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THE EQUIVALENT ENERGY METHOD FOR CALCULATING
ELASTIC-PLASTIC FRACTURE

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HEAVY SECTION STEEL TECHNOLOGY PROGRAM

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FOREWORD

This paper was presented at the Fourth National Symposium on Fracture Mechanics, Pittsburgh, Pennsylvania, August 24-26, 1970. Considerable amounts of pertinent data have become available since the original presentation. Most of the data generally support the views presented herein; however, some data suggest that complete generality may not be realized. Two notations have been placed in the body of the paper and these are discussed in the appendix.

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Nomenclature

- a: crack depth as defined in linear elastic fracture mechanics (in.)
- $a()$: crack depth in a model of thickness () of the structure of interest
- b,c: volumetric energy ratios as defined in text
- C: shape and constraint factor for the flaw configuration used in linear elastic fracture mechanics
- $E()$: energy absorbed in structure as defined (in. lb/in.³ or psi)
- K_{Ic} : fracture toughness at a particular temperature (psi $\sqrt{\text{in.}}$)
- K_{Icd} : fracture toughness for a particular thickness d at a given temperature (psi $\sqrt{\text{in.}}$)
- P_Q, P_{Q1} : load at fracture of a specimen either real or hypotherized (lb or psi)
- $r_{f,d}$: ratio of a dimension, f, of a structure and the corresponding dimension, d, of a geometrically similar structure
- $s_{d,f}$: volumetric energy ratio between a model of thickness d and the structure of thickness, f
- σ_f : the equivalent stress at maximum load, that is, a point on the linear extrapolation of the load-deflection curve of the structure in the locality of the flaw but with the flaw not present. The area (normalized energy) so defined is the area under the load-deflection curve up to maximum load on the structure.

THE EQUIVALENT ENERGY METHOD FOR CALCULATING ELASTIC-PLASTIC FRACTURE*

F. J. Witt

Abstract

The equivalent energy method is set forth for relating flaw size, stress (that is, maximum load to fracture) and temperature for frangible, transitional and tough behavior of materials. The basic method depends on testing a flawed model of the structure of interest at the temperature of interest and knowing the volumetric energy ratio between model and prototype. The usefulness of the method depends substantially on the generalizations that can be made from volumetric energy ratio curves.

Using the equivalent energy method as a means of comparing results from a number of fracture toughness investigations on ASTM A533, grade B, class 1 steel has led to the hypothesis that "the volumetric energy ratio for ASTM A533, grade B, class 1 steel is dependent primarily on thickness and temperature with little or no dependence on geometry or method of loading." A correlation of data from compact tension tests, dynamic tear tests, flawed tensile specimen tests and gross strain tests partially substantiate this hypothesis. However, the correctness of the hypothesis and the applicability of the equivalent energy method must await evaluations from additional investigations.

An elastic-plastic fracture mechanics procedure is set forth of which linear elastic fracture mechanics is a special case. Based on the above hypothesis, a new fracture toughness parameter, K_{Icd} , is introduced which is a property of the material in a particular thickness. K_{Icd} is easily obtained at any temperature.

Examples of the application of this method to the A533 steel are presented. Current research will establish the feasibility and general applicability of the method.

Background and Introduction

As a result of the growth of the nuclear power industry and the limited operating experience with nuclear power systems, the United States Atomic Energy Commission is sponsoring extensive research to assess the safety of

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these plants. One of the larger research programs is the Heavy Section Steel Technology (HSST) Program^{1,2} which is being carried out under the direction of Oak Ridge National Laboratory (ORNL). The emphasis in the HSST program is on the structural behavior of the thick reactor pressure vessels, particularly in the presence of flaws. The material of interest is the ASTM A 533, grade B, class 1 steel and welds typical for this material which are currently used in the fabrication of reactor pressure vessels.

One of the major objectives of the HSST program is to determine what information small laboratory type specimens actually reveal about the behavior of thick-section reactor pressure vessels (from 6 to 12 in. thick) typical of the boiling water and pressurized water reactor power plants. The first step in this direction is to ascertain what small specimen behavior reveals about large specimen behavior. Three activities of the HSST program have been completed which relate to this problem. They are (1) the dynamic tear (DT) test investigations at Naval Research Laboratory,^{3,4,5} (2) the testing of heavy section compact tension (CT) fracture toughness specimens at Westinghouse Electric Corporation,⁶ and (3) the investigation of section size on transition temperature in steel by Martin Marietta Corporation.⁷

On the basis of the above investigations it has been shown that, using the laws of geometric similitude, significant size effects in the impact energy-absorbing capacity of the ASTM A 533, grade B, class 1 material exists even at shelf temperature.^{8,9} In particular a size effect in energy-absorbing capacity of around 5 exists in 12-in. thicknesses at shelf temperatures. Figure 1 is taken from Refs. 8 and 9 and serves to summarize the shelf behavior. Essentially this figure shows that if an impact specimen in thicknesses from about 1/4 to 1 in. absorbs a certain amount of energy, then a 12-in.-thick specimen geometrically similar absorbs about 1/5 the energy one would expect by multiplying the energy absorbed in fracturing the smaller specimen by the cube of the scale factor.

