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**PHOTOPRODUCTION OF CHARGED PIONS
FROM DEUTERIUM NEAR THRESHOLD**

by

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APPROVED FOR PUBLIC RELEASE

**Ph.D. Thesis
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PHOTOPRODUCTION OF CHARGED PIONS
FROM DEUTERIUM NEAR THRESHOLD

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University of Illinois, 1964

An experiment has been performed in order to improve existing information on the behavior of the π^- -photoproduction cross section near threshold. To do this an X-ray beam from the University of Illinois betatron was passed through a 14 inch liquid deuterium bubble chamber.

Each X-ray pulse ($E_{\text{max}} = 230 \text{ MeV}$) contained a total energy of about 10^6 MeV . The X-ray beam was hardened by means of a standard beam hardening technique to reduce the electron background. The X-ray beam monitor was calibrated with an uncertainty of $\pm 2.4\%$.

A total of 140,000 usable photographs were taken. About 5400 π^- -photoproduction events and about 2700 π^+ -photoproduction events were found. Each π^- -event was identified by a kinematic fitting of the vertex, and each π^+ -event was identified by a characteristic $\pi-\mu$ decay signature.

Results were compared with Baldin's impulse approximation calculations in order to obtain the matrix element for the π^- -photoproduction process from a free neutron. The results agree with the previous experiment by Adamovich et al. and with the photoproduction theory of Chew, Goldberger, Low and Nambu. This result, however, was found to be sensitive to the so-called "cut-off" parameter.

The π^- to π^+ ratio has been determined for seven pion laboratory energy and angle bins, using the π^+ -events observed in deuterium.

Coulomb corrections have been applied to these results.

A Chew-Low "unstable target" extrapolation procedure was used in order to obtain the cross section for the π^- -photoproduction process from a free neutron. A correction factor has been calculated to eliminate the effects of the Coulomb interaction and specific nuclear force interaction of the recoiling protons in the final state. The result of this analysis agrees with the CGLN theory evaluated with a coupling constant $f^2 = 0.073 \pm .003$ and $N^{(-)}$ being set to zero.

The extrapolation to threshold was carried out using the cross section shape predicted by CGLN. The threshold value of " $a_0^{(-)}$ " was found to be 26.0 ± 1.0 $\mu\text{b/ster}$. Comparing this with the result of the latest π^+ experiment at the University of Illinois, the threshold σ^-/σ^+ ratio was found to be $1.23 \pm .06$.

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I. INTRODUCTION

A. Historical Background

The photoproduction of negative pions in the process

$$\gamma + n \rightarrow \pi^- + p$$

(hereafter identified as $\gamma n \pi^-$ process) is closely related to the positive pion photoproduction process

$$\gamma + p \rightarrow \pi^+ + n$$

($\gamma p \pi^+$).

In fact, if one assumes that the mass of the nucleon is infinite and thus that the nucleon current does not contribute to the process, charge symmetry requires that the cross sections for the two processes be equal. The difference between the two processes has been interpreted classically in terms of the difference in nucleon recoil currents,^{1/} or, naively in terms of the different electric dipole moments of the two different pion-nucleon systems.^{2/}

The dispersion theory of Chew, Goldberger, Low and Nambu^{3/} (hereafter referred to as CGLN) has enjoyed a fair degree of success in its description of pion-nucleon scattering and of positive and neutral photopion processes. It is not unnatural to expect this theory to hold equally well for the $\gamma n \pi^-$ process. The "minus to plus ratio" (hereafter referred to as σ^-/σ^+ ratio) as given by CGLN theory is a fairly smooth function of energy and angle and approaches 1.3 at the threshold. The σ^-/σ^+ ratio obtained from the previously mentioned classical arguments is in good agreement with the CGLN value.

Aside from the details of any theory, there is a well-known connection between low energy charge exchange pion scattering and the $\gamma n \pi^-$ process via the Panofsky ratio p :

$$p = \frac{\sigma(\pi^- + p \rightarrow n + \pi^0)}{\sigma(\pi^- + p \rightarrow n + \gamma)}$$

at zero pion energy. Using a detailed balancing argument one gets

$$p = \frac{q_{\pi^-}^2}{2k_\gamma^2} \frac{\sigma(\pi^- + p \rightarrow n + \pi^0)}{\sigma(n + \gamma \rightarrow \pi^- + p)}$$

The discrepancy that appeared to exist between the directly measured Panofsky ratio and the one calculated by the above relationship was tentatively resolved by Cini et al.^{4/} They used extrapolation techniques suggested by theory instead of empirically suggested procedures. These techniques were applied to the charge exchange scattering process, the $\gamma p \pi^+$ photoproduction process and the σ^-/σ^+ photoproduction ratio. For this last number, $\sigma^-/\sigma^+ = 1.3$ at threshold was the value chosen. Although this value of σ^-/σ^+ was confirmed by the emulsion experiment of Adamovich et al.,^{5,6/} who obtained $R = 1.3 \pm .15$, a more thorough investigation was clearly required.

The experiment to be described here was performed to obtain reliable data on the photoproduction of charged pions from deuterium, i.e.,

$$\gamma + d \rightarrow \pi^- + 2p$$

$(\gamma d \pi^-)$ and

$$\gamma + d \rightarrow \pi^+ + 2n$$

$(\gamma d \pi^+)$ near the threshold by means of the bubble chamber technique.

These data are analyzed in an attempt to establish the matrix element for the $\gamma n \pi^-$ process, the photoproduction of π^- mesons from free neutrons.

B. Review of Previous Experiments

Experimental studies of the $\gamma n \pi^-$ process have not been carried out as extensively as that of the $\gamma p \pi^+$ process. This is mainly due to the lack of a condensed free neutron target. Although the study of the inverse process $\pi^- + p \rightarrow \gamma + n$ circumvents this difficulty, it is experimentally difficult and has been carried out only recently, and has not yet yielded any conclusive results.^{1/}

As it is necessary to use bound neutrons for a target in the study of the $\gamma n \pi^-$ process, deuterium is the obvious choice because of its simplicity and low binding energy. However, any attempt to study the $\gamma n \pi^-$ process by observing the $\gamma d \pi^-$ process with or without simultaneous observation of the $\gamma d \pi^+$ process should take into account the following effects:

- i) the effect of the deuteron binding on the "primary interaction," i.e., the effect of the momentum distribution of the neutron in the deuteron,
- ii) the "exclusion effect" and the final state nucleon-nucleon scattering effect,
- iii) the final state Coulomb scattering effects.

All of these have been studied in the course of the analysis of the present experiment.

In attempting to measure the cross section for the $\gamma n \pi^-$ reaction, two experimental approaches can be used.

1. Study of the σ^-/σ^+ ratio in deuterium^{8-24/}

Here the relative yields of the $\gamma d \pi^-$ and $\gamma d \pi^+$ processes are measured and their ratio at a given meson angle and energy is evaluated.

This approach has been popular, since it is so amenable to counter techniques and since many of the specific deuteron effects cancel out, giving a ratio, R_d , which should be close to the σ^-/σ^+ ratio from free nucleons. For example, corrections i) and ii) for the $\gamma d \pi^-$ process should be very close to the similar corrections for the $\gamma d \pi^+$ process. The Coulomb corrections for the $\gamma d \pi^-$ process have frequently been ignored, but Baldin^{25/} has pointed out that the Coulomb corrections near the threshold region can amount to $10 \sim 15\%$.

Internal motion in the deuteron manifests itself in the form of kinematic smearing, so that the ratio, R_d , is an average over a finite spectrum of the γ -ray energy and the pion production angle in the frame of reference of the photon-neutron system. Since the σ^-/σ^+ ratio is believed to be a slowly varying function of the photon energy and the pion angle, one can argue that the kinematic smearing will have very little effect,^{14/} but this argument requires experimental verification.

A direct analysis of R_d becomes rather meaningless near the threshold region, since large final state corrections cannot be applied rigorously without knowing completely the kinematics of the $\gamma d \pi^-$ process.

A comprehensive experimental review of measurements of the ratio R_d has been given by Hogg.^{22/}

2. Complete study of the $\gamma d\pi^-$ process^{5,19,26/}

Since there are three particles in the final state of $\gamma d\pi^-$ reaction, the determination of at least six momentum components is necessary in order to identify the kinematics uniquely. Fortunately the three final state particles are all charged, so any one having sufficient energy to be of consequence will be visible in a bubble chamber. This is the important advantage offered by the bubble chamber technique. When the complete kinematics are known, the final state corrections can be applied in a rigorous manner. It is also obvious that this method can explore the region very close to the threshold, whereas counter techniques are ineffective in that region. The disadvantage is, of course, common to all visual techniques, that it takes a long period of patient work to accumulate enough statistics.

There are different approaches in the analysis of data, once the kinematics have been established.

a. Impulse approximation

Baldin,^{25/} starting with an arbitrary $\gamma n\pi^-$ interaction, has made an impulse approximation calculation of the $\gamma d\pi^-$ process, taking into account the final state nuclear and Coulomb interactions. Adamovich et al.^{26/} have obtained the $\gamma n\pi^-$ matrix element between $E_\gamma = 155$ and 190 MeV by comparing their experimental cross sections to the Baldin calculation.

b. Chew-Low extrapolation technique^{27/}

This technique, in evaluating the $\gamma n \pi^-$ process through the analysis of the $\gamma d \pi^-$ process, utilizes the fact that the S-matrix for the $\gamma d \pi^-$ process has a pole corresponding to the single neutron intermediate state, and that the residue of this pole is a product of the $\gamma n \pi^-$ process amplitude and the d-N-N vertex function.

In terms of the momentum of the "spectator proton" p_1 , the pole is located at

$$p_1^2 = -\alpha^2,$$

where α is the inverse deuteron radius and is equal to $\sqrt{\text{binding energy} \times M}$.

This becomes plausible when it is realized that the Fourier transform of the asymptotic wave function of the deuteron is proportional to $(p_1^2 + \alpha^2)^{-1}$.

