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MATHEMATICAL MODEL FOR PREDICTING HEAT TRANSFER
CHARACTERISTICS OF A BAYONET TUBE

B. R. Dickey and C. L. Bendixsen

IDAHO NUCLEAR CORPORATION
NATIONAL REACTOR TESTING STATION
IDAHO FALLS, IDAHO

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By

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IDAHO NUCLEAR CORPORATION

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A B S T R A C T

High temperature NaK circulating through a tube bundle provides the heat required for fluidized bed calcination of radioactive liquid wastes in the Waste Calcining Facility. The hot NaK flows through a central core tube before reversing flow through a surrounding annulus. Any significant transfer of heat from the core to the annulus (short-circuiting) potentially reduces the overall rate of heat transfer from the exchanger to the fluidized bed. A mathematical model is developed to: (1) predict the magnitude of the effect of short-circuiting on the heat duty of the exchanger, (2) determine if any significant increase in exchanger duty can be realized by decreasing the short-circuiting of heat at the existing operating conditions, and (3) calculate any increase in exchanger duty resulting from a flow reversal of the NaK through the bayonet tube (ie, hot NaK entering the annulus and exiting through the core). Numerical examples illustrate the heat transfer characteristics of a typical bayonet tube for a specified set of operating conditions and tube dimensions.

SUMMARY

Large volumes of liquid waste generated in the processing of spent nuclear fuel elements at the Idaho Chemical Processing Plant (ICPP) are being converted to granular solids by fluidized bed calcination in the Waste Calcining Facility (WCF). Heat required for this endothermic process is supplied from high temperature NaK circulating through a tube bundle within the bed.

Future processing requirements dictate an increase in the feed rate to the calciner and an accompanying increase in the quantity of heat required for the fluidized bed. Increasing either the flow rate or operating temperature of the NaK is not feasible because of mechanical and metallurgical problems.

The existing tube bundle is constructed of bayonet type tubes having a central core separated by an argon-filled gas gap from the surrounding annulus. High temperature NaK flows through the core tube and then reverses flow through the annulus from which heat is transferred to the bed. Any significant heat transfer from the core to the annulus (short-circuiting) potentially reduces the rate of heat transfer to the bed.

A study has been made to: (1) evaluate the quantitative effects of short-circuiting on the heat duty of the existing exchanger, (2) determine if a significant increase in duty could be obtained by minimizing conduction and radiation across the gas gap (eg, operating with a partial vacuum and/or using a low emissivity metal for the tube walls), and (3) calculate any increase in exchanger duty as a result of reversing the flow of NaK within the bayonet tubes. A mathematical model for heat transfer within and from a bayonet tube was proposed, and differential equations describing the NaK temperature within the core and annulus were derived. If radiation is neglected in the model, an analytical solution is obtained for the outlet temperature of the NaK leaving the exchanger. Inclusion of radiation in the model necessitates

the use of a finite difference technique for calculating the steady-state temperature of the NaK leaving the annulus or core.

The effect of the following factors on the calculated exchanger heat duty was investigated: (1) thermal conductivity across the gas gap, (2) emissivity of the tube walls, (3) absence of a gas gap, and (4) flow reversal of the NaK through the exchanger. At the current temperature levels in the NaK exchanger, no significant increase in the overall heat transfer rate to the fluidized bed is realized by increasing the thermal resistance between the core and annulus. The heat duty of the exchanger is independent of the direction of flow of NaK through the bayonet tube.

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I. INTRODUCTION

Radioactive wastes generated during the reprocessing of spent nuclear fuel elements at the ICPP are being converted to granular solids by a fluidized bed calcination process. The granular solids are subsequently stored within steel bins inside a concrete vault buried in the ground. Calcination of the liquid wastes is conducted at approximately 750°F, and the endothermic process requires that heat be supplied continuously to the fluidized bed.

Heat required for the calcination is obtained from high temperature NaK (78 percent potassium and 22 percent sodium by weight) circulating through a tube bundle within the fluidized bed. The NaK, after being heated to approximately 1350°F in an oil-fired furnace, is circulated through the heating loop by an electromagnetic pump at a rate of 120 gallons per minute. The entire heat transfer loop is of welded construction using Type 316 stainless steel.

The NaK heat exchanger contains 42 bayonet tubes, which are inserted horizontally into the calciner vessel⁽⁵⁾. Inner and outer bulkheads, as shown in Figure 1, permit the NaK to flow through the tubes in parallel. Each bayonet tube has a core tube and two outside "duplex" tubes as shown in Figure 2. The NaK enters the outer bulkhead, flows through the core tube, reverses flow through the annulus surrounding the core tube, and passes into the inner bulkhead for transfer back to the furnace.