The relevance of the behavior summarized above and discussed in Refs. 8 and 9 to the fracture behavior of a flawed thick-wall pressure vessel is raised. In this paper we are concerned with the frangible, transitional and tough behaviors, that is, we are concerned with fracture resulting

Fig. 1. Size Effects on Energy-Absorbing Capacity of Impact Specimens Tested at Upper Shelf Temperature (Refs. 8 and 9).

from elastic-plastic deformation as well as elastic deformation only. It is the intention here to propose a method for relating flaw size and stress level which should be applicable at any temperature including that for frangible behavior.

Before developing the topic of interest the following section is presented so as to set properly the stage for the discussion to follow.

Size Effects and Volumetric Energy Ratios

The size effects to be discussed are those related to structural behavior. Since we are asking in this discussion what small specimen tests tell us about large specimen behavior, the simplest and most direct way to obtain this information is by modeling. In this case dimensional analysis¹⁰ may be applied directly to the data without recourse to any additional approximations involving either empirical or theoretical formulations. Thus if two structures, exactly similar and loaded similarly but different in size, behave differently, then we shall say that there exists a size effect between the two structures. We shall determine the size effect numerically by dividing the value for the observed parameter of interest for the larger or prototype specimen into the corresponding value for the smaller or model specimen. The quantities being so compared must be compared on the basis of dimensional analysis; for example, comparable loads have a ratio equal to the square of the ratio of any two similar dimensions. The energies vary as the ratio of the cube of the dimensions. We shall use the term normalizing to indicate such scaling procedure as will allow comparison of the behaviors of interest.

Using the principles of dimensional analysis and assuming the material is the same for both model and prototype we know that for normalized loads the two will behave in the same manner both for the elastic behavior and the plastic behavior at least up to maximum load on one or the other of the structures assuming there is no flaw growth. Indeed this is the very basis of modeling techniques. Note that to maintain similitude when flaws are present the dimensions of the flaws are to the same scale as the model and prototype. Thus absolute flaw size as used in fracture mechanics is not yet a parameter under consideration.

Based on the above discussion we conclude that if fracture is the end result in both model and prototype tests then any gross differences in measured quantities reflecting the fracture mode must be due to a size effect related to the fracture mechanism. This discussion then is directed toward size effects in fracture mechanisms based on gross structural behavior and not on micromechanisms.

Size effects in fracture are very common in steels especially at low temperature. Perhaps the simplest example is that of determining the linear-elastic plane-strain fracture toughness, K_{Ic} .¹¹ K_{Ic} is a material constant, and if properly obtained, satisfies

$$K_{Ic} = C \sigma_f \sqrt{\pi a} , \quad (1)$$

where C is a shape factor, σ_f is fracture stress and a is crack depth. For two geometrically similar specimens having a scale factor, b (ratio of a dimension of larger to corresponding dimension of smaller) from which K_{Ic} is obtained at a given temperature we find by substituting in Eq. (1) that

$$\sigma_{f1} = \frac{\sigma_{f2}}{\sqrt{b}} , \quad (2)$$

where σ_{f1} is the fracture stress in the larger specimen and σ_{f2} is the fracture stress in the smaller one. The size effect in strength in Eq. (2) is $\sqrt{b}$. For linear behavior as expressed by Eq. (2) energies to fracture geometrically similar specimens (that is the area under the load deflection curve in the case of bending tests) are related by

$$E_1 = b^2 E_2 , \quad (3)$$

where E_1 is the energy to fracture in the larger specimen and E_2 is the energy to fracture in the smaller specimen. Another way to say this is that there is a size effect in energy to fracture equal to the scale factor since by similitude we should have b^3 in Eq. (3). In summary, we should note that two geometrically similar flawed structures tested at the same temperature which fail under conditions for which linear-elastic

fracture mechanics applies differ in normalized energy-absorbing capacity to fracture by a factor equal to the ratio of similar dimensions.

