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AN INTERFEROMETRIC DISPLACEMENT
TRANSDUCER

G. Wayne Dutton

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AN INTERFEROMETRIC DISPLACEMENT TRANSDUCER

G. Wayne Dutton

Abstract. An interferometric displacement transducer capable of measuring linear displacement up to 1.5 inches with an accuracy of about 6×10^{-6} inches is described. The transducer employs a modified Michelson polarization interferometer with the movable mirror mounted on a stylus shaft supported by a piston-controlled cylindrical air bearing. Details of the air bearing, air-control system, interferometer, and fringe detection system are given. Fringe-to-inch conversion and error analysis are discussed.

INTRODUCTION

In the field of dimensional metrology, a need exists for high-resolution displacement transducers with sufficient ranges to eliminate the use of the *stepping procedure*¹ currently employed with conventional inductive² or capacitive gauge heads³ of small range. From the standpoint of accuracy and efficiency, linear measurements in one continuous operation have preference.

Commercially-available laser interferometer measuring systems⁴ could cover a range of a few inches, but their movable element, a corner-cube (triangular) reflector, is of necessity too large and massive to be used as an orientable gauge head in physical contact with an object being inspected.

As a result, a slide must be furnished to support and transport the corner cube relative to the rest of the interferometer. In addition, a null indicator is usually required to establish a reference on the object being inspected.

¹The term *stepping procedure* relates to the process whereby a small-range transducer is repositioned to zero by means of an independent micrometer when the maximum range has been reached.

²For example, the contacting Electrojet ® of Bendix Corporation, Automation and Measurement Division, Dayton, Ohio.

³For example, the noncontacting Dynagage ® of Whittaker Corporation, North Hollywood, California.

⁴For instance, the systems manufactured by OPTOMECHANISMS, Incorporated, Long Island, New York; Hewlett-Packard, Palo Alto, California.

Except for certain optical and mechanical modifications, the transducer described has similarities to one reported on by the author in 1968.⁵ The current optical transducer is a small compact unit capable of inspecting the surface contour of an object, as well as being able to continuously measure a direct linear displacement of up to 1.5 inches (1 inch = 2.54 centimeters). Its inherent accuracy has the same capability as that of a commercial laser interferometer system, if a light source of similar wavelength were used. A conventional monochromatic light source (e.g., mercury-198 or krypton-86) could be used as long as its coherence length and intensity are sufficient to give good interference fringes over a 1.5-inch path difference. However, because of modest cost and ease of operation, a 3-milliwatt helium-neon laser⁶ is used to obtain a maximum continuous interferometric path difference of about 4 inches (10 centimeters).

DISCUSSION

Air Bearing:

A cross-sectional view of the gauge head is shown in Figure 1. The gauging stylus (a) is a cylindrical shaft (runner) supported by a grooved porous-carbon cylindrical bearing pad (b). Air pressure of 50 pounds per square inch (psi) or 350 kilopascals is introduced at the outer surface of the porous pad via a tube (c) and passes through to the shaft causing it to float with about a 0.0001-inch radial clearance. To improve the flotation qualities of the cylindrical air bearing, circumferential exhaust grooves were cut along the bearing pad. Four equally-spaced longitudinal grooves were also cut to connect the circumferential grooves. The assembly constitutes an air bearing with low friction and high stiffness. A transverse cross-sectional view of the bearing

⁵G. Wayne Dutton. "An Interferometric Displacement Transducer." 1968 *Instrument Society of America Conference*, Paper No. 68-565. New York. October 28-31, 1968.

⁶Model-122 Laser of Spectra-Physics, Incorporated, Mountain View, California.

FIGURE 1. Sectional View of Optical Transducer.

FIGURE 2. Cross-Sectional View of Air Bearing Showing Pressure Profile.

showing the longitudinal exhaust grooves and the pressure distribution is given in Figure 2. A 0.1563-inch diameter gauging ball is fixed to the outer end of the stylus shaft to act as the gauge-contacting element.