The core of each bayonet tube is a sandwich-type tube with an argon-filled gas blanket, 0.025 inches thick, which reduces heat transfer from the hot NaK in the core to the cooler NaK in the surrounding annulus. Helium gas, contained within the space between the inner and outer "duplex" tubes, serves as a NaK leak detector.

Future processing requirements at ICPP require an increase in the feed rate to the Waste Calcining Facility (WCF). Heat can be supplied

FIGURE 1
BAYONET TUBE HEAT EXCHANGER BUNDLE

FIGURE 2
BAYONET TUBE DETAILS

to the fluidized bed from: (1) in-bed combustion, a process currently under development, or (2) a NaK tube bundle similar to that presently being used. Any design of a new tube bundle justifies consideration of the significance of heat transfer from the core to the annulus (short-circuiting) in reducing the exchanger heat duty.

Present operations employ an argon-filled gas gap to reduce the heat transfer between the core and annulus. If short-circuiting of heat from the core to the annulus is significant, the rate of heat transfer to the fluidized bed is substantially reduced. Use of a partial vacuum in the gap and/or a reduction in radiative transfer by using low emissivity metals for the walls of the gap could improve the overall rate of heat transfer from the tube bundle. Another potential method for improving the heat transfer to the bed would be to reverse the flow of NaK through the bayonet tube (ie, enter the annulus and exit through the core). Since thermocouples are not present at the ends of the bayonet tubes, reliable experimental data cannot be obtained, thus a mathematical analysis of the heat transfer characteristics of the bayonet tube is required.

The remainder of this report is concerned with the development of a mathematical model which describes the heat transfer within and from a "duplex" tube. Equations are derived, based on the model for computing the temperature of the NaK leaving the exchanger (exchanger duty) for a set of specified conditions. An analytical solution was obtained for the case of conductive heat transfer through the metal walls and across the gas gap. Numerical solutions, calculated by a digital computer program, were required to compute the outlet NaK temperature if radiative transfer across the gas gap was included. Illustrative examples are used to show the effect on the outlet NaK temperature of short-circuiting of heat from the core to the annulus.

II. MATHEMATICAL MODELS FOR BAYONET TUBE HEAT TRANSFER

Any mathematical model for predicting the rate of heat transfer from the core to the annulus of a "duplex" tube and from the tube surface

to the fluidized bed must have the capability of: (1) predicting radiative and conductive heat transfer across a gas gap, and (2) predicting conductive and convective heat transfer from the outer tube surface to the fluidized bed. The calculation of the radiative heat transport across the gas gap separating the core tube from the outer annulus requires that the temperature within the core and annulus of the bayonet tube be approximated by a number of perfectly mixed "cells." If radiation is neglected in the mathematical model, an analytical solution to the resulting differential equations can be obtained. Inclusion of radiation across the gas gap requires the solution of a set of finite difference equations to calculate the NaK temperature leaving the annulus. The following assumptions were used in the proposed mathematical models:

1. Negligible heat conduction through the NaK in both radial and axial directions. (See Appendix A)
2. Negligible resistance between NaK and tube walls.
3. Negligible heat transfer from the end of the bayonet tube to the fluidized bed.
4. Constant heat transfer coefficient between the tube surface and the fluidized bed.
5. Perfectly mixed fluidized bed with respect to temperature.
6. Negligible convective heat transfer across the gas gap.
7. Radiation view factors from a given "cell" in the core tube to the opposite cell in the outer annulus are assumed to be unity⁽¹⁾.

Detailed derivations of the equations based on the proposed models are given in Appendix B. Each model, the resulting equation(s), and the calculational procedures are discussed in the following paragraphs.

1. Perfect Insulation Between Core Tube and Annulus

For the case of perfect insulation between the core tube and the annulus, the model reduces to that for predicting the rate of heat transfer to the surrounding medium from a hot fluid flowing in a pipe. Physically, this implies that the temperature of the NaK, $T_i(L)$, leaving

the core tube is identical to the inlet NaK temperature, $T_i(o)$. The differential equation describing the temperature in the annulus, $T_o(X)$, as a function of distance in the annulus reduces to

$$\frac{dT_o(X)}{dX} + \frac{Ua_o}{WC_p} [T_o(X) - T_B] = 0 \quad (1)$$

with the boundary condition $T_o(L) = T_{in}$.

