WCAP-1615

SHOCK LOADING ANALYSIS OF CORE SUB-ASSEMBLY - APDA CORE,

by

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August 1960

MASTER

S.O. # XAR-90044-AR } *Key word line*

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I. INTRODUCTION:

This report summarizes an analysis of the stresses and deflections in the APDA core sub-assembly "C" under conditions of shock imposed by handling.

The approach of this study is to assume that the sub-assembly and its components respond to an impact load in a manner similar to their response under a statically applied load. Acceleration figures from previous drop tests on the shipping containers are used to determine this equivalent static loading. Maximum deflections and stresses in the assembly are determined from this static loading.

This report considers load application perpendicular to the assembly axis and the grid tab surfaces for the following reasons:

(1) Handling during shipment primarily subjects the assembly to impact loads perpendicular to the assembly axis; (2) since the grid tab spring rates are lowest when the applied load is perpendicular to the tab surface, maximum fuel pin and grid tab deflection occur when loaded in this mode.

cont'd
The analysis section of this report considers shock effects on the entire sub-assembly and on an 11 grid (type "A") fuel assembly within the central core region. Significant results for other grid arrangements are plotted in figure 6. Load deflection characteristics of the fuel element support tabs in the type "A" grid (see figure 3) are also included in Section IV since this information is necessary for the shock analysis.

II. SUMMARY OF RESULTS:

The following values of bending stress and deflections are based on the maximum acceleration recorded in the shipping container drop tests, i.e. 24 G's.

The sub-assembly wrapper tube, under impact, deflects .307 in. at the center and is subjected to 34,560 psi bending stress in the outer fibres.

The fuel pins in an 11 grid core assembly are deflected a maximum of .006 in. with respect to the support grids. This deflection is a composite of pin deflection and grid tab deflection. The maximum bending stress in the pins is 2828 psi.

Under impact, the support grid tabs are deflected .0033 in. for an 11 grid type "A" assembly. Deflections for other grid arrangements are plotted in Figure 6.

The impact load on the most critical grid structural parts is 6% of the allowable load before buckling and develops 2310 psi stress at this cross-section.

III. CONCLUSIONS AND RECOMMENDATIONS:

Deflections and bending stresses encountered under rough handling are not sufficient to permanently bend the fuel pins or to buckle the grid structural elements. However, sub-assembly bending imposes high stresses in the wrapper tube and may impose high local stresses in the grid areas attached to the tie rods.

Since the grid tabs are stressed initially to the yield point, any further deflections due to impact loading will produce permanent set in the grid tabs. While permanent set may be taken up by the opposing tabs, misalignment of the pins with respect to the grids may result. Reduction of the maximum acceleration to 10 G's reduces the tab impact deflection in the 11 grid assembly to .014 in. which would insure negligible permanent deflection.

In view of the above conclusions, it is recommended that:

- (1) The shipping container supports be moved toward the center of the container to produce higher negative bending moments on the ends of the sub-assembly and reduce the high bending stresses in the wrapper tube.
- (2) The shipping container cushioning be "softened" as much as clearance will allow to reduce the high accelerations on the sub-assembly.

IV. ANALYSIS:

This analysis investigates deflections and stresses produced in the APDA core sub-assembly during handling. The areas considered are:

- (1) the sub-assembly wrapper tube,*
- (2) the core region fuel pins,
- (3) the core support grid tabs, and
- (4) critical grid structural elements.

A. Wrapper Tube Bending Stresses and Deflections

The core sub-assembly is supported at both ends when enclosed in the shipping container (see Figure 1a). Since the container is normally handled in the horizontal position, the sub-assembly's main structural envelope, the wrapper tube, is subjected to bending stresses. Considering the sub-assembly as uniformly loaded over various sections and simply supported at the center of the cushioned areas (see Figure 1b), the reaction forces and bending moments along the wrapper tube due to the sub-assembly weight alone may be determined (Figure 1c).

