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FOLLOWING μ CAPTURE IN COMPLEX NUCLEI

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THE MEASUREMENT OF NUCLEAR EXCITATION FOLLOWING μ^- CAPTURE IN COMPLEX NUCLEI*

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Abstract

The techniques and results of experiments designed to measure the nuclear excitation spectrum following muon capture are reviewed. Measurements of this excitation are achieved principally by counting or measuring the energy spectrum of emitted neutrons, identifying prompt γ -ray transitions, or identifying resultant radionuclides, with any single measurement giving only a partial picture of the excitation process. Various measurements are compared and where they are complementary, or overlap, they are found to be in good agreement. Special excitation characteristics peculiar to some nuclides (such as considerable charged-particle emission or pronounced resonance characteristics in the neutron spectrum) are observed or inferred consistently by the various measurement techniques. Recent measurements of neutron and γ spectra following muon capture demonstrate the potential importance of muons as a tool for studying giant resonances and shell-model hole levels of nuclei.

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The Measurement of Nuclear Excitation
Following μ^- Capture in Complex Nuclei

I. Introduction

I have exercised some care in choosing the title for this review — allowing myself sufficient breadth to fill the hour but, at the same time, providing myself with the protective shield of the word "measurement." There are too many experts here on the theory of muon capture for me to dare more than a talk on nuts and bolts.

What I shall do is briefly describe the measurement techniques used to study this excitation and review the present status of the various types of measurements, in particular indicating how they interrelate and how well they agree with one another in regions of overlap. I shall concentrate on the kinds of measurements that give information about the principal characteristics of the distribution of nuclear excitations. I shall confine myself primarily to the range of Z where nuclear capture is the predominant mode of muon disappearance, and shall omit or treat only incidentally that class of experiments that measures partial transition rates with the intention of determining particular weak-interaction coupling constants. Charged-particle emission I shall also treat only incidentally since it represents a small fraction of the total excitation and was already discussed in some detail by Singer.¹

Let me begin with a simple review of the sequence of events subsequent to nuclear capture,

$$\mu^- + {}^A_ZX \rightarrow {}^A_{Z-1}Y^* + \nu. \quad (1)$$

First the excited product nucleus may break up by emission of one or more nucleons, primarily neutrons:

The secondary product nucleus, Y' , is, in general, in an excited state and decays to its ground state by emission of one or more γ rays

If this ground state is radioactive it will β decay

With a neutrino spectrometer one could measure the excitation spectrum directly in the initial reaction, (1). Lacking this, what one must do instead is learn as much as he can piecemeal from the secondary and tertiary decays (2), (3), and (4).

It is almost like a blind man with limited access to an elephant's hide trying to convert the tactile information into a picture of the beast. Actually, various pictures of the "beast" have been modeled² and the practice is generally to test whether the feel of the hide is consistent with the picture from the model.

II. The Measurement Techniques

Basically, four types of measurements have been used for obtaining this information: residual radioactivity, neutron counting, neutron spectroscopy, and γ -spectroscopy.

The measurement of residual radioactivity accounts for the total number of muon captures leading to a particular final nuclear ground

state. It accounts for all associated particle emission but gives no information about either particle kinetic energies or γ -ray energies. It gives no information whatsoever if the product nucleus is stable.

Since most of the energy comes off in the form of neutrons (in fact, half or more is in the form of neutron binding energy), one can learn a great deal by simply counting neutrons. If one knows, or can estimate, the neutron kinetic energy distribution and the nuclear- γ -ray energy distribution, one can, with neutron-multiplicity-distribution data, on a very coarse grid, determine the relative probabilities of several excitations, that is those leading to zero, one, two, etc. neutron emissions.

By measuring γ -ray spectra one can, in principal, identify both the final product nucleus and its state of excitation. There are several limitations peculiar also to this type of measurement. Ground-state transitions are unobservable. Probably, even more important, numerically speaking, there may be a number of levels excited, none of which individually is excited sufficiently often to be observable above background, but, which taken altogether, constitute a significant fraction of the total captures. Furthermore, in many possible final states γ transitions are not sufficiently well known to allow identification of all γ rays observed.

Neutron-energy-spectrum measurements test, or determine, the kinetic-energy-correction methods used in interpreting the results of the above three methods. The high-energy tail of the neutron energy spectrum, furthermore, gives information directly about the high-energy tail of the excitation distribution. Finally, if the muon capture occurs into an energetically well defined, but particle —

unstable, resonance level this will be observable directly as a peak in the neutron energy spectrum.