The solution to the boundary value problem given by Equation (3) is

$$\theta(X) = [T_{in} - T_B] \text{EXP} [-U_o(L-X)]$$

$$\text{where } \theta(X) = T_o(X) - T_B \quad (2)$$

$$U_o = \frac{Ua_o}{WC_p}$$

For a specified set of conditions, Equation (2) can be used to compute the maximum amount of heat that can be transferred from a single bayonet tube to the fluidized bed. The heat transferred from the "duplex" tube to the fluidized bed under actual conditions will be less than that calculated by Equation (2), since some heat is short-circuited from the core tube to the annulus by conduction and radiation. However, in the limit as radiative and conductive heat transfer across the gas gap is minimized, the outlet temperature of the NaK, $T_o(o)$, should approach the value of $T_o(o)$ calculated from Equation (2).

2. Conduction from Core Tube to Annulus

Within the actual duplex tube, heat can be transferred from the core tube to the outer annulus by a combination of conduction through metal walls and radiation and conduction across the gas gap. For the conditions considered herein, the distance across the gas gap is so small that convection can be considered negligible⁽³⁾. If radiation across the gas gap is neglected, the mathematical model for heat transfer within and from the "duplex" tube is shown in Figure 3. Heat is transferred from a perfectly mixed section of the core tube, through a metal wall, across the gas gap by conduction, through a metal wall into a perfectly mixed section of the annulus, and finally through a metal wall to the perfectly mixed fluidized bed.

FIGURE 3
MODEL FOR HEAT TRANSFER FROM CORE TUBE TO ANNULUS, RADIATION NEGLECTED

The differential equations describing the steady-state temperatures as a function of distance along the core tube and annulus are derived by making a heat balance around the core tube and annulus between the sections X and $X+\Delta X$. Details of the derivation and the solution of the resulting boundary value problem are given in Appendix B; the solution for the temperatures within the core tube and annulus are given respectively by Equations (3) and (4).

$$\begin{aligned}
 T_i(X) = & T_{in} e^{-U_c X} + U_c T_o \left[\frac{-U_c e^{-U_c X}}{(S_1 + U_c)(S_2 + U_c)} + \frac{S_1 e^{S_1 X}}{(S_1 + U_c)(S_1 - S_2)} \right. \\
 & \left. + \frac{S_2 e^{S_2 X}}{(S_2 + U_c)(S_2 - S_1)} \right] + [U_c^2 T_o - U_c^2 T_{in} - U_c U_o T_B] \\
 & \left[\frac{e^{-U_c X}}{(S_1 + U_c)(S_2 + U_c)} + \frac{e^{S_1 X}}{(S_1 + U_c)(S_1 - S_2)} + \frac{e^{S_2 X}}{(S_2 + U_c)(S_2 - S_1)} \right]
 \end{aligned}$$

$$-U_c^2 U_o T_B \left[\frac{1}{U_c s_1 s_2} + \frac{e^{-U_c X}}{(-U_c)(s_1 + U_c)(s_2 + U_c)} + \frac{s_1 X}{s_1 (s_1 + U_c)(s_1 - s_2)} + \frac{s_2 X}{s_2 (s_2 + U_c)(s_2 - s_1)} \right] \quad (3)$$

$$T_o(X) = [U_c (T_o - T_{in}) - U_o T_B] \left[\frac{s_1 X}{(s_1 - s_2)} - \frac{s_2 X}{(s_1 - s_2)} \right] + T_o \left[\frac{s_1 e^{s_1 X} - s_2 e^{s_2 X}}{(s_1 - s_2)} \right] - U_o U_c T_B \left[\frac{1}{s_1 s_2} + \frac{s_1 X}{s_1 (s_1 - s_2)} + \frac{s_2 X}{s_2 (s_2 - s_1)} \right] \quad (4)$$

As shown in Figure 3, heat is transferred from the core to the annulus through a series of resistances. These series of resistances to the flow of heat are defined in the quantity, U_c , given by

$$U_c = \frac{1}{WC_p \left[\frac{d_1}{k_m a_{m1}} + \frac{d_g}{k_g a_{mg}} + \frac{d_2}{k_m a_{m2}} \right]} \quad (5)$$

If no gas gap exists, the denominator of Equation (5) will have only one term corresponding to the resistance of a single metal wall. The effect of the thermal conductivity of the gas in the gap on reducing the rate of heat transfer from the core tube to the annulus is governed by the magnitude of the resistance corresponding to the second term in the denominator of Equation (5). The temperature of the NaK leaving the bayonet tube, $T_o(o)$, is determined by equating the temperatures $T_i(L)$ and $T_o(L)$ calculated using Equations (3) and (4) respectively.