The procedure is to calculate the cross section for the $\gamma d \pi^-$ process at constant energy in the γn system for various values of p_1 . The resulting curve can then be extrapolated back to the pole at $p_1^2 = -\alpha^2$ and thus the cross section for the $\gamma n \pi^-$ process can be evaluated.

Swanson et al.^{19,20/} applied this extrapolation procedure to their bubble chamber data. Although their statistics were poor, they showed the feasibility of this approach.

In the present experiment a 14" bubble chamber has been used instead of the 4" chamber used by Swanson et al. It was planned so that the resulting increase in statistics and in the range of the extrapolated data would provide a qualitative improvement in the results.

II. THEORETICAL BACKGROUND

A. Basic Photoproduction Process

The calculation of the cross sections for the photoproduction of charged pions can be approached by means of a partial wave expansion.^{2/} In the low energy region ($E_\gamma \leq 400$ MeV) it is found that electric dipole, magnetic dipole and electric quadrupole absorptions leading to s and p wave pion-nucleon final states are expected to predominate in the photoproduction process. Table 1 lists the absorbed multipoles, the possible final states (only s and p states are considered), the resulting angular distribution and the usual notation for the corresponding amplitude.

Table 1

Absorbed Multipoles	Pion-nucleon Final State	Angular Distribution	Amplitude
E_1	$S_{1/2}$	1	$E_1(1/2)$
M_1	$P_{1/2}$	1	$M_1(1/2)$
M_1	$P_{3/2}$	$5 - 3 \cos^2 \theta$	$M_1(3/2)$
E_2	$P_{3/2}$	$1 + \cos^2 \theta$	$E_2(3/2)$

Since we have four basis photoproduction processes 16 complex amplitudes are necessary for a complete description. Charge independence arguments reduce the number of basic processes to three. They are the isotopic vector process leading to isotopic spin $I = 3/2$ and $I = 1/2$ states, and the isotopic scalar process leading to an $I = 1/2$ state. Watson's

theorem, based on the requirement of unitarity and time reversal invariance, equates the phase of any photoproduction amplitude to the corresponding pion-nucleon scattering phase shift at the same energy. Thus, photoproduction can be described by 12 "real" amplitudes.

The well-known (3,3) resonance at $E_\gamma \sim 330$ MeV has been known to dominate photoproduction, and has resulted in a simplified phenomenological description of photoproduction processes.^{2,28/} In this "(3,3) enhancement model" one can ignore all non-resonant multipole amplitudes to the extent that only the interference between the non-resonant and resonant amplitudes need be kept.

Near threshold ($E_\gamma < 200$ MeV), however, photoproduction of charged pions is dominated by the electric dipole transition to S-wave pion-nucleon state. The electric dipole transition occurs predominantly through the so-called "gauge" invariance term in the interaction Hamiltonian, but, as pointed out by Cini et al.,^{4/} the effect of the "direct interaction term" cannot be ignored.

The inclusion of the "gauge invariance term" and the direct interaction terms alone leads to the following expression for the charged pion photoproduction cross section,^{4/}

$$\frac{d\sigma^\pm}{d\Omega} = 2e^2 f^2 \frac{g_\pi^2 \omega}{(1+\omega/m)^2} \sum \left| \frac{1}{\sqrt{k\omega}} \left[(\vec{\sigma} \cdot \vec{\epsilon}) \left(1 + \frac{1}{2} R(\omega) \right) + \frac{\vec{\sigma} \cdot (\vec{k} - \vec{q}_\pi) \vec{q}_\pi \cdot \vec{\epsilon}}{(\vec{k} - \vec{q}_\pi)^2 + 1} \right] \right|^2 \quad (2.1)$$

where the sum implies the average over photon polarization and nucleon spin, and $\sigma^\pm$ refers to positive and negative pion cross section,

respectively. The first term in the square bracket is the gauge invariance term and the second term is the direct interaction term. $+ 1/2 R(w) \vec{\sigma} \cdot \vec{\epsilon}$ is the nucleon recoil term and gives rise to the difference between $\gamma\pi^+$ process and $\gamma\pi^-$ process. Corrections of the order $1/M$ to the renormalized Born approximation calculation give $R(w) = (g_p + g_n) w/M$, where g_p and g_n are the proton and neutron gyromagnetic ratio.

This expression evaluated at 90° cm for $\gamma\pi^+$ process is shown to agree^{29/} within 3% with Robinson's calculation^{30/} of the complete CGLN theory from threshold up to $E_\gamma = 180$ MeV. Recent π^+ experiments^{6,31,32/} agree reasonably well with the above expression in the same energy region.

The first successful dispersion theory of pion-nucleon scattering and photoproduction was derived by Chew and Low^{33/} with a cut-off model. The relativistic dispersion theory of CGLN generally reproduces the predictions of the cut-off theory with refinements of order $1/M$.

Various authors have calculated dispersion theory without resorting to an expansion in powers of $1/M$.^{34,35/} The results, however, do not materially differ from CGLN results if the same parameters and the same scattering phase shifts are used in the evaluations.

A number of authors,^{35-37/} have evaluated the effect of the γ - ρ - π interaction on photoproduction. In the CGLN notation the σ^-/σ^+ ratio can be expressed,

$$R = \frac{\sigma^-}{\sigma^+} = \left| \frac{F^- - F^0}{F^- + F^0} \right|^2 \quad (2.2)$$

where the isovector amplitude F^- dominates the isoscalar amplitude F^0 .

The γ - ρ - π interaction affects only the F^0 amplitude and hence its effect appears most strongly in the σ^-/σ^+ ratio. Ball's theory^{35/} gives, for example,

$$R = 1.28 (1 - .14 \Lambda/e) ,$$

at threshold, where Λ is γ - ρ - π coupling constant. However, theories do not agree and there appears, as yet, to be no strong argument as to which theory is correct.

B. π^- Photoproduction from Deuterium

1. Impulse approximation

The validity and limitations of the impulse approximation in general have been extensively discussed by Chew and Wick,^{38/} and by Chew and Goldberger.^{39/} It is generally conceded that $\gamma d \pi^\pm$ processes should lend themselves well to the application of the impulse approximation. Detailed arguments in support of this were given, for example, by Beneventano et al.^{14/}

The impulse approximation calculation has been carried out many times in the past. The latest calculation by Baldin^{25/} is considered to be the most thorough treatment of the problem, and will be discussed here.

Using units in which $\hbar = c = \mu = 1$, the photon energy and momentum will be denoted by $(k_0, \vec{k})$, and the pion energy and momentum by $(w, \vec{q}_\pi)$. The momenta for protons are denoted by p_1 and p_2 . Following Chew,^{40/} the dependence of scattering matrix T on the nucleon position $\vec{r}$, spin $\vec{\sigma}$ and isotopic spin $\vec{\tau}$ may be made explicit by writing

$$T = (\vec{\sigma} \cdot \vec{K} + L) \exp (i(\vec{k} - \vec{q}_\pi) \cdot \vec{r}) \vec{\tau} \cdot \vec{\tau} . \quad (2.3)$$

This form is required by momentum conservation and by the general properties of the Pauli spin operator. $\vec{K}$ and L are customarily referred to as "spin-flip" and "non-spin-flip" amplitudes, respectively. The cross section can be expressed

$$d\sigma = \frac{8}{(2\pi)^2} \sum |\langle f|T|i\rangle|^2 \delta(E_f - E_i) d^3q_n, d^3q, \quad (2.4)$$

where by $\sum$ sum over final and average over initial nucleon spin is implied, and $\vec{q} = 1/2 (\vec{p}_1 + \vec{p}_2)$. The momentum δ -function has been taken out of the matrix element $\langle f|T|i\rangle$.

The final state consists of two protons so the Pauli exclusion principle requires separate consideration of the two final states: symmetric and anti-symmetric states. When spin sum and average is carried out the cross section becomes

$$d\sigma = \frac{16}{(2\pi)^2} \left\{ |\vec{K}|^2 \left[\frac{2}{3} |I_a|^2 + \frac{1}{3} |I_s|^2 \right] + L^2 |I_a|^2 \right\} \times \delta(E_f - E_i) d^3q_n, d^3q \quad (2.5)$$

$$= 32 p^2 q \frac{W}{k} \left\{ \right\} dp dq d(\cos\theta_{pq}^{\wedge}) \quad (2.6)$$

where $\vec{p} = (\vec{p}_1 - \vec{p}_2)/2 \quad (2.7)$

$$I_{a,s} = \int \varphi_{fa,s}^* e^{i\vec{q}\cdot\vec{r}} \varphi_d(r) d^3r$$

$$\varphi_d = \frac{1}{\beta - \alpha} (2\alpha\beta(\alpha + \beta))^{\frac{1}{2}} (e^{-\alpha r} - e^{-\beta r})/r$$

$$\varphi_{fa} = (2\pi)^{-3/2} \frac{1}{2} (e^{i\vec{p}\cdot\vec{r}} - e^{-i\vec{p}\cdot\vec{r}})$$

$$\varphi_{fs} = (2\pi)^{-3/2} \frac{1}{2} (e^{i\vec{p}\cdot\vec{r}} + e^{-i\vec{p}\cdot\vec{r}} + \frac{1 - e^{2i\delta}}{ip} \frac{e^{-i\alpha r}}{r})$$

The suffix "a" and "s" refers to anti-symmetric and symmetric nature of the final state two-nucleon wave functions, respectively.

Coulomb corrections between the two protons can be taken into account by the following substitutions:

$$e^{i\vec{p}\cdot\vec{r}} \rightarrow |T(1+i\eta)| e^{-\frac{\pi}{2}\eta} F(i\eta/2 | 1 | -i(p_1 + \vec{p}\cdot\vec{r})) e^{i\vec{p}\cdot\vec{r}} \quad (2.8)$$

$$e^{ipr}/r = 2 \frac{T(1+i\eta)}{|T(1+i\eta)|} e^{-\frac{\pi}{2}\eta} e^{-ipr} W_1(-i\eta + 1/2 | 2ipr) ip \quad (2.9)$$

where $\eta = e^2/(2p/M)$, T is the Gamma-function, F is the confluent hyper-geometric function, and W_1 is the Whittaker function.