Since the moving reflector consists of a flat mirror and not a corner cube, the angle between the plane of this mirror and the optical axis must not change over the entire 1.5-inch range of the gauge stylus, to insure a constant fringe pattern. This requires a highly uniform and stiff air bearing. A bearing with sufficient accuracy and stiffness to prevent variances in the fringe pattern has been constructed.

In addition to air-bearing accuracy, the plane of the stylus mirror must be flat and perpendicular to the air-bearing axis for the generation of stable fringes of good contrast. Flat mirrors are easy to obtain, but if the stylus shaft has freedom to rotate, exigent perpendicularity is of prime importance and extremely difficult to obtain. Even though both detectors observe the same type of fringe pattern, it is still desirable for reasons of electronic detection to adjust the fringes for high-linear dispersion (i.e., for maximum spacing) and, hence, maximum light-level change. To achieve such fringe stability, the air-bearing stylus had to be keyed to prohibit rotation of the stylus mirror and thus eliminate the resulting variations in the fringe pattern caused by nonperpendicularity of the mirror and air-bearing axis. This

FIGURE 3. Air-Actuation System for Optical Transducer.

was accomplished by machining a flat on one side of the air-actuation piston and cementing a matching 1.5-inch long key in the cylinder. (Refer to next section.) Both key and flat were polished to eliminate drag with about 0.002-inch clearance. The clearance was sufficiently small to prevent any noticeable change in fringe spacing or orientation as the stylus moved.

Construction was completed ultimately of a gauge possessing good fringe stability, provided the laser input beam could be accurately aligned along the bearing axis. The fringe pattern remained unchanged for the full 1.5-inch travel of the stylus.

Air-Actuation System:

In order to extend and retract the gauging stylus, a piston-cylinder arrangement was incorporated at the mirror-end of the shaft, shown as (d) in Figure 1. An air duct connects the portion of the cylinder in front of the piston to an external actuation tube (e). A small pressure (about 30-millimeter water gauge) applied to the tube via a

flow restrictor (needle) valve will cause the shaft to retract at a constant rate. Conversely, a small vacuum of comparable negative gauge pressure will extend the shaft. A diagram of the air-actuation system is shown in Figure 3. The rate at which the shaft extends or retracts is governed by the setting of the flow-restrictor valves. A toggle valve was installed to selectively extend or retract the gauging stylus.⁷ If the gauge is oriented other than horizontally, the flow valves can be adjusted to compensate for gravity. In any given gauge position, the valves were set so that the stylus speed did not exceed the maximum fringe counting rate of the photodetection circuitry as explained under *Electronics*, (see Page 8).

Optics:

Referring to Figure 1, the entering collimated beam (f) of monochromatic laser light is directed by the

⁷Initially, a third-flow restrictor valve was used to select the gauging tip pressure, but was not necessary: a convenient 10-gram static tip force resulted when the toggle valve was in the extend position with the flow valve set at about one inch-per-second stylus rate.

FIGURE 4. Diagram of Interferometer Showing Light Passage.

FIGURE 5. Polarization of the Beams in the Interferometer: A = Linear, B = Circular, and C = Combined Beam.

mirror (g) into the interferometer beamsplitting unit (h). Part of the light is reflected up to mirror (i) and part transmitted to the movable stylus mirror (j). (A detailed description of the various optical components in the beamsplitting unit and their functions in the interference phenomenon follows below.) Eventually, two beams containing the interference patterns exit at the bottom of the beamsplitting unit, pass through polarizing materials to achieve a 90-degree phase difference for directional counting and impinge upon the respective photodetectors (k). The photodetector holding fixture (l) is rotatable for signal adjustment.

Difficulty encountered in maintaining a constant 90-degree phase difference between the optical signals at the fringe photodetectors prompted a search for a phase-generation technique that would be independent of the relative spatial location of the photodetectors in the fringe pattern. One such method was discovered, proving to be extremely stable for long periods of time and simple in construction. It utilizes polarization effects on the laser light for phase-difference generation. With this method, both detectors view the entire fringe pattern which is adjusted so that only one fringe covers the field of view. The method also eliminates the necessity of using a beam-expanding telescope on the laser since the beam does not need to be any larger than an individual photodetector.

