MASTER

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THE
AUTOMATIC BIAS AND FLUX-RESET
OF A MAGNETIC MODULATOR

by

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LIST OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>
A_{ci}	Cross sectional area of i th reactor core
A_j	j th coefficient (defined when used)
A_{total}	Cross sectional area of reactor winding
a_i	Turns ratio of i th transformer
α	Phase angle of power supply in seconds
B	Flux density unless otherwise noted
B_i	Flux density of i th reactor
B_m	Saturation flux density
B_{Ti}	Flux density of i th transformer
ΔB	Change in flux density
C	Capacitor
C_n	n th capacitor
D_i	i th diode
e	Base of the natural system of logarithms
$\mathcal{E}$	Energy
f	Frequency in cycles per seconds or farads
h	Henries
I	Per-unit current
i	Instantaneous current in amperes
k_0	Initial value of k th parameter
L_i	Inductance of i th reactor
L_{PTi}	Primary inductance of i th transformer

<u>Symbol</u>	<u>Definition</u>
L_{STi}	Secondary inductance of ith transformer
ℓ_i	Mean magnetic path length of ith reactor
N_i	Turns of ith reactor
Q	A function of damping ratio, defined when used
R_i	Resistance of ith resistor
R_B, R_b	Bias resistor
S	Laplace transform operator
T_i	ith transformer
t	Time, in seconds
τ	Per-unit time or dummy integration variable
V	Per-unit voltage
V_{mi}	Maximum per-unit input voltage
v	Instantaneous voltage in volts
V_m	Maximum instantaneous voltage
Z	Impedance in ohms
η_i	Efficiency factor of ith stage or section
ω_i	Angular power supply frequency
ν_i	Angular undamped resonant frequency of ith "transfer loop"
μ	Relative permeability
μ_{is}	Effective saturated relative permeability of ith reactor
μ_m	Relative saturated permeability of core material

<u>Symbol</u>	<u>Definition</u>
ζ	Damping ratio of a LRC circuit
θ	Per-unit phase angle of power supply
ϕ	Instantaneous flux in webbers
$\bar{\phi}$	Per-unit flux

Note: The MKS rationalized system of units is used throughout.

CHAPTER I

INTRODUCTION

The magnetic modulator is a device that stores energy during one cycle of the power supply, then delivers it to a load in the form of a short pulse. This is accomplished by utilizing the non-linear characteristics of saturable reactors and takes the form of recurrent transients. The two basic modes of operation of magnetic modulators are bipolar and unipolar. In bipolar operation, the modulator delivers the pulse energy to the load after each half cycle of the power supply. The output pulses alternate in polarity; and, hence, the term bipolar. Unipolar operation results when the modulator delivers the pulse energy to the load after each cycle of the power supply. The output pulses are of the same polarity as is indicated by the term "unipolar." The magnetic modulator is most generally driven by a sinusoidal generator; however, it may be driven by a square wave generator.

Magnetic core switching as a means for generating large peak power pulses was probably first suggested by Mr. W. S. Melville in 1951 when he published a complete circuit and analysis with some design data.¹ For this reason, Mr. Melville is sometimes considered to be the "father" of magnetic modulators. Mr. Busch and associates of Bell Telephone Laboratories published a very complete analytical study of magnetic modulators and some discussion of flux reset with external bias applied to the first saturable reactor.² This report is excellent for obtaining a fundamental understanding of magnetic modulators. Mr. E. J. Smith and associates of Polytechnic Institute of Brooklyn published four reports from 1955 to 1959.^{3,4,5,6}

Magnetic modulators have the capability of generating large peak power pulses in the order of several megawatts and whose average power is of the order of tens of kilowatts. Output pulse widths from 100 nanoseconds to several microseconds are also possible. It is quite possible that magnetic modulators have a considerably higher upper limit on peak pulse power than is obtainable by use of thyratrons. Magnetic devices, being passive, are more reliable circuit elements than thyratrons. Low power modulators (in the order of tens of kilowatts peak power) usually require less volume than an equivalent thyatron modulator. Magnetic modulators have been built for modulation of both magnetron and klystron oscillators.

Both klystron and magnetron oscillators require unipolar modulation. For this application, it is necessary to either "clip" the unwanted pulse of a bipolar modulator output (which is wasteful) or use a unipolar modulator. In this study, the writer investigated the operation of a low power magnetic modulator using two solid state diodes and a resistor to provide the bias necessary for unipolar operation. The non-linear modulator input characteristics were studied theoretically by means of a piece-wise linear analysis. Per-unit input current, first capacitor voltage, and first saturable reactor flux wave forms were calculated with the aid of a digital computer. The computed wave forms were compared to experimental wave forms.

CHAPTER II

FUNDAMENTALS OF MAGNETIC MODULATOR OPERATION

An understanding of magnetic modulator operation may be obtained by examining the operation of the several different parts of the modulator separately, then combining these parts into the complete modulator. In order to understand magnetic modulator operation, it is necessary to have some knowledge of saturable reactors.

Saturable Reactors

The saturable reactors used in the magnetic modulator are used as magnetic switches; i.e., in the unsaturated state, the saturable reactor must act as a blocking impedance and in the saturated state be a very low fixed inductance. This is achieved by using a "square" hysteresis loop magnetic material. The desired features of the magnetic core are that the change from the unsaturated state to the saturated state occur very rapidly and that the ratio of unsaturated permeability to the saturated permeability be very large. Thus, a perfectly square hysteresis loop core would be ideal for this application. It is difficult, however, to discuss the hysteresis loop of a core unless the exact excitation of the core is known. The same core, for instance, may have a nearly perfect square hysteresis loop when excited by a rectangular current pulse but have a decided "hourglass" shape when excited by a 60 cycle square wave voltage.⁷ The point is that use of the hysteresis loop as a design parameter is not desirable.

There are several different ferromagnetic alloys that can be classed as having "square" hysteresis loops. Two types have desirable characteristics for use in magnetic modulators. One type is the 50 per

cent nickel-iron alloy which has a very rectangular hysteresis loop when it is grain-oriented. The other is the 4 per cent molybdenum, 78 per cent nickel, and 18 per cent iron alloy. The annealing process of the latter material determines significantly the "squareness" of the hysteresis loop. Several companies make these core materials and have their own trade names for them. Two of the core manufacturers and their trade names for the two core materials are given in Table 1.

TABLE 1
"SQUARE LOOP" CORE MATERIALS

CORE MATERIAL	MANUFACTURER	TRADE NAME
50 per cent Ni - 50 per cent Fe Grain Oriented	Arnold Engineering Co.	Deltamax
	Magnetics, Inc.	Orthonol
4 per cent Mo - 78 per cent Ni - 18 per cent Fe Annealed for Maximum Squareness	Arnold Engineering Co.	Square Permalloy
	Magnetics, Inc.	Square Hy-Mu 80
4 per cent Mo - 78 per cent Ni - 18 per cent Fe Annealed for High Initial Permeability	Arnold Engineering Co.	Mo-Permalloy
	Magnetics, Inc.	Round Hy-Mu 80

Round Hy-Mu 80 is not well suited for use in a magnetic modulator because its change from high permeability to a very low permeability is too gradual. Round Hy-Mu 80 was not used in this study. Square Hy-Mu 80 has one-half the saturation flux density (B_m) and a less coercive force (H_c) than does Orthonol material.

The core parameters necessary for design information are the saturated permeability, saturation flux density, and the core geometry. During operation in the unsaturated state, the change in flux density is determined from Equation 2.1.

$$\Delta B = \frac{1}{NA_c} \int_{t_1}^{t_2} V_L dt \quad (2.1)$$

The integral of Equation 2.1 is often referred to as the volt-time integral applied across the saturable reactor. Solving Equation 2.1 for the number of turns yields a useful design equation. Equation 2.2 gives the number of turns required for the saturable reactor core to be driven from one saturated state to the other by a given volt time integral.

$$N = \frac{1}{2B_m A_c} \int_{t_1}^{t_2} V_L dt \quad (2.2)$$

Most of the saturable reactors used in magnetic modulators are toroidal in form. Toroid cores are now available in hermetically sealed cases, which makes it possible to vacuum impregnate the saturable reactors without contaminating the core. Magnetics, Incorporated uses an aluminum case around three sides of the core and a non-conducting washer to seal the fourth side. The Arnold Engineering Company uses an all-aluminum case but with one side having an aluminum washer coated with a thin layer of insulating oxide. The writer has noticed failure of the oxide insulation of several Arnold Engineering cores under high voltage short pulse operation, resulting in an effective shorted turn on of the saturable reactor. The saturable reactors used in magnetic modulators are wound both as inductors and transformers, depending upon the particular application in the modulator circuit.

Fundamental Parts of the Magnetic Modulator

The magnetic modulator consists of three basic parts. The first part is called the input or charging stage which is typically designed to be resonant at the power supply frequency. The second part, the magnetic compression stage, may consist of one or more individual sections. The third part, the output stage, couples the modulator to its load. Figure 1 is a block diagram of a magnetic modulator and Figure 2 is a complete modulator schematic. Each of the three parts is discussed in detail below.

The Input Stage - The input stage receives energy from the power supply during the "charge" cycle; then, at the end of the "charge" cycle, the energy is delivered to the next stage of the modulator. The "charge" cycle is one-half the power supply period for bipolar operation or one complete power supply period for unipolar operation. The primary components of the input stage are reactive and consist of a linear inductor and capacitor in series.

Most of the magnetic modulator designs use the capacitor for the energy storage element; however, a few of the first magnetic modulators used the inductor for the energy storage element. This discussion will be restricted to operation wherein the capacitor is the energy storage element. The size and voltage rating of the capacitor is dependent upon both the amount of energy required by the modulator load and the modulator efficiency. The value of the linear inductor is determined by the desired series resonant frequency of the input stage, which is typically equal to the power supply frequency.

MAGNETIC MODULATOR

BLOCK DIAGRAM

FIGURE 1.

MAGNETIC MODULATOR SCHEMATIC
FIGURE 2

The equivalent circuit of the input stage is a series inductor, capacitor, and small resistance. Referring to the modulator schematic of Figure 2, it is not immediately apparent which components are in the "input stage." The "construction" of the equivalent modulator input circuit from the complete schematic requires several assumptions. These necessary assumptions will be stated here but the validity of which will not be apparent until the other parts of the modulator have been discussed. The assumptions are as follows:

- (1) The capacitive reactance of C_2 and C_3 at the power supply frequency is very much larger than the inductive reactance of saturable reactors L_3 , L_4 , and L_{PT2} .
- (2) The combined series inductance of the saturable reactors L_3 , L_4 , and primary inductance of the output transformer L_{PT2} are very much smaller than the inductance of the charging inductor L_1 when reflected to the primary of T_1 .
- (3) The autotransformer saturable reactor T_1 is unsaturated during the charge cycle and the primary inductive reactance is very much larger than the reflected capacitive reactance of C_1 at the power supply frequency.
- (4) The biasing circuit, which includes the two diodes D_1 and D_2 and resistor R_3 , does not affect the charging circuit operation appreciably.

Application of assumption (1) yields the equivalent circuit of Figure 3 where L_t represents the series inductance of L_3 , L_4 , and

L_{PT2} (C_3 is neglected). The reactance of L_t is typically very much smaller than the reactance of C_1 at the power supply frequency.

The application assumptions (2), (3), and (4) results in Figure 4 when referred to the primary of T_1 . The dotted lines in the equivalent circuits represent circuit elements wherein the current flow during the "charge" cycle is negligible in comparison to the charging current.

The input stage equations may now be written.

The damping effect of the series inductor resistor is neglected in the input equations because, to include it here, would detract somewhat from visualizing the circuit operation. The damping effect of the resistor and circuit losses, as well as operation at other than resonance, will be discussed in detail in a later chapter. The differential equation for the input stage, neglecting damping, is given by Equation 2.3.

$$L_i \frac{di}{dt} + \frac{1}{a_i^2 C_i} \int i dt = V_{mi} \sin \omega_i t \quad (2.3)$$

Assuming that there is no initial energy storage in the circuit, Equation 2.4 is the Laplace transformer of Equation 2.3 solved for the transform current.

$$i(s) = \frac{V_{mi} S}{L_i \sqrt{L_i a_i^2 C_i} (s^2 + \frac{1}{L_i a_i^2 C_i}) (s^2 + \omega_i^2)} \quad (2.4)$$

The condition for resonant charging is given by Equation 2.5.

$$\omega_i^2 = \frac{1}{L_i a_i^2 C_i} \quad (2.5)$$

Combining Equations 2.4 and 2.5 yields Equation 2.6.

$$i(s) = \frac{V_{mi} \omega_i S}{L_i (s^2 + \omega_i^2)^2} \quad (2.6)$$

MAGNETIC MODULATOR

APPROXIMATE EQUIVALENT INPUT CIRCUIT

FIGURE 3

NOTE $R = R_G + R_B$ FOR i_i POSITIVE

$R = R_G$ FOR i_i NEGATIVE

T_i IS UNSATURATED

SIMPLIFIED APPROXIMATE EQUIVALENT
INPUT CIRCUIT

FIGURE 4

The inverse Laplace transform of Equation 2.6 gives the input current time function of Equations 2.7 and 2.8.

$$i(t) = \frac{V_{mi}}{2L} t \sin \omega_i t \quad (2.7)$$

$$i(t) = \frac{1}{2} V_{mi} \sqrt{\frac{a_i^2 C_i}{L}} \omega_i t \sin \omega_i t \quad (2.8)$$

The voltage on the capacitor is proportional to the time integral of the input current, which is given by Equations 2.9 and 2.10.

$$V_{c_i} = \frac{1}{a_i^2 C_i} \int_0^t i(\tau) d\tau \quad (2.9)$$

$$V_{c_i}(t) = \frac{a_i}{2} V_{mi} (\sin \omega_i t + \omega_i \cos \omega_i t) \quad (2.10)$$

Similarly, the flux in the autotransformer saturable reactor T_1 , which is in shunt with the capacitor, is proportional to the volt-time integral of the voltage across the capacitor. Equations 2.11 and 2.12 give the flux density of the autotransformer T_1 .

$$B_{T_1} = \frac{1}{N_1 A_{c_i}} \int_0^t V_{c_i}(\tau) dt \quad (2.11)$$

$$B_{T_1}(t) = \frac{V_{mi}}{\omega_i N_1 A_{c_i}} (1 - \cos \omega_i t - \frac{1}{2} \omega_i t \sin \omega_i t) + B(0^+) \quad (2.12)$$

Because the voltage on C_1 is proportional to the current-time integral, the maximum or minimum voltage on the capacitor occurs at current "zeros." Since the energy storage on the capacitor is proportional to the square of the voltage on the capacitor, the times of maximum energy on the capacitor occur at each time the input current goes to zero. For most efficient operation, it is necessary to transfer the capacitor energy to the next stage at the instant the peak energy on

the capacitor is reached. Referring to Equation 2.7, we see that the current zeros occur as shown by Equation 2.13.

$$t_o = \frac{n\pi}{\omega_i} \quad (2.13)$$

For bipolar operation, n equals 1. Making this substitution into Equations 2.13 and 2.10 for time, we find that the maximum voltage on C_1 is given by Equation 2.14.

$$V_{C_1(max)} = a_1 \frac{\pi}{2} V_{mi} \quad (2.14)$$

Here it is necessary to note that when the capacitance is discharged at time equal to π/ω_1 , the differential equation for the input circuit for the second half cycle of the power supply is different from Equation 2.3. The difference is that there is an initial phase angle of the power supply voltage of π radians. Recalling a trigonometric identity, we see that this phase angle simply changes the algebraic sign of the driving function. Therefore, we can write the current, voltage, and flux density equations for the second half cycle as shown by Equations 2.15, 2.16, and 2.17.

$$i(t) = -\frac{1}{2} V_{mi} \sqrt{\frac{a_1^2 C_1}{L_1}} \omega_i t \sin \omega_i t \quad (2.15)$$

$$V_{C_1}(t) = -\frac{a_1}{2} V_{mi} (\sin \omega_i t - \omega_i t \cos \omega_i t) \quad (2.16)$$

$$B_{T_1}(t) = -\frac{V_{mi}}{\omega_i N_1 A_{C_1}} (1 - \cos \omega_i t - \sin \omega_i t) + B_{T_1}(0^+) \quad (2.17)$$

At the time the first energy peak is reached on the capacitor, we see that the peak voltage is given by Equation 2.18.

$$V_{C_1(max)} = -a_1 \pi V_{mi} / 2 \quad (2.18)$$

It is now apparent that bipolar operation results in alternate polarity pulses that occur each half cycle of the power supply. The change in the flux density each half cycle has the same magnitude but is of opposite polarity and is given by Equation 2.19.

$$\Delta B_n = \frac{2 V_{mi}}{\omega_i N_i A_{ci}} \quad (2.19)$$

Unipolar operation occurs when n equals 2 in Equation 2.13 as shown in Equation 2.20.

$$t_o = \frac{2\pi}{\omega_i} \quad (2.20)$$

The capacitor voltage at the time peak capacitor energy occurs is given by substituting the value of time obtained from Equation 2.20 into Equation 2.10, as shown by Equation 2.21.

$$V_{ci(max)} = -\alpha_i \pi V_{mi} \quad (2.21)$$

The flux density at this time is $B(0^+)$.

Therefore, for unipolar operation, we see that the absolute peak voltage on the capacitor is π times the magnitude of the input voltage and occurs at the end of one complete cycle of the power supply. It is also apparent that the flux density has returned to its initial value. Hence, the assumption that there was no initial stored energy in the input circuit was valid. The necessary criterion here is that the capacitor voltage be discharged when the maximum energy is reached on the capacitor. This is accomplished by saturation of the saturable reactor T_1 , which is in shunt with the capacitor. The saturation of T_1 initiates the magnetic compression stage operation.

Magnetic Compression - Magnetic compression is used here to refer to the saturable reactor circuit that effects the transfer of energy from an "input" capacitor to an "output" capacitor in a much shorter time (time compression) that was required to charge the input capacitor. Such circuits may be cascaded wherein the output capacitor of one section is the input capacitor of the succeeding section. There are two types of compression stages. One is the series capacitors and shunting saturable reactors and the other is the series saturable reactors and shunting capacitors. Although the basic operation of each type is similar, the shunt capacitor and series saturable reactor operation is easier to visualize.

A general magnetic compression section with series saturable reactors and shunt capacitors is shown in Figure 5A. The operation of one magnetic compression section can be analyzed by reference to Figure 5B, where Z represents the impedance of the rest of the modulator to the right of the $n + 1$ st reactor. The component subscript " n " shown in Figure 5 designates the components of the n th magnetic compression section. The capacitor C_{n+1} is the output capacitor of the n th magnetic compression section and the input capacitor of the $n + 1$ st magnetic compression section.

The analysis of the operation of the magnetic compression section involves the transient analysis of the circuit of Figure 5B. The analysis at this point neglects the circuit losses. The initial conditions are assumed as follows:

- (1) Capacitor C_n charged to voltage V_{cm} .
- (2) Capacitor C_{n+1} contains no charge.

MAGNETIC COMPRESSION STAGE AND OUTPUT STAGE

FIGURE 5.A

MAGNETIC COMPRESSION SECTION

FIGURE 5 B

- (3) Current in saturable reactor L_n is 0.
- (4) The saturable reactor L_n saturates at time $t = 0^+$.
- (5) The saturable reactor L_{n+1} is unsaturated during transfer of energy from C_{n+1} and very little current flows through L_{n+1} compared to the current flow in C_{n+1} .

The differential equation for the magnetic compression section is given by Equation 2.22.

$$L_{sn} \frac{di}{dt} + \frac{C_n + C_{n+1}}{C_n C_{n+1}} \int i dt = 0 \quad (2.22)$$

The Laplace transform of Equation 2.22 is given by Equation 2.23.

$$\left(S L_{sn} + \frac{C_n + C_{n+1}}{S C_n C_{n+1}} \right) i(S) = \frac{V_{cm}}{S} \quad (2.23)$$

Solving Equation 2.23 for the transform current yields Equation 2.24.

$$i(S) = \frac{V_{cm} / L_{sn}}{S^2 + \frac{C_n + C_{n+1}}{L_{sn} C_n C_{n+1}}} \quad (2.24)$$

Taking the inverse transform of Equation 2.24 results in Equation 2.25.

$$i(t) = V_{cm} \sqrt{\frac{L_{sn}(C_n + C_{n+1})}{C_n C_{n+1}}} \sin \sqrt{\frac{C_n + C_{n+1}}{L_{sn} C_n C_{n+1}}} t \quad (2.25)$$

Let the angular resonant frequency be ν_n given by Equation 2.26.

$$\nu_n = \sqrt{\frac{C_n + C_{n+1}}{L_{sn} C_n C_{n+1}}} \quad (2.26)$$

The Laplace transform of the voltage on C_{n+1} is shown by Equation 2.27.

$$V_{C(n+1)}(S) = \frac{V_{cm} / L_{sn} C_{n+1}}{S(S^2 + \gamma_n^2)} \quad (2.27)$$

The voltage on C_{n+1} as a function of time is given by Equation 2.28, which is the inverse transform of Equation 2.27.

$$V_{C(n+1)}(t) = \frac{C_n}{C_n + C_{n+1}} V_{cm} (1 - \cos \gamma_n t) \quad (2.28)$$

From Equation 2.28 it is seen that the first peak voltage on C_{n+1} occurs at time equal to π/γ_n and is given by Equation 2.29.

$$V_{C(n+1) \max} = \frac{2C_n}{C_n + C_{n+1}} V_{cm} \quad (2.29)$$

When capacitor C_n is equal to C_{n+1} , the peak voltage on C_{n+1} is equal to V_{cm} .

