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INFORMAL REPORT

Optimization of the Superconducting
Switch Design in a Superconducting Magnetic
Energy Storage System

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OPTIMIZATION OF THE SUPERCONDUCTING SWITCH DESIGN IN A SUPERCONDUCTING MAGNETIC ENERGY STORAGE SYSTEM

BY

D. M. Weldon, J. D. G. Lindsay, H. L. Laquer

ABSTRACT

A superconducting magnetic energy storage system switched by normal-going superconductors was recently proposed by Ribe as part of an experiment demonstrating scientific feasibility of a thermonuclear reactor. In this proposal heating of the plasma was to be accomplished in 2 stages with a capacitor bank providing the initial shock heating and a magnetic energy storage system switched in later providing most of the heating by adiabatic compression. Each superconducting switch used with the magnetic energy storage system was to be placed in parallel with a protection resistor, operated at room temperature, whose resistance was much smaller than the normal state resistance of the switch. This was to prevent excessive energy loss in the cryogenic environment. It was later suggested by Garwin that if the protection resistor were removed and the mass of the switch reduced, the temperature change of the switch could be made very large so that the subsequent recooling of the switch could be done with higher thermodynamic efficiency. The relative merits of these two modes of operating a superconducting switch have been analyzed including the cost of superconducting materials, operating power for the refrigerator, and capital investment in the refrigerator. In general if both modes of operation are equally acceptable as far as operation of the switch is concerned, the net economic advantage, if any, to allowing the switch to make a large temperature excursion is highly dependent on the physical design of the switch. If proper opening and latching of the switch requires that its mass be small so that the first mode of operation is not acceptable, some form of refrigerator staging can reduce the overall cost of the system.

I. MAGNETIC ENERGY STORAGE

Figure 1 is a schematic representation of the magnetic energy storage system proposed by Ribe¹ as part of an experiment demonstrating scientific feasibility of a thermonuclear reactor. Energy is stored in this system by building up a persistent current in the loop $L_p R_{sw}$ with the switch R_{sw} in its superconducting state. Energy is released from storage by forcing R_{sw} normal, causing the flux decay in L_p to induce a current in secondary L_s and load coil L_c . It can be easily shown that under the condition $L_c = L_s$, the sum of the energy lost in resistors R_{sw} and R_p is given by

$$W_R = \frac{2-k^2}{2} W_{po} \quad (1)$$

where k is the coupling coefficient between L_p and L_s , ($M = k\sqrt{L_p L_s}$), and W_{po} is the energy originally stored in L_p .

Fig. 1. Schematic representation of a magnetic energy storage system. Cryogenic regions are enclosed in dashed lines.

Since $k \approx 1$, roughly half of the energy stored in L_p is lost during the transfer process. Details of the fusion experiment are not important to this discussion, but the general shape of the experiment is that of two tori with circumferences of about 180 meters; one torus comprises the energy storage system and the other the plasma containment device. Figure 2 shows a cross section of the experiment, and Table I lists the parameters of the energy storage system as dictated by the requirements of the fusion experiment. The storage system has been modularized in units 32 cm long each storing about 1.73 MJ. Since 562 of the modules described by Fig. 1 are required, relevant quantities have been listed for each module, for each meter of experiment and for the entire storage system.

Fig. 2. Conceptual view of a separated-shock θ -pinch.

TABLE I

PARAMETERS OF THE ENERGY STORAGE SYSTEM

k	= Coupling constant between L_s and $L_p = 0.92$
X	= $L_c/L_s = 1.0$
I_{po}	= Current in primary before transfer = 25,000 A
τ_s	= Transfer time of energy to secondary = 2×10^{-3} seconds
τ_p	= Time between each pulse of experiment = 15 minutes

PER METER OF LENGTH

W_{po}	= Energy stored in primary = 5.41 MJ
W_R	= Energy dissipated in R_{sw} and $P_p = 3.11$ MJ
W_c	= Energy delivered to load coil $L_c = 1.15$ MJ
N_m	= Number of modules in each meter = 3.125

PER MODULE

W'_{po}	= 1.73 MJ
W'_R	= 0.995 MJ
W'_c	= 0.368 MJ
R	= Resistance of R_{sw} and R_p in parallel = 1.59 ohms
l_p	= Length of each module = 32 cm
L_p	= 5.54 mH

FOR ENTIRE SYSTEM

W''_{po}	= 972 MJ
W''_R	= 559 MJ
W''_c	= 207 MJ
D_R	= Length of storage system = 180 meters
N_R	= Number of 32 cm modules in storage system = 562

