ACRA — A COMPUTER PROGRAM FOR THE ESTIMATION OF RADIATION DOSES CAUSED BY A HYPOTHETICAL REACTOR ACCIDENT

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MATHEMATICS DIVISION
and
OPERATIONS DIVISION

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ABSTRACT

The radioactive material released to the atmosphere on account of a hypothetical reactor accident is transported away by the wind and is dispersed to a 3-dimensional normal distribution. The fission product activity is described throughout the space domain as a function of time. Presently the code calculates two types of exposure from the passing cloud of finite size: internal exposure from inhalation of the radionuclides in the air, and external dose from gamma radiation. The cloud depletion terms by washout and dry deposition have not been incorporated. However, this option can be easily implemented.

Up to 100 receptor locations and 20 time intervals can be handled in one run. The program accepts information for up to 300 isotopes. Computer time is about 2 seconds per receptor and time interval. This time has to be doubled if inversion layers are present.

1. Formulation of the Problem

1.1.1 Mathematical Analysis

The mathematical analysis of this report is based on an earlier report by F. T. Binford, J. Barish and F. B. K. Kam.¹ Its basic assumptions can be stated briefly as follows: In a hypothetical reactor accident, a quantity of volatile fission products is released into the reactor containment and instantly mixed with the surrounding air. This mix is emitted at a constant rate into the atmosphere and the air lost within the building by this process is replaced by clean air. Part of the radioactive material may be retained in filters which are placed in the stack through which the radioactive mix is released. After leaving the stack, the fission products are dispersed and carried away by wind.

It is the purpose of this computer program to evaluate the radiation dose delivered by such radioactive clouds to receptors at given locations and over prescribed time intervals. We consider first the transport and dispersion of the fission products after leaving the reactor building. We shall assume that the locations of the receptors and of the reactor building are given as points in a Cartesian coordinate system with x, y as the horizontal coordinates. The vertical coordinates, i.e., height above the receptor are the z coordinates. Let the origin of the coordinate system be at the receptor location and let the positive x -axis be along the prevailing wind direction. A unit of material which leaves the stack at time t is carried away by wind in τ minutes to a point with coordinates $x_o(\tau), y_o(\tau), z_o(\tau)$, the horizontal movement of course governed by the

¹F. T. Binford et al., Estimation of Radiation Doses Following a Reactor Accident, ORNL-4086.

wind velocity u (m/min), (hence, if x_r, y_r are the horizontal coordinates of the reactor building $x_o = x_r + u \tau$, $y_o = y_r$.) The vertical distance from the receptor is $z_o = h_s - h$ where h is the height of the receptor above ground and h_s the effective stack height. An estimate for h_s is given on p. 28 of the previous report. The material is dispersed in a cloud whose density distribution is assumed to be Gaussian. Thus, the concentration at a point (x, y, z) is given by the formula:

$$(1.1) \quad d(x, y, z; t) = \frac{\exp - \left[\frac{(x - x_o)^2}{2\sigma_x^2} + \frac{(y - y_o)^2}{2\sigma_y^2} + \frac{(z - z_o)^2}{2\sigma_z^2} \right]}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z}$$

We shall assume that the horizontal dispersion is symmetric, $\sigma_x = \sigma_y = \sigma_\rho$. Another widely accepted model which is also used in the previous report disregards the dispersion in the wind direction so that (1.1) is replaced by the 2-dimensional distribution

$$(1.1a) \quad \hat{d}(y, z; t) = \frac{\exp - \left[\frac{(y - y_o)^2}{2\sigma_y^2} + \frac{(z - z_o)^2}{2\sigma_z^2} \right]}{2\pi \sigma_y \sigma_z}$$

The main reason for introducing this model is that it eliminates one integration step. However this integration step can be carried out explicitly by means of a fast and efficient evaluation of the error function. We prefer, therefore, the more realistic model (1.1).

The values of σ_x , σ_y , σ_z depend on the prevailing meteorological conditions and must be given to the program, for instance from the

tables on p. 29 of the earlier report. To account for the influence of the ground and perhaps an inversion layer in the upper atmosphere on the dispersion, virtual clouds will be added which are mirror images of the original cloud to the ground and the inversion layer respectively.

The damage to the receptor is due to two different types of exposure to radiation. The "internal dose" is received by direct inhalation of the fission products and subsequent exposure. Suppose the inhalation rate is β cm³/min and the dose delivered by a unit of radioisotope i is δ_i rem/ μ c. The dose rate R_{int} in rem/min per unit (curies) of material distributed as in (1.1) is then given by

$$(1.2) \quad R_{int}(\tau) = \beta \delta_i d(0, 0, 0; \tau)$$

assuming that the receptor is at the origin of the coordinate system.

The second type of radiation damage, the external dose, is due to γ -radiation. The amount of radiation per unit area normal to the source-receptor line received at a distance r from the source is given by the formula

$$(1.3) \quad I(r, E) = \frac{1 + C\mu e^{D\mu r}}{4\pi r^2} e^{-\mu r}$$

where C , D , and μ are functions of E , the energy of the γ -radiation considered. If the source is distributed as in (1.1) the quantity has to be multiplied by d and integrated to yield

$$(1.4) \quad I_t(E, \tau) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I(r, E) d(x, y, z; t) dx dy dz$$

where $r^2 = x^2 + y^2 + z^2$ assuming the same receptor location as in (1.2). This integration constitutes a major part of the computational effort of the program. A detailed description will be given later. To obtain the dose rate I_t is multiplied by the energy E , an absorption factor σ , which depends also on E , and a proportionality constant of 3.556 whose derivation is given on p. 17-18 of the earlier report. Since a given isotope i may radiate at different energy levels E_k with relative intensities $f_i(E_k)$, these different contributions must be added to obtain a dose rate for a unit of activity of a given isotope i ; that is

$$(1.5) \quad R_{\text{ext}}(i, \tau) = 3.556 \cdot \sum_k E_k \sigma(E_k) \cdot I_t(E_k, \tau) f_i(E_k) \text{ (rem/min)} .$$

Finally, the dose rates R_{ext} and R_{int} are integrated in time to obtain the total dose delivered to the receptor. Let $\rho_i(t)$ be the total amount of isotope i which leaves the stack during the time of the accident until time t . Let further $S_{i,j}(\tau)$ be the amount of isotope j generated through radioactive decay from a unit of isotope i during a time interval τ so that the sum

$$(1.6) \quad \sum_i \rho_i(t - \tau) S_{i,j}(\tau)$$

is the total amount of isotope j arriving at the cloud (1.1) during the first t min after the accident. After integration over τ the total internal and external doses become

$$(1.7) \quad D_{int} = \int_0^t \sum_j \sum_i \rho_i(t - \tau) S_{ij}(\tau) R_{int}(\tau) d\tau$$

and

$$(1.8) \quad D_{ext} = \int_0^t \sum_j \sum_i \rho_i(t - \tau) S_{ij}(\tau) R_{ext}(j, \tau) d\tau .$$

The summations are performed over all isotopes and their possible radioactive offsprings present in the reactor. Note that in (1.8) the dose rate R_{ext} depends on the isotope and contains also a sum over different energy levels, each necessitating a separate integration (1.4). For this reason the energy levels will be normalized to not more than 20 for all isotopes and the integration will be carried out at each energy level for all isotopes simultaneously. The integration in formula (1.7) can be avoided if the dispersion is disregarded in the direction of the wind. Using (1.1a) and considering the movement of the cloud d in time we obtain for the concentration at a point (x, y, z) within the "plume."

$$(1.1b) \quad d^*(y, z; \tau) = \frac{\exp \frac{(y - y_0)^2}{2\sigma_y^2} + \frac{(z - z_0)^2}{2\sigma_z^2}}{2\pi \sigma_y \sigma_z u}$$

and accordingly (1.2) is replaced by

$$(1.2a) \quad R_{int}^*(\tau) = \beta \delta d^*(0, 0; \tau)$$

so that (1.7) becomes

$$(1.7a) \quad D_{int}^* = \sum_j \sum_i \phi_i(t - \tau_o) S_{ij}(\tau_o) R_{int}^*(\tau_o)$$

where $\tau_o = |x_r|/u$ that is the time needed for the wind to transport the radioactive material to the receptor. This mode of computation is implemented in the program as an option. The difference between D_{int} and D_{int}^* becomes significant near (< 1000 m) and behind the reactor where D_{int} can be as much as several orders of magnitude larger than D_{int}^* .

The values of ρ_i and S_{ij} are solutions of certain linear differential equations. Explicit solutions have the form of a sum of exponential functions containing the decay constants. Numerical instabilities can occur if the decay constants are nearly equal. Programs to circumvent that difficulty have been designed for instance by H. H. Van Tuyl, ISOGEN - A Computer Program Code for Radioisotope Generation Calculations, Report HW-83785. A similar approach modified to handle the specific situation at hand is used as a part of this program.

In its present form the program can handle only linear decay sequences, that is without branching. However, if branching occurs the sequence can be split in a number of independent linear chains, one for each branch. In preparation of the data for these chains, some isotopes may have to be renamed in order to have different isotopes in different chains.

2. Organization of the Program

2.0.0 Introduction

The program consists of a main program which will be referred to as MAIN and a large number of subprograms which will be described in detail later. The following major computational and data handling tasks will be performed.

1. Input and preparation of data. This is accomplished by the subroutine SETUP.
2. Computation of $\rho_i(t)$ and $S_{ij}(t)$. The basis for these computations is the solution of certain differential equations which are obtained by a subroutine DIFF. In the case of $\rho_i(t)$ the values are computed and stored by a subroutine RELEASE. The values of $S_{ij}(t)$ are computed directly in the main program MAIN each time they are needed because otherwise the storage space required would be prohibitively large.

The values of $\rho_i(t)$ and $S_{ij}(t)$ are computed at a specified number n ($- NTAU$ in the program) of equidistant time intervals $t_i = t_0 + i \cdot \Delta t, i = 1, n$ ($\Delta t = TAU$ in the program). Intermediate values are found by linear interpolation. This corresponds to a subdivision of the x axis into interval of length $u \cdot \Delta t$, i.e., the distance traveled by wind during the time Δt . This subdivision is independent of the distribution of integration nodes for the integrations (1.7) and (1.8) and thus not affected by the number of receptor locations. In other words the time spent for the computation of $S_{ij}(t)$ and $\rho_i(t)$ depends only on the number of isotopes and is roughly proportional to it. The time for the integrations (1.4), (1.7), and (1.8) on the other hand depends only on the number of receptor locations and not on the

number of isotopes, the total computer time being the sum of the two. A detailed estimate for the computer time will be given at the end of this report.

3. The integrations (1.7) and (1.8) are also performed in MAIN together with summation over the different isotopes and energy levels.
4. The integration (1.4) is performed in the subroutine INT.

Great care has to be exercised to economize the computational effort particularly in calculations 3. and 4. Since the functions involved have essentially the behavior of a Gaussian curve, conventional numerical integration methods fail unless the nodes of integration are carefully placed within an area where the function values are significant. Failure to observe this precaution led to disastrous results in the past.

Communication between programs is accomplished via several blocks of COMMON storage which are described in detail later.

2.1 The Program MAIN

2.1.1 Introduction

The primary task of this program is to initiate and supervise all processing and computing. It performs also two specific computations, namely the evaluation of the radioactive decay within the atmosphere given by the coefficients $S_{ij}(t)$ and the summations and integration given in formulas (1.7) and (1.8) to obtain the final radiation dose.

The program starts by calling SETUP which provides the necessary input data. A subsequent call for RELEASE initiates the computation of the amount of radioisotopes released from the stack during given intervals. The coefficients S_{ij} are computed next. The program then prepares and carries out the evaluation of D_{ext} according to formula (1.8). The

nodes of integration and weights are provided by subroutine NEXTRHO. The values of $I_t(E, \tau)$ are found by the subroutine INT.

After D_{ext} has been computed, a similar procedure is used to find the values of D_{int} . The results are then printed and the program asks through SETUP for new sets of data or instruction to terminate processing.

2.1.2 Mathematical Development

The computational procedure for finding S_{ij} is very similar to the one performed in subroutine RELEASE. Both require the solution of certain simple linear differential equations which are integrated in subroutine DIFF.

Let $X_i(t)$ be the amount of radioisotope i present at time t . This isotope may decay into isotope $(i+1)$ at a rate λ_i . This process is described by the system of differential equations

$$(2.1.1) \quad \dot{X}_i = -\lambda_i X_i + \lambda_{i-1} X_{i-1}$$

where i ranges over all isotopes in a given chain. The solution of this system in matrix form

$$(2.1.2) \quad S_{ij}(t) = e^{At}$$

is described in detail in 2.3. This matrix contains the desired coefficients $S_{ij}(t)$ and is a direct result from the computations in DIFF given the entries of (2.1.1).

The matrix $S_{ij}(t)$ is computed at equidistant timepoints $t = i \cdot \Delta t$, $i = 1, n$ using the formula

$$(2.1.3) \quad S_{ij}((i+1) \Delta t) = \sum_k S_{ik}(\Delta t) S_{kj}(i \Delta t)$$

which requires only one integration of the system (2.1.1) by DIFF for the value Δt . The matrix multiplication (2.1.3) is combined with one summation in (1.8) namely

$$(2.1.4) \quad \sum_j S_{ij}(i \Delta t) f_j = v_i(i)$$

and (2.1.3) reduces to a vector multiplication

$$(2.1.5) \quad v_i(i+1) = \sum_k S_{ik}(\Delta t) v_k(i) .$$

This process is guided by the subroutine NEXT.