The various optical components and their relative positions for the polarization method are shown in Figure 4. In the actual system, some of the elements were cemented together for rigidity and convenience, but the basic principles remained unchanged (see Figure 1). Figure 5 shows the various kinds of polarization of the light beams in the interferometer. The monochromatic, linearly polarized laser light is divided, as usual, in the interference beamsplitter. The reflected portion goes to the reference mirror and reflects back to the beamsplitting interface still linearly polarized. The transmitted portion passes through an eighth-wave ($\lambda/8$) retardation plate thus becoming elliptically polarized, reflects back from the stylus mirror and again passes through the eighth-wave plate becoming fully polarized, circularly. The returning circularly-polarized beam then recombines (in the fashion of an ordinary Michelson interferometer) with the returning

linearly polarized reference beam. The combined beam, containing the fringe pattern, is then divided once more by the fringe-pattern beamsplitter, and the respective portions are sent through separate polarizing materials to the photodetectors. The 45 and 90-degree prism is present merely to divert the reflected beam parallel to the transmitted beam so that the detectors may be mounted parallel in a common fixture.

To understand the operation of the method of phase generation, the various polarized beams must be expressed as vectors in the complex plane. Refer to the polarization vector of the incident laser light as $\vec{\xi}_l(t)$, directed along the x axis, as shown in Figure 6. Since the laser is linearly polarized, $\vec{\xi}_l(t)$ is given by:

$$\begin{aligned} (Xi) \vec{\xi}_l(t) &= \text{Re} (a e^{i\omega t}) \\ &= a \cos \omega t \end{aligned} \quad (1)$$

In Equation 1:

$$\begin{aligned} \text{Re} &= \text{real part of} \\ a &= \text{plane-wave amplitude} \\ (\text{omega}) \omega &= \text{angular frequency} \\ t &= \text{time} \\ i &= \sqrt{-1} \end{aligned}$$

Equation 1 then represents the beam labeled as A (linear) in Figure 5. The circularly polarized beam (B of Figure 5) can then be represented by the vector, $\vec{\xi}_c(t)$, as shown in Figure 6.

$$\begin{aligned} \vec{\xi}_c(t) &= a e^{i(\omega t + \delta\phi)} \\ &= a \cos(\omega t + \delta\phi) \\ &\quad + i a \sin(\omega t + \delta\phi) \end{aligned} \quad (2)$$

In Equation 2, $\delta\phi$ (delta phi) is a phase difference caused by an arbitrarily assumed path difference in the interferometer. And $\delta\phi$, a function of the position of the gauging stylus and mirror, is given by:

$$(\text{delta-phi}) \delta\phi = \frac{4\pi}{\lambda} \delta D$$

FIGURE 6. Light Vector and Polaroid Orientations.

Here λ (lambda) is the laser wavelength and δD is one-half the optical path difference in the interferometer (i.e., δD is the distance the stylus has moved from the first-order fringe).

To find the combined beam C of Figure 5 [denoted by $\vec{\xi}(t)$], it is necessary to vectorially add $\vec{\xi}_l(t)$ and $\vec{\xi}_c(t)$:

$$\vec{\xi}(t) = \vec{\xi}_l(t) + \vec{\xi}_c(t)$$

In the complex plane, this is simply:

$$\vec{\xi}(t) = a \cos \omega t + a \cos(\omega t + \delta\phi) + i a \sin(\omega t + \delta\phi)$$

