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COMPUTING METHODS FOR ROVER REACTORS*

by

C. B. Mills

University of California
Los Alamos Scientific Laboratory
Los Alamos, New Mexico

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ABSTRACT

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The numerical approximations to the diffusion and transport equations for neutron flux in a reactor are written as intervals in space and angle for each set of connected energy intervals. The coarse-grained mesh that results is adequate for a wide variety of reactor calculations if sufficient care is used for energy-group averaged neutron absorption and transfer coefficients; and if the equations, as functions of space and angle, are expressed in intervals and with boundary conditions appropriate to the experimental assembly. Simple linear averaging of neutron flux and space intervals is satisfactory for many cases as is the use of equivalent simple spatial forms for the diffusion approximation. The greatest simplification is that of using a standard set of energy intervals, narrow enough that a simple average suffices for the energy variable in the integration of reaction rates over energy and volume. The conventional transport forms are also reviewed for anisotropic scattering cross sections and resonance integrals.

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Reducing an actual reactor experiment to a mathematically computable form requires the combination of theoretical and experimental approximations in ways specific to the experimental assembly. This procedure involves three steps: 1) homogenization and symmetrization of the geometrically coarse-grained assemblies, with changes in neutron cross section to effective values where appropriate; 2) weighting of moderator volume and reactor dimensions for equivalent neutron transport and slowing down; and 3) comparison of a systematic and simple experimental display to evaluate errors.

The computing system was applied to a high-power reactor experiment and proved to be entirely adequate in the area interest. Measurements of fission density as a function of space at high temperature corresponded well with the expected distribution.

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INTRODUCTION

In conjunction with the development of reactors for the Rover program, computing methods and critical experiments were prepared to evaluate the reactor systems. Extensive studies were made of the theoretical neutronics aspects, including diffusion, cross sections, and Doppler broadening. Then, a wide variety of previous, "computable" reactor experiments were analyzed. Essentially, such analyses consisted of describing each experiment in terms that are computable and then inspecting the integral results of such computations. Data from critical assemblies were adapted for use in the program; and, finally, the computing methods were applied to actual Rover reactors, including the KIWI A (discussed in the last section of this article) and subsequent reactors in the program.

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STATIC REACTOR NEUTRONICS

The equations of static reactor physics are connected with actual reactor experiments, by neutron cross sections, on one hand, and by experimental descriptions, on the other. Because neutron cross section data are neither complete nor exact, many approximations are required in the formation of constants for the equations of neutron flow in a medium. Also, reactor critical experiments are generally not described in ways consistent with the boundary conditions of reactor physics equations. Nevertheless, satisfactory computing systems and reactor experiments can be prepared as a part of reactor development programs.

The Equation of Neutron Diffusion

The neutron conservation equations equate integral neutron sources, S/k , and sinks, S , and compute the neutron multiplication, k , in each generation as

$$k = S/S \quad (1)$$

where S and S are operators; and k is the eigenvalue. If k is greater than unity, the total number of neutrons increases by this fraction for each integration. It is generally assumed that the number of neutrons is large, so that number fluctuation does not play a part in the analysis. Then, the simple number conservation equation is :

$$\Omega \cdot \text{grad } N + \sigma N = S \quad (2)$$

in which an eigenvalue equal to k , the neutron multiplication per

generation, is now in the source term to establish a time-independent form. ⁽¹⁾ Here, the neutron flux, N , is a function of energy, angle, and space, as is the integral source term. For spherical geometry, the form of the transport approximation to Eq. (2) is:

$$(\mu D_r + \frac{1 - \mu^2}{r} D_\mu + \sigma) N(E, r, \mu) = S(E, r) \quad (3)$$

where $N(E, r, \mu)$ is the neutron flux (neutrons/cm² sec) in energy dE at radius r in the direction θ ($\mu \equiv \cos \theta$) with respect to the positive r direction. ⁽²⁾ The neutron source term is independent of μ in the transport approximation and is represented by

$$S(E, r) = \sum_{E'} \sigma(E' \rightarrow E) N(E', r) + S(\text{fission}) \quad (4)$$

which indicates that fission and scattering from other energy points contribute to the total source at (E, r) . Eq. (3) relates the total source $S(E, r)$ to geometrical and nuclear effects (scattering and absorption). Note that if σ , the macroscopic total cross section, changes from generation to generation, the transport equation (Eq. 2) is nonlinear. The static condition is that of linearity.

The numerical approximations to μ and r are: ⁽²⁾

$$N(r, \mu) = \frac{\mu - \mu_{j-1}}{\mu_j - \mu_{j-1}} N(r, \mu_j) + \frac{\mu_j - \mu}{\mu_j - \mu_{j-1}} N(r, \mu_{j-1}) \quad (5)$$

for $j = 1, 2, \dots$, n intervals in $-1 < \mu < +1$. With $n = 4$ and $\mu = -1$, there are five equations. The μ intervals are chosen to be equal.

The integrals of $D_r N$ and N over the interval (r_{i-1}, r_i) are given by

$N_i - N_{i-1}$ and $\frac{1}{2} \Delta_i (N_i + N_{i-1})$ where $\Delta_i = r_i - r_{i-1}$. The various transport codes treat the flux rays (N) by somewhat different methods to generate satisfactory numerical approximations. Carlson⁽²⁾ assumed these simple forms

$$\int N(r, \mu) d\mu = \frac{1}{2} \ell_j [N(j) + M(j)] \quad (6)$$

$$\int \mu N(r, \mu) d\mu = \frac{1}{2} \ell_j [a_j N(j) + \bar{a}_j M(j)] \quad (7)$$

$$\int (1 - \mu^2) D_\mu N(r, \mu) d\mu = \frac{1}{2} \ell_j b_j [N(j) - M(j)] \quad (8)$$

where

$$a_j = (2\mu_j + \mu_{j-1})/3, \quad \bar{a}_j = (\mu_j + 2\mu_{j-1})/3 \quad (9)$$

and

$$b_j = \left(\frac{2}{3}\right) (3 - \mu_j^2 - \mu_j \mu_{j-1} - \mu_{j-1}^2) / (\mu_j - \mu_{j-1}), \quad (10)$$