The energy on capacitor C_{n+1} must be transferred to the succeeding modulator stage at the time the first peak energy is reached on C_{n+1} . This transfer is accomplished by the saturation of L_{n+1} at this time. Since the voltage-time function applied across L_{n+1} is known (Equation 2.28), it is possible to calculate the change in flux density by use of Equation 2.1, as shown by Equation 2.30.

$$\Delta B_{n+1} = \frac{1}{N_{n+1} A_{C(n+1)}} \int_0^{\pi/\gamma} \frac{C_n V_{cm}}{C_n + C_{n+1}} (1 - \cos \gamma_n t) dt = \frac{\pi C_n V_{cm}}{\gamma_n N_{n+1} A_{C(n+1)} (C_n + C_{n+1})} \quad (2.30)$$

To reset a saturable reactor core to some exact value of flux density other than a saturated flux density (B_m) is very difficult. For this reason it is desirable to "switch" the core from one saturated state

to the other during the charging of capacitor C_{n+1} . Resetting a saturable reactor core is the process of returning the core to a desired initial state sometime prior to the start of the next cycle of operation. The actual resetting process will be discussed in another section. The need for "resetting" the saturable reactor core to an exact value is obvious when repetitive operation is desired. When reset to saturation flux density, Equation 2.30 may be written in terms of twice the saturation flux density of the core and solved for the turns-core-cross-sectional-area product, as shown by Equation 2.31.

$$N_{n+1} A_{c(n+1)} = \frac{\pi C_n V_{cm}}{2 B_{m(n+1)} (C_n + C_{n+1})} \quad (2.31)$$

Equation 2.31 is a useful design equation.

In the above discussions, nothing was said about what would happen if the saturable reactor L_{n+1} did not saturate when the peak voltage on C_{n+1} was reached. Since the series circuit of C_n , C_{n+1} , and L_n is oscillatory, the current will tend to reverse polarity in L_n at the time the peak voltage on capacitor C_{n+1} is reached. As soon as a very small reverse current starts to flow in L_n , the saturable reactor L_n becomes unsaturated. The unsaturated inductance of L_n is much larger than that of L_{n+1} ; and, therefore, C_{n+1} discharges primarily through L_{n+1} . The result is that the volt-time integral will build up very rapidly on L_{n+1} and it will saturate if designed near the proper value.

The first magnetic compression section of the magnetic modulator shown in Figure 2 is not readily apparent. Figure 6 shows the first and second magnetic compression stages of Figure 2. Comparing Figure

5B to Figure 6, the following equalities may be made:

$$\begin{aligned} C_n &= C_1 \\ C_{n+1} &= C_2 \\ L_n &= L_{sT1} \\ L_{n+1} &= L_3 \end{aligned} \tag{2.32}$$

During the transfer of energy from C_1 to C_2 , the autotransformer T_1 is saturated. The inductive reactance of L_1 is very large at the "frequency" of energy transfer and effectively blocks any current flow back to the power supply. In the saturated state, the autotransformer T_1 is nearly the same as if the core were air.

Bipolar operation requires that the autotransformer T_1 be designed to be driven into both "positive" and "negative" saturation. Equation 2.19 would be used for the design of the autotransformer T_1 , primary turns, with ΔB replaced by twice the saturation flux density of its core. The turns ratio would depend upon the desired voltage step-up requirements.

The second magnetic compression section includes C_2 , L_3 , and C_3 (Figure 6). Capacitor C_2 is the input capacitor and capacitor C_3 is the output capacitor. The operation is the same as described above except the discharge resonant frequency is much higher than the previous section. When C_3 is charged to its peak voltage, saturable reactor L_4 becomes saturated. Upon saturation of L_4 , the output stage operation is initiated.

Output Stage - Two types of output stages have been used. One type uses a pulse forming network and the other operates without a "pulse former." For output pulse width requirements of less than one-half

microsecond, the line type pulse forming network loses its effectiveness due to winding capacities and leakage inductances.

A schematic of an output stage using a pulse forming network is shown in Figure 7. For this application, the unsaturated inductance of saturable reactor L_n must provide sufficient current blocking during the charge of the line capacitors that no significant current flows through the load. When saturated, the saturable reactor L_n must have a low enough saturated inductance to be the first inductance of the pulse forming network. This design is not practical for narrow output pulse widths because the above requirements become nearly impossible to satisfy.

An output stage with no pulse forming is shown in Figure 8. Figure 8A shows the output stage with a set-up output transformer. The equivalent output stage is shown in Figure 8B with the load resistance reflected to the primary of the output transformer. Equation 2.33 is the differential equation for the output stage.

$$L_4 \frac{di}{dt} + \frac{R_L}{a_2^2} i + \frac{1}{C_3} \int i dt = 0 \quad (2.33)$$

Equation 2.34 is the Laplace transform of Equation 2.33.

$$(L_4 S + \frac{R_L}{a_2^2} + \frac{1}{SC_3}) i(S) = \frac{V_{cm3}}{S} \quad (2.34)$$

Solving Equation 2.34 for the current function yields Equation 2.35.

$$i(S) = \frac{V_{cm3}/L_4}{S^2 + \frac{2}{\sqrt{L_4 C_3}} \left(\frac{R_L}{2a_2^2} \sqrt{\frac{C_3}{L_4}} \right) S + \frac{1}{L_4 C_3}} \quad (2.35)$$

MAGNETIC COMPRESSION STAGE

FIGURE. 6

MAGNETIC MODULATOR OUTPUT STAGE

WITH PULSE FORMING NETWORK

FIGURE 7

APPROXIMATE EQUIVALENT OUTPUT STAGE

FIGURE 8 A

SIMPLIFIED EQUIVALENT OUTPUT STAGE

FIGURE 8 B

For a critically damped output stage, Equation 2.36 must be satisfied.

$$\frac{1}{2} \frac{R_L}{a_2} \sqrt{\frac{C_3}{L_4}} = 1 \quad (2.36)$$

Making the substitution of Equation 2.36 into Equation 2.35 gives Equation 2.37.

$$i(s) = \frac{V_{cm3} / L_4}{(s + 1/\sqrt{L_4 C_3})^2} = \frac{V_{cm3} / L_4}{(s + \gamma_4)^2} \quad (2.37)$$

Taking the inverse Laplace transform of Equation 2.37 results in the output time function of Equation 2.38.

$$i(t) = V_{cm3} \sqrt{\frac{C_3}{L_4}} \gamma_4 t e^{-\gamma_4 t} \quad (2.38)$$

Since the modulator load is resistive, the output voltage is R_L times the output current and is given by Equation 2.39.

$$V_L(t) = 2 a_2 V_{cm3} \gamma_4 t e^{-\gamma_4 t} \quad (2.39)$$

Differentiating Equation 2.38 and solving for the value of time when the derivative vanishes gives the time at which the output pulse is maximum. The time of occurrence of output pulse peak amplitude is given by Equation 2.40.

$$t_m = \frac{1}{\gamma_4} = \sqrt{L_4 C_3} \quad (2.40)$$

The peak voltage is given by Equation 2.41.

$$V_L(t_m) = 2 V_{cm3} a_2 e^{-1} = 0.736 a_2 V_{cm3} \quad (2.41)$$

Composite Magnetic Modulator Operation

In the preceding discussion of the operation of the various parts of the magnetic modulator, assumptions were made that enabled us to contain the energy in one modulator section at a time. The validity of these assumptions will be investigated in looking at the overall modulator operation. The overall magnetic modulator operation is discussed first neglecting the mechanism of "flux-reset" in the saturable reactor cores, then investigating the "flux-reset" process which concludes this chapter.

Modulator Operation Neglecting Flux-Reset

Composite magnetic modulator operation results in the transfer of energy from one capacitor to another. As the energy progresses from the power supply to the modulator load, each energy transfer is accomplished in a shorter interval of time. For efficient energy transfer, the energy must be contained within the transfer loop. (The transfer loop consists of the components contained within the primary current flow loop during the transfer of energy from one capacitor to another within the loop.) With the exception of the input stage and output stage, the transfer loop is a magnetic compression section. The components preceding and following the transfer loop must have characteristics which make the current flow out of the transfer loop negligible in comparison to the current flow within the transfer loop during the energy transfer. This requirement can be met by using saturable reactors for all the inductors except the input charging inductor. The necessary requirement is that the saturable reactors preceding and following the transfer loop be the current blocking elements; and, hence, must be unsaturated during the energy transfer within the transfer loop. The saturable reactor within

the transfer loop must be saturated during the energy transfer interval and then become unsaturated at the end of the energy transfer interval. This latter requirement is met as described in the magnetic compression section above. The integral equation of 2.42 shows the inductor current as a function of the applied voltage.

$$i_L = \frac{1}{L} \int V_L dt \quad (2.42)$$

Since the inductance is directly proportional to the permeability, the current is inversely proportional to the permeability. The ratio of unsaturated to saturated permeability of these saturable reactors is of the order of 100,000:1. Thus, if the saturable reactor in the "transfer loop" is saturated and the adjoining saturable reactors are unsaturated, the unsaturated reactors provide good current blocking.

The time interval of energy transfer is given by Equation 2.43 where L is the effective saturated inductance and C is the effective series capacitance of the transfer loop.

$$T = \pi \sqrt{LC} \quad (2.43)$$

Equation 2.43 satisfies Equation 2.28, Page 19, for the first peak voltage (after the start of energy transfer) on the output capacitor of a compression section. The time interval of energy transfer is measured from the time the saturable reactor saturates to the completion of the energy transfer through the saturated inductance.

Time compressions of the order of 10:1 are typical for the magnetic compression sections. Therefore, the saturated inductance ratio of adjoining saturable reactors is of the order of 100:1. (The capacitors

of the magnetic modulator are approximately the same size in order to obtain maximum efficiency of energy transfer.) The time compression from the input stage to the first magnetic compression section is typically of the order of 100:1. Thus, the charging inductance of L_1 , referred to the secondary of T_1 , is of the order of 10,000 times larger than the saturated inductance of the first saturable reactor T_1 .

Similarly, the saturated inductance of T_1 is of the order of one hundred times larger than the saturated inductance of L_3 and the saturated inductance of L_3 is of the order of one hundred times larger than the saturated inductance of L_4 . Therefore, the inductance of L_1 , referred to the secondary of T_1 , is of the order of one million times larger than the saturated inductance of L_3 and one hundred million times larger than the saturated inductance of L_4 .

Since the ratio of unsaturated inductance to the saturated inductance of the saturable reactors normally used in magnetic modulators is typically of the order of 100,000:1, the inductance of L_1 , referred to the secondary of T_1 , is of the order of ten times larger than the unsaturated inductance of L_3 and one thousand times larger than the unsaturated inductance L_4 . Similarly, the unsaturated inductance of T_1 is of the order of ten times larger than the inductance of L_1 . During most of the charge cycle of C_1 , the saturable reactors L_3 , L_4 , and T_2 are saturated, as will be shown in the following section. Therefore, the assumptions that the charging current for C_1 flows through L_3 , L_4 , and T_2 (refer to Figure 11, Page 33) and that the voltage drop across these components is small compared to the voltage across the secondary of T_1 are valid assumptions.

The composite modulator voltage wave forms for unipolar operation are shown in Figure 9. The composite voltage wave forms are shown on two time scales. The curves on the left are plotted on a scale to show two complete cycles of operation. The curves on the right side are an expanded time scale showing the transfer of energy from C_1 to the output. Since the energy is transferred from capacitor to capacitor, only the capacitor voltage and output voltage wave forms are shown. Figure 10 is the theoretical per-unit plot of the output voltage. Equations 2.44 and 2.45 give the scale factors for Figure 10.

$$V_L = v_L / a_{T2} v_{c3 \max} \quad (2.44)$$

$$\tau = t \sqrt{L_{s4} C_3} \quad (2.45)$$

A schematic representation of the modulator operation is shown in Figures 10 through 15, which shows pictorially one complete cycle of modulator operation. The portions of the circuit that are of heavy lines represent the major current flow during that portion of modulator operation. The dotted current arrows represent small current flow through the saturable reactors as the flux builds up within their cores. Heavy lines on the saturable reactor core representations (idealized hysteresis loops) represent the flux excursion of the B-H loop during the portion of the modulator operation being shown. The dots on the B-H loops represent the start and stop points of the flux excursion. The time designations on the time axis of Figure 9 (voltage wave forms) is referenced on the schematic Figures 11 through 15 so that the corresponding voltage wave forms may be observed in conjunction with the schematic representation.

MAGNETIC MODULATOR VOLTAGE WAVEFORMS
FIGURE 9

TIME SCALE EXPANDED FROM
LEFT SIDE WAVEFORMS

APPROXIMATELY AS FOLLOWS:

t_2 TO t_3	100 : 1
t_3 TO t_4	300 : 1
t_4 TO t_5	600 : 1

PER UNIT OUTPUT PULSE

FIGURE 10

REFER TO FIGURE 9 TIME INTERVAL t_0 TO t_1 FOR VOLTAGE WAVEFORMS

POSITIVE PORTION OF C₁ CHARGE CYCLE
FIGURE II

REFER TO FIGURE 9 TIME INTERVAL t_1 TO t_2 FOR VOLTAGE WAVEFORMS

NEGATIVE PORTION OF C_1 CHARGE CYCLE

FIGURE 12

REFER TO FIGURE 9 TIME INTERVAL t_2 TO t_3 FOR VOLTAGE WAVEFORMS

FIRST COMPRESSION SECTION OPERATION
FIGURE 13

REFER TO FIGURE 9 TIME INTERVAL t_3 TO t_4 FOR VOLTAGE WAVEFORMS

SECOND COMPRESSION SECTION OPERATION
FIGURE 14

REFER TO FIGURE 9 TIME INTERVAL t_4 TO t_5 FOR VOLTAGE WAVEFORMS

OUTPUT STAGE OPERATION
FIGURE 15

Modulator Flux-Reset Operation

In the past, DC bias windings have been used to reset the saturable reactors in magnetic modulators into saturation. The disadvantage of this method of reset is that a DC power supply is required; also, an additional winding is required on each saturable reactor along with a large choke.

The T_1 biasing arrangement and the flux-reset method used as shown in Figure 11 is termed by the writer as "automatic bias and flux-reset" as opposed to the biasing and flux-reset method using an external DC power supply and bias windings. To the best of the writer's knowledge, this method of flux-reset has not appeared in the literature.

The flux-reset of the saturable reactors in the magnetic modulator of Figure 11 is provided by the charging current of the input capacitor C_1 . The C_1 charging current through the saturable reactors just prior to the saturation of T_1 is of opposite polarity than it is during the transfer of energy through the compression stage. (Compare Figure 12 to Figures 13, 14, and 15.) Hence, the charging current of C_1 resets L_3 , L_4 , and T_2 into saturation. The bias of T_1 is provided for by the resistor R_B , which is in the charging circuit during only the positive current flow, as shown in Figure 11. This provides for saturation of T_1 on the negative C_1 voltage swing. The overall effect of R_B is to give a DC level to the input current, which provides for the bias of T_1 . The value of R_B is set to just prohibit saturation of T_1 at the maximum positive flux swing. The output pulse polarity is opposite to the polarity of the C_1 voltage at which T_1 saturates; therefore, if positive pulse output is desired, the bias resistor would be in the circuit for

positive current flow. Magnetron oscillators use a negative pulse from the modulator, in which case the bias resistor would be in the negative input current circuit.

A negative input current flowing through L_1 at the time of saturation of T_1 will continue to flow, forced by the energy storage in L_1 , until this energy is given up. The result is that of maintaining T_1 in negative saturation. While T_1 is saturated, it is nearly equivalent to an air-core transformer and requires a large excitation current to sustain any secondary voltage. However, a small amount of secondary voltage could cause the saturable reactors L_3 , L_4 , and T_2 to be driven out of saturation. This effect is more pronounced for L_3 than L_4 or T_2 because of the relatively larger number of turns on L_3 . This mode of operation is indicated in Figure 16. Because of the possibility of L_3 , L_4 , and T_4 being driven out of saturation after the energy has been delivered to the load but prior to the start of the positive C_1 charge cycle, the start of the flux excursion of these saturable reactors is somewhat indefinite, as is shown in Figure 11. This concludes the discussion of fundamental magnetic modulator operation.

REFER TO FIGURE 9 TIME INTERVAL t_5 TO t_6 FOR VOLTAGE WAVEFORMS

DECAY OF ENERGY STORED IN L_1

FIGURE 16

CHAPTER III

THEORETICAL ANALYSIS OF AN AUTOMATIC-BIASED MAGNETIC MODULATOR

Magnetic modulator design requires a fairly accurate determination of the input capacitor voltage at the time of saturation of the first saturable reactor T_1 . The reason for this requirement is more apparent after the equations relating the saturable reactor design parameters to "input" and "output" specifications have been reviewed. These equations are outlined in Chapter IV and are derived in Appendix C. It is also necessary that the input current be either zero or of the same polarity as the C_1 capacitor voltage at the time of saturation of the first saturable reactor T_1 . If the input current were to reverse polarity, the saturable reactors that are reset to saturation by this current would be driven out of saturation and very possibly saturated in the wrong direction. Since Equations 2.7 and 2.8 are not sufficiently accurate for design purposes when automatic-bias and flux-reset are used for unipolar operation, more accurate equations are developed in the next section.

Solution of Input Circuit Differential Equations

Solving the general differential equations for the input circuit requires the solution of a set of simultaneous non-linear differential equations. The non-linearity is a result of the diodes in the bias circuit and the saturation of the first saturable reactor T_1 . Because of the nature of the non-linearity, the non-linear differential equations may be reduced to piece-wise linear differential equations. When the saturable reactor T_1 is not saturated, the circuit of Figure 17A is the equivalent input circuit. The value of resistor R is one value for

positive current and another value for negative current. The circuit of Figure 17B is the equivalent circuit during the time that T_1 is saturated. The polarity of the current is easily determined; unfortunately, the state of the flux of T_1 is not so simply determined. The time at which T_1 saturates can be determined by the volt-time integral and the initial value of the flux of the saturable reactor T_1 . The duration of the saturation of T_1 can be determined by the polarity of the input current and the polarity of voltage on C_1 at the time T_1 saturates. If the input current and capacitor voltage are of opposite polarity at the time of saturation of T_1 , the saturable reactor will remain saturated for the discharge time of C_1 , which is considered negligible in comparison to the input period (typically 100:1). If the input current and capacitor voltage are of the same polarity at the time of saturation of T_1 , the input current will hold T_1 saturated until the input current polarity reverses.