3. Conductive and Radiative Transfer Across the Gas Gap

If radiation across the gas gap is superimposed on the conduction, a system of non-linear differential equations results. The model from which the differential equations are obtained is identical to the model shown in Figure 3 with the exception that radiation is included. The

FIGURE 4
MODEL FOR CONDUCTIVE AND RADIATIVE HEAT TRANSFER ACROSS THE GAS GAP

resulting differential equations are solved by a finite difference approximation, a model of which is shown in Figure 4.

The temperatures within the core and annulus are approximated by considering the tube to be divided into a number of perfectly mixed "cells." Radiation and conduction from the core tube is visualized as occurring from a given "cell" in the core tube to its opposite "cell" in the annulus (eg, cell m to cell n). A heat balance around each "cell" results in the following system of equations relating the temperatures of the "cells" within the core and annulus:

$$T_1 + \frac{U_R \Delta X}{(1+U_c \Delta X)} T_1^4 = \frac{T_{in}}{(1+U_c \Delta X)} + \frac{U_c \Delta X}{(1+U_c \Delta X)} T_N + \frac{U_R \Delta X}{(1+U_c \Delta X)} T_N^4 \quad (6)$$

$$T_m + \frac{U_R \Delta X}{(1+U_c \Delta X)} T_m^4 = \frac{T_{m-1}}{(1+U_c \Delta X)} + \frac{U_c \Delta X}{(1+U_c \Delta X)} T_n + \frac{U_R \Delta X}{(1+U_c \Delta X)} T_n^4 \quad (7)$$

$$(m = 2, 3, \dots M)$$

$$(n = M+1, M+2, \dots N-1)$$

$$T_{M+1} + \frac{U_R \Delta X}{(1+U_R \Delta X + U_c \Delta X)} T_{M+1}^4 = \frac{1 + U_c \Delta X}{(1+U_c \Delta X + U_R \Delta X)} T_M + \frac{U_R \Delta X}{(1+U_c \Delta X + U_R \Delta X)} T_M^4 + \frac{U_o \Delta X}{(1+U_c \Delta X + U_R \Delta X)} T_B \quad (8)$$

$$T_n + \frac{U_R \Delta X}{(1+U_R \Delta X + U_c \Delta X)} T_n^4 = \frac{T_{n-1}}{(1+U_c \Delta X + U_R \Delta X)} + \frac{U_R \Delta X}{(1+U_c \Delta X + U_R \Delta X)} T_m + \frac{U_R \Delta X}{(1+U_c \Delta X + U_R \Delta X)} T_m^4 + \frac{U_o \Delta X}{(1+U_c \Delta X + U_R \Delta X)} T_B \quad (9)$$

$$(n = M + 2, \dots, N-1)$$

$$(m = 2, 3, \dots, M-1)$$

$$T_N + \frac{U_R \Delta X}{(1+U_R \Delta X + U_c \Delta X)} T_N^4 = \frac{T_{N-1}}{(1+U_c \Delta X + U_R \Delta X)} + \frac{U_c \Delta X}{(1+U_c \Delta X + U_R \Delta X)} T_1 + \frac{U_R \Delta X}{(1+U_c \Delta X + U_R \Delta X)} T_1^4 + \frac{U_o \Delta X}{(1+U_c \Delta X + U_R \Delta X)} T_B \quad (10)$$

The system of Equations (6) through (10) is solved using the following algorithm:

1. The steady-state temperature of each "cell" is assumed.
2. A new temperature is calculated for each cell using the method of false position ("regula falsi").⁽⁶⁾
3. The new "cell" temperatures are used to calculate new values of the functions on the right hand side of Equations (6) through (10).
4. A new set of "cell" temperatures is then calculated and the entire process repeated until convergence of the calculated temperatures is obtained.

A digital computer program, written in FORTRAN, to solve Equations (6) through (10) is discussed in Appendix C.

4. Effect of Flow Reversal of NaK on Exchanger Duty

The effect on the outlet NaK temperature of reversing the flow of NaK through the bayonet tube requires consideration of the system shown in Figure 5.