where

$$N(j) \equiv N(r, \mu_j) \text{ and } M(j) \equiv N(r, \mu_{j-1}). \quad (11)$$

These transformations result in the stationary transport equation

$$(a D_r + \frac{b}{r} + \sigma)N + (\bar{a} D_r - \frac{b}{r} + \sigma)M = cS \quad (12)$$

where a , $\bar{a}$, and b depend on j , $j = 1, 2, \dots, n$; N and M , on j and r ; and S , on r , where $c \equiv 2$. Also, the flux at r is

$$N(r) = \frac{1}{2} \int_{-1}^1 N(r, \mu) d\mu = \sum_{j=0}^n p_j N(j) \quad (13)$$

where

$$p_0 = p_n = \frac{1}{2n} \text{ and } p_1 = \frac{1}{n}.$$

Transformation of Eq. (3) by numerical methods is the first step in reducing the continuity equation to a manageable form. This is done by, firstly, separating the energy variable, using a set of equations connected by neutron source terms, one for each small energy interval. Secondly, in the same way, the angle variable is converted to a set of equations. And, thirdly, the space variable is reduced to equation form by using an approximation for the flux gradient. A major, and satisfactory, simplification is the use of standard sets of energy and angle intervals: six to twenty-five energy intervals, depending on the range of $N(E)$; and one to sixteen angle intervals depending on flux anisotropy. The space variable is set to intervals specific to the geometry describing the boundary conditions required for neutron flow equations. The neutron energy range for reactor physics is $0 < E < 14$ Mev. Few problems require more than four angle intervals ($-1 < \mu < +1$). The number of space intervals necessary is on the order of 10 to 100. The numerical approach to the transport equation, discussed by many authors (3 - 7), is treated separately from the reactor physics problem.

The diffusion approximation for neutron transport and slowing down is treated in essentially the same way as the transport approximation. Inasmuch as the numerical multigroup diffusion approximation has received less attention than the transport approximation, the diffusion equations will be set down in greater detail for heuristic reasons. (See reference 8 for a more complete study.) The equation, as a function of energy and space, is:

$$-D \nabla^2 \varphi + \sigma_a \varphi = -\frac{\partial q}{\partial u} + \nu f \int_0^{u^{th}} \sigma_f \varphi du \quad (15)$$

Here D is the diffusion coefficient ($D = \frac{1}{3\sigma_{tr}}$); σ_a and σ_f are macroscopic absorption (removal from lethargy interval du) and fission cross sections, respectively; q is the slowing down density ($q = \xi \sigma_s \varphi$); ν is the eigenvalue ($k \equiv \nu_0/\nu$ where ν_0 is the neutron/fission integral); f is the fraction of the fission neutrons in du at lethargy u ($u = \ln E_0/E$ where $E_0 = 10^7$ ev); and φ is neutron flux per unit lethargy. Integrating over lethargy, with $u_2 - u_1 = U^1$, establishes group averaged neutron cross sections, σ^1 , for each lethargy group 1:

$$D \nabla^2 \varphi(r) - a\varphi(r) + b = 0 \quad (16)$$

where a sums the averaged neutron sinks at r in U^1 ; and b , the neutron sources. Then, with

$$\nabla^2 = \frac{1}{r^c} \frac{d}{dr} r^c \frac{d}{dr} \quad (17)$$

to include slabs, cylinders, and spheres ($c = 0, 1$, and 2 , respectively) in the geometrical part of the equation, and with the assumptions that for a small space interval, Δr ,

$$\frac{\varphi_{n+1} - 2\varphi_n + \varphi_{n-1}}{(\Delta r)^2} = \frac{d^2 \varphi(r)}{dr^2} \quad (18)$$

and

$$\frac{\varphi_{n+1} - \varphi_{n-1}}{2 \Delta r} = \frac{d\varphi(r)}{dr}, \quad (19)$$

the equation

$$\varphi_{n+1} = \frac{2 + a(\Delta r)^2 D}{1 + c \Delta r / 2r} \varphi_n - \frac{2r - c \Delta r}{2r + c \Delta r} \varphi_{n-1} - \frac{(\Delta r)^2}{1 + c \Delta r / 2r} \frac{b}{D} \quad (20)$$

describes the numerical form of $\varphi(r)$. The equation

$$\varphi_{n+1} = M_n \varphi_n - N_n \varphi_{n-1} - R_n \quad (21)$$

can be integrated from $\varphi_B = 0$ (the "extrapolated" boundary B) by assuming the form: (2)

$$\varphi_n = \alpha_n \varphi_{n+1} + \beta_n \quad (22)$$

at any space point n. Substituting and collecting terms in φ_n

$$\alpha_n = \left(M_n - N_n \alpha_{n-1} \right)^{-1} \quad (23)$$

and

$$\beta_n = \alpha_n \left(N_n \beta_{n-1} + R_n \right). \quad (24)$$

The boundary crossing conditions

$$\varphi(r) = \varphi'(r) \quad (25)$$

and

$$D \frac{d\varphi(r)}{dr} = \frac{D' d\varphi'(r)}{dr} \quad (26)$$

permit the formation of appropriate equations when crossing material interfaces. A Taylor expansion of the form

$$\varphi_1 = \varphi(\Delta r) = \varphi(0) + \Delta r \frac{d\varphi(0)}{dr} + \frac{(\Delta r)^2}{2} \frac{d^2\varphi(0)}{dr^2} \quad (27)$$

permits the formation of starting conditions to second order at the origin, with cylinders and spheres. The boundary conditions are φ (boundary of reactor) = 0; and $d\varphi(0)/dr = 0$ at the reactor center. With these equations, the numerical equations can be solved with one sweep across E and r (outward in r for α and β , inward for φ) for a given fission source distribution. A source depending dependent on $\varphi(u, r)$ requires several iterations, in order to stabilize on the integral fission source at r :

$$S(r) = \int_0^{u^{th}} \varphi(u, r) v \sigma_f(u, r) du. \quad (28)$$

The above procedure, briefly illustrating numerical methods of solving the neutron transport problem, shows that the converged solution (fully iterated over flux and fission neutron source density) is discrete and assumed to be first-order linear in space and intervals, and, in the case of the transport equations, also in angle intervals.