If the polarizing materials are now placed with their axes along the ± 45 -degree lines as shown by the unit vectors, Polaroid 1, $\hat{\eta}_1$, and Polaroid 2, $\hat{\eta}_2$, in Figure 6, it can be shown that the signals (σ_1 and σ_2) received at the respective photodetectors as the stylus moves will be out of phase by 90° . The vectors $\hat{\eta}_1$ and $\hat{\eta}_2$ should be orthonormal such that $\hat{\eta}_1 \cdot \hat{\eta}_2 = 0$. Hence,

$$(\eta) \hat{\eta}_1 = \frac{\sqrt{2}}{2} (1 + i) \text{ and}$$

$$\hat{\eta}_2 = \frac{\sqrt{2}}{2} (1 - i)$$

Then the light transmitted through Polaroid 1 (Figure 6) is represented by the projection of $\vec{\xi}$ onto $\hat{\eta}_1$, which is just the scalar product of these vectors:

$$\begin{aligned} \vec{\xi} \cdot \hat{\eta}_1 &= \xi_x \hat{\eta}_{1x} + \xi_y \hat{\eta}_{1y} \\ &= \frac{a\sqrt{2}}{2} [\cos \omega t + \cos(\omega t + \delta\phi) + \sin(\omega t + \delta\phi)] \quad (3) \end{aligned}$$

The intensity of the signal at Detector 1, (σ_1), in Figure 7, is then:

$$(\sigma) \sigma_1 = \text{avg} (\vec{\xi} \cdot \hat{\eta}_1)^2$$

In the above, *avg* means *time-average value of* which can be represented by the integral quotient:

FIGURE 7. Plot of Signal Intensities (σ_1 and σ_2) Showing the 90-Degree Phase Difference.

$$\sigma_1 = \frac{\int_0^{2\pi} (\vec{\xi} \cdot \hat{\eta}_1)^2 d(\omega t)}{\int_0^{2\pi} d(\omega t)} \quad (4)$$

Combining Equations 3 and 4 and integrating, Equation 5 gives:

$$\sigma_1 = \frac{a^2}{2} \left(\frac{3}{2} + \cos \delta \phi + \sin \delta \phi \right) \quad (5)$$

Similarly, for the signal at Detector 2, (σ_2) in Figure 7, the equation becomes:

$$\begin{aligned} \vec{\xi} \cdot \hat{\eta}_2 &= \xi_x \hat{\eta}_{2x} + \xi_y \hat{\eta}_{2y} \\ &= \frac{a\sqrt{2}}{2} [\cos \omega t + \cos(\omega t + \delta \phi) - \sin(\omega t + \delta \phi)] \end{aligned} \quad (6)$$

Then,

$$\begin{aligned} \sigma_2 &= \text{avg} (\vec{\xi} \cdot \hat{\eta}_2)^2 \\ &= \frac{\int_0^{2\pi} (\vec{\xi} \cdot \hat{\eta}_2)^2 d(\omega t)}{\int_0^{2\pi} d(\omega t)} \end{aligned} \quad (7)$$

Combining Equations 6 and 7 and integrating, as before, the result is:

$$\sigma_2 = \frac{a^2}{2} \left(\frac{3}{2} + \cos \delta \phi - \sin \delta \phi \right) \quad (8)$$

It can now be shown that σ_1 is 90° out of phase with σ_2 by adding $\pi/2$ to the argument of Equation 5:

$$\begin{aligned} \sigma_1 \left(\delta \phi + \frac{\pi}{2} \right) &= \left[\frac{3}{2} + \cos \left(\delta \phi + \frac{\pi}{2} \right) + \sin \left(\delta \phi + \frac{\pi}{2} \right) \right] \\ &= \frac{a^2}{2} \left(\frac{3}{2} + \cos \delta \phi - \sin \delta \phi \right) = \sigma_2(\delta \phi) \end{aligned}$$

Thus in Figure 7, the signal at Detector 1 is 90° out of phase with the signal at Detector 2, which was the desired result of this optical system. Figure 7 is a superimposed graph of σ_1 (Equation 5) and σ_2 (Equation 8).