Using appropriate integral representations of F and W_1 , Baldin proceeds to obtain for the $\beta \rightarrow \infty$ case,

$$I_a = 4\pi C(\eta) (I(p, q, p, \eta) - I(p, q, p_2, \eta)) \quad (2.10)$$

$$I_s = 4\pi C(\eta) [I(p, q, p, \eta) + I(p, q, p_2, \eta) + i \frac{1}{C^2(\eta)} \frac{e^{i\delta}}{pq} \sin \delta (J(p, q) - J(p, -q))] \quad (2.11)$$

where

$$C^2(\eta) = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \quad (2.12)$$

$$J(p, q) = \int_0^1 dt \frac{1}{t + \frac{p-q+i\alpha}{p+q-i\alpha}} t^{i\gamma} \quad (2.13)$$

$$I(p, q, p_1, \gamma) = \frac{1}{p_1^2 + \alpha^2} e^{\frac{i\gamma}{2}} \ln \frac{(p^2 + \alpha^2)^2}{(q^2 + \alpha^2 - p^2)^2 + 4\alpha^2 p^2} \\ \times e^{i\gamma \tan^{-1} \frac{2\alpha p}{q^2 + \alpha^2 - p^2}} \quad (2.14)$$

The contribution from small r involves two other effects which are difficult to evaluate. If the distance between the two nucleons is within $1/\mu$, photopions produced will be subsequently reabsorbed.^{41/} The available phase space is then much greater for emission of two nucleons than for emission of two nucleons plus one meson. The other effect is due to nuclear multiple scattering of the meson. Baldin points out that since both effects decrease the wave function for $r \leq 1/\mu$, it is reasonable to choose $\varphi_{a,s}$ to be zero for $r \leq 1/\mu$.

Inclusion of this correction results in the following replacements.^{42/}

$$I(p, q, p_1, \gamma) \rightarrow I(p, q, p_1, \gamma) - \frac{1}{\mu^2} \left\{ 1 - \frac{2}{3\mu} (\alpha - \frac{1}{2}p) \right. \\ \left. + \frac{1}{4} \frac{\alpha^2}{\mu^2} - \frac{1}{12} \frac{q^2}{\mu^2} - \frac{\alpha \gamma p}{2\mu^2} \right\} \quad (2.15)$$

$$J(q) \rightarrow \int_0^1 dt \frac{t^{i\gamma}}{t + \frac{p-q+i\alpha}{p+q-i\alpha}} e^{-\frac{2i\gamma}{\mu} \frac{t}{t-1}} e^{\frac{i}{\mu} (p-q+i\alpha)} \quad (2.16)$$

There are two different Coulomb scattering effects for π^- -mesons. First, the Coulomb scattering between the spectator proton and the

π^- -meson; Second, between the proton which was converted from the neutron and the π^- -meson. They are of different nature and hence it appears to be appropriate to treat them separately.

The first effect is taken into account by calculating what amounts to an overlap integral of initial deuteron wave function and the final π^- -proton Coulomb wave function. The factor, F_π , to be multiplied to the amplitude is (omitting the normalization factor for the time being)

$$F_\pi(p_1, g_\pi, \zeta) = I\left(\left|\frac{\vec{p}_1 - m\vec{g}_\pi}{m+1}\right|, \frac{m}{m+1} \left|\vec{p}_1 + \vec{g}_\pi\right|, p_1, \zeta_1\right) \quad (2.17)$$

where, $\zeta_1 = e^2 / |\vec{v}_{p_1} - \vec{v}_\pi|$, and the function I has been defined by Eq. (2.14). The second effect can be taken into account by the Coulomb penetration factor $c(\zeta_2)$, where $\zeta_2 = e^2 / |\vec{v}_{p_2} - \vec{v}_\pi|$. Since we do not know which proton is the spectator, a proper procedure is to use an effective average of the two possible corrections. Such a procedure is suggested by the form $2/3|I_a|^2 + 1/3|I_s|^2$. The enhancement factor, which is to be applied to the cross section formula is then,

$$\begin{aligned} & \left\{ 2 \left| c(\zeta_2) F_\pi(p_1, g_\pi, \zeta_1) - c(\zeta_1) F_\pi(p_2, g_\pi, \zeta_2) \right|^2 \right. \\ & \left. + \left| c(\zeta_2) F_\pi(p_1, g_\pi, \zeta_1) + c(\zeta_1) F_\pi(p_2, g_\pi, \zeta_2) \right|^2 \right\} / \\ & \left\{ 2 \left| F_\pi(p_1, g_\pi, 0) - F_\pi(p_2, g_\pi, 0) \right|^2 + \left| F_\pi(p_1, g_\pi, 0) + F_\pi(p_2, g_\pi, 0) \right|^2 \right\} \quad (2.18) \end{aligned}$$

This is not valid if we assume that $L^2/|\vec{K}|^2 \neq 0$, but there are strong reasons to believe that below 200 MeV $L^2/|\vec{K}|^2 \neq 0$ is a good approximation.

To obtain $|\vec{K}|^2$ it is only necessary to compare experimental cross section for $\gamma\pi^-$ process at a fixed γ -ray energy to formula (2.5), assuming $L^2/|\vec{K}|^2 = 0$.

2. Chew-Low extrapolation technique^{27/}

Application of the Chew-Low extrapolation technique to obtain $\gamma n \pi^-$ cross section from observation of the $\gamma \pi^-$ interaction has been discussed previously by Swanson et al.^{20/} and by Moravcsik.^{43/} The basic idea behind and the actual procedures have been briefly mentioned in I-B.

The extrapolation procedure can be summarized by the following equation:

$$\sigma(\gamma n \rightarrow \pi^- + p) = \lim_{P^2 \rightarrow -\alpha^2} 8\pi m \frac{1-\alpha^2}{2\alpha} \frac{(P^2 + \alpha^2)^2}{W^2 - m^2} k^2 \frac{d\sigma(\gamma d \rightarrow \pi^- + 2p)}{dP^2 dW^2} \quad (2.19)$$

where r_0 is the triplet effective range, (P - n scattering), W^2 is the invariant mass squared for the π^- - p system, and m is the mass of nucleon.

There are the following considerations which were not incorporated in the previous application of Chew-Low technique:

- i) unreliability in identification of the "spectator",
- ii) nucleon-nucleon scattering, and proton-proton Coulomb scattering,
- iii) pion-spectator proton Coulomb scattering,

iv) pion-proton Coulomb scattering.

R. Schult has suggested^{44/} a procedure which effectively takes into account the first two effects.

Using the notations defined in connection with the impulse approximation calculations, we define a quantity, F_{pp} , by:

$$F_{pp} = \{2|I_a|^2 + |I_s|^2\} / \lim_{\substack{p^2 \rightarrow -\alpha^2 \\ \lambda \rightarrow 0 \\ \delta \rightarrow 0}} \{2|T_a|^2 + |I_s|^2\} \quad (2.20)$$

The observed cross section $\frac{\partial^2 \sigma}{\partial p_1^2 \partial w^2}$ is now divided by F_{pp} to obtain the correction (i.e., corrections i and ii) free contribution to the extrapolation. In this way, events are counted in two ways, i.e., first one proton is considered to be the spectator, and second, the other proton is considered to be the spectator. It is believed that although we have no way of distinguishing the spectator, if only the wrong choice of spectators were used in the extrapolation, the extrapolation would lead to zero.

The effect (iii and iv) is taken into account by $F_{\pi}(p_1, q_{\pi}, \lambda_1) \cdot C(\lambda_2)$, properly normalized: i.e.,

$$F_{\pi}(p_1, q_{\pi}, \lambda_1) C(\lambda_2) / F_{\pi}(p_1, q_{\pi}, 0). \quad (2.21)$$

III. APPARATUS AND TECHNIQUE

A. General Features

The 14" Illinois bubble chamber, filled with liquid deuterium, was placed in the X-ray beam of the University of Illinois betatron. The peak energy of the X-ray beam was set at 230 MeV. Reactions occurring in the chamber were photographed with three cameras. Out of 220,000 sets of such pictures taken, 140,000 sets are used in this analysis.

To decrease the electron background caused principally by the Compton effect, roughly three radiation lengths of CH_2 and LiH beam hardener were used. This procedure also removed much of the proton background due to the low energy photodisintegration of the deuteron. The hardened beam spectrum was calibrated by using a pair spectrometer technique.

The X-ray beam was monitored by a high pressure argon chamber. The response of the argon chamber was calibrated against a standard National Bureau of Standards (NBS) P-2 chamber.

Fig. 1 shows the experimental arrangement of betatron, collimators, beam hardener assembly, the bubble chamber and the ion chamber.

B. The Bubble Chamber

The bubble chamber used was built by the high energy physics group of the University of Illinois in 1961. It was used earlier for the positive pion photoproduction experiment,^{32/} and was found to be quite reliable. A detailed description of this bubble chamber has been given

EXPERIMENTAL LAYOUT FOR 14 INCH BUBBLE CHAMBER RUNS

Figure 1.

by Simpson.^{32/} The following measures were taken to reduce the background: First, the beam window and the exit window were made of 30 mill Be, to reduce the number of pair electrons. Second, a vacuum extension pipe was placed between the beam hardener and the bubble chamber to eliminate the background due to six feet of air path. Electrons produced at the window of this pipe were swept away by a magnetic sweeping field which was effective inside the vacuum region. Third, the operating condition of the bubble chamber was adjusted so that the minimum ionizing tracks were barely visible. Fourth, the size of the beam was made as large as possible, 6 cm in diameter, to diffuse the background.

With an average beam intensity of $.7 \times 10^6$ MeV/pulse, about 40 analyzable $\gamma d\pi^-$ reactions per 1000 frames resulted, and the background did not cause any serious difficulty in scanning and measuring.