The type of size effects to be discussed in this paper deals with the size effects in normalized energy-absorbing capacity. This quantity is affected by both thickness and temperature. If we take a ratio of corresponding normalized energy-absorbing capacities up to maximum load of two geometrically similar specimens, we obtain a volumetric energy ratio. In the section that follows the specific details of which normalized energy is used in defining the volumetric energy ratio are set forth.

The Equivalent Energy Method

The equivalent energy method conceptually is very simple. One only needs to know the volumetric energy ratio ($s_{m,p}$) between model and prototype and the energy-absorbing capacity of the model (E_m). The energy-absorbing capacity of the prototype (E_p) can therefore be expressed by

$$E_p = \frac{E_m}{s_{m,p}} . \quad (4)$$

One has a choice here on which energy he wishes to calculate. In particular we do not propose to use the total energy introduced into the structure at maximum load to fracture. The normalized energies defined above will be discussed both in terms of local energy (energy measured in the locality of the flaw) at maximum load to fracture in the presence of the flaw and nominal energy at maximum load to fracture with the flaw not present. Specifically we shall concern ourselves with energy under load-deflection curves (or stress-strain behavior) local to the flawed region.

As an example of the behavior of geometrically similar structures let us consider the series of compact tension specimens tested in thicknesses of 2 to 12 in. at temperatures of 50°F and 75°F.⁶ The load deflection curves for eleven specimens are shown in Fig. 2. We note that, allowing for experimental scatter, the specimens all behave the same up to maximum load.

Fig. 2. Comparison of Load Displacement (COD) Curves for CT Specimens Tested at 50°F and 75°F Normalized to a 1 T Specimen.

The normalized crack length of the various specimens are given in Fig. 2 so that the apparent anomalous behavior of the 2 T (2 in. thick) specimens can be explained. Since theoretically the deflection varies as the cube of the lever arm length, the specimen with the longer than normal arm deflects more for a given loading. This compares quite well with the behavior exhibited. Comparison at higher temperatures are reported in Ref. 12.

We now ask how one would calculate the fracture load (maximum load to fracture) in a structure using the equivalent energy method. The first step in calculating the fracture load is to test a model geometrically similar to the flawed prototype structure. For the discussion in this section we would obtain two load-deflection curves. One curve should be measured so as to reflect the fracture process. For instance, a gross strain or crack opening displacement measurement across the flaw should suffice. The second curve is obtained in the same region of the structure but without the flaw. Thus an unflawed model may have to be tested up to the fracture load in the presence of the flaw. One now applies the same procedure to both curves.

First for each curve we measure the areas up to maximum load and divide the areas by the volumetric energy ratio for the thickness and at the temperature of the prototype structure. That portion of the load-deflection curve which encompasses the normalized area defines the normalized energy to maximum load for the prototype structure. One simply applies the laws of similitude to the energy and, using the load-deflection curve for the prototype structure, determines the maximum load.

We shall now use the equivalent energy method to obtain the maximum load to fracture of a 6-in.-thick tensile specimen tested at Southwest Research Institute.¹³ The test conditions were:

Gage: Cross section 18 in. by 6 in. thick with 24 in. gage length.

Orientation: Longitudinal

Test Temperature: 215°F nominally

Flaw Size: Semielliptical part through, 2.37 in. deep, 6 in. long, machine sharpened by electrical discharge machining with 3 to 5 mils root radius.

We shall first demonstrate the equivalent energy method by considering the load-deflection curve obtained from a flawed model. A 1-in.-thick model similar to the specimen was tested under similar conditions except the flaw in the model was equivalent to a 2.5 in. deep flaw 7 in. long. For purposes here we shall ignore these minor differences in flaw dimensions. The load-deflection curve for the model is given in Fig. 3. The only available volumetric energy ratio data (that is, normalized size effects data) were those for the impact specimens shown in Fig. 1. Thus using these results the volumetric energy ratio was chosen to be between 2.35 and 3.1 for a thickness of 6 in. We now find those portions of the load-deflection curve of the model which contain from $1/3.1$ to $1/2.35$ of the area to maximum load. This is noted in Fig. 3 by points A and B. We find a load of between 206,000 lb and 212,000 lb. Multiplying these loads by 36 (scale factor squared) we would predict the maximum load to be between 7,425,000 lb and 7,650,000 lb. The actual fracture load on the 6-in. specimen was 7,550,000 lb.