FIGURE 5
MODEL FOR FLOW REVERSAL OF NaK IN THE BAYONET TUBE

A steady-state heat balance between the sections X and $X + \Delta X$ (see Figure 3) results in the following equations for the temperatures within the core and the annulus. For the core:

$$\begin{aligned}
 T_i(X) = & T_o e^{\frac{U_c X}{S_3 - S_4}} - U_c T_{in} \left[\frac{e^{\frac{S_3 X}{S_3 - S_4}}}{(S_3 - S_4)} + \frac{e^{\frac{S_4 X}{S_4 - S_3}}}{(S_4 - S_3)} \right] \\
 & - U_c U_o T_B \left[\frac{1}{S_3 S_4} + \frac{e^{\frac{S_3 X}{S_3 - S_4}}}{S_3 (S_3 - S_4)} + \frac{e^{\frac{S_4 X}{S_4 - S_3}}}{S_4 (S_4 - S_3)} \right] \\
 & - U_c^2 T_o \left[\frac{e^{\frac{U_c X}{(U_c - S_3)(U_c - S_4)}}}{(U_c - S_3)(U_c - S_4)} + \frac{e^{\frac{S_3 X}{(S_3 - U_c)(S_3 - S_4)}}}{(S_3 - U_c)(S_3 - S_4)} + \frac{e^{\frac{S_4 X}{(S_4 - U_c)(S_4 - S_3)}}}{(S_4 - U_c)(S_4 - S_3)} \right]
 \end{aligned} \tag{11}$$

For the annulus:

$$\begin{aligned}
 T_o(X) = & T_{in} \left[\frac{(S_3 - U_c) e^{S_3 X}}{(S_3 - S_4)} + \frac{(S_4 - U_c) e^{S_4 X}}{(S_4 - S_3)} \right] \\
 & + U_o T_B \left[\frac{-U_c}{S_3 S_4} + \frac{(S_3 - U_c) e^{S_3 X}}{S_3 (S_3 - S_4)} + \frac{(S_4 - U_c) e^{S_4 X}}{S_4 (S_4 - S_3)} \right] \\
 & + U_c T_o \left[\frac{e^{S_3 X}}{(S_3 - S_4)} + \frac{e^{S_4 X}}{(S_4 - S_3)} \right]
 \end{aligned} \tag{12}$$

where

$$S_3 = \frac{-U_o + (U_o^2 + 4U_c U_o)^{1/2}}{2}$$

$$S_4 = \frac{-U_o - (U_o^2 + 4U_c U_o)^{1/2}}{2}$$

The unknown outlet NaK temperature, $T_i(o)$, leaving the core tube is calculated by equating Equations (11) and (12) evaluated at $X = L$.

III. RESULTS

Operating conditions and specifications for the NaK heat exchanger are shown in Table I. Outlet NaK temperatures for several operating conditions are shown in Table II. The temperatures, T_o , for the conditions shown; represent the extremes between which the outlet temperature varies in an actual bayonet tube. Outlet temperatures for the case of a perfectly insulated core tube and a core tube separated from the annulus by a metal wall and gas gap were calculated using Equations (2) and (3), respectively. The outlet temperature calculated using a gas gap with a thermal conductivity of 0.03 Btu/hr-ft-°F corresponds closely to the present operating conditions.

Table I	
OPERATING CONDITIONS AND SPECIFICATIONS FOR NaK HEAT EXCHANGER	
Inlet NaK Temperature	1320°F
Total NaK Feed Rate to Exchanger	42,000 lb/hr
Type of Tube	Bayonet
Hot NaK Inlet	To Core Tube
Length of a Tube	52 Inches
Overall Heat Transfer Coefficient From Annulus to Fluidized Bed, U	75 Btu/hr-ft ² -°F
Thermal Conductivity of Metal	10 Btu/hr-ft-°F
Number of Tubes in Bundle	42

Table II		
TEMPERATURE OF NaK LEAVING ANNULUS FOR INDICATED CONDITIONS		
Resistance to Heat Flow	Thermal Conductivity of Insulating Gap, Btu/hr-ft-°F	Temperature of NaK Leaving Annulus, T _o , °F
Perfectly Insulated Core	k _g = 0.0	1061.0
Metal Wall With Argon- Filled Gas Gap (Exist- ing Conditions)	k _m = 10.0, k _g = 0.03	1066.7
Solid Metal Wall	k _m = 10.0	1138.9

When radiative transfer across the gas gap was included, finite difference Equations (6) through (10) were required to calculate the temperatures in the core and annulus. The numerical procedure was validated by comparing numerical results with a known analytical solution as shown in Table III.