The differential equation for the equivalent circuit of Figure 17A is given by Equation 3.1. Since T_1 is unsaturated, i_1 is negligible compared to i_{Lc} . (See Figure 17A)

$$L_1 \frac{di_{Lc}}{dt} + R i_{Lc} + \frac{1}{a_1^2 C_1} \int i_{Lc} dt = V_m \sin [\omega_i (t + \alpha)] \quad (3.1)$$

The transform current of Equation 3.2 is obtained from the Laplace transform of Equation 3.1 considering the initial current i_0 and capacitor voltage.

$$i_{Lc}(s) = \frac{\frac{V_m}{L_1} (\omega_i \cos \omega_i \alpha + s \sin \omega_i \alpha) s}{\left(s^2 + \frac{R}{L_1} s + \frac{1}{a_1^2 L_1 C_1}\right) (s^2 + \omega_i^2)} + \frac{s i_0 - V_c / L_1}{s^2 + \frac{R}{L_1} s + \frac{1}{a_1^2 L_1 C_1}} \quad (3.2)$$

SIMPLIFIED EQUIVALENT INPUT STAGE

WITH T_1 UNSATURATED

FIGURE 17 A

SIMPLIFIED EQUIVALENT INPUT CIRCUIT

WITH T_1 SATURATED

FIGURE 17 B

The undamped resonant frequency is defined by γ and the damping ratio is defined by ξ as shown in Equations 3.3 and 3.4, referred to the primary of T_1 .

$$\gamma^2 = \frac{1}{a_1^2 L_1 C_1} \quad (3.3)$$

$$\xi = \frac{a_1}{2} R \sqrt{\frac{C_1}{L_1}} \quad (3.4)$$

The combination of Equations 3.2, 3.3, and 3.4 results in Equation 3.5.

$$i_{Lc}(s) = \frac{\frac{V_{mi}}{L_1} [(\cos \omega_i \alpha) \omega_i + (\sin \omega_i \alpha) s] s}{[(s + \xi \gamma)^2 + \gamma^2 (1 - \xi^2)] (s^2 + \omega_i^2)} + \frac{s i_o - V_o / L_1}{[(s + \xi \gamma)^2 + \gamma^2 (1 - \xi^2)]} \quad (3.5)$$

The capacitor voltage can be obtained from integration (with respect to time) of the input current as shown by Equation 3.6. The flux equation can be obtained from integration (with respect to time) of the voltage equation as shown by Equation 3.7.

$$V_{c1} = \frac{1}{a_1 C_1} \int_0^t i_{Lc}(\tau) d\tau \quad (3.6)$$

$$\phi = \frac{1}{N_1} \int_0^t V_{c1}(\tau) d\tau \quad (3.7)$$

The differential equation for the equivalent circuit of Figure 17B, during the saturation of T_1 , is given by Equation 3.8.

$$L \frac{di_L}{dt} + R i_L = V_{mi} \sin [\omega_i (t + \alpha)] \quad ; \quad L = (L_1 + L_{T1(sat)}) \quad (3.8)$$

The Laplace transform of Equation 3.8 solved for the transform current is shown by Equation 3.9.

$$i_L(s) = \left(\frac{V_{mi}}{L} \right) \frac{\omega_i \cos \omega_i \alpha + s \sin \omega_i \alpha}{(s + \frac{R}{L}) (s^2 + \omega_i^2)} + \frac{i_o}{s + \frac{R}{L}} \quad (3.9)$$

The solution of Equations 3.5, 3.6, 3.7, and 3.9 is shown in Appendix A. The results are converted to per-unit values for generality. The general input equations for current, flux, and voltage in the time domain are given by Equations 3.10, 3.11, 3.12, and 3.13. Equations 3.14 through 3.31 are the constants used in Equations 3.10 through 3.13.

$$I(\tau) = A_1 \{ A_2 \cos [B(\tau + \theta)] + A_3 \sin [B(\tau + \theta)] \} + A_1 [A_4 \sin(Q\tau) + A_5 \cos(Q\tau)] e^{-\tau} \quad (3.10)$$

$$V(\tau) = A_6 \sin [B(\tau + \theta)] + A_7 \cos [B(\tau + \theta)] + [A_8 \sin(Q\tau) + A_9 \cos(Q\tau)] e^{-\tau} \quad (3.11)$$

$$\Phi(\tau) = A_{11} \cos [B(\tau + \theta)] + A_{12} \sin [B(\tau + \theta)] + [A_{13} \sin(Q\tau) + A_{14} \cos(Q\tau)] e^{-\tau} + A_{15} \quad (3.12)$$

$$I_L(\tau) = -A_{16} \cos [B(\tau + \theta)] + A_{17} \sin [B(\tau + \theta)] + [A_{18} \cos(Q\tau) - A_{19} \sin(Q\tau) + I_0] e^{-\tau} \quad (3.13)$$

$$B = \omega / \nu \quad (3.14)$$

$$Q = \sqrt{1 - \zeta^2} \quad (3.15)$$

$$A_1 = \frac{B}{B^4 + 2(2\zeta^2 - 1)B^2 + 1} \quad (3.16)$$

$$A_2 = 1 - B^2 \quad (3.17)$$

$$A_3 = 2\zeta B \quad (3.18)$$

$$A_4 = \frac{B(2\zeta^2 - 1) + 1/B}{Q} \sin(B\theta) - \frac{\zeta(B^2 + 1)}{Q} \cos(B\theta) - \frac{\zeta I_0 + V_c}{A_1 Q} \quad (3.19)$$

$$A_3 = -A_2 \cos B\theta - A_5 \sin B\theta + I_o / A_1 \quad (3.20)$$

$$A_6 = A_1 A_2 / B \quad (3.21)$$

$$A_7 = -A_1 A_3 / B \quad (3.22)$$

$$A_8 = A_1 (QA_5 - \zeta A_4) \quad (3.23)$$

$$A_9 = -A_1 (QA_4 + \zeta A_5) \quad (3.24)$$

$$A_{11} = -A_6 / B \quad (3.25)$$

$$A_{12} = A_7 / B \quad (3.26)$$

$$A_{13} = QA_9 - \zeta A_8 \quad (3.27)$$

$$A_{14} = -(QA_8 + \zeta A_9) \quad (3.28)$$

$$A_{15} = \Phi_c - (A_{11} \cos B\theta + A_{12} \sin B\theta + A_{14}) \quad (3.29)$$

$$A_{16} = B / (B^2 + 4\zeta^2) \quad (3.30)$$

$$A_{17} = 2\zeta / (B^2 + 4\zeta^2) \quad (3.31)$$

Equations 3.10 through 3.31 for per-unit, time, current, voltage, and flux are converted to actual time, current, voltage, and flux by Equations 3.32 through 3.41.

$$i(\nu t) = \nu_{mi} \sqrt{a_1^2 C / L_1} I(\tau) \quad (3.32)$$

$$i(0^+) = \nu_{mi} \sqrt{a_1^2 C / L_1} I_o \quad (3.33)$$

$$v_c(\nu t) = \nu_{mi} V(\tau) \quad (3.34)$$

$$V_c(0^+) = v_{mi} V_c \quad (3.35)$$

$$B(\nu t) = \frac{v_{mi}}{N_i A_c \nu} \Phi(\tau) \quad (3.36)$$

$$B(0^+) = \frac{v_{mi}}{N_i A_c \nu} \Phi_0 \quad (3.37)$$

$$t = \tau / \nu \quad (3.38)$$

$$\nu = 1 / \sqrt{a_i^2 C_i L_i} \quad (3.39)$$

$$\xi = \frac{1}{2} R \sqrt{a_i^2 C_i / L_i} \quad (3.40)$$

$$\alpha = \theta / \nu \quad (3.41)$$

Digital Computer Program

The purpose of the computer program is to evaluate the solutions of the input equations with provision for the switching of the parameters to account for the non-linearities. For details of the Fortran source program and flow chart, see Appendix B.

The initial value of input current, capacitor voltage, and power supply phase angle at the start of the computation for each set of input parameter values is set to zero. The initial value of flux density is set at the negative saturated value. After each computation of data points for per-unit capacitor voltage, per-unit input current, and per-unit flux density, the computer must test for a reversal of current and for saturation of flux density. For example, if current reverses, the computer must change to the appropriate value of damping ratio, then set the following: tau back to zero, power supply phase angle to the last value of tau, initial current, capacitor

voltage and flux density set to the last value computed for current, capacitor voltage, and flux density, respectively. All the coefficients given by Equations 3.15 to 3.31 must be re-calculated before computations are continued.

The parameter "M" in the flow chart, Appendix B, is stepped each time current reverses polarity and is used to stop the computer after five cycles of operation have been computed. The "test blocks" have the quantity being tested in the upper half and the lower half indicates the possible result of the test. The result of the test determines the sub-routine to which the computer will go. For example, referring to the flow chart on Page (Appendix B), the first test is for M equal to ten. If M is equal to or less than ten, M is increased by one and the computer goes to the alpha₁ routine. If M is greater than ten, then the computer halts.

The computer program and one sample run are given in Appendix B. All of the computer sub-routines were used at least once, although no one "run" used all of the sub-routines.

Conditions of Modulator Operation Investigated

The parameters that were varied are shown in Table 2. Damping ratio, ζ_1 , is the damping ratio for positive current and ζ_2 is the damping ratio for negative current. The damping ratio ζ_1 was varied from 0 to 0.350 and ζ_2 was varied from 0 to .040. The frequency ratio ω/γ , was varied from 0.749 to 1.249 in two steps. Instead of making a computer run for frequency ratio of 1, the frequency ratio was 0.999. This was done to eliminate the need for another set of equations for

equal to 1. Per-unit saturation flux density was varied from 1.5 to 2.0. When driven at resonance with no damping, the maximum per-unit flux density excursion is approximately 1.71.

TABLE 2

COMPUTER VARIABLE^a INPUT DATA

PARAMETER	INITIAL VALUE	CHANGE IN VALUE	TOTAL CHANGE
ζ_1	Variable	Variable	Variable
ζ_2	Variable	Variable	Variable
Φ_s	Variable	Variable	Variable
ω/ν	Variable	Variable	Variable
τ	0	Variable	Indefinite ^b

a * Variable Data Controlled by Computer Input Information

b * Determined by Five Complete Cycles of Operation for Each Condition Tested

Results of Computation

Three sample curves are shown in Figures 20, 21, and 22 on Pages 69, 71, and 72. These curves are discussed in Chapter V.

The information required for design as listed on Page 41 were satisfied by the computer data. The computer data indicated also that, if the current reversed polarity immediately prior to saturation of T_1 , the modulator would have a period slightly larger than the power supply period, thus causing the positive flux density swing to increase in

amplitude each cycle until positive saturation was reached. This then returns the modulator back in phase with the power supply and the process is then repeated. (The computer program did not account for any variations in flux-reset of the magnetic compression stage or output transformer cores.) From the above, it is seen that the value of positive current damping ratio ζ_1 that insures no positive saturation of T_1 also insures negative current at the time of negative saturation of T_1 . For a value of ζ_2 equal to 0.015, ζ_1 must equal at least 0.150. It was noted from the computer data that the desired value of ζ_1 depends upon the value of ζ_2 but not enough data was taken to determine the exact relationship. Results of the computation are discussed in detail in Chapter VI.

CHAPTER IV

DESIGN OF A LOW POWER EXPERIMENTAL MAGNETIC MODULATOR

The information obtained from the solution of the input equations (from Chapter III) supplemented by some preliminary design considerations provides the necessary information for the development of explicit design equations. In this chapter the explicit design equations are developed for a magnetic modulator having a schematic as shown in Figure 18. Using these equations, a low power experimental magnetic modulator is designed from a set of input and output requirements.

Preliminary Design Considerations

The design equations developed in Appendix C assume that the parameters (1) through (4) are known and that the capacitor values have been determined. The validity of this assumption is demonstrated later.

- (1) Efficiency of energy transfer per stage; η_1^2 , η_2^2 , η_3^2 ,
 η_4^2 .
- (2) Saturated permeability of the saturable reactors; μ_{sr1} ,
 μ_{sr2} , μ_{sr3} , μ_{sr4} .
- (3) Output requirements and power supply characteristics;
peak load voltage, output pulse width, pulse repetition rate, power supply frequency, and voltage.
- (4) Number of magnetic compression sections required.
- (5) Peak capacitor voltage or capacitor size; C_1 , C_2 ,
and C_3 .

MAGNETIC MODULATOR SCHEMATIC
FIGURE 18

Design Equations for a Magnetic Modulator

The design equations developed in Appendix C are for a two section magnetic compression stage with a critically damped output stage. The input stage is for resonant charging; i.e., resonant frequency of the input stage is equal to the power supply frequency. The design equations are developed by starting at the output and working back to the input using the equations developed in Chapter II. The design equations are shown in Equations 4.1 through 4.10.

$$a_2 = \sqrt{\frac{\tau_p}{t_p} \left(\frac{C_3 R_L}{2} \right)} \quad (4.1)$$

$$a_1 = \left(\frac{e}{8\pi} \right) \frac{(C_1 + C_2)(C_3 + C_2)}{\eta_1 \eta_2 \eta_3 \eta_4 C_1 C_2} \left(\frac{V_{im}}{V_{mi}} \right) \sqrt{\frac{2\tau_p}{\tau_p C_3 R_L}} \quad (4.2)$$

$$N_4 = \left[\frac{\tau_p R_L l_2 10^7}{\tau_p 2a_2^2 4\pi A_{c4} \mu_{s4}} \right]^{1/2} \quad (4.3)$$

$$N_3 = \frac{N_4 B_{m4} A_{c4} (C_1 + C_2)(C_2 + C_3)}{2\pi^2 \eta_1 \eta_2 \eta_3 C_1 C_2 a_1 V_{mi}} \left[\frac{(C_1 + C_2) l_3 10^7}{C_2 C_3 \pi A_{c3} \mu_{s3}} \right]^{1/2} \quad (4.4)$$

$$N_{ST1} = \frac{N_3 B_{m3} A_{c3} (C_1 + C_2)}{\pi^2 \eta_1 \eta_2 a_1 V_{mi} C_1} \left[\frac{(C_1 + C_2) l_2 10^7}{\pi A_{c1} C_2 \mu_{s1}} \right]^{1/2} \quad (4.5)$$

$$N_{PT1} = N_2 / a_1 \quad (4.6)$$

$$L_i = \frac{1}{a_i^2 \omega_i^2 C_i} \quad (4.7)$$

$$N_i = \left[\frac{L_i l_i 10^7}{4\pi A_{ci} \mu_i} \right]^{1/2} = \left(\frac{L_i}{L_j} \right)^{1/2} N_j \quad (4.8)$$

$$N_{r2} \geq \frac{\pi V_m a_1 a_2 \eta_1 \eta_2 \eta_3 \eta_4 t_p}{B_{mr2} A_{cr2} C_p} \quad (4.9)$$

$$N_{pr2} = N_{r2} / a_2 \quad (4.10)$$

Inspection of the above equations reveals the dependence of the saturable reactor design upon the other components and parameters. Therefore, if one saturable reactor is designed incorrectly, the others will not function properly.

Determination of the Required Quantities

Efficiency - The efficiency is a difficult quantity to determine accurately because of the nature of the saturable reactor core losses. The saturable reactor core loss depends upon the speed at which it is "switched" and the shape of the driving voltage wave form. A considerable amount of effort has been expended, and reported in the literature, in attempting to determine the core losses.^{10,11} Unfortunately, much of the core loss data is not very useful for magnetic modulator design. This situation arises because much of the saturable reactor core data was taken when not operated in a manner similar to magnetic modulator uses. One of the most significant contributions to the core-loss problem was published in a report by E. J. Smith, J. Antin, and K. T. Lian in 1955.⁵ E. J. Smith and associates made some theoretical analysis and experimental measurements of core losses under pulsed operation which more nearly approximates the conditions of use in a magnetic modulator. The best data, at present, appears to be the actual efficiencies measured from working modulators.

The reported magnetic compression and output stage efficiencies do not vary over as wide a range as does the reported input stage efficiencies. C. E. Carter of Westinghouse Electric Corporation reported magnetic compression stage efficiencies of 85 to 90 per cent per magnetic modulator section.⁸ The reports from E. J. Smith and associates of Polytechnic Institute of Brooklyn state that they have achieved 80 to 95 per cent efficiencies per section with output stage efficiency somewhat less than the magnetic compression sections.^{3,4,5,6} From the above data, the magnetic compression stage efficiency was assumed to be 80 per cent. Equations 4.11 and 4.12 give these efficiencies.

$$\eta_1^2 = \eta_2^2 = 0.9 \quad (4.11)$$

$$\eta_4^2 = 0.8 \quad (4.12)$$

The input stage efficiencies reported in the literature vary from 50 to 85 per cent, depending upon the input configuration and power supply impedance. Since the input configuration of the magnetic modulator using automatic bias does not appear in the literature, it is necessary to use the data from Chapter III to determine the input efficiency. The damping ratio used in the input equations includes the power supply impedance, input core, and copper losses. Saturable reactor core flux reset energy loss is not rigorously incorporated into ζ because of the difficulty in evaluating the reset energy loss. Mr. C. E. Carter, Jr. reported a 5 to 10 per cent energy loss due to resetting of one or more saturable reactor cores.

Review of the "computer" results of Chapter III indicates that a ζ_1 of 0.150 and a ζ_2 of 0.015 insured no current reversal just prior

to saturation of T_1 . Since the rate of change of current is nearly maximum at this point and also near zero, it was decided to use a larger value for ξ_1 . The next larger value used in the computer run was 0.250 but was considered larger than practical. A value of 0.200 was chosen for ξ_1 , and the input capacitor C_1 voltage was interpolated from the computer data for ξ_1 of 0.150 and 0.250. The interpolated per unit capacitor voltage referred to the primary of T_1 is given by Equation 4.13.

$$V_{C1max} = 2.1 \quad (4.13)$$

The efficiency factor of the input stage is defined here as the square of the ratio of the voltage of the capacitor C_1 at the time of saturation of T_1 to the maximum value obtainable with no losses. The efficiency factor defined this way is a lower value than is the actual efficiency; however, defining the input stage efficiency factor in this manner is more practical for the design equations. Equation 4.14 then gives the input efficiency factor as defined above.

$$\eta_i^2 = \left(\frac{2.1}{\pi}\right)^2 = 0.45 \quad (4.14)$$

Saturated Permeability - The saturated permeability of the saturable reactors is a quantity that is difficult to determine. Mr. C. E. Carter, Jr. noted this problem as follows:

μ_s cannot be calculated until the final reactor is designed. If some means of estimating μ_s were not known, an appreciable amount of extra work would have to be done, since the design of the second stage reactor would have to wait until the first stage design were complete. However, it is possible to estimate μ_s for materials used to within

operable limits. Almost all of the reactors built in the performance of the contract had a μ_s of between 5 and 7. (It is to be noted that the value of μ_s given is for the reactor and includes leakage paths not in the core material. The μ_s for most core materials used is between 2 and 3.) Using a value of 6 for μ_s has given results which were usable. The estimate is probably as close as calculations would be, due to the difficulty of determining all leakage paths accurately.⁸

Mr. E. J. Smith and associates derived an approximate formula for calculating the effective saturated permeability as related to the core saturated permeability, taking into consideration the flux leakage path, and is given by Equation 4.15.

$$\mu_s = \mu_m + \left(\frac{A_{TOTAL}}{A_{CORE}} - 1 \right) \quad (4.15)$$

In his "summary," Mr. E. J. Smith wrote:

The saturated (relative permeability for several tape thicknesses of Orthonol over the range of flux build-up frequencies indicated previously was measured to be approximately 4. The saturated permeability was determined by the time of discharge of one condenser into another through the saturated core and then calculating back from the known values of circuit parameters. The energy transferred must be appropriate for the weight and size of the core tested.⁶

Calculations of μ_s from core data published by Magnetics, Incorporated gave values of 1.6 for Orthonol (.50 - .50 Ni-Fe) and 4.0 for Hy My 80 (.04 Mo - .78 Ni - .18 Fe) at a magnetizing force of 640 ampere turns per meter. A value of three for the saturated permeability of Orthonol material was chosen on the basis of the published data. There was not sufficient reported data to make a similar determination of the saturated permeability of Hy Mu 80 material. Therefore, four Hy My 80 cores were wound and tested in the manner suggested by E. J. Smith.

Figure 19 shows the oscilloscope traces for the discharge current pulse.

Current Discharge Pulse
for 4% Mo, 18% Fe, 78% Ni Core
(Magnetics P/N 52000-1D).
150 turns on Core

SATURATED PERMEABILITY EXPERIMENTAL

DETERMINATION DATA

FIGURE 19

The relative saturated permeability of six was then determined for the Hy Mu 80 core material. The effective saturated permeability was then calculated from Equation 4.15 for the particular core and case geometry.

Output Requirements and Power Supply Capability - The output requirements assumed for the experimental magnetic modulator are a 3250 volt peak pulse amplitude with a half amplitude width of 0.30 microsecond into a 1000 ohm resistor load. The PRF is equal to the power supply frequency of 1600 cycles per second. The power supply is assumed to have a 115 volt RMS output with an internal impedance of less than 30 ohms. The peak power output is then 11.6 kilowatts and total load energy is given by Equation 4.16.

$$\mathcal{E}_L = \frac{1}{R_L} \int_0^{\infty} (2a_2 V_{cm} \gamma_4 t e^{-\gamma_4 t})^2 dt = \frac{1}{R_L} \int_0^{\infty} (2V_{cm} \gamma_4 t e^{-\gamma_4 t})^2 dt \quad (4.16)$$

The argument of the integral of Equation 4.16 is for one output pulse (one cycle). Solution of Equation 4.16 is given in terms of the known quantities by Equation 4.17.

$$\mathcal{E}_L = \frac{t_p}{R_L} V_{cm}^2 (0.754) \doteq 2.4 \text{ m JOULES} \quad (4.17)$$

Capacitor Size and Voltage Rating - The peak voltage on the capacitors was chosen to be 2000 volts or less. Equation 4.18 relates output capacitor value to load energy and output stage efficiency.

$$C_3 = \frac{2\mathcal{E}_L}{\eta_4^2 V_{cm}^2} = \frac{2(2.4)10^{-3}}{0.8(2000)^2} = 1500 \text{ pf} \quad (4.18)$$

Because the efficiencies of energy transfer are less than one, the first capacitors will require higher peak voltage than 2000 volts if the capacitor values are all 1500 pf. If C_1 , C_2 , and C_3 are chosen to be

2000 pf (the next larger standard size), the peak voltages on C_1 , C_2 , and C_3 are shown by Equations 4.19, 4.20, and 4.21.

$$V_{C3m} = \frac{2\epsilon_L}{\eta_4^2 C_3} \doteq 1710 \text{ Volt} \quad (4.19)$$

$$V_{C2m} = \frac{V_{C3m}}{\eta_3} \doteq 1800 \text{ VOLT} \quad (4.20)$$

$$V_{C1m} = \frac{V_{C3}}{\eta_1 \eta_3} \doteq 1900 \text{ VOLT} \quad (4.21)$$

In order to keep the capacitor peak voltage nearly equal, the value of C_1 was chosen to be the next larger standard size as shown in Equation 4.22.

$$C_1 = 2500 \text{ pf} \quad (4.22)$$

The peak voltage on C_1 is given by Equation 4.23.

$$V_{C1m} = \frac{V_{C3}}{\eta_1 \eta_3} \doteq 1700 \text{ VOLT} \quad (4.23)$$

Core Size and Material - Equations for determining total core volume have been developed in the literature and are based on minimum core volume designs. For low power magnetic modulator designs, the minimum core volume could very probably be too small to wind and insulate properly.⁹ The design Equations 4.1 through 4.10 could be solved for core volumes; however, the writer believes that the form in which they are given is adequate. The design procedure is as follows:

- (1) Calculate the turns ratios for T_1 and T_2 .
- (2) Select a core material and geometry for L_4 and calculate the number of turns required.