Next we consider the procedure applied to an unflawed specimen. For the specimen at hand we need to consider only a load-deflection curve for the specimen as shown in Fig. 4. Applying the same procedure as above to the curve in Fig. 4 which is plotted as a normalized load deflection curve we obtain fracture stresses (points A and B on Fig. 4) on the gross section of 68,500 and 70,300 psi which gives fracture loads of 7,400,000 lb and 7,600,000 lb.

Since both procedures as demonstrated above appear to be adequate to predict maximum load to fracture, we may wonder which is preferable. If all similar structures behaved as the compact tension specimens depicted in Fig. 2, then undoubtedly the load-deflection curve which reflects the behavior in the presence of the flaw could be used. However, this curve may be difficult to obtain and it may be equally difficult to demonstrate convincingly that model and prototype behave in the same manner up to maximum load on the prototype. These difficulties do not exist for load-deflection curves obtained for unflawed specimens. In fact the unflawed load-deflection curves for many complex structures can be obtained by analytical or numerical methods without recourse to experiments. One must of course know where on the curve the model containing the flaw reached

Fig. 3. Load-Deflection Curve for 1-in. Flawed Tensile Specimen.

Fig. 4. Normalized Load Deflection Curve at 200°F for Unflawed Tensile Specimen in the Locality of the Flaw.

maximum load. In summary, of the two curves one could use, preference is given to the curve for the flawed region but with the flaw not present. This is the load-deflection curve to which we shall refer in the remainder of this paper unless otherwise specified.

Some Volumetric Energy Ratio Data

Although several efforts conducted under the auspices of the HSST program have studied the behavior of large and small fracture toughness type specimens, no research has specifically been oriented toward obtaining volumetric energy ratios. However, there is a sufficient amount of data available for the ASTM A 533, grade B, class 1 steel to indicate the applicability of a volumetric energy ratio curve obtained from one specimen to a geometrically different specimen. A summary of the available data is shown in Fig. 5. Note that we have plotted volumetric energy ratio data as a function of thickness and also as a function of temperature. We have considered energy to maximum load for the statically tested specimens and total energy to fracture for the impact specimens. In dealing with these data no corrections have been made. In particular no accounting for variations in crack depth have been made. All of the data are normalized to the energy-absorbing capacity at 180°F for a 1-in.-thick specimen of the specimen series under consideration. There are two exceptions to this. Since gross strain specimens (see Ref. 14) were not tested at 180°F, the data for the specimens of each of these two series which were tested at 32°F have been placed on the curve for the compact tension specimens. Thus a volumetric energy ratio of about ten was taken at this temperature. The other specimens of each of these series were then normalized to the energy absorbed at 32°F.

The most significant observation is that the curves for all the 1-in.-thick specimens practically coincide at least above 30°F. The exception is the gross strain data from materials which exhibit a yield plateau, that is, it has an instability from yield strain (about 0.2%) to about 1.5% strain. If at essentially the same temperature and for a given load the strain can be either of these quantities, then the volumetric energy ratio curve exhibits a discontinuity. However, even in this case, a

Fig. 5. Volumetric Energy Ratios as a Function of Thickness and Temperature for Different Specimens of ASTM A 533, Grade B, Class 1 Steel Normalized to the Measured Energy at 180°F for Each Specimen Series (non-surface, longitudinal orientation).

prediction of maximum load to fracture of such specimens using the equivalent energy method would be correct. In fact the data imply that the volumetric energy ratio of the material with a yield plateau "waits" at yield load while the specimens exhibiting no yield plateau "catch up". The type of instability behavior on yield plateau has not been observed in pressurizing small vessels fabricated from the A 533 steel.

The data scatter at low temperatures look quite large; however, this is not the temperature of interest. Testing of more nearly similar specimens should reduce the scatter considerably. The data could be adjusted but it is not the object of this paper to discuss how this should be done. Actually the volumetric energy ratio for 1 T compact tension specimens tested at -75°F and -100°F were obtained from K_{Ic} data on 1 T specimens tested at these temperatures using the method developed later in this paper [see Eq. (13)]. The material tested was obtained from quarter thickness, center and three-quarter thickness of the plate. The scatter represents the square of the variation of the K_{Ic} as obtained at -75°F and -100°F . The details of the remaining 1-in. compact tension specimen data are discussed in depth in Ref. 12.