Table III
AGREEMENT BETWEEN ANALYTICAL AND NUMERICAL TEMPERATURES
 $k_g = 0.03 \text{ Btu/hr-ft-}^\circ\text{F}$, $\epsilon_m = 0.1$

Location Within Core, Ft	Temperature, $^\circ\text{F}$		Location Within Annulus, Ft	Temperature, $^\circ\text{F}$	
	Analytical	Numerical		Analytical	Numerical
X = 0.0	1320.0	1320.0	X = 4.3	1319.9	1318.9
.2	1320.0	1319.9	4.2	1311.6	1310.6
.4	1320.0	1319.8	4.0	1295.3	1294.4
.6	1320.0	1319.7	3.8	1279.4	1278.6
.8	1320.0	1319.7	3.6	1263.9	1263.3
1.0	1320.0	1319.6	3.4	1252.0	1250.4
1.2	1320.0	1319.5	3.2	1241.4	1241.1
1.4	1320.0	1319.4	3.0	1227.1	1226.8
1.6	1320.0	1319.4	2.8	1213.0	1212.9
1.8	1320.0	1319.3	2.6	1199.4	1199.4
2.0	1320.0	1319.3	2.4	1188.1	1186.3
2.2	1319.9	1319.2	2.2	1176.5	1173.6
2.4	1319.9	1319.1	2.0	1166.9	1166.3
2.6	1319.9	1319.1	1.8	1152.5	1150.2
2.8	1319.9	1319.0	1.6	1140.5	1139.4
3.0	1319.9	1319.0	1.4	1130.8	1129.2
3.2	1319.9	1319.0	1.2	1120.6	1120.4
3.4	1319.9	1318.9	1.0	1113.9	1113.8
3.6	1319.9	1318.9	0.8	1100.1	1100.6
3.8	1319.9	1318.9	0.6	1092.2	1093.7
4.0	1319.9	1318.9	0.4	1080.1	1082.5
4.2	1319.9	1318.9	0.2	1071.2	1073.6
4.3	1319.9	1318.9	0.0	1062.4	1064.0

Based on the conditions specified, the effect of heat transfer from the core to the annulus (short-circuiting) on the exchanger duty is given in Table IV.

Table IV
EFFECT OF SHORT-CIRCUITING ON EXCHANGER HEAT DUTY

Material Separating Core From Annulus	Calculated Exchanger Duty, Millions Btu/hr	%Efficiency (Based on Perfectly Insulated Core Tube
Plain Metal Wall $k_m = 10 \text{ Btu/hr-ft-}^\circ\text{F}$	1.598	69.4
Metal Walls Separated By Gas Gap $k_g = 0.1 \text{ Btu/hr-ft-}^\circ\text{F}$ $\epsilon_m = 0.6$	2.223	96.8
*Metal Walls Separated By Gas Gap $k_g = 0.03 \text{ Btu/hr-ft-}^\circ\text{F}$ $\epsilon_m = 0.6$	2.250	97.8
Evacuated Gas Gap $c_m = 0.1$	2.200	99.2
Perfectly Insulated Core Tube	2.300	100.0

* Present operating conditions.

The effect of emissivity of the metal walls enclosing the gas gap and of the thermal conductivity of the gas on the outlet NaK temperature, T_o , is shown in Figure 6. As expected, the radiative transfer superimposed on the conduction resulted in an increase in the outlet NaK temperatures for a given value of the thermal conductivity of the gas.

FIGURE 6
DEPENDENCE OF OUTLET NaK TEMPERATURE ON EMISSIVITY OF TUBE WALLS AND THERMAL CONDUCTIVITY OF GAS GAP

FIGURE 7
EFFECT OF SOLID METAL WALL BETWEEN CORE AND ANNULUS
ON THE TEMPERATURE DISTRIBUTION

For the case of a single metal wall separating the core from the annulus, the NaK temperature along the flow path through the core and annulus is shown in Figure 7. Because of the large rate of heat transfer from the core to the annulus, the temperature of the NaK passes through a minimum downstream from the inlet to the outer annulus.

Temperatures within the bayonet tube, calculated on the basis of NaK fed to the core or annulus (Equations (3) and (4) and Equations (11) and (13) respectively) are compared in Figure 8. Since identical quantities of heat were transferred to the fluidized bed in each case, the heat duty of the exchanger was shown to be independent of the direction of flow of the NaK through the exchanger.

FIGURE 8
COMPARISON OF TEMPERATURE DISTRIBUTION
WITHIN BAYONET TUBE FOR HOT NaK FED TO CORE OR ANNULUS

IV. CONCLUSIONS

1. The maximum rate of heat transfer from the NaK tube bundle to the fluidized bed occurs when the core tube is perfectly insulated from the annulus.
2. At the current temperature levels in the NaK exchanger, no significant increase in the overall heat transfer rate to the fluidized bed can be realized by increasing the thermal resistance between the core tube and annulus.
3. For a given set of operating conditions and exchanger dimensions, the exchanger duty is independent of the direction of flow of NaK through the exchanger.