- (3) Using the information from (1) and (2) above, select a core material and geometry for L_3 and calculate the number of turns required.
- (4) Similarly calculate the secondary turns required for T_1 ; then, from (1) above, calculate the primary turns for T_1 .
- (5) Similarly calculate the turns required for L_1 .
- (6) Select a core material and geometry for T_2 (usually the same core parameters used for L_4 may be used for T_2) and calculate the number of turns required.

Design of Experimental Magnetic Modulator - The quantities determined in the preceding section are given in Table 3. Equations 4.24 through 4.34 show the use of Equations 4.1 through 4.10 in completing the modulator design.

$$a_1 = \frac{e (4.5)(4.0)(20)}{8\pi (1.67)(.95)^2 (.90)(5.0)} \sqrt{\frac{(2)(0.3)}{(2.45)(2)}} \doteq 5.0 \quad (4.24)$$

$$a_2 = \sqrt{\frac{(2.45)(2)}{(0.3)(2)}} \doteq 2.87 \quad (4.25)$$

$$N_4 = \left[\frac{(1.23)(5)(7.96)(4.92)10^3}{(6.48)(2.81)^2} \right]^{1/2} \doteq 67 \text{ TURNS} \quad (4.26)$$

$$N_3 = \frac{(6.7)(7)(8.1)(3.6)10^2}{(6.03)(5)(1.97)(1.63)} \left[\frac{(4.92)10^{-7}}{\pi(8.1)(5)} \right]^{1/2} \doteq 286 \text{ TURNS} \quad (4.27)$$

$$N_2 = \frac{(2.86)(1.4)(8.1)(9)10^3}{(2.5)(1.63)(9.87)(6.37)} \left[\frac{(7.98)(9)}{(4.53)4\pi} \right]^{1/2} \doteq 1275 \text{ TURNS} \quad (4.28)$$

$$N_{PT1} = \frac{1275}{5} = 255 \text{ TURNS} \quad (4.29)$$

$$L_1 = \frac{1}{(25)(25)} \doteq 157 \text{ mh.} \quad (4.30)$$

$$N_1 = \left(\frac{157}{157} \right)^{1/2} (1000) \doteq 1000 \text{ TURNS} \quad (4.31)$$

$$N_{T2} > \frac{(1.625)(\pi)(5)(2.87)(0.54)(0.3)10^2}{(0.7)(8.1)(2.45)} \doteq 86 \text{ TURNS} ; N_{T2} = 115 \text{ TURNS} (4.32)$$

$$N_{PT2} = \frac{115}{2.87} \doteq 40 \text{ TURNS} \quad (4.33)$$

$$V_{Lm} = (1710)(.736)(2.87)(0.9) \doteq 3260 \text{ VOLT} \quad (4.34)$$

Table 4 gives the completed design and capacitor voltages.

TABLE 3

EXPERIMENTAL MAGNETIC MODULATOR PRELIMINARY DESIGN

INITIAL DESIGN PARAMETER DEFINITION

Circuit Parameters

Item	Value	Item	Value	Item	Value
η_1	0.67	C_1	2500 pf	V_{mi}	162.5 Volt
η_2	0.95	C_2	2000 pf	V_{Lm}	3200 Volt
η_3	0.95	C_3	2000 pf	τ_p	2.446
η_4	0.89	ω_1	10^4 Rad/Sec	t_p	$0.3/\mu$ Sec

Reactor Parameters

Powdered Iron (1) 2% Mo, 81% Ni, 17% Fe		S. R. (2) 50% Ni, 50% Fe		S. R. (3) 4% Mo, 78% Ni, 18% Fe	
B_m	0.7 Web/m ²	B_m	1.4 Web/m ²	B_m	0.7 Web/m ²
μ_u	125	μ_m	3	μ_m	6

TABLE 3 Continued

Selected Reactor Core Parameters

Reactor	(a) Mfg.	Core P/N	ℓ (M)	A_c (M ²)	A Total (M ²)	μ_s	(c) Mat'l.
T ₁	M	52188-1A	$7.98 \cdot 10^{-2}$	$4.53 \cdot 10^{-5}$	$9.35 \cdot 10^{-5}$	4	(2)
T ₂	M	52000-1D	$4.92 \cdot 10^{-2}$	$8.1 \cdot 10^{-6}$	$2.25 \cdot 10^{-5}$	8	(3)
L ₃	M	52000-1A	$4.92 \cdot 10^{-2}$	$8.1 \cdot 10^{-6}$	$2.25 \cdot 10^{-5}$	5	(2)
L ₄	M	52000-1D	$4.92 \cdot 10^{-2}$	$8.1 \cdot 10^{-6}$	$2.25 \cdot 10^{-5}$	8	(3)
L ₁	A	A-930157-2	$6.95 \cdot 10^{-2}$	—	$7.11 \cdot 10^{-5}$	(b)	(1)

(a) Manufacturer: M - Magnetics, Incorporated.; A - Arnold Engineering Company.

(b) Inductance for 1000 turn winding is nominally 157 mh.

(c) See designation in table above for material.

TABLE 4

EXPERIMENTAL MAGNETIC MODULATOR FINAL DESIGN

Saturable Reactor Turns

Saturable Reactor	Turns	Additional Taps for Experimental Purposes (a)
T ₁ Secondary	1275	1150, 1400, 1525
T ₁ Primary	255	230, 280, 305
T ₂ Secondary	115	44
T ₂ Primary	40	127
L ₃	286	257, 325
L ₄	67	60, 74
Charging Inductor		
L ₁	157 mh ^(b)	145 mh, 165 mh, 187 mh

(a) Experimental data presented in this thesis does not include any data for operation at any of these values.

(b) The actual value was 154 mh in the experimental modulator.

TABLE 4 Continued

Modulator Capacitor Design Values

Capacitor	Size	Voltage Rating	Expected Peak Voltage
C ₁	2500 pf	4000	1700
C ₂	2000 pf	3000	1800
C ₃	2000 pf	3000	1710

R_B : 250 Ohm, 2 Watt Potentiometer

D₁ and D₂ : 1N645

CHAPTER V

EXPERIMENTAL RESULTS

The experimental results are considered in good agreement with the theoretically predicted results. Table 5 shows a comparison of design values with experimental results. Figures 20, 21, and 22 give a comparison of experimental wave forms with computed wave forms. These results are discussed in detail below.

Comparison of Computed Wave Forms with Experimental Results

The computed wave forms show an overall transient from the initial value of current and power supply phase angle of zero to values different from zero at the time of saturation of T_1 . The photographs of experimental modulator wave forms do not show such a transient because the transient takes a very few cycles of operation to be completed. The computed wave form transient is influenced somewhat by the flux overshoot at the saturated flux density limit due to the size of the time increment used. The capacitor voltages shown in the experimental wave forms when shown simultaneously with other wave forms is the summation of the voltage on both C_1 and C_2 . This was necessary because of common grounding of the oscilloscope inputs.

Figure 20 compares the computed wave forms for operation at resonance with damping ratios of 0.150 for positive input current and 0.015 for negative input current. It is to be noted that the modulator was designed using a "positive" damping ratio of 0.200; however, the wave forms compare quite well. Comparison of the current maximum and minimum values indicates that the "negative" damping ratio of the experimental modulator was larger than 0.015. Figure 21 shows operation with excessive "positive"

TABLE 5

COMPARISON OF DESIGN VALUE WITH EXPERIMENTAL RESULTS

(Operation with All Windings at Designed Value)			Input Stage Resonance (a)	
Item	Design Value	Experimental Value	L ₁ Value (mh)	Resonant Frequency
η_1	0.67	0.685	144.6	1652 cps
η_2	0.95	0.976	154.0	1602 cps
η_3	0.95	0.862 (b)	(154.5) ^(c)	(1600 cps) ^(c)
η_4	0.90	0.99 (b)	(157.0) ^(c)	(1587 cps) ^(c)
μ_{ST1}	4	3.85	165.3	1542
μ_{L3S}	5	5.99	187.0	1451
V _{Im}	3250 Volt	3450 Volt	-----	-----
Pulse Width	0.30 Sec	0.295 Sec	-----	-----
PRF, f _i	1600 cps	1600 cps	-----	-----
v _{c1} Max.	1700 Volt	1750 Volt	-----	-----
v _{c2} Max.	1800 Volt	1900 Volt	-----	-----
v _{c3} Max.	1710 Volt	1640 Volt ^(b)	-----	-----
L ₁	157.0 mh	154.0 mh	-----	-----

Note: (a) The input stage series resonance was measured by adjusting the power supply frequency (f_i) to cause the input current and voltage to be in phase. These measurements were made with C₂ and R_B shorted and the power supply voltage set at 44 volts p-p to eliminate saturation of T₁.

(b) There appears to be an error in the measurement of v_{c3} max.

(c) These values are interpolated from the experimental data.

FIGURE 20 A

EXPERIMENTAL WAVEFORMS FOR $\omega/\gamma = 1.0, \zeta_1 = 0.200$

FIGURE 20 B

input current damping ratio. The computed wave form transient is slower with the larger damping ratio than with the lower value.

Operation at a frequency of 1.25 times the resonant frequency is shown in Figure 22. The "smoother" capacitor voltage wave form is due to the saturable reactor L_3 not being driven through the unsaturated state (reset cycling) twice during the positive half of the input charge cycle. The reset cycling of L_3 during the input charge cycle is discussed in a following section.

Comparison of Modulator Operation with Design Requirements

The design requirements were satisfied quite well with no need for empirical (Bench) design as is indicated in Table 5. The experimental efficiency factors η_1 and η_2 agreed with the assumed values; however, η_3 and η_4 did not agree very closely. One contributing cause for this disagreement is the fact that the output saturable reactor L_4 was saturating somewhat prematurely. It is also possible that the voltage measurement of V_{C3} was in error. The saturated permeability of L_3 was almost 20 per cent higher than assumed, which accounts for the premature saturation of L_4 . Premature saturation is referred to here as the saturation prior to the time peak capacitor voltage would have been reached. (See Figure 24B, Page 74.)

The experimental modulator wave forms for operation at resonance with R_b set just large enough to prevent positive saturation of T_1 are shown in Figures 23, 24, and 25. The capacitor voltage of C_1 is shown in Figure 23A for three complete cycles of operation. This wave form can be compared to the capacitor voltage of Figure 20B. The "saw tooth" effect just after saturation of T_1 is caused by the flux-reset cycling

COMPUTED WAVEFORMS FOR $\omega/\nu = 1, \zeta_1 = 0.350, \zeta_2 = 0.015$

FIGURE 21 A

EXPERIMENTAL WAVEFORMS FOR $\omega/\nu = 1, \zeta_1 = 0.35$

FIGURE 21 B

COMPUTED WAVEFORMS FOR $\omega/p = 1.25, \zeta_1 = 0.15, \zeta_2 = 0.015$

FIGURE 22 A

EXPERIMENTAL WAVEFORMS FOR $\omega/p = 1.25, \zeta_1 = 0.200$

FIGURE 22 B

Refer to Figure 9 (Page 31) for Time Relationship

FIGURE 23 A

FIGURE 23 B

Refer to Figure 9 (Page 31) for Time Relationship

CAPACITOR C₂ VOLTAGE

FIGURE 24 A

CAPACITOR C₃ VOLTAGE

FIGURE 24 B

OUTPUT VOLTAGE PULSE

FIGURE 25 A

OUTPUT VOLTAGE PULSE

FIGURE 25 B

of the magnetic compression stage saturable reactors. (Flux-reset cycling of this type is treated at some length by K. J. Busch and associates of Bell Systems Laboratories.²) Figure 23B shows the sum of the capacitor voltages of C_1 and C_2 (during the interval of energy transfer through the compression section) with the oscilloscope sweep starting at the time of saturation of T_1 . The time scale has been expanded one hundred times from that of Figure 23A. The C_2 capacitor voltage is shown in Figure 24A with the oscilloscope sweep starting at the same point as in Figure 23B, except the time scale has been expanded two times. The oscilloscope sweep was started at the time of saturation of L_3 (peak of the C_2 voltage wave form) for Figure 24B, which shows the capacitor C_3 voltage wave form. The time scale of Figure 24B has been expanded five times that of Figure 24A. The output saturable reactor L_4 saturates at the peak of V_{C3} (Figure 24B) and starts the oscilloscope sweep for Figures 25A and 25B. The output pulse voltage is shown in Figures 25A and 25B where the time scale has been expanded by a factor of two from Figure 24B and a factor of 2000 times from Figure 23A. Figure 25B shows the pulse width of the output pulse, the vertical scale, is not calibrated. Additional photographs of the experimental modulator operation with various conditions of biasing resistor and frequency ratio are in Appendix D.

Flux-Reset Cycling of L_3 - The flux-reset cycling of L_3 during the positive half of the C_1 charge cycle was noted earlier. This flux-reset cycling is in addition to the flux-reset cycling as noted in Figure 23A and mentioned by Mr. Busch and associates.² This problem is shown in Figures 26 and 27. The time scale of Figures 26 and 27 are 100 micro

Refer to Figure 9 (Page 31) for Time Relationship

NO RESET CYCLING OF L_3 , R_B TOO SMALL FOR UNIPOLAR OPERATION

FIGURE 26 A

RESET CYCLING OF L_3 JUST STARTING,
 R_B INCREASED FOR UNIPOLAR OPERATION

FIGURE 26 B

Refer to Figure 9 (Page 31) for Time Relationship

RESET CYCLING OF L_3 WELL ESTABLISHED
 R_B LARGE ENOUGH TO INSURE UNIPOLAR OPERATION

FIGURE 27 A

RESET CYCLING OF L_3 WITH R_B MUCH LARGER
 THAN NECESSARY TO INSURE UNIPOLAR OPERATION

FIGURE 27 B

seconds per division (expanded by a factor of 2 from Figure 23A).

Figure 26A indicates a tendency for the core to be driven out of saturation about the reference time t_6 . The biasing resistor was increased from zero to the point where this indication was first apparent (180 ohms). The modulator input current was approximately zero at the time of negative saturation of T_1 . Transformer T_1 was saturating on the positive half cycle just prior to the capacitor voltage going to zero. The result of positive saturation can be seen by the rate at which L_3 was switched from negative saturation to positive saturation as compared to Figure 26B (reference time t_1). Also, evidence of positive saturation of T_1 is shown by the negative going C_3 voltage pulse at this time. (The negative voltage pulse is off scale.) When the bias resistor is further increased (to 190 ohms), the condition of Figure 26B occurs. (Note the flux going positive from the negative saturated point, then returning back to negative saturation, between reference time t_3 and just past t_6 .) The flux excursion of L_3 into the unsaturated region during this interval occurs when the input current is negative at the time of saturation of T_1 . The negative input current flow is coupled through the saturated inductance of T_1 (nearly an air-core transformer) into the compression stage. This current is in such a direction as to drive L_3 toward positive saturation. When the input current goes positive, L_3 is driven back to negative saturation. The L_3 core is actually reset when the input current swings negative during the second half cycle of operation.

The flux excursion of L_3 during the first half cycle of the input charging current cycle causes an additional energy loss due primarily to the unusable energy placed on C_2 at this time. The result is an apparent

increase in R_p . The biasing resistor R_p can now be reduced to about 150 ohms without causing positive saturation of T_1 . The overall effect is a "hysteresis" characteristic for the value of R_p .

Modulator Stability

The experimental magnetic modulator with automatic bias was stable under all conditions of operation investigated when biased to eliminate saturation of T_1 while the capacitor voltage on C_1 was positive. Some instability was observed when the modulator was operated under the conditions listed below.

Instability with Improper Bias - When the bias resistor was adjusted so that the input current was positive at the time of saturation of T_1 , some instability was experienced. The cause of this instability was two-fold. The saturable reactor L_3 was being driven out of saturated reset causing it to saturate prematurely during the energy transfer interval. This caused incomplete transfer of energy and loss in output pulse amplitude. The amount that L_3 was driven out of saturated reset was not constant for each cycle, which resulted in variations in output pulse amplitude.

The other cause of instability was that the normal modulator output period was slightly longer than the power supply period. This caused the positive flux swing of T_1 to increase as the modulator fell out of phase until positive saturation was reached. At this point a small negative output pulse occurred and the modulator was forced back in phase with the power supply. The result was two different periods between output pulses, the difference between the periods being somewhat propor-

tional to the magnitude of the positive current at the time of saturation of T_1 .

When the bias resistor was adjusted so that T_1 was never saturated while the capacitor voltage on C_1 was positive, no instability was observed regardless of the power supply frequency. At low frequencies (below 0.75, the undamped resonant frequency), there was not sufficient bias resistance available in the experimental modulator circuit to prohibit positive saturation (250 ohm potentiometer).

Hysteresis Affect of Bias Resistor - This problem was discussed in some detail in Chapter V. This problem is not actually one of instability but one of adjustment of R_b . There is a change in output amplitude as the modulator changes from operation without "reset cycling of L_4 " to that with "reset cycling of L_4 " as R_b is increased from a low value. Once this change is made, the modulator is stable until the bias resistor is changed downward. When the saturable reactor L_4 is "reset cycled," small amplitude output pulses of both polarities occur during the period between regular output pulses.

The proper method for setting the value of R_b is to increase it from a value low enough to cause saturation of T_1 during positive voltage on C_1 to a value high enough that just eliminates "positive saturation" of T_1 . At this point, L_3 is being reset cycled, as can be observed by the small output pulses occurring in the inter-pulse period. Reduction in R_b from this value may result in the modulator reverting back to operation with insufficient bias by a disturbance of some unwanted transient, such as interrupting the power supply. In the absence of any such disturbance, the value of R_b can be reduced about 15 per cent before the

modulator will revert back to operation with insufficient bias. The small output pulses occurring in the inter-pulse period always result when the bias resistor is the proper value for stable operation, when the modulator is operated at resonance.

Magnetic Modulator Operation Off-Resonance - Operation of the automatic biased magnetic modulator at either low frequency (25 per cent below resonance) or high frequency (over 50 per cent above resonance) results in stable operation but a loss in output amplitude.

Low frequency operation results in multiple inter-period output pulses due to increased reset cycling of the compression stage saturable reactors.

The experimental modulator was operated at a frequency as low as 50 per cent below resonance. At frequencies lower than 25 per cent below resonance, there was insufficient bias resistance in the circuit.

Operation at a frequency about 25 per cent above resonance results in stable operation without the reset cycling of the magnetic compression stage saturable reactors. This is due to the input current approaching zero faster after saturation of T_1 than when operated at resonance. The reason for this is because the power supply voltage is approximately 50 per cent of its positive peak value at the time of saturation of T_1 , whereas at resonant operation the power supply voltage is near zero at the time of saturation of T_1 . Operation at a frequency 25 per cent above resonance results in a 10 per cent loss in peak amplitude. Some of the loss is due to a loss in input stage voltage gain rather than loss in efficiency. The peak amplitude may be regained by increasing the turns ratio of T_1 and reducing the value of L_1 to maintain the desired undamped resonant frequency.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

Conclusions

The computer study provided the information about the operation of the input stage which is necessary for accurate design. The design procedure developed yielded a magnetic modulator that required much less empirical adjustment than devices designed by procedures used previously by the author and others. The only adjustment required for proper operation was the value of the biasing resistor R_B . This adjustment compensates for losses which cannot be determined in advance.

The automatic bias and flux-reset feature requires less components and provides for more frequency tolerance than does the DC type bias. Because the bias resistor R_B does not exhibit a hysteresis effect and the magnetic compression stage cores are not reset cycled, the author concludes that operation at a power supply frequency 25 per cent above input stage resonance results in a very desirable magnetic modulator design.

Recommendations of Further Study

High Voltage Limiting - The high voltage resulting from an unloaded output stage may possibly be limited by proper design of the output transformer using a saturable reactor. If the unsaturated primary inductance of the output transformer is near the same value as the unsaturated inductance of L_4 , the volt-time integral applied across the series combination of the output transformer and L_4 during the charge of C_3 will divide, thus saturating both L_4 and T_2 near the same time (T_2 unloaded).

When T_2 is properly loaded, the load resistance referred to the primary of T_2 must be small in comparison to the impedance of the unsaturated inductance of L_4 presented to the charging voltage of C_2 .

A more detailed study of the flux-reset in the compression and output stage saturable reactors when more than two sections of magnetic compression are employed is in order. With more sections of magnetic compression, the flux-reset cycling problem may be more pronounced.

Refinements of Computer Program - Some error in computed data resulted from flux overshoot of the saturated limit. This error could be reduced to any desired value by using smaller time increments; however, the amount of computations and data print-out would increase proportionally. A change in the computer program that would allow for smaller increments in time to be used when the value of flux was approaching a saturated limit would be desirable. A change such as this would allow for a much larger time increment than was used for the computed data presented herein. Modification of the computer program to allow for variation in the number of computed current cycles and variation of initial conditions would give desirable versatility. A theoretical study of the steady state magnetic modulator operation would be more accurate if these two computer program changes were made.