A. Radioactivity

Let me begin with the first technique, the measurement of residual radioactivity. I think this was really the first technique that was employed very seriously to try to probe into the excitation distribution. This was done quite a while ago, in 1954, at Chicago, by Winsberg.³ He studied iodine, which has also been studied by other techniques. The actual results of the measurements are shown in column 3 of Table I. The only products that could be accounted for in their entirety were ^{127}Te , ^{122}Sb , and ^{124}Sb . The first yield estimate is for ^{125}Te where it is assumed that the branching between the isomer and the stable ground state is the same as for ^{127}Te , where both are observable. The evidence for $4n + 1p$ emission to ^{122}Sb appears to be reasonably strong. The alternative explanation for this final state, pion contamination, a potential problem in all of these experiments, is ruled out by comparison of the relative yields of ^{122}Sb and ^{118}Te . Their relative production by pion capture was approximately equal. The assignments for other yields are based on very qualitative interpolations and extrapolations.

I think this example illustrates nicely both the possibilities for and the limitations of this technique.

B. Neutron Counting

The next method that was used was simply counting the neutrons emitted. In order for this type of measurement to be meaningful the neutrons must be counted with sufficiently high efficiency so that multiplicity distribution information is obtained. The technique used

is one borrowed from the people who measured such multiplicities from fission. One uses a large tank of liquid scintillator that has dissolved in it a high-cross-section neutron capturer. Originally a compound of Cd was used for this purpose. Now Gd is used. An example of such a tank is shown in Fig. 1. Under operating conditions a phototube would be mounted on each window. The tube passing through the tank center is 8 in. in diameter. In a complete experimental setup the tank subtends almost 4π about the target (Fig. 2).

Such a detecting technique although highly efficient requires that neutrons be thermalized before undergoing capture. The detector must be sufficiently large both to thermalize the neutron and to contain a reasonably large fraction of the neutron-capture gamma rays. The average time for the entire process is about 10 microseconds, during which time this enormous tank is sensitive to all background with energy as low as about one - two Mev. Where such a tank provides 4π detection about a small-diameter well, neutron-detection efficiencies in excess of 80% are quite feasible. However, we can probably expect a limiting efficiency in the neighborhood of 60% for muon capture work because of the larger beam tube required, the practical tank-size limits, and the sensitive-time limits as determined by accelerator beam and background. The experimental data in the last measurements of this sort⁴ were collected before the introduction of the high-duty-cycle stretched beam at Berkeley. Considerable improvement in the statistics of the measurements is therefore possible.

The results of these neutron-multiplicity distribution measurements⁴ are shown in Table II. I should note that these results have not before been presented in their entirety in this form but only as the "observed" distribution after background subtraction, f_n . The distribution

functions f_n and p_v transform into one another by the equations (Appendix 1)

$$f_n = \sum_{v=n}^{vmax} p_v \epsilon^n (1-\epsilon)^{v-n} \binom{v}{n}$$

$$p_v = \sum_{n=v}^{vmax} f_n (-1)^{n-v} (1-\epsilon)^{n-v} \epsilon^{-n} \binom{n}{v}$$

where ϵ is the neutron-detection efficiency. For comparison with theoretical predictions it was quite easy to convert the predicted multiplicity distribution to our detection efficiency. Furthermore there was no necessity to worry about the problems of error propagation that would attend any manipulation of experimental data. Inasmuch as comparisons must now be made with other types of experimental data and since our old results have, on several occasions, lent themselves to misinterpretation I have performed the mathematical transformations. The handling of error propagation is approximately correct - it would be exact if there were no background. (See Appendix 2.)

C. Neutron-Energy Spectra

There are a number of techniques for measuring neutron energy spectra in the MeV range. They all have the common feature that the detection process involves an n-p collision. The technique giving the highest precision and having the best dynamic range characteristics is probably time of flight. Unfortunately in the case of muon capture there is not a satisfactory signal to initiate this time measurement.⁵ Energy measurements employed then have required determining the