The magnitudes of angle, energy, and space intervals must be consistent

with the neutron flux gradients in these three variables. In addition, increased angular interval approximations must be compensated for by decreased space intervals in cases of more extreme flux anisotropy in angle.

The elementary number conservation equations of mathematical physics serve as the foundation for reactor physics. For static reactor, the forms are linear, simple integral-differential equations for which numerical methods provide direct solution on fast computing machines. The solutions are in the form of values of neutron flux in intervals ΔE , Δr , and $\Delta \mu$, in which the flux may vary in a first-order linear fashion. Flux weighting of the neutron cross sections provides eigenvalues directly useful in reactor evaluation, as well as a detailed knowledge of neutron sources and sinks.

Neutron Cross Sections

A knowledge of absorption and scattering cross sections⁽¹⁰⁾ as a function of energy, E , permits the integration of the reaction rate in a medium and, from this, the formation of average reaction cross sections in an energy interval appropriate to the multigroup equations for neutron diffusion. In the reactor systems of interest, neutrons are slowed down by moderator atoms in a discontinuous way. The energy change, $E_0 \rightarrow E$, due to elastic scattering is given as a function of the scattering angle, θ , by

$$E/E_0 = \frac{1}{(A + 1)^2} \left(1 + A^2 + 2A \cos \theta \right) \quad (29)$$

This equation results from the vector addition of velocities in the c-m system. The maximum fractional energy loss in the c-m system is $\alpha = (A - 1)^2 / (A + 1)^2$. Each isotropic elastic scattering of a neutron at energy E' is into an interval dE , with the probability for isotropic scattering in the c-m system being $\frac{dE}{E' - \alpha E'}$, where $E' - \alpha E'$ is the width of the energy interval into which the neutron is scattered. Accordingly, the interaction rate in a medium of atomic density N at energy E is $\sum_t \varphi$, where $\sigma_t = \sigma_a + \sigma_s$, and $\Sigma = N\sigma$, for moderator scattering only, is:

$$\sum (E) \varphi(E) = \int_E^{E/\alpha} \frac{\sum_s(E') \varphi(E')}{(1 - \alpha)E'} dE' \quad (30)$$

The form of the reaction rate $\sum_s \varphi$ is:

$$\sum_s \varphi(E) = Q/E \quad (31)$$

as may be seen by substituting this form into Eq. (30). Here, Q is clearly the neutron source strength. Then the interaction rate is:

$$N [\sigma_s(E) + \sigma_a(E)] \varphi(E) = N \sigma_s Q/E \quad (32)$$

where σ_s is the off-resonance scattering (above E). The effective reaction cross section is the total reaction rate divided by the neutron flux:

$$\bar{\sigma} = \frac{\int_{E_1}^{E_2} \frac{\sigma(E) dE}{\sum_t(E) E}}{\int_{E_1}^{E_2} \frac{dE}{\sum_t(E) E}} \simeq \frac{1}{\Delta u} \int_{E_1}^{E_2} \sigma(E) dE \quad (33)$$

The fission process results in a neutron energy distribution of the form: (11)

$$f(E) = aE^{\frac{1}{2}} \exp(-bE) \quad (34)$$

where, for U^{235} fission, $a = 1.872$ and $b = 1/1.290 \text{ Mev}^{-1}$. Generally, however, neutron scattering is not isotropic; the probability of scattering is specific to $\sigma(E, \mu)$ and has the form:

$$d \sigma_s(E \rightarrow E'; \mu) = \varphi(E) \sigma_s \left[\mu, E - E' \left(1 - \frac{2A(1 - \mu')}{(A + 1)^2} \right) \right] dE d\mu \quad (35)$$

where $\sigma_s(E \rightarrow E'; \mu)$ is the probability of an elastic scattering from E to E' ; $\varphi(E) = \frac{1}{E} \int f(E) dE$ is the neutron flux weighting function integrated over the fission spectrum; and $\sigma_s(\mu, \delta)$ is the angle elastic scattering function. Thus, the total $\sigma(E \rightarrow E')$ must be integrated over $E_1 - E_2$, (a source group width), into $E_1^1 - E_2^1$, the lower energy groups 1, with the sum of the scattering at E equal to the integral

$$\sigma_s(e) = \int_{-1}^1 \sigma_s(E, \mu) d\mu \quad (36)$$

The transport approximation assumes that

$$\sigma_{tr} = (1 - \bar{\mu}) \sigma_s \quad (37)$$

where

$$\bar{\mu} = \frac{1}{N} \int_{\Delta E} \int_{-1}^{+1} \mu \sigma_s^j(E, \mu) \varphi(E) d\mu dE \quad (38)$$

in which N is the normalizing integral, and $j = \pm 1$, depending on whether or not a Rosselind mean is desired for λ_g , the mean free path. Because of neutron flux variation in each group, it is more accurate to use the Wigner correction $\sigma_{tr} = (1 - \bar{\mu})(\sigma_s + \sigma_a)$. There are several different methods for making this correction, of which this is only one.

If anisotropic scattering effects are significant, the transport equation is solved by expanding the neutron flux and the cross sections in Legendre polynomials, according to a method developed by Lathrop.⁽¹²⁾ The corresponding neutron cross section is computed by using the successive Legendre polynomials as additional weights, with $p = 0, 1, \dots, 9$ for scattering in the c-m system. The result is a table of scattering probability, $\sigma_l^{i,j}$ for Legendre polynomial l , from group i into j . Note that all nuclei have significant scattering anisotropy above 1 Mev; the light elements, e.g. hydrogen and dueterium have this property at all energies. However, the transport approximation (a corrected isotropic scattering assumption) suffices for most cases of interest, with the exception of small, water-moderated or fast reactors.