A typical operating condition for the bubble chamber was: vapor pressure, 86.0 psia; chamber pressure, 96.0 psia; chamber pressure at the top of the expansion 46 psia; percent volume expansion, $\sim 1\%$.

C. Beam Formation

The X-ray beam was produced by the circulating electron beam in the betatron striking an internal platinum target about 0.002" thick. X-rays filtered through the porcelain donut were collimated by a 3/8" primary lead collimator. Preceding the primary and following the same were two polyethylene tubes, each 50 cm long. A secondary lead collimator slightly larger than the beam cleaned the edge of the beam. The beam then went through another 50 cm of polyethylene tube, a clearing field and 1.5 meters of LiH beam hardener, in that order.

A third collimator was placed immediately ahead of the second clearing magnet and inside the vacuum extension pipe to provide a stop for charged particles swept out of the beam by the clearing field magnet. A fourth collimator was placed just ahead of the bubble chamber to further clean the edges of the beam.

An argon chamber was placed 3 meters beyond the bubble chamber to monitor the X-ray beam.

The alignment of collimators, beam hardeners, the bubble chamber and the ion chamber were done systematically by means of X-ray pictures of removable cross hairs centered on each component in turn.

The direction of the X-rays with respect to the bubble chamber fiducials was obtained by measuring the relative orientations of the bubble chamber beam windows with respect to the fiducials, and the X-ray beam direction with respect to the beam windows.

At the exit window of the bubble chamber the beam was 3 cm in radius according to the X-ray pictures taken before the run. Later analysis of the origins of the $\gamma\pi^-$ reactions shows (see Fig. 2) that the beam intensity was almost uniform up to 2.0 cm and slowly decreased to zero at 3 cm. A similar analysis of event origins was used to obtain the direction of the γ -ray with respect to the fiducials. The value thus obtained was $0.90 \pm .2^\circ$ as compared to the direct measurement of $1.15 \pm .1^\circ$.

The latest energy calibration of the betatron^{45/} believed to be accurate to 0.25%, was obtained by measuring the field at the target crotch using an electron spin resonance probe and by measuring the

Figure 2.

target position relative to the magnetic center of the betatron. These measurements place our maximum γ -ray energy at 230 ± 0.6 MeV.

D. Beam Hardener

A major portion of the charged particle background from a normal bremsstrahlung beam consists of Compton electrons produced by photons of energy less than a few MeV. This background can be reduced by using an X-ray beam filtered by an absorber made of low Z elements.^{46/} The effect of the beam hardener is to eliminate practically all X-rays below 10 MeV.

The beam hardener consists of three pieces of polyethylene (CH_2) tubes followed by two air-tight tubes filled with powdered LiH. Table 2 gives the composition of the beam hardener.

Table 2
Composition of the Beam Hardener

Material	Amount (gm/cm^2)	Element	Amount (gm/cm^2)
CH_2	148.5	H	31.50
LiH	77.5	C	129.5
$\text{H}_8\text{C}_5\text{O}_2$	3.81	Li	69.67
Al	0.69	O	1.22
		Al	0.69

The relative shape of the hardened spectrum was determined^{47/} by measuring the ratio of the number of photons per monitor in a narrow energy bin with the beam hardener in place to the number with the beam hardener out. These data were obtained by using a pair spectrometer technique to count photons with the beam hardener both in and out. The result of this measurement is shown in Fig. 3. Also shown is the theoretically expected transmission normalized to six experimental points between 135 and 230 MeV. The normalization of this curve is such that unity on this graph corresponds to transmission coefficient of 0.054.

E. Monitor

Since the bubble chamber had to be operated at relatively low beam intensity, and since the monitor has to integrate for a long period of time (~ 25 min.) a sensitive and low leakage chamber had to be used. The ion chamber we used was a specially designed thick-walled chamber filled with five atmosphere of argon and methane gas mixture (90% argon).

To calibrate the argon chamber, the calibration of the NBS P2 ionization chamber was extended to 149, 199, 229 and 258 MeV maximum bremsstrahlung energy.^{48/} This is an extension to the previous calibration by NBS in the maximum photon energy range $6 \sim 170$ MeV. The Illinois replica (P2-11) of the P2 chamber was found to have a response of 3.847×10^{-13} $\mu\text{C}/\text{MeV}$ in the beam of maximum photon energy 229 MeV. The ratio of the response of the argon chamber to that of P2 chamber, as determined by transmission chamber comparisons, was $8.340 \pm .4\%$. Thus, the response of the argon chamber was determined to be 3.208×10^{-12} $\mu\text{C}/\text{MeV} \pm 1.9\%$.

Figure 3.

The charge collected per full monitor was obtained by calibrating the meter (3.0425 volts/full scale) and the charging condensor ($1.032 \times 10^{-3} \mu\text{f}$). This is $3.140 \times 10^{-3} \mu\text{c} \pm .3\% \mu\text{c}/\text{full scale}$. Thus, the total energy of the spectrum per full scale monitor is $.979 \times 10^{-9} \text{ MeV} \pm 1.9\%$.

F. Beam Spectrum

The bremsstrahlung spectrum as seen by the bubble chamber was obtained by multiplying a theoretical spectrum by the following quantities: First, the beam hardener transmission (as described in III-D); second, the donut transmission; third, the transmission through the beam window plus 7" of liquid deuterium. The term "transmission" as used above refers to the ratio of number of photons per monitor with a material in the beam to the number with the material out of the beam; i.e., this is not an absolute transmission coefficient.

There are various theoretical bremsstrahlung spectra differing in the approximations used in calculation.^{49/} Three integrated-over-angle spectra, all in extreme relativistic approximation, are most commonly used: 1) The Schiff Spectrum uses Born approximation assuming an approximate (and convenient) screening potential. Because it is given in a relatively simple analytical form it has been used extensively in the past. 2) The screened Bethe-Heitler spectrum uses Born approximation with Thomas-Fermi screening. 3) The screened Davies-Bethe-Maximom spectrum uses Thomas-Fermi screening but Coulomb corrections are included by using the Sommerfeld-Maue wave functions. The D-B-M

formulation when applied to the pair-production interaction removed a 10% discrepancy between theory and experiment. It is believed to be correct in absolute magnitude up to $(Z/137)^2 \ln E/E$, where E is equal to $E_{\gamma\text{max}} - E_{\gamma}$ in electron mass units. This uncertainty ranges, in our case, from 1.1% at 160 MeV to 3% at 210 MeV.

The choice of the spectrum is dictated by the correctness of the shape rather than the absolute magnitude, and we chose to use the Bethe-Heitler spectrum in the belief that this represents the shape better than the others.^{50/} Actually, there is very little difference in 150 - 210 MeV region between the Bethe-Heitler spectrum and the D-B-M spectrum, the difference ranging from 1% at 160 MeV to 3% at 210 MeV.

The "transmission" through the donut has been calculated using the attenuation coefficients given by Atkinson and Willis,^{51/} and Grodstein.^{52/} This is a small correction ($\sim 1.4\%$) in the region of interest.

The transmission through the beryllium window and 2.36 gm/cm^2 of liquid deuterium were obtained from the Klein-Nishina formula, Bethe-Heitler partially screened pair production cross sections and experimental values for electron triplet production, for the region $E_{\gamma} = 130 \sim 230 \text{ MeV}$. Below 130 MeV, Grodstein's tabulations were used. The correction due to this ranges from $1 \sim 1.2\%$ in our region of interest.

The X-ray beam spectrum thus obtained is presented in Fig. 4. The ordinate is the number of photons per MeV per monitor x photon energy. Also included in this figure is the effect of "spray" due to the bubble chamber (+ 1%).

Figure 4.

G. Density of Deuterium

The density of deuterium is required in calculating cross sections as well as in obtaining the energy of a particle of given range. Since we have a large number of μ -mesons with a definite energy the density can be easily obtained using a range-energy table. One could also measure the index of refraction of the liquid by means of an apparent depth measurement of the chamber, and compute the density using Lorentz-Lorenz relation.

Fig. 5 shows the range distribution of 2700 μ^+ -mesons from decay at rest of π^+ mesons. The solid curve is a Gaussian fit to the distribution. The range of μ -mesons is obtained to be

$$R_{\mu} = 0.959 \pm 0.001 \text{ cm} ,$$

where the error is the mean error.

The range energy table of Whaling^{53/} was used for proton energy less than 10 MeV. Aron's table^{54/} was joined to this smoothly for proton energy greater than 10 MeV. Using this range-energy table the density of deuterium was found to be

$$\rho_D = .1352 \pm .002 \text{ gm/cm}^3 ,$$

where the error is due to the uncertainty of the range-energy table ($\sim 1.5\%$).

The range momentum curve scaled to this density is given in Fig. 6.

Apparent depth measurement of the chamber yielded the following index of refraction.

$$n_D = 1.108 \pm .002 .$$

Figure 5.

Fig 6

Application of Lorentz-Lorenz relation using this value for n_D yields,

$$\rho_D = .135 \pm .003 \text{ gm/cm}^3 ,$$

consistent with the previously obtained value.

IV. ANALYSIS PROCEDURES

A. Introduction

An event due to a $\gamma d\pi^-$ reaction may appear as a three, two or a one prong event. This is because some particle may not carry enough energy to be visible, while some may be too lightly ionizing to be seen. Analyses have been made on only those events in which kinematics can be completely determined. This requirement excludes all one-prong events and all two-prong events in which either prong does not end in the fiducial volume. Typical $\gamma d\pi^-$ events are shown in Fig. 7.

In $\gamma d\pi^+$ event the identifying features are 1) the typical $\pi^+ - \mu^+ - e^+$ decay chain with a μ^+ track of well-defined range ($0.958 \pm .046$ cm in liquid D_2 , see III-G), and 2) the difference in ionization between the stopping π^+ and the outgoing μ^+ track. Positive electron tracks from decays of μ^+ mesons were seen in 30% of π^+ events. A typical π^+ event is shown in Fig. 8.