neutron energy from the energy deposition in an n-p collision. Energies of individual neutrons have been measured from recoil direction and track length in both hydrogen bubble⁶ and spark⁷ chambers. These methods seem to have a practical threshold of 4-5 MeV. Several experiments have been performed that determine the neutron-energy spectrum from the light spectrum produced in an organic phosphor by n-p collision recoils.⁸⁻¹¹ Results by this last technique cover the greatest dynamic range, and report the greatest measurement precision over the regions of most interest. I shall therefore concentrate on describing this technique and results therefrom. The process producing these energy spectra from the raw data is quite formidable and in order that you may appreciate both the effort involved in producing final data and the possible sources of apparent inconsistencies I would like to say a few words about the technique, working backwards from the resultant neutron-energy spectrum. The final step is quite straightforward. Given a proton-energy spectrum, $P(E)$, that is the result of one-collision, isotropic n-p scattering we may find the spectrum of interacting neutrons by the simple differential relationship,

$$N(E) = E \frac{dP(E)}{dE}$$

How does one obtain this clean proton-energy spectrum? First, one must be sure to count only protons. And one can indeed distinguish protons from minimum-ionizing particles in certain organic scintillators because of a property of these phosphors that causes them to emit proportionally more "late light" from highly-ionizing particles. A simple circuit that provides some separation of the "early" and "late"

parts of a phototube signal is shown in Fig. 3 together with an x-y oscilloscope display comparing the pulse height ratios from the two circuit branches. Circuits that perform this function are commercially available.

Do these selected pulses give the proton spectrum? Alas no, because the scintillator response is nonlinear in this proton-energy range. One must therefore find a suitable way to calibrate the scintillator. Finally, one must avoid, or correct for, multiple collisions, non-hydrogen collisions, and scattering anisotropies. These compensations and corrections are all made either in hardware or software and a computer program turns the scintillator pulse-height spectrum into a neutron-energy spectrum. Some examples of such spectra are shown in Fig. 4.

D. Prompt γ Rays

Finally, we come to the detection and analysis of nuclear gamma-ray spectra. This kind of measurement is esthetically very attractive because of the extremely high precision with which these energies can be measured with a Ge(Li) spectrometer. Several measurements of this sort have been reported and some data taken by us with this technique will be described later in this conference.¹² The principal detector limitations in this application are its comparatively small size and its sensitivity to fast-neutron damage. The omnipresent fast neutrons gradually cause the quality of this fine energy resolution to deteriorate. Recent developments in Ge-detector technology show promise of yielding pure Ge detectors that are both larger and less sensitive to neutron damage.¹³

An example of the neutron-multiplicity-type inferences one may make from γ -ray data¹⁴ is shown in Table III where the authors infer

from their results that:

- 1) On the order of 70% of the captures can be accounted for.
- 2) No-neutron emission appears quite unlikely.
- 3) No proton emission is observed.

III. Comparison of Results

Now let us begin with this last piece of data and see how the various measurements interrelate and how well they agree with one another.

First, it should be pointed out that, although covering a number of nuclei, this is not a general sampling. It consists of odd-Z, odd-A nuclei with $Z_{\min} > 20$.¹⁵ If we look at direct neutron multiplicity measurements (Table II) we see similar results for odd-Z nuclei but, on the other hand, considerable no-neutron emission for some even-Z nuclei.

Where a direct comparison can be made, in iodine, these results are in remarkably good agreement with those from neutron-multiplicity measurements. Similarly in ^{110}Ag the results are well within the assigned errors (Table IV).

The early radioactivity measurements on iodine indicated no-particle emission for 8% of the μ^- captures (Table I). These results are not necessarily inconsistent with either the γ -ray- or neutron-multiplicity data if we remember that the γ -ray result gives only a lower limit to the yield of a particular final nuclide.

More recent radioactivity measurements (Table V) are in excellent agreement with neutron multiplicity results - particularly with regard to the no-neutron emission from Si. The radioactivity evidence for

single-proton emission is complimented by both γ -ray^{12,16,17} and charged-particle-spectrum¹⁸ results indicating an addition d and presumably n+p emission.

All in all then, although we don't have as complete a picture as we might wish, the radioactivity, prompt γ -ray spectra, and neutron multiplicity measurements have given a consistent picture of the final nuclear states following μ^- capture. Mutual agreement is, at best, excellent, and, at worst, not inconsistent in describing both average effects and individual differences.

What about the neutron-spectrum measurements? At first glance, what might appear difficult to compare, or inconsistent, I think are really neither. For example, let us look at the two measurements of the ^{16}O -capture spectra that are represented as showing resonance structure^{10,11} (Fig. 5.) Thanks to a recently reported normalization¹⁹ of the results of Ref. 9 we may compare the two results directly and absolutely. I think the agreement looks rather good, especially in consideration of the data-reduction problems I've previously described. Perhaps this amount of disagreement may really demonstrate the limitations on this spectrum-measuring technique. The resonance character of the spectrum seems unmistakable. The details of the differences in the measurements represent, unfortunately, a factor-of-two difference in the inferred resonance contribution.