The cross section for the resonant interaction of neutron and atom is given by :⁽¹³⁾

$$\sigma_a(E) = \frac{\pi \lambda_n^2 \Gamma_n \Gamma_\gamma}{(E - E_0)^2 + \Gamma^2/4} = \frac{2.606 \times 10^6 g}{E_0} \frac{\Gamma_n}{\Gamma^2} \sqrt{E/E_0} \frac{1}{4(E - E_0)^2/\Gamma^2 + 1} \quad (39)$$

$$\sigma_{rs} = \pi \lambda^2 \left| \frac{i\Gamma_n}{(E - E_0) + \frac{1}{2} i\Gamma} + 2ikR \right| = \sigma_0 \frac{\Gamma_n}{\Gamma} \frac{1}{4(E - E_0)^2/\Gamma^2 + 1} \quad (40)$$

$$+ \left(\sigma_0 \sigma_p g \frac{\Gamma_n}{\Gamma} \right)^{\frac{1}{2}} \frac{2(E - E_0)/\Gamma}{4(E - E_0)^2/\Gamma^2 + 1}$$

$$\sigma_r = \sigma_a + \sigma_{rs}; \quad (40)$$

where a is absorption, and rs is scattering, with $\sigma_0 = \frac{2.606 \times 10^6 g}{E_0 \Gamma} \Gamma_n$; and $\Gamma = \Gamma_n + \Gamma_\gamma$. The first expression in Eq. (40) is resonance scattering; the second, interference scattering. The integral over E of these functions may be changed significantly by temperature and other processes. Because neutron interactions with various materials are of interest, a detailed presentation of multigroup-averaged values, for many energy intervals, has been found to be most directly useful in the field of reactor physics.

The physical separation of absorber and moderator also reduces the absorption rate in resonances. If the geometry is that of a periodic lattice with wide separation ($d \gg \lambda$, where λ is the mean free path for scattering) the absorber may be called isolated. Neutrons are slowed

down in both regions, although with very different rates. For example, the logarithmic energy loss for hydrogen is 1; for carbon, 0.16 and for uranium 0.0084. The width of an absorption resonance may be small in comparison to the energy loss per collision (e.g., for U^{235} , $\Gamma_\gamma \approx 35$ mv; but $5 < \Gamma_f < 100$ mv, and $\overline{E} - E' \geq 8$ mv; therefore, a distinction can be made between narrow $\Gamma < (1 - \alpha)E$ and wide $\Gamma \gtrsim (1 - \alpha)E$ neutron absorption and scattering resonances. In the following equations, the narrow resonance analysis is designated NR; and the wide resonance, IM ("for infinite mass of the scatterer"). The effect of heterogeneity on light atom scattering resonance has been described by Bell.⁽¹⁴⁾ He separates the probability for neutron scattering, P_1 , into absorber ($i = 0$) and moderator ($i = 1$) and computes an effective resonance absorption cross section, $\tilde{\sigma}_a$, defined by

$$\tilde{\sigma}_a = \int \sigma_{a0}(E) \varphi_0(E) dE / \int \varphi_1(E) dE \quad (41)$$

where $\varphi_1(E) \approx Q/E$ above the resonance. Noting that a neutron that does not collide in absorber volume V_0 does collide in V_1 ,

$$(1 - P_0) \sum_{t0} V_0 \approx (1 - P_1) \sum_{s1} V_1 \quad (42)$$

and using Eq. 30 above for moderator and absorber separately, Bell presents a simple form for $\tilde{\sigma}_a$, the effective absorption:

$$\tilde{\sigma}_{ao} = \frac{1}{\Delta u} \int_{E_1}^{E_2} \frac{1 - P_o(E) \left(1 - \frac{\sigma_o}{\sigma_{to}(E)}\right)}{1 - P_o(E) \frac{\sigma_{so}^{IM}(E)}{\sigma_{to}(E)}} \sigma_{ao}(E) \frac{dE}{E} \quad (43)$$

where σ_o is the absorber narrow resonance off-resonance scattering cross section; σ_{so}^{IM} is the absorber infinite mass-scattering cross section; and σ_{to} is the total absorber cross section. Using the Wigner approximation for isolated lumps

$$P_o(E) = \frac{\sum_{to}(E)}{s + \sum_{to}(E)} \quad (44)$$

where s equals the lump surface divided by four times the lump volume, we find that:

$$\tilde{\sigma}_{ao} = \frac{1}{\Delta u} \int_{E_1}^{E_2} \frac{s/N_o + \sigma_o^{NR}}{s/N_o + \sigma_{to}^{NR}(E)} \sigma_{ao}(E) dE/E \quad (45)$$

where

$$\sigma_{to}^{NR}(E) = \sigma_{to}(E) - \sigma_{so}^{IM}(E). \quad (46)$$

Thus, an effective cross section, $\tilde{\sigma}$, can be used in a medium that treats the lumping effect as a change in absorber cross sections. In further work along this line, Bell derives a foil interaction effect on s for dense lattices, replacing s by the expression

$$\tau = \frac{s \sum_m}{\sum_m + s(v_o/v_1)} \quad (47)$$

where subscript m indicates moderator scattering, and (v_o/v_1) is the ratio of absorber to moderator volumes.

Doppler Broadening

Doppler broadening of the resonance lines has been studied by Devaney⁽¹⁵⁾ in a manner consistent with Bell's methods. The neutron velocity distribution function has the form

$$f(w_z, w_\rho) = \frac{2w_\rho}{\sqrt{\pi} T^{3/2}} \exp\left(-\frac{w_z^2 - w_\rho^2}{T}\right) \quad (48)$$

where $w_z = \sqrt{\frac{M}{2}} v_z$, and $w_\rho = \sqrt{\frac{M}{2}} v_\rho$. Here M is the mass of the atom; v is neutron velocity; T is the temperature of the atom, in electron volts; v is the laboratory velocity of the target atom, with v_z in the direction of the neutron velocity and v_ρ perpendicular to that direction.