II. THE SUPERCONDUCTING SWITCH

A. Case I $R_{sw} = 24.1$ ohms, $R_p = 1.70$ ohms

These are the parameter values suggested in the original proposal¹ with a large value of R_{sw} compared to R_p to cause most of the energy dissipated during switching to be deposited in R_p rather than R_{sw} . However, making $R_{sw} \gg R_p$ brings on other difficulties: The cost of the switch goes up, and it becomes more difficult to open it. The simplest way to open the switch is by discharging a small high voltage capacitor through it to exceed the critical current of the superconductor. Figure 3 shows a plot of the resistance developed in the first increase in resistance, (which takes place in 10 to 200 μ sec), against the voltage on the trigger capacitor for several representative materials. The resistance has been normalized to the full normal state resistance. From this graph it is clear that to develop a resistance of 90% of maximum, a voltage of 0.3 $I_C R_{sw}$ would be required in the trigger capacitor. Since R_{sw} is taken here as 24.1 ohms and from Table I, I_C can be taken as 25 kA, the voltage required on the trigger capacitor is 180 kV. Several things can be done to avoid the insulation problems brought about by these high voltages. The simplest solution is to put taps on the switch at equally spaced points along its length and trigger each section of switch from a separate capacitor bank. This adds some complexity to the construction of the switch,

Fig. 3. Normalized switch resistance plotted against the normalized applied trigger voltage for several switch materials.

but each section can be triggered by a lower voltage. Those sections triggered by a complete reversal of the current direction can be made to open only a fraction of a microsecond later than those in which the capacitor discharge adds to the previously flowing transport current. Other methods of opening the switch would be the application of an external magnetic field or radio-frequency heating of the switch. Not much is known of the problems associated with radio-frequency heating, but switching by application of a magnetic field has been used very successfully by Solé and his colleagues in France.^{3,4} It should be noted that even if the switch were triggered by current from a single capacitor, it could still be opened with smaller voltage than 180 kV, but that the switching time would be governed by thermal propagation and could be comparable to the required 2 msec transfer time. Table II lists the parameters of the 24.1 ohm switch. All of the other switch designs use the same material and are only changed in length to change resistance. The temperature change brought about by switching is calculated by assuming a mean density of 7.0 g/cm³ for the switch but using the enthalpy of copper to calculate the temperature rise. Inspection of Table V, which gives enthalpy as a function of temperature for various materials, shows that this is an acceptable approximation. If W_R is the total energy lost during switching in the resistance R_{sw} and R_p , the fraction lost in R_{sw} alone is given by,

$$W_{sw} = \frac{R_p}{R_p + R_{sw}} W_R \quad (1)$$

B. Case II $R_{sw} = 5.07$ ohms, $R_p = 2.32$ ohms

This case has been designed so that the final switch temperature is room temperature (300 K). Allowing the switch temperature after switching to rise up to room temperature or above was suggested to us by R. L. Garwin.² In this case as well as case III the temperature variation of the switch resistivity has been ignored since it is less than 20% and only the length changed to reduce the resistance. With the additional constraint set by the energy transfer time into the fusion experiment that the total resistance of R_{sw} and R_p in parallel is 1.59 ohms, scaling of the switch length

TABLE II

SWITCH DESIGN OF CASE I

R_{sw}	= Full normal state switch resistance = 24.1 ohms
R_p	= Resistance of protection resistance = 1.70 ohms
l_{sw}	= Length of switch = 357 meters
T_f	= Final temperature of switch = 66.1 K
h	= Specific enthalpy change due to switching = 3.52 J/g
ρ_N	= Resistivity of switch = 50 $\mu\text{ohm-cm}$
a	= Area ratio of matrix plus superconductor to superconductor = 1.5 (superconductor is NbTi and matrix is CuNi(30 w/o))
T_i	= Initial temperature of switch = 4K
δ	= Approximate mean density of switch 7 gm/cm ³
α	= Cost of superconductor per cm ³ = 1.9 \$/cm ³ *
j_{sc}	= Critical current density = 5×10^5 A/cm ² (in superconductor)
V_{sc}^{sw}	= Volume of superconductor in switch = 5.60×10^3 cm ³
V_{sw}	= Volume of switch = 8.40×10^3 cm ³
M_{sw}	= Mass of switch = 58.8 kg
C_{sw}	= Cost of switch = 10.64 k\$
W_{sw}	= Energy dissipated in switch = 2.06×10^5 J
$V_{sc}^{sw'}$	= 1.79×10^3 cm ³
V_{sw}'	= Volume of switch = 2.69×10^3 cm ³
M_{sw}'	= 18.82 kg
C_{sw}'	= 3.40 k\$
W_{sw}'	= 6.59×10^4 J
	PER METER OF EXPERIMENT
	PER MODULE (32 cm)
	FOR ENTIRE SYSTEM (180 meters, 562 modules)
$V_{sc}^{sw''}$	= 1.01×10^6 cm ³

* The cost of \$1.90 per cm² of NbTi contained in the fabricated conductor corresponds to \$150 per pound of contained NbTi and represents the best prices paid for relatively small quantities of conductor at the present time. Powell⁹ predicts large quantity costs lower by a factor of 4 or 5.