Neglecting strength of internal components, the maximum bending stress produced in the wrapper tube without shock loading is

$$\sigma_{\max.} = \frac{M_{\max.} C}{I}$$

where

$$M_{\max.} = 1156 \text{ in-lb. (see Figure 1c)}$$

$$I = \frac{(2.646)^4 - (2.454)^4}{12}$$

$$= 1.066 \text{ in.}^4$$

$$C = \frac{2.646}{2}$$

$$= 1.323 \text{ in.}$$

$$\sigma_{\text{max.}} = \frac{(1156) (1.323)}{(1.066)}$$

$$= 1440 \text{ psi}$$

Using moment area methods, the deflection at the center of the sub-assembly without shock loading is:

$$y = \sum \frac{A}{EI} \bar{x}$$

$$\approx \left(\frac{2}{3} a b\right) \frac{3}{8} a / EI \quad (\text{see Figure 1c})$$

$$= \frac{a^2 b}{4 EI}$$

$$E = 29 \times 10^6 \text{ psi (347 SS)}$$

$$I = 1.066 \text{ in}^4 \text{ (see page 6)}$$

$$y = \frac{(37)^2 (1156)}{4 (29 \times 10^6) (1.066)}$$

$$= .013 \text{ in.}$$

The effects of shock loading on the wrapper tube may be conservatively estimated as the product of the maximum acceleration measured and the sub-assembly weight. Since deflection and bending stresses are directly proportional to the applied load, the wrapper tube deflec-

tions and stresses are therefore directly proportional to acceleration.

Under 24G horizontal loading,

$$\sigma_{\max.} = (24) (1440)$$

$$= 34,560 \text{ psi}$$

$$y_{\max. \text{ center}} = (24) (.0128)$$

$$= .307 \text{ in.}$$

B. Fuel Pin Bending Stresses and Deflections

Since the fuel pins in the core region are free to move within the support grids, bending stresses resulting from wrapper tube deflection primarily occur in the tie rods and grid corners, and should not damage the fuel pins. This and the following section will consider, however, deflections and stresses resulting from fuel pin movement under impact perpendicular to the support grid tabs.

A fuel pin in an 11 grid (type "A") fuel assembly is supported at a minimum of six places. This analysis will consider one fuel pin, subjected to a 24-G loading, as a continuous beam simply supported at six points and loaded uniformly with a static load equivalent to the shock loading. (see Figure 2) A conservative estimate of the equivalent static loading, w , is the product of the maximum deceleration measured and the fuel pin weight.

$$w = \frac{F_{\text{static}}}{L} = \frac{W_{\text{pin}} G}{L}$$

W_{pin} = weight of fuel pin

$$= .347 \text{ lb}_f$$

G = maximum acceleration in g's =
24

L = length of fuel carrying
portion of fuel pin = 31.1 in.

$$= \frac{(.347)}{(31.1)} \quad (24)$$

$$= .268 \text{ lb}_f/\text{in.}$$

For a uniformly loaded continuous beam with six supports and equal spans, the maximum bending stress in the pin is:

$$\sigma = \frac{4}{38} w L^2 \frac{C}{I}$$

where:

$$C = \text{radius} = \frac{.1575}{2} = .07875 \text{ in.}$$

I = moment of inertia with respect
to pin neutral axis = $3.02 \times 10^{-5} \text{ in.}^4$

L = distance between supporting
grids = 6.2 in.

$$\sigma = \frac{4}{38} (.268) (6.2)^2 \frac{(.07875)}{(3.02 \times 10^{-5})}$$

$$= 2828 \text{ psi}$$

This fuel pin bending stress occurs at a grid section.

Fuel pin deflections relative to the support grids can be found by employing moment area methods. Maximum deflections in the end and center spans have been calculated (see below) using Figure 2 and the following values:

$$E_{\text{pin}} = 19 \times 10^6 \text{ psi}$$

$$I = 3.02 \times 10^{-5} \text{ in.}^4$$

<u>LOCATION</u>	<u>MAXIMUM DEFLECTION</u>
End Span	.002 in.
Center Span	.0015 in.

The maximum deflection of the fuel pin with respect to the sub-assembly is approximately the sum of the above pin deflection and the grid tab deflection calculated in Section IV C.

