

RECEIVED 1971

CONF-710609--9

MASTER

SOME NEW APPLICATIONS OF INDEFINITE METRIC
TO QUANTUM ELECTRODYNAMICS AND WEAK INTERACTIONS

T. D. Lee

Columbia University, New York, N.Y.

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

An invited talk given at the Amsterdam International Conference on Elementary Particles,
July 1971.

This research was supported in part by the U. S. Atomic Energy Commission.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

J49

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

Several topics related to possible physical applications of indefinite metric are discussed; among these are the neutron-proton mass difference, the production cross-section of the heavy photon and the possible existence of the spin 0 charged intermediate boson.

The use of indefinite metric for the Hilbert space in quantum theory was first introduced by Dirac¹ in 1942. In subsequent years, the indefinite metric has been applied successfully by Gupta² and Bleuler³ to the longitudinal and scalar modes of the photon field; it has also been used (at least implicitly) in connection with the Feynman regulator⁴ in the renormalization problem. More recently, it was pointed out^{5,6,7} that there exists a large class of theories with indefinite metric in which the unitarity of the S-matrix remains valid, and therefore can be used for physical applications. Furthermore, as is well known, the indefinite metric makes it extremely simple to remove the otherwise persistent divergence difficulties that are present in the usual theory. In this talk, I shall concentrate only on some new possible applications of such indefinite metric theories to quantum electrodynamics and weak interactions.

I. Applications to Electrodynamics

In the conventional quantum electrodynamics, one encounters serious divergence difficulties in connection with both electromagnetic mass differences between hadrons of the same isospin multiplet and radiative corrections to weak decays. In order to remove such difficulties, which are usually in the form of logarithmic infinities, the simplest method is to assume⁶ the possible existence of a heavy photon field B_μ . One replaces, in all electromagnetic interactions, the usual zero-mass photon field A_μ by a complex field $A_\mu + iB_\mu$ where B_μ is of negative metric and mass m_B .

The electromagnetic interaction is, then,

$$H_{\gamma} = e j_{\mu} (A_{\mu} + iB_{\mu}) \quad (1)$$

where j_{μ} is the usual electromagnetic current. The free propagator of the modified photon field $(A_{\mu} + iB_{\mu})$ is

$$\frac{1}{k^2} - \frac{1}{k^2 + m_B^2}$$

which is $O(k^{-4})$ as $k^2 \rightarrow \infty$.

1. Neutron-proton mass difference⁸

As a first application, we may consider the mass difference δm between the neutron and the proton. In the present theory, δm is given by (to first order in the fine structure constant α)

$$\delta m = m_n - m_p = \frac{\alpha}{8\pi^3} \int \frac{m_B^2 d^4 q}{q^2 (q^2 + m_B^2)} \left[(T_{\mu\mu})_n - (T_{\mu\mu})_p \right] \quad (2)$$

where $(T_{\mu\nu})_n$ and $(T_{\mu\nu})_p$ are, respectively, the forward Compton scattering amplitude of n and of p . In the following, we shall assume, apart from the above change in the photon propagator, the validity of the usual Cottingham formula⁹ which relates the difference $(T_{\mu\mu})_n - (T_{\mu\mu})_p$ to the corresponding difference between the structure functions in en and ep scatterings. As we shall see, this assumption leads to some definite conditions on the structure functions, which can be tested experimentally.

Let k_μ be the 4-momentum of the nucleon and q_μ that of the virtual photon. It is convenient to express $(T_{\mu\nu})_n - (T_{\mu\nu})_p$ in terms of two scalar functions t_1 and t_2 which depend only on the invariants q^2 and $k \cdot q$:

$$\begin{aligned} (T_{\mu\nu})_n - (T_{\mu\nu})_p = & i [q^2 \delta_{\mu\nu} - q_\mu q_\nu] t_1 \\ & + i m_N^{-2} [(k \cdot q)^2 \delta_{\mu\nu} - (k \cdot q) (k_\mu q_\nu + q_\mu k_\nu) + q^2 k_\mu k_\nu] t_2 \end{aligned} \quad (3)$$

