IMPACT PICTURE OF pp , $\bar{p}p$, $\pi^\pm p$, AND
 $K^\pm p$ ELASTIC SCATTERING FROM 20 TO 5000 GEV

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Abstract

We give a simple model of hadron scattering from 20 to 5000 GeV which is based on the impact picture derived from quantum field theory. Numerical results, consistent with existing data are given and in addition we predict the general behavior of high energy hadron-hadron elastic scattering.

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During the last five years, we have learned a great deal about high-energy quantum electrodynamics.¹ We now have in our possession the asymptotic high-energy behavior of all elastic² and diffractive, dissociation³ processes. Although such results are extremely interesting in themselves, much of the enthusiasm to achieve this understanding was motivated by the hope that a study of the best field theory available might help us to understand high energy scattering in general. The solution in electrodynamics satisfies unitarity, dispersion relations, and crossing symmetry. The general qualitative features of the sum of all Feynman diagrams have been extracted and have led to a consistent picture of how particles behave at high energy insofar as their electromagnetic interactions are concerned. We have termed this description the impact picture. It is of great interest to know whether the impact picture is relevant to hadron-hadron scattering. The test must come from experimental data. The purpose of this letter is to present, based on the impact picture, a simple model of pp , $\bar{p}p$, $\pi^\pm p$, and $K^\pm p$ elastic scattering. With the parameters chosen properly, the results from the model are given for incident energies between 20 and 5000 GeV and compared with existing experimental data in this range.

In the impact picture each of the two incident particles in a high energy collision process can be visualized as a superposition of multi-particle virtual states.⁴ By the uncertainty principle, each virtual

state can exist a finite length of time in its own center-of-mass system. By time dilation, such a virtual state can exist for a long time in any frame where it is extremely relativistic. On the other hand, in the frame of each of the incident particles, the other incident particle is Lorentz contracted into a thin pancake. This pancake can be roughly separated into two regions:

- 1) A black core with a radius R which expands logarithmically with s , i. e. ($R \sim R_0 \ln s$). In this core, absorption is essentially complete. This black core owes its existence to the production of relatively low energy particles in the center-of-mass system.
- 2) A grey or partially absorptive fringe with a width of order unity (independent of s). Diffractive dissociation occurs in this fringe.

The particles in the two virtual states constituting the pancakes thus interact independently and simultaneously. After the interaction, the new multi-particle virtual states recombine to contribute to the scattered states.

We list some of the immediate predictions of the impact picture for hadron interactions at extremely high energies.

- a) Hadrons that are slow in the CM system are produced according to the distribution^{5, 6}

$$\frac{d^3 p}{E} f(p_{\perp}^2) . \quad (1)$$

This distribution holds not only for the inclusive process but also for each individual exclusive process with suitable quantum numbers.⁵

b) All total cross sections rise indefinitely with energy in such a way that²

$$\lim_{s \rightarrow \infty} \sigma_{\text{total}}(h_1 h_2) / \sigma_{\text{total}}(h_3 h_4) = 1 , \quad (2)$$

where h_1 , h_2 , h_3 , and h_4 are any four hadrons. The Pomeranchuk theorem⁷ is a special case of (2).

c) All total elastic cross sections are one half of total cross sections:²

$$\lim_{s \rightarrow \infty} \sigma_{\text{el}} / \sigma_{\text{total}} = \frac{1}{2} . \quad (3)$$

d) In elastic cross sections, the diffraction peak shrinks indefinitely, and a dip exists and moves to arbitrarily small values of momentum transfer with increasing energy.²

e) The total cross sections for diffractive inelastic processes rise indefinitely with energy.³

At Serpukhov energies, a comparison of pp and K^+p channels shows that (2) is still off by more than a factor of 2. Similarly (3) is off by a factor of about 2 for pp scattering. Since the dependence on energy is logarithmic, neither (2) nor (3) can be expected to be accurate

even at ISABELLE energies ($s \sim 2 \times 10^5 \text{ GeV}^2$). In order to make a meaningful comparison with data from the National Accelerator Laboratory and the CERN Intersecting Storage Rings, we present in this letter a model which, on the one hand, possesses all properties demanded by the impact picture at extremely high energies, and, on the other hand, is sufficiently realistic to fit data for incident laboratory energy above 20 GeV.

