On the Mariscotti Description of Ground State Bands
of Even-Even Nuclei*

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Abstract

Mariscotti has shown that almost all ground state bands in even-even nuclei may be fit within the framework of the higher-order-cranking model. Three types of bands are observed; some follow the γ trajectory ($\mathcal{J}_0 > 0, C > 0$), others the α and α - β trajectories ($\mathcal{J}_0 < 0, C > 0$). We show that the first two trajectories are unique in that they yield bands with no upper cut-off for which the angular momenta and angular velocities are parallel, whereas the 2^+ state of the α - β trajectory has its angular momentum opposite to its angular velocity.

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In an earlier letter, Mariscotti¹ carefully examined all solutions of the higher-order-cranking equations²

$$E = \frac{1}{2} \omega^2 (\mathcal{J}_0 + 3C\omega^2) \quad (1)$$

and

$$[I(I+1)]^{\frac{1}{2}} = \omega(\mathcal{J}_0 + 2C\omega^2) \quad (2)$$

$\mathcal{J}_0$ and C are parameters characterizing a particular nucleus. ω is the semi-classical angular velocity of the nucleus (rotating as a whole). For a given spin I in the ground state band, (2) may be solved for ω . This value of ω is then substituted into (1) to yield E , the energy associated with the state I . (ω may be eliminated from (1) and (2) to yield a single equation which is cubic in E .) In Ref. 2, we originally limited ourselves to the analysis of strongly deformed nuclei for which $\mathcal{J}_0$ and C are both apparently positive. Although (2) is cubic in ω , only one real solution for ω exists with this choice of parameters, and a unique value of E is associated with each I .

Mariscotti considered the possibility that $\mathcal{J}_0$ may be negative for certain nuclei.³ When $\mathcal{J}_0$ and C have opposite signs, it is possible that three distinct values of ω will correspond to a given I , each leading to a different E . Figures 1 and 2 exhibit the dependence of E and ω on the parameter $d = 27 I(I+1)C/2\mathcal{J}_0^3$.⁴ In Ref. 1, the E vs. d curve is divided into three segments (trajectories). The α and β trajectories join at the point $d = 0$, $E = \mathcal{J}_0^2/8C$. The β and γ trajectories join at $d = -1$, $E = -\mathcal{J}_0^2/24C$. The γ trajectory passes through the origin. If $\mathcal{J}_0$ and C have the same sign, then $d \geq 0$ and there is a unique relation between E and I which may be determined from the γ trajectory. For ground state bands of even-even nuclei which satisfy the relation $2.231 \leq E(4)/E(2) \leq 3.333$, good

fits to the experimental spectra are obtained with $\mathfrak{J}_0$ and C positive.⁵

If $\mathfrak{J}_0$ and C have opposite signs, then three distinct solutions for E (one on each trajectory) are found for $-1 < d < 0$. For $d < -1$, only one solution exists (α trajectory). Mariscotti shows that the "vibrational" nuclei with $1.825 \leq E(4)/E(2) \leq 2.231$ seem to follow the α trajectory while singly and doubly closed-shell nuclei with $1 \leq E(4)/E(2) \leq 1.825$ seem to follow a hybrid trajectory called $\alpha\text{-}\beta$.⁶ In both cases, it is necessary to choose $\mathfrak{J}_0 < 0$ and $C > 0$. The $\alpha\text{-}\beta$ trajectory (or more precisely $\alpha\text{-}\beta\text{-}\gamma$) is made up of a 0^+ γ state, a 2^+ β state, and all higher spin states (4^+ , 6^+ , ...) on the α trajectory. We wish to point out in this note that some of the possible solutions correspond to states for which the angular momentum is anti-parallel to the angular velocity.

In deriving (1) and (2), we tacitly assume that $\langle J_x \rangle = [I(I+1)]^{\frac{1}{2}}$ so that the angular momentum direction is arbitrarily chosen. For a physical system it would seem reasonable to require that $\omega \geq 0$. This condition corresponds to the requirement that angular momentum and angular velocity be parallel.⁷ By comparing Figs. 1 and 2, it can be seen that any trajectory for which ω is negative has the undesirable property that E decreases as I increases. We see from Fig. 2 that for positive C , ω_γ is positive for positive $\mathfrak{J}_0$, and the only positive solution for negative $\mathfrak{J}_0$ is given by ω_α . Consequently, the physically observed α and γ trajectories both satisfy the condition of parallel angular momentum and angular velocity. When we consider Mariscotti's $\alpha\text{-}\beta$ trajectory, we see that the 2^+ state which lies on the β trajectory has negative ω , so that angular momentum and angular velocity are anti-parallel. If we define moment of inertia to be the ratio of angular momentum to angular velocity, we see that this state has a negative moment of inertia whereas all others

previously considered are positive. Such a state seems difficult to visualize, but understanding of this point is necessary if one is to have faith in this model.

The remaining possible choices of parameters have never been used in data fitting. If $\mathcal{J}_0$ and C are both negative, the only real solution leads to negative ω_Y . Positive $\mathcal{J}_0$ and negative C yields positive ω_B and ω_Y . In this case, however, solutions only exist for $-1 < d < 0$ so that any band corresponding to this choice of parameters would have a cut-off at some $I_{\max}$ unless states with negative ω are admitted. The β trajectory in this case has the peculiar property that ω decreases as I increases.

Footnotes

1. M. A. J. Mariscotti, Phys. Rev. Letters 24, 1242 (1970).
2. Samuel M. Harris, Phys. Rev. Letters 13, 663 (1964) and Phys. Rev. 138, B509 (1965).
3. This possibility was apparently first suggested by G. Scharff-Goldhaber (unpublished). See also G. Scharff-Goldhaber and A. S. Goldhaber, Phys. Rev. Letters 24, 1349 (1970).
4. This definition of d differs from that of Ref. 1 by the introduction of a factor of $27/2$.
5. The most extensive analysis of experimental bands with J_0 and C positive was performed within the framework of the VMI model, the results of which are equivalent to those of the higher-order-cranking picture. M. A. J. Mariscotti, G. Scharff-Goldhaber, and B. Buck, Phys. Rev. 178, 1864 (1969).
6. In the region $1.825 < E(4)/E(2) < 2.231$, some bands seem to lie between the α and the α - β trajectories.
7. These considerations can easily be seen to be valid in the classical analogue of P. Thieberger, Phys. Rev. Letters 25, 1664 (1970).

Figure Captions

Fig. 1. E (in units of $\mathcal{J}_0^2/8C$) vs. $d = 27 I(I+1)C/2\mathcal{J}_0^3$.

Fig. 2. ω (in units of $|\mathcal{J}_0/6C|^{1/2}$) vs. $d = 27 I(I+1)C/2\mathcal{J}_0^3$ for positive

C . For negative C , the vertical axis must be inverted.

Fig. 1

Fig. 2