The Breit-Wigner formula for resonance shape is

$$\sigma_a^i(e) = \pi g \lambda_1^2 \frac{e_i}{e} \frac{\Gamma_n^i \sqrt{\frac{e}{e_i}} \Gamma_a^i}{(e - e_i)^2 + \frac{1}{4} \left(\sqrt{\frac{e}{e_i}} \Gamma_n^i + \sum_a \Gamma_a^i \right)^2} \quad (49)$$

where e is energy, and i labels the resonance. The spin weight factor is $g = \frac{1}{2} \frac{2J + 1}{2I + 1}$, where I is the target atom spin; and $J = I \pm \frac{1}{2}$ is the

level spin, for $l = 0$ neutrons.

The average of the reaction cross section σ_r over the target atom velocity is expressed as

(50)

$$\sigma_r(E_n) = \frac{1}{v_n} \int_{-\infty}^{\infty} dv_z \int_0^{\infty} dv_{\rho} \cdot v \cdot f \cdot \left[\sigma_{roT} \sqrt{\frac{e_{oT}}{e}} + \sum_{i=1}^N \sigma_r^i(e) \right]$$

The first term in the bracket is the $1/v$ thermal variation; the second, the sum over N levels. Substituting

$$e = \frac{\mu}{M} \left[w_{\rho}^2 + \left(w_z - \sqrt{\frac{M}{m}} E_n^{1/2} \right)^2 \right] \quad (51)$$

into Eq. (50) (where μ is the reduced mass, and E_n is the laboratory energy of the neutrons) integrating over w_z where possible, and writing

$$A = \sqrt{\frac{ME_n}{mT}}, \quad B = \sqrt{\frac{M}{\mu T}}, \quad y = \sqrt{e}, \quad v_n = \sqrt{\frac{2E_n}{m}}, \quad \text{and} \quad \chi_1^2 = \frac{\hbar^2}{2\mu e_1} \quad (52)$$

yields:

$$\sigma_r = \sigma_{roT} \sqrt{\frac{e_{oT}}{e}} + \frac{\sqrt{\pi} g \hbar^2 T B^2}{2MA^2} \sum_{i=0}^N \int_0^{\infty} y dy \left[\exp \left(-(A - By)^2 \right) \right. \\ \left. - \exp \left(1 - (A + By)^2 \right) \times \frac{\Gamma_n^{oi} \Gamma_r^i}{\left(y^2 - e_1 \right)^2 + \frac{1}{4} \left(y \Gamma_n^{oi} + \sum_a \Gamma_a \right)^2} \right] \quad (53)$$

where $\Gamma_n^{oi} = \Gamma_n^i / e_i^{1/2}$. Converting to the laboratory system by $\frac{m}{\mu} e = E_L$,

$\frac{m}{\mu} \Gamma_n = \Gamma_{nL}$, $\frac{m}{\mu} \Gamma_r = \Gamma_{rL}$, $\sqrt{\frac{m}{\mu}} y = x$, and $\sqrt{\frac{\mu}{m}} B = B_L$ and using numerical values for the constants, yields:

(54)

$$\sigma_r(E_n) = \sigma_{roT} \sqrt{\frac{E_{oT}}{E_n}} + 3.67252 \times 10^5 g\left(\frac{m}{\mu}\right)^2 \frac{B_L^3}{A^2} \times \left\{ \int_0^\infty x dx \left[\exp\left(-(A - B_L x)^2\right) \right] \right. \\ \left. - \exp\left(A + B_L x^2\right) \times \sum_{i=1}^N \Gamma_{nL}^{oi} \Gamma_{rL}^i / \left[(x^2 - E_i)^2 + \frac{1}{4} \left(x \Gamma_{nL}^{oi} + \sum_a \Gamma_{aL}^i \right) \right] \right\}$$

where $B_L = \sqrt{\frac{M}{mT}}$. Devaney has noticed that the factor $\left(\frac{m}{\mu}\right)^2$ is frequently omitted in experimental analyses.

Tabulations of $\sigma_r(E_n)$ as a function of temperature ⁽¹⁷⁾ are used in conjunction with Bell's integral methods with Eqs. (57 through 62), following, where, for foils:

$$\sigma_p = \frac{\text{moderator scattering } (\Sigma_s)}{\text{absorber atomic density}} \quad (55)$$

and

$$\sigma_p \simeq \frac{\text{surface}}{4 \cdot \text{Volume}} \cdot \frac{1}{N_a} \quad (56)$$

The integral forms ⁽¹³⁾, where $F_i(\sigma_p)$ is the integral of F over energy interval $E_i - E_{i+1}$; and RI is the total resonance integral are:

$$F_{NRi}(\sigma_p) = \sigma_p \int_{E_i}^{E_{i+1}} \frac{\sigma_F(E)}{\sigma_T(E) + \sigma_p} \frac{dE}{E} \quad (57)$$

$$F_{IMi}(\sigma_p) = \sigma_p \int_{E_i}^{E_{i+1}} \frac{\sigma_F(E)}{\sigma_a(E) + \sigma_p} \frac{dE}{E} \quad (58)$$

$$F_i(\infty) = \int_{E_1}^{E_{i+1}} \sigma_F(E) \frac{dE}{E} \quad (59)$$

$$(RI)_F^{NR}(\sigma_p) = \sum_{i=1}^{I-1} F_{NRi}(\sigma_p) \quad (RI)_F^{IM}(\sigma_p) = \sum_{i=1}^{I-1} F_{IMi}(\sigma_p) \quad (60)$$

$$(RI)_\gamma^{NR}(\sigma_p) = \sum_{i=1}^{I-1} G_{NRi}(\sigma_p) \quad (RI)_\gamma^{IM}(\sigma_p) = \sum_{i=1}^{I-1} G_{IMi}(\sigma_p) \quad (61)$$

For all integrals, the trapezoidal rule has been used, i.e.

$$\int_{x_1}^{x_N} f(x) dx = \sum_{j=1}^{N-1} \frac{1}{2} \left[f(x_j) + f(x_{j+1}) \right] (x_{j+1} - x_j) \quad (62)$$