TABLE II cont.

$$V_{sw}' = 1.51 \times 10^6 \text{ cm}^3$$

$$M_{sw}'' = 1.06 \times 10^4 \text{ kg}$$

$$C_{sw}'' = 1.92 \times 10^3 \text{ k$}$$

$$W_{sw}'' = 3.71 \times 10^7 \text{ J}$$

(and hence resistance, mass, and cost) is given by

$$l_{sw} \approx \frac{\text{constant}}{\sqrt{h}} \quad (2)$$

where h is the specific enthalpy of the switch after switching. Table III lists the parameters for this switch except that the parameters which were identical to those of case I have been omitted.

To attain the maximum limiting current in the superconducting switch, it is important that the switch be held rigid, since motion of the conductor in a magnetic field leads to instabilities which drive the switch normal. Therefore the switch may have to be at least partly potted in plastic and held tightly to its winding form, which implies a considerable mass of material is in intimate contact with the switch. The question arises whether the heat transfer to the plastic will be so high that the temperature of the switch will be low during most of the refrigeration cycle. This would lead to removal of the heat at lower thermodynamic efficiency. The rate of heat transfer (Q) may be estimated from the equation,

$$Q = \frac{A}{L} \int_{4^\circ\text{K}}^{T_2} \lambda dT,$$

where A is the area of contact between the switch and winding form, T_2 is the highest temperature attained by the switch, λ is the thermal conductivity of the material of the winding form, and L is the distance across which the temperature gradient is developed. For L a distance equal to the switch thickness may be selected because if an amount of heat comparable to the heat initially deposited in the switch is transferred over this distance, it may be assumed that there is enough mass of supports

TABLE III

SWITCH DESIGN OF CASE II

R_{sw}	= Full normal state switch resistance = 5.07 ohms
R_p	= Resistance of protection resistance = 2.32 ohms
l_{sw}	= Length of switch = 75.1 m
T_f	= Final temperature of switch = 300 K
h	= Specific enthalpy change due to switching = 79.6 J/g

PER METER OF EXPERIMENT

V_{sc}^{sw}	= Volume of superconductor in switch = $1.17 \times 10^3 \text{ cm}^3$
V_{sw}	= Volume of switch = 1.77×10^3
M_{sw}	= Mass of switch = 12.4 kg
C_{sw}	= Cost of switch = 2.24 k\$
W_{sw}	= Energy dissipated in switch = $9.76 \times 10^5 \text{ J}$

PER MODULE (32 cm)

$V_{sc}^{sw'}$	= $3.76 \times 10^2 \text{ cm}^3$
V_{sw}'	= $5.66 \times 10^2 \text{ cm}^3$
M_{sw}'	= 3.96 kg
C_{sw}'	= 0.715 k\$
W_{sw}'	= $3.12 \times 10^5 \text{ J}$

FOR ENTIRE SYSTEM (180 meters)

$V_{sc}^{sw''}$	= $2.12 \times 10^5 \text{ cm}^3$
V_{sw}''	= $3.18 \times 10^5 \text{ cm}^3$
M_{sw}''	= $2.23 \times 10^3 \text{ kg}$
C_{sw}''	= $4.04 \times 10^2 \text{ k$}$
W_{sw}''	= $1.76 \times 10^8 \text{ J}$

included to lower the switch temperature significantly. From Table III the switch length is $7.51 \times 10^3 \text{ cm}$, and if the width is taken as 2 cm, the

area of contact with the form is $1.50 \times 10^4 \text{ cm}^2$.

This also implies that the cross-sectional area of the switch is $7.54 \times 10^{-2} \text{ cm}^2$ and hence $L = 3.77 \times 10^{-2} \text{ cm}$. Using $T_2 = 300^\circ \text{K}$ and Teflon as the form material gives

$$\int_4^{300} \lambda dT = .895 \frac{\text{watts}}{\text{cm}}$$

Then $Q = 3.56 \times 10^5$ watts and in a time of 450 seconds, (half the cycle time of the experiment), a total of $1.60 \times 10^8 \text{ J}$ is conducted from the switch. Also from Table II, the energy deposited in a single switch is only $3.12 \times 10^5 \text{ J}$. The calculation of heat transfer to the coil form could be too large because the temperature difference between the switch and its winding form is not 296 K for a full 450 seconds, the area of contact with the form might be overestimated, and the mass of form in a thickness L is less than that of switch in the same thickness because of the density difference. It is clear that heat transfer to the plastic form holding the switch and perhaps to trapped helium in the cooling channels surrounding it will be a major factor in designing the switch and refrigerator. The helium could be especially troublesome because of its relatively high specific heat at low temperatures. In later sections some attempt will be made to quantitatively estimate the effect of heat transfer from the switch to surrounding material.