where m_N is the nucleon mass. Following Cottingham⁹, one may first perform the Wick rotation in the complex $(k \cdot q)$ plane, keeping all other components of q_μ fixed. In the integral (2), q^2 becomes then restricted only to real and positive values, and $(k \cdot q)$ is purely imaginary. The values of t_1 and t_2 at such imaginary values of $(k \cdot q)$ are then assumed to be given by the dispersion integral (at fixed real positive q^2)

$$t_i(q^2, k \cdot q) = [t_i]_{\text{pole}} + \int_{v_0}^{\infty} [v^2 - m_N^{-2} (k \cdot q)^2 - i\epsilon]^{-1} 2v I_i dv \quad (4)$$

where $\epsilon = 0+$, the lower limit v_0 is related to q^2 by

$$v_0 = (2m_N)^{-1} (q^2 + 2m_N m_\pi + m_\pi^2) \quad (5)$$

$[t_i]_{\text{pole}}$ refers to the nucleon pole contribution¹⁰ and I_1, I_2 are functions of q^2 and v , related to the difference of the usual structure functions W_1 and W_2 in inelastic ep and en scatterings¹¹ by

$$I_1 = q^{-2} [(W_1 - q^{-2} v^2 W_2)_n - (W_1 - q^{-2} v^2 W_2)_p] \quad (6)$$

and

$$I_2 = q^{-2} [(W_2)_n - (W_2)_p] , \quad (7)$$

It is useful to separate out the longitudinal parts $(W_L)_n$ and $(W_L)_p$:

$$(W_L)_N \equiv [1 + q^{-2} v^2] (W_2)_N - (W_1)_N \quad (8)$$

where the subscript N can be either n or p . Eq. (8) may then be written as

$$I_1 = q^{-2} [(W_2)_n - (W_2)_p] - q^{-2} [(W_L)_n - (W_L)_p] . \quad (9)$$

Experimentally, W_2 is most easily measurable, while W_L is the least accessible quantity; Eq. (9) is therefore a more convenient expression to use than Eq. (6).

At present, there exist only some measurements¹¹ on $(W_2)_n - (W_2)_p$ over a fairly wide range of q^2 and v , but the difference $(W_L)_n - (W_L)_p$ is as yet unknown.

By using Eqs. (3), (4), (7) and (9), one can readily decompose the integral (2) for δm into a linear sum of three terms:

$$\delta m = (\delta m)_{\text{pole}} + (\delta m)_{W_2} + (\delta m)_{W_L} \quad (10)$$

in which these three terms $(\delta m)_{\text{pole}}$, $(\delta m)_{W_2}$ and $(\delta m)_{W_L}$ depend, respectively, only on $(t_i)_{\text{pole}}$, $(W_2)_n - (W_2)_p$ and $(W_L)_n - (W_L)_p$. Among these, the single nucleon-pole contribution to the mass difference has been calculated by Cini, et al.¹⁰; they obtained

$$(\delta m)_{\text{pole}} \cong -0.66 \text{ MeV} \quad (11)$$

which, by itself, would make the proton heavier than the neutron.

To analyse the remaining two terms $(\delta m)_{W_2}$ and $(\delta m)_{W_L}$, it is useful to introduce in the inelastic eN scattering the usual scaling variable ω and the invariant mass M of the final hadron system:

$$\omega = 2m_N v/q^2 \quad (12)$$

and

$$2m_N v = q^2 + M^2 - m_N^2 . \quad (13)$$

The recent SLAC data¹¹ show that at finite M^2 and q^2 , so long as M is $\gtrsim 2$ GeV, the difference $(\nu W_2)_p - (\nu W_2)_n$ satisfies, at least approximately, the scaling property, i. e.,

$$(\nu W_2)_p - (\nu W_2)_n \cong F_2(\omega) . \quad (14)$$

For $M < 2$ GeV, the most important πN resonance is the $M \cong 1236$ MeV $(\frac{3}{2}, \frac{3}{2})$ state. For electro-production, isospin conservation requires that only the $|\Delta \vec{I}| = 1$ electromagnetic current operator has a non-zero matrix element, and charge symmetry implies that the corresponding structure function for the proton must be the same as that for the neutron; therefore, the $(\frac{3}{2}, \frac{3}{2})$ resonance state does not lead to any mass difference between n and p . It seems, then, reasonable to expect that the integrals for $(\delta m)_{W_2}$ and $(\delta m)_{W_L}$ should be dominated by the relatively higher mass region $M \gtrsim 2$ GeV.