V. NOMENCLATURE

a_o = area per foot of outer surface of bayonet tube, ft

a_{mi} = area per foot of inner surface of gas gap, ft

a_{mo} = area per foot of outer surface of gas gap, ft

a_{ml}, a_{mg}, a_{m2} = log mean areas of inner tube wall, gas gap, and outer tube wall respectively, ft

C = the quantity $\left(U_o + U_c \right)$, ft^{-1}

C_p = specific heat of NaK, Btu/lb-°F

d_1, d_g, d_2 = thickness of inner tube wall, gas gap, and outer tube wall respectively, ft

$\bar{F}$ = the quantity $\frac{0.173}{\frac{1}{\epsilon_m} + \left(\frac{a_{mi}}{a_{mo}} \right) \left[\frac{1}{\epsilon_m} - 1 \right]}$, $\frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{R}^4}$

h = enthalpy of fluid, Btu/lb

k_m, k_g = thermal conductivities of metal and gas respectively, Btu/hr-ft-°F

L = length of bayonet tube, ft

$m, M, MP1, n, N$ = designation of constant temperature cells.

S = the Laplace Transform parameter

S_1, S_2, S_3, S_4 = constants in the partial fraction expansion of the Laplace Transform Equations (3) and (4) and Equations (11) and (12) respectively.

$T_i(X), T_o(X)$ = temperature at position X in core tube and annulus, respectively, in °F or, °R depending on context

T_{in}, T_o = inlet and outlet temperature of NaK respectively, °F

T_B = temperature of a perfectly mixed fluidized bed, $^{\circ}\text{F}$

U_c = the quantity $\frac{1}{WC_p \left[\frac{d_1}{k_m a_{m1}} + \frac{d_g}{k_g a_{mg}} + \frac{d_2}{k_m a_{m2}} \right]}$, ft^{-1}

U_o = the quantity $\frac{Ua_o}{WC_p} = \text{ft}^{-1}$

U_R = the quantity $\frac{\bar{F} a_{mi}}{WC_p}$, $\frac{^{\circ}\text{F}}{\text{ft } ^{\circ}\text{R}^4}$

U = tube surface to fluidized bed heat transfer coefficient,
 $\text{Btu/hr-ft}^2\text{-}^{\circ}\text{F}$

W = NaK flow rate, lb/hr

X = distance from inlet of core tube, ft

Greek Letters

Δ = denotes an incremental change in a variable

θ = the temperature scale defined by $\theta(X) = T_o(X) - T_B$

ϵ_m = emissivity of tube walls

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APPENDIX A

VERIFICATION OF NEGLIGIBLE AXIAL HEAT CONDUCTION WITHIN NaK

The most questionable assumption used in the development of the mathematical models is that of negligible heat conduction through the NaK in the axial direction. Constant temperature in the radial direction can be safely assumed because of the small tube size and the relatively high thermal conductivity of the NaK (ie, 16 Btu/hr-ft-°F). However, significant heat conduction could occur in the axial direction at a given flow velocity of the NaK. A rigorous analysis of the effect of axial conduction on the calculated temperatures within the bayonet tube requires the derivation and solution of the appropriate differential equations. The significance of axial conduction within the NaK was determined by calculating the temperature drop resulting from axial conduction within a perfectly insulated core tube. The model for heat transfer within a perfectly insulated core tube and the surrounding annulus is shown in Figure A-1.

FIGURE A-1

HEAT TRANSFER WITHIN BAYONET TUBE SUBJECT TO AXIAL CONDUCTION THROUGH THE NaK

A heat balance within the core tube between X and X + ΔX yields the following:

$$\left[q_f + q_c A_c \right] \Big|_X - \left[q_f + q_c A_c \right] \Big|_{X + \Delta X} = 0 \quad (1a)$$

Rate of heat input @ X by flow and con- duction.	Rate of heat output @ X + ΔX by flow and con- duction.
---	--