For intermediate values a linear interpolation is used. Since the entries in the matrix (2.1.2) are exponential functions which are smooth and convex such linear interpolation is permissible. At worst it may lead to some overestimation of the radiation dose. The value of Δt is chosen to fit the subsequent integration that is the time needed to transport the isotopes from stack to beyond the farthest receptor should be covered by less than 100 time intervals Δt . This may result in decreased accuracy for receptors close to the reactor if fast decaying isotopes are present. In this case the radiation dose for receptors in the vicinity of the reactor should be evaluated in a separate run.

As mentioned in 2.0.0 the integrands in (1.7) and (1.8) have the shape of a Gaussian curve, and care has to be exercised to use only the significant portion of such curves for numerical integration. The subroutines NEXTRHO and NEXTRHOL respectively are designed to select the proper nodes and weights for this procedure. This is described in detail in 2.6.1. The integration (1.8) is performed separately for each of the given energy levels (up to 20 levels may be used). Integration (1.7) requires of course only one run.

2.2 The Subprogram SETUP

2.2.1 This program directs the acquisition of input data and the subsequent processing by means of control parameters which are read from control cards. These parameters are numbered from 1 through 9 initiating the following processes:

1. Read data concerning the desired levels of gamma energy
2. Read data with space and time coordinates
3. Read meteorological data
4. Read isotope data
5. Read reactor data
6. Read constants for numerical integration
7. Initiate data processing
8. Precision constants are changed
9. Terminate processing

The input data may be read and changed at any time subject only to the following constraints:

- i Data #2-5 must be read at least once before any attempt of data processing can be made. A standard set of data #1, #6, and #8 resides in the program which can be changed as an option.
- ii Data #1 take precedent over data #4 and data #4 over #5 that is data #4 must be read prior to data #5 and a change in #4 necessitates a new reading of #5. Similarly #4 and #5 have to be read anew after change in #1.
- iii Any change in the breathing rate β must be followed by new sets of data #4 and #5.

Violations of these rules result in an error message and termination of the program.

As many runs as desired may be performed, each to be initiated by a control card containing control parameter 7. Processing is terminated by control parameter 9. A detailed instruction for the preparation of input cards is given in Chapter 3 of this report.

In addition to the reading of data an extensive data preparation is performed. Data #1 and #3 are rearranged in ascending order. The isotopes are arranged in families and the γ -energies are fit into the levels defined by #1. A coordinate transform puts the reactor location at the origin and the wind direction into the positive x-axis. The program determines also the value of Δt to be large enough to cover the whole range of receptor locations and finds parameters which determine the grid spacing for integration given in NEXTRHO. It computes the starting time t_0 for the subprogram RELEASE and decides how many different runs are required to cover all time coordinates.

The data are printed immediately after the set is read. If inconsistencies are detected, an error message is given and the processing stops.

2.3 The Subprogram DIFF

2.3.1 Mathematical Development

Consider the system of differential equations

$$(2.3.1) \quad \dot{X} = AX$$

where A is a lower triangular matrix and X is a column vector, i.e., the system (2.3.1) has the form

$$(2.3.1a) \quad \dot{X}_1 = u_{11} X_1$$

$$\dot{X}_2 = u_{21} X_1 + u_{22} X_2$$

ect. with $\dot{X}_i = \frac{dX_i}{dt}$. The general solution of (2.3.1) is

$$(2.3.2) \quad X(t) = e^{At} X(0)$$

where $X(0)$ is the initial value of $X(t)$ at $t = 0$.

An explicit solution of (2.3.1a) and thus (2.3.2) can be found by the well known method of solving the first line for $X_1(t)$ and substituting this solution in the lines below and solving for $X_2(t)$ and so on. This method leads, however, to numerical instabilities if some of the diagonal terms u_{ii} of A are nearly equal. For this reason the matrix e^{At} is computed

in a more direct fashion based on a decomposition of the matrix A . For any arbitrary square matrix A , the matrix e^{At} is defined by the power series

$$(2.3.3) \quad e^{At} = U + At + \frac{A^2 t^2}{2!} + \dots$$

convergent for all t (U is the unit matrix). The following theorems are easy consequences of (2.3.3):

- i. If $AB = BA$ then $e^{(A+B)t} = e^{At} e^{Bt}$.
- ii. If $AB = BA = 0$ then $e^{(A+B)t} = U + (e^{At} - U) + (e^{Bt} - U)$.
- iii. If I is an idempotent matrix, $I^2 = I$ then $e^{It} = U + I(e^t - 1)$.

Let $R(X)$ be the minimal polynomial of A , that is the polynomial of lowest degree such that $R(A) = 0$. We have

$$(2.3.4) \quad R(X) = \prod_{i=1}^k (X - \lambda_i)^{n_i}$$

where λ_i , $i=1, k$ are the k different eigenvalues of A . Consider a factoring of $R(X)$ into two relatively prime and nonconstant polynomials $P(X)$ and $Q(X)$, $R(X) = P(X) Q(X)$. We shall call $(X)_P$ and $(Y^T)_P$ the right and left null spaces of $P(A)$ respectively, that is the spaces of column vectors X and row vectors Y^T for which

$$(2.3.5) \quad P(A) X = Y^T P(A) = 0.$$

It is well known that the spaces $(X)_P$ and $(Y^T)_Q$ as well as $(X)_Q$ and $(Y^T)_P$ are orthogonal complements of each other and that there exist two unique idem potent matrices I_P and I_Q such that the following equations are satisfied:

$$\begin{aligned}
 (2.3.6) \quad & I_P + I_Q = U \\
 & I_P X = X \quad X \in (X)_P \\
 & \quad = 0 \quad X \in (X)_Q \\
 & Y^T I_P = Y^T \quad Y^T \in (Y^T)_P \\
 & \quad = 0 \quad Y^T \in (Y^T)_Q
 \end{aligned}$$

and corresponding equations for I_Q . The idem potent I_P can be constructed in the following manner: The vector spaces $(X)_P$ and $(Y^T)_P$ have the same dimension, say m . We can then select two basis, $X_1, X_2, \dots, X_m$ and $Y_1^T, Y_2^T, \dots, Y_m^T$, which are mutually orthonormal, that is

$$(2.3.7) \quad Y_i^T X_j = \begin{cases} 0 & \text{for } i \neq j \\ 1 & \text{for } i = j \end{cases}$$

$$\text{Also } Y_i^T X = 0 \text{ for } X \in (X)_Q$$

$$\text{and } Y^T X_j = 0 \text{ for } Y^T \in (Y^T)_Q$$

Thus if we set

$$(2.3.8) \quad I_P = \sum_{i=1}^m X_i Y_i^T$$

and consider that $(X)_P$ and $(X)_Q$ as well as $(Y^T)_P$ and $(Y^T)_Q$ span the whole vector space, then (2.3.6) follows easily. We have also

$$(2.3.9) \quad I_P I_Q = I_Q I_P = 0$$

$$I_P A = A I_P = A_P$$

$$I_Q A = A I_Q = A_Q$$

$$A_P + A_Q = A$$

$$A_P A_Q = A_Q A_P = 0$$

These formulas lead to the desired decomposition of A . In particular let $P_i(X) = (X - \lambda_i)^{n_i}$; then

$$(2.3.10) \quad A = \sum_{i=1}^k \lambda_i I_{P_i} + N$$

where N commutes with all I_{P_i} , $I_{P_i} N = N I_{P_i}$ and is nilpotent, i.e., $N^\ell = 0$ for sufficiently large ℓ . Thus the matrix e^{At} becomes

$$(2.3.11) \quad e^{At} = \sum_{s=0}^{\ell-1} N^s \frac{t^s}{s!} \left[U + \sum_{i=1}^k I_{P_i} (1 - e^{\lambda_i t}) \right].$$

In a lower triangular matrix the eigenvalues are the diagonal elements a_{ii} and the related idempotents can easily be constructed. This construction is however numerically unstable if the λ_i 's are nearly equal. Such λ_i 's are better combined into one polynomial, say

$$(2.3.12) \quad P_j(X) = \prod_{|\lambda_i - c_j| < \epsilon} (X - \lambda_i)^{n_i}$$

that is $P_j(X)$ contains all roots of $R(X)$ which are sufficiently close to a suitably chosen value c_j . This leads to a decomposition

$$(2.3.13) \quad A = \sum_{j=1}^{k^*} c_j I_{P_j} + N^*.$$

The matrix N^* commutes again with all I_{P_j} but is no longer nilpotent. However all eigenvalues of N are smaller than the value ϵ in (2.3.10) which allows an efficient computation of e^{N^*t} by means of the power series (2.3.3). Thus

$$(2.3.14) \quad e^{At} = \sum_{s=0}^{\infty} \frac{N^{*s}}{s!} \left[U + \sum_{j=1}^{k^*} I_{P_j} \left(1 - e^{c_j t} \right) \right].$$

The computation in the program are based on this formula. The idempotents I_{P_j} are constructed according to formula (2.3.7). In order to find the basis for $(X)_{P_j}$ and $(Y^T)_{P_j}$ consider the matrix $P_j(A)$. This is a lower triangular matrix whose diagonal elements are 0 at the places

corresponding to λ_i , say $a_{ii} = 0$. Then the i -th row vector is a linear combination of the row vectors 1 through $i-1$ in $P_j(A)$, say

$$\begin{aligned}
 (2.3.15) \quad & (u_{i1}, u_{i2}, \dots, u_{i,i-1}, 0, 0, \dots, 0) \\
 &= X_1(u_{11}, 0, 0, \dots, 0) + X_2(u_{21}, u_{22}, 0, 0, \dots, 0) \\
 &+ \dots + X_{i-1,1}(u_{i-1,1}, u_{i-1,2}, \dots, u_{i-1,i-1}, 0, 0, \dots, 0)
 \end{aligned}$$

and the column vector

$$X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_{i-1} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{belongs to } (X)_{P_j}.$$

In this manner we obtain a set of linearly independent vectors of $(X)_P$, one for each vanishing diagonal element in $P_j(A)$. The same method applied to $[P_j(A)]^T$ gives a basis for $(Y^T)_{P_j}$. Equation (2.3.15) can be combined with (2.3.7) to obtain an orthonormal system. It should be noted that I_P in (2.3.8) does not depend on the specific basis chosen for $(X)_P$ and $(Y^T)_Q$.

2.3.2 Programming Considerations

All matrices in this program are triangular. Economy in storage and computational efficiency make it desirable to store only the significant entries. A set of subprograms has been designed to perform matrix operations within this environment. These are:

MULT for multiplication of a vector by a matrix.
 MMULT for multiplication of two matrices.
 SOLVE for solving the linear equation $AX = B$.
 TRANSP for forming the transpose A^T from a matrix A .

It is generally assumed that all vectors and matrices are contained in the same array. The positions of the first entries within this array are the parameters of these subprograms.

The program accepts a matrix A stored in the manner just outlined and a real value t and returns the matrix e^{At} computed according to (2.3.14) on a specified location. The new matrix e^{At} may overlap the original matrix A . Of course the original values of A are then destroyed.

2.4 The Subprogram RELEASE

2.4.1 Mathematical Development

Consider a family of radioisotopes n numbered according to the decay sequence, that is the i -th isotope decays into the $(i+1)$ -th isotope at a rate λ_i . Let $X_i(t)$ be the amount of isotope i present at time t in the reactor containment. This isotope escapes through the stack at a rate α , the lost material being replaced by fresh air from the outside. Within the stack a fraction ϕ_i of the isotope i may be trapped in filter. The total amount of isotope i retained in the filter at time t may be called $Y_i(t)$. As time passes the isotope i in the filter decays into isotope $(i+1)$, and we shall assume that a fraction ψ_{i+1} of this isotope is also retained in the filter. The remainder is released into the air, and the total amount of isotope i released into the air from the time of the accident up to time t shall be called $Z_i(t)$. (This is not the total amount present at time t since we disregard subsequent decay which is handled in a different program.)

The functions $X_i(t)$, $Y_i(t)$, $Z_i(t)$ satisfy the following system of differential equations $\left[\dot{X}_i(t) = \frac{dX_i}{dt} \text{ ect.} \right]$:

$$(2.4.1a) \quad \dot{X}_i = -(\lambda_i + \alpha) X_i + \lambda_{i-1} X_{i-1}$$

$$(2.4.1b) \quad \dot{Y}_i = \alpha \varphi_i X_i - \lambda_i Y_i + \lambda_{i-1} \psi_i Y_{i-1}$$

$$(2.4.1c) \quad \dot{Z}_i = \alpha(1 - \varphi_i) X_i + \lambda_{i-1}(1 - \psi_i) Y_{i-1}$$

i ranging over all isotopes of the family.

The initial conditions for the time $t=0$ when the accident has occurred are

$$(2.4.2a) \quad X_i(0) = q_i$$

$$(2.4.2b,c) \quad Y_i(0) = Z_i(0) = 0$$

where q_i is the amount of isotope i leaving the core due to the accident.

The system (2.4.1) may be written in vector form

$$(2.4.3) \quad \dot{U}(t) = A U(t)$$

and solved by the procedure described in 2.3, the general solution being

$$(2.4.4) \quad U(t) = e^{At} U(0) .$$

The vector $U(t)$ contains the set of values $\rho_i^{(t)}$ with i varying over the isotopes of one family. For the integrals (1.7), (1.8) the values of $\rho_i(t - \tau)$ are needed where t is the length of time after the accident during which the receptor is exposed and τ ranges from 0 to $n \cdot \Delta t$ as explained in 2.0.0. Let $t_0 = t - n \cdot \Delta t$. Equation (2.4.4) is first solved for this value

$$(2.4.5) \quad U_0 = e^{At_0} U(0)$$

and U_0 is used as a new initial value. The subsequent values $U(t_0 + i \cdot \Delta t)$ are computed by the same technique as in (2.1.4), i.e., using only U_0 and the matrix $e^{A \Delta t}$.