Another related problem is the heating of the inside of the cryostat by the gas used to cool the higher temperature switch. A concrete design of the switch and cryostat is really needed to take this effect into account and for the present we ignore it, only noting that it is necessary to keep the mass of channels carrying the cooling gas around the switch to a minimum.

C. Case III. $R_{sw} = 3.29 \text{ ohms}$, $R_p = 3.08 \text{ ohms}$

In this case the temperature of the switch after switching is 573°K (300°C) which has the advantage that cooling down to room temperature can be done without any refrigerator whatsoever. If it were certain that the temperature of the superconductor never went above 300°C , there would be no danger of damaging it because the heat treatment

for increasing the critical current is in that neighborhood e.g. 370° C for 96 hours. However, somewhere above 400° C there is a metallurgical change which is deleterious to the critical current density, and localized heating could cause part of the switch to reach this temperature. A temperature of 300° C would also preclude the use of low melting point solders in connecting the switch. Table IV gives switch parameters for this case. It is worth noting that the total cost of the switch superconductor in this case is less than 14% of the cost of the superconductor in case I.

TABLE IV

SWITCH DESIGN OF CASE III

R_{sw}	= Full normal state switch resistance = 3.29 ohms
R_p	= Resistance of protection resistance = 3.08 ohms
l_{sw}	= Length of switch = 48.7 m
T_f	= Final temperature of switch = 573 K
h	= Specific enthalpy change due to switching = 189 J/g

TABLE IV cont.

PER METER OF EXPERIMENT	
V_{sc}^{sw}	= Volume of superconductor in switch = $7.64 \times 10^2 \text{ cm}^3$
V_{sw}	= Volume of switch = $1.15 \times 10^3 \text{ cm}^3$
M_{sw}	= Mass of switch = 8.02 kg
C_{sw}	= Cost of switch = 1.45 k\$
W_{sw}	= Energy dissipated in switch = $1.50 \times 10^6 \text{ J}$

PER MODULE (32 cm)

$V_{sc}^{sw'}$	= $2.44 \times 10^2 \text{ cm}^3$
V_{sw}'	= $3.67 \times 10^2 \text{ cm}^3$
M_{sw}'	= 2.57 kg
C_{sw}'	= 0.464 k\$
W_{sw}'	= $4.80 \times 10^5 \text{ J}$

FOR ENTIRE SYSTEM (180 meters)

$V_{sc}^{sw''}$	= $1.38 \times 10^5 \text{ cm}^3$
V_{sw}''	= $2.06 \times 10^5 \text{ cm}^3$
M_{sw}''	= $1.45 \times 10^3 \text{ kg}$
C_{sw}''	= $2.62 \times 10^2 \text{ k$}$
W_{sw}''	= $2.7 \times 10^8 \text{ J}$

TABLE V

ENTHALPY OF VARIOUS MATERIALS IN JOULES/g

T (°K)	Cu	Ni	Ti	Nb(Normal)	α-Fe	Teflon
3	6.4×10^{-5}	5.46×10^{-4}	3.29×10^{-4}	4.0×10^{-4}	4.12×10^{-4}	
4	1.3×10^{-4}	9.8×10^{-4}	5.99×10^{-4}	7.3×10^{-4}	7.42×10^{-4}	
6	4.4×10^{-4}	2.28×10^{-3}	1.45×10^{-3}	1.86×10^{-3}	1.73×10^{-3}	.006
8	1.12×10^{-3}	4.28×10^{-3}	2.81×10^{-3}	3.93×10^{-3}	3.23×10^{-3}	.02
10	2.4×10^{-3}	7.1×10^{-3}	4.89×10^{-3}	7.4×10^{-3}	5.37×10^{-3}	0.047
20	0.034	0.041	0.04	0.066	0.0316	0.52
30	0.195	0.145	0.182	0.28	0.110	1.54
40	0.610	0.413	0.581	0.80	0.31	2.99
50	1.40	0.937	1.36	1.63	0.73	4.83
60	2.58	1.79	2.59	2.76	1.43	7.02
70	4.13	3.00	4.27	4.2	2.46	9.59
80	6.02	4.56	6.37	5.8	3.84	12.5
100	10.6	8.63	11.69	9.6	7.56	19.5
120	16.1	13.76	18.24	13.8	12.4	27.9
140	22.1	19.7	25.69	18.3	18.2	37.7
160	28.5	26.3	33.84	23.1	24.6	49.0

TABLE V CONT.

T (°K)	Cu	Ni	Ti	Nb(Normal)	α-Fe	Teflon
180	35.3	33.4	42.54	28.0	31.7	61.7
200	42.4	40.8	51.66	33.1	39.2	75.9
260	64.4	65.0	80.82	48.6	63.6	126.
300	79.6	82.4	101.39	59.2	81.1	168.
400	119.					
500	159.					
600	200.					