Without further ado, the model for elastic scattering is as follows. (Generalization to diffractive inelastic scattering without spin-parity exchange is immediate, but will not be discussed here.) For an elastic channel j , the amplitude at high energies is

$$M_j(s, \vec{\Delta}) = \frac{is}{2\pi} \int d\vec{x}_\perp e^{-i\vec{\Delta} \cdot \vec{x}_\perp} D_j(s, \vec{x}_\perp), \quad (3)$$

where $\vec{\Delta}$ is the momentum transfer and D_j is taken to be

$$D_j(s, \vec{x}_\perp) = 1 - \exp \left\{ -f_j(E e^{-i\pi/2})^c \exp \left[-\lambda (x_\perp^2 + x_{oj}^2)^{1/2} \right] \right\}. \quad (4)$$

Here E is the laboratory energy of the incident particle, c and λ are universal constants (independent of j), and f_j and x_{oj} are parameters that are, however, the same if the incident particle is replaced by its anti-particle. Equation (4) holds when the target particle is a nucleon,

and the phase factor $e^{-i\pi/2}$ is necessary in order to satisfy crossing symmetry.

The differential cross section is given by

$$\frac{d\sigma(j)}{dt} = \left| \frac{M_j(s, \vec{\Delta})}{s} \right|^2 \quad (5)$$

where $t = -\vec{\Delta}^2$. By the optical theorem, the total cross section is related to the imaginary part of $M_j(s, 0)$. Here we introduce a background term:

$$\sigma_{\text{total}}(j) = A_j s^{-1/2} + \frac{4.893}{s} \text{Im } M_j(s, 0), \quad (6)$$

where the factor 4.893 is needed to convert to mb.

In this model, there are fourteen parameters for pp , $\bar{p}p$, $\pi^\pm p$, and $K^\pm p$ scattering: c , λ , $x_0(pp)$, $x_0(\pi p)$, $x_0(Kp)$, $f(pp)$, $f(\pi p)$, $f(Kp)$, $A(pp)$, $A(\bar{p}p)$, $A(\pi^+ p)$, $A(\pi^- p)$, $A(K^+ p)$, and $A(K^- p)$. Of these, the first eight are needed for very high energies, and the remaining six are introduced to fit, for example, Serpukhov data.

The asymptotic energy scale of hadron interactions is controlled by the parameter c .⁸ In order to determine the value of c , one accurate piece of data is required at very high energy. Very recently,^{9b} this most important piece of information has been provided by two beautiful experiments at the ISR in the form of a total pp cross section.

Once the value of c is determined we can make a number of predictions on pp scattering and predict the general high energy behavior of πp and Kp elastic scattering.

Specifically the above fourteen parameters are determined from the following fourteen pieces of data:

- 1) At $E = 20$ BeV, the pp total cross section is 39.0 mb.^9
- 2) At $E = 60$ BeV, the pp total cross section is 38.4 mb.^9
- 3) At $E = 1485$ BeV, the pp total cross section is 43.2 mb. This is the crucial piece of data obtained at the ISR to which we have previously referred.
- 4) At $E = 60$ BeV, the dip in the pp differential cross section occurs at $|t| = 1.8 (\text{BeV}/c)^2$.¹⁰
- 5) At $E = 505$ BeV, the average slope, between $|t|$ of 0.14 and $0.24 (\text{BeV}/c)^2$, for $\ln d\sigma/dt$ is $11.5 (\text{BeV}/c)^{-2}$ for pp elastic scattering.¹¹
- 6) At $E = 20$ BeV, the total $K^+ p$ cross section is 17.5 mb.^9
- 7) At $E = 60$ BeV, the total $K^+ p$ cross section is 18.3 mb.^9
- 8) At $E = 60$ BeV, the average slope, between $|t|$ of 0.1 and $0.28 (\text{BeV}/c)^2$, for $\ln d\sigma/dt$ is $7.4 (\text{BeV}/c)^{-2}$ for $K^+ p$ elastic scattering.¹¹
- 9) At $E = 47.5$ BeV, the $\bar{p}p$ total cross section is 44.1 mb.^9
- 10) At $E = 52.5$ BeV, the $K^- p$ total cross section is 20.3 mb.^9

- 11) At $E = 20$ BeV, the $\pi^+ p$ total cross section is 23.5 mb.⁹
- 12) At $E = 60$ BeV, the $\pi^+ p$ total cross section is 23.15 mb.⁹
- 13) At $E = 54.2$, the average slope, between $|t|$ of 0.05 and 0.53 (BeV/c)², for $\ln d\sigma/dt$ is 9.5 (BeV/c)⁻² for $\pi^+ p$ elastic scattering.¹¹
- 14) At $E = 60$ BeV, the $\pi^- p$ total cross section is 24.2 mb.⁹