The computer program written for the input circuit simulated a system having one saturable reactor. The same method could be used to simulate the complete modulator having several saturable reactors.

LIST OF REFERENCES

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1. W. S. Melville, "The Use of Saturable Reactors as Discharge Devices for Pulse Generators." Proceedings of the I.E.E., Volume 98, Part III, Pages 185-207, (1951).
2. K. J. Busch, A. D. Hasley, Carol Nitzert, "Magnetic Pulse Modulators." Bell Systems Technical Journal, Volume 34, No. 5, Pages 943-993, (September, 1955).
3. E. J. Smith, J. Antin, K. T. Lian, "Magnetic Modulators for Radar Applications." Polytechnic Institute of Brooklyn, Research Report R-419-55, Pages 18-351, R.A.D.C. Contract No. AF-30 (602)-984, (April, 1955).
4. E. J. Smith, K. T. Lian, "Magnetic Pulse Modulator with Biased Diode Load." Polytechnic Institute of Brooklyn, Phase Report R-501-56, Pages 18-431, R.A.D.C. Contract No. AF-30 (602)-984, (October, 1956).
5. Joseph Antin, "A Two-Megawatt Magnetic Modulator." Polytechnic Institute of Brooklyn, Research Report No. R-551-57, Pages 18-470, R.A.D.C. Contract No. AF-30 (602)-1619, (February, 1957).
6. E. J. Smith, "Studies of Pulse-Type Magnetic Modulators -- A Summary." Polytechnic Institute of Brooklyn, Final Report R-588-57, Pages 18-516, R.A.D.C. Contract No. AF-30 (602)-984, (July, 1957).
7. Richard C. Barker, "Non-Linear Magnetics." Electro-Technology, Pages 95-114, (March, 1963).
8. C. E. Carter, Jr., "Saturable Pulse Reactors and Radar Modulators." Westinghouse Electric Corporation, Final Development Report, Signal Corps, Contract No. DA-36-039 SC63124, (February, 1956).
9. Sheldon Plotkin, "A Feasibility Study of High-Power Magnetic Modulators." Levinthal Electronic Products, Inc., Final Report R.A.D.C. Contract No. AF-30 (602)-1177, (October, 1956).
10. J. E. Sunderline, "Core Volume Derivation for Magnetic Pulse Modulators." AIEE Conference Paper, 56-228, (1956).
11. B. M. Wolfframm, "The Efficiency of Magnetic Pulse Generators." AIEE Conference Paper, CP 56-729.

BIBLIOGRAPHY

BIBLIOGRAPHY

Books

- Ku, Y. H. Analysis and Control of Non-Linear Systems. The Ronald Press Company, New York, 1958.
- Wyllie, C. R. Jr. Advanced Engineering Mathematics. McGraw Hill Book Company, Inc., New York, 1951.

Periodicals

- Barker, Richard C. "Non-Linear Magnetics." Electro-Technology, Pages 95-114, (March, 1963).
- Busch, K. J., Hasley, A. D., Neitzert, Carol "Magnetic Pulse Modulators." Bell Systems Technical Journal, Volume 34, No. 5, Pages 943-993, (September, 1955).
- Grossner, N. R. "Pulse-Transformer Circuits and Analysis - 1." Electro-Technology, Pages 71-80, (February, 1962).

Technical Reports

- Antin, Joseph "A Two-Megawatt Modulator." Polytechnic Institute of Brooklyn, Research Report No. R-551-57, Pages 18-470, R.A.D.C. Contract No. AF-30 (602)-1619, (February, 1957).
- Carter, C. E. Jr. "Saturable Pulse Reactors and Radar Modulators." Westinghouse Electric Corporation, Final Development Report, Signal Corps, Contract No. DA-36-039 SC63124, (February, 1956).
- Kusda, T. "Flux-Reset Mechanism in Magnetic Modulator Circuits." Polytechnic Institute of Brooklyn, Research Report R-759-59, Pages 18-687, R.A.D.C. Contract No. AF-30 (602)-1619, (September, 1959).
- Melville, W. S. "The Use of Saturable Reactors as Discharge Devices for Pulse Generators." Proceedings of the I.E.E., Volume 98, Part III, Pages 185-207, (1951).
- Plotkin, Sheldon "A Feasibility Study of High-Power Magnetic Modulators." Levinthal Electronic Products, Inc., Final Report R.A.D.C. Contract No. AF-30 (602)-1177, (October, 1956).
- Smith, E. J. "Studies of Pulse-Type Magnetic Modulators -- A Summary." Polytechnic Institute of Brooklyn, Final Report R-588-57, Pages 18-516, R.A.D.C. Contract No. AF-30 (602)-984, (July, 1957).

Smith, E. J., Antin, J., Lian, K. T. "Magnetic Modulators for Radar Applications." Polytechnic Institute of Brooklyn, Research Report R-419-55, Pages 18-351, R.A.D.C. Contract No. AF-30 (602)-984, (April, 1955).

Smith, E. J., Lian, K. T. "Magnetic Pulse Modulator with Biased Diode Load." Polytechnic Institute of Brooklyn, Phase Report R-501-56, Pages 18-431, R.A.D.C. Contract No. AF-30 (602)-984, (October, 1956).

Sunderline, J. E. "Core Volume Derivation for Magnetic Pulse Modulators." AIEE Conference Paper 56-228, (1956).

Wolfframm, B. M. "The Efficiency of Magnetic Pulse Generators." AIEE Conference Paper CP 56-729.

APPENDIX A

DERIVATION OF INPUT STAGE EQUATIONS

Input Stage Equations With T_1 Unsaturated

The differential Equation is given by Equation 3.1, page 42.

$$L_1 \frac{di_{Lc}}{dt} + R i_{Lc} + \frac{1}{a_1^2 C_1} \int i_{Lc} dt = v_{mi} \sin[\omega_i(t + \alpha)] \quad (A-1)$$

where L_1 includes the charging inductance and the leakage inductance of T_1 . Taking the Laplace transform of Equation A-1 we have

$$\left[L_1 S + R + \frac{1}{a_1^2 C_1} \right] i_{Lc}(s) - S L i_0 + N_c = \frac{v_{mi} (\omega_i \cos \omega_i \alpha + S \sin \omega_i \alpha) S}{S^2 + \omega_i^2} \quad (A-2)$$

Solving for $i_{Lc}(s)$ gives Equation A-3.

$$i_{Lc}(s) = \frac{\frac{v_{mi}}{L_1} (\omega_i \cos \omega_i \alpha + S \sin \omega_i \alpha) S}{(S^2 + \frac{R}{L_1} S + \frac{1}{a_1^2 L_1 C_1})(S^2 + \omega_i^2)} + \frac{S i_0 - N_c / L_1}{S^2 + \frac{R}{L_1} S + \frac{1}{a_1^2 L_1 C_1}} \quad (A-3)$$

Set

$$\gamma^2 = \frac{1}{a_1^2 L_1 C_1} \quad (A-4)$$

$$\xi = \frac{a_1}{2} R \sqrt{\frac{C_1}{L_1}} \quad (A-5)$$

then

$$i_{Lc}(s) = \frac{\frac{v_{mi}}{L_1} [\omega_i \cos \omega_i \alpha + S \sin \omega_i \alpha] S}{[(S + \xi \gamma)^2 + \gamma^2 (1 - \xi^2)][S^2 + \omega_i^2]} + \frac{S i_0 - N_c / L_1}{[(S + \xi \gamma)^2 + \gamma^2 (1 - \xi^2)]} \quad (A-6)$$

The roots of S are

$$S_{1,2} = \pm j \omega_i \quad S_{3,4} = -\xi \gamma \pm j \gamma \sqrt{1 - \xi^2} \quad (A-7)$$

where

$$j = \sqrt{-1}$$

The inverse transform is obtained by use of Heaviside's Expansion Theorems. The general inverse transform of the term

$$\frac{P(s)}{(s+a)^2 + b^2}, \text{ where } P(s) \text{ is a polynomial in } s,$$

is given by

$$f(t) = \frac{e^{-at}}{b} (P_j \cos bt + P_r \sin bt) \quad (\text{A-8})$$

P_j and P_r are defined by Equation A-9.

$$P(-a+jb) = P_r + jP_j \quad (\text{A-9})$$

The inverse transform of Equation A-3 is obtained by adding the terms obtained by use of Heaviside's Expansion Theorems.

The first term of $\dot{L}_c(t) = \dot{L}_1(t)$, is obtained for the roots of $s = \pm j\omega_i$.

Then

$$P(s) = \frac{\frac{N_{mi}}{L_1} [w_i \cos w_i \alpha + s \sin w_i \alpha] s}{(s + \gamma)^2 + \gamma^2 (1 - \xi^2)}$$

and

$$P(-0 + j\omega_i) = \frac{\frac{N_{mi}}{L_1} [w_i \cos w_i \alpha + j w_i \sin w_i \alpha] j \omega_i}{(j \omega_i + \gamma)^2 + \gamma^2 (1 - \xi^2)} \quad (\text{A-10})$$

$$= \frac{j \omega_i \frac{N_{mi}}{L_1} \cos w_i \alpha - \frac{N_{mi}}{L_1} w_i \sin w_i \alpha}{-\omega_i^2 + \gamma^2 + \gamma^2 (1 - \xi^2) + j 2 \gamma \omega_i}$$

$$= \frac{\left[j \omega_i \frac{N_{mi}}{L_1} \cos w_i \alpha - \frac{N_{mi}}{L_1} w_i^2 \sin w_i \alpha \right]}{[\gamma^2 - \omega_i^2 + j 2 \gamma \omega_i]}$$

$$\left\{ \frac{(\gamma^2 - \omega_i^2) - j 2 \gamma \omega_i}{(\gamma^2 - \omega_i^2) - j 2 \gamma \omega_i} \right\}$$

$$\begin{aligned}
 P(0+j\omega_i) = & \frac{-\frac{N_m}{L_1} \omega_i^2 (\gamma^2 - \omega_i^2) \sin \omega_i \alpha + 2\zeta \gamma \omega_i^2 \frac{N_m}{L_1} \cos \omega_i \alpha}{(\gamma^2 - \omega_i^2)^2 + 4\zeta^2 \gamma^2 \omega_i^2} \\
 & + j \frac{\frac{N_m}{L_1} \omega_i^2 (\gamma^2 - \omega_i^2) \cos \omega_i \alpha + \frac{N_m}{L_1} 2\zeta \gamma \omega_i^3 \sin \omega_i \alpha}{(\gamma^2 - \omega_i^2)^2 + 4\zeta^2 \gamma^2 \omega_i^2}
 \end{aligned}
 \quad (A-11)$$

Using Equation A-8 with Equation A-11, we may write

$$\begin{aligned}
 i_1(t) = & \frac{N_m \omega_i / L_1}{[(\gamma^2 - \omega_i^2)^2 + 4\zeta^2 \gamma^2 \omega_i^2]} \left\{ (\gamma^2 - \omega_i^2) \cos \omega_i \alpha \cos \omega_i t \right. \\
 & + 2\zeta \gamma \omega_i \sin \omega_i \alpha \cos \omega_i t - (\gamma^2 - \omega_i^2) \sin \omega_i \alpha \sin \omega_i t \\
 & \left. + 2\zeta \gamma \omega_i \cos \omega_i \alpha \sin \omega_i t \right\} \\
 = & \frac{N_m \omega_i / L_1}{[(\gamma^2 - \omega_i^2)^2 + 4\zeta^2 \gamma^2 \omega_i^2]} \left\{ (\gamma^2 - \omega_i^2) (\cos \omega_i \alpha \cos \omega_i t \right. \\
 & - \sin \omega_i \alpha \sin \omega_i t) + 2\zeta \gamma \omega_i (\sin \omega_i \alpha \cos \omega_i t \\
 & \left. + \cos \omega_i \alpha \sin \omega_i t) \right\} \\
 = & \frac{N_m \omega_i / L_1}{(\gamma^2 - \omega_i^2)^2 + 4\zeta^2 \gamma^2 \omega_i^2} \left\{ (\gamma^2 - \omega_i^2) \cos [\omega_i (t + \alpha)] \right. \\
 & \left. + 2\zeta \gamma \omega_i \sin [\omega_i (t + \alpha)] \right\} \\
 = & \frac{N_m \sqrt{\frac{a_1^2 c_1}{L_1}} \left(\frac{\omega_i}{\gamma} \right)}{\left[1 - \left(\frac{\omega_i}{\gamma} \right)^2 \right]^2 + 4\zeta^2 \left(\frac{\omega_i}{\gamma} \right)^2} \left\{ \left[1 - \left(\frac{\omega_i}{\gamma} \right)^2 \right] \cos \left[\left(\frac{\omega_i}{\gamma} \right) (\gamma t + \gamma \alpha) \right] \right. \\
 & \left. + 2\zeta \left(\frac{\omega_i}{\gamma} \right) \sin \left[\left(\frac{\omega_i}{\gamma} \right) (\gamma t + \gamma \alpha) \right] \right\} .
 \end{aligned}
 \quad (A-12)$$

Equation A-12 may also be written

$$i_1(t) = \frac{V_{mi} \sqrt{\frac{a^2 c_1}{L_1}} \left(\frac{\omega_i}{\nu} \right)}{\left(\frac{\omega_i}{\nu} \right)^4 + 2(2\zeta^2 - 1) \left(\frac{\omega_i}{\nu} \right)^2 + 1} \left\{ \left[1 - \left(\frac{\omega_i}{\nu} \right)^2 \right] \cos \left[\left(\frac{\omega_i}{\nu} \right) (\nu t + \nu \alpha) \right] \right. \\ \left. + 2\zeta \left(\frac{\omega_i}{\nu} \right) \sin \left[\left(\frac{\omega_i}{\nu} \right) (\nu t + \nu \alpha) \right] \right\} \quad (A-12)$$

The second term of i_{1c} is obtained for corresponding quadratic factor of $S = (s + \zeta\nu)^2 + \nu^2(1 - \zeta^2)$ as follows:

$$P(s) = \frac{\frac{V_{mi}}{L_1} [\omega_i \cos \omega_i \alpha + \sin \omega_i \alpha] s}{s^2 + \omega_i^2} + (s \zeta_0 - \frac{V_0}{L_1}) \\ = P_1(s) + P_2(s) \quad (A-13)$$

Then

$$\frac{L_1}{V_{mi}} P_1(-\zeta\nu + j\nu\sqrt{1-\zeta^2}) = \left\{ \frac{(-\zeta\nu + j\nu\sqrt{1-\zeta^2}) \omega_i \cos \omega_i \alpha + (-\zeta\nu + j\nu\sqrt{1-\zeta^2})^2 \sin \omega_i \alpha}{\zeta^2 \nu^2 + \omega_i^2 - \nu^2(1-\zeta^2) - j2\zeta\nu^2\sqrt{1-\zeta^2}} \right\} \\ = \frac{(-\zeta\nu + j\nu\sqrt{1-\zeta^2}) \omega_i \cos \omega_i \alpha + (-\zeta\nu + j\nu\sqrt{1-\zeta^2})^2 \sin \omega_i \alpha}{\nu^2(2\zeta^2 - 1) + \omega_i^2 - j2\zeta\nu^2\sqrt{1-\zeta^2}} \\ \left\{ \frac{\nu^2(2\zeta^2 - 1) + \omega_i^2 + j2\zeta\nu^2\sqrt{1-\zeta^2}}{\nu^2(2\zeta^2 - 1) + \omega_i^2 + j2\zeta\nu^2\sqrt{1-\zeta^2}} \right\} \\ = \frac{-\zeta\nu\omega_i[\zeta^2\nu^2 + \nu^2(1-\zeta^2) + \omega_i^2] \cos \omega_i \alpha}{[\nu^2(2\zeta^2 - 1) + \omega_i^2]^2 + 4\zeta^2\nu^2(1-\zeta^2)} \\ + \frac{\{[\zeta^2\nu^2 + \nu^2(1-\zeta^2)]^2 + \omega_i^2[\zeta^2\nu^2 - \nu^2(1-\zeta^2)]\} \sin \omega_i \alpha}{\nu^4(2\zeta^2 - 1) + 2\nu^2\omega_i^2(2\zeta^2 - 1) + \omega_i^4 + 4\zeta^2\nu^4(1-\zeta^2)} \\ + j\omega_i\nu\sqrt{1-\zeta^2} \frac{[\zeta^2\nu^2 + \nu^2(1-\zeta^2) - \omega_i^2] \cos \omega_i \alpha + 2\zeta\nu\omega_i \sin \omega_i \alpha}{\omega_i^4 + 2\omega_i^2\nu^2(2\zeta^2 - 1) + \nu^4} \\ = \frac{-\zeta\nu\omega_i(\nu^2 + \omega_i^2) \cos \omega_i \alpha + \nu^2[\nu^2 + \omega_i^2(2\zeta^2 - 1)] \sin \omega_i \alpha}{\omega_i^4 + 2\omega_i^2\nu^2(2\zeta^2 - 1) + \nu^4} \\ - j\nu\omega_i\sqrt{1-\zeta^2} \frac{(\nu^2 - \omega_i^2) \cos \omega_i \alpha + 2\zeta\nu\omega_i \sin \omega_i \alpha}{\omega_i^4 + 2\omega_i^2\nu^2(2\zeta^2 - 1) + \nu^4} \quad (A-14)$$

Also

$$P_{r1} = \left(\frac{V_{mi}}{L_1} \right) \frac{\left[\left(\frac{\omega_i}{\gamma} \right)^2 (2\zeta^2 - 1) + 1 \right] \sin \left[\left(\frac{\omega_i}{\gamma} \right) \gamma \alpha \right] - \zeta \left(\frac{\omega_i}{\gamma} \right) \left[\left(\frac{\omega_i}{\gamma} \right)^2 + 1 \right] \cos \left[\left(\frac{\omega_i}{\gamma} \right) \gamma \alpha \right]}{\left(\frac{\omega_i}{\gamma} \right)^4 + 2(2\zeta^2 - 1) \left(\frac{\omega_i}{\gamma} \right)^2 + 1}, \quad (A-15)$$

and

$$P_{j1} = \left(\frac{V_{mi}}{L_1} \right) \left(-\sqrt{1 - \zeta^2} \right) \left(\frac{\omega_i}{\gamma} \right) \frac{2\zeta \left(\frac{\omega_i}{\gamma} \right) \sin \omega_i \alpha - \left[\left(\frac{\omega_i}{\gamma} \right)^2 - 1 \right] \cos \omega_i \alpha}{\left(\frac{\omega_i}{\gamma} \right)^4 + 2(2\zeta^2 - 1) \left(\frac{\omega_i}{\gamma} \right)^2 + 1}. \quad (A-16)$$

is determined below.

$$P_2 (-\zeta \gamma + j\sqrt{1 - \zeta^2}) = -(\zeta \gamma L_0 + \frac{V_0}{L_1}) + j\gamma \sqrt{1 - \zeta^2} L_0 = P_{r2} + jP_{j2}. \quad (A-17)$$

Now

$$P_r = P_{r1} + P_{r2} \quad \text{and} \quad P_j = P_{j1} + P_{j2}. \quad (A-18, 19)$$

Therefore

$$i_2(t) = \left(\frac{P_j}{\gamma \sqrt{1 - \zeta^2}} \cos \sqrt{1 - \zeta^2} \gamma t + \frac{P_r}{\gamma \sqrt{1 - \zeta^2}} \sin \sqrt{1 - \zeta^2} \gamma t \right) e^{-\zeta \gamma t} \quad (A-20)$$

where

$$\begin{aligned} \frac{P_r}{\gamma \sqrt{1 - \zeta^2}} &= V_{mi} \sqrt{\frac{a_1^2 c_1}{L_1}} \frac{\left[\left(\frac{\omega_i}{\gamma} \right)^2 (2\zeta^2 - 1) + 1 \right] \sin \omega_i \alpha - \zeta \left(\frac{\omega_i}{\gamma} \right) \left[\left(\frac{\omega_i}{\gamma} \right)^2 + 1 \right] \cos \omega_i \alpha}{\sqrt{1 - \zeta^2} \left[\left(\frac{\omega_i}{\gamma} \right)^4 + 2(2\zeta^2 - 1) \left(\frac{\omega_i}{\gamma} \right)^2 + 1 \right]} \\ &= \frac{\zeta L_0 + V_0 \sqrt{\frac{a_1^2 c_1}{L_1}}}{\sqrt{1 - \zeta^2}}. \end{aligned} \quad (A-21)$$

Also,

$$\begin{aligned} \frac{P_{j1}}{\gamma \sqrt{1 - \zeta^2}} &= -V_{mi} \sqrt{\frac{a_1^2 c_1}{L_1}} \left(\frac{\omega_i}{\gamma} \right) \frac{\left[2\zeta \left(\frac{\omega_i}{\gamma} \right) \sin \omega_i \alpha - \left[\left(\frac{\omega_i}{\gamma} \right)^2 - 1 \right] \cos \omega_i \alpha \right]}{\left[\left(\frac{\omega_i}{\gamma} \right)^4 + 2(2\zeta^2 - 1) \left(\frac{\omega_i}{\gamma} \right)^2 + 1 \right]} \\ &+ L_0. \end{aligned} \quad (A-22)$$