We obtain in this manner all the necessary values $\rho_j(t - i \cdot \Delta t)$, $i=1, \dots, n$ as $Z_j(t_0 + (n-i) \cdot \Delta t)$. These values are retained and stored for further use in MAIN. The procedure is performed for all families of isotopes.

2.5 The Subprogram INT

2.5.0 Introduction

This program computes the amount of γ radiation received from a cloud of radioisotopes. It takes into account only the instantaneous distribution, that is the isotopes are dispersed symmetrically around a fixed center as described in (1.1). The movement of the cloud and development of the isotopes in time is handled in MAIN.

For convenience some of the definitions and symbols introduced here are slightly different from the ones used in 1.1.1.

2.5.1 Mathematical Development

We use a x-y-z coordinate system in which the receptor is located at the origin and the center of the cloud is at (x_o, y_o, z_o) . The distribution of the isotopes is given by the formula [see (1.1)]

$$(2.5.1) \quad S(x, y, z) = \frac{\exp - \frac{(x - x_o)^2}{2\sigma_x^2} - \frac{(y - y_o)^2}{2\sigma_y^2} - \frac{(z - z_o)^2}{2\sigma_z^2}}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z}$$

denoting the relative strength of the γ - radiation at the point (x, y, z) .

The dose D received at the origin from this distribution is then

$$(2.5.2) \quad D = \int_{-h}^{h_1-h} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} S(x, y, z) \frac{1 + C_{pre} D_{pr}}{4\pi r^2} e^{-\mu r} dx dy dz$$

$$(r^2 = x^2 + y^2 + z^2)$$

We assume here, that the receptor is at a height h above the ground and that the atmosphere contains an inversion layer at height h_1 which prevents the cloud to disperse above this layer. In order to compensate for the influence of ground and inversion layer 3 "virtual" clouds are added which are mirror images of the original cloud in respect to ground or inversion layer. More specifically one of the virtual clouds will be a mirror image of the original cloud in respect to the inversion layer. The two other virtual clouds are mirror images of the original cloud and the first virtual cloud in respect to the ground.

An exact computation would require infinitely many virtual clouds, but the influence of the other virtual clouds may be disregarded as long as σ_z is not substantially larger than the height of the inversion layer since the concentration decreases like $e^{-\frac{s^2}{2\sigma_z^2}}$ where s is the distance between the virtual cloud and receptor. If σ_z is large then the concentration is nearly uniform between ground and inversion layer and the error caused by discarding virtual clouds can be rectified by applying a factor which represents the ratio between the real and the assumed concentration of radioactive material (i.e., resulting from the real and the 3 virtual clouds). This factor is roughly equal to the ratio between the integral

$$\int_{-\infty}^{+\infty} e^{-\frac{(z-z_0)^2}{2\sigma_z^2}} dz$$

and same integral taken between $-2h_1 - h$ and $+2h_1 - h$, that is the part of the cloud (2.5.1) which is covered by our computation.

The same integral (2.5.1) is used for the real and virtual clouds. If the receptor is on the ground, i.e., $h = 0$, the computation of the 2 virtual clouds below the ground can be eliminated by changing the lower limit of the outer integral in (2.5.2) from $-h$ to $-h_1$. Of course h_1 is set to $h_1 = \infty$ if no inversion layer is present.

We assume that the dispersion is symmetric in the horizontal direction

$$\sigma_x = \sigma_y = \sigma_\rho$$

and adjust the coordinate system in such a way that

$$x_0 = \rho$$

$$y_0 = 0$$

$$z_0 = h_s - h = h_0$$

that is the x-axis points in horizontal direction from the receptor to the center of the cloud where ρ is the horizontal distance. h_s is the stack height adjusted according to the vertical movement of the cloud.

We introduce a spherical coordinate system (r, φ, θ) with

$$x = r \cos \varphi \cos \theta$$

$$y = r \sin \varphi \cos \theta$$

$$z = r \sin \theta$$

and obtain

$$(2.5.3) \quad S(r, \varphi, \theta) = \frac{\exp \left[-\frac{(r \cos \varphi \cos \theta - \rho)^2}{2\sigma_\rho^2} - \frac{(r \sin \theta - h_0)^2}{2\sigma_z^2} \right]}{(2\pi)^{3/2} \sigma_\rho^2 \sigma_z}$$

$$(2.5.4) \quad D = \frac{1}{4\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \int_0^{2\pi} \int_0^{r_1} S(r, \varphi, \theta) (1 + C_{\mu} r e^{D\mu r}) e^{-\mu r} dr d\varphi \cos \theta d\theta$$

where r_1 is the distance from the receptor to the ground or inversion layer respectively in the θ -direction, i.e.,

$$r_1 = \frac{-h}{\sin\theta}, \text{ for } \theta < 0, \text{ and } r_1 = \frac{h_i - h}{\sin\theta} \text{ for } \theta > 0, r_1 = \infty \text{ for } \theta = 0.$$

As mentioned before $-h$ will be replaced by $-h_i - h = -h_i$ if $h = 0$ for $\theta < 0$. The inner integral in (4) can be split into 2 parts of the form

$$(2.5.5) \quad D_1 = \gamma_1 \int_0^{r_1} e^{-Ar^2 - 2B_1 r - C} dr$$

$$D_2 = \gamma_2 \int_0^{r_1} r e^{-Ar^2 - 2B_2 r - C} dr$$

with

$$\gamma_1 = 2^{-\frac{7}{2}} \pi^{-\frac{5}{2}} \sigma_\rho^{-2} \sigma_z^{-1}$$

$$\gamma_2 = C\mu \gamma_1$$

$$A = \frac{\cos^2 \theta}{2\sigma_\rho^2} + \frac{\sin^2 \theta}{2\sigma_z^2}$$

$$B_1 = \frac{\mu}{2} - \frac{\rho \cos \theta \cos \varphi}{2\sigma_\rho^2} - \frac{h_0 \sin \theta}{2\sigma_z^2}$$

$$B_2 = B_1 - \frac{D\mu}{2}$$

$$C = \frac{\rho^2}{2\sigma_\rho^2} + \frac{h_0^2}{2\sigma_z^2}$$

The concentration at the receptor is

$$S(0,0,0) = (2\pi)^{-\frac{3}{2}} \sigma_\rho^{-2} \sigma_z^{-1} e^{-C}$$

and this value can be used to determine the internal dose [see formulas (1.2) and (1.7)].

In order to evaluate (2.5.5) we consider the integrals

$$(2.5.6) \quad I_k(x) = \int_0^\infty t^k e^{-(x+t)^2 + x^2} dt$$

we have

$$(2.5.7) \quad I_0(x) = e^{x^2} \operatorname{erfc}(x)$$

$$I_1(x) = \frac{1}{2} - x e^{x^2} \operatorname{erfc}(x)$$

After some elementary transforms we obtain

$$(2.5.8) \quad D_1 = A^{-\frac{1}{2}} \gamma_1 e^{-C} (I_0(x) - e^{-a(2x+a)} I_0(x+a))$$

$$D_2 = A^{-1} \gamma_2 e^{-C} (I_1(x) - e^{-a(2x+a)} (I_1(x+a) + a I_0(x+a)))$$

with

$$x = \frac{B_{1,2}}{\sqrt{A}}, \quad a = r_1 \sqrt{A}$$

The functions $I_k(x)$ are computed using approximations by 3rd and 4th degree polynomials. For negative values of x the relations

$$(2.5.9) \quad I_0(-x) = e^{x^2 \sqrt{\pi}} - I_0(x)$$

$$I_1(-x) = x e^{x^2 \sqrt{\pi}} + I_1(x)$$

are used.

For integration over φ a trapezoidal rule is used because for periodic functions nothing can be gained by using integration formulas of higher order. The procedure is made adaptive by doubling the number of nodes in the integration until no significant change occurs. The maximum number of nodes which can be used in this procedure is 64.

For the outermost integral the transformation

$$(2.5.10) \quad t = \sin \theta$$

$$dt = \cos \theta \cdot d\theta$$

is performed and a 19 point Gauss-Legendre formula used for the transformed integral.

The integration becomes unreliable if the dispersion parameter σ is small compared to the distance between cloud and receptor because then the function $S(r, \varphi, \theta)$ changes too rapidly with φ and θ . In this case $S(r, \varphi, \theta)$ possesses a sharp maximum outside of which it falls rapidly to 0. We can therefore extend the limits of integration from $-\infty$ to $+\infty$ and use a Taylor expansion in the neighborhood of the maximum of $S(r, \varphi, \theta)$.

For the inner integrals we obtain

$$\begin{aligned}
 (2.5.11) \quad D_1 &\cong \gamma_1 \int_{-\infty}^{+\infty} e^{-Ar^2 - 2B_1 r - C} dr \\
 &= \gamma_1 A^{-\frac{1}{2}} \sqrt{\pi} e^{-(C - \frac{B_1^2}{A})} \\
 D_2 &\cong \gamma_2 \int_{-\infty}^{+\infty} r e^{-Ar^2 - 2B_2 r - C} dr \\
 &= B_2 A^{-\frac{3}{2}} \sqrt{\pi} e^{-(C - \frac{B_2^2}{A})}
 \end{aligned}$$

Let θ_0 be the value of θ which maximizes the exponential terms in (2.5.11).

We then introduce a new (x', y', z') coordinate system.

$$\begin{aligned}
 (2.5.12) \quad x &= x' \cos \theta_0 - z' \sin \theta_0 \\
 y &= y' \\
 z &= x' \sin \theta_0 + z' \cos \theta_0 .
 \end{aligned}$$

From this we obtain new spherical coordinates r, φ, θ' by

$$(2.5.13) \quad x' = r \cos \varphi' \cos \theta'$$

$$y' = r \sin \varphi' \cos \theta'$$

$$z' = r \sin \theta' \quad ;$$

we set

$$\sin \theta' = u, \quad \sin \varphi' = v$$

and obtain a Taylor expansion of (2.5.13)

$$x' = r \left(1 - \frac{u^2 + v^2}{2} + \dots \right)$$

$$y' = r(v + \dots)$$

$$z' = r(u + \dots)$$

This leads to a Taylor expansion of A and B

$$A = A_0 + A_u u + A_{uu} \frac{u^2}{2} + A_{vv} \frac{v^2}{2}$$

$$B_{1,2} = B_{1,2,0} + B_u u + B_{uu} \frac{u^2 + v^2}{2}$$

with

$$A_0 = \frac{\cos^2 \theta_0}{2\sigma_\rho^2} + \frac{\sin^2 \theta_0}{2\sigma_z^2}$$

$$A_u = \sin 2\theta_o \left(\frac{1}{2\sigma_z^2} - \frac{1}{2\sigma_\rho^2} \right)$$

$$A_{uu} = 2 \cos 2\theta_o \left(\frac{1}{2\sigma_z^2} - \frac{1}{2\sigma_\rho^2} \right)$$

$$A_{vv} = 2 \sin^2 \theta_o \left(\frac{1}{2\sigma_z^2} - \frac{1}{2\sigma_\rho^2} \right)$$

$$B_{1,o} = \frac{\mu}{2} - \frac{\rho \cos \theta_o}{2\sigma_\rho^2} - \frac{h_o \sin \theta_o}{2\sigma_z^2}$$

$$B_{2,o} = B_{1,o} - \frac{D\mu}{2}$$

$$B_u = \frac{\rho \sin \theta_o}{2\sigma_\rho^2} - \frac{h_o \cos \theta_o}{2\sigma_z^2}$$

$$B_{uu} = \frac{\rho \cos \theta_o}{2\sigma_\rho^2} + \frac{h_o \sin \theta_o}{2\sigma_z^2}$$

Thus we obtain an approximation of D_1 and D_2 of the form

$$(2.5.14) \quad D_1 \cong \gamma_1 A_o^{-\frac{1}{2}} \sqrt{\pi} (1 + \dots + \beta_1 u^2 + \beta_2 v^2) *$$

$$* \exp \left[- \left(C - \frac{B_{1,o}^2}{A_o} \right) - \frac{B_{1,o}^2}{A_o} (\alpha_1 u^2 + \alpha_2 v^2) \right]$$

$$D_2 \cong \gamma_2 B_{2,0} A_0^{-\frac{3}{2}} \sqrt{\pi} (1 + \dots + \delta_1 u^2 + \delta_2 v^2) \times$$

$$\exp \left[- \left(C - \frac{B_{2,0}^2}{A_0} \right) - \frac{B_{2,0}^2}{A_0} (\alpha_1 u^2 + \alpha_2 v^2) \right]$$

The linear terms in $(1 + \dots + \beta_1 u^2 + \dots)$ and $(1 + \dots + \delta_1 u^2 + \dots)$ are left out because these do not contribute to the final integral which is computed from

$$(2.5.15) \quad D_1^* = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} D_1(u, v) \, du \, dv$$

$$= \gamma_1 \pi^{\frac{3}{2}} A_0^{-\frac{1}{2}} B_{1,0}^{-2} (\alpha_1 \alpha_2)^{-\frac{1}{2}} \exp \left(\frac{D_{1,0}^2}{A_0} - C \right)$$

$$\times \left[1 + A_0^{-\frac{1}{2}} B_{1,0}^{-1} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) \right]$$

$$D_2^* = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} D_2(u, v) \, du \, dv$$

$$= \gamma_2 \pi^{\frac{3}{2}} A_0^{-\frac{1}{2}} B_{2,0}^{-1} (\alpha_1 \alpha_2)^{-\frac{1}{2}} \exp \left(\frac{B_{2,0}^2}{A_0} - C \right)$$

$$\times \left[1 + A_0^{-\frac{1}{2}} B_{2,0}^{-1} \left(\frac{\delta_1}{\alpha_1} + \frac{\delta_2}{\alpha_2} \right) \right].$$