Another place where we may compare absolute yields is in Ca (Fig. 6) where the normalized low-energy results of Evseev et al.¹⁰ may be compared with the higher-energy results of Sundelin et al.⁹ Again the quality of agreement seems not unreasonable.

If we now combine neutron spectrum and multiplicity data we find the yield information given in Table VI. There are again no clear internal inconsistencies.

Finally, with regard to spectra, I would like to say a few words about the low energy neutrons - the boil-off neutrons. In the heavy elements especially, most of the neutrons are emitted at energies lower than the energy-measurement threshold.

Experimentally, we look only at the tip of an iceberg and attempt a one-parameter fit to what we see, and which will, hopefully, describe also what we do not see. This one parameter is called the temperature. Unfortunately, the value of this parameter depends upon the form of the equation in which it is used and different experimentors have used different equations. I prefer the simple mathematical form

$$N(E) = \bar{v}(E/\theta^2) \exp[-(E/\theta)]$$

because it fits data about as well as anything else and is easily integrable. If we fit this mathematical expression to the Pb data of Evseev et al.¹⁰ between 2 and 7 MeV, we obtain for θ , 0.99 MeV. Then, assuming all neutrons are in this evaporation spectrum we obtain

$$\bar{v} = 1.35 \pm 0.08$$

accounting for 80% of the directly-measured neutron emission results of Macdonald et al.⁴ (1.71 ± 0.07). Considering the extent of the extrapolation into the low-energy-neutron region. The agreement is not unreasonable, but quantifying any difference in terms of direct emission would, I think, be difficult.

In summary then, so far as the general characteristics of nuclear excitation are concerned I would say that the various experimental results are quite consistent in their regions of overlap. Measured differences in these excitation characteristics are quite clearly due to differences in individual nuclei and not to differences in the measurements. Some nuclei almost invariably emit at least one neutron; some have very significant charged-particle emissions, and at least one, Si, is particle stable a large fraction of the time. Some neutron spectra are almost entirely characteristic boil-off spectra (e.g. Pb) whereas others have considerable high-energy tails, and yet others show quite strikingly the peaks characteristic of resonance capture.

IV. Nuclear Studies Using γ Spectroscopy

Finally, I would like to say a bit more about some of the detailed results from γ spectroscopy, particularly the kinds of excitations and transitions exemplified by the 6.3 MeV level of ^{15}N , produced copiously in μ^- capture on ^{16}O ²⁰ (Fig. 7.) This is a $3/2^-$ proton-hole level, an intermediate level in the capture process predicted in resonance-model calculations^{21,22} and also what you would expect in the most simple-minded intuitive picture because of the nature of the μ^- capture mechanism - the creation of a neutron in an excited state while leaving a proton hole. The peak is very broad, 120 keV at its base, testifying to the nature of the nuclear-recoil velocity - the effect of recoil from both the neutrino and a fast neutron are clearly seen.²³ The observation of any poppler broadening suffices to set a lower limit for the level lifetime. In this particular case much of the wealth of information contained in this single γ transition was more corroboration than discovery.

A similar situation obtains in Ca with the analogous excitation of the $d\ 5/2^-$ proton-hole level. Here the excitation is not unique but shared by several spectroscopic levels.²⁴

Does, perhaps, Ca constitute an upper limit in Z where one may expect to see and identify such special kinds of excitations? Most certainly not. G. A. Bartholomew²⁵ has shown us a structural feature in the γ -ray spectra from μ^- capture in radiogenic lead — also characterized by a broad peak. There, of course, the Doppler effect has nothing to do with the broadening. But the effect may be more closely related to the multiplicity of levels in Ca where great numbers of transitions have a common origin in some gross shell-structure feature.

V. Conclusions

In conclusion let me say that I think, although improved measurements are possible everywhere, that the experimental data on the gross features of nuclear excitation are mutually consistent and in reasonably good order. Further nuclear excitation study by γ spectroscopy appears to promise much in the way of fruitful results — particularly from the standpoint of nuclear-structure studies. Of special interest is identifying and understanding the broad peaks and giant-resonance levels. Perhaps more mundane but also worthwhile is the standard energy-level reconstruction from γ spectroscopy on nuclei not easily produced otherwise — with the added bonus of built-in lifetime-limit determination. With the advent of high-current accelerators, muons may, in fact, become at least as useful tools for such studies as are thermal neutrons.