Aside from the general background of electron tracks which obscures the view, most background tracks were those of protons which were produced by photodisintegration of deuterium or by neutron-deuteron scattering. These proton tracks if scattered by deuterons can simulate $\gamma d\pi^-$ reactions as well as $\gamma d\pi^+$ reactions. Stringent kinematic requirements, together with ionization considerations, make it virtually impossible for a p-d scattering event to pass as a $\gamma d\pi^-$ event. However, for short π^+ events careful examinations of events in all three views were necessary to eliminate background events.

Figure 7. A typical two-prong π^- -event and a three-prong π^- -event.

132441

Figure 7.

Figure 8. A typical π - μ event.

Figure 8.

The space reconstruction and kinematic analysis of all events were carried out in the IBM 7094 computer at the University of Illinois Digital Computer Laboratory.

B. Scanning and Measuring Procedures

"Recordak" film readers were used to scan the film. The magnification of the projected picture was about 1.5 times the life size. Two views of each roll were scanned independently, and the results were combined.

All three-prong and two-prong events whose vertex was inside the beam, and all possible π - μ events which originated inside the beam were recorded. Standard scanning sheets were used to record roll number, frame number, and check marks in appropriate columns to specify purely topological features of possible events.

About 16,000 frames, out of a total of 140,000 frames, with possible events were listed, but eventually only 50% of them were found to have analyzable events.

Random sampling of 23 rolls showed that 6% of accepted π^+ - events and 5% of accepted π^- - events were missed by one scanner. Assuming that events were missed randomly, one gets overall scanning efficiency of 99.6% and 99.7% for π^+ and π^- - events, respectively.

Events were measured on a Hydel measuring machine connected to an IBM026 card punch. There are three projection lenses mounted on a movable microscopic stage and three corresponding film hold-down slots fixed to the machine. Pictures were projected on a 24" x 24" translucent screen one view at a time. The stage was connected to two "Datex"

encoders and upon activation of the data read-out switch, the coordinate pair X and Y, each five digits long, were punched out automatically. Each projection lens has its own illuminated cross hair attached to the lens by means of a prism arrangement. The accuracy of the machine is about $2\ \mu$ and any operator after a few hours of training was able to measure a well-defined point with an accuracy of $5\ \mu$. To reduce operator errors in measuring sequence a programmed data control circuit was used to indicate the next steps in the measuring sequence.

The following additional switches were provided to facilitate the input of pertinent parameters and additional information: they are event word, track word, end word, error word and spare word. Each word was ten digits long, and was either wired permanently or was easily resettable.

Events were divided into four event types: π^+ events, two-prong π^- events, three-prong π^- events and two-prong questionable events. These were labeled as types 1, 2, 3 and 4, respectively. Event types 1 and 4 were measured as one track and type 2 and type 3 as 2 and 3 tracks, respectively. Up to six data points were measured on each track. For π^+ events the last two points of a track measurement were beginning and end points of the μ -track.

The measurement of an event proceeded as follows: after frames with an event were found, the operator set the frame number and the event type on "event word switches" and pushed the "event word button," which resulted in punching out of these preset numbers, then 2 fiducial points each in views 1, 2 and 3 were measured. One, two or three tracks

were measured in each view according to the event type. After all tracks were measured in all three views measurement was terminated by pushing the "end" button.

On the average, about seven events were measured per hour.

C. Event Reconstruction

Since an event is measured in all three views there are three possible view combinations that can be used to reconstruct an event. Space reconstruction was done in all possible view combinations and the resulting angles and ranges of each track were averaged. Only when the individual result did not deviate appreciably from the average, was the space reconstruction deemed satisfactory.

To outline the space reconstruction procedures used, we first study the relationship between the coordinates of a bubble (B) and the coordinates of its image (I). Fig. 9 shows the relative arrangement of a bubble in liquid deuterium, combined glass windows, the lens and the film. We assumed that all six refracting surfaces (three glass windows) and the film surface were plane and parallel to each other. The principal ray $B\theta PI$ lies then in a plane perpendicular to windows. The intersection point, $P(U,V)$, of the principal ray and the outer surface of the windows will be called the reprojected point.

Using measured film coordinates of image I and two fiducials the reprojected coordinates $P(U,V)$ of the point P is obtained by a rotation, a translation and a magnification.

Using notations made obvious by Fig. 9, the magnification M becomes

Fig. 9

Space Reconstruction Optics

$$M = \frac{R}{\gamma} = 1 + \frac{x_g}{L} \frac{\tan \theta_g}{\tan \theta} + \frac{Z}{L} \frac{\tan \theta_b}{\tan \theta}$$

$$= \sigma + \frac{\rho}{L} Z \quad (4.1)$$

where

$$\sigma = 1 + \frac{x_g}{L} \cdot \frac{1}{\sqrt{n_g^2 + (n_g^2 - 1) \tan^2 \theta}} \quad (4.2)$$

$$\rho = \frac{1}{\sqrt{n_b^2 + (n_b^2 - 1) \tan^2 \theta}} \quad (4.3)$$

Thus, if the depth Z is known, X and Y can be obtained simply by:

$$X = MU$$

$$Y = MV$$

1) Reconstruction of a point: Let the two views being used be A and B . We choose the X - Y coordinate system so that the intersect of camera axis A with the inner surface of the bubble chamber window is the origin and the direction AB is the X -axis. Then,

$$X = M_A U_A = M_B U_B + D \quad (4.4)$$

$$Y = M_A V_A = M_B V_B$$

where

$$M_{A,B} = \sigma_{A,B} + \rho_{A,B} \frac{Z}{L_{A,B}}, \quad (4.5)$$

and $U_{A,B}$ is measured from the respective camera axis. D is the distance between A and B .

Solving (4.4) for Z one gets

$$Z = \frac{\sigma_B U_B - \sigma_A U_A + D}{\frac{\rho_A}{L_A} U_A - \frac{\rho_B}{L_B} U_B} \quad (4.6)$$

X and Y coordinates can now be obtained using (4.3).

ii) Reconstruction of a line: Given a point in view A and a line in view B one has to find a point in view B that corresponds to the given point in view A. To do this we first map a point in view A into a line in view B. Eliminating Z from (4.4), one gets

$$C_1 U_B - C_2 V_B + C_3 = 0 \quad (4.7)$$

where

$$\begin{aligned} C_1 &= (\sigma_B - \sigma_A \cdot f) V_A \\ C_2 &= (\sigma_B - \sigma_A \cdot f) U_A + f \cdot D \\ C_3 &= V_A \cdot D \\ f &= \rho_B L_A / \rho_A L_B \end{aligned} \quad (4.8)$$

Approximating the measured points in view B as a straight line near the approximate value of U_B , one can obtain the intersect of this line with the line given by (4.7). This intersection will give a better value for U_B and V_B . This interaction procedure is repeated until a self-consistent solution of U_B and V_B is obtained.

Once U_B and V_B are determined, X, Y and Z are obtained using the same method as is used in the reconstruction of a point.

D. Event Analysis

Each event that was successfully reconstructed was tested further to see if it could be a π - μ event or a π^- -event. Any successful interpretation was then stored on magnetic tape for future processing. All information regarding each event was also printed out. Figs. 10 and 11 show a sample output format.

All events were visually checked to see if a computer interpretation was reasonable or not. Teams consisting of one physicist and one scanner inspected events in two or three views and decided on one of three possible courses: to accept a possible interpretation, to reject the event as a background or to ask for a remeasurement. Eventually all but 60 out of 5000 π^- -events were accepted.

Processing procedures and acceptance criteria for π^+ and π^- -events will be discussed separately.

1) π - μ event: All events measured as type 1, 2 and 4 were tested for a possible π - μ interpretation. If the range of any track was shorter than 1.15 cm and longer than 0.8 cm and if the beginning point of the corresponding π^+ -track was within 4 cm of the center of X-ray beam, then this event was considered to be a possible π^+ event.

Visual identification of a long π^+ event (π -range > 2 cm) did not pose any problem. Ionization difference between the slowing down π^+ and the outgoing μ^+ track was sufficiently obvious.

Short (π -range $< .5$ cm) π^+ events were occasionally hard to separate from a p-d scattering event. In such cases the event was

Figure 10. Typical computer outputs for a $\pi^- \mu$ event,
and a two prong π^- -event.

.....
 INPUT TYPE I, IV 536985 50 NPTS=22727200000000000000000000 LVE=000000000 NBGIN=001 NSCAT=000 MCODE= 810112

 PH1 DPH1 TAN DIA

```
INPT TYPE I, IV 535885 50 NPTS=2222270000350000C0000000000 LIVE=000000000 N8GIN=001 NSCAT=000 MCODE= 810112
```

	HANDE	OF	ABTS	YRGS	ZRES	RE	YE	ZE	PHI	DPMI	TAN	DIAN
1	1.00	.012	-2.71	2.61	19.22	.22	6.43	9.30	35.2	.047	-.295E 00	.005
2	1.00	.034	-2.24	4.42	9.22	.11	3.71	7.64	-63.6	1.832	.540E 00	.158

CORNER=0 COUT=0 LEAVING T= 0 J

2000-01-01
 21.54 9.06 10.68 4.50

KINEMATIC FITTINGS IF A PRONG EVENT FOLLOW

KINEMATIC FITTINGS IF 2 PRONG EVENT FOLLOW														
IA	P	E	LOS	PSI	PCM	ECM	ECOSM	PHI	TANL	K	RANGE	DR	DP	CH
1	GAMA	190.8	190.8	1.000	.00	173.9	173.9	1.000	1.15	.000	.5241E-02	.00	1.1	0
2	PI-	21.1	21.1	-.785	-117.24	62.7	13.4	-.863	-143.73	.796	.7040E-01	3.22	.11	.5
3	P	142.0	10.7	-.375	-120.86	136.1	9.8	-.253	-63.53	.540	.8035E-02	.89	.03	1.0
4	P	241.2	30.5	-.736	59.34	185.7	18.2	-.477	32.62	-.352	.4408E-02	6.09	.41	3.9
** MISSING P RANGE 6.09 LONG AND DARK ** DELETED FROM HINARY														
RANGE DR DP CH														

```

*** MISSING RANGE 6.09 LONG AND DARK ** DELETED FROM TAPE
TR      P      E      COS      PSI      PCN      ECM      COSM      PHI      TANL      K      RANGE      DR      DP      CH
1  GAMMA  264.2  254.2  1.000  -.00  233.4  233.4  1.000  1.15  .000  -.3785E-02  .00  .00  1.2  0
2  P      207.2  21.5  -.785  -117.24  305.2  48.7  -.912  -143.78  .236  -.5157E-02  3.22  .05  .7  1
3  PI-    35.7  4.5  .375  -120.86  33.4  3.9  -.132  -63.59  .543  .2381E-01  .89  .04  .4  0
4  P      439.1  97.7  .333  62.00  325.0  54.7  -.873  20.00  -.172  .2311E-02  50.97  1.07  2.7  3
*** GAMMA RAY ENERGY TOO LARGE #GAMMA=.2642E 03 DEGAMMA=.1243E 01 DELETED FROM BINARY TAPE

```

ILCLIT REJECT 13 .EVENT 536485.FIT 1

.....