ANALYSIS OF EXPERIMENTS

Reactor physics equations, essentially equivalent for any one experiment, have accuracies dependent upon the constants of the equations, the group-averaged neutron cross sections, and the boundary conditions. Accordingly, experimental analysis consists of specifying each of these factors in terms that are computable and then inspecting the integral results of the computation, i.e., the solution of the equations. Described below are potentially the major sources of error in the calculations: cross sectional data and experimental description. To minimize error, the neutron cross sections are evaluated for accuracy by using them in computations of a series of reactors that are geometrically closely equivalent, having differences only in one parameter: the quantity of the element being studied. The same procedure is followed in evaluating geometrical effects. If effects in a small energy interval are of interest, adjoint equation methods are used; these are not described here.⁽¹⁰⁾ Because of the nature of experimental reactor studies, our primary concern in each experiment is the extent to which k_{eff} differs from unity. If, in a wide variety of experiments, k_{eff} is approximately equal to unity, the implication in the integral sense is that both cross sections and critical experiments have been correctly treated. The mathematics are assumed (with some cross-checks) to be exact, as that condition can be approached merely with increasingly accurate numerical approximations and extended computing times. This procedure is equi-

valent to partial differentiation of the elements of Eq. (1) with respect to the parameters of the reactor.

Experimental Studies

Inasmuch as any discussion of the many experimental analyses requires a large volume of descriptive material, it is preferable to illustrate the points discussed previously with a series of figures. Finally, the treatment of a specific reactor experiment will be described in some detail.

A comparison of computed and experimental critical radii for water- and graphite-moderated reactors is shown in Figs. 1 and 2, respectively. Two things are immediately noticed: first, the experimental radii fall on the computed curves of moderator-to-fuel atomic ratio versus bare reactor radius; and, second, there are no experimental points for small hydrogen-to-uranium ratios. The low-radius asymptotes (not shown) do approach the correct radii for zero moderator (pure metal spheres). Figure 3 illustrates the large differences between homogeneous and heterogeneous experiments. The former were D_2O - or BeO - moderated experiments, the beryllium-moderated experiments having 10-mil-thick U^{235} foils. As one result the critical beryllium-to-uranium atomic ratio is reduced, reflecting the increase in critical mass consistent with 'lumped' fissionable material.

A comparison of computed with experimental fission density is meaningful only for more complex systems, because it is too simple a function

for one-region experiments. Figure 4 illustrates fission density versus radius for D_2O -moderated and graphite-reflected experiments.⁽¹⁸⁾ Figures 5 through 8 show the same thing for graphite-moderated, beryllium-reflected 2-dimensional experiment and diffusion approximation calculations.⁽¹⁹⁾ The results of foil activation studies for the same reactor experiment are shown in Figures 9 through 12, which exhibit normal scatter of experimental data. These studies illustrate the kind of neutron flux separation (into above-thermal and total- both weighted by $\sigma_a(E)$) that may be done by foil activity measurements.

Rather than elaborating further upon the agreement between theoretically and experimentally determined values for k_{eff} and fission density, it is sufficient to state that the correspondence shown by the above examples was well within the allowable experimental error when the studies were made (in 1958 and before) and has improved as experimental techniques have improved. An extensive study has been made of a wide variety and large number of "computable" moderated reactor experiments as the basis for a development program at Los Alamos Scientific Laboratory for very high-power reactors.⁽²⁰⁾

Description of Critical Experiments

Reactor critical experiments have provided the basic elements of reactor physics data from the earliest days. For analytical purposes, the discrepancies and approximations of theoretical studies are, first, made consistent with the essentially geometrical transformation from

the critical experiment and, then, refined further to fit the description of the experiment in computable terms. The reactor structure, if visualized in units of neutron mean free paths, may be described as many small regions usually with rectilinear boundaries, or as an equivalent spherical and homogeneous medium containing all the major structural components. The minor components may sometimes be summed as a simple $(1/v)$ absorber with many trace impurities added and replaced, mathematically, by their equivalent in boron to account for absorption effects.

The major components of typical heterogeneous experimental reactor assembly are: the fissionable material (usually as 1- to 10-mil-thick foils), the core moderator (blocks of beryllium or graphite), and the reflector (generally a moderating material); these components are supported in a very light lattice, usually fabricated of aluminum. Homogeneous experiments are, generally, water solutions of U^{235} or Pu^{239} , with or without a reflector, contained in a cylindrical vessel. Fast spectrum assemblies may be simple homogeneous spheres or more complex geometries.

The minimum data derived from a critical assembly are: critical mass, material density, and geometry. Further data that may be obtained include: density; foil activity as a function of position, with and without a thermal neutron absorber; reactivity (the value in terms of changes in reactor period of small changes in reactor assembly composition); and, from measurements at several positions, reactivity distribution. Less frequently, the prompt neutron lifetime and the delayed neutron value are measured. All the data are specific to the assembly from which they

are measured. Occasionally, in the case of solid configurations, more frequently for water solutions, a set of data is obtained relating critical mass to the hydrogen-to-fuel atomic ratio or geometry.

Reduction of Experiments

Idealization of the many different geometries described above into computable forms consists essentially of averaging thicknesses, evaluating corner and edge effects, and determining impurities. Simple geometries, such as tanks of solution, may be computed directly with two-dimensional (r,z) transport codes, using the actual atomic densities of all elements in the solution. Close attention must be paid to the material outside the tank that provides extra reflection. Because two-dimensional calculations are in order of magnitude more time-consuming than one-dimensional, it is usual to convert two-dimensional boundaries to one-dimensional form (e.g. right cylinders to infinite cylinders of smaller radii) by using geometrical buckling ideas to assert that

$$B^2(2 - d) = \left(\frac{\pi}{R + d} \right)^2_{\text{sphere}} = \left(\frac{2.405}{R + d} \right)^2_{\text{cylinder}} = \left(\frac{\pi}{L + 2d} \right)^2_{\text{slab}},$$

where d is the extrapolation distance. Within reasonable limits these simple formulae suffice, if a flux spectrum-weighted value for d is used. It is true that B^2 equivalence is not applicable to fast spectrum reactors because of the very large dependence of extrapolation distance on energy; therefore, spherical geometry is used whenever possible.