The constants α_1, α_2 , ect. are

$$\alpha_1 = \frac{1}{2} \frac{A_{uu}}{A_0} - \frac{B_{uu}}{B_0} - \left(\frac{B_u}{B_0} \right)^2$$

$$\alpha_2 = - \frac{B_{uu}}{B_o} - \frac{1}{2} \frac{A_{vv}}{A_o}$$

$$\beta_1 = \frac{3}{8} \left(\frac{A_u}{A_o} \right)^2 - \frac{1}{4} \frac{A_{uu}}{A_o}$$

$$\beta_2 = \frac{1}{4} \frac{A_{vv}}{A_o}$$

$$\delta_1 = \frac{1}{2} \frac{B_{uu}}{B_o} - \frac{3}{4} \frac{A_{uu}}{A_o} + \frac{9}{2} \left(\frac{B_u}{B_o} \right)^2$$

$$\delta_2 = \frac{1}{2} \frac{B_{uu}}{B_o} - \frac{3}{4} \frac{A_{uu}}{A_o}$$

As stated before the value of θ_o is obtained by maximizing the exponential terms in (2.5.11). Since these terms are different for D_1 and D_2 , the values of θ_o will be different for the 2 integrals. In order to find this maximum it is easier to go back to the original (x,y,z) coordinate system and to minimize

$$(2.5.16) \quad Ar^2 - 2Br - C =$$

$$= \frac{(x - \rho)^2 + y^2}{2\sigma_\rho^2} + \frac{(z - h_o)^2}{2\sigma_z^2} + kr$$

where $r^2 = x^2 + y^2 + z^2$ and $k = \mu$ or $(1-D) \mu$ respectively.

Differentiation of (2.5.16) gives the conditions

$$(2.5.17) \quad \frac{x - \rho}{\sigma_\rho^2} + k \frac{x}{r} = 0$$

$$\frac{y}{\sigma_\rho^2} + k \frac{y}{r} = 0$$

$$\frac{z - h_0}{\sigma_z^2} + k \frac{z}{r} = 0$$

for the maximum. The second equation in (2.5.17) yields $y = 0$ and we obtain for x and z the equations:

$$x = r \cos \theta_0 = \frac{r \rho}{r + k \sigma_\rho^2}$$

$$z = r \sin \theta_0 = \frac{r h_0}{r + k \sigma_z^2}$$

and finally

$$(2.5.18) \quad \cos \theta_0 = \frac{\rho}{r + k \sigma_\rho^2}$$

$$\sin \theta_0 = \frac{h_0}{r + k \sigma_z^2}$$

From these equations follows that

$$(2.5.19) \quad \frac{\rho^2}{(r + k\sigma_\rho^2)^2} + \frac{h_o^2}{(r + k\sigma_z^2)^2} = 1$$

which gives a condition for the value of r at which the minimum of (2.5.16) occurs. Solving (2.5.19) for r and substituting this value in (2.5.18) gives the desired values of $\cos \theta_o$ and $\sin \theta_o$. Equation (2.5.19) leads to a 4th degree polynomial equation which can be solved without difficulty by Newton's method.

Approximation (2.5.15) is valid if the coefficients for the quadratic terms in the exponent of (2.5.14) are large; in particular the term B_o^2/A_o should be large. If the solution of (2.5.19) is $r = r_o$ then this term becomes approximately

$$(2.5.20) \quad \frac{B_o^2}{A_o} \cong \frac{r_o^2}{2\sigma^2}$$

where σ is some value between σ_ρ and σ_z . This means that r_o should be large compared to σ . The value r_o is the distance between the center of the cloud and the receptor modified by the absorption factor, i.e., the distance as it would appear to the receptor if there is no absorption of the γ rays. This distance should be large compared to the dispersion coefficient in order that the cloud can be approximated by a point source. Numerical experiments have shown that direct integration and approximation agree within less than 5% if $r_o \cong 7\sigma_\rho$, so that direct integration is chosen if $r_o < 7\sigma_\rho$ and approximation by a point source otherwise.

2.5.2 Program Organization

In this program the coefficients which depend only on the dispersion parameters σ are computed first followed by a computation of the remaining coefficients and the internal dose. A test is performed to determine whether direct approximation or numerical integration should be used followed by the appropriate computational procedure. A function subroutine EVAL provides the values $I_0(x)$ and $I_1(x)$ respectively. Tests are used to suppress the computation of values which are too small to be of interest and may cause exponent underflow in the computer.

2.6 Description of the Auxiliary Programs

2.6.0 Introduction

A description of the auxiliary routines is given here primarily in order to enable the user of the program to understand the FORTRAN coding and to trace possible malfunctions. This information is dispensable for routine usage and interpretation of results.

2.6.1 The Subroutines NEXTRHO and NEXTRHOL

These programs are designed to find the nodes and weights for the numerical quadrature used in the evaluation of the integrals (1.7) and (1.8) respectively. The integrands behave in both cases similar to a Gaussian curve. In order to obtain sufficient precision a sufficiently large number of nodes has to fall into the region where the integrand is close to the maximum value. We shall call this interval the "middle part" of the integral; the other parts where the integrand goes rapidly to zero shall be called the "tail ends." In the integral (1.8) the middle part consists of an interval centered at the time point when the cloud is closest to the receptor and has length

$$\frac{4\sigma_r}{u}, \text{ or } \frac{3}{u} \sqrt{\frac{y}{\mu}}$$

whichever is larger, where

σ_r = the horizontal dispersion at the center of the cloud closest to the receptor

y = the shortest distance between the receptor and the center of the closest cloud

μ = the absorption coefficient for γ -rays at the particular energy level

u = velocity of wind

These values were obtained by observing a large number of integrals (1.4) for various values of r and E . A 7 point Gauss-Legendre formula is used for numerical integration over this interval. The integrals over the tail end are obtained from a 4 point Gauss-Laguerre formula owing to the fact that the integral (1.4) behaves like $e^{-\mu r}$ for large values of r .

Only one tail end exists if the receptor is located close to or behind the stack relative to the wind direction. In this case the middle part may be shortened if necessary. If a small interval of integral (1.8) still exists in front of the designated middle part but is not long enough to accommodate all nodes of the Gauss-Laguerre formula, this formula is then replaced by a 4 point Gauss-Legendre formula.

The integral (1.7) is evaluated by an 11 point Gauss-Hermite formula which assumes that the integrand behaves asymptotically like

$$e^{-\frac{\tau^2 \mu^2}{2\sigma_r^2}}.$$

The integration formulas can be changed to ones containing more (up to 20) or fewer points by reading in the appropriate nodes and weights. If the receptor is near or behind the stack the integration must be modified since in these cases the maximum of the density R_{int} is no longer the point where the receptor is closest to the cloud. The dispersion at the receptor location may be so small that a significantly larger amount of radioisotopes comes from clouds which are farther away but whose dispersion is larger. For this reason the true maximum of R_{int} is estimated and the integration nodes are positioned in reference to this maximum as center and using the dispersion at this point for σ_r . The integral (1.7) is changed to (1.7a) by formally changing the 11 point Gauss-Hermite formula to a 1 point formula with the appropriate weight function.

2.6.2 The Subroutines MULT, MMULT, SOLVE and TRANSP

These subroutines are designed to effect certain matrix operations for lower triangular matrices. All matrices and vectors are assumed to be stored in the same large one-dimensional array in compact form. That is the elements a_{ik} of a lower triangular matrix are stored in the sequence $a_{11}, a_{21}, a_{22}, a_{31}, a_{32}, a_{33}, \dots, a_{n,n-1}, a_{nn}$ starting at a specified location. Column vectors are stored as usual.

The subroutine $MULT(V, N, N1, N2, N3)$ performs the multiplication of a $N \times N$ lower triangular matrix which is stored in array V beginning at location $N1$ with a column vector stored in the same array starting at location $N2$ and stores the resulting vector again in V beginning at location $N3$. The resulting vector may replace or overlap either the input vector or part of the input matrix.

The subroutine MMULT (V,N,N1,N2,N3) multiplies two matrices at N1 and N2 and stores the result at N3.

The subroutine SOLVE (V,N,N1,N2,N3) finds the solution of the linear equation $AX = B$ where A is a $N \times N$ matrix stored at N1 and B is a column vector stored at N2. The solution vector X is then stored at N3. The original vectors and matrices are not changed in MULT, MMULT, and SOLVE except perhaps through overlapping with the output.

The subroutine TRANSP (V,N,N1) changes the sequence of the elements of a $N \times N$ matrix which is stored in V beginning at location N1 as described above. This sequence is changed to the sequence $a_{nn}, a_{n,n-1}, a_{n,n-2}, \dots, a_n, a_{n-1,n-1}, a_{n-1,n-2}, \dots, a_{n-1,1}, \dots, a_{22}, a_{21}, a_{11}$. This converts effectively the matrix into its transpose. The original matrix is destroyed in the process.

2.6.3 The Subroutines XMULT, XMMULT, and XDIFF

These subroutines perform the same operations as MULT, MMULT, and DIFF respectively but they do this for a string of matrices and vectors instead of a single matrix. The information about the string is contained in an integer array IND which contains the orders n of the vectors in the first N locations and the number of elements in the matrices, i.e., $n(n+1)/2$ in locations $N + 1$ through $2N$. A zero or negative integer in IND terminates the process. All matrices and vectors are located in the same array W and N1, N2, N3 signifies the first elements in the strings of matrices and/or vectors.

2.6.4 Subroutine NEXT

The subroutine NEXT (W,IND,IDX,N,K,NMAT,NVEC) performs successive multiplications of a string of vectors by a string of matrices. These

matrices are fundamental solutions of a differential equation as obtained by DIFF for a given time interval Δt and the vectors represent given initial conditions. The subroutine NEXT converts the solution already obtained for the time t into a solution for $t + \Delta t$. To hold the loss of precision due to repeated matrix multiplications to a minimum, fundamental solutions are obtained for time intervals $2\Delta t$, $4\Delta t$, ..., $2^K \Delta t$ by repeated squaring of the original matrix and these matrices are used to reduce the number of matrix multiplication. The array IDX directs the program to select the appropriate matrix multiplication. The array IND enables the program to handle strings of matrices and vectors as in XMULT, ect. The numbers NMAT and MVEC represent the first elements of the strings of matrices and vectors which are all located in the array W. The numbers NMAT and NVEC indicate the total storage space set aside for one full string of matrices or vectors respectively.

2.6.5 The Function Subroutine EVAL

This function subroutine evaluates the integrals $I_1(X)$ and $I_0(X)$ introduced in formula (6), (7), and (8) in 2.2.5. Here the parameter RO in EVAL(RO,J) describes the value of X in $I_i(X)$ and J describes the value of a in (2.5.8) selected from an array named RI(J). These and all other parameters of the function as well as the coefficients for the approximating polynomial are communicated to the program via COMMON/COEFF/.

2.7 COMMON-Storage and BLOCK DATA

2.7.0 Introduction

In order to accommodate communication between the programs and because of the large number of variables involved, all but local variables are stored in four different blocks of COMMON variables. These blocks

are named GEO, INTEGR, COEFF, and ISOTOP. A detailed list of these variables is given below. Some of the space allotted to certain variables is used as working space in other parts of the program in order to save storage space.

2.7.1 Contents of COMMON-Storage

COMMON-BLOCK GEO:

H = height of the receptor above ground.

HI = height of inversion layer.

IH = number of real and virtual clouds:

IH = 1: Receptor on ground, no inversion layer present.

IH = 2: Receptor above ground or inversion layer present.

IH = 4: Receptor above ground and inversion layer present.

(The preceding variables are temporary storage locations for use as input in INT.)

IHI = indicator for inversion layer. Inversion layer present if
IHI $\neq$ 0.

HS = adjusted stack height, i.e., actual height of cloud at specified location.

UWIND = wind velocity.

IELT = initial time t_0 as used in RELEASE in number of units TAU.

SIGMAR = σ_r , horizontal dispersion on specified locations.

SIGMAZ = σ_z , vertical dispersion.

DEXT = external dose.

DINT = internal dose.

(DEXT and DINT are temporary storage locations used as output for INT)

NTAU = number of time intervals Δt to be computed in MAIN and RELEASE.

TAU = Δt , time interval.

NSPACE = number of receptor locations.

NTIME = number time points.

X,Y = coordinates of the receptor.

NT = time points with TAU as unit.

HX,IHX = same as H and IH for permanent storage.

XTAU = TAU * UWIND

RHO = ρ , horizontal distance between receptor and cloud.

WRHO = weight function, temporary storage.

ITIN = counter for integration nodes.

SIG = dispersion measure at receptor location, used for determining the nodes of integration.

DC = external dose.

DIC = internal dose.

(DC and DIC are permanent storage.) These locations serve also as working space for RELEASE prior to the integration in MAIN.

COMMON-BLOCK COEFF:

CA,CB,C = coefficients to approximate the integrals (2.5.5) by polynomials in EVAL.

R1 = integration limit r_1 in (2.5.5).

IR = indicator, IR = 0 if $r_1 = \infty$.

IND = index, IND = 1: D_1 is computed.

IND = 2: D_2 is computed.

[See formula (2.5.5).]