C. Deflection of Type "A" Grid Tabs

Under impact, the fuel assembly support grids are subjected to high reaction loadings which tend to deflect the grid tabs. For a uniformly loaded continuous beam, (see Figure 2), the maximum reaction load on a grid is:

$$P = \frac{43}{38} w L$$

$$= \frac{43}{38} (.268) (6.2)$$

$$= 1.88 \text{ lb}_f$$

Grid deflection, y , is a function of the impact load and the effective spring rate of the grid. Although the grid tabs are initially loaded to the yield point, experimental data indicates that the tab spring rate remains nearly constant well into the plastic range.

$$y = P/\bar{K}_{eq}$$

where:

y = maximum grid tab deflection

$\bar{K}_{eq}$ = equivalent spring rate of one grid.

Since grid tabs oppose one another,

$$\bar{K}_{eq} = 2 \quad \bar{K}_{tab}$$

$$y = \frac{P}{2 \bar{K}_{tab}}$$

Spring rates for type "A" grids can be found by employing the formulas developed in Section IV E. For the modified type "A" grid, the following dimensions and physical constants apply: (see Figure 3)

$$L_1 = .312 \text{ in.}$$

$$L_2 = .1875 \text{ in.}$$

$$a = .047 \text{ in.}$$

$$b = .00575 \text{ in.}$$

$$E = 29 \times 10^6 \text{ psi}$$

$$G = 11.3 \times 10^6 \text{ psi}$$

$$I_1 = 1.01 \times 10^{-8} \text{ psi}$$

$$I_2 = 1.187 \times 10^{-8} \text{ psi}$$

Substituting into equation (4) of Section IV E,

$$K = (.047) (.00575)^3 \left[\frac{16}{3} - 3.36 \frac{(.00575)}{(.047)} \left(1 - \frac{(.00575)^4}{12(.047)^4} \right) \right]$$

$$= 43.98 \times 10^{-9}$$

Substituting the above values into equation (10) of Section IV E,

$$\beta = \frac{(.312)^2(43.98)(11.3)(10^{-3}) - 2(29)(1.01)(.047)(.1875)(10^{-2})}{8(43.98)(11.3)(.312)(10^{-3}) + 4(29)(1.01)(.1875)(10^{-2})}$$

$$= .0296$$

Solving for the tab spring rate, $\bar{K}$ (equation 12 - Section IV E),

$$\frac{1}{\bar{K}_{\text{tab}}} = \frac{(.312)^2}{48 (29)(1.01)(10^{-2})} + \frac{6 (.029598) + .312}{4 (43.98)(11.3)(10^{-3})} + \frac{(.0296 + .0235)(.1875)(.047)}{4 (43.98)(11.3)(10^{-3})}$$

$$+ \frac{(.1875)^3}{384 (29)(1.187)(10^{-2})}$$

$$\overline{K}_{\text{tab}} = 272.2 \text{ lb}_f/\text{in.}$$

Solving for the grid tab deflection under 24 G loading,

$$y = \frac{P}{2 \overline{K}}$$

$$= \frac{(1.88)}{2(272.2)}$$

$$= .0033 \text{ in.}$$

Experimental data indicates that the yield point of the modified type "A" grid tab is reached after approximately .004 in. deflection. This indicates that the upper tab, initially deflected to the yield point, would still hold the fuel pin against the lower tab if any permanent set of the lower tab resulted from the .0033 in. impact deflection. Furthermore, since the spring rate of the tab does not decrease rapidly beyond the yield point, the elasticity remaining in the lower tab should prevent large permanent deflection from occurring.

Maximum grid tab impact deflections for various support grid arrangements are plotted in Figure 6.

D. Type "A" Support Grid Buckling

In addition to local stresses in the grid corners due to sub-assembly bending, the lower sections of the grids are subjected to the full impact load of the pins supported above. Structural grid elements, acting as columns, therefore, may be subjected to buckling loads. At the outside of the grid, the critical load required to cause buckling is: (see Figure 5).

$$P_{\text{critical}} = \frac{\pi^2 E I}{L^2}$$

where:

L = length of column = .199 in.