Very few data on large specimens are available for this discussion. We note in particular that the 12 T data for both compact tension and dynamic tear specimens agree quite well. Also at about 200°F we know the volumetric energy ratio between 1- and 6-in.-thick flawed tensile specimens is about the same as between 1- and 6-in.-thick dynamic tear specimens as discussed in the previous section. At 50°F the volumetric energy ratio between a 6-in.-thick flawed tensile specimen and a 6-in.-thick compact tension specimen is the same. As previously noted for frangible behavior where linear elastic fracture mechanics is applicable the volumetric energy ratio at a given temperature is always equal to the ratio of the dimensions. Along this line we also note that the volumetric energy ratio at temperatures below 150°F between a 1-in.-thick and a similar thicker specimen is about equal to the ratio of their dimensions with the exception of the 12-in.-thick specimens. The 12-in.-thick specimens contain quenched and tempered surface material which reaches tough behavior at about 0°F , thus reducing the volumetric energy ratio when compared with thinner specimens containing only non-surface material.

Let us now summarize our observations as obtained from the available data illustrated in Fig. 5. First volumetric energy ratios for the dynamic tear specimens are obtained from total energy to fracture tested dynamically. The volumetric energy ratios for the compact tension, gross strain, and flawed tensile specimens are obtained from energy to maximum load tested statically. Thus a rather wide gamut of testing conditions and specimen configurations have, for the 1-in. thickness, produced the same volumetric energy ratio curve at least at temperatures above 30°F. A similar correspondence is indicated for all the thicknesses investigated. One should realize that the data used to obtain Fig. 5 were treated in a rather arbitrary manner which the author has found most useful for this discussion. We should not lose sight of the fact however that other treatments, when properly examined, would yield the same conclusions as reached in this section. The scatter band for all the data is considered to be $\pm 10\%$.

In summary, the correlations presented in this section to a considerable degree support the hypothesis that volumetric energy ratios depend substantially on thickness and temperature with little or no dependence on specimen geometry. Thus with this tentative conclusion we see that the equivalent energy method becomes a very valuable tool. After further verification, it is hoped that one may obtain volumetric energy ratios of interest from conveniently tested specimens such as the compact tension specimens and may use the data for predicting the behavior of a pressure vessel containing a flaw having also tested a small model of the flawed prototype.

Formulation of the Equivalent Maximum Load Stress Using the Equivalent Energy Method

Up to now we have dealt only with energy considerations. We shall now develop the expression for the equivalent stress at maximum load.

First let us consider Fig. 6. Here we have extended the elastic portion of a load-deflection curve (or stress-strain curve) of a specimen until the areas under the curve (energy to maximum load) equals the area under the real load-deflection curve. We denote the maximum load (or equivalent stress) obtained by this linearizing procedure by σ_m . Using the equivalent energy method for a larger specimen which has a volumetric

energy ratio of $s_{m,p}$ we would deduce that the normalized maximum load (or equivalent stress), σ_p , for the larger specimen is given by

$$\sigma_p = \frac{\sigma_m}{\sqrt{s_{m,p}}} . \quad (5)$$

Now obviously σ_p may be erroneously high but the area under the line up to σ_p is the correct area which is to be found under the real load-deflection curve.

One now compares Eq. (5) with Eq. (2) and notes that they are similar. In each case the radicand is numerically the volumetric energy ratio (that is, size effect). Thus one wonders if there exists an elastic-plastic fracture mechanics formulation which uses a formula similar to Eq. (1) and which is premised on the equivalent energy method discussed above.

An Elastic-Plastic Fracture Mechanics Formulation for Predicting Fracture

In this section we shall develop an equivalent energy elastic-plastic fracture mechanics procedure which as a special case reduces to linear-elastic fracture mechanics. The only assumption we shall make is to the generality of the volumetric energy ratio curve. In particular we shall assume, based on the prior discussion, that the volumetric energy ratio depends primarily on thickness and temperature and to a much lesser degree on the configuration and method of loading. The extent to which a material behaves in this manner will determine the accuracy and usefulness of the formulation which is set forth below.

First let us state some useful relationships between volumetric energy ratios for the following development. In particular, by definition, we have

$$s_{m,p} = \frac{1}{s_{p,m}} . \quad (6)$$

Also for similar specimens of relative sizes m , n , and p

$$s_{m,p} = s_{m,n} \cdot s_{n,p} . \quad (7)$$

This follows from

$$\frac{E_m}{E_p} = \frac{E_m}{E_n} \cdot \frac{E_n}{E_p} \quad (8)$$

where E is the normalized energy to maximum load.