Data in this table came from references 6 and 7.

TABLE VI
DENSITY AND ENTHALPY OF HELIUM GAS
AT 1 ATMOSPHERE PRESSURE

T (°K)	Density (milligrams/cm ³)	Enthalpy (J/g)
4.22	16.9	30.3
6	9.17	42.5
8	6.44	53.9
10	5.04	64.9
20	2.44	118.
30	1.62	170.
40	1.22	222.
50	0.973	274.
60	0.813	326.
70	0.697	378.
80	0.608	430.
100	0.488	534.
120	0.404	638.
140	0.348	742.
160	0.305	846.
180	0.271	950.
200	0.244	1053.
260	0.187	1365.
300	0.163	1573.

Data in this table comes from reference 8.

III. STAGING OF REFRIGERATORS

A. Operating Power.

Consider the situation in which N refrigerators operate between room temperature T_R (300K) and some lower temperature T_1 , each refrigerator cooling the switch in turn until T_N (4 K) is reached. Temperature T_0 and specific enthalpy h_0

correspond to switch condition just after switching. The total electrical power, Q (watts), required to cool the switch after each use can be written as,

$$Q = \frac{M_{sw}''}{\tau_p} \sum_{i=1}^N \frac{(h_{i-1} - h_i)}{\mu_i} \frac{(T_R - T_i)}{T_i}, \quad (1)$$

where M_{sw}'' is the mass of switches for the entire experiment in grams, h_i is the specific enthalpy of the switch in joules/gram at temperature T_i , τ_p is the cycle time of the experiment (900 seconds), and μ_i is the mechanical efficiency of the refrigerator. Strobbridge⁵ has given a graph of μ as a function of refrigerator capacity in watts for a number of large refrigerators. In the range,

$$10^2 \leq \frac{(h_{i-1} - h_i) M_{sw}''}{\tau_p} \leq 10^6, \quad ,$$

an adequate parametrization of this curve for our purposes is

$$\mu_i = 0.0738 \left[\frac{(h_{i-1} - h_i) M_{sw}''}{\tau_p} \right]^{0.11}. \quad (2)$$

Each refrigerator would work full time cooling a regenerator, and each regenerator would be connected in turn to the switch. By using a regenerator a smaller refrigerator is needed for each stage than if cooling had to be done during that fraction of 15 minutes allowed for cooling from temperature T_i to T_{i+1} .

B. Capital Investment.

1. Refrigerators. For estimating purposes, the cost of refrigerators operating in the range 1.8 to 90 K has been given by Strobridge⁵ as

$$C = 6000 P^{0.7}, \quad (3)$$

where C is the cost in dollars, and P is the installed input power in kilowatts. By estimating P for the i^{th} cooling stage we can write this equation as

$$C_i = 6000 \left[10^{-3} \times \frac{M_{sw}}{\mu_i} (h_{i-1} - h_i) \times \frac{(T_{R-i} - T_i)}{T_i} \right]^{0.7}. \quad (4)$$

Although the upper limit of equations 2 and 4 is 90K, we will use them at higher temperatures for lack of other readily available information for cost estimation.

2. Regenerators. To roughly estimate the costs and volumes involved for regenerators, we have found for each stage (i) the mass of iron whose temperature would change by 10% of the temperature difference $T_i - T_{i+1}$ if it absorbed the heat released by the switch in cooling from T_i to T_{i+1} . In Tables VII through XII this mass and the associated volume and cost is tabulated, with the cost figured on the basis of 0.06 \$/lb. for boiler plate. Metals are preferred for use in the regenerators because of their high thermal conductivity. Space for gas flow channels has been ignored in figuring the volume. The temperature of each stage was selected to minimize the total cost of the refrigerators ignoring the cost of the cold chest and regenerators. While this is a valid approximation when heat transfer to the winding form is neglected it sometimes fails badly when this effect is included since the cost of the regenerators might become even larger than the refrigerator cost. The main contribution to the high cost is the lowest temperature regenerators where plastics and helium gas have much higher specific heats than iron and huge masses of iron would therefore be required. Tables VIII, X, and XII exhibit this problem for the 3 switch lengths including an estimate of the mass of the winding form since the temperature selection of each stage was done ignoring the cost of

regenerators. There are several ways to reduce the high cost of regenerators. Temperature selection of each stage could be made to minimize the total cost of regenerators and refrigerators, or else the lowest temperature stage could be made a liquefier so cooling capacity would be stored in liquid helium. Alternatively, if the time allowed for the final stage cooling were known, the final stage refrigerator could be made large enough to cool the switch in this time.