The problem still remains of determining the contrast of the fringes in the interferometer. The best quantitative indication of the contrast is given by the following ratio, defined as fringe visibility, V :⁸

$$V = \left(\frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{\sigma_{\text{max}} + \sigma_{\text{min}}} \right) \quad (9)$$

In the present interferometer:

$$\sigma_{\text{maximum}} = (5.83) \frac{a^2}{4}$$

$$\sigma_{\text{minimum}} = (0.17) \frac{a^2}{4}$$

$$\text{Hence, } V = 0.94$$

Note that the maximum possible value of V given by Equation 9 is unity.

⁸A. A. Michelson. *Studies in Optics*. University of Chicago Press, Chicago, Illinois. 1927. Page 31.

Legend

- Resistors — Units in ohms (Ω), K = 1000, M = mega (10^6).
 Capacitors — Units in picofarads (pf) or microfarads (μ f).
 BNC — Cable connector.
 V — Volts.
 W — Watts.

FIGURE 8. Photodetector-Preamplifier Circuit.

Electronics:

As indicated earlier, the two photodetectors⁹ are located such that each observes one of the interference patterns generated by the fringe-pattern beamsplitter. They are mounted in a common holding fixture (see Figure 1) with small polarizing windows cemented immediately in front of the separate detectors. Each detector and polarizing unit can be rotated about the optical axes to obtain the correct 90° phase difference. Then with suitable logic circuitry, the direction of motion of the gauging stylus can be determined. For example, if a particular fringe passes over Detector 1 before Detector 2, corresponding to an extension of the stylus, the fringe counter will subtract; and if

it passes 2 before 1 (shaft retracting), the counter will add.

A diagram of the photodetector amplifier circuit is shown in Figure 8. The displacement transducer uses one such amplifier for each photodetector. The amplifiers were designed so that they were driven into saturation by a bright (reinforcement) fringe and were cut off by a dark (cancellation) fringe. The DC voltage was adjusted so that the electronic transition occurred when the corresponding detector saw the average light intensity of a bright and dark fringe.

Since the two detector signals were adjusted for quadrature, each quarter-fringe movement of the stylus induced a logic transition in one or the other amplifiers. The system was thus able to

⁹Fotofet®-600 made by Crystalonics, Incorporated, Cambridge, Massachusetts.

resolve a minimum displacement of $1/4 \times \lambda/2 \approx 3 \times 10^{-6}$ inches, where λ is the wavelength of the laser light.

The outputs of the detector amplifiers were fed into a bidirectional counter¹⁰ which registered the quarter-fringe pulse count on a six-integer readout. The maximum pulse-counting rate depends on the maximum frequency response of the photodetector and counter circuitry. This was about 800 kilohertz (kHz). In actual operation, the stylus shaft was moved with the air-actuation system at about one inch per second (320 kHz).

Fringe-to-Inch Conversion:

Fringe to inch conversion was initially performed manually on a desk calculator. Eventually, this was accomplished automatically and continuously by a small computer executing binary multiplication. The derivation and discussion of the actual fringe-to-inch multiplier follows.

The number of inches per interference fringe, $(\lambda)/2$, for the interferometer depends on the wavelength of the helium-neon laser light traversing the air path between the beamsplitter and the movable mirror of the stylus. The wavelength then depends on the index of refraction, and the index, in turn, depends on the temperature, pressure, and water vapor content of the air.¹¹

The particular gauge described herein was used in an environmentally controlled room where the temperature was held at 20 ± 1 °C, and the water-vapor pressure (humidity control) at 5 ± 1 millimeters of mercury (mm Hg). Air pressure was the only variable that was not controlled. It varied as 613 ± 18 mm Hg over the course of a year.¹² A mercury barometer was read prior to each use of the gauge. The correct conversion factor was then determined and fed into the computer as a constant multiplier.¹³ The error caused by wavelength uncertainty was kept arbitrarily to less

than the resolution of the gauge (i.e., less than $\pm 3 \times 10^{-6}$ inches) over the entire 1.5 inches of travel, as noted below.