CX,CX1 = exponential functions used in EVAL. [See formula (2.5.8).]

$$SQPI = \sqrt{\pi}$$

EXT = underflow limit for exponential function.

NINT1, NINT2, NTAUIN, NTAUIN1 = number of points used for integration (1.7)

and (1.8) in MAIN.

XINT1, XINT2, XINT3, XINT4 = nodes of integration.

WINT1, WINT2, WINT3, WINT4 = weight functions for the above integrals.

COMMON-BLOCK INTEGR

$$PI = \pi$$

$$SQ2 = 1/\sqrt{2}$$

$$TPI = (2\pi)^{-3/2}$$

NB, NE = first and last number of the array of values for integration over the elevation angle Θ in INT. NE-NB+1 is the number of points used in this integration (see formula 2.5.10).

NTHETA = number of positive nodes in the above integration.

$$THES = \sin \Theta$$

$$THEC = \cos \Theta \text{ [see (2.5.10)]}$$

$$SQS = \sin^2 \Theta$$

$$SQC = \cos^2 \Theta$$

W = weight function for the above integration.

PHIC = $\cos \varphi$, used for integration over the azimuth angle φ .

DEPS = precision limit, used for terminating iteration procedures.

$$EPSPHI = 1 + DEPS$$

COMMON-BLOCK ISOTOP

$$CE = C, \text{ formula (1.3).}$$

$$DE = D, \text{ formula (1.3).}$$

$$MU = \mu, \text{ formula (1.3).}$$

(The above are temporary storage locations.)

INL = indicator, INL $\neq$ 0 directs the program to compute only the internal dose.

NEG = number of gamma energy levels.

ACE,ADE,AMU = permanent storage for CE,DE,MU.

ASIGMA = $\sigma(E_k)$ in (1.5).

AFE = $f_i(E_k)$ in (1.5).

ALPHA = exhaust rate α , see (2.4.1).

NFAM = array containing the number of members in each family of isotopes.

REL = array containing the results of subroutine RELEASE for given time intervals.

Q = initial amount of radioactive material, see (2.4.2).

XLAMDA = decay rate λ_i , see (2.4.1).

PHI,PSI= filter constants φ_i , ψ_i , see (2.4.1).

DELTA = radiation dose δ_i , see (1.2).

SDEV = working space used in MAIN for computing S_{ij} , see (2.1.2) through (2.1.5).

2.7.2 List of Block DATA

Several sets of data are given initially through BLOCK DATA statement. Some of them can be overridden through card input. For details see the users guide 3.1. Specifically the following data are given:

1. Energy levels and related constants: NEG,AMU,ASIGMA,ACE,ADE,AERG (AERG in SETUP).
2. Time intervals: TAU,NTAU.
3. Coefficients for EVAL: CA,CB.
4. Integration constants: NB,NE,NTHETA,THES,W,NINT1,NINT2,NTAUIN,NTAUIN1,XINT1,XINT2,XINT3,XINT4,WINT1,WINT2,WINT3,WINT4.

5. Precision constants: EXT, DEPS, EPSPH1.

6. PI, SQPI.

3. User's Guide

3.1 Preparation of Input Data

3.1.0 Introduction

Data are transmitted to the program via input cards. There are several different sets of data which will be described in detail later. Each data set is preceded by an ID-card which identifies the set. Thus the sets need not to be read in a specific sequence although set 1 has to be read before set 4 and 5 and set 4 before set 5. ID-card 7 indicates that a full data set has been read and initiates processing. After each processing step some or all of the data sets may be changed for the next processing step. A new set 1 necessitates new sets 4 and 5 and a new set 4 must be followed by a new set 5. The processing is terminated by ID-card 9. All ID-cards carry their respective numbers in column 5. The card format for real values (decimal with or without exponent) is E15.7, i.e., 15 columns are reserved for each such value. All values are adjusted to the right.

The following options which affect the processing may be used:

- i. Suppress integration in the evaluation of the internal dose, i.e., use formula (1.7a) instead of (1.7). See 3.1.6 set 3.
- ii. Suppress computation of the external dose. See 3.1.1.

3.1.1 Energy Levels (Optional)

A standard set of energy levels is part of the program (see 2.7.2). A different set may be read but this must precede the reading of isotope and reactor data since energy levels are needed to prepare these data.

With each energy level E_γ a set μ , C, D of formula (1.3) and $\sigma(E_\gamma)$ as in formula (1.5) is read.

Card Format:

ID-Card: 1 in column 5.

Data-Card 1:

NEG = number of energy levels (≤ 20) col. 1-10.

Data-Cards 2: (there are NEG such cards)

AERG = E_γ = Energy level. Col. 1-15

ACE = C in (1.3). Col. 16-30

ADE = D in (1.3). Col. 31-45

AMU = μ in (1.3). Col. 46-60

ASIGMA = $\sigma(E_\gamma)$ in (1.5). Col. 61-75

Data-cards 2 may be read in any arbitrary sequence. The program rearranges the values E_γ in ascending sequence and eliminates duplications.

A zero in col. 10 of data card 1 indicates to the program that no external doses need to be computed. Data cards 2 may not be given in this case. This special set (i.e., ID-card with 1 in col. 5 followed by a card containing 0 in col. 10) does not interfere with the processing of isotope data and may therefore be read after set 4 or 5.

3.1.2 Coordinates

This set contains the locations of the reactor and receptors plus the desired time intervals.

Card Format:

ID-Card: 2 in column 5.

Data Card 1:

NSPACE = number of receptor locations (≤ 100) col. 1-5.

NTIME = number of time intervals (≤ 20) col. 6-10.

XR = x-coordinate of reactor, col. 11-25.

YR = y-coordinate of reactor, col. 26-40.

Data Card 3: (there are NSPACE such cards)

X = x-coordinate of receptor col. 1-15.

Y = y-coordinate of receptor col. 16-30.

HX = height of receptor above ground, col. 31-45.

Data Cards 4: (there are NTIME values, 5 per card)

XT = elapsed time starting from the time of accident, 5 values per card
in col. 1-15, 16-30, 31-45, 46-60, 61-75.

These values will be destroyed during processing. A new coordinate system is computed with x-axis in wind direction and origin at the reactor location.

3.1.3 Meteorological Data

These data contain the meteorological conditions at various distances from the reactor in the direction of wind. Intermediate values will be obtained by interpolation.

Card Format:

ID-Card: 3 in col. 5.

Data-Card 1:

MET = number of points on which meteorological information is given^{*}
(≤ 100) col. 1-10.

VEL = wind velocity, col. 11-25.

DIR1 = wind direction in degrees, counterclockwise from the positive
x-axis. Col. 26-40

* If MET = -1 only data card 1 is read; if MET = 0 data cards 1 and 2 are read but not 2.

Data-Cards 2: (there are MET such cards)

DIST = distance from reactor, col. 1-15.

SH = horizontal dispersion σ_r , col. 16-30.

SV = vertical dispersion σ_z , col. 31-45.

HSS = adjusted stack height, col. 46-60.

Data-Card 3:

IHI = 0 if no inversion layer,

1 if inversion layer, col. 5.

HT = height of inversion layer (if any), col. 6-20.

Data-cards 2 can be read in any sequence. Program arranges the DIST-values in ascending order.

3.1.4 Isotope Data

There is a set of cards for each isotope which may be read in any sequence. Up to 300 isotopes may be considered at any one processing run. A blank card terminates the reading of isotope data, unless there are exactly 300 isotopes.

Only straight chains of isotopes can be handled by the program like $A \rightarrow B \rightarrow C \rightarrow \dots$, i.e., B is the daughter of A and C is the daughter of B, ect. If branching occurs, say

$$\begin{array}{c} A \rightarrow B \rightarrow \begin{array}{l} \nearrow C_1 \rightarrow \\ \searrow C_2 \rightarrow \end{array} \end{array}$$

then the chain must be separated into 2 chains like $A_1 \rightarrow B_1 \rightarrow C_1 \rightarrow \dots$

and $A_2 \rightarrow B_2 \rightarrow C_2$ and the isotopes A_1, A_2 , ect, must be given different identification, even if they represent the same isotope. The same is true if an isotope appears in two different chains.

Card Format:

ID-Card: 4 in col. 5.

Data Card 1: (one for each isotope)

JNAME = identification symbol for the isotope consisting of up to 8 characters in col. 1-8 starting at col. 1 (e.g., I-138).

Blanks in col. 1-8 terminate reading of isotope data.

JDAUGH = identification symbol of radioactive daughter of isotope JNAME, up to 8 characters in col. 11-18, starting at col. 11. Space may be left blank if element is stable or radiation is negligible.

YLAMDA = rate of decay, col. 21-35.

YDELTA = δ_i in formula (1.2) col. 36-50.

TERG = total energy of γ -radiation, col. 51-65.

NERG = number of different energy levels at which γ -rays are emitted, col. 66-75.

Data-Cards 2: (there are NERG cards for each isotope)

ERG = energy level E_k , col. 1-15.

YFE = $f_i(E_k)$ in formula (1.5), col. 16-30.

The partial energies given in data cards 2 must be consistent with the total energy on card 1. Otherwise an error message is printed. Duplication of isotope data is also rejected.

3.1.5 Reactor Data

Card Format:

ID-Card: 5 in column 5.

Data-Card 1:

ALPHA = exhaust rate α (see 2.4.1), col. 1-15.

FR = fraction of isotopes released from core into the reactor containment,
col. 16-30.

Data-Cards 2: (one and only one card for each isotope given in data set 4)

JNAME = identification symbol for isotope, up to 8 characters in col. 1-8
starting at col. 1. Blanks in col. 1-8 terminates reading of data
set 5.

YQ = amount of material in curies/kw, col. 11-25.

YPHI = filter constant ϕ_i (see 2.4.1), col. 26-40.

YPSI = filter constant ψ_i (see 2.4.1), col. 41-55.

Data cards 2 may be read in any sequence but each isotope JNAME which appears on a data card 1 of set 4 must also appear on the corresponding place in data card 2 of set 5 and vice versa. Otherwise an error message is given. Isotopes which are not generated in the reactor but are decay products of the primary fission products are included by setting the value of YQ in col. 11-25 to zero. If branching occurs and an isotope appears with different identifications in set 4 then a separate card must be given for each identification with YQ values distributed according to the branching ratios. The last card of set 5 must be blank in col. 1-8. As pointed out before, set 5 can be read only after set 4 is completed.

3.1.6 Integration Constants

The nodes and weights for numerical integration are given initially to the program via BLOCK DATA statements. These may be changed if desired for either higher accuracy or increased computing speed. There are three set of such constants. Set #1 contains the constants for integration over the elevation angle in the subroutine INT. Set #2 contains the constants

for evaluating the external dose as used in subroutine NEXTRHO and set #3 the corresponding values for evaluating the internal dose used in NEXTRH01. Each set must be read separately starting with an ID-card containing the number 6 in column 5 followed by another ID-card which contains either 1, 2, or 3 in column 5 depending which set is to be read.

The nodes x_i and weights w_i can be obtained from tables for instance A. H. Stroud and Don Secrest, Gaussian Quadrature Formulas, Prentice Hall 1966, which was used for our initial data. The values are normalized as follows:

Gauss-Legendre:

$$\int_{-1}^{+1} f(x) dx \approx \sum_{i=1}^n w_i f(x_i)$$

Gauss-Laguerre:

$$\int_0^{\infty} e^{-x} f(x) dx \approx \sum_{i=1}^n w_i f(x_i)$$

Gauss-Hermite:

$$\int_{-\infty}^{+\infty} e^{-x^2} f(x) dx = \sum_{i=1}^n w_i f(x_i)$$

These values will be modified in the program for the specific application. Of course, only nonnegative values x_i and related w_i need to be given for Gauss-Legendre and Gauss-Hermite formulas.

Card Format:

Set #1:

1st ID-card: 6 in col. 5

2nd ID-card: 1 in col. 5

Data Card 1:

NTHETA = order of integration (≤ 9), col. 10.

In this Gauss-Legendre formula NTHETA is the number of positive nodes. These and related weights are read from data cards 3. This set is preceded by data card 2 which contains the information about the midpoint; if this is a node, i.e., $x_i = 0$, odd number of nodes, then the data card 2 contains the corresponding weight w_i . Otherwise, i.e., even number of nodes, $w_i = 0$. The latter should be avoided because of computational inefficiency. The format is:

Data Card 2:

THES(10) = 0. (Midpoint of integration, value is not used), col. 1-15.

W(10) = w_i at midpoint or 0, col. 16-30.

Data Cards 3: (There are NTHETA such cards)

THES(I) = x_i , positive nodes in ascending order, col. 1-15

W(I) = w_i , related weights, col. 16-30.

Set #2:

1st ID-card: 6 in col. 5

2nd ID-card: 2 in col. 5

Data-Card 1:

K1 = number of nodes in the Gauss-Legendre formula applied to the
"middle part," (≤ 20), col. 9-10.

K2 = number of nodes in the "tail end," (≤ 20), col. 19-20.

There are three different integration formulas and consequently data sets to be read. The first contains nodes and weights for the Gauss-Legendre formula applied to the middle part. There are $n_1 = [(K1+1)/2]$ nonnegative nodes to be read in descending order. The next set contains $n_2 = K2$ nodes of the Gauss-Laguerre formula for the tail ends to be read in ascending order. Finally there are the $n_3 = [(K2+1)/2]$ nonnegative nodes of the Gauss-Legendre formula which replaces the Gauss-Laguerre formula at the tail end close to the reactor to be read in descending order. The card format for all values is the same namely:

Data Cards 2: (There are $n = n_1 + n_2 + n_3$ such cards)

XINT 1,2,3 = nodes x_i , col. 1-15.