Let us first discuss the evaluation of $(\delta m)_{W_2}$ by assuming (14) to be a good approximation in the entire region of $\omega \gtrsim 1$ and $M \gtrsim 2$ GeV. One would like to find the answers to the following three questions:

- (i) Is the relevant dispersion integral convergent [i.e., the convergence of the part of (4) that depends only on $(W_2)_p - (W_2)_n$] ?
- (ii) Is the subsequent integration over d^4q for $(\delta m)_{W_2}$ convergent?
- (iii) Is the sign of $(\delta m)_{W_2}$ positive?

As we shall see, the answers to these three questions are all affirmative.

Firstly, we note that the relevant dispersion integral is convergent if $F_2(\omega)$ is finite; this is certainly consistent with the present experimental data. Any reasonable extrapolation from the available SLAC data¹¹ would lead to $F_2(\omega)$ to be finite and small (perhaps zero). It can then be readily verified that because of the modified photon propagator the subsequent d^4q integral is also convergent. The resulting $(\delta m)_{W_2}$ has a logarithmic dependence on the heavy photon mass m_B . For simplicity, we give here only the explicit expression of $(\delta m)_{W_2}$ for $m_B \gg m_N$:

$$(\delta m)_{W_2} = (2\pi)^{-1} 3\alpha m_N \ln(m_B/m_N) \int_1^\infty \omega^{-2} F_2(\omega) d\omega + O(1) \quad (15)$$

where the $O(1)$ term remains finite in the limit $m_B \rightarrow \infty$. The present SLAC data indicate that $F_2(\omega)$ is positive, and at $\omega \cong 12$, its value may have already decreased to $\cong 0$. The approximate value of the integral in (15) may then be estimated:

$$\int_1^\infty \omega^{-2} F_2(\omega) d\omega \sim \int_1^{12} \omega^{-2} F_2(\omega) d\omega \sim +0.05 \quad (16)$$

Thus, $(\delta m)_{W_2}$ is finite and positive. Recalling that $\delta m = m_n - m_p$, one sees that the fact that $(W_2)_p$ is larger than $(W_2)_n$ yields a positive contribution to the mass difference $m_n - m_p$, in contrast to the sign of the single nucleon pole contribution $(\delta m)_{\text{pole}}$. The precise value of $(\delta m)_{W_2}$ depends, of course, on the heavy photon

mass m_B . [In the conventional quantum electrodynamics, without the existence of B^0 , $(\delta m)_{W_2}$ would be logarithmically divergent.]

Next, we consider the evaluation of $(\delta m)_{W_L}$. From the existing inelastic ep scattering data, it is known that $(W_L)_p$ is quite small and, for large ν values, the scaling law is approximately applicable to $(W_L)_p$. However, at present, nothing is known about $(W_L)_n$. For definiteness, we shall assume that it is a good approximation to represent both $(W_L)_p$ and $(W_L)_n$ in the entire region of $\omega \geq 1$ and $M \geq$ a few GeV by ($N = n$ or p)

$$(W_L)_N = [F_L(\omega)]_N + \nu^{-1} [F'_L(\omega)]_N ; \quad (17)$$

$[F_L(\omega)]_N$ is then the value of $(W_L)_N$ in the scaling limit [i. e., $\nu \rightarrow \infty$ at fixed ω], and $\nu^{-1} [F'_L(\omega)]_N$ denotes the remainder.

In order to have the relevant dispersion integral convergent, we must require, assuming that $[F'_L(\infty)]_N$ is finite,

$$[F_L(\infty)]_n = [F_L(\infty)]_p . \quad (18)$$

Furthermore, in order to have the subsequent d^4q integration convergent, we require the stronger condition that $(W_L)_p \rightarrow (W_L)_n$ in the scaling limit; i. e.,

$$[F_L(\omega)]_p = [F_L(\omega)]_n . \quad (19)$$

Such a condition can be satisfied in many specific parton models¹² [e. g., by constructing models in which $F_L(\omega) = 0$ for both n and p ; such models are, of course, consistent with the limited amount of presently available experimental information]. Eqs. (17) and (19)

then imply that, for $M \geq$ a few GeV, it is a good approximation to represent the difference $(W_L)_p - (W_L)_n$ by

$$(\nu W_L)_p - (\nu W_L)_n = [F'_L(\omega)]_p - [F'_L(\omega)]_n \equiv F'_L(\omega) . \quad (20)$$