Using an observed average air pressure at the Plant location of 613 mm Hg (at 20 °C and 5 mm Hg water-vapor pressure), the laser wavelength is found to be:

$$(\lambda) \lambda = 0.63285292 \text{ micrometers} \quad (10)$$

The figure corresponds to a fringe interval of:

$$\frac{\lambda}{2} = \frac{0.63285292 \times 10^{-4}}{2 \times 2.54} = 12.45773 \times 10^{-6} \text{ inches} \quad (11)$$

Or, a quarter-fringe interval of:

$$\frac{\lambda}{8} = 3.11443 \times 10^{-6} \text{ inches} \quad (12)$$

If no changes were made in the conversion factor because of environmental fluctuations, the value of $\lambda/8$ in Equation (12) would be the best choice. However, the error induced in a length measurement by using a constant factor could conceivably far exceed the resolution of the transducer, as discussed in the following section.

Error Analysis:

In addition to the effects of environment on the laser's vacuum wavelength, a small error also exists in the effective vacuum wavelength itself. The error occurred because the laser used with the optical transducer possessed, simultaneously, three closely spaced wavelengths in its output beam, each separated by a small frequency difference of 517 megahertz (MHz). The maximum magnitude of this error in a 1.5-inch travel of the gauging stylus is $\pm 0.25 \times 10^{-6}$ inches.

Within the experimental environment, the variations in wavelength per unit wavelength of small changes in the average water-vapor pressure (f), temperature (t), and air pressure (p), respectively, are:¹⁴

¹⁰ Janus Control Corporation, Waltham, Massachusetts.

¹¹ Variations in other constituents of the air (carbon dioxide, etc.) were neglected.

¹² The Rocky Flats Plant in Colorado is at 6000 feet of elevation.

¹³ See Appendix A, "Environmental Effects on Wavelength,"

Page 11.

¹⁴ The coefficients in Equation 13 were derived from the tables referred to in Appendix A. The units are °C and millimeters of mercury throughout.

$$\left(\frac{\Delta\lambda}{\lambda}\right)_f = \Delta f (0.057) \times 10^{-6} \quad (13a)$$

$$\left(\frac{\Delta\lambda}{\lambda}\right)_t = \Delta t (0.747) \times 10^{-6} \quad (13b)$$

$$\left(\frac{\Delta\lambda}{\lambda}\right)_p = \Delta p (-0.356) \times 10^{-6} \quad (13c)$$

Since the uncertainty in wavelength per unit wavelength is the same as the uncertainty in a measured length (L) per unit length, then:

$$\frac{\Delta L}{L} = \frac{\Delta\lambda}{\lambda} \quad (14)$$

The noted variations can be used in the environmental conditions to predict the corresponding accumulated measurement errors, ΔL , over a distance of $L = 1.5$ inches:

$$\begin{aligned} (\Delta L)_f &= \left(\frac{\Delta\lambda}{\lambda}\right)_f \times L \quad (15a) \\ &= 0.057 \times 10^{-6} \times \Delta f \times L \\ &= 0.057 \times 10^{-6} \times (\pm 1 \text{ mm Hg}) \\ &\quad \times 1.5 \text{ inches} \\ &= \pm 0.09 \times 10^{-6} \text{ inches} \end{aligned}$$

$$\begin{aligned} (\Delta L)_t &= \left(\frac{\Delta\lambda}{\lambda}\right)_t \times L \quad (15b) \\ &= 0.747 \times 10^{-6} \times \Delta t \times L \\ &= 0.747 \times 10^{-6} \times (\pm 1^\circ \text{C}) \\ &\quad \times 1.5 \text{ inches} \\ &= \pm 1.12 \times 10^{-6} \text{ inches} \end{aligned}$$

$$\begin{aligned} (\Delta L)_p &= \left(\frac{\Delta\lambda}{\lambda}\right)_p \times L \quad (15c) \\ &= -0.356 \times 10^{-6} \times \Delta p \times L \\ &= -0.356 \times 10^{-6} \times (\pm 18 \text{ mm Hg}) \\ &\quad \times 1.5 \text{ inches} \\ &= \pm 9.61 \times 10^{-6} \text{ inches} \end{aligned}$$