INPUT TYPE 2, IN 536494 50 NPIS=22222200000000000000000000000000 LVE=000000000 NBGIN=000 NSCAT=000 MCJDE= 810112

INPUT TYPE 2, IN 535494 60

[illegible]

NONE COULD BE MJ-TRACK #1: 5.336 42= 4.211
ENERGY 32.72 12.22 24.72

KINEMATIC FITTINGS OF 2 PRONG EVENT FOLLOW

KINEMATIC FITTINGS OF 2 PRONG EVENT FOLLOWUP											RANGE	DR	DP	CR
TR	P	E	COS	PSI	PCM	ECM	COSECM	PHI	TANL	K	RANGE	DR	DP	CR
1	DAMA	174.4	177.4	1.000	.00	164.8	164.8	1.000	1.15	.000	.5580E-02	.00	.00	.3
2	PI-	53.5	13.4	.541	-195.05	73.1	16.0	-.744	-129.67	.233	.1603E-01	6.93	-.23	.6
3	P	217.7	24.7	.994	42.17	144.6	4.6	.987	5.04	-.075	.4606E-02	4.21	.36	.5
4	P	37.7	.6	.122	95.25	85.0	3.8	-.023	54.13	.109	.3055E-01	.00	.00	2.7
TR	P	E	COS	PSI	PCM	ECM	COSECM	PHI	TANL	K	RANGE	DR	DP	CR
1	DAMA	275.4	276.4	1.000	.00	241.9	241.9	1.000	1.15	.000	.3635E-02	.00	.00	1.3

TR	P	E	POS	PSI	PCM	LCM	DOSEN	PHI	TANL	K	RANGE	DR	DP	CH
1	GAMA	275.1	175.1	11.00	241.7	241.9	1.000	1.15	1.000	1.3635E-02	.00	.00	1.3	1
2	P	241.7	32.7	1.441	105.05	145.1	61.4	1.831	129.67	.203	1.4083E-02	5.92	.09	.0
3	PHI	22.2	10.5	1.435	42.17	36.4	4.7	1.488	5.04	1.075	1.819E-01	4.21	.14	.0
4	P	423.7	01.3	1.897	75.87	312.4	50.8	1.531	25.54	1.138	1.2373E-02	45.17	1.14	2.3

... GAMMA RAY ENERGY 100 LARGE - GAMMA- .2751E 03 DESGAMMA= .140E 01 DELETED FROM BINARY TAPE

ILLFIT SUBJECT IS .EVENT 740494.FIT 2

Figure 11. A typical output for a three prong π^- -event.

TR L S I	RANGE	DR	XBEG	YBEG	ZBEG	XE	YE	ZE	PHI	DPHI	IAY	DIAY
1 3 0 3	11.34	.000	-5.11	.67	10.71	-13.92	-2.28	4.22	-161.5	.024	-.699E 00	.003
2 0 0 0	.27	.031	-5.10	.68	10.73	-4.90	.63	10.90	-13.0	3.957	.840E 00	.706
3 0 0 0	2.74	.020	-5.10	.68	10.73	-2.44	1.17	10.32	10.5	.230	-.152E 00	.034
RESULT OF LEAVING TEST,			CORNER=0		COUT=0		LEAVING I= 1 0 0					
ENERGY	14.48	6.13	5.57	2.35	19.72	8.30						

EVENT	FIT	TMASS	MODE	MV	CHISQ	ST	CUT	DEL	DELX	1 MASS	DIM	XMASS	DXM	DELTA E
553779	1	1875.5	4	2	3.02	3	0	.27E-06	.51E-02	.0	.2	.0	.0	.0

TK	OUTPUT ERROR MATRIX										STRETCHES		
1	.88E-04	-.00E 00	-.81E-08	-.00E 00	1.00E-04	.00E 00	-.81E-08	.00E 00	.74E-09	-.52	.00	.00	
2	.23E-05	-.19E-08	-.56E-08	-.19E-08	.30E-04	-.18E-06	-.56E-08	-.18E-06	.96E-07	-.03	.01	.00	
3	.12E-02	-.31E-03	-.22E-05	-.31E-03	.41E-02	-.22E-04	-.22E-05	-.22E-04	.15E-06	2.37	4.07	3.44	
4	.82E-04	.60E-05	-.28E-08	.60E-05	.38E-03	-.23E-06	-.28E-08	-.23E-06	.55E-09	.51	.13	-.06	

EVENT	FIT	TMASS	MODE	MV	CHISQ	ST	CUT	DEL	DELX	1 MASS	DIM	XMASS	DXM	DELTA E	
553779	2	1	1875.5	4	2	16.26	4	2	.77E-04	.66E-01	.0	.0	.0	.0	.0

TK	OUTPUT ERROR MATRIX										STRETCHES		
1	.96E-04	-.00E 00	.23E-08	-.00E 00	1.00E-04	.00E 00	.23E-08	.00E 00	1.00E-10	-.53	.00	.00	
2	.23E-05	-.10E-11	-.24E-06	-.10E-11	.30E-04	-.47E-05	-.24E-06	-.47E-05	.51E-01	-.00	-.00	.00	
3	.29E-03	.40E-04	-.56E-06	.40E-04	.12E-03	-.69E-06	-.56E-06	-.69E-06	.17E-07	10.39	-67.26	-28.78	
4	.36E-03	.14E-05	-.25E-08	.14E-05	.69E-03	-.19E-05	-.25E-08	-.19E-05	.40E-07	.31	-.16	.03	

*** CHI-SQ TOO LARGE, THIS INTERPRETATION DELETED FROM BINARY TAPE

studied carefully in all three views. To make the results unambiguous π^+ events with π^+ range of less than 1 cm were not used in the final analysis.

ii) Three prong π^- event: A three prong event is kinematically over-determined unless all three tracks are leaving the chamber. This requires use of a kinematic fitting process in which all measured variables are varied until a best possible kinematic fit is obtained. A general purpose vertex fitting program "ILLFIT," which was developed by Professor G. Ascoli of the University of Illinois, was used for this purpose.

The input to Illfit consists of three measured variables and a corresponding 3 by 3 error matrix for each track. ϕ , $\tan\lambda$, and $1/k\cos\lambda$ were used as measured variables, where ϕ , λ and k are projected angle, dip angle and momentum, respectively. The error matrix was constructed by assuming that the measured variables are functions of measured coordinates. Range straggling as well as multiple scattering were included in the calculation of the error matrix.

All three possible mass assignments were tried in fitting the vertex. In about 90% of the cases only one possible fit was found by "ILLFIT". When there were two or three possible fits all but one were usually rejected because of its large χ^2 (>10), or an impossible γ -ray energy.

If all three tracks of a π^- -event left the fiducial volume, the kinematics were solved by using energy-momentum conservation laws. A similar procedure such as Swanson et al.^{19/} used were employed.

Visual checks of all 3-prong events were done to eliminate all bad measurements and spurious fits. Ionization and multiple scattering considerations were sufficient to identify π^- -track.

iii) 2-prong π^- -event: All events measured as type 1, 2, and 4 were checked to see if this could be a π^- -event. To qualify as a two-prong π^- -event, the vertex has to be in the beam and both tracks have to stop in the fiducial volume. If these conditions were satisfied, "Illfit" was used again to obtain the kinematics of the event for all three possible mass assignments.

Unlike a 3-prong π^- -event, the kinematics are now unique rather than over-determined. What "Illfit" does in this case is to use the energy-momentum conservation rules to calculate the γ -ray energy and the energy and momentum of the missing particle. Input error matrix is used to calculate the error on the calculated variables.

If the calculated γ -ray energy was too large (> 235 MeV), or if the calculated range of missing particle was long enough ($> .4$ cm) to be found easily but short enough (< 20 cm) to be dark, then the interpretation was temporarily rejected.

A two prong π^- -event usually had a short spectator proton. Although it was too short to be measured reliably, it was of some help in visual identification of ambiguous cases.

E. Efficiency Calculations

1. Geometrical efficiency

The visible region, or the fiducial volume, of the chamber is chosen to be defined by a truncated cone of radius 15 cm at the

illumination window and 12.4 cm at the camera window. This is about .4 cm smaller in radius than the "actual visible region," so that any three prong events can be analyzed if it originates in the above defined fiducial volume.

However, any two prong π^- -event or any $\pi-\mu$ event cannot be analyzed if it leaves the visible region of the chamber. In these cases, therefore, the efficiency of observing an event has to be calculated.

We have calculated the efficiency for each observed event using a Monte-Carlo method. Complicated geometry of the chamber and the diversity of the event configuration virtually eliminate a straightforward integration method.