I = moment of inertia of smallest cross-section = I' (see Figure 5)

$$I' = \frac{bh^3}{12}$$

$$= \frac{(2)(.094)(.0115)^3}{12}$$

$$= 23.8 \times 10^{-9} \text{ in.}^4$$

$$P_{\text{critical}} = \frac{\pi^2 (29 \times 10^6) (23.8 \times 10^{-9})}{(.199)^2}$$

$$= 172 \text{ lb}_f$$

The impact load from the pins above this section of grid is:

$$P_{\text{impact}} = \frac{11}{2} \times P_{\text{impact/pin}} \quad (\text{see Figure 5})$$

$$= \frac{11}{2} (1.88)$$

$$= 10.3 \text{ lb}_f$$

The ratio of the maximum load on the column to the load necessary to cause buckling is:

$$\frac{P_{\text{impact}}}{P_{\text{critical}}} = \frac{10.3}{172}$$

$$= .06$$

The compressive stress developed at the narrowest cross-section is:

$$\sigma_c = P/A$$

A = cross-sectional area

$$= 2 (.094)(.0115)$$

$$= 4.46 \times 10^{-3} \text{ in}^2$$

$$\sigma_c = \frac{10.3}{4.46 \times 10^{-3}}$$

$$= 2310 \text{ psi}$$

E. Determination of Spring Rate for Type "A" Grid

A load acting on the center of the type "A" grid (see Figures 3 and 4) deflects the central strip or tab, and produces a reaction load and moment on the edge of the end sections. This reaction moment, M, twists this section through an angle θ deflecting the outside edges through δ_1 , (see Figure 4c). The reaction load, P/2, further deflects the end area through a distance δ_2 . This analysis attempts to incorporate the end effects due to the reaction load and moment into the equations describing the behavior of the central strip. The spring rate equation developed, therefore, reflects the response of the entire grid tab to a central loading.

Considering Section (1), (figure 4b)

$$EI_1 y'' = M - \frac{Px}{2} \quad 0 \leq x \leq \frac{L_1}{2} \quad (1)$$

integrating,

$$EI_1 y' = Mx - \frac{Px^2}{4} + C_1$$

$$\text{Since } y' = 0 \text{ at } x = \frac{L_1}{2}$$

$$C_1 = -\frac{M L_1}{2} + \frac{P L_1^2}{16}$$

and

$$EI_1 y' = Mx - \frac{Px^2}{4} - \frac{M L_1}{2} + \frac{P L_1^2}{16} \quad (2)$$

integrating again,

$$EI_1 y = \frac{Mx^2}{2} - \frac{Px^3}{12} - \frac{M L_1 x}{2} + \frac{P L_1^2 x}{16} + C_2 \quad (3)$$

Evaluation of C_2 requires knowledge of y at $X = 0$

Considering Section 2, (figure 4c)

$$y = \delta_1 + \delta_2 \text{ at } x = 0$$

$$\delta_1 = \theta a$$

$$\theta = \frac{TL}{KG}$$

where

$$K = ab^3 \left[\frac{16}{3} - 3.36 \frac{b}{a} \left(1 - \frac{b^4}{12 a^4} \right) \right] \quad (4)$$

(See Roark, Formulas for Stress and Strain, p. 170)

$$G = \frac{E}{2(1-\mu)}$$

μ = Poisson's Ratio

$$L = \frac{L_2}{2}$$

$$T = \left[M + \frac{P}{2} a \right] / 2$$

$$\delta_1 = \frac{\left[M + \frac{P}{2} a \right] L_2 a}{4 K G} \quad (5)$$

$$\delta_2 = \frac{\frac{P}{2} L_2^3}{192 EI_2} \quad \begin{array}{l} \text{Deflection of Centrally} \\ \text{Loaded Beam with Fixed Ends} \end{array} \quad (6)$$

and

$$y = \delta_1 + \delta_2 = \frac{(M + \frac{Pa}{2}) L_2 a}{4 KG} + \frac{P L_2^3}{384 EI_2} \quad \text{at } x = 0 \quad (7)$$