Hence, the water-vapor and temperature variations individually cause errors less than the ± 3 -microinch resolution of the system. However, the air-pressure error indicates that a fringe-to-inch multiplication factor must be used that varies with air pressure such that the total error caused by environmental changes will be less than $\pm 3 \times 10^{-6}$ inches. Since the combined error caused by Δf , Δt and the inherent multimode laser error of $\pm 0.25 \times 10^{-6}$ inches are about $\pm 1.46 \times 10^{-6}$ inches, the error caused by Δp must be limited to less than $\pm 1.54 \times 10^{-6}$ inches. Then the greatest Δp to be tolerated before a corrected multiplier can be inserted is given by Equation 15c:

$$\pm 1.54 \times 10^{-6} = 0.356 \times 10^{-6} \times \Delta p_{\max} \times 1.5$$

Or,

$$\Delta p_{\max} = \pm 2.88 \text{ millimeters mercury} \quad (16)$$

During the actual operation of the gauge, a new multiplier is used for each ± 2 mm Hg change in air pressure as indicated by the barometer. A data table, giving the correct multiplier for each 2 mm Hg air-pressure increment from 595 to 631 mm Hg at 20°C and 5 mm Hg vapor pressure; was made for use as a reference by the authors.

CONCLUSIONS

From the analyses, it is apparent that the error caused by environmental wavelength fluctuations will be less than $\pm 3 \times 10^{-6}$ inches over the entire 1.5-inch range of the gauge stylus. In addition, as noted under *Electronics*, the position of the stylus is resolved to an accuracy of about $\pm 3 \times 10^{-6}$ because of the uncertainty in the logic circuitry of the fringe counter. Thus, the total theoretical error for the interferometric displacement transducer does not exceed $\pm 6 \times 10^{-6}$ inches.

APPENDIX A. Environmental Effects on Wavelength.

The refractive index formulae were taken from Bengt Edlen in the article entitled *The Refractive Index of Air*, published in *Metrologia*, Volume 2, 1966.

The procedure is outlined for calculating the wavelength (λ) in air, in terms of temperature (t), pressure (p), and watervapor pressure (f) for a given vacuum wavelength, (λ_{vac}). The refractivity of standard air, n_s , (i.e., dry air containing 0.03 percent by volume of carbon dioxide at 760 mm Hg pressure and 15 °C) is given by:

$$(n_s - 1) \times 10^8 = 8342.13 \times \frac{2,406,030}{130 - (\sigma)^2} + \frac{15,997}{38.9 - (\sigma)^2} \quad (1)$$

In Equation 1, sigma (σ) is the vacuum wavenumber in microns⁻¹. The refractivity of dry air at pressure, p (mm Hg), and temperature t (°C), in terms of n_s is:

$$n_{tp} - 1 = (n_s - 1) \frac{p}{720.775} \frac{1 + p(0.817 - 0.0133 t) \times 10^{-6}}{1 + 0.0036610 t} \quad (2)$$

The additional effect of water vapor of pressure, f (mm Hg), is expressed by:

$$n_{tpf} = n_{tp} - f [5.722 - 0.0457 (\sigma)^2] \times 10^{-8} \quad (3)$$

The wavelength in air is then:

$$\lambda_{air} = \lambda_{vac} n_{tpf} \quad (4)$$

Using the above relations and the vacuum wavelength^{A-1} of an isotropic helium-neon laser oscillating at the Lamb-dip of 0.63299147 microns, a table was constructed via computer of laser wavelength in air for convenient ranges of t , p , and f for quick reference.

^{A-1} K. D. Mielenz *et al.*, "Measurement of the 633 nm Wavelength of He-Ne Lasers." *Applied Physics Letters*, 7:10. 1965.