Solids, such as beryllium, beryllium oxide, or graphite moderators,

are treated in essentially the same way for one-region assemblies, using the concept of equal moderator volume for more complex shapes. Two-region assemblies (e.g. core and reflector) are more difficult to idealize: the results of one- and two-dimensional transport theory studies, in which an actual geometry was used for the two-dimensional analysis, and an "equal moderator volume" was assumed for the one-dimensional treatment show that core corners must be weighted in reactivity value. The effect with this geometrical approximation, is: $k(\text{cyl}) \simeq 1.03$ for $k[2-d(x,y)] = 1.00$. This is a relatively large effect (10% to 30% in critical mass) and has required renormalization of some experiments by approximately the same amount.

Multiregion solid assemblies are no more difficult to treat than two-region assemblies, although the lumping effects of absorbers or moderators must be treated in more detail. If the transport mean free path is longer than the size of the lump, the effects on flux amplitude are small; if slowing down length is shorter than the dimension of the different material, first-order effects on flux result. These effects are generally obvious from experimental geometries. Less obvious is the treatment of absorbers, particularly those with resonances. It has been found that, unless the effect is first-order in terms of Δk_{eff} , a simple foil surface-to-volume approximation can be made for resonances; and the simple exponential attenuation treatment of the $(1/v)$ region of absorption is adequate.⁽¹⁾ Normally, very thick absorbing foils must be treated as separate regions,

because of very large flux-depression effects both in the foil itself and in the nearby regions. In general, surprisingly small effects result from the simple treatment of neutron absorbing foils in reactor critical assemblies. Much larger, in terms of fission density changes, are the effects of streaming along void channels from thermalizing reflectors into strongly absorbing core regions. The magnitudes of all the above approximations are very strongly dependent on the flux spectrum in the reactor assembly. Clearly, thermal and epi-thermal spectrum assemblies are affected very little by fast transport, resonance absorption, and geometrical transformations, as they are generally large systems with very small concentrations of absorber elements. Also, fast spectrum systems have very little resonance and no thermal absorption; and the mean free paths for scattering are large, so there are only very small effects resulting from the heterogeneity of symmetrical systems. However, neutron flux anisotropy effects are large with fast systems and small with thermal systems usually, given no strongly absorbing layers. In summary, the treatment of large homogeneous thermal systems must emphasize thermalization effects; that of small, fast systems must emphasize anisotropic scattering and flux effects; that of heavily absorbing, large systems must emphasize resonance absorption effects. Most reactors do not fit any simple category because of the interaction of the several parts of the assembly.

Computations for the Kiwi A Reactor

Kiwi A was one of the first high-power reactors developed at Los Alamos Scientific Laboratory for Rover, the nuclear powered rocket program. The reactor has a cylindrical configuration, consisting of an axial D_2O "island", an annular heat-exchanger region made of graphite and enriched uranium, and a relatively thick graphite reflector. For safety and control, boron-filled metal rods are placed along the axis; void spaces in the core (the heat-exchanger region) permit hydrogen to flow axially in this annulus while being heated from 100° to approximately 2000° K.

Figure 13, a critical experiment summary sheet, describes the first rough mockup assembly A-16 in terms of radii and atomic densities. Also shown is second assembly using a varying concentration of U^{235} to make the fission absorption more constant radially. The information gained from this first assembly was used to construct an assembly for experimental studies at zero power, closely approximating the final high-temperature experiment. This third, with its constant radial power densities is tabulated in Fig. 14. Figures 15.- 19 show the correspondence between computed and experimental fission density values for these assemblies. The differences are within the experimental error. Figure 19 shows the final fission density for two control rod positions, when the fissionable isotope was distributed to generate a radially constant power density at high temperature and power condition.⁽²¹⁾ Figure 20 confirms the constant radial power density distribution.

This experiment is interesting from two standpoints: first, neutron

cross sections ⁽²²⁾ satisfactory for simple, separate D₂O- and graphite-moderated experiments also gave good results for relatively complex assemblies of D₂O and graphite together. Second, simple numerical methods yield good results for varying temperatures and fissionable isotope densities, with the absolute value of k_{eff} approximately equal to unity.

The experimental reactor configurations illustrated in Figures 21 through 24 were idealized according to the procedure discussed above and then computed by one- and two-dimensional diffusion and transport codes. The results in terms of k_{eff} and fission density distribution, were very nearly the same for both the one- and two-dimensional techniques.

SUMMARY

The static reactor physics methods used in the study of nuclear-powered rocket reactors have been briefly summarized. Emphasis was placed on the numerical solution of the transport equation, on the determination of neutron resonance integrals, and on the reduction of reactor critical assemblies to computable forms. The low resolution and uncertain magnitude of neutron cross sections forced reliance on the interpretation of a wide variety of simple reactor critical experiments. Nevertheless, these studies permit confidence in the analytical results for relatively complex, high-power and -temperature rocket reactors. A display of the eigenvalues of a wide variety of critical experiments in equivalent to differentiating the integral operations consisting of transport equations

with neutron cross sections as energy- and space-dependent factors. Once the changes in the eigenvalues are known, both experiments and the differential cross section and their treatment (values used are specific to the critical assemblies) can be studied. Then adjustments of these numbers can be made, within the uncertainty of these energy- and reactor-design- dependent values. The methods used in transforming experiments to computable numbers are discussed, consistent with the equations and neutron cross sections; and an application to a rocket reactor illustrates the form and accuracy of the results, as well as the experimental arrangements. Two major simplifications were employed in the analyses: 1) the choice of standard list of group-averaged neutron cross sections that cover the neutron energy range in sufficient detail to show the form of the neutron flux $N(r, \theta, E)$; and 2) the description of a wide variety of simple reactor critical assemblies, to establish the magnitude of error in neutron cross sections and then their treatment specific to the separate experiments, and to determine integral errors in reactor calculations.

FIGURES 1 THROUGH 24
FOLLOWS.

Fig. 1. Computed and experimental critical radii, water-moderated reactors.

Fig. 2. Computed and experimental critical radii, graphite-moderated reactors.

Fig. 3. Critical radii, homogeneous and heterogeneous reactor experiments.