WINT 1,2,3 = related weights w_i , col. 16-30.

Set #3:

1st ID-card: 6 in col. 5

2nd ID-card: 3 in col. 5

Data Card 1:

MN = number of nodes in the Gauss-Hermite formula (≤ 18), col. 9-10.

There are $n = [(MN+1)/2]$ nonnegative nodes to be read in descending order from the subsequent set of cards. The format is as before namely:

Data Cards 2: (There are n such cards)

XINT⁴ = nodes x_i , col. 1-15.

WINT⁴ = related weights w_i , col. 16-30.

If formula (1.7a) is desired for computing the internal dose, the data card 1 should contain a 1 for MN in col. 10 and the values for XINT⁴ and WINT⁴ in data card 2 are arbitrary (e.g., 0).

3.1.7 Miscellaneous Constants

An ID-card with an 8 in col. 5 initiates the reading of various constants whose change may be desired at one time or another. There is only one data card in this set. The format is:

Data Card:

ID = identification number, col. 5.

DATA1, DATA2 = data specified by ID in col. 6-20 and 21-35.

If ID = 1 then DATA1 = TAUX and DATA2 = NTAU. The value TAUX is the minimum value for Δt as explained in formula (2.1.3), (2.1.4) and the related values in 2.4. NTAU $\leq$ 100 is the maximum number of intervals Δt for which the radioactive decay is computed. Small values of Δt may be desirable to obtain higher accuracy for receptors near the reactor.

If ID = 2 the breathing rate β is changed, DATA1 = β . This change must be made before isotope and reactor data are read in.

If ID = 3 the precision constants DATA1 = DEPS and DATA2 = EXT are changed. The present value of DEPS is 10^{-6} and is used for the precision limit in an adaptive integration in INT. It is also used to terminate an iteration procedure for direct approximation of the integral DEXT in INT. The value EXT, which is 100 at present is used to avoid underflow in the computation of the exponential function, that is e^{-x} is set to 0 if $x > EXT$. A larger value for DEPS and a smaller one for EXT may speed up the computation at some sacrifice in accuracy.

3.2 Interpretation of Output Data and Error Messages

3.2.1 Confirmation of Input

All input data are confirmed by printouts either immediately after they are received by the computer or after the first sorting as in set #1 (energy levels) and set #3 (meteorological data). Prints are also made after each significant processing step to give the user additional checks for the validity of his data. The two most important prints are a list of isotopes arranged in families and the distribution of gamma energy for each isotope in the fixed set of energy levels. Integration constants and the dispersion measures on which the distribution of integration nodes is based are also printed.

3.2.2 Error Messages

Most error messages from the computer are self-explanatory. The only exception is the string of status variables which is printed in connection with "DATA INCOMPLETE," indicating a request for more data. To each data set I there is assigned a status indicator INIT(I) whose values are -1 if no data have been received, 0 if data are received but not processed and +1 if data are received and processed. In connection with the error message a -1 in any of the sets #2, 3, 4, or 5 means that these data sets must be read before the next processing run. For instance the output (1) -1, (2) 1, (3) 1, (4) 0, (5) -1, etc., would mean that a new set of isotope data (#4) has been read but the corresponding reactor data (#5) are missing. The 1's in (2) and (3) indicate that coordinates and meteorological data are already processed in a previous run and the -1 in (1) indicates that the original levels of gamma energy are used. Since data other than #2-5 are optional, a -1 for any other set does not indicate a request for such data.

Any of the error messages concerning input data may also be caused by an incorrect number of cards in any of the input sets. An identification card may then be interpreted as a data card and vice versa, producing an error message.

3.2.3 Interpretation of Results

In discussing the results of this program the primary concern will be of course the question about the reliability. There are two sources of error which could lead to inaccurate results. The first is due to the limitations in the precision of the computational process and the second is caused by the limitations of the mathematical model itself and the inaccuracy of the input values.

Inaccuracies in the computation can be determined with relative confidence. Although strict theoretical limits are hard to obtain numerical experiments indicate that the combined errors will rarely exceed 5% and never cause any dangerous underestimation of radiation by about more than 20%.

In contrast the limitation of the model in this computation is a much more serious matter. The weakest link in this respect is the meteorological model. It must remain doubtful whether the distribution within the cloud is Gaussian and if so what the correct dispersion coefficients are. Empirical data on this are sparse. (See for instance: "Meteorology and Atomic Energy," AECU-3066, USAEC Publication 1455.) If the receptor is at some distance from the reactor, say more than 5 km, then the concentration in the neighborhood of the receptor can be considered constant and is about inversely proportional to the square of the dispersion constant. Thus a reasonable selection of the dispersion constant will give a reasonable result regardless of whether the distribution is Gaussian or not.

2

The dispersion is, however, very uneven near the reactor and almost certainly not Gaussian because of the influence of air turbulence. This is of little consequence for gamma radiation since the bulk of it comes from an area of high concentration which may be considered a point source. (This does not take into account the radiation which originates in the interior of the reactor.) For the internal dose, however, the knowledge of the exact distribution of the radioactive material is of outmost importance. Even if we assume that the true distribution can be reasonably approximated by a Gaussian, a small change in the dispersion coefficients may lead to a change of the internal dose by several orders of magnitude. To obtain reasonable results for the internal dose of receptors close (say less than 500 m) to the reactor a realistic set of dispersion constants is necessary, combined with change of $\lambda\delta$ to about .1 min as described in 3.1.7 in order to assure full utilization of these dispersion constants. No receptors distant from the reactor may then be entered in this particular run of the program since otherwise the value of $\lambda\delta$ would automatically be enlarged to cover the whole range of receptor locations. Even so, no reliable data can be obtained by this program for receptors which are closer to the stack exit than about 100 m, and this is true also for the external dose.

APPENDIX: SAMPLE PROBLEM

On the following pages a listing of the computer output for a sample problem is given. The receptor positions 1. to 7 are at distances of 100 m, 200 m, 500 m, 1,000 m, 2,000 m, 5,000 m, and 10,000 m respectively. Exposure times are 1 hour and 24 hours. This is listed in the first output sheet as a confirmation of the input data. The second sheet lists dispersions and other meteorological data. Next, a list of isotope data is given in the same sequence as they have been fed into the computer. This is followed by the amount of each isotope present at the time of the accident in curies/kw. The filter constants PHI and PSI are assumed to be zero, i.e. no filters are present.

The sequence of outputs lists the results of source data preparation, starting with a list of parameters used in numerical integration. Next is a list of the isotopes with the corresponding parameters modified for internal use. The gamma energy levels for each isotope are listed next after normalization to the 14 fixed levels supplied by the program. The last list concerning isotopes gives the distribution of the isotopes to families.

The actual output is preceeded by a list of a dispersion measures, for each receptor location, which is a measure for the distribution of nodes for the numerical integration. This is followed by the actual dose rates separate for each exposure time.

The value of $\text{TAU} = \Delta t$ for this computation is 15 min. which translates into space intervals of $15 \cdot 240 = 3,600$ m. That is the dispersion

measures listed under "meteriological data" are linearly interpolated over these distances, effectively eliminating the information for closer distances given in this table. In order to utilize this information the value of TAU was reduced to .6 min. as indicated in the next listing and the receptors which are farther from the receptor than 1,000 m were eliminated. The ensuing computation gave substantially larger values for the internal dose which is primarily due to the increased vertical dispersion at close distances. The external dose is less affected since gamma rays travel over longer distances. The numbers 59 and 1439 instead of 60 and 1440 min are round off errors in the display due to the smaller time increments.

COORDINATES

REACTOR LOCATION	X=	0.0	Y=	0.0	
RECEPTOR COORDINATES		X		Y	HEIGHT
		0.1000000E 03		0.0	0.0
		0.2000000E 03		0.0	0.0
		0.5000000E 03		0.0	0.0
		0.1000000E 04		0.0	0.0
		0.2000000E 04		0.0	0.0
		0.5000000E 04		0.0	0.0
		0.1000000E 05		0.0	0.0

TIME COORDINATES	TIME AFTER ACCIDENT
	0.6000000E 02
	0.1440000E 04

METEOROLOGICAL DATA

WIND VELOCITY=	0.2400000E 03	WIND DIRECTION=	0.0	
DISTANCE FROM REACTOR		HOR. DISP.	VERT. DISP.	ADJUSTED STACK HEIGHT
0.1000000E 03	0.1200000E 02	0.7500000E 01	0.1000000E 03	
0.2000000E 03	0.2300000E 02	0.1400000E 02	0.1000000E 03	
0.3000000E 03	0.3400000E 02	0.2100000E 02	0.1000000E 03	
0.5000000E 03	0.5500000E 02	0.3400000E 02	0.1000000E 03	
0.7000000E 03	0.7800000E 02	0.4600000E 02	0.1000000E 03	
0.1000000E 04	0.1050000E 03	0.6500000E 02	0.1000000E 03	
0.2000000E 04	0.2000000E 03	0.1180000E 03	0.1000000E 03	
0.3000000E 04	0.2900000E 03	0.1650000E 03	0.1000000E 03	
0.5000000E 04	0.4550000E 03	0.2550000E 03	0.1000000E 03	
0.7000000E 04	0.6250000E 03	0.3350000E 03	0.1000000E 03	
0.1000000E 05	0.8600000E 03	0.4500000E 03	0.1000000E 03	
0.2000000E 05	0.1590000E 04	0.7650000E 03	0.1000000E 03	

NO INVERSION LAYER

ISOTOPE DATA

KR83M DECAYS INTO KR83

LAMDA= 0.6210998E-02 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.2695170E-02 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.2695170E-01
	0.1500000E 00	0.0
	0.2000000E 00	0.0
	0.3000000E 00	0.0
	0.4000000E 00	0.0
	0.5000000E 00	0.0
	0.6000000E 00	0.0
	0.8000000E 00	0.0
	0.1000000E 01	0.0
	0.1250000E 01	0.0
	0.1500000E 01	0.0
	0.2000000E 01	0.0
	0.3000000E 01	0.0
	0.4000000E 01	0.0

KR85M DECAYS INTO KR85

LAMDA= 0.2650000E-02 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.1616390E 00 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.7717699E 00
	0.2000000E 00	0.1623000E-01
	0.3000000E 00	0.1337300E 00
	0.4000000E 00	0.6269999E-02
	0.5000000E 00	0.0
	0.6000000E 00	0.0
	0.8000000E 00	0.0
	0.1000000E 01	0.0
	0.1250000E 01	0.0
	0.1500000E 01	0.0
	0.2000000E 01	0.0
	0.3000000E 01	0.0
	0.4000000E 01	0.0

KR87 DECAYS INTO RB87
 LAMDA= 0.9072997E-02 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.7754239E 00 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.2200000E-03
	0.1500000E 00	0.3099998E-03
	0.2000000E 00	0.0
	0.3000000E 00	0.0
	0.4000000E 00	0.4699600E 00
	0.5000000E 00	0.1369000E-01
	0.6000000E 00	0.1185000E-01
	0.8000000E 00	0.7121998E-01
	0.1000000E 01	0.2321000E-01
	0.1250000E 01	0.1362000E-01
	0.1500000E 01	0.2005000E-01
	0.2000000E 01	0.9007999E-01
	0.3000000E 01	0.8083999E-01
	0.4000000E 01	0.1400000E-02

KR88 DECAYS INTO RB88
 LAMDA= 0.4125997E-02 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.2030669E 01 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.7571995E-01
	0.2000000E 00	0.3720599E 00
	0.3000000E 00	0.1301000E-01
	0.4000000E 00	0.2587000E-01
	0.5000000E 00	0.4339997E 02
	0.6000000E 00	0.2200000E-03
	0.8000000E 00	0.1171800E 00
	0.1000000E 01	0.5884000E-01
	0.1250000E 01	0.3798000E-01
	0.1500000E 01	0.1245700E 00
	0.2000000E 01	0.4829800E 00
	0.3000000E 01	0.1918200E 00
	0.4000000E 01	0.0

RB88 DECAYS INTO SR88
 LAMDA= 0.3894000E-01 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.6923820E 00 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.0
	0.2000000E 00	0.0
	0.3000000E 00	0.0
	0.4000000E 00	0.0
	0.5000000E 00	0.0
	0.6000000E 00	0.0
	0.8000000E 00	0.7340997E-01
	0.1000000E 01	0.7058996E-01
	0.1250000E 01	0.4009999E-02
	0.1500000E 01	0.8194995E-01
	0.2000000E 01	0.1725900E 00
	0.3000000E 01	0.2586000E-01
	0.4000000E 01	0.3090000E-02

KR89 DECAYS INTO RB89
 LAMDA= 0.2180000E 00 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.2068020E 01 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.8019999E-02
	0.1500000E 00	0.9839997E-02
	0.2000000E 00	0.2018800E 00
	0.3000000E 00	0.1034600E 00
	0.4000000E 00	0.1078300E 00
	0.5000000E 00	0.1779300E 00
	0.6000000E 00	0.2909300E 00
	0.8000000E 00	0.1580499E 00
	0.1000000E 01	0.1340899E 00
	0.1250000E 01	0.8767998E-01
	0.1500000E 01	0.2948400E 00
	0.2000000E 01	0.1781400E 00
	0.3000000E 01	0.1115100E 00
	0.4000000E 01	0.4613000E-01