Appendix 1 Inversion of the Multiplicity-Distribution Function²⁶

A simple but elegant mathematical method for inverting the relationship between f_n , the observed distribution, and p_v , the actual distribution, may be performed as follows:

Beginning with

$$f_n = \sum_{v=n}^{vmax} p_v \epsilon^n (1-\epsilon)^{v-n} \binom{v}{n}$$

we multiply by the arbitrary factor g^n and sum over n

$$\begin{aligned} \sum_{n=0}^{vmax} g^n f_n &= \sum_{n=0}^{vmax} g^n \sum_{v=n}^{vmax} p_v \epsilon^n (1-\epsilon)^{v-n} \binom{v}{n} \\ &= \sum_{v=0}^{vmax} p_v \sum_{n=0}^{vmax} g^n \epsilon^n (1-\epsilon)^{v-n} \binom{v}{n}. \end{aligned}$$

We may evaluate the first sum by making the substitution $\frac{g\epsilon}{1-\epsilon} = \frac{\lambda}{1-\lambda}$.

Then

$$\begin{aligned} \sum_{n=0}^{vmax} g^n f_n &= \sum_{v=0}^{vmax} p_v \left(\frac{1-\epsilon}{1-\lambda} \right)^v \sum_{n=0}^{vmax} \left(\frac{\lambda}{1-\lambda} \right)^n (1-\lambda)^v \binom{v}{n} \\ &= \sum_{v=0}^{vmax} p_v (1-\epsilon+g\epsilon)^v \sum_{n=0}^{vmax} \lambda^n (1-\lambda)^{v-n} \binom{v}{n} \\ &= \sum_{v=0}^{vmax} p_v (1-\epsilon+g\epsilon)^v. \end{aligned}$$

Defining $e^{i\phi} = (1-\epsilon+g\epsilon)$, where ϕ is a real number, and substituting into the above expression we obtain

$$\sum_{n=0}^{\nu\max} \left[\frac{e^{i\phi} + (\epsilon-1)}{\epsilon} \right]^n f_n = \sum_{\nu=0}^{\nu\max} p_{\nu} e^{i\nu\phi}.$$

Performing a binomial expansion of the expression in brackets, gives

$$\sum_{n=0}^{\nu\max} \frac{f_n}{\epsilon^n} \sum_{j=0}^n e^{ij\phi} (\epsilon-1)^{n-j} \binom{n}{j} = \sum_{\nu=0}^{\nu\max} p_{\nu} e^{i\nu\phi}.$$

Multiplying both sides by $e^{i\nu'\phi}$, where ν' is an integer, and integrating over ϕ from 0 to 2π gives

$$\sum_{n=0}^{\nu\max} \frac{f_n}{\epsilon^n} \sum_{j=0}^n (\epsilon-1)^{n-j} \binom{n}{j} 2\pi \delta_{\nu',j} = \sum_{\nu=0}^{\nu\max} p_{\nu} 2\pi \delta_{\nu\nu'}.$$

$$\sum_{n=\nu'}^{\nu\max} \frac{f_n}{\epsilon^n} (\epsilon-1)^{n-\nu'} \binom{n}{\nu'} = p_{\nu'},$$

and finally

$$p_{\nu'} = \sum_{n=\nu'}^{\nu\max} f_n (1/\epsilon)^{\nu'} (1-1/\epsilon)^{n-\nu'} \binom{n}{\nu'}.$$

Appendix 2. Calculation of the Multiplicity-Distribution Values in Table II from the Experimental Results in Table IV of Ref. 4²⁶

If f_n is the probability of observing n neutrons from a capture event, then we may express the probability of observing a particular distribution as

$$F_{N_0, \dots, N_n, \dots, N_{vmax}} = Z! \prod_{n=0}^{vmax} \frac{f_n^{N_n}}{N_n!} \quad (1)$$

where $vmax$ is the maximum number of neutrons observable in a single event, Z is the total number of events, and N_n is the number of events in which n neutrons were observed. Therefore the f_n 's and N_n 's are not linearly independent but are related by

$$\sum_{n=0}^{vmax} N_n = Z, \quad (2)$$

and

$$\sum_{n=0}^{vmax} f_n = 1. \quad (3)$$

The averages and errors associated with such a multinomial distribution are calculated in the same manner as for binomial distributions and the quantities of interest are

$$\overline{N_n} = \sum_{N_{vmax}} \dots \sum_{N_n} \dots \sum_{N_0} N_n F \delta \left(Z - \sum_{n=0}^{vmax} N_n \right) = Z f_n. \quad (4)$$