Evaluating Equation 3 at $x = 0$

$$EI_1 y = C_2$$

substituting in for y from equation (7),

$$EI_1 \left[\frac{(M + \frac{Pa}{2}) L_2 a}{4 KG} + \frac{P L_2^3}{384 EI_2} \right] = C_2$$

substituting back into equation (3),

$$EI_1 y = \frac{Mx^2}{2} - \frac{Px^3}{12} - \frac{M L_1 x}{2} + \frac{P L_1^2 x}{16} + \left[\frac{(M + \frac{Pa}{2}) L_2 a}{4 KG} + \frac{P L_2^3}{384 EI_2} \right] EI_1 \quad (8)$$

to evaluate M in terms of P ,

$$0 = y' \text{ at } x = 0$$

$$EI_1 y' = Mx - \frac{Px^2}{4} - \frac{ML_1}{2} + \frac{PL_1^2}{16} \quad (2)$$

and

$$EI_1 \theta = -\frac{ML_1}{2} + \frac{PL_1^2}{16}$$

since,

$$\theta = \frac{(M + \frac{Pa}{2}) L_2}{4 KG}$$

$$EI_1 \frac{(M + \frac{Pa}{2}) L_2}{4 KG} = -\frac{ML_1}{2} + \frac{PL_1^2}{16}$$

Solving for M,

$$\frac{EI_1 M L_2}{4 KG} + \frac{M L_1}{2} = -\frac{EI_1 Pa L_2}{8 KG} + \frac{PL_1^2}{16}$$

$$M = P \left[\frac{\frac{L_1^2}{16} - \frac{EI_1 a L_2}{8 KG}}{\frac{L_1}{2} + \frac{EI_1 L_2}{4 KG}} \right] \quad (9)$$

$$\text{Let } \beta = \left[\frac{\frac{L_1^2}{16} - \frac{EI_1 a L_2}{8 KG}}{\frac{L_1}{2} + \frac{EI_1 L_2}{4 KG}} \right] = \frac{L_1^2 KG - 2 EI_1 a L_2}{KG 8 L_1 + 4 EI_1 L_2} \quad (10)$$

$$M = \beta p$$

Substituting into Equation 8 and evaluating at $x = \frac{L_2}{2}$

$$EI_1 y = \frac{\beta P L_1^2}{8} - \frac{P L_1^3}{96} - \frac{\beta P L_1^2}{4} + \frac{P L_1^3}{32} \quad (11)$$

$$+ EI_1 \left[\frac{(\beta P + \frac{Pa}{2}) L_2 a}{4 KG} + \frac{P L_2^3}{384 EI_2} \right]$$

Simplifying:

$$y = \frac{\beta P L_1^2}{8 EI_1} + \frac{P L_1^3}{48 EI_1} + P \left[\frac{(\beta + \frac{a}{2}) L_2 a}{4 KG} + \frac{L_2^3}{384 EI_2} \right]$$

Solving for Spring Rate, $\bar{K}$,

$$\bar{K} = \frac{P}{y} = \frac{1}{\frac{L_1^2(6\beta + L_1)}{48 EI_1} + \frac{(\beta + \frac{a}{2}) L_2 a}{4 KG} + \frac{L_2^3}{384 EI_2}} \quad (12)$$

where β and K can be determined from equations (10) and (4).

Load deflection data from tests on various size grids provide spring rates within 5% of the values calculated from equation 12.

(a)

ANALYSIS MODEL

(b)

MOMENT DIAGRAM

(c)

NOTE: DIMENSIONS & WEIGHT DISTRIBUTION ARE APPROXIMATE

APDA CORE SUBASSEMBLY "C"
IN SHIPPING CONTAINER

ED. SK. 287931 C

Figure 1

TYPICAL M/EI DIAGRAM (SIX SUPPORTS)

IMPACT LOADING OF APDA FUEL PIN CONSIDERED AS UNIFORMLY LOADED
CONTINUOUS BEAM WITH SIMPLE SUPPORTS

ED. Sk. 287930 C

Figure - 2

APDA FUEL ASSEMBLY
MODEL A
SUPPORT GRID STRAP

Figure - 3

ED. Sk. 287930 C

ANALYSIS MODEL AND FREE BODY DIAGRAMS
FOR APDA TYPE "A" GRID

ED. Sk. 207929 C

Figure - 4

APDA FUEL ASSEMBLY
 GRID AREAS SUBJECTED TO MAXIMUM BUCKLING LOADS IN IMPACT
 ED. SK. 287929 C

Figure - 5

DEFLECTION OF TYPE "A" GRID TABS UNDER
IMPACT LOADS IMPOSED BY HANDLING

Figure 6

ED. SK. 287931 C