It is then straightforward to verify that the resulting $(\delta m)_{W_L}$ is finite, and it has a logarithmic dependence on m_B . For $m_B \gg m_N$, one has, similarly to (15),

$$(\delta m)_{W_L} = -(3\alpha/\pi) m_N \ln(m_B/m_N) \int_1^\infty \omega^{-2} F'_L(\omega) d\omega + 0(1) . \quad (21)$$

[Again, in the conventional quantum electrodynamics, $(\delta m)_{W_L}$ would be infinite¹³.]

We emphasize that the validity of (19) can be directly tested by further experiments. As shown by (15), the result that $(\delta m)_{W_2}$ is positive encourages our supposition that the observed mass difference $m_n - m_p$ may be simply calculable by assuming the existence of the heavy photon and the validity of the Cottingham formula, but without further subtraction constants¹⁴ for the dispersion integral (4). In such a case, if m_B is known, then one can predict a value for the integral in (21), which can then be compared with further experimental results. For example, if m_B is arbitrarily set at ~ 40 GeV, then Eqs. (15) and (16) indicate that the order of magnitude of $(\delta m)_{W_2}$ is given by

$$(\delta m)_{W_2} \approx +0.6 \text{ MeV} . \quad (22)$$

By using the observed value of $m_n - m_p$, one expects $F'_L(\omega)$ to satisfy

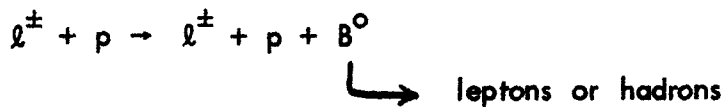
$$\int_1^\infty \omega^{-2} F'_L(\omega) d\omega \approx -0.1 . \quad (23)$$

Conversely, if $F_L^1(\omega)$ is known and is, indeed, of the desired sign, then one can predict the heavy boson mass m_B , based on the observed mass difference $m_n - m_p$.

2. Production of B^0

The heavy photon B^0 , if it exists, can be produced by any process involving the electromagnetic interaction. The unitarity of the S-matrix is insured by the property that B^0 is unstable; it can decay into either a lepton pair or other hadron modes. In the S-matrix, both its initial and final states consist only of the usual positive metric stable particles. The existence of B^0 can be observed through resonance phenomena just like any other unstable particles [except that B^0 is represented by a complex pole on the "first sheet", instead of the usual pole on the "second sheet"].

The production of B^0 by high energy charged leptons has been recently calculated by R. Linsker¹⁵. The cross-section for the production



is given in the Figure. If m_B is not much heavier than 10 GeV, then one may expect to see its production at NAL in the near future.

At present, one knows that $m_B > 5$ GeV from the $(g - 2)$ measurement¹⁶ of the muon and from the lepton pair production in the $p + \text{uranium}$ experiment¹⁷. If one assumes the usual scaling properties of the structure functions due to the strong interaction, the presence of the heavy photon would introduce a change in the scaling properties of the observed W_1 and νW_2 functions:

$$[W_1]_N = \left(\frac{m_B^2}{q^2 + m_B^2} \right)^2 [F_1(\omega)]_N$$

and

$$[vW_2]_N = \left(\frac{m_B^2}{q^2 + m_B^2} \right)^2 [F_2(\omega)]_N .$$
(24)

From the absence of such a correction term $[m_B^2/(q^2 + m_B^2)]^2$ in the present SLAC data¹¹, one may then deduce indirectly

$$m_B \gtrsim 9 \text{ GeV} .$$
(25)

II. Applications to Weak Interaction¹⁸

Let the Lagrangian density of the semiweak interaction be given by

$$\mathcal{L}_{wk} = g J_{\mu} W_{\mu} + \text{adjoint}$$

where J_{μ} is the usual weak interaction current. All presently observed weak transitions are second order in g^2 , transmitted by the covariant W-propagator

$$D_{\mu\nu}(k^2) = \delta_{\mu\nu} \int \frac{\sigma_1 dM}{k^2 + M^2} + k_{\mu} k_{\nu} \int \frac{(\sigma_1 + \sigma_0) M^{-2} dM}{k^2 + M^2} \quad (26)$$