Now let us consider the load-deflection curve for a compact tension specimen tested at any temperature. For example, let us consider the curve for a specimen tested at 250°F such as shown in Fig. 7. The specimen was actually 1-in.-thick, but this is not important for the discussion here. The point of maximum load is A. We choose B as any point on the elastic portion of the curve. We now divide the area to point A by the area to point B and call this ratio b . We then ask the question, "How thick would a specimen have to be to fail at point B?" Without testing we don't know. However, using the equivalent energy method we do know what the volumetric energy ratio is. It is b . We now scale the specimen tested (call its thickness d) by the factor b . Thus we have a specimen of equivalent thickness bd which will fail at point B. We now define a fracture toughness parameter K_{Icd} which by definition, we shall calculate in the same manner the K_Q is calculated from compact tension specimens.¹⁵ That is,

$$K_{Icd} = \frac{P_Q f\left(\frac{a}{W}\right)}{B \cdot \sqrt{W}} \quad (9)$$

where $P_Q = P_{Q1} \cdot b^2$, $B = bd$, $W = 2bd$. P_{Q1} is the load at point B. By substituting these values into Eq. (9) we obtain

$$K_{Icd} = \frac{P_{Q1} \sqrt{b} f\left(\frac{a}{W}\right)}{d \cdot \sqrt{2d}} \quad (10)$$

The subscript d denotes the thickness of the specimen actually tested. For example K_{Icd} for a thickness of 2 in. ($d = 2$) is expressed as K_{Ic2} .

While validity (see Ref. 15) is not a question here, we should note that K_{Icd} is independent of where we pick point B on the elastic portion

Fig. 7. Load-Front Face Displacement Curves for a 1 T Compact Tension Specimen Tested at 250°F Illustrating How K_{Ic1} is Calculated (Ref. 12).

of the curve. Thus we can always pick point B so as to satisfy the validity criteria.

Next we shall derive a relationship between K_{Icd} values obtained from geometrically similar specimens of different thicknesses tested at the same temperature. We let the thicknesses be d and p . We normalize the load-deflection curve of the specimen with thickness p to the one of thickness d and choose the same point B with load P_{Q1} (refer to Fig. 7). The energy ratio for the specimen of thickness p at the maximum load to point B is now designated as c . As before, we take b as the corresponding energy ratio for the specimen of thickness d . Thus we have

$$K_{Icd} = \frac{P_{Q1} \sqrt{b} \cdot f\left(\frac{a}{W}\right)}{d \cdot \sqrt{2d}} \quad (11a)$$

which may be rewritten as

$$K_{Icd} = \frac{P_{Q1} \left(\frac{p}{d}\right)^2 \cdot \sqrt{c} \cdot f\left(\frac{a}{W}\right)}{p \cdot \sqrt{2p}} \sqrt{\frac{bd}{pc}} \quad (11b)$$

From Eq. (7) we have

$$b = s_{d,p} c.$$

This gives, by applying the definition in Eq. (10)

$$K_{Icd} = K_{Icp} \cdot \sqrt{\frac{s_{d,p} d}{p}} \quad (12)$$

If $r_{p,d}$ is the scale factor between the two specimens ($r_{p,d} = p/d$) then Eq. (12) becomes

$$K_{Icd} = K_{Icp} \cdot \sqrt{\frac{s_{d,p}}{r_{p,d}}} \quad (13)$$

Applying the same procedure as above for similar specimens of the same thickness but tested at different temperatures (say we obtain K_{Icd} at temperature T_0 and K_{Icp} at temperature T_1 with $s_{d,p}$ the energy obtained at T_0 divided by the energy obtained at T_1) we obtain the same relationship as given in Eq. (13) with $r_{p,d} = 1$. Thus $s_{d,p}$ can be used to relate behavior as a function of temperature.

The above procedure applied to results from a bend test¹⁵ produces the same relationship as given in Eq. (13) but will not be presented here.

We shall now show that K_{Icd} is unique and is a property of the material at a given temperature for thickness d . Let us assume we test two specimens in the same thickness but not geometrically similar and obtain two values for K_{Icd} . We call them K_{Icd_1} and K_{Icd_2} . Now let us choose a thickness f such that both types of specimens in thickness f would yield a valid fracture toughness value K_{Ic} . Applying Eq. (13) we have

$$K_{Ic} = K_{Icd_1} \sqrt{\frac{s_{f,d_1}}{r_{d_1,f}}} \quad (14)$$

and

$$K_{Ic} = K_{Icd_2} \cdot \sqrt{\frac{s_{f,d_2}}{r_{d_2,f}}} \quad (15)$$

Since the specimens are of the same thickness

$$s_{f,d_1} = s_{f,d_2} \quad (16)$$

and it follows that

$$K_{Icd_1} = K_{Icd_2} \quad (17)$$

That is, we have shown that K_{Icd} is uniquely determined. Indeed it is a material property for the thickness, d , of the material at a given temperature. We should recall, however, that this holds only to the degree that the hypothesis set forth at the beginning of this section is applicable.