Similarly,

$$\overline{N_n(N_n - 1)} = Z(Z-1) f_n^2 \quad (5)$$

and

$$\overline{N_n N_m} = Z(Z-1) f_n f_m \quad n \neq m. \quad (6)$$

Combining the results of (4), (5) and (6) we find

$$\overline{N_n N_m} = Z(Z-1) f_n f_m + Z f_n \delta_{mn}. \quad (7)$$

If we define $\Delta f_n = N_n/Z - f_n$,

then

$$\begin{aligned} \overline{\Delta f_n \Delta f_m} &= \overline{\left(\frac{N_n}{Z} - f_n \right) \left(\frac{N_m}{Z} - f_m \right)} \\ &= \frac{\overline{N_n N_m}}{Z^2} - \frac{f_n \overline{N_m}}{Z} - \frac{f_m \overline{N_n}}{Z} + f_n f_m. \end{aligned}$$

Substituting from (4) and (7) we obtain

$$\overline{\Delta f_n \Delta f_m} = \frac{f_n \delta_{mn} - f_n f_m}{Z}. \quad (8)$$

If the probability of emission of v neutrons is

$$p_v = \sum_{n=v}^{v_{\max}} f_n (1/\epsilon)^v (1-1/\epsilon)^{n-v} \binom{n}{v}, \quad (9)$$

where ϵ is the detector efficiency, then

$$\begin{aligned} (\Delta p_v)^2 &= \left[\sum_{n=v}^{v_{\max}} \Delta f_n (1/\epsilon)^v (1-1/\epsilon)^{n-v} \binom{n}{v} \right]^2 \\ &= \sum_{n=v}^{v_{\max}} \sum_{m=v}^{v_{\max}} \overline{\Delta f_n \Delta f_m} (1/\epsilon)^{2v} (1-1/\epsilon)^{m+n-2v} \binom{n}{v} \binom{m}{v}. \end{aligned}$$

Substituting from (8) gives

$$\overline{(\Delta p_v)^2} = \sum_{n=v}^{v_{\max}} \frac{f_n}{Z} \left[(1/\epsilon)^v (1-1/\epsilon)^{n-v} \binom{n}{v} \right]^2 - \left\{ \sum_{n=v}^{v_{\max}} f_n (1/\epsilon)^v (1-1/\epsilon)^{n-v} \binom{n}{v} \right\}^2. \quad (10)$$

If v_0 is the largest value of v for which p_v is aesthetically pleasing (i.e. is still larger than $\sqrt{(\Delta p_v)^2}$) we may wish to combine terms for the higher multiplicities and find

$$\begin{aligned} \sum_{v_0+1}^{v_{\max}} p_v &= \sum_{v=v_0+1}^{v_{\max}} \sum_{n=v}^{v_{\max}} f_n (1/\epsilon)^v (1-1/\epsilon)^{n-v} \binom{n}{v} \\ &= \sum_{n=v_0+1}^{v_{\max}} f_n \sum_{v=v_0+1}^n (1/\epsilon)^v (1-1/\epsilon)^{n-v} \binom{n}{v} \\ &= \sum_{n=v_0+1}^{v_{\max}} f_n \left[1 - \sum_{v=0}^{v_0} (1/\epsilon)^v (1-1/\epsilon)^{n-v} \binom{n}{v} \right] \end{aligned} \quad (11)$$

$$\equiv \sum_{n=v_0+1}^{v_{\max}} f_n C(v_0, n) \quad (12)$$

Then, by comparison with (9) and (10)

$$\overline{\left(\sum_{v_0+1}^{v_{\max}} \Delta p_v \right)^2} = \sum_{n=v_0+1}^{v_{\max}} \frac{f_n}{Z} [C(v_0, n)]^2 - \left[\sum_{n=v_0+1}^{v_{\max}} f_n C(v_0, n) \right]^2 \quad (13)$$

where $C(v_0, n)$ is defined by (11) and (12).