RB89 DECAYS INTO SR89

LAMBDA= 0.4560000E-01 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.2381109E 01 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.0
	0.2000000E 00	0.5559999E-02
	0.3000000E 00	0.2144000E-01
	0.4000000E 00	0.0
	0.5000000E 00	0.0
	0.6000000E 00	0.7613999E-01
	0.8000000E 00	0.6061000E-01
	0.1000000E 01	0.6080300E 00
	0.1250000E 01	0.5397200E 00
	0.1500000E 01	0.2877000E-01
	0.2000000E 01	0.2456200E 00
	0.3000000E 01	0.1361099E 00
	0.4000000E 01	0.1350000E-01

SR89 DECAYS INTO Y89

LAMBDA= 0.9527999E-05 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.1363650E-03 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.0
	0.2000000E 00	0.0
	0.3000000E 00	0.0
	0.4000000E 00	0.0
	0.5000000E 00	0.0
	0.6000000E 00	0.0
	0.8000000E 00	0.6999999E-04
	0.1000000E 01	0.8000000E-04
	0.1250000E 01	0.0
	0.1500000E 01	0.0
	0.2000000E 01	0.0
	0.3000000E 01	0.0
	0.4000000E 01	0.0

1131 DECAYS INTO XE131M
 LAMDA= 0.5972000E-04 DELTA= 0.1473000E 01 TOTAL GAMMA ENERGY= 0.3794840E 00 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.1964000E-01
	0.1500000E 00	0.1400000E-02
	0.2000000E 00	0.1083000E-01
	0.3000000E 00	0.3451400E 00
	0.4000000E 00	0.5330800E 00
	0.5000000E 00	0.3240000E-02
	0.6000000E 00	0.6382996E-01
	0.8000000E 00	0.2307000E-01
	0.1000000E 01	0.0
	0.1250000E 01	0.0
	0.1500000E 01	0.0
	0.2000000E 01	0.0
	0.3000000E 01	0.0
	0.4000000E 01	0.0

XE131M DECAYS INTO XE131
 LAMDA= 0.4024999E-04 DELTA= 0.0 TOTAL GAMMA ENERGY= 0.2787490E-02 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.1225000E-01
	0.2000000E 00	0.4749998E-02
	0.3000000E 00	0.0
	0.4000000E 00	0.0
	0.5000000E 00	0.0
	0.6000000E 00	0.0
	0.8000000E 00	0.0
	0.1000000E 01	0.0
	0.1250000E 01	0.0
	0.1500000E 01	0.0
	0.2000000E 01	0.0
	0.3000000E 01	0.0
	0.4000000E 01	0.0

I132 DECAYS INTO XE132

LAMBDA= 0.5066998E-02 DELTA= 0.5314000E-01 TOTAL GAMMA ENERGY= 0.2236580E 01 NO. OF GAMMAS= 14

ENERGY LEVELS

E	FE
0.9999996E-01	0.1300000E-03
0.1500000E 00	0.2270000E-02
0.2000000E 00	0.1910000E-02
0.3000000E 00	0.1178000E-01
0.4000000E 00	0.1539000E-01
0.5000000E 00	0.1896500E 00
0.6000000E 00	0.1055140E 01
0.8000000E 00	0.1246830E 01
0.1000000E 01	0.1912900E 00
0.1250000E 01	0.1084999E 00
0.1500000E 01	0.7594997E-01
0.2000000E 01	0.2828000E-01
0.3000000E 01	0.1060000E-02
0.4000000E 01	0.0

I133 DECAYS INTO XE133M

LAMBDA= 0.5553998E-03 DELTA= 0.3960000E 00 TOTAL GAMMA ENERGY= 0.6188090E 00 NO. OF GAMMAS= 14

ENERGY LEVELS

E	FE
0.9999996E-01	0.0
0.1500000E 00	0.0
0.2000000E 00	0.2090000E-02
0.3000000E 00	0.5229998E-02
0.4000000E 00	0.4430000E-02
0.5000000E 00	0.6418400E 00
0.6000000E 00	0.2807900E 00
0.8000000E 00	0.5618000E-01
0.1000000E 01	0.2792000E-01
0.1250000E 01	0.3530000E-01
0.1500000E 01	0.5507777E-02
0.2000000E 01	0.2100000E-03
0.3000000E 01	0.0
0.4000000E 01	0.0

XE133M DECAYS INTO XE133
 LAMDA= 0.2130000E-03 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.2311650E-01 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.0
	0.2000000E 00	0.6582999E-01
	0.3000000E 00	0.3316000E-01
	0.4000000E 00	0.0
	0.5000000E 00	0.0
	0.6000000E 00	0.0
	0.8000000E 00	0.0
	0.1000000E 01	0.0
	0.1250000E 01	0.0
	0.1500000E 01	0.0
	0.2000000E 01	0.0
	0.3000000E 01	0.0
	0.4000000E 01	0.0

XE133 DECAYS INTO CS133
 LAMDA= 0.9134000E-04 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.2964410E-01 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.2964399E 00
	0.1500000E 00	0.0
	0.2000000E 00	0.0
	0.3000000E 00	0.0
	0.4000000E 00	0.0
	0.5000000E 00	0.0
	0.6000000E 00	0.0
	0.8000000E 00	0.0
	0.1000000E 01	0.0
	0.1250000E 01	0.0
	0.1500000E 01	0.0
	0.2000000E 01	0.0
	0.3000000E 01	0.0
	0.4000000E 01	0.0

I1338 DECAYS INTO XE1338

LAMBDA= 0.5553998E-03 DELTA= 0.3960000E 00 TOTAL GAMMA ENERGY= 0.6188090E 00 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.0
	0.2000000E 00	0.2090000E-02
	0.3000000E 00	0.5229998E-02
	0.4000000E 00	0.4430000E-02
	0.5000000E 00	0.6418400E 00
	0.6000000E 00	0.2807900E 00
	0.8000000E 00	0.5618000E-01
	0.1000000E 01	0.2792000E-01
	0.1250000E 01	0.3530000E-01
	0.1500000E 01	0.5509999E-02
	0.2000000E 01	0.2100000E-03
	0.3000000E 01	0.0
	0.4000000E 01	0.0

XE1338 DECAYS INTO CS1338

LAMBDA= 0.9134000E-04 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.2964410E-01 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.2964399E 00
	0.1500000E 00	0.0
	0.2000000E 00	0.0
	0.3000000E 00	0.0
	0.4000000E 00	0.0
	0.5000000E 00	0.0
	0.6000000E 00	0.0
	0.8000000E 00	0.0
	0.1000000E 01	0.0
	0.1250000E 01	0.0
	0.1500000E 01	0.0
	0.2000000E 01	0.0
	0.3000000E 01	0.0
	0.4000000E 01	0.0

I134 DECAYS INTO XE134

LAMBDA= 0.1325000E-01 DELTA= 0.2488000E-01 TOTAL GAMMA ENERGY= 0.2464629E 01 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.1091000E-01
	0.1500000E 00	0.3172000E-01
	0.2000000E 00	0.1710000E-01
	0.3000000E 00	0.6180000E-02
	0.4000000E 00	0.1068299E 00
	0.5000000E 00	0.1180100E 00
	0.6000000E 00	0.3283100E 00
	0.8000000E 00	0.1273930E 01
	0.1000000E 01	0.7547100E 00
	0.1250000E 01	0.9977996E-01
	0.1500000E 01	0.9289998E-01
	0.2000000E 01	0.5844000E-01
	0.3000000E 01	0.0
	0.4000000E 01	0.0

I135 DECAYS INTO XE135M

LAMBDA= 0.1724000E-02 DELTA= 0.1227000E 00 TOTAL GAMMA ENERGY= 0.1918349E 01 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.4179999E-02
	0.2000000E 00	0.2474000E-01
	0.3000000E 00	0.4058000E-01
	0.4000000E 00	0.5394000E-01
	0.5000000E 00	0.1790000E 00
	0.6000000E 00	0.8656996E-01
	0.8000000E 00	0.7356995E-01
	0.1000000E 01	0.2861900E 00
	0.1250000E 01	0.5626600E 00
	0.1500000E 01	0.2778400E 00
	0.2000000E 01	0.1356200E 00
	0.3000000E 01	0.3999998E-03
	0.4000000E 01	0.0

XE135M DECAYS INTO XE135
 LAMDA= 0.4443000E-01 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.4214399E 00 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.0
	0.2000000E 00	0.0
	0.3000000E 00	0.0
	0.4000000E 00	0.0
	0.5000000E 00	0.5856000E 00
	0.6000000E 00	0.2144000E 00
	0.8000000E 00	0.0
	0.1000000E 01	0.0
	0.1250000E 01	0.0
	0.1500000E 01	0.0
	0.2000000E 01	0.0
	0.3000000E 01	0.0
	0.4000000E 01	0.0

XE135 DECAYS INTO CS135
 LAMDA= 0.1265000E-02 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.2495330E 00 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.2180000E-02
	0.2000000E 00	0.4638700E 00
	0.3000000E 00	0.4578000E 00
	0.4000000E 00	0.4600000E-02
	0.5000000E 00	0.2899999E-03
	0.6000000E 00	0.2564000E-01
	0.8000000E 00	0.2070000E-02
	0.1000000E 01	0.5999999E-04
	0.1250000E 01	0.1000000E-04
	0.1500000E 01	0.0
	0.2000000E 01	0.0
	0.3000000E 01	0.0
	0.4000000E 01	0.0

I1358 DECAYS INTO XE135B
 LAMDA= 0.1724000E-02 DELTA= 0.1227000E 00 TOTAL GAMMA ENERGY= 0.1918349E 01 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.4179999E-02
	0.2000000E 00	0.2474000E-01
	0.3000000E 00	0.4058000E-01
	0.4000000E 00	0.5394000E-01
	0.5000000E 00	0.1790000E 00
	0.6000000E 00	0.8656996E-01
	0.8000000E 00	0.7356995E-01
	0.1000000E 01	0.2861900E 00
	0.1250000E 01	0.5626600E 00
	0.1500000E 01	0.2778400E 00
	0.2000000E 01	0.1356200E 00
	0.3000000E 01	0.3999998E-03
	0.4000000E 01	0.0

XE135B DECAYS INTO CS135B
 LAMDA= 0.1265000E-02 DELTA= 0.0 TOTAL GAMMA ENERGY= 0.2495330E 00 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.2180000E-02
	0.2000000E 00	0.4638700E 00
	0.3000000E 00	0.4578000E 00
	0.4000000E 00	0.4600000E-02
	0.5000000E 00	0.2899999E-03
	0.6000000E 00	0.2564000E-01
	0.8000000E 00	0.2070000E-02
	0.1000000E 01	0.5999999E-04
	0.1250000E 01	0.1000000E-04
	0.1500000E 01	0.0
	0.2000000E 01	0.0
	0.3000000E 01	0.0
	0.4000000E 01	0.0

XE137 DECAYS INTO CS137
 LAMDA= 0.1805000E 00 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.1877559E 00 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.0
	0.2000000E 00	0.0
	0.3000000E 00	0.9999999E-04
	0.4000000E 00	0.1348000E 00
	0.5000000E 00	0.1668400E 00
	0.6000000E 00	0.9699999E-03
	0.8000000E 00	0.5639996E-02
	0.1000000E 01	0.6839998E-02
	0.1250000E 01	0.4979998E-02
	0.1500000E 01	0.6369997E-02
	0.2000000E 01	0.6449997E-02
	0.3000000E 01	0.3020000E-02
	0.4000000E 01	0.1800000E-03

XE138 DECAYS INTO CS138
 LAMDA= 0.4881000E-01 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.2183390E 01 NO. OF GAMMAS= 14

ENERGY LEVELS	E	FE
	0.9999996E-01	0.0
	0.1500000E 00	0.2433600E 00
	0.2000000E 00	0.4895500E 00
	0.3000000E 00	0.6286899E 00
	0.4000000E 00	0.5578400E 00
	0.5000000E 00	0.1665600E 00
	0.6000000E 00	0.0
	0.8000000E 00	0.0
	0.1000000E 01	0.0
	0.1250000E 01	0.0
	0.1500000E 01	0.1801800E 00
	0.2000000E 01	0.6357700E 00
	0.3000000E 01	0.4049998E-02
	0.4000000E 01	0.0

CS138 DECAYS INTO BA138
 LAMDA= 0.2153000E-01 DELTA= 0.0

TOTAL GAMMA ENERGY= 0.2386570E 01 NO. OF GAMMAS= 14

ENERGY LEVELS

E	FE
0.9999996E-01	0.6719999E-02
0.1500000E 00	0.2304000E-01
0.2000000E 00	0.3014000E-01
0.3000000E 00	0.1005000E-01
0.4000000E 00	0.1958600E 00
0.5000000E 00	0.2822400E 00
0.6000000E 00	0.5464000E-01
0.8000000E 00	0.3620000E-01
0.1000000E 01	0.3229100E 00
0.1250000E 01	0.2242900E 00
0.1500000E 01	0.5933900E 00
0.2000000E 01	0.1580800E 00
0.3000000E 01	0.9183997E-01
0.4000000E 01	0.1790000E-02