The next step is to define a quantity C as follows:

$$C = \frac{K_{Icd}}{\sqrt{\pi a_d} \sigma_d} \quad (18)$$

where a_d is the flaw depth in some structure of interest scaled to a thickness of d . σ_d is the normalized load (stress) obtained by extrapolating the elastic portion of the load deflection curve of a model of

thickness d of the flawed structure of interest until the triangle so defined encompasses the energy up to maximum load on the real load-deflection curve of the model. In other words, σ_d is obtained in the same manner as σ_m in Fig. 6. We use of course the nominal load-deflection curve in the region of the flaw with the flaw not present.

We shall now show that C is independent of thickness. Consider any two thicknesses d and p . By applying Eqs. (5) and (13) with $r_{p,d} = a_p/a_d$, we have

$$C = \frac{K_{Icd}}{\sqrt{\pi a_d} \cdot \sigma_d} = \frac{K_{Icp} \cdot \sqrt{\frac{s_{d,p} a_d}{a_p}}}{\sqrt{\pi a_d} \cdot \sigma_p \cdot \sqrt{s_{d,p}}} \quad (19)$$

which gives

$$C = \frac{K_{Icp}}{\sqrt{\pi a_p} \cdot \sigma_p} \quad (20)$$

Thus at a given temperature C is a constant.

Next we wish to show that C is independent of temperature. We consider two specimens of the same thickness tested at different temperatures, T_0 and T_1 . We take the size effect as $s_{d,p}$ where at T_0 we have K_{Icd} and σ_d . At T_1 we have K_{Icp} and σ_p . Thus using Eqs. (5) and (13) again we have

$$C = \frac{K_{Icd}}{\sqrt{\pi a_d} \cdot \sigma_d} = \frac{K_{Icp} \sqrt{s_{d,p}}}{\sqrt{\pi a_p} \cdot \sigma_p \cdot \sqrt{s_{d,p}}} = \frac{K_{Icp}}{\sqrt{\pi a_p} \cdot \sigma_p} \quad (21)$$

Here of course $a_d = a_p$.

Thus C is independent of thickness and temperature and depends only on the geometry and loading on the structure. In particular we shall now show that C is the same quantity as that used in linear elastic fracture mechanics for strictly frangible behavior. This follows immediately since, for sufficiently large thickness f , $K_{Icf} = K_{Ic}$. If σ_f is frangible fracture, then we have

$$C = \frac{K_{Ic}}{\sqrt{\pi a_f} \sigma_f} . \quad (22)$$

If in Eq. (18) we let σ_d be the results from a model test [that is, $\sigma_d = \sigma_m$ in Eq. (5)], then substituting for σ_d from Eq. (5) (using $\sigma_f = \sigma_p$) in Eq. (18) we obtain

$$K_{Icd} = C \sigma_f \sqrt{\pi a_d \cdot s_{d,f}} . \quad (23)$$

It remains only to show that σ_f is unique. This is done by using Eqs. (7) and (13) as applied to thicknesses d and p . Thus

$$\sigma_f = \frac{K_{Icd}}{C \sqrt{\pi a_d \cdot s_{d,f}}} = \frac{K_{Icp} \cdot \sqrt{\frac{s_{d,p}}{r_{p,d}}}}{C \sqrt{\pi a_d \cdot s_{d,p} \cdot s_{p,f}}} \quad (24)$$

which gives

$$\sigma_f = \frac{K_{Icp}}{C \sqrt{\pi a_p \cdot s_{p,f}}} . \quad (25)$$

Here of course we again use $r_{p,d} = a_p/a_d$.

Thus we have in Eq. (23) the basic fracture calculation formula for the equivalent energy elastic-plastic fracture mechanics method. It is to be remembered however that in fact, Eq. (23) and Eq. (5) are equivalent.