These results are based purely on the statistics of counting and give for $\overline{\Delta f^2}$

$$\overline{\Delta f_n^2} = f_n (1-f_n) / Z.$$

Because of some corrections for additional errors the quoted errors⁴ $\overline{\Delta f_n^2}$ are somewhat larger, so for the errors given in Table II, f_n in equations (10) and (13) was approximated as

$$f_n \approx \sqrt{2 \overline{(\Delta f_n)^2}} \quad (\text{as quoted in ref. 4})$$

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24. See Fig. 4 of reference 12 (these proceedings).
25. G. A. Bartholomew, these proceedings, Fig. 11.
26. Although these are presumably text-book-type calculations I am not familiar with a suitable reference. I have therefore attached the calculations, in full, as appendices.

Table IParticle Emission Inferred from Residual Radioactivity^a $^{127}_{53}\text{I}$ Target

Nucleons emitted	Product (half-life)	Yield (%)	Inferred yield (%)
0	$^{127}\text{Te}(9.4\text{h})$ $^{127}\text{Te}^{\text{M}}(110\text{d})$	5.2, 3 ± 0.2	8.2
1n	^{126}Te (stable)	--	43
2n	^{125}Te (stable), $^{125}\text{Te}^{\text{M}}(58\text{d})$	-- , 12.6 ± 2.5	34
3n	^{124}Te (stable)		11
4n	$^{123}\text{Te}(10^{13}\text{y})$, $^{123}\text{Te}^{\text{M}}(117\text{d})$	-- , < 5	2 - 3
:			
9n	$^{118}\text{Te}(9\text{d})$	< 0.03	
1p1n	$^{125}\text{Sb}(2.7\text{y})$	--	
1p2n	$^{124}\text{Sb}(60\text{d})$	0.23 ± 0.07	0.23
1p3n	^{123}Sb (stable)	--	0.4
1p4n	$^{122}\text{Sb}(2.8\text{d})$	0.16 ± 0.02	0.16
:			
2pxn	Sn	--	

^a Ref. 3

Table II

Neutron Multiplicity Distributions (%)

Nucleus	P_0	P_1	P_2	ν_{max}	ν_{max}
				$\frac{1}{2} P_\nu$	$\frac{1}{3} P_\nu$
Al	7.7±3.7	82.2±7.7	-6.8±15.6	10.1±5.6	16.9±11.7
Si	36.3±5.7	45.7±10.8	24.5±21.1	18.0±7.6	-6.5±15.8
Ca	37.0±2.9	54.1±5.6	9.3±7.3	9.0±3.6	-0.3±4.6
Fe	19.0±2.5	59.9±5.5	9.0±9.2	21.1±4.2	12.0±6.5
Ag	6.5±3.1	48.9±8.1	30.7±16.0	44.6±6.8	14.0±11.4
I	4.4±3.1	70.9±7.7	9.3±13.9	24.7±6.1	15.5±9.8
Au	11.1±2.2	37.7±5.8	40.1±12.7	51.2±5.0	11.1±9.3
Pb	-0.5±3.4	59.2±8.5	23.7±13.4	41.2±7.1	17.6±8.3

Table IIIFinal-State Distributions from Nuclear γ Rays^a

Target Nuclide	Neutron-emission probability (%)				Total captures seen (%)
	P ₀	P ₁	P ₂	P ₃	
⁴⁵ Sc	≥0	≥63.5±6.0	≥3.6±1.0		67.1±6.1
⁵⁵ Mn	≥0	≥44.0±4.0	≥14.1±2.9	≥7.3±3.0	65.3±5.8
⁵⁹ Co	≥1.9±0.3	≥47.3±5.2	≥14.5±1.7	≥7.1±1.5	72.2±5.7
⁹³ Nb	≥1.0±0.3	≥52.5±4.0	≥18.0±2.5		71.5±4.7
¹²⁷ I	≥0	≥53.7±5.1	≥1.7±0.5	≥16.2±1.6	72.8±5.5
²⁰⁹ Bi	≥0	≥43.0±7.0	≥19.3±3.5	≥7.7±1.3	70 ±8

^aRef. 14.