IV. COMPARISON OF SYSTEMS

A. Case I Switch.

Table VII shows the switch of case I cooled by 1 to 3 stages of refrigeration and no other material in thermal contact with the switch. The temperatures selected are close to those giving an absolute minimum of capital investment in the refrigerators; the minimum itself is very broad so an integral number of degrees was chosen. As anticipated, there is a decrease in cost of the system as staging is used, going from 4.5×10^6 \$ for 1 stage to 1.6×10^6 \$ for 3. One point which is not obvious from the table is the decrease in operating power as soon as staging is used. For example, if we were to assume an operating time of 1000 hours per year and an electrical power cost of 0.02 \$/kw-hr, the power bill for 1 stage of refrigeration would be \$250,000 versus \$50,000 for 2 stages.

In Table VIII some attempt has been made to estimate the effect of the coil forms and trapped helium. A mass of plastic, taken here as teflon, equal to 3 times the mass of the switch and a mass of helium gas at 4.2 °K which would occupy 10 times the volume of the switch is assumed to be in such close contact with the switch that the equilibrium temperature is reached before any refrigeration can be accomplished. The specific heat of helium at constant pressure is used as a worst case. These are of course, pessimistic assumptions. In Table VIII, the tabulated quantity Δh is the enthalpy change in the metal conductor only, since the simplest way of performing the calculation is to compute an effective enthalpy including winding form and helium for the given mass of conductor. In comparing Tables VII and VIII it can be seen that although there is a substantial increase in the cost of the refrigerators when the extra mass is included, the

main increase is in the regenerators. Indeed the cost of regenerators becomes so high that it no longer pays to use them and direct refrigeration will be needed.

B. Case II Switch.

Table IX shows results for 1 to 4 stages of refrigeration used on the switch of case II with the capital costs minimized as in the previous section and no material in contact with the switch. Table X includes the plastic winding form and trapped helium in the same proportions used for the case I switch: 3 times the mass of the switch of plastic and 10 times its volume in helium at 4.2 K. In comparing the cost of case II with that of case I, the volume of superconducting material in the switches must also be taken into account. The cost of superconductor in case I is 1.92×10^6 \$ whereas for case II it is only 0.4×10^6 \$. Economic comparison of case I and case II switches depends critically on the amount of material in contact with the switch. With no extra material Tables VII and IX show refrigerators with regenerators will give substantial savings. Including a rough estimate for the winding form as in Tables VIII and X shows the case I switch with one stage of refrigeration has a distinct advantage over the case II switch even including cost of superconductor. On the other hand if proper switch operation were to require the less massive switch of case II, Table X shows staged refrigerators even with regenerators at all stages is cheaper.

C. Case III.

Table XI gives results for the case III switch with 1 to 4 stages of refrigeration and no extra mass in contact with the switch. No cost estimate has been made for the equipment needed to cool the switch from 573K to 300K. The savings in going from case II to case III with staged refrigerators are substantial but not as large as those from case I to case II. Total cost of superconductor for case III switches is 0.26×10^6 \$ versus 0.45×10^6 \$ for case II switches. Costs of total refrigeration systems for the same number of stages run 25% less for case III compared to case II.

When the winding form effect is included, (cf. Table XII), the case III switch still has an advantage over case II if 2 or more stages are used.

Moreover the cost, including superconductor, of case III with 4 stages about equals that of the case I switch with 1 stage. For 3 or fewer stages the case III switch still costs more than the case I switch with 1 stage of refrigeration.

V. ALTERNATIVE DESIGNS

As mentioned, detailed switch design greatly affects the efficiency of various cooling schemes. The present study was primarily motivated by problems associated with potting braided switches in plastic materials. Alternatives associated with braids immobilized by high resistivity solders or intermetallic diffusion bonds¹⁰ have not as yet been fully explored.

Another option would be to operate a switch made from Nb₃Sn or similar high temperature superconductors at elevated temperatures, say 14 K. In this case one would trade higher costs of superconducting materials due to lower but possibly more stable critical current densities against greatly reduced refrigerator costs due to higher Carnot efficiencies.

VI. CONCLUSION

Although the cost estimates made for various refrigerator systems have been very approximate, and more refined calculations must be made when the requirements of the switch design are better known, certain conclusions can still be drawn. If the resistance of each switch in the proposed θ -pinch experiment were reduced to a point where the temperature excursion after opening went to room temperature or above, there would be several immediate advantages. The cost of superconductor for the switches would go down and it would be easier to make them operate. Reducing the switch resistance while keeping the energy transfer time constant means more energy is lost in the switch, but the cost of the refrigeration system will not increase proportionately and may even decrease. This is because of the increased efficiency of refrigerators at higher temperatures. The exact cost difference for the entire experiment between using high and low resistance switches is very dependent on the mass of material in close thermal contact with the switch.

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TABLE VII

Refrigeration of case I switch. Temperature of switch after opening is 66.1 K.