In order that such a theory may correspond to a renormalizable one, we require

$$\int (\sigma_1 + \sigma_0) M^{-2} dM = 0 \quad (27)$$

The simplest solution is

$$\sigma_1 = \pm \delta(M - m_1)$$

and

$$\sigma_0 = \mp (m_0/m_1)^2 \delta(M - m_0) ; \quad (28)$$

therefore,

$$D_{\mu\nu}(k) = \pm \left[\frac{\delta_{\mu\nu}}{k^2 + m_1^2} + \frac{k_{\mu} k_{\nu}}{m_1^2} \left(\frac{1}{k^2 + m_1^2} - \frac{1}{k^2 + m_0^2} \right) \right] \quad (29)$$

which corresponds to a charged spin 1 boson W_1^{\pm} of mass m_1 and a charged spin 0 boson W_0^{\pm} of mass m_0 . These two bosons are of opposite metric. The upper signs in (28) and (29) imply that W_1^{\pm} is of positive metric and W_0^{\pm} of negative metric, and the lower signs imply the opposite.

The masses m_0 and m_1 are independent parameters of the theory. We note that if, among these masses, the lowest one is of spin 1, then, as is well known, its decay into leptons offers a particularly useful tool in the experimental search for intermediate bosons. However, if the intermediate boson with the lowest mass is of spin 0, and if the mass difference $(m_1 - m_0)$ is not too small, then the electromagnetic transition

$$W_1^\pm \rightarrow W_0^\pm + \gamma \quad (28)$$

may become the dominant decay mode of W_1^\pm . Furthermore, if one neglects charged lepton masses as compared to m_0 , then

$$W_0^\pm \not\rightarrow \ell^\pm + \nu_\ell \text{ (or } \bar{\nu}_\ell \text{)} , \quad (29)$$

and therefore W_0^\pm decays mainly only into hadrons. In such a case, an effective way to search for these bosons would be through their non-leptonic decay modes, rather than through their leptonic decay modes. In order to observe W_0^\pm (or W_1^\pm), one may consider, for example,

$$\nu_\mu + p \rightarrow \mu^- + p + W_J^\pm \quad (30)$$

where the subscript J can be either 0 or 1. The cross-section for W_1^\pm -production has been discussed extensively in the literature¹⁹. The cross-section for W_0^\pm -production has been calculated recently²⁰, and it is found to be more than an order of magnitude smaller than that of W_1^\pm -production in almost all cases of physical interest.

Another interesting consequence of indefinite metric is that the sign of the usual

weak interaction Fermi constant G may now be positive or negative, depending on whether W_1^\pm is of positive or negative metric. In principle, the sign of the Fermi constant can be measured by observing interference terms between, say, strong and weak interactions, such as the parity-violation experiments in nuclear γ transition²¹. However, at present, our knowledge of strong interactions has not advanced to a level to make this determination unambiguous.

In this connection, it may be worthwhile to emphasize that the sign of the Fermi constant G is a well-defined physical quantity in the context of the usual (current \times current) theory, independently of whether intermediate bosons exist or not. If one assumes the existence of intermediate bosons, then the sign of G becomes connected with the metric of the spin 1 intermediate boson, as is discussed above.