Combining Equations A-12, A-20, A-21, and A-22 gives the current Equation of A-23.

$$\begin{aligned}
i(t) = & \frac{V_{mi}}{\sqrt{\frac{L_1}{a^2 c_1}}} \frac{\left(\frac{\omega_i}{\nu}\right)}{\left[\left(\frac{\omega_i}{\nu}\right)^4 + 2(2\tau^2 - 1)\left(\frac{\omega_i}{\nu}\right)^2 + 1\right]} \left\{ \left[1 - \left(\frac{\omega_i}{\nu}\right)^2\right] \cos\left[\left(\frac{\omega_i}{\nu}\right)(\nu t + \nu\alpha)\right] \right. \\
& + 2\tau\left(\frac{\omega_i}{\nu}\right) \sin\left[\left(\frac{\omega_i}{\nu}\right)(\nu t + \nu\alpha)\right] + \left\{ \left[\frac{\left(\frac{\omega_i}{\nu}\right)(2\tau^2 - 1) + \left(\frac{\omega_i}{\nu}\right)}{\sqrt{1 - \tau^2}} \sin\left(\frac{\omega_i}{\nu}\right)\nu\alpha \right. \right. \\
& - \frac{\tau\left[\left(\frac{\omega_i}{\nu}\right)^2 + 1\right]}{\sqrt{1 - \tau^2}} \cos\left(\frac{\omega_i}{\nu}\right)\nu\alpha - \frac{\tau \frac{L_0}{V_{mi}} \sqrt{\frac{L_1}{a^2 c_1}} + \left(\frac{V_0}{V_{mi}}\right) \left[\left(\frac{\omega_i}{\nu}\right)^4 + 2(2\tau^2 - 1)\right]}{\left(\frac{\omega_i}{\nu}\right) \sqrt{1 - \tau^2}} \\
& \left. \left. \left(\frac{\omega_i}{\nu}\right)^2 + 1\right] \sin\sqrt{1 - \tau^2}\nu t + \left[\left(\frac{\omega_i}{\nu}\right)^2 - 1\right] \cos\left(\frac{\omega_i}{\nu}\right)\nu\alpha \right. \right. \\
& - 2\tau\left(\frac{\omega_i}{\nu}\right) \sin\left(\frac{\omega_i}{\nu}\right)\nu\alpha + \frac{L_0}{V_{mi}} \sqrt{\frac{L_1}{a^2 c_1}} \frac{\left[\left(\frac{\omega_i}{\nu}\right)^4 + 2(2\tau^2 - 1)\left(\frac{\omega_i}{\nu}\right)^2 + 1\right]}{\left(\frac{\omega_i}{\nu}\right)} \\
& \left. \left. \cos\sqrt{1 - \tau^2}\nu t \right\} e^{-\tau\nu t} \right\} \quad (A-23)
\end{aligned}$$

For generality, Equation A-23 is converted to per unit values as follows:

$$\begin{aligned}
I &= \frac{i}{V_{mi}} \sqrt{\frac{L_1}{a^2 c_1}} = \text{per unit current} \\
V_c &= \frac{V_0}{V_{mi}} = \text{per unit voltage} \\
\tau &= \nu t = \text{per unit time} \\
\theta &= \nu\alpha = \text{per unit phase angle}
\end{aligned} \quad (A-24)$$

In order to simplify notation, the following constants are defined:

$$B = \omega_i / \nu \quad (A-25)$$

$$Q = \sqrt{1 - \zeta^2} \quad (\text{A-26})$$

$$A_1 = \frac{(\frac{\omega_i}{\gamma})}{(\frac{\omega_i}{\gamma})^4 + 2(2\zeta^2 - 1)(\frac{\omega_i}{\gamma})^2 + 1} = \frac{B}{B^4 + 2(2\zeta^2 - 1)B^2 + 1} \quad (\text{A-27})$$

$$A_2 = 1 - (\frac{\omega_i}{\gamma})^2 = 1 - B^2 \quad (\text{A-28})$$

$$A_3 = 2\zeta(\frac{\omega_i}{\gamma}) = 2\zeta B \quad (\text{A-29})$$

$$\begin{aligned} A_4 &= \frac{(\frac{\omega_i}{\gamma})(2\zeta^2 - 1) + (\frac{\gamma^2}{\omega_i})}{\sqrt{1 - \zeta^2}} \sin(\frac{\omega_i}{\gamma})\theta - \frac{\zeta[(\frac{\omega_i}{\gamma})^2 + 1]}{\sqrt{1 - \zeta^2}} \cos(\frac{\omega_i}{\gamma})\theta \\ &\quad - (\zeta I_o + V_c) \frac{[(\frac{\omega_i}{\gamma})^4 + 2(2\zeta^2 - 1)(\frac{\omega_i}{\gamma})^2 + 1]}{(\frac{\omega_i}{\gamma})\sqrt{1 - \zeta^2}} \\ &= \frac{B(2\zeta^2 - 1) + 1/B}{Q} \sin B\theta - \frac{\zeta(B^2 - 1)}{Q} \cos B\theta - \frac{\zeta I_o + V_c}{A_1 Q} \end{aligned} \quad (\text{A-30})$$

$$\begin{aligned} A_5 &= -[1 - (\frac{\omega_i}{\gamma})^2] \cos B\theta - 2\zeta(\frac{\omega_i}{\gamma}) \sin B\theta + \frac{I_o[(\frac{\omega_i}{\gamma})^4 + 2(2\zeta^2 - 1)(\frac{\omega_i}{\gamma})^2 + 1]}{(\frac{\omega_i}{\gamma})} \\ &= -A_2 \cos B\theta - A_3 \sin B\theta + I_o/A_1 \end{aligned} \quad (\text{A-31})$$

Making these substitutions into Equation A-23 gives A-32.

$$\begin{aligned} I(\tau) &= A_1 [A_2 \cos B(\tau + \theta) + A_3 \sin B(\tau + \theta)] \\ &\quad + A_1 [A_4 \sin Q\tau + A_5 \cos Q\tau] e^{-\zeta\tau} \end{aligned} \quad (\text{A-32})$$

Equations A-32 and 3.10, page 45, are identical. Equations A-25 through A-31 are equal to Equations 3.14 through 3.20, pages 45 and 46.

The per unit voltage on C_1 referred to the primary of T_1 is given by

$$\begin{aligned}
 V(\tau) &= \frac{v_{mi}(\nu t)}{v_{mi}} = \frac{1}{v_{mi} a_1^2 c_1} \int i(\nu t) dt = \frac{v_{mi} \sqrt{\frac{a_1^2 c_1}{L_1}}}{v_{mi} a_1^2 c_1 \nu} \int \frac{i'(\nu t)}{\sqrt{\frac{a_1^2 c_1}{L_1}}} d\nu t \\
 &= \int I(\tau) d\tau \\
 &= A_1 \int [A_2 \cos B(\tau + \theta) + A_3 \sin B(\tau + \theta)] d\tau \\
 &\quad + A_1 \int [A_4 \sin Q\tau + A_5 \cos Q\tau] e^{-\xi\tau} \\
 &= \frac{A_1 A_2}{B} \sin B(\tau + \theta) - \frac{A_1 A_3}{B} \cos B(\tau + \theta) \\
 &\quad + \frac{A_1}{\xi} [(A_4)(Q A_5 - \xi A_4) \sin Q\tau + (-A_1)(Q A_4 + \xi A_5) \cos Q\tau] e^{-\xi\tau}
 \end{aligned} \tag{A-33}$$

Set

$$A_6 = \frac{A_1 A_2}{B} \tag{A-34}$$

$$A_7 = -\frac{A_1 A_3}{B} \tag{A-35}$$

$$A_8 = A_1 (Q A_5 - \xi A_4) \tag{A-36}$$

$$A_9 = -A_1 (Q A_4 + \xi A_5) \tag{A-37}$$

Making these substitutions into Equation A-33 yields

$$\begin{aligned}
 V(\tau) &= A_6 \sin B(\tau + \theta) + A_7 \cos B(\tau + \theta) \\
 &\quad + [A_8 \sin Q\tau + A_9 \cos Q\tau] e^{-\xi\tau}
 \end{aligned} \tag{A-41}$$

Equations A-34 through A-37 are the same as Equations 3.21 through 3.24, pages 45 and 46. Equation 3.11 is identical to Equation A-41.

The per unit flux is given by Equation A-42.

$$\begin{aligned}\Phi(\tau) &= \frac{N_1 V}{N_{mi}} \phi(v\tau) = \left(\frac{N_1 V}{N_{mi}}\right) \frac{1}{N_1} \int N(v\tau) d\tau = \left(\frac{V}{N_{mi}}\right) \left(\frac{N_{mi}}{V}\right) \int \frac{N(v\tau)}{N_{mi}} dV\tau \\ &= \int V(\tau) d\tau \\ &= -\frac{A_6}{B} \cos B(\tau + \theta) + \frac{A_7}{B} \sin B(\tau + \theta) + K_2 \\ &\quad + [(QA_9 - \zeta A_8) \sin Q\tau - (QA_8 + \zeta A_9) \cos Q\tau] e^{-\zeta\tau} \quad (A-42)\end{aligned}$$

Solving for K, using the initial conditions, we have

$$\Phi(0) = \Phi_0 = -\frac{A_6}{B} \cos B\theta + \frac{A_7}{B} \sin B\theta + K - (QA_8 + \zeta A_9) \quad (A-43)$$

$$K = \frac{A_6}{B} \cos B\theta - \frac{A_7}{B} \sin B\theta + (QA_8 + \zeta A_9) + \Phi_0 \quad (A-44)$$

For simplification, set

$$A_{11} = -A_6/B \quad (A-45)$$

$$A_{12} = A_7/B \quad (A-46)$$

$$A_{13} = QA_9 - \zeta A_8 \quad (A-47)$$

$$A_{14} = -(QA_8 + \zeta A_9) \quad (A-48)$$

$$A_{15} = \Phi_0 - (A_{11} \cos B\theta + A_{12} \sin B\theta + A_{14}) \quad (A-49)$$

Making these substitutions into Equation A-42, we have

$$\begin{aligned}\Phi(\tau) = & A_{11} \cos B(\tau + \theta) + A_{12} \sin B(\tau + \theta) \\ & + [A_{13} \sin Q\tau + A_{14} \cos Q\tau] e^{-\tau} + A_{15}.\end{aligned}\quad (A-50)$$

Equation A-50 is the same as Equation 3.12, page 45, and Equations A-45 through A-49 are the same as Equations 3.25 through 3.29, page 46.

INPUT EQUATIONS FOR T_1 SATURATED

For T_1 saturated the capacitor C_1 voltage and T_1 flux is given by

$$V(t) = 0 \quad ; \quad \Phi(t) = \pm \Phi_s \quad (A-51)$$

The differential equation for the equivalent circuit is shown by Equation 3.8, page 44, and is repeated below.

$$L \frac{di_L}{dt} + Ri = N_m \sin[\omega_i(t + \alpha)] \quad ; \quad L = L_T + L_{T1(sat)} \quad (A-52)$$

Taking the Laplace transform of Equation A-53 gives

$$(LS + R) i_L(s) = N_m \frac{\omega_i \cos \omega_i \alpha + s \sin \omega_i \alpha}{s^2 + \omega_i^2} + L i_0 \quad (A-53)$$

$$i_L(s) = \frac{N_m}{L} \frac{\omega_i \cos \omega_i \alpha + s \sin \omega_i \alpha}{(s + R/L)(s^2 + \omega_i^2)} + \frac{i_0}{s + R/L} \quad (A-54)$$

The roots of s are

$$s_1 = -R/L \quad ; \quad s_{2,3} = \pm j\omega_i \quad (A-55)$$

The inverse Laplace transform is taken by use of Heaviside's Expansion Theorems, and is shown below.

$$i_{L1} = \left[i_0 + \left(\frac{N_m}{L} \right) \frac{\omega_i \cos \omega_i \alpha - \frac{R}{L} \sin \omega_i \alpha}{(R/L)^2 + \omega_i^2} \right] e^{-\frac{R}{L}t} \quad (A-56)$$

$$i_{L2,3} = \frac{1}{\omega_i} [P_i \cos \omega_i t + P_r \sin \omega_i t] \quad (A-57)$$

where

$$P(-0 + j\omega) = P_r + jP_i$$

and

$$\begin{aligned} P(s) &= \frac{V_{mi}}{L} \frac{\cos \omega_i \alpha + \sin \omega_i \alpha}{s + R/L} \\ &= \frac{V_{mi}}{L} \frac{\frac{R}{L} \omega_i \cos \omega_i \alpha + 2\omega_i \sin \omega_i \alpha}{(R/L)^2 + \omega_i^2} \\ &\quad + j \frac{V_{mi}}{L} \frac{\frac{R}{L} \omega_i \sin \omega_i \alpha - \omega_i^2 \cos \omega_i \alpha}{(R/L)^2 + \omega_i^2} \end{aligned} \quad (A-58)$$

Combining Equations A-56, A-57, and A-58 gives

$$\begin{aligned} i(t) &= \frac{R V_{mi}}{L^2 \left[\left(\frac{R}{L} \right)^2 + \omega_i^2 \right]} \left\{ \left[\frac{L}{R} \omega_i \cos \omega_i \alpha - \sin \omega_i \alpha \right] e^{-\frac{R}{L} t} \right. \\ &\quad \left. + \sin \omega_i (t + \alpha) - \frac{L}{R} \omega_i \cos (t + \alpha) \right\} + i_0 e^{-\frac{R}{L} t} \end{aligned} \quad (A-59)$$

When T_1 is unsaturated L_1 includes the inductance of the charging reactor and the leakage inductance of T_1 . Because of the large ratio of unsaturated to saturated permeability of T_1 , the saturated inductance of T_1 is of the same order of magnitude as the leakage inductance of the unsaturated state. Therefore L of Equation A-59 is approximately equal to L_1 in Equation A-2. Using this approximation, we may write

$$\zeta = \frac{1}{2} R \sqrt{\frac{a_i^2 c}{L}} \quad ; \quad \nu = \frac{1}{\sqrt{a_i^2 c L}} \quad (A-60)$$

or

$$\frac{R}{L} = 2 \zeta \nu \quad ; \quad L \nu = \sqrt{L / a_i^2 c} \quad (A-61)$$

Then

$$\begin{aligned} \frac{R V_{mi}}{L^2 \left[\left(\frac{R}{L} \right)^2 + \omega_i^2 \right]} &= \frac{V_{mi} 2\zeta \gamma}{L [4\zeta^2 \gamma^2 + \omega_i^2]} = \frac{2\zeta \gamma V_{mi}}{L \gamma \left[\left(\frac{\omega_i}{\gamma} \right)^2 + 4\zeta^2 \right]} \\ &= \frac{2\zeta \gamma V_{mi} / \sqrt{L/a_1^2 C_1}}{\left(\frac{\omega_i}{\gamma} \right)^2 + 4\zeta^2} \end{aligned} \quad (A-62)$$

Equation A-59 becomes

$$\begin{aligned} i_L(t) &= \frac{2\zeta \gamma V_{mi} \sqrt{a_1^2 C_1}}{\left(\frac{\omega_i}{\gamma} \right)^2 + 4\zeta^2} \left\{ \left[\frac{\left(\frac{\omega_i}{\gamma} \right)}{2\zeta} \cos \left(\frac{\omega_i}{\gamma} \right) \gamma \alpha - \sin \left(\frac{\omega_i}{\gamma} \right) \gamma \alpha \right] e^{-2\zeta \gamma \tau} \right. \\ &\quad \left. + \sin \left[\left(\frac{\omega_i}{\gamma} \right) (\gamma \tau + \gamma \alpha) \right] - \frac{\left(\frac{\omega_i}{\gamma} \right)}{2\zeta} \cos \left[\left(\frac{\omega_i}{\gamma} \right) (\gamma \tau + \gamma \alpha) \right] \right\} \\ &\quad + I_0 e^{-2\zeta \gamma \tau} \end{aligned} \quad (A-63)$$

Converting to per unit values using Equation A-24, gives

$$\begin{aligned} I_L(\tau) &= -\frac{\left(\frac{\omega_i}{\gamma} \right)}{\left(\frac{\omega_i}{\gamma} \right)^2 + 4\zeta^2} \cos \left(\frac{\omega_i}{\gamma} \right) (\tau + \theta) + \frac{2\zeta}{\left(\frac{\omega_i}{\gamma} \right)^2 + 4\zeta^2} \sin \left(\frac{\omega_i}{\gamma} \right) (\tau + \theta) \\ &\quad + \left[\frac{\left(\frac{\omega_i}{\gamma} \right)}{\left(\frac{\omega_i}{\gamma} \right)^2 + 4\zeta^2} \cos \left(\frac{\omega_i}{\gamma} \right) \theta - \frac{2\zeta}{\left(\frac{\omega_i}{\gamma} \right)^2 + 4\zeta^2} \sin \left(\frac{\omega_i}{\gamma} \right) \theta + I_0 \right] e^{-2\zeta \tau} \end{aligned} \quad (A-64)$$

Set

$$A_{16} = \frac{\left(\frac{\omega_i}{\gamma} \right)}{\left(\frac{\omega_i}{\gamma} \right)^2 + 4\zeta^2} = \frac{B}{B^2 + 4\zeta^2} \quad (A-65)$$

and

$$A_{17} = \frac{2\zeta}{\left(\frac{\omega_i}{\gamma} \right)^2 + 4\zeta^2} = \frac{2\zeta}{B^2 + 4\zeta^2} \quad (A-66)$$

Making these substitutions in Equation A-64 yields

$$I_L(\tau) = -A_{16} \cos B(\tau + \theta) + A_{17} \sin B(\tau + \theta) \\ + [A_{16} \cos B\theta - A_{17} \sin B\theta + I.] e^{-2\tau} \quad (A-67)$$

Equation A-67 is the same as Equation 3.13 , page 45, and Equations A-65 and A-66 are the same as Equations 3.30 and 3.31, page 46. Equations 3.32 through 3.41, pages 46 and 47 are obtained from Equations A-4, A-5, and A-24. This concludes the derivation of the general input equations.

APPENDIX B

APPENDIX B

Pages 106 through 111 is the flow diagram for the computer program. Pages 112 through 116 is the computer Fortran statement, and pages 117 through 127 is a sample calculation. The sample calculation is for $\omega/\nu=0.999$, $\zeta_1=0.150$ and $\zeta_2=0.015$.