Stage	T (K)	Refrigeration (W/W)	Efficiency (% of Carnot)	Δh (J/g)	Capacity (kW)	Input Power (MW)	Refrigerator Cost (M\$)	Regenerator Iron (m ³)	Iron Mass (Mg)	Iron Cost (k\$)
1	4.	74.0	23.8	3.52	41.2	12.8	4.51			
1	20.	14.0	23.7	3.486	40.8	2.41	1.40	129.	1020.	201
2	4.	74.0	14.3	.034	0.401	.208	.251	57.4	452	89.4
Totals				3.52	41.2	2.61	1.65	187.	1470.	291.
1	40.	6.5	23.3	2.91	34.1	.952	.730	35.6	280.	55.5
2	20.	14.0	19.5	.576	6.75	.485	.455	49.2	387.	76.7
3	4.	74.0	14.3	.034	.397	.206	.250	57.4	452.	89.4
Totals				3.52	41.2	1.64	1.44	142.	1120.	222.

TABLE VIII

Refrigeration of case I switch including winding form and trapped helium. Initial temperature = 19.3 K.

Stage	T (K)	Refrigeration (W/W)	Efficiency (% of Carnot)	Δh (J/g)	Capacity (kW)	Input Power (MW)	Refrigerator Cost (M\$)	Regenerator Iron (m ³)	Iron Mass (Mg)	Iron Cost (k\$)
1	4	74.0	23.8	3.52	41.2	12.8	4.51			
1	10	29.0	22.9	2.55	29.9	3.78	1.92	1390	11000	2170
2	4	74.0	20.6	.969	11.3	4.07	2.02	4380	34500	6820
Totals				3.52	41.2	7.85	3.93	5770	45400	8990

TABLE IX

Refrigeration of case II switch. Temperature of switch after opening is 300 K.

Stage	T (K)	Refrigeration (W/W)	Efficiency (% of Carnot)	Δh (J/g)	Capacity (kW)	Input Power (MW)	Refrigerator Cost (M\$)	Regenerator Iron (m ³)	Iron Mass (Mg)	Iron Cost (k\$)
1	4.	74.0	28.2	79.6	196.	51.3	11.9			
1	40.	6.5	28.2	79.0	194.	4.48	2.16	12.8	100.	19.9
2	4.	74.0	16.5	<u>0.610</u>	<u>1.50</u>	<u>.672</u>	<u>.572</u>	<u>78.3</u>	<u>616.</u>	<u>122.</u>
Totals				79.6	196.	5.15	2.73	91.1	717.	142.
1	100.	2.0	27.8	69.0	170.	1.22	.869	4.01	31.5	6.24
2	30.	9.0	22.5	10.4	25.6	1.02	.766	20.9	164.	32.5
3	4.	74.0	14.6	<u>.195</u>	<u>.479</u>	<u>.243</u>	<u>.281</u>	<u>38.1</u>	<u>300.</u>	<u>59.3</u>
Totals				79.6	196.	2.49	1.92	63.0	495.	98.1
1	130.	1.3	27.4	60.5	149.	0.711	0.595	3.31	26.0	5.15
2	50.	5.0	23.9	17.7	43.5	0.910	0.707	8.88	69.9	13.8
3	20.	14.0	18.0	1.37	3.36	0.261	0.295	16.3	128.	25.4
4	4.	74.0	12.0	<u>.034</u>	<u>.083</u>	<u>0.051</u>	<u>0.094</u>	<u>12.0</u>	<u>94.7</u>	<u>18.8</u>
Totals				79.6	196.	1.93	1.69	40.5	319.	63.2

TABLE X

Refrigeration of case II switch including winding form and trapped helium. Initial temperature = 98.9 K.

Stage	T (K)	Refrigeration (W/W)	Efficiency (% of Carnot)	Δh (J/g)	Capacity (kW)	Input Power (MW)	Refrigerator Cost (M\$)	Regenerator Iron (m ³)	Iron Mass (Mg)	Iron Cost (k\$)
1	4.	74.0	28.2	79.6	196	51.3	11.9			
1	23.	12.0	28.0	74.6	183	7.88	3.20	314.	2470.	490.
2	4.	74.0	20.8	<u>5.04</u>	<u>12.4</u>	<u>4.40</u>	<u>2.13</u>	<u>1510.</u>	<u>11900.</u>	<u>2350.</u>
Totals				79.6	196.	12.3	5.34	1820.	14300.	2840.
1	40.	6.5	27.6	65.4	161.	3.79	1.92	74.3	585.	116.
2	20.	14.0	22.6	10.5	25.7	1.60	1.05	499.	3930.	777.
3	4.	74.0	20.1	<u>3.70</u>	<u>9.09</u>	<u>3.34</u>	<u>1.76</u>	<u>1170.</u>	<u>9220.</u>	<u>1830.</u>
Totals				79.6	196.	8.73	4.72	1750.	13700.	2720.