The resulting parameters are:

$$\begin{aligned}
 c &= 0.082925 \\
 \lambda^3 &= 0.60071 \\
 x_0(pp) &= 3.8750 \\
 f(pp) &= 6.5812 \\
 A(pp) &= 47.372 \\
 x_0(Kp) &= 1.4295 \\
 f(Kp) &= 1.3452 \\
 x_0(\pi p) &= 3.8457 \\
 f(\pi p) &= 3.6355 \\
 A(\pi^+ p) &= 29.220 \\
 A(\bar{p}p) &= 101.81 \\
 A(K^+ p) &= 8.2543 \\
 A(K^- p) &= 29.384 \\
 A(\pi^- p) &= 40.416
 \end{aligned}$$

In Fig. 1, we show the total cross sections from the present model for the six processes under consideration together with the data from Serpukhov,⁹ NAL,¹² and ISR.¹² All these cross sections are predicted to increase significantly.

In Fig. 2, we give the differential cross sections for pp scattering and compare them with the existing ISR data.^{12a} It is seen that the agreement is very good except in the region of the dip. Not easily visible from these curves is the change in slope near $|t| = 0.1 \text{ (GeV/c)}^2$, as first observed by Carrigan.¹³ We emphasize that this break is not put in, but is a prediction of the model. Another result of the model is that there is no dip for Kp, and the dip for πp occurs near $|t| \approx 3 \text{ (GeV/c)}^2$, consistent with the measurement of Orear.¹⁴

In Fig. 3, we show our results on slope parameters for pp elastic scattering for $|t|$ both above and below 0.15 (GeV/c)^2 . The slope parameter¹⁵ varies rather slowly with energy. Since no background term is used, this result cannot be expected to be accurate at lower energies. This result on slope parameters is in contrast with a conventional Regge fit to Serpukhov and ISR data, but is consistent with recent preliminary measurements at NAL¹⁶ which indicates that the slope parameter for $|t| < 0.1$ is not a linear function of $\ln s$.

Finally, we have calculated the ratio of the real to imaginary part of the pp forward elastic scattering amplitude. The results are in agreement with all of the existing data.

More detailed presentation and discussion of our results will be given elsewhere.

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REFERENCES AND FOOTNOTES

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⁸A preliminary form of the present model was submitted as a preprint to the XVI International Conference at Chicago in September 1972. Due to the lack of data on the pp total cross section at the ISR at that time, we reduced the number of unknown parameters by making a simplifying assumption that $A(K^+p) = 0$. Note also that for simplicity we have here used a power of $(E e^{-i\pi/2})$ without dividing by factors involving $\ln E$, which appear in field theoretical models. Thus our c here is an effective value. The introduction of such logarithmic factors make c larger, and a more complete discussion of this point will be presented later.

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^{9b}The ISR data was kindly provided to us in a private communication from G. Cocconi and G. Finochiaro.

¹⁰J. V. Allaby et al., Phys. Letters 34B, 431 (1971). From this reference it is clear that the position of the structure in the pp elastic cross section at low energies ($s < 100$) is in the range $1 \leq |t| \leq 2$. There is also a large s dependence in the region of the structure indicating the presence of background terms in the amplitude. We have chosen the value $|t| = 1.8$ because it tends to improve the overall fit.

¹¹The experimental values for the logarithmic derivatives of $d\sigma/dt$ are slightly below the values chosen for the fit. This is in an attempt to

take account of small residual background terms in the scattering amplitude which diminish as a power of the energy.

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- ¹⁶ D. Gross, contributed paper to the New York Meeting of the American Physical Society, January 1973.

FIGURE CAPTIONS

- Fig. 1 Total hadron-proton cross sections in millibarns are shown as a function of s (the square of the center-of-mass energy). The curves are predictions of the impact picture with the parameterization given in the text.
- Fig. 2 Proton-proton differential elastic scattering cross section $d\sigma/dt$ as a function of $|t|$ for various values of s . The solid curves are predictions of the impact picture with the parameterization given in the text.
- Fig. 3 Values of the slope parameter b at small momentum transfer ($0.05 \leq |t| \leq 0.15$) and intermediate momentum transfer ($0.15 \leq |t| \leq 0.35$) are shown as a function of s . The curves are the predictions of the impact picture with the parameterization given in the text.

Fig 1

$b(\text{GeV}/c)^2$

PP ELASTIC SCATTERING SLOPE PARAMETER

$0.05 \leq |\eta| \leq 0.15$

(0.06)

(.072)

(.07)/(.12)

(0.35)

(0.07)

(.07)

(0.12)

(.09)

(.30)

(.19)

(.19)

(0.27)

(.24)

$0.15 \leq |\eta| \leq 0.35$

(.19)

10

100

$S(\text{GeV})^2$

1000

10,000

14

13

12

11

10

9