./\text{in.}^2$,

β = diameter ratio, D_2/D_1 , and

k = ratio of specific heats.

The density, ρ_1 , was determined from an upstream temperature measurement and P_1 . With mass flows known, volume flows and the various flow parameters could be calculated.

Speed. The compressor speed was measured by a Hewlett-Packard Model 521ER Electronic Counter which converted the signal from a magnetic pickup at the compressor shaft. This speed was included as input data for computation of speed parameters.

Clearances. Impeller-to-casing clearances were at first measured by a Bentley-Nevada Corporation Model D-152 Distance Detector system based on the inductance principle. A better method proved to be the Wayne-Kerr capacitance system which uses the Distance Meter DM100 to amplify the signal from a capacitance probe. Oscilloscopes were employed to display the signals from the meters.

Flow angles. The 0.125-inch diameter cylindrical yaw probe used to measure flow angles in the compressor channel was traversed and yawed manually. A pointer and protractor indicated the flow angle when the two side pressures were balanced on a water manometer.

Electrical input. Volts, amperes, power factor, and watts input to the drive motor were measured by a Weston Model 639 Industrial Analyzer. More accurate watt measurements were obtained on a Sensitive Research Corporation high frequency polyphase wattmeter.