EXHAUST RATE= 0.7999998E-01		ESCAPE RATE= 0.1000000E 01	
KR83M	Q= 0.2500000E 06	PHI= 0.0	PSI= 0.0
KR85M	Q= 0.7800000E 06	PHI= 0.0	PSI= 0.0
KR87	Q= 0.1400000E 07	PHI= 0.0	PSI= 0.0
KR88	Q= 0.1900000E 07	PHI= 0.0	PSI= 0.0
RB88	Q= 0.0	PHI= 0.0	PSI= 0.0
KR89	Q= 0.2400000E 07	PHI= 0.0	PSI= 0.0
R889	Q= 0.0	PHI= 0.0	PSI= 0.0
SR89	Q= 0.0	PHI= 0.0	PSI= 0.0
I131	Q= 0.3000000E 06	PHI= 0.0	PSI= 0.0
XE131M	Q= 0.1600000E 05	PHI= 0.0	PSI= 0.0
I132	Q= 0.4600000E 06	PHI= 0.0	PSI= 0.0
I133	Q= 0.1970000E 05	PHI= 0.0	PSI= 0.0
XE133M	Q= 0.8300000E 05	PHI= 0.0	PSI= 0.0
XE133	Q= 0.3400000E 07	PHI= 0.0	PSI= 0.0
I133B	Q= 0.6603000E 06	PHI= 0.0	PSI= 0.0
XE133B	Q= 0.0	PHI= 0.0	PSI= 0.0
I134	Q= 0.7800000E 06	PHI= 0.0	PSI= 0.0
I135	Q= 0.9050000E 05	PHI= 0.0	PSI= 0.0
XE135M	Q= 0.9300000E 06	PHI= 0.0	PSI= 0.0
XE135	Q= 0.3200000E 07	PHI= 0.0	PSI= 0.0
I135B	Q= 0.5295000E 06	PHI= 0.0	PSI= 0.0
XE135B	Q= 0.0	PHI= 0.0	PSI= 0.0
XE137	Q= 0.3100000E 07	PHI= 0.0	PSI= 0.0
XE138	Q= 0.2800000E 07	PHI= 0.0	PSI= 0.0
CS138	Q= 0.0	PHI= 0.0	PSI= 0.0

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LIST OF ISOTOPES

PARENT	DAUGHTER	LAMBDA	DELTA	Q	PHI	PSI
KR83M	KR83	6.211E-03	0.0	4.025E 07	0.0	0.0
KR85M	KR85	2.650E-03	0.0	2.943E 08	0.0	0.0
KR87	KR87	9.073E-03	0.0	1.543E 08	0.0	0.0
KR88	KR88	4.126E-03	0.0	4.605E 08	0.0	0.0
RB88	SR88	3.894E-02	0.0	0.0	0.0	0.0
KR89	KR89	2.180E-01	0.0	1.101E 07	0.0	0.0
RB89	SR89	4.560E-02	0.0	0.0	0.0	0.0
SR89	Y89	9.528E-06	0.0	0.0	0.0	0.0
I131	XE131M	5.972E-05	1.230E 00	5.023E 09	0.0	0.0
XE131M	XE131	4.025E-05	0.0	3.975E 08	0.0	0.0
I132	XE132	5.067E-03	3.764E 00	9.078E 07	0.0	0.0
I133	XE133M	5.554E-04	3.075E 00	3.547E 07	0.0	0.0
XE133M	XE133	2.130E-04	0.0	3.897E 08	0.0	0.0
XE133	CS133	9.134E-05	0.0	3.722E 10	0.0	0.0
I133B	XE133B	5.554E-04	3.075E 00	1.189E 09	0.0	0.0
XE133B	CS133B	9.134E-05	0.0	0.0	0.0	0.0
I134	XE134	1.325E-02	4.609E 00	5.887E 07	0.0	0.0
I135	XE135M	1.724E-03	2.957E 00	5.249E 07	0.0	0.0
XE135M	XE135	4.443E-02	0.0	2.093E 07	0.0	0.0
XE135	CS135	1.265E-03	0.0	2.530E 09	0.0	0.0
I135B	XE135B	1.724E-03	2.957E 00	3.071E 08	0.0	0.0
XE135B	CS135B	1.265E-03	0.0	0.0	0.0	0.0
XE137	CS137	1.805E-01	0.0	1.717E 07	0.0	0.0
XE138	CS138	4.881E-02	0.0	5.737E 07	0.0	0.0
CS138	BA138	2.153E-02	0.0	0.0	0.0	0.0

ENERGY SPECTRA

	0.1000	0.1500	0.2000	0.3000	0.4000	0.5000	0.6000	0.8000	1.0000	1.2500	1.5000	2.0000	3.0000	4.0000
KR83M	0.0270	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KR85M	0.0	0.7718	0.0162	0.1337	0.0063	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KR87	0.0002	0.0003	0.0	0.0	0.4700	0.0137	0.0119	0.0712	0.0232	0.0135	0.0200	0.0989	0.0808	0.0015
KR88	0.0	0.0757	0.3721	0.0130	0.0259	0.0043	0.0002	0.1172	0.0588	0.0380	0.1246	0.4830	0.1918	0.0
RB88	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0734	0.0706	0.0040	0.0819	0.1726	0.0259	0.0029
KR89	0.0080	0.0098	0.2019	0.1025	0.1078	0.1779	0.2909	0.1580	0.1341	0.0877	0.2948	0.1781	0.1115	0.0461
RB89	0.0	0.0	0.0056	0.0214	0.0	0.0000	0.0761	0.0606	0.6080	0.5397	0.0288	0.2456	0.1361	0.0135
SR89	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0001	0.0001	0.0	0.0	0.0	0.0	0.0
I131	0.0196	0.0014	0.0108	0.3451	0.5331	0.0022	0.0638	0.0231	0.0	0.0	0.0	0.0	0.0	0.0
XE131M	0.0	0.0122	0.0047	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I132	0.0001	0.0023	0.0019	0.0113	0.0154	0.1897	1.0551	1.2468	0.1913	0.1085	0.0759	0.0284	0.0011	0.0
I133	0.0	0.0	0.0021	0.0052	0.0044	0.6418	0.2808	0.0562	0.0279	0.0353	0.0055	0.0002	0.0	0.0
XE133M	0.0	0.0	0.0658	0.0332	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
XE133	0.2964	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I133B	0.0	0.0	0.0021	0.0052	0.0044	0.6418	0.2808	0.0562	0.0279	0.0353	0.0055	0.0002	0.0	0.0
XE133B	0.2964	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I134	0.0109	0.0317	0.0171	0.0062	0.1068	0.1180	0.3283	1.2739	0.7547	0.0998	0.0929	0.0584	0.0	0.0
I135	0.0	0.0042	0.0247	0.0405	0.0539	0.1790	0.0866	0.0736	0.2862	0.5627	0.2778	0.1356	0.0004	0.0
XE135M	0.0	0.0	0.0	0.0	0.0	0.5856	0.2144	0.0	0.0	0.0	0.0	0.0	0.0	0.0
XE135	0.0	0.0022	0.4639	0.4578	0.0046	0.0003	0.0256	0.0021	0.0001	0.0000	0.0	0.0	0.0	0.0
I135B	0.0	0.0042	0.0247	0.0405	0.0539	0.1790	0.0866	0.0736	0.2862	0.5627	0.2778	0.1356	0.0004	0.0
XE135B	0.0	0.0022	0.4639	0.4578	0.0046	0.0003	0.0256	0.0021	0.0001	0.0000	0.0	0.0	0.0	0.0
XE137	0.0	0.0	0.0030	0.0001	0.1348	0.1668	0.0010	0.0056	0.0068	0.0050	0.0064	0.0064	0.0030	0.0002
XE138	0.0	0.2434	0.4696	0.6287	0.5578	0.1666	0.0	0.0	0.0	0.0	0.1802	0.6358	0.0040	0.0
CS138	0.0067	0.0230	0.0301	0.0100	0.1959	0.2822	0.0546	0.0362	0.3229	0.2243	0.5934	0.1581	0.0818	0.0018

FAMILIES OF ISOTOPES

FAMILY NUMBER 1 1 ISOTOPES

KR83M

FAMILY NUMBER 2 1 ISOTOPES

KR85M

FAMILY NUMBER 3 1 ISOTOPES

KR87

FAMILY NUMBER 4 2 ISOTOPES

KR88 RB88

FAMILY NUMBER 5 3 ISOTOPES

KR89 RB89 SR89

FAMILY NUMBER 6 2 ISOTOPES

II131 XE131M

FAMILY NUMBER 7 1 ISOTOPES

II132

FAMILY NUMBER 8 3 ISOTOPES

II133 XE133M XE133

FAMILY NUMBER 9 2 ISOTOPES

II133B XE133B

FAMILY NUMBER 10 1 ISOTOPES

II134

FAMILY NUMBER 11 3 ISOTOPES

II135 XE135M XE135

FAMILY NUMBER 12 2 ISOTOPES

II135B XE135B

FAMILY NUMBER 13 1 ISOTOPES

XE137

FAMILY NUMBER 14 2 ISOTOPES

XE138 CS138

LOCATION NO.	DISPERSION MEASURE
1	0.6789995E 03
2	0.6789995E 03
3	0.6789995E 03
4	0.6789995E 03
5	0.6789995E 03
6	0.1281333E 04
7	0.1836800E 04

STABILITY CONDITION=C WIND SPEED= 240.00 M/MIN WIND DIRECTION= 0.0 DEGREES TAU= 15.00 MIN LEAK RATE= 9.00E-07 /MIN

TIME AFTER ACCIDENT (MIN)	RECEPTOR LOCATION			EXTERNAL	INTERNAL
	X (M)	Y (M)	Z (M)	DOSE (REM)	DOSE (REM)
60	1.000E 02	0.0	0.0	9.868E 00	1.091E-14
	2.000E 02	0.0	0.0	1.053E 01	6.109E-07
	5.000E 02	0.0	0.0	9.850E 00	1.939E 01
	1.000E 03	0.0	0.0	1.162E 01	7.098E 02
	2.000E 03	0.0	0.0	6.655E 00	4.676E 02
	5.000E 03	0.0	0.0	1.726E 00	1.149E 02
	1.000E 04	0.0	0.0	4.047E-01	2.783E 01

STABILITY CONDITION=C WIND SPEED= 240.00 M/MIN WIND DIRECTION= 0.0 DEGREES TAU= 15.00 MIN LEAK RATE= 9.00E-07 /MIN

TIME AFTER ACCIDENT (MIN)	RECEPTOR LOCATION			EXTERNAL	INTERNAL
	X (M)	Y (M)	Z (M)	DOSE (REM)	DOSE (REM)
1440	1.000E 02	0.0	0.0	9.917E 00	1.102E-14
	2.000E 02	0.0	0.0	1.059E 01	6.169E-07
	5.000E 02	0.0	0.0	9.913E 00	1.960E 01
	1.000E 03	0.0	0.0	1.171E 01	7.192E 02
	2.000E 03	0.0	0.0	6.735E 00	4.764E 02
	5.000E 03	0.0	0.0	1.795E 00	1.211E 02
	1.000E 04	0.0	0.0	5.287E-01	3.783E 01

TIME FOR THIS RUN=23 SECONDS.

TAU= 0.9999996E-01 NTAU= 100

LOCATION NO.	DISPERSION MEASURE
1	-0.1499999E 02
2	-0.1499999E 02
3	-0.1499999E 02
4	0.2115197E 03
5	0.4028794E 03
6	0.9167944E 03
7	0.1731655E 04

STABILITY CONDITION=C WIND SPEED= 240.00 M/MIN WIND DIRECTION= 0.0 DEGREES TAU= 0.60 MIN LEAK RATE= 3.00E-02 /MIN

TIME AFTER ACCIDENT (MIN)	RECEPTOR LOCATION			EXTERNAL	INTERNAL
	X (M)	Y (M)	Z (M)	DOSE (REM)	DOSE (REM)
59	1.000E 02	0.0	0.0	9.688E 00	1.004E-08
	2.000E 02	0.0	0.0	1.080E 01	3.708E-03
	5.000E 02	0.0	0.0	1.117E 01	1.291E 02
	1.000E 03	0.0	0.0	1.041E 01	6.642E 02
	2.000E 03	0.0	0.0	6.062E 00	4.395E 02
	5.000E 03	0.0	0.0	1.716E 00	1.149E 02
	1.000E 04	0.0	0.0	4.120E-01	2.853E 01

TIME FOR THIS RUN=27 SECONDS.

LOCATION NO.	DISPERSION MEASURE
1	-0.149999E 02
2	-0.149999E 02
3	-0.149999E 02
4	0.211519E 03
5	0.402879E 03
6	0.916794E 03
7	0.173165E 04

STABILITY CONDITION=C WIND SPEED= 240.00 M/MIN WIND DIRECTION= 0.0 DEGREES TAU= 0.60 MIN LEAK RATE= 8.00E-02 /MIN

TIME AFTER ACCIDENT (MIN)	RECEPTOR LOCATION			EXTERNAL DOSE (REM)	INTERNAL DOSE (REM)
	X (M)	Y (M)	Z (M)		
1439	1.000E 02	0.0	0.0	9.736E 00	1.013E-08
	2.000E 02	0.0	0.0	1.086E 01	3.740E-03
	5.000E 02	0.0	0.0	1.124E 01	1.304E 02
	1.000E 03	0.0	0.0	1.048E 01	6.717E 02
	2.000E 03	0.0	0.0	6.126E 00	4.464E 02
	5.000E 03	0.0	0.0	1.774E 00	1.201E 02
	1.000E 04	0.0	0.0	5.231E-01	3.749E 01

TIME FOR THIS RUN=28 SECONDS.

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June 15, 1973

To: Recipients of Subject Report

Report No.: ORNL-TM-4082 Classification: Unclassified

Author(s): F. W. Stallmann and F. B. K. Kam

Subject: ACRA - A Computer Program for the Estimation of Radiation

Doses Caused by a Hypothetical Reactor Accident.

Request compliance with indicated action:

Please make the following correction in your copy(ies) of the subject report.

On page 47 add one line of input description (last line on page should be)

"SCØND = Stability condition (A-G) Col. 41-45."

W. C. Brown
Laboratory Records Department
Information Division

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