Fig. 4. Relative Fission density vs radius in D_2O -reflected and -moderated reactors.

Fig. 5. Radial power distribution from axis of graphite-moderated, beryllium-reflected cylindrical reactor.

Fig. 6. Axial power distribution from midplane of graphite-moderated, beryllium-reflected cylindrical reactor.

Fig. 7. Radial fission traverses.

Fig. 8. Axial fission traverses.

Fig. 9. Indium detector traverse, radially, in graphite-moderated, beryllium-reflected cylindrical reactor.

Fig. 10. Indium detector traverse, axially, in graphite-moderated, beryllium-reflected cylindrical reactor.

Fig. 11. Radial traverses for indium, gold, and uranium foils.

Fig. 12. Axial traverses for indium, gold, and uranium foils.

CRITICAL EXPERIMENT SUMMARY

Cylinder, length 119.7cm (extrap.)

Material Thickness (in.)

Assembly A-16 (18)			
D ₂ O	7.76		
Dural	0.57	$\rho(\text{C in fuel}) = 1.42 \text{ gm/cc}$	$\text{Mc} = 25.4 \text{ kg U}^{235}$
Gap	0.27	$\rho(\text{D}_2\text{O}) = 1.01 \text{ gm/cc}$	$\text{C/Oy} = 237$
C	0.76	$\rho(\text{C refl}) = 1.54 \text{ gm/cc}$	
Fuel	6.06	$B^2(\text{axial}) = 6.888 \times 10^{-4}$	A-18 peak U^{235}
C	0.73	$\Sigma_0(\text{Al 60-61}) = 15.4 \times 10^{-3} \text{ cm}^{-1}$	$\text{Oy conc.} = 236 \text{ mg/cc}$
Gap	0.19	$\Sigma_0(\text{Al 25}) = 14.5 \times 10^{-3} \text{ cm}^{-1}$	of plate
C	16.79	$\Sigma_0(\text{C+imp.}) = 3.78 \times 10^{-4} \text{ cm}^{-1}$	$= 189 \text{ mg/cc}$
			of core

Elements in each region

Region		1	2	3	4	5	6	...
Element								
Name	Atomic density							
D	1	.0608						
O	2	.0304						
Al	3	.0037	.0215	.0037	.0037	.0037		
C	4		.0366	.0712	.0611	.0772		
B	5	29×10^{-8}	73×10^{-8}			14×10^{-8}		
U-235	6			.00028				
D DB ²	7	1.13×10^{-2}						
C DB ²	8		1.88×10^{-2}	$.967 \times 10^{-2}$	1.12×10^{-2}	$.891 \times 10^{-2}$		
Outer radius for each region		19.71	23.77	39.17	41.50	84.07		

Σ_0 in C due to impurities and carbon $3.78 \times 10^{-4} \text{ cm}^{-1}$
 " " Al " " " " Al $15.40 \times 10^{-3} \text{ cm}^{-1}$

Radius (cm)	*A-18 N(U ²³⁵)
24.54	1.10×10^{-4}
25.31	1.74
26.08	2.12
26.85	2.41
27.62	8.83
29.92	3.60
32.24	4.66
33.93	4.70
36.09	4.13
36.86	3.39
37.63	3.00
38.40	2.63
39.17	2.29

NOTE: A-16 has a uniform fuel loading using, on the average, 4 mil foils of U-235. A-18 used foils 1 mil and thicker.

Fig. 13. Critical Experiment Summary Sheet for Assemblies A-16 and A-18.

CRITICAL EXPERIMENT SUMMARY

Z. E. P. O.

A cylinder, end reflected, with shells
of D₂O, Al, fuel, reflector.

Peak 250 mg/cc of fuel element (fuel/void=5)

Axial $B^2 \approx 6.58 \times 10^{-4}$

Axial void for control rod 2 1/2 in. O.D.

$\Sigma_0(C) - 4.51 \times 10^{-4}$ inch impurities

Elements in each region

Region		1	2	3	4	5	6	7	8
Element									
Name	Atomic density								
Al	1	.006		.0366					.0603
B	2	10^{-8}	6×10^{-8}	122×10^{-8}				87×10^{-8}	197×10^{-8}
D	3		.0668	.0262					
O	4		.0334	.0131					
C	5				.0448	.0722	.0763	.0866	
U-235	6					*			
D DB ²	7		$.985 \times 10^{-2}$	2.51×10^{-2}	1.47×10^{-2}	$.911 \times 10^{-2}$	$.862 \times 10^{-2}$	$.760 \times 10^{-2}$	
C DB ²									
Outer radius for each region		3.175	19.76	21.59	24.74	40.39	47.94	85.09	88.90

*Fuel: radii	$N(U^{235})^1$ ($\times 10^{-5}$)	Final	r	N	Final
25.52	7.97	14.10	37.25	43.80	56.90
26.30	10.37	18.88	38.04	35.80	51.60
27.09	12.95	22.94	38.82	31.80	47.30
27.87	16.35	27.24	39.60	26.80	42.40
28.65	20.33	32.27	40.39	21.30	36.60
31.00	29.83	44.00			
33.34	45.80	57.40			
34.90	49.80	59.80			

¹Includes high temperature effect at
300/mg cc Oy in the fuel element:
LAMS-2259

NOTE: $N(U^{235}) \times 1/2$ also run as an experiment.

Fig. 14. Critical Experiment Summary Sheet for Zepo.

Fig. 15. Radial power distribution in Assembly A-16.

Fig. 16. Radial power distribution in Assembly A-16.

Fig. 17. Radial fuel and power distribution in first Zepo loading.

Fig. 18. Radial fuel and power distribution in full Zepo loading.

Fig. 19. Fissioning distribution in Zepo Whim 3, loaded for flat-power KIWI-A.

Fig. 20. Fission density in KIWI-A at high Temperature.

Fig. 21. D₂O island components.

Fig. 22. Transverse cross section Assemblies A-16 and A-18.

Fig. 23. Longitudinal cross section of Assembly A-18.

Fig. 24. Transverse cross section of Zepo-A.

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