H	NAME TAYLOR	
H	EQUIP=PRINTE,CARDRE:	
	DIMENSION A(17)	0010
	READ 1000,RPHI,DP,NP,RC1,DC1,NC1,RC2,DC2,NC2,RB,DB,NB,DT	0020
	PHIS=RPHI	0030
	DO 775 IP=1,NP	0040
	B=RB	0050
	DO 760 IB=1,NB	0060
	C1=RC1	0070
	DO 725 IC1=1,NC1	0080
	C2=RC2	0090
	DO 700 IC2=1,NC2	0100
	GO TO 300	0110
100	Q1=SQRTF(1.-C**2)	0120
	Q2=2.*C**2-1.	0130
	Q3=SINF(THETA*B)	0140
	Q4=COSF(THETA*B)	0150
	A(1)=B/((B**2)*(B**2+2.*Q2)+1.)	0160
	A(2)=1.-B**2	0170
	A(3)=2.*C*B	0180
	A(4)=((B*Q2+1./B)*Q3-C*(B**2+1.)*Q4-(C*CUR+VC)/A(1))/Q1	0190
	A(5)=-A(2)*Q4-A(3)*Q3+CUR/A(1)	0200
	A(6)=A(1)*A(2)/B	0210
	A(7)=-A(1)*A(3)/B	0220
	A(8)=A(1)*(Q1*A(5)-C*A(4))	0230
	A(9)=-A(1)*(Q1*A(4)+C*A(5))	0240
	A(11)=-A(6)/B	0260
	A(12)= A(7)/B	0270
	A(13)=- (C*A(8)-A(9)*Q1)	0280
	A(14)=- (C*A(9)+A(8)*Q1)	0290
	A(15)=PHI0-(A(11)*Q4+A(12)*Q3+A(14))	0300
	GO TO KKK,(350,430)	0310
200	Q5=COSF((THETA+TAU)*B)	0320
	Q6=SINF((THETA+TAU)*B)	0330
	Q7=COSF(Q1*TAU)	0340
	Q8=SINF(Q1*TAU)	0350
	Q9=EXPF(-C*TAU)	0360
	A1=A(1)*(A(2)*Q5+A(3)*Q6+(A(4)*Q8+A(5)*Q7)*Q9)	0370
	AV=A(6)*Q6+A(7)*Q5+(A(8)*Q8+A(9)*Q7)*Q9	0380
	APHI=A(11)*Q5+A(12)*Q6+(A(13)*Q8+A(14)*Q7)*Q9+A(15)	0390

	P=TAU*THETA	0395
	PRINT 900,P,AL,AV,APHI	0400
	GO TO KKK,(360,440)	0410
300	TAU=DT	0420
	M=0	0430
	CUR=0	0440
	THETA=0	0450
	VC=0	0460
	PHIO=PHIS	0470
	C=C1	0480
320	IF(M=10)330,330,700	0490
C	BEGIN ALPHA ROUTINE *****	0500
330	M=M+1	0510
335	PRINT 910	0520
340	ASSIGN 350 TO KKK	0530
	PRINT 920,M,CUR,THETA,PHIO,C,B,VC	0540
	PRINT 925	0545
	GO TO 100	0550
350	ASSIGN 360 TO KKK	0560
	GO TO 200	0570
360	IF(AL)400,370,370	0580
370	IF(APHI-PHIS)380,380,480	0590
380	IF(APHI+PHIS)530,390,390	0600
390	TAU=TAU+DT	0610
	GO TO 200	0620
400	IF(M=10)410,410,700	0630
C	BEGIN BETA ROUTINE *****	0640
410	M=M+1	0650
	THETA=THETA+TAU	0660
	CUR=AL	0665
	VC=AV	0670
	PHIO=APHI	0680
	C=C2	0690
	TAU=DT	0700
420	PRINT 930	0710
425	PRINT 920,M,CUR,THETA,PHIO,C,B,VC	0720
	PRINT 925	0725
	ASSIGN 430 TO KKK	0730
	GO TO 100	0740
430	ASSIGN 440 TO KKK	0750

	GO TO 200	0760
440	IF(AI)450,450,540	0770
450	IF(APHI+PHIS)560,460,460	0780
460	IF(APHI-PHIS)470,470,600	0790
470	TAU=TAU+DT	0800
	GO TO 200	0810
C	BEGIN GAMMA ROUTINE *****	0820
480	THETA=THETA+TAU	0830
	TAU=DT	0840
	CUR=AI	0850
	VC=0	0851
	PHIO=PHIS	0852
	C=C1	0860
	Q10=B**2+4.*C**2	0870
	A(16)=B/Q10	0880
	A(17)=2.*C/Q10	0890
	PRINT 940	0900
	PRINT 920,M,CUR,THETA,PHIO,C,B,VC	0910
	PRINT 925	0915
490	AIL=(A(16)*COSF(B*THETA)-A(17)*SINF(B*THETA)*CUR)*EXPF(-2.*CTAU)	0920
	1A(16)*COSF((THETA+TAU)*B)+A(17)*SINF((THETA+TAU)*B)	0930
	P=TAU+THETA	0935
	PRINT 900,P,AIL,VC,PHIO	0940
	IF(AIL)510,510,500	0950
500	TAU=TAU+DT	0960
	GO TO 490	0970
510	THETA=THETA+TAU	0980
	VC=0	0990
	CUR=AIL	1005
	PHIO=PHIS	1010
	C=C2	1020
	TAU=DT	1030
	IF(M-10)520,520,700	1040
520	M=M+1	1050
	GO TO 420	1060
C	BEGIN DELTA ROUTINE *****	1070
530	THETA=THETA+TAU	1080
	VC=0	1090
	PHIO=-PHIS	1100
	C=C1	1110

	CUR=AI	1120
	TAU=DT	1130
	PRINT 960	1140
	GO TO 340	1150
C	BEGIN ZEIA=ZETA ROUTINE *****	1160
540	THETA=THETA+TAU	1170
	C=C1	1180
	VC=AV	1190
	PHIO=APHI	1200
	CUR=AI	1205
	TAU=DT	1220
	IF(M-10)550,550,700	1230
550	M=M+1	1235
	PRINT 970	1240
	PRINT 920,M,CUR,THETA,PHIO,C,B,VC	1250
	PRINT 925	1255
	ASSIGN 350 TO KKK	1260
	GO TO 100	1270
C	BEGIN EPISLON ROUTINE *****	1280
560	THETA=THETA+TAU	1290
	CUR=AI	1300
	VC=0	1304
	PHIO=PHIS	1306
	TAU=DT	1310
	Q10=B**2+4.*C**2	1320
	A(16)=B/Q10	1330
	A(17)=2.*C/Q10	1340
	PRINT 980	1350
	PRINT 920,M,CUR,THETA,PHIO,C,B,VC	1360
	PRINT 925	1365
570	AIL=(A(16)*COSF(B*THETA)-A(17)*SINF(B*THETA)*CUR)*EXPF(-2.*CTAU)-	1370
1A(16)*COSE((THETA+TAU)*B)+A(17)*SINE((THETA+TAU)*B)		1380
	P=TAU+THETA	1385
	PRINT 900,P,AIL,VC,PHIO	1390
	IF(AIL)580,590,590	1400
580	TAU=TAU+DT	1410
	GO TO 570	1420
590	THETA=THETA+TAU	1430
	VC=0	1440
	PHIO=PHIS	1450

	C=C1	1460
	CUR=AIL	1470
	TAU=DT	1480
	GO TO 320	1490
C	BEGIN ETA ROUTINE *****	1500
600	THETA=THETA+YAL	1510
	CUR=AI	1520
	VC=0	1530
	PHIQ=PHIS	1540
	TAU=DT	1550
	PRINT 990	1560
	GO TO 425	1570
700	C2=C2+DC2	1580
725	C1=C1+DC1	1590
750	B=B+DB	1600
	IF(B-1.)760,75C,760	1610
760	CONTINUE	1620
775	PHIS=PHIS+DP	1630
900	FORMAT(4F14.6)	1640
910	FORMAT(1H1,18H ALPHA ROUTINE***)	1660
920	FORMAT(X,3H= ,12,3X,6H1(0)= ,F10.6,3X,7HTHETA= ,F10.6,3X,8HPHI(0)	1670
	1= ,F10.6,/,9X,6HZETA= ,F7.3,3X,10HOMEGA/NU= ,F10.6,7X,4HV= ,F10.	1675
	26,///)	1680
925	FORMAT(6X,4HTIME,10X,7HCURRENT,7X,7HVOLTAGE,7X,4HFLUX,///)	1690
930	FORMAT(1H1,18H BETA ROUTINE****)	1700
940	FORMAT(1H1,18H GAMMA ROUTINE***)	1710
960	FORMAT(1H1,18H DELTA ROUTINE***)	1730
970	FORMAT(1H1,19H ZETA-ZETA ROUTINE)	1740
980	FORMAT(1H1,18H EPISLON ROUTINE*)	1750
990	FORMAT(1H1,18H ETA ROUTINE*****)	1760
1000	FORMAT(2F4.2,12,2F3.3,12,2F3.3,12,2F4.3,12,F5.4)	1770
	END	

ALPHA ROUTINE***

M= 1 I(0)= 0.000000 THETA= 0.000000 PHI(0)= -1.700000

ZETA= 0.150 OMEGA/NU= 0.999000 VCB= 0.000000

TIME CURRENT VOLTAGE FLUX

0.100000	0.004937	0.000165	-1.699996
0.200000	0.019456	0.001307	-1.699934
0.300000	0.042983	0.004357	-1.699671
0.400000	0.074779	0.010179	-1.698970
0.500000	0.113949	0.019558	-1.697516
0.600000	0.159463	0.033181	-1.694917
0.700000	0.210174	0.051624	-1.690719
0.800000	0.264835	0.075347	-1.684416
0.900000	0.322122	0.104679	-1.675463
1.000000	0.380659	0.139813	-1.663287
1.100000	0.439033	0.180805	-1.647305
1.200000	0.495824	0.227567	-1.626933
1.300000	0.549626	0.279870	-1.601606
1.400000	0.599065	0.337347	-1.570787
1.500000	0.642828	0.399494	-1.533981
1.600000	0.679677	0.465682	-1.490753
1.700000	0.708473	0.535161	-1.440735
1.800000	0.728193	0.607074	-1.383639
1.900000	0.737944	0.680467	-1.319271
2.000000	0.736983	0.754306	-1.247531
2.100000	0.724722	0.827487	-1.168431
2.200000	0.700746	0.898859	-1.082094
2.300000	0.664813	0.967237	-0.988759
2.400000	0.616868	1.031421	-0.888786
2.500000	0.557037	1.090215	-0.782655
2.600000	0.485633	1.142443	-0.670962
2.700000	0.403151	1.186972	-0.554423
2.800000	0.310267	1.222726	-0.433861
2.900000	0.207824	1.248706	-0.310204
3.000000	0.096830	1.264006	-0.184476
3.100000	-0.021560	1.267825	-0.057785

BETA ROUTINE***

M= 2 I(0)= -0.021560 THETA= 3.100000 PHI(0)= -0.057785

ZETA= 0.015 OMEGA/NU= 0.999000 VC= 1.267825

TIME	CURRENT	VOLTAGE	FLUX
3.200000	-0.148609	1.259390	0.068681
3.300000	-0.283736	1.237834	0.193655
3.400000	-0.425462	1.202423	0.315786
3.500000	-0.572147	1.152576	0.433658
3.600000	-0.722011	1.087887	0.545807
3.700000	-0.873156	1.008131	0.650733
3.800000	-1.023585	0.913280	0.746929
3.900000	-1.171233	0.803507	0.832892
4.000000	-1.313988	0.679196	0.907146
4.100000	-1.449722	0.540943	0.968266
4.200000	-1.576318	0.389556	1.014897
4.300000	-1.691700	0.226054	1.045773
4.400000	-1.793861	0.051657	1.059744
4.500000	-1.880891	-0.132214	1.055789
4.600000	-1.951009	-0.323957	1.033039
4.700000	-2.002583	-0.521798	0.990794
4.800000	-2.034163	-0.723807	0.928540
4.900000	-2.044499	-0.927922	0.845962
5.000000	-2.032567	-1.131964	0.742958
5.100000	-1.997585	-1.333666	0.619647
5.200000	-1.939033	-1.530695	0.476380
5.300000	-1.856663	-1.720679	0.313743
5.400000	-1.750515	-1.901235	0.132559
5.500000	-1.620917	-2.070000	-0.066111
5.600000	-1.468495	-2.224657	-0.280971
5.700000	-1.294172	-2.362968	-0.510497
5.800000	-1.099160	-2.482802	-0.752948
5.900000	-0.884961	-2.582161	-1.006375
6.000000	-0.653349	-2.659213	-1.268637
6.100000	-0.406358	-2.712317	-1.537419
6.200000	-0.146267	-2.740048	-1.810254

EPISILON ROUTINE*

M= 2 I(0)= -0.146267 THETA= 6.200000 PHI(0)= -1.700000

ZETA= 0.015 OMEGA/NU= 0.999000 VC= 0.000000

TIME	CURRENT	VOLTAGE	FLUX
6.300000	-0.149143	0.000000	-1.700000
6.400000	-0.142045	0.000000	-1.700000
6.500000	-0.125069	0.000000	-1.700000
6.600000	-0.098410	0.000000	-1.700000
6.700000	-0.062359	0.000000	-1.700000
6.800000	-0.017301	0.000000	-1.700000
6.900000	0.036290	0.000000	-1.700000

ALPHA ROUTINE***

M= 3 I(0)= 0.036290 THETA= 6.900000 PHI(0)= -1.700000

ZETA= 0.150 OMEGA/NU= 0.999000 VC= 0.000000

TIME	CURRENT	VOLTAGE	FLUX
7.000000	0.095862	0.006563	-1.699722
7.100000	0.160136	0.019329	-1.698481
7.200000	0.227659	0.038698	-1.695635
7.300000	0.296924	0.064919	-1.690512
7.400000	0.366389	0.098089	-1.682420
7.500000	0.434505	0.138152	-1.670665
7.600000	0.499740	0.184894	-1.654567
7.700000	0.560602	0.237954	-1.633475
7.800000	0.615658	0.296821	-1.606782
7.900000	0.663564	0.360847	-1.573939
8.000000	0.703078	0.429254	-1.534466
8.100000	0.733082	0.501146	-1.487971
8.200000	0.752600	0.575521	-1.434154
8.300000	0.760810	0.651289	-1.372821
8.400000	0.757060	0.727284	-1.303889
8.500000	0.740878	0.802286	-1.227397
8.600000	0.711977	0.875035	-1.143507
8.700000	0.670263	0.944254	-1.052508
8.800000	0.615840	1.008664	-0.954816
8.900000	0.549003	1.067008	-0.850977
9.000000	0.470243	1.118067	-0.741658
9.100000	0.380238	1.160682	-0.627645
9.200000	0.279844	1.193768	-0.509839
9.300000	0.170089	1.216338	-0.389242
9.400000	0.052158	1.227513	-0.266951
9.500000	-0.072623	1.226541	-0.144145

BETA ROUTINE***

M= 4 I(0)= -0.072623 THETA= 9.500000 PHI(0)= -0.144145

ZETA= 0.015 OMEGA/NU= 0.999000 VC= 1.226541

TIME	CURRENT	VOLTAGE	FLUX
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9.600000	-0.206757	1.212640	-0.022074
9.700000	-0.348231	1.184945	0.097923
9.800000	-0.495392	1.142804	0.214433
9.900000	-0.646436	1.085737	0.325986
10.000000	-0.799436	1.013451	0.431073
10.100000	-0.952359	0.925852	0.528166
10.200000	-1.103100	0.823052	0.615737
10.300000	-1.249501	0.705377	0.692280
10.400000	-1.389386	0.573369	0.756334
10.500000	-1.520584	0.427789	0.806502
10.600000	-1.640963	0.269613	0.841472
10.700000	-1.748457	0.100026	0.860044
10.800000	-1.841097	-0.079583	0.861143
10.900000	-1.917039	-0.267636	0.843845
11.000000	-1.974592	-0.462378	0.807393
11.100000	-2.012243	-0.661891	0.751211
11.200000	-2.028684	-0.864119	0.674924
11.300000	-2.022830	-1.066884	0.578369
11.400000	-1.993845	-1.267914	0.461605
11.500000	-1.941153	-1.464863	0.324922
11.600000	-1.864457	-1.655344	0.168848
11.700000	-1.763747	-1.836954	-0.005851
11.800000	-1.639311	-2.007302	-0.198168
11.900000	-1.491733	-2.164044	-0.406858
12.000000	-1.321901	-2.304907	-0.630447
12.100000	-1.131000	-2.427722	-0.867238
12.200000	-0.920504	-2.530453	-1.115322
12.300000	-0.692166	-2.611227	-1.372596
12.400000	-0.448007	-2.668359	-1.636779
12.500000	-0.190294	-2.700377	-1.905430

EPISLON ROUTINE*

M= 4 I(0)= -0.190294 THETA= 12.500000 PHI(0)= -1.700000

ZETA= 0.015 OMEGA/NU= 0.999000 VC= 0.000000

TIME	CURRENT	VOLTAGE	FLUX
12.600000	-0.191360	0.000000	-1.700000
12.700000	-0.182462	0.000000	-1.700000
12.800000	-0.163713	0.000000	-1.700000
12.900000	-0.135323	0.000000	-1.700000
13.000000	-0.097600	0.000000	-1.700000
13.100000	-0.050943	0.000000	-1.700000
13.200000	0.004158	0.000000	-1.700000

ALPHA ROUTINE***

M= 5 I(0)= 0.004158 THETA= 13.200000 PHI(0)= -1.700000

ZETA= 0.150 OMEGA/NU= 0.999000 VC= 0.000000

TIME	CURRENT	VOLTAGE	FLUX
13.300000	0.066126	0.003468	-1.699878
13.400000	0.132910	0.013386	-1.699091
13.500000	0.203015	0.030161	-1.696972
13.600000	0.274895	0.054048	-1.692822
13.700000	0.346975	0.085146	-1.685922
13.800000	0.417670	0.123397	-1.675554
13.900000	0.485419	0.168582	-1.661011
14.000000	0.548700	0.220332	-1.641619
14.100000	0.606058	0.278125	-1.616744
14.200000	0.656128	0.341300	-1.585814
14.300000	0.697649	0.409065	-1.548330
14.400000	0.729492	0.480507	-1.503878
14.500000	0.750669	0.554608	-1.452140
14.600000	0.760353	0.630258	-1.392905
14.700000	0.757891	0.706273	-1.326076
14.800000	0.742808	0.781415	-1.251679
14.900000	0.714825	0.854405	-1.169865
15.000000	0.673853	0.923947	-1.080913
15.100000	0.620005	0.988746	-0.985234
15.200000	0.553590	1.047529	-0.883365
15.300000	0.475113	1.099062	-0.775970
15.400000	0.385267	1.142172	-0.663833
15.500000	0.284926	1.175765	-0.547853
15.600000	0.175137	1.198842	-0.429031
15.700000	0.057104	1.210518	-0.308465
15.800000	-0.067829	1.210033	-0.187333

BETA ROUTINE****

M= 6 I(0)= -0.067829 THETA= 15.800000 PHI(0)= -0.187333

ZETA= 0.015 OMEGA/NU= 0.999000 VC= 1.210033

TIME	CURRENT	VOLTAGE	FLUX
15.900000	-0.202015	1.196609	-0.066889
16.000000	-0.343559	1.169385	0.051529
16.100000	-0.490789	1.127707	0.166506
16.200000	-0.641886	1.071098	0.276572
16.300000	-0.794905	0.999266	0.380218
16.400000	-0.947801	0.912121	0.475915
16.500000	-1.098455	0.809780	0.562135
16.600000	-1.244698	0.692577	0.637375
16.700000	-1.384342	0.561061	0.700173
16.800000	-1.515212	0.416001	0.749135
16.900000	-1.635170	0.258382	0.782954
17.000000	-1.742146	0.089400	0.800433
17.100000	-1.834173	-0.089549	0.800502
17.200000	-1.909408	-0.276875	0.782244
17.300000	-1.966166	-0.470814	0.744906
17.400000	-2.002940	-0.669442	0.687924
17.500000	-2.018434	-0.870693	0.610930
17.600000	-2.011574	-1.072383	0.513771
17.700000	-1.981538	-1.272235	0.396515
17.800000	-1.927769	-1.467900	0.259463
17.900000	-1.849986	-1.656988	0.103154
18.000000	-1.748199	-1.837096	-0.071635
18.100000	-1.622717	-2.005837	-0.263886
18.200000	-1.474148	-2.160870	-0.472345
18.300000	-1.303400	-2.299927	-0.695527
18.400000	-1.111681	-2.420850	-0.931726
18.500000	-0.900487	-2.521614	-1.179025
18.600000	-0.671596	-2.600358	-1.435315
18.700000	-0.427046	-2.655411	-1.698307
18.800000	-0.169126	-2.685321	-1.965559

EPISLON ROUTINE*

M= 6 I(0)= -0.169126 THETA= 18.800000 PHI(0)= -1.700000

ZETA= 0.015 OMEGA/NU= 0.999000 VC= 0.000000

TIME	CURRENT	VOLTAGE	FLUX
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18.900000	-0.168577	0.000000	-1.700000
19.000000	-0.158076	0.000000	-1.700000
19.100000	-0.137752	0.000000	-1.700000
19.200000	-0.107833	0.000000	-1.700000
19.300000	-0.068641	0.000000	-1.700000
19.400000	-0.020592	0.000000	-1.700000
19.500000	0.035811	0.000000	-1.700000

ALPHA ROUTINE***

M= 7 I(0)= 0.035811 THETA= 19.500000 PHI(0)= -1.700000

ZETA= 0.150 OMEGA/NU= 0.999000 VC= 0.000000

TIME	CURRENT	VOLTAGE	FLUX
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19.600000	0.097962	0.006646	-1.699720
19.700000	0.164497	0.019739	-1.698456
19.800000	0.233927	0.039642	-1.695545
19.900000	0.304714	0.066569	-1.690293
20.000000	0.375297	0.100578	-1.681994
20.100000	0.444110	0.141570	-1.669944
20.200000	0.509616	0.189290	-1.653456
20.300000	0.570322	0.243333	-1.631876
20.400000	0.624804	0.303147	-1.604597
20.500000	0.671733	0.368042	-1.571077
20.600000	0.709888	0.437201	-1.530846
20.700000	0.738180	0.509690	-1.483525
20.800000	0.755667	0.584476	-1.428832
20.900000	0.761568	0.660437	-1.366591
21.000000	0.755273	0.736383	-1.296745
21.100000	0.736360	0.811071	-1.219356
21.200000	0.704591	0.883226	-1.134615
21.300000	0.659929	0.951559	-1.042838
21.400000	0.602531	1.014787	-0.944473
21.500000	0.532749	1.071653	-0.840093
21.600000	0.451129	1.120942	-0.730395
21.700000	0.358405	1.161508	-0.616195
21.800000	0.255487	1.192283	-0.498420
21.900000	0.143454	1.212302	-0.378097
22.000000	0.023537	1.220711	-0.256347
22.100000	-0.102892	1.216792	-0.134366