Alternate ways of writing Eq. (23) are

$$\frac{K_{Icd}}{\sqrt{\frac{s_{d,f}}{r_{f,d}}}} = C \sigma_f \sqrt{\pi a_f} \quad (26)$$

and

$$K_{Icf} = C \sigma_f \sqrt{\pi a_f} . \quad (27)$$

In summary we define the terms in Eqs. (23), (26), and (27) as follows:

- K_{Icd} - a fracture toughness property of the material in thickness d . This value is determined from any fracture toughness specimen from which valid K_{Ic} values may be obtained. An additional restriction is given in the definition of $s_{d,f}$ below.
- C - the constraint or shape factor for the flaw configuration used in linear elastic fracture mechanics for strictly frangible behavior.
- σ_f - the equivalent stress at maximum load, that is, a point on the linear extrapolation of the load-deflection curve of the structure in the locality of the flaw but with the flaw not present. The area (normalized energy) so defined by σ_f is the area under the load-deflection curve up to maximum load on the structure.
- a_d - the depth of the flaw in a model of thickness d of the structure of interest.
- $s_{d,f}$ - volumetric energy ratio between a model of thickness d and the structure of thickness f ; the temperature of the model and that for the K_{Icd} used is the same. The volumetric energy ratios of the specimen from which K_{Icd} is obtained and the structures of interest must be the same within acceptable limits for the applicable thickness.

Discussion and Conclusion

The equivalent energy method is a procedure for predicting fracture of a flawed structure at any temperature if a model test has been performed and the applicable volumetric energy ratio is known. The method is limited by the as yet incompletely verified generality of the volumetric energy ratio curve obtained from particular specimens. Based on available data a tentative conclusion has been reached that for ASTM A 533, grade B, class 1 steel the volumetric energy ratio depends only on thickness and temperature. If this is born out in additional tests then many of the safety problems posed concerning the behavior of nuclear reactor pressure vessels can be resolved in a quantitative manner. If on the other hand, complete generality is not obtained than one must be more discerning in the manner in which the volumetric energy ratio data and K_{Icd} data are obtained. For instance, since flawed tensile specimens are more like a flawed pressure vessel wall, volumetric energy ratio and K_{Icd} data from such specimens may be applicable to pressure vessels while similar data

from compact tension specimens may not be applicable. The reverse could be true for nozzle corners.

The equivalent energy elastic-plastic fracture mechanics formulation has several advantages but does require that the volumetric energy ratio for the specimens from which K_{Icd} is obtained is the same as that for the structure of interest. If this is true then the vast experience in testing fracture toughness specimens and in the field of linear elastic fracture mechanics can be brought into play. For instance if the shape factor C can be calculated analytically then a model test is not even required. Essentially the method eliminates the necessity of obtaining K_{Ic} . In its place one must obtain K_{Icd} and the volumetric energy ratio of interest. These values however are always obtainable and one is not plagued with validity requirements for obtaining K_{Ic} which require specimens of unusual thickness.

If a linear load-deflection curve is obtained for a fracture toughness specimen then the K_Q (see Ref. 15) value calculated is numerically equal to K_{Ic} . This follows from applying the method of obtaining K_{Icd} to the linear curve. We also note that in Eq. (13), if the volumetric energy ratio between the specimens is equal to the ratio of the dimensions, then $K_{Icd} = K_{Icp}$. In particular the volumetric energy ratio data for 1 in. and larger DT specimens below about 150°F appear to be about equal to the ratio of the dimensions when the surface material is discounted. Assuming the same volumetric energy ratio behavior for 1 T CT and the larger CT specimens from which K_{Ic} values were determined by Westinghouse⁶ one may conclude that the K_{Icd} values obtained from 1 T specimens should be about equal to K_{Ic} .

The applicability of the equivalent energy method has been demonstrated for only one grade of steel, ASTM A 533, grade B, class 1. This naturally raises the question of the generality of the method. This in turn depends on the degree of variation for a given steel of the volumetric energy ratio curves obtained from different specimens. In other words, if for a steel or any other material, the volumetric energy ratio depends only on thickness and temperature the method should be applicable. There is no reason to suspect that the A 533 steel is unique in satisfying this type of behavior. Thus one might hope that the equivalent energy method will find general applicability to any structure for which fracture in the presence of flaws presents a hazard.

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APPENDIX

Note 1, Page 13. The recent flawed tensile test results performed by Southwest Research Institute (see Paper 22 of this Conference) indicate that the interpolated volumetric energy ratio curve between 75°F and 200°F for six-inch-thick specimens may be overly conservative. At 100°F a value of 10 for the volumetric energy ratio appears to be more applicable.

Note 2, Page 21. Recent data obtained by Begley and Mager of Westinghouse Electric Corporation tend to refute the claim of the uniqueness of K_{Icd} . The data were obtained from non-standard compact tension specimens (thickness not commensurate with length and width). Until a more definitive resolution of the results is forthcoming it is recommended that the uniqueness not be assumed. The implication here is that for greatly different constraint conditions for the different specimens of the same thickness, the volumetric energy ratios may differ considerably. This possibility is discussed throughout the paper and in particular in the concluding section.