Table IVComparison of neutron-multiplicity and γ -ray spectra

Target Nuclide	P(0)	P(1)	P($\nu > 1$)	
^{27}Al	$\geq 6.8 \pm 0.7$	$\geq 70 \pm 6$	$\geq 4.5 \pm 0.5$	81.3 $\pm$ 6% of capture accounted for ^a
	8.3 ± 0.85	85.5 ± 7.3	5.5 ± 0.6	Normalized ^b
	7.7 ± 3.7	82.2 ± 7.7	10.1 ± 5.6	Direct observation of neutrons
^{127}I	≥ 0	$\geq 53.7 \pm 5.1$	$\geq 25 \pm 2$	78.9 $\pm$ 5.5% of captures accounted for ^c
	≥ 0	68.1 ± 6.5	31.7 ± 2.5	Normalized ^b
	4.4 ± 3.1	70.9 ± 7.7	24.7 ± 6.1	Direct observation of neutrons

^aRef. 12.^bAssuming proportionate distribution of unobserved excitations^cRef. 14

Table V

Partial probabilities of muon capture without nucleon emission and with single-proton emission as determined from radioactivity measurements. The value, W , is the ratio of the partial capture rate to the total capture rate.

Reaction	W(%)	Reaction	W(%)
<u>No-nucleon emission^a</u>			
$^{27}\text{Al}(\mu^-, \nu)^{27}\text{Mg}$	10 ± 1	$^{51}\text{V}(\mu^-, \nu)^{51}\text{Ti}$	10 ± 1
$^{28}\text{Si}(\mu^-, \nu)^{28}\text{Al}$	28 ± 4	$^{56}\text{Fe}(\mu^-, \nu)^{56}\text{Mn}$	16 ± 3
<u>Single-proton emission^b</u>			
$^{28}\text{Si}(\mu^-, \nu p)^{27}\text{Mg}$	5.3 ± 0.1	$^{39}\text{K}(\mu^-, \nu p)^{39}\text{Cl}$	3.2 ± 0.6

^aG. Bunation, V. Evseev, L. Nikityuk, V. Pokrovsky, V. Ribakov, and I. Yutlandov in High-Energy Physics and Nuclear Structure, S. Devons, editor (Plenum Press, New York, 1969) p. 182.

^bL. Vil'gel'mova, V. Evseev, L. Nikituk, L. Pokrovskii and I. Yutlandov, Yadern. Fiz. **13**, 551 (1971).

Table VI

Summary of absolute neutron yields in various energy ranges from neutron-spectra and neutron-multiplicity results

Nuclide	E_1 (MeV)	E_2 (MeV)	$\bar{v}(E_1 \rightarrow E_2)^a)$	$\bar{v}(E > E_2)^b)$	$\bar{v}(\text{total})$	$\bar{v}(E < E_1)$
O	2.5	7.5	0.526 ± 0.097		$0.8 \pm 0.2^c)$	
Si		7.73		0.232 ± 0.041	$0.86 \pm 0.07^d)$	
S	1.5	10	0.414 ± 0.035	0.154 ± 0.028		
Ca	2	10	0.265 ± 0.033	0.135 ± 0.024	$0.75 \pm 0.03^d)$	0.35 ± 0.05
Pb	2	7	0.536 ± 0.031		$1.71 \pm 0.07^d)$	

^a Evseev et al. (Ref. 10).

^b Sundelin et al. (Ref. 9).

^c Plett & Sobottka (model fit) (Ref. 11).

^d Macdonald et al. (Ref. 4).

Figure Captions

1. Large liquid-scintillator tank with 8-in.-diameter beam tube.
2. Experimental arrangement of muon telescope and liquid-scintillator tank (from Ref. 4).
3. Distinguishing proton recoils from electrons in an organic phosphor using pulse-shape discrimination. On the left is a schematic diagram of a circuit that will produce some separation of early and late phototube current. On the right is a negative image of an oscilloscope dot display of stretched x and y pulses.
4. Examples of neutron-energy spectra measured by pulse-shape discrimination and pulse-height analysis. Note that the results cover a range of from 2 to 50 MeV.
5. Neutron-energy spectra from ^{16}O - a comparison of the resonance-structure observations of Evseev et al (solid histogram - ref. 10) and Plett and Sobottka (dot histogram - Ref. 11).
6. Neutron-energy spectra from Ca showing the absolute agreement, in the region of overlap, between the low-energy results of Evseev et al (solid histogram - ref. 10) and the high-energy results of Sundelin et al. (shaded histogram - Ref. 9).
7. Energy spectrum of γ rays following μ^- capture in ^{16}O (from Ref. 20).

GPR-1467

Fig. 1

MU-17656

Fig. 2

N - γ Discrimination

MU - 20662

Fig. 3

Fig. 4

XBL723-2553

Fig. 5

XBL723-2552

Fig. 6

Fig. 7