References

1. P. A. M. Dirac, Proc. Roy. Soc. (London) A180, 1 (1942).
2. S. N. Gupta, Proc. Phys. Soc. 63, 681 (1950); 64, 850 (1951).
3. K. Bleuler, Helv. Phys. Acta 23, 567 (1950).
4. R. P. Feynman, Phys. Rev. 76, 749 (1949).
5. T. D. Lee and G. C. Wick, Nucl. Phys. B9, 209 (1969); B10, 1 (1969).
6. T. D. Lee and G. C. Wick, Phys. Rev. D2, 1033 (1970); D3, 1046 (1971).
7. R. E. Cutkosky, P. V. Landshoff, D. Olive and J. C. Polkinghorne, Nucl. Phys. B12, 281 (1969).
8. The discussion of this section is based on some unpublished results of work done in collaboration with G. C. Wick.
9. W. N. Cottingham, Ann. Phys. (N.Y.) 25, 424 (1963).
10. There is an ambiguity as to whether the single nucleon pole term $(t_1)_{\text{pole}}$ should be exactly given by the pole factor $[(q^2)^2 - 4(k \cdot q)^2]^{-1}$ times the appropriate residue in the $(k \cdot q)$ plane, or one that is evaluated by using a phenomenological Feynman diagram technique, such as the calculation made by Cini et al., Phys. Rev. Lett. 2, 7 (1959). These two expressions are, in principle, quite different, since by using the Feynman diagram for the single nucleon intermediate state, one would obtain a $(t_1)_{\text{pole}}$ term which does not approach zero as $k \cdot q \rightarrow \infty$ at fixed q^2 . Thus, the Feynman diagram method leads to a final expression which, in the language of the usual dispersion theory, would correspond to a dispersion integral (4) for t_1 (but not for t_2) that has an explicitly calculable subtraction constant; this subtraction constant is a function of q^2 and is proportional to the

difference between the square of the anomalous magnetic moment of the proton and that of the neutron, which is, however, quite small. Both methods, therefore, result in an almost identical numerical value for $(\delta m)_{\text{pole}}$, given by Eq. (11) below.

11. For a summary of the present experimental status, see E. D. Bloom et al., to be published in the Proceedings of the XV International Conference on High Energy Physics, Kiev, 1970. The W_1 and W_2 functions are the same as those used by Bloom et al. The W_L function used here is the same as $R W_1 = (\sigma_S/\sigma_T) W_1$ in the paper by Bloom et al.
12. See, e.g., J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).
13. The (unlikely) possibility remains that the integral $\int_1^{\infty} \omega^{-2} F_2(\omega) d\omega$ may happen to be exactly equal to $2 \int_1^{\infty} \omega^{-2} F'_L(\omega) d\omega$, in which case the sum $(\delta m)_{W_2} + (\delta m)_{W_L}$ would be finite even in the conventional quantum electrodynamics. See the discussions by Jackiw, Van Royen and West, Phys. Rev. D2, 2473 (1970). Cf. also H. Pagels, Phys. Rev. 185, 1990 (1969).
14. In the literature, there have been suggestions that perhaps these dispersion integrals should be divergent and therefore additional subtractions are needed. [See, e.g., H. Harari, Phys. Rev. Lett. 17, 1303 (1966); W. N. Cottingham, Proceedings of the Scottish Universities Summer School, 1970 (Academic Press); H. Goldberg and Y. N. Scrivera, Northeastern University preprint.] Our present analysis is based on the assumption that this is not the case.
15. R. Linsker, Phys. Rev. Lett. (to be published), Columbia Report NYO-1932(2)-198.
16. J. Bailey et al., Phys. Lett. 28B, 287 (1968); see also E. Picasso, in Proceedings of the Third International Conference on High-Energy Physics and Nuclear Structure (Plenum, New York, 1970), p. 615.

17. J. Christenson, G. Hicks, P. Limon, L. M. Lederman, B. Pope, and E. Zavattini, Phys. Rev. Lett. 25, 1523 (1970).
18. For further details see T. D. Lee, Phys. Rev. Lett. 25, 1144 (1970).
19. T. D. Lee, P. Markstein and C. N. Yang, Phys. Rev. Lett. 7, 429 (1961).
J. S. Bell and M. Veltman, Phys. Lett. 5, 94 (1963). C. T. Wu et al., Phys. Rev. Lett. 12, 57 (1964). R. W. Brown, A. K. Mann and J. Smith, Phys. Rev. Lett. 25, 257 (1970).
20. R. Linsker, Phys. Rev. Lett. (to be published), Columbia Report NYO-1932(2)-199.
See also J. Reiff, Nucl. Phys. B28, 495 (1971).
21. V. M. Lobashov et al., Zh. Eksp. Teor. Fiz., Pis'ma Red. 5, 73 (1967) [JETP Lett. 5, 59 (1967)].

Figure Caption

B^0 -production cross-section in $l^{\pm}p$ collisions for $m_B = 7$ and 10 GeV [calculated by R. Linsker¹⁵].