Given an event, its origin was chosen randomly inside the X-ray beam region, and its azimuthal angle around the beam direction was chosen randomly. Then, a simple test was performed to see if the event was leaving the chamber. The ratio of number of successful trials to the total number of trials gives the efficiency of observing the event, and the inverse of this is the probable number of events expected if the chamber was infinitely large. Two hundred trials per event gave sufficient accuracy in efficiency.

2. The physical efficiency

There is a basic limitation in the observability of an event. This is due to the fact that for certain kinematic configurations an event may have more than four unmeasurable momentum components and hence unanalyzable. For example, an event with two unmeasurably

short (range less than 0.17 cm) tracks, or an event with an unmeasurably long track (26 cm in the forward direction and 15 cm in the orthogonal direction) and a short track cannot be analyzed no matter where it begins in the chamber.

In obtaining the matrix element for the $\gamma n \pi^-$ process through an impulse approximation, this difficulty is circumvented by simply evaluating the impulse integral only in the analyzable kinematical region. However, in applying the Chew-Low extrapolation technique there appears to be no such simple procedure. In this case, one needs to deduce the differential cross section $\partial^2 \sigma / \partial p_1^2 \partial W^2$ for the complete kinematic region from the observed value. One can do this if one knows a priori the pion angular distribution of the $\gamma n \pi^-$ process. Although a precise knowledge of this is lacking, one can safely assume it to be isotropic near threshold.

With an isotropic angular distribution an evaluation of this "physical efficiency" has been carried out by means of a Monte Carlo method. A kinematically allowed event was randomly generated in a given bin of photon energy, the spectator momentum squared and the invariant mass squared of π^- -p system. Then the angle of π^- -meson is chosen randomly in the γ -n center of mass system and the resulting momenta were transformed to the lab system. The resulting random event is now fixed in space except for the over-all azimuthal angle around the X-ray beam direction. A set of simple tests were performed to see if this event was measurable or not. The ratio of the number of successful tries to the total number of tries gives a physical

efficiency of the bin. The number of trials was varied roughly according to the number of events in that bin, maintaining the statistical error in the efficiency evaluation at less than one-third of the experimental error.

The "physical" efficiency varied from 0.3 to 1.0, but in heavily contributing regions it was seldom smaller than 0.8.

F. Corrections to Data

A two prong π^- -event cannot be analyzed if 1) the π^- -meson decays in flight, ii) the π^- -meson is scattered by a deuteron, iii) a proton is scattered by a deuteron. Each of these effects will be discussed briefly in the following.

1) Decay in flight correction: The decay in flight correction can be expressed by $\exp(t/\tau)-1$, where t is the time of flight and τ is the mean life time of π -meson, 2.55×10^{-8} sec. A simple evaluation of the time of flight as a function of range of π -meson resulted in a correction factor ranging from 0.9% at 2 cm to 4.8% at 15 cm. This correction was applied to all two prong π^- -events except when the σ^-/σ^+ ratio is evaluated.

ii) π^- -d scattering correction: There appears to be no published experimental data on the π^- -d total cross section below 30 MeV. Empirical extrapolation of total cross section measurements at higher energies indicates that the cross section ranges from about 16 mb at 30 MeV to about 3 mb at 5 MeV. At very low energies, however, Rutherford scattering is expected to dominate. For example,

at 5 MeV the total cross section due to Rutherford scattering is about 30 mb (for a pion C.M. scattering angle greater than 15°). A simple order of magnitude estimate of the π^- -d scattering correction is obtained by assuming the cross section to be constant throughout the region and equal to 20 mb. The result is 2.1, 1.0 and 0.3% for a pion of range 27, 13 and 4 cm, respectively. However, real corrections are believed to be smaller than these and were ignored. The over-all effect of this correction is estimated to be less than .7%.

iii) p-d scattering correction: An empirical evaluation of this correction was obtained by analyzing three prong π^- -events in which a proton track was scattered. Since the initial proton momentum was computed in the kinematic fitting process the probability of a proton being scattered before it comes to rest (a residual range of 3 mm was considered to be at rest) can be obtained as a function of the proton momentum. This is done by computing the ratio of the number of scattered protons in a given initial momentum bin to the total number of protons in the same bin. Using about one third of the available data these ratios were evaluated and the result is presented in Fig. 12. The solid line represents a crude estimate of the same correction obtained by using the existing data on p-d scattering.

The elastic proton-deuteron cross sections are well known in the region of our interest ($5 \sim 40$ MeV).⁵⁵⁻⁶¹ However, there are no published inelastic cross sections between 15 and 40 MeV. Available measurements of the inelastic process between 7 and 14 MeV indicate that the energy behavior of the cross section is very nearly linear.^{62/}

Fig. 12

Figure 12.

Using this as a guide, the inelastic cross sections at higher energies were estimated by means of a linear extrapolation technique. The combined elastic and inelastic cross sections were 1400, 540, 400 and 360 mb at 5, 15, 25 and 35 MeV, respectively. The scattering probabilities were computed in a straightforward manner using these cross sections.

The agreement between the result of this experiment and the solid line appears to be reasonable. However, the systematic tendency of the lowest three points led us to believe that p-d scattering at low energy may not be detected efficiently in the bubble chamber pictures. In view of this, it is felt that a compromise between the result of this experiment and the solid line would be more realistic. The broken line in Fig. 12 represents the actual correction we used.

V. RESULTS AND CONCLUSIONS

A. Comparison of Experiment with Impulse Approximation Calculations

Negative pion events with a pion range of less than 13 cm were sorted into bins of γ -ray energy and relative proton momentum $p = 1/2 |\vec{p}_1 - \vec{p}_2|$. There were 3174 such events between $E_\gamma = 150$ and 200 MeV. Each event was weighted according to geometrical efficiency (see IV-E), and was corrected for p-d scattering effects (see IV-F) and for decay in flight of π^- (see IV-F). The resulting distributions are plotted in Figs. 13 through 16. The ordinate in these figures is the ratio $\frac{d\sigma}{dp}/\sigma$.

In order to compare experimental results with impulse approximations, we assumed that the non-spin flip matrix element, L , was negligible compared to the spin-flip matrix element, $|K|$, in our energy range. Empirical proof of this has been obtained earlier by Adamovich et al.^{2/} Impulse calculations (see II-B) were performed with a $1/m_\pi$ cut-off and the results are presented by solid lines in Figs. 13-16.

The agreement between this experiment and the impulse calculations appears to be reasonable, as far as the relative shape of $d\sigma/dp$ is concerned. It turns out, however, that the impulse calculation depends strongly on the choice of the cut-off. Evaluations with no cut-off were performed using the Hulthén wave function for the deuteron. The relative shape of $d\sigma/dp$ predicted by this calculation is very similar to that of the "with cut-off" calculations, but the absolute magnitudes differ by as much as 40 percent. Although there is reason to believe that a cut-off of $1/m_\pi$ is quite reasonable

Figure 13.

Figure 14.

Figure 15.

Figure 16.

(see II-B), the evaluation of the matrix element for the $\gamma n \pi^-$ process through the impulse approximation is somewhat uncertain.

Nevertheless, the matrix elements squared, $|\vec{K}|^2$, were obtained by comparing the experimental cross sections with the cut-off calculations. These are given in Fig. 17.

$|\vec{K}|^2$ is related to the total cross section of the $\gamma n \pi^-$ process through the following relation,

$$\sigma^{\text{total}}(\gamma + n \rightarrow \pi^- + p) = \frac{1}{\pi} |\vec{K}|^2 \frac{q_{\pi} w}{(1+k/E_n)(1+w/E_p)}, \quad (5.1)$$

where the notation is the same as in II-B. An evaluation of CGLN theory was carried out assuming $N^{(-)}$ to be zero and using $r^2 = 0.081$. The result of this calculation is represented by the solid line in Fig. 17.

B. σ^-/σ^+ Ratio from Deuterium

About 2700 π^+ events and about 3600 stopping π^- events were found in the entire sample of 140,000 sets of pictures. Each event was weighted according to the geometrical efficiency for observing it. Each two prong π^- -event was corrected for p-d scattering. Three prong stopping π^- -events were weighted by the efficiency of observing the stopping π^- track alone. These weighting and correction procedures have been discussed previously in II-E and II-F.

Each π^- -event was corrected for the p-p and π^- -2p Coulomb scattering effects. p-p Coulomb corrections were obtained by evaluating the ratio of the impulse integral with Coulomb scattering to that without

Figure 17.

Coulomb scattering. These impulse calculations were performed with a "cut-off" of $1/m_\pi$. The Coulomb correction for the π^- -meson was calculated according to the formula (2.18).

Events were sorted into bins of lab cosines and ranges of π -mesons, and the ratios σ^-/σ^+ , were obtained. The difference in threshold of the $\gamma p \pi^+$ and the $\gamma n \pi^-$ reactions requires a correction for the difference in the effective photon beam intensity at their thresholds. This amounts to $\sim -1\%$ correction to the σ^-/σ^+ ratio.

The resulting σ^-/σ^+ ratios are summarized in table 3. The results are surprising in one respect. The p-p Coulomb interaction is repulsive. Hence, the corrections have usually been assumed to be positive. However, our calculations indicate that if the pion leaves in the forward direction, the correction becomes negative. This seems to be due to the factor, $c^2(\eta) \exp(2\eta \cdot \tan^{-1}(2\alpha p/q^2 + \alpha^2 - p^2))$, which results from squaring formula (2.10). If the pion is going in the forward direction, p^2 is usually greater than $q^2 + \alpha^2$ and hence the above factor becomes greater than 1.

Those bins with a laboratory cosine greater than 0.2 are not observed with a 100 % efficiency. This is due to the fact that if a pion goes forward, then the protons sometimes do not carry enough energy to be measurable.

The results agree reasonably well with previous experiments in the same energy region. However, the experimental errors for all experiments in this region are such that it is difficult to reach any definite conclusions.