BETA ROUTINE****

M= 8 I(0)= -0.102892 THETA= 22.100000 PHI(0)= -0.134366

ZETA= 0.015 OMEGA/NU= 0.999000 VC= 1.216792

TIME	CURRENT	VOLTAGE	FLUX
22.200000	-0.239129	1.199755	-0.013425
22.300000	-0.382314	1.168734	0.105118
22.400000	-0.530744	1.123118	0.219835
22.500000	-0.682573	1.062472	0.329241
22.600000	-0.835835	0.986555	0.431820
22.700000	-0.988472	0.895326	0.526041
22.800000	-1.138352	0.788953	0.610380
22.900000	-1.283307	0.667820	0.683339
23.000000	-1.421150	0.532528	0.743472
23.100000	-1.549717	0.383899	0.789400
23.200000	-1.666884	0.222965	0.819841
23.300000	-1.770606	0.050970	0.833624
23.400000	-1.858943	-0.130643	0.829714
23.500000	-1.930087	-0.320245	0.807229
23.600000	-1.982393	-0.516032	0.765459
23.700000	-2.014400	-0.716047	0.703882
23.800000	-2.024859	-0.918194	0.622178
23.900000	-2.012753	-1.120266	0.520245
24.000000	-1.977317	-1.319967	0.398204
24.100000	-1.918053	-1.514935	0.256410
24.200000	-1.834744	-1.702775	0.095455
24.300000	-1.727464	-1.881084	-0.083828
24.400000	-1.596588	-2.047481	-0.280365
24.500000	-1.442787	-2.199637	-0.492849
24.600000	-1.267036	-2.335307	-0.719743
24.700000	-1.070605	-2.452355	-0.959290
24.800000	-0.855052	-2.548790	-1.209526
24.900000	-0.622210	-2.622789	-1.468299
25.000000	-0.374174	-2.672725	-1.733282

EPYSLON ROUTINE

M= 8 I(0)= -0.374174 THETA= 25.000000 PHI(0)= -1.700000

ZETA= 0.015 OMEGA/NU= 0.999000 VC= 0.000000

TIME	CURRENT	VOLTAGE	FLUX
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25.100000	-0.381305	0.000000	-1.700000
25.200000	-0.378453	0.000000	-1.700000
25.300000	-0.365665	0.000000	-1.700000
25.400000	-0.343087	0.000000	-1.700000
25.500000	-0.310963	0.000000	-1.700000
25.600000	-0.269631	0.000000	-1.700000
25.700000	-0.219523	0.000000	-1.700000
25.800000	-0.161155	0.000000	-1.700000
25.900000	-0.095128	0.000000	-1.700000
26.000000	-0.022118	0.000000	-1.700000
26.100000	0.057128	0.000000	-1.700000

ALPHA ROUTINE***

M= 9 I(0)= 0.057128 THETA= 26.100000 PHI(0)= -1.700000

ZETA= 0.150 OMEGA/NU= 0.999000 VC= 0.000000

TIME	CURRENT	VOLTAGE	FLUX
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26.200000	0.138801	0.009781	-1.699579
26.300000	0.221435	0.027792	-1.697769
26.400000	0.303304	0.054043	-1.693746
26.500000	0.382705	0.088371	-1.686691
26.600000	0.457980	0.130446	-1.675813
26.700000	0.527540	0.179776	-1.660360
26.800000	0.589890	0.235714	-1.639638
26.900000	0.643649	0.297468	-1.613023
27.000000	0.687570	0.364115	-1.579981
27.100000	0.720562	0.434617	-1.540072
27.200000	0.741698	0.507832	-1.492967
27.300000	0.750236	0.582537	-1.438455
27.400000	0.745626	0.657441	-1.376453
27.500000	0.727518	0.731212	-1.307005
27.600000	0.695768	0.802490	-1.230293
27.700000	0.650440	0.869912	-1.146636
27.800000	0.591808	0.932134	-1.056484
27.900000	0.520349	0.987846	-0.960426
28.000000	0.436741	1.035799	-0.859174
28.100000	0.341852	1.074818	-0.753564
28.200000	0.236729	1.103828	-0.644544
28.300000	0.122585	1.121863	-0.533164
28.400000	0.000786	1.128089	-0.420565
28.500000	-0.127171	1.121815	-0.307963

BETA ROUTINE***

M= 10 I(0)= -0.127171 THETA= 28.500000 PHI(0)= -0.307963

ZETA= 0.015 OMEGA/NU= 0.999000 VC= 1.121815

TIME	CURRENT	VOLTAGE	FLUX
28.600000	-0.265106	1.102262	-0.196645
28.700000	-0.409418	1.068581	-0.087982
28.800000	-0.558283	1.020226	0.016582
28.900000	-0.709749	0.956838	0.115562
29.000000	-0.861753	0.878258	0.207443
29.100000	-1.012158	0.784540	0.290709
29.200000	-1.158771	0.675953	0.363855
29.300000	-1.299377	0.552986	0.425420
29.400000	-1.431769	0.416352	0.473997
29.500000	-1.553775	0.266979	0.508265
29.600000	-1.663293	0.106013	0.527006
29.700000	-1.758316	-0.065196	0.529126
29.800000	-1.836964	-0.245104	0.513676
29.900000	-1.897511	-0.431986	0.479872
30.000000	-1.938414	-0.623952	0.427110
30.100000	-1.958334	-0.818969	0.354980
30.200000	-1.956160	-1.014882	0.263286
30.300000	-1.931033	-1.209435	0.152049
30.400000	-1.882356	-1.400303	0.021522
30.500000	-1.809815	-1.585111	-0.127809
30.600000	-1.713385	-1.761469	-0.295219
30.700000	-1.593342	-1.927000	-0.479742
30.800000	-1.450263	-2.079369	-0.680180
30.900000	-1.285028	-2.216314	-0.895102
31.000000	-1.098818	-2.335676	-1.122857
31.100000	-0.893103	-2.435427	-1.361583
31.200000	-0.669637	-2.513704	-1.609226
31.300000	-0.430438	-2.568830	-1.863552

EPISLON ROUTINE*

M= 10 I(0)= -0.430438 THETA= 31.300000 PHI(0)= -1.700000

ZETA= 0.015 OMEGA/NU= 0.999000 VC= 0.000000

TIME	CURRENT	VOLTAGE	FLUX
31.400000	-0.435726	0.000000	-1.700000
31.500000	-0.431033	0.000000	-1.700000
31.600000	-0.416423	0.000000	-1.700000
31.700000	-0.392057	0.000000	-1.700000
31.800000	-0.358196	0.000000	-1.700000
31.900000	-0.315194	0.000000	-1.700000
32.000000	-0.263497	0.000000	-1.700000
32.100000	-0.203636	0.000000	-1.700000
32.200000	-0.136226	0.000000	-1.700000
32.300000	-0.061954	0.000000	-1.700000
32.400000	0.018422	0.000000	-1.700000

ALPHA ROUTINE***

M= 11 I(0)= 0.018422 THETA= 32.400000 PHI(0)= -1.700000

ZETA= 0.150 OMEGA/NU= 0.999000 VC= 0.000000

TIME	CURRENT	VOLTAGE	FLUX
32.500000	0.102286	0.006019	-1.699769
32.600000	0.187259	0.020494	-1.698514
32.700000	0.271582	0.043449	-1.695387
32.800000	0.353519	0.074731	-1.689547
32.900000	0.431381	0.114017	-1.680174
33.000000	0.503548	0.160817	-1.666493
33.100000	0.568499	0.214486	-1.647782
33.200000	0.624828	0.274229	-1.623393
33.300000	0.671268	0.339121	-1.592764
33.400000	0.706706	0.408116	-1.555432
33.500000	0.730201	0.480064	-1.511042
33.600000	0.740996	0.553733	-1.459361
33.700000	0.738532	0.627821	-1.400282
33.800000	0.722450	0.700985	-1.333828
33.900000	0.692601	0.771852	-1.260161
34.000000	0.649048	0.839048	-1.179580
34.100000	0.592062	0.901214	-1.092519
34.200000	0.522125	0.957029	-0.999549
34.300000	0.439916	1.005229	-0.901367
34.400000	0.346310	1.044632	-0.798796
34.500000	0.242360	1.074147	-0.692771
34.600000	0.129289	1.092800	-0.584329
34.700000	0.008470	1.099746	-0.474601
34.800000	-0.118590	1.094285	-0.364794

APPENDIX C

APPENDIX C

DEVELOPMENT OF DESIGN EQUATIONS

The following Equations were developed in Chapter II, and are listed here for reference.

$$V_{ci \max} = -a_1 \pi V_{mi} \quad (2.21)$$

$$\omega_n = \sqrt{\frac{C_n + C_{n+1}}{L_{sn} C_n C_{n+1}}} \quad (2.26)$$

$$V_{c(n+1)}(t) = \frac{C_n}{C_n + C_{n+1}} V_{cm} (1 - \cos \omega_n t) \quad (2.28)$$

$$V_{c(n+1) \max} = \frac{2 C_n}{C_n + C_{n+1}} V_{cm} \quad (2.29)$$

$$N_{n+1} A_{c(n+1)} = \frac{\pi C_n V_{cm}}{2 B_{m(n+1)} (C_n + C_{n+1}) \omega_n} \quad (2.31)$$

$$\frac{1}{2} \frac{R_L}{a_2} \sqrt{\frac{C_3}{L_{s4}}} = 1 \quad (2.36)$$

$$V_L(t) = 2 a_2 V_{cm3} \omega_4 t e^{-\omega_4 t} \quad (2.39)$$

$$\omega_4 = 1 / \sqrt{L_{s4} C_3} \quad (2.40)$$

$$V_L(t_m) = 2 V_{cm3} a_2 e^{-1} \quad (2.41)$$

$$\eta_1 = \frac{V_{ci \max}}{a_1 \pi V_{mi}} \quad (\text{Defined in Chapter VI, p56) (C-0)}$$

The per unit time of the output pulse is given by

$$\tau = \omega_4 t = t / \sqrt{L_{s4} C_3} \quad (C-1)$$

The half-amplitude pulse width is defined by t_p . Then the corresponding per unit pulse width is obtained from Equation C-1.

$$\tau_p = \sqrt{L_3 C_3} t_r$$

or

$$\frac{\tau_p}{\tau_r} = \sqrt{L_3 C_3} \quad (C-2)$$

where t_p is specified and τ_p is obtained from the per unit plot of the output pulse. An iteration formula may be used for a more accurate determination of τ_p , developed as follows:

set

$$\tau_p = \tau_2 - \tau_1 \quad ; \quad t_p = t_2 - t_1$$

where t_2 is the time at which the trailing edge of the output pulse decreases to $\frac{1}{2}$ amplitude, and t_1 is the time at which the leading edge reaches $\frac{1}{2}$ amplitude. The times are measured from the start of the output pulse. From Equation 2.41, converted to per unit values we may write

$$e^{-1} = 2\tau_1 e^{-\tau_1} = 2\tau_2 e^{-\tau_2} \quad (C-3)$$

and

$$f(\tau_n) = \tau_n e^{-\tau_n} - \frac{e^{-1}}{2} = 0 \quad (C-4)$$

For iteration, we have

$$f_{n+1} = f_n - f_n / f'_n \quad (C-5)$$

Thus

$$\begin{aligned} \tau_{n+1} &= - \frac{\tau_n - 0.5 e^{-1} e^{-\tau_n}}{1 - 0.5 e^{-1} e^{-\tau_n}} \\ &\approx \tau_n - \frac{\tau_n - 0.18394 e^{-\tau_n}}{1 - 0.18394 e^{-\tau_n}} \end{aligned} \quad (C-6)$$

Initial values of τ_n can be obtained from Figure 10, page 32. Two iterations gave

$$\tau_1 = 0.2320 \quad \text{and} \quad \tau_2 = 2.6783$$

Then

$$\tau_p = 2.4463 \approx 2.45$$

Solving Equation C-2 for L_{s4} gives

$$L_{s4} = \left(\frac{\epsilon_r}{\tau_r} \right) \frac{1}{c_3} \quad (\text{C-7})$$

Using Equation C-7 and solving Equation 2.36 for a_2 yields

$$a_2 = \left[\frac{R_L}{2} \sqrt{\frac{c_2}{L_{s4}}} \right]^{1/2} = \sqrt{\left(\frac{c_p}{\epsilon_p} \right) \left(\frac{c_3 p_L}{2} \right)} \quad (\text{C-8})$$

From Equation C-0, we may write

$$V_{e1 \text{ MAX}} = a_1 \pi \eta_1 V_{mi} \quad (\text{C-9})$$

Also, from Equation 2.29 and the energy transfer efficiencies we have

$$V_{e2 \text{ MAX}} = a_1 \pi \eta_1 \eta_2 V_{mi} \frac{2c_1}{c_1 + c_2} \quad (\text{C-10})$$

$$V_{e3 \text{ MAX}} = a_1 \pi \eta_1 \eta_2 \eta_3 V_{mi} \frac{4c_1 c_2}{(c_1 + c_2)(c_2 + c_3)} \quad (\text{C-11})$$

and using Equation 2.41, we get

$$V_{Lm} = a_1 \pi \eta_1 \eta_2 \eta_3 \eta_4 V_{mi} \frac{8c_1 c_2 a_2}{\epsilon (c_1 + c_2)(c_2 + c_3)} \quad (\text{C-12})$$

Solving Equation C-12 for a_1 and combining it with Equation C-8 gives

$$a_1 = \left(\frac{e}{8\pi} \right) \frac{(C_1 + C_2)(C_2 + C_3)}{\eta_1 \eta_2 \eta_3 \eta_4 C_1 C_2} \left(\frac{V_{mi}}{V_{mi}} \right) \sqrt{\frac{2 \tau_P}{C_2 C_3 Z_1}} \quad (C-13)$$

Equations C-8 and C-13 corresponds to Equations 4.1 and 4.2, page 53.

The inductance of a reactor is given by

$$L = \frac{4\pi A_c N^2 \mu 10^{-7}}{\ell} \quad (C-14)$$

Combining Equations C-14 and C-2 with 2.36, and solved for the turns gives

$$\begin{aligned} 1 &= \frac{R_L}{2a_2^2} \sqrt{\frac{C_2}{L_{s4}}} \frac{\left(\frac{\tau_P}{C_P} \right)}{\sqrt{L_{s4} C_3}} = \frac{R_L}{2a_2^2} \left(\frac{\tau_P}{C_P} \right) \frac{1}{L_{s4}} \\ \frac{1}{2} \frac{R_L}{a_2^2} \left(\frac{\tau_P}{C_P} \right) &= \frac{4\pi A_{c4} N_4^2 \mu_{s4} 10^{-7}}{\ell_4} \\ N_4 &= \left[\frac{\tau_P R_L \ell_4 10^7}{C_P 2a_2^2 4\pi A_{c4} \mu_{s4}} \right]^{1/2} \end{aligned} \quad (C-15)$$

From Equation 2.31 we may write

$$N_4 = \frac{\pi C_2 \eta_1 \eta_2 \eta_3}{2B_{m4}(C_2 + C_3) \eta_3} = \frac{a_1 \pi^2 \eta_1 \eta_2 \eta_3 V_{mi} C_1}{B_{m4}(C_1 + C_2)(C_2 + C_3)} \sqrt{\frac{C_2 + C_3}{L_{s3} C_2 C_3}} \quad (C-16)$$

Combining Equation C-14 with C-16 and solving for the turns of L_3 gives

$$N_3 = \frac{N_4 B_{m4} A_{c4} (C_1 + C_2)(C_2 + C_3)}{2\pi^2 \eta_1 \eta_2 \eta_3 C_1 C_2 a_1 V_{mi}} \left[\frac{(C_1 + C_2) \ell_3 10^7}{C_2 C_3 \pi A_{c3} \mu_{s3}} \right]^{1/2} \quad (C-17)$$

Similarly we may write

$$N_3 = \frac{2\pi C_1 \eta_1 \eta_2}{2B_{m3}(C_1 + C_2) \eta_2} = \frac{a_1 \pi^2 \eta_1 \eta_2 V_{mi}}{2B_{m3}(C_2 + C_1)} \sqrt{\frac{C_1 + C_2}{L_{s1}(\text{sw}) C_1 C_2}} \quad (C-18)$$

Combining Equations C-18 and C-14, and solving for N_{sT1} gives

$$N_{ST1} = \frac{N_3 B_{m3} A_{c2} (C_1 + C_2)}{\pi^2 \eta_1 \eta_2 a_1 v_{m1} C_1} \left[\frac{(C_1 + C_2) \mathcal{L}_2 10^7}{\pi A_{CT1} C_1 C_2 \mu_{ST1}} \right] \quad (C-19)$$

Then

$$N_{PT1} = N_{ST1} / a_1 \quad (C-20)$$

For resonant charging

$$\omega_i = \frac{1}{\sqrt{L_1 a_1^2 C_1}} \quad (C-21)$$

Solving for L_1 we obtain

$$L_1 = \frac{1}{a_1^2 C_1 \omega_i^2} \quad (C-22)$$

Also, from Equation C-14

$$N_1 = \left[\frac{L_1 \mathcal{L}_1 10^7}{4\pi A_{c1} \mu_1} \right]^{1/2} \quad (C-23)$$

Powdered Iron core manufactures usually list the inductance of a core with a given number of turns. If L_j is the inductance for N_j turns, then

$$N_1 = \left(\frac{L_1}{L_j} \right)^{1/2} N_j \quad (C-24)$$

For no saturation of the output transformer T_2 during the output pulse it is required that

$$N_2 > \frac{1}{2 B_{mT2} A_{CT2}} \int_0^\infty \mathcal{N}_L dt \quad (\text{one pulse}) \quad (C-25)$$

but

$$\mathcal{N}_L = 2 \eta_4 a_2 \eta_{c3} \gamma_4 t e^{-\gamma_4 t}$$

Therefore

$$N_{ST2} > \frac{2\pi a_1 a_2 \eta_1 \eta_2 \eta_3 \eta_c N_{mi}}{2B_{MT2} A_{CT2} \gamma_+} \int_0^{\infty} \gamma_+ \tau e^{-\gamma_+ \tau} d\gamma_+ \tau$$

$$N_{ST2} > \frac{\pi a_1 a_2 \eta_1 \eta_2 \eta_3 \eta_c \tau_p}{B_{MT2} A_{CT2} \tau_p} \quad (C-26)$$

Also

$$N_{PT2} = N_{ST2} / a_2 \quad (C-27)$$

Equations C-15, C-17, C-19, C-20, C-22, C-23, C-24, C-26, and C-27 correspond to Equations 4.3 through 4.10, pages 53 and 54.

The calculation of saturated permeability of the saturable reactor when connected in a discharge circuit may be done as follows:

- (1) Assume that the discharge current pulse approximates a $\frac{1}{2}$ sine wave, and that the "sine wave" period is inversely proportional to the resonant frequency of the discharge circuit.
- (2) Calculate the "sine wave" period from the $\frac{1}{2}$ amplitude discharge current pulse width, and from this calculate the resonant frequency of the discharge circuit.
- (3) From the known circuit parameters, calculate the saturated inductance of the saturable and the saturated permeability.

If the discharge circuit consists of two capacitors and the saturable reactor, and the $\frac{1}{2}$ amplitude current pulse width is τ_s , then the saturated permeability of the saturable reactor is given by Equations C-29 and C-30.

$$\mu_s = \frac{9 \epsilon_s^2 \ell \cdot 10^{-7}}{16 \pi^3 A_c \left(\frac{C_n C_{n+1}}{C_n + C_{n+1}} \right)} \quad (C-29)$$

and

$$\mu_m = \mu_s + i - \frac{A_{TOTAL}}{A_{CORE}} \quad (C-30)$$

APPENDIX D

APPENDIX D

Modulator operation at other conditions than given in Chapter V is given in Figures 29 through 32. Figure 29 shows waveforms of operation at a frequency ratio of 0.75 compared to computed waveforms. The computed waveforms are in per unit values, one unit per major division of the graph. All the experimental waveforms have the same calabrations and are as follows:

Trace (from Top)	Quantity	Vertical Calibration
1	B_{T1}	1.66 Webber/meter ² /division
2	V_1	325 Volts p-p
3	$V_{C1} + V_{C2}$	2000 Volt/div.
4	i_1	200 ma/div.
-	time	200 Sec/div.

The conditions of test are listed with each Figure.

Figure 33 is a Photograph of The Experimental Modulator.

TEST EQUIPMENT

EPUT METER: Beckman Model 5360

AUDIO OSCILLATOR: Hewlett Packard Model 200 CD

POWER AMPLIFIER: McIntosh 60 Watt Model MC-60 (Modified)

DUAL BEAM OSCILLOSCOPE: Tektronix Type 551

OPERATIONAL AMPLIFIED PLUG-IN: Tektrnix Type O

4 TRACE PREAMPLIFIER: Tektronix Type M

LOW VOLTAGE PROBES: Tektronix Type P 6006

HIGH VOLTAGE PROBES: 6 KV, 1000:1, Tektronix DC Probe

ISOLATION TRANSFORMER: $N_p = 300$ turns; $N_s = 340$ turns

NOTE: Power Amplifier feed back was adjusted to obtain an effective Generator impedance of 16 ohms at 115 volts RMS output.

EXPERIMENTAL TEST SET-UP

FIGURE 28

$$\zeta_1 = 0.25, \zeta_2 = 0.015$$

WAVEFORMS FOR $\omega/\nu = 0.75$
FIGURE 29

$R_B = 0, w/\gamma = 1$

R_B SLIGHTLY LESS THAN REQUIRED FOR UNIPOLAR OPERATION

EXPERIMENTAL WAVEFORMS OF BIPOLAR OPERATION

FIGURE 30

$\omega/\nu = 0.5$; $R_B = 250 \text{ ohm}$

$\omega/\nu = 1.50$, $R_B = 150 \text{ ohm}$

EXPERIMENTAL WAVEFORMS OF OPERATION OFF RESONANCE

FIGURE 31

TOP VIEW

BOTTOM VIEW

EXPERIMENTAL MAGNETIC MODULATOR

FIGURE 32