TABLE X (continued)

Stage	T (K)	Refrigeration (W/W)	Efficiency (% of Carnot)	Δh (J/g)	Capacity (kW)	Input Power (MW)	Refrigerator Cost (M\$)	Regenerator Iron (m ³)	Iron Mass (Mg)	Iron Cost (k\$)
1	47.	5.38	27.3	60.1	148.	2.91	1.59	60.5	476.	94.3
2	20.	14.0	23.6	15.8	38.8	2.30	1.35	210.	1650.	327.
3	10.	29.0	19.5	2.73	6.71	1.00	0.755	292.	2300.	456.
4	4.	74.0	17.4	<u>.969</u>	<u>2.38</u>	<u>1.01</u>	<u>0.763</u>	<u>918.</u>	<u>7230.</u>	<u>1430.</u>
Totals				79.6	196.	7.22	4.47	1480.	11700.	2310.

TABLE XI

Refrigeration of case III switch from 300 K. Temperature of switch after opening in 573 K.

Stage	T (K)	Refrigeration (W/W)	Efficiency (% of Carnot)	Δh (J/g)	Capacity (kW)	Input Power (MW)	Refrigerator Cost (M\$)	Regenerator Iron (m ³)	Iron Mass (Mg)	Iron Cost (k\$)
1	4.	74.0	26.9	79.6	126.	34.8	9.06			
1	40.	6.5	26.8	79.0	125.	3.04	1.64	8.25	64.9	12.9
2	4.	74.0	15.7	<u>.610</u>	<u>.968</u>	<u>.456</u>	<u>.436</u>	<u>50.6</u>	<u>398.</u>	<u>78.8</u>
Totals				79.6	126.	3.49	2.08	58.8	463.	91.7
1	100.	2.0	26.4	69.0	110.	0.828	0.662	2.59	20.4	4.03
2	30.	9.0	21.5	10.4	16.5	0.692	0.584	13.5	106.	21.0
3	4.	74.0	13.9	<u>.195</u>	<u>.309</u>	<u>0.165</u>	<u>0.214</u>	<u>24.6</u>	<u>194.</u>	<u>38.3</u>
Totals				79.6	126.	1.69	1.46	40.7	320.	63.4
1	130.	1.3	26.1	60.5	96.0	0.482	0.453	2.14	16.8	3.33
2	50.	5.	22.8	17.7	28.1	0.617	0.539	5.74	45.1	8.94
3	20.	14.	17.2	1.37	2.17	0.177	0.224	10.5	83.0	16.4
4	4.	74.	11.4	<u>.034</u>	<u>.0538</u>	<u>0.035</u>	<u>0.072</u>	<u>7.78</u>	<u>61.2</u>	<u>12.1</u>
Totals				79.6	126.	1.31	1.29	26.2	206.	40.8

TABLE XII

Refrigeration of case III switch including winding form and trapped helium. Initial temperature = 157.1

Stage	T (K)	Refrigeration (W/W)	Efficiency (% of Carnot)	Δh (J/g)	Capacity (kW)	Input Power (MW)	Refrigerator Cost (M\$)	Regenerator Iron (m ³)	Iron Mass (Mg)	Iron Cost (k\$)
1	4.	74.0	29.5	189.	300.	75.1	15.5			
1	30.	9.00	29.4	180.8	287.	8.79	3.46	105.	823.	163.
2	4.	74.0	20.9	<u>8.16</u>	<u>13.0</u>	<u>4.58</u>	<u>2.19</u>	<u>1030.</u>	<u>8110.</u>	<u>1600.</u>
Totals				189.	300.	13.4	5.65	1130.	8930.	1770.
1	60.	4.00	29.0	158.	251.	3.47	1.80	28.7	226.	44.8
2	20.	14.	23.9	27.1	43.0	2.52	1.44	157.	1230.	244.
3	4.	74.0	19.2	<u>3.70</u>	<u>5.87</u>	<u>2.27</u>	<u>1.34</u>	<u>850.</u>	<u>6690.</u>	<u>1320.</u>
Totals				189.	300.	8.25	4.59	1030.	8140.	1610.
1	72.	3.17	28.7	145.	231.	2.55	1.45	22.4	176.	34.9
2	30.	9.00	24.6	35.5	56.3	2.06	1.25	75.9	597.	118.
3	10.	29.0	20.6	7.19	11.4	1.60	1.05	249.	1960.	388.
4	4.	74.0	16.5	<u>.969</u>	<u>1.54</u>	<u>.688</u>	<u>.581</u>	<u>593.</u>	<u>4670.</u>	<u>925.</u>
Totals				189	300.	6.90	4.34	941.	7400.	1470.