Table 3

 σ^-/σ^+ ratio from deuterium

Range of π (cm)	E_π CM (MeV)	$\cos\theta$ CM	Lab cosine	Raw ratio	P-P correction %	π -pp correction %	Corrected ratio
1 - 5	173	-.85	-1.0 ~ -.6	1.48	0.0	- 7.6	1.37 $\pm$ 9 %
	169	-.70	-.6 ~ -.2	1.47	+2.8	- 8.2	1.40 $\pm$ 9 %
	165	-.37	-.2 ~ -.2	1.55	+1.0	- 7.6	1.43 $\pm$ 9 %
	160	-.05	.2 ~ .6	1.43	-3.2	- 9.8	*1.25 $\pm$ 9 %
	157	+.50	.6 ~ 1.0	1.15	-6.2	-11.3	* .95 $\pm$ 10 %
5 - 17	187	-.90	-1.0 ~ -.6	1.56	+3.8	- 5.4	1.53 $\pm$ 10 %
	180	-.62	-.6 ~ -.2	1.54	+9.2	- 6.4	1.59 $\pm$ 10 %
	175	-.30	-.2 ~ +.2	1.35	+1.4	- 5.8	1.28 $\pm$ 9 %
	168	+.05	.2 ~ .6	1.51	-4.0	- 7.0	*1.34 $\pm$ 10 %
	163	+.60	.6 ~ 1.0	1.06	-6.6	- 7.3	* .91 $\pm$ 10 %
11 - 17	197	-.90	-1.0 ~ -.6	1.38	+4.9	- 4.2	1.38 $\pm$ 15 %
	168	+.65	.6 ~ 1.0	.80	-8.2	- 6.1	* .69 $\pm$ 13 %

*These are lower limits on σ^-/σ^+ ratio.

C. Chew-Low Extrapolation

There were 4754 π^- -events observed between $E_\gamma = 150$ and 210 MeV. These events were weighted by the geometrical efficiency for observing them and were corrected for decay in flight and p-d scattering. Events were sorted into bins of E_γ , p_1^2 (p_1 is the spectator proton momentum) and w^2 (invariant mass squared of π^- -p system), each event being counted twice for the two different spectator interpretations.

For each spectator interpretation, the corrected number of events was multiplied by the F_{pp} factor (see Sec. II-B), by the π^- -spectator Coulomb correction factor (2.21) and by the kinematical factor $(p_1^2 + \alpha^2)^2 k^2 / (w^2 - m^2)$. Each $E_\gamma - p_1^2 - w^2$ bin was divided by the corresponding "physical efficiency" (see IV-E). The average over photon energy was carried out by adding up all bins of same p_1^2 and w^2 , and dividing this result by the corresponding sum of $\int dp_1^2 dw^2 dN_k$ evaluated for each $E_\gamma - p_1^2 - w^2$ bin, where dN_k is the number of photons between k and $k + dk$.

Each bin was then multiplied by the following normalization factor:

$$8\pi m \frac{1 - \alpha^2}{2\alpha} \frac{A}{N_0 \rho_d t}$$

where

$A = 2.015$ AMU, deuteron mass.

$\rho_d = 0.1352$ gm/cm³, density of deuterium,

$t = 27.35$ cm, target length,

$N_0 = 6.025 \times 10^{23}$, Avogadro's number.

As was mentioned in Sec. V-A, the result of the impulse approximation calculation depends on the choice of a cut-off. Since the factor F_{pp} is

proportional to the impulse integral, the extrapolation will also be dependent on the cut-off choice. To see if the results of these differing extrapolations were unambiguous, the Chew-Low extrapolation was carried out for a variety of F_{pp} factors. These were: A) calculated with $1/m_\pi$ cut-off, B) calculated with $.8/m_\pi$ cut-off, C) $1/2m_\pi$ cut-off, D) with no cut-off but using the Hulthen wave function. The differences among these various approaches are illustrated in Fig. 18. It is noted that although a different choice of F_{pp} factors leads to a different slope of the extrapolation line, but the value at the pole is always the same within experimental errors.

The extrapolated results for different choices of F_{pp} factors are summarized in Table 4. These were obtained using a linear extrapolation.

As can be seen in Table 4, the pole value in each w^2 bin is insensitive to the choice of cut-off. However, there is reason to believe that the best extrapolation choice is the one that gives the line with the least slope. Fig. 18 shows that this can be achieved with a cut-off of $0.8/m_\pi$. The data to be extrapolated, for this choice of cut-off, are given in Figs. 19 through 22. What is not included in these Figs. 18 - 22 is the $\pi^- - p$ (parent) Coulomb penetration factor, which lowers all data points belonging to the same w^2 bin by an equal proportion.

The result for the bin $w^2 = 59.7 \sim 60.2$ is less reliable than the others. This is due to the fact that most of the events contributing to this bin are of such small size that we are near the limit of human capability of detection. A survey of about 100 events contributing to

Table 4

W^2 bin		59.7 -60.2	60.2 -60.7	60.7 -61.2	61.2 -61.7	61.7 -62.2	62.2 -62.7	62.7 -63.2	63.2 -63.7	63.7 -64.2
E_γ		152.0	157.2	162.4	167.6	172.8	177.9	183.1	188.3	193.5
No. of degrees of freedom		12	10	13	15	12	12	14	14	14
Method A (cut-off 1.0)	σ in μb	34.7	74.3	83.2	97.2	95.5	119.1	109.1	132.0	146.8
	$\delta\sigma$ in μb	3.7	5.1	5.3	6.3	7.2	12.6	15.5	18.7	27.7
	χ^2	7.6	9.0	7.9	26.0	28.3	5.1	25.5	15.5	10.9
Method B (cut-off .8)	σ in μb	36.8	77.3	85.2	97.2	94.3	118.7	110.0	127.4	147.8
	$\delta\sigma$	3.5	4.8	5.0	6.0	6.8	11.8	14.6	17.7	26.3
	χ^2	7.6	7.4	7.8	21.6	24.2	3.7	24.0	13.6	9.6
Method C (cut-off .5)	σ in μb	36.0	75.1	83.5	95.3	94.8	119.0	104.6	129.5	149.0
	$\delta\sigma$	3.1	4.2	4.4	5.2	6.1	10.3	12.5	15.0	21.5
	χ^2	7.5	9.2	7.3	22.8	22.8	4.5	23.7	12.3	9.4
Method D (No cut-off)	σ in μb	33.8	71.3	82.6	97.7	102.5	128.5	127.1	140.1	156.7
	$\delta\sigma$	3.1	4.2	4.4	5.3	6.3	10.9	15.0	20.7	23.0
	χ^2	9.4	14.0	9.2	34.9	27.0	5.8	15.2	11.0	14.5

Figure 18:

Figure 19.

Chew-Low Extrapolation Cut-off = $0.8 m_\pi$

Figure 20.

Chew-Low extrapolation

Cut-Off = $.8/m_\pi$

$W^2 = 62.2 \sim 62.7$

$W^2 = 62.7 \sim 63.2$

Figure 21.

Figure 22.

FIG 23

this bin indicates that the scanning inefficiency was indeed higher than the average by a factor of 2. For this reason, we consider the result at $w^2 = 59.7 \sim 60.2$ only to indicate lower limit of the cross section in that bin.

The matrix elements squared, $|\vec{K}|^2$, have been evaluated using the relation (5.1) and are presented in Fig. 23. The results appear to be systematically 10% lower than the CGLN prediction evaluated with a commonly accepted coupling constant of .081.

D. Conclusions

The σ^-/σ^+ ratios obtained in (V-B) appear to be consistent with previous experiments in this energy range. One surprising feature found in this connection is that p-p Coulomb interaction enhances $\gamma d\pi^-$ process in certain kinematic regions. This point merits further theoretical study.

The $\gamma n\pi^-$ matrix elements obtained through the impulse approximation were found to be sensitive to the choice of a "cut-off". Although our results are in agreement with the CGLN theory and with the previous experiment^{2/} analyzed in a similar manner, the interpretation of this result is somewhat uncertain due to the "unknown" parameter of the impulse approximation.

The Chew-Low extrapolations were found to be insensitive to the choice of a cut-off. As can be seen in Fig. 23, our results follow the curve predicted by CGLN, and hence, it is justifiable to extrapolate to threshold using the CGLN energy dependence. The result of this

extrapolation shows that our data are consistent with the CGLN theory if the coupling constant is taken to be $.073 \pm .003$. The extrapolated value of $|\vec{K}|^2$ was found to be $1.03 \pm .04 \times 10^{-27} \text{ cm}^2$. In terms of "a" coefficient, this is equivalent to "a₀⁻" of $2.0 \pm 1.0 \mu\text{b/ster}$ at the threshold. Comparing this with the latest π^+ experiment^{32/} performed at the University of Illinois gives the following threshold σ^-/σ^+ ratio:

$$(\sigma^-/\sigma^+)_{\text{threshold}} = 1.23 \pm .06 .$$

It is noted that this is somewhat smaller than the theoretically predicted value of 1.3. However, this is in good agreement with the result of Watson et al.^{28/} who obtained $\sigma^-/\sigma^+ = 1.24$ at threshold using a phenomenological model and σ^-/σ^+ ratios at higher energies.

One can obtain the threshold σ^-/σ^+ ratio from a detailed-balance argument. The latest measurement^{63/} of the Panofsky ratio gives, $p = 1.53 \pm 0.02$. The threshold behavior of the S-wave charge exchange scattering phase shift has been discussed extensively by Cini et al.^{4/} and by Hamilton and Woolcock.^{64/} The consensus is that, at the threshold, $(\delta_1 - \delta_3)/q = 0.24 \pm 0.1$. Using these values one obtains from the detailed-balance argument the following matrix element squared:

$$|\vec{K}|^2 = 1.12 \times 10^{-27} \text{ cm}^2 \pm 0.09,$$

consistent with the CGLN prediction, but somewhat higher than our result.

This discrepancy between our result and the CGLN prediction can be reconciled by including a γ - ρ - π interaction. For example, if we use Ball's theory^{35/} the required coupling constants would be:

$$\begin{aligned} f^2 &= .076 \\ \Lambda/e &= 0.4 \pm .4 . \end{aligned}$$

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