In the limit as ΔX → 0, the following differential equation is obtained from Equation (1a)

$$\frac{d^2 T_i(X)}{dX^2} - \left(\frac{WC_p}{k_l A_c} \right) \frac{dT_i(X)}{dX} = 0 \quad (2a)$$

subject to the following boundary conditions

- (1) $T_i(0) = T_{in}$
- (2) $T_i(L) = T_o(L)$

where q_f = rate of heat flow @ X or X + ΔX resulting from fluid flow,
Btu/hr

q_c = heat flux at X or X + ΔX due to conduction, Btu/hr-ft²

A_c = cross-sectional area of core tube, ft²

k_l = thermal conductivity of NaK, Btu/hr-ft-°F

C_p = heat capacity of NaK, Btu/lb-°F

$T_i(X)$ = temperature at point X in core tube, °F

T_{in} = inlet NaK temperature, °F

The solution of Equation (2a) giving the temperature within the core tube resulting from axial conduction is

$$T_i(X) = T_{in} \frac{(e^{bL} - e^{bX})}{(e^{bL} - 1)} + T_o(L) \frac{[e^{bX} - 1]}{(e^{bL} - 1)} \quad (3a)$$

A heat balance on the annulus between X and $X + \Delta X$ results in the following equation

$$\left[Wh + q_c A_c \right] \Big|_{X + \Delta X} - \left[Wh + q_c A_c \right] \Big|_X - \int_X^{X + \Delta X} U a_o \left[T_o(X) - T_B \right] dX = 0 \quad (4a)$$

Rate of heat input
@ $X + \Delta X$ by flow
and conduction

Rate of heat
output @ X
by flow and
conduction

Rate of heat output from
tube surface

Taking the limit as $\Delta X \rightarrow 0$ and rearranging Equation (4a) results in the following differential equation for the temperature in the annulus as a function of the distance X

$$\frac{d^2 T_o(X)}{dX^2} - \left(\frac{WC_p}{k_1 A_c} \right) \frac{dT_o(X)}{dX} - \left(\frac{U a_o}{k_1 A_c} \right) \left[T_o(X) - T_B \right] = 0 \quad (5a)$$

with boundary conditions

$$(1) \quad T_o(L) = T_i(L)$$

$$(2) \quad T_o(0) = T_o$$

If $\theta(X)$ is defined as $\theta(X) = T_o(X) - T_B$, the solution to Equation (5a) can be written as

$$\theta(X) = \left[\frac{T_i(L) - T_B + (T_o - T_B) \cosh m_2 L}{\sinh m_1 L} \right] \sinh m_1 X + (T_o - T_B) \cosh m_2 X \quad (6a)$$

where:

T_o = temperature of NaK leaving bayonet tube, °F

T_B = perfect mixing temperature of fluidized bed, °F

U = heat transfer coefficient at tube surface, Btu/hr-ft²-°F

a_o = area of outside surface per foot of tube, ft

b = the quantity, $\frac{WC_p}{k_1 A_c}$, ft⁻¹

c = the quantity, $\frac{U a_o}{k_1 A_c}$, ft⁻²

The value of $T_i(L) = T_o(L)$ was calculated from the fact that the heat flux must be continuous across the plane at $X = L$; thus it is required that

$$\left. \frac{dT_i(X)}{dX} \right|_{X=L} = \left. \frac{dT_o(X)}{dX} \right|_{X=L} \quad (7a)$$

$$m_1 = \text{the quantity, } \frac{-b + (b^2 + 4c)^{1/2}}{2}, \text{ ft}^{-1}$$

$$m_2 = \text{the quantity, } \frac{-b - (b^2 + 4c)^{1/2}}{2}, \text{ ft}^{-1}$$

Evaluation of the derivatives of Equations (3a) and (6a) with respect to X and substitution into Equation (7a) yields the following relationship

$$\frac{b [T_o(L) - T_{in}] e^{bL}}{e^{bL} - 1} = m_1 \left\{ \frac{T_o(L) - T_B - (T_o(L) - T_B) \cosh m_2 L}{\sinh m_2 L} \right\} \cosh m_1 L \quad (8a)$$

At the operating conditions of the heat exchanger, the following parameters can be calculated

$$b = .410$$

$$m_1 = 0.065$$

$$m_2 = .410$$

For these values, Equation (8a) simplifies to

$$.410 [T_o(L) - 1320] = .065 [- (T_1(L) - 720)] 1.029 \quad (9a)$$

The value of $T_o(L)$ required to satisfy Equation (9a) is essentially 1320°F, thus the temperature drop due to axial conduction through the NaK is negligible.

APPENDIX B - DERIVATION OF EQUATIONS

1. Perfect Insulation Between Core Tube and Annulus

For this case, no short-circuiting of heat between the inner and outer tubes will occur. The fluid temperature entering the outer tube is identical to the temperature of the hot fluid entering the exchanger. The model for heat transfer within and from a "duplex" tube to a fluidized bed is shown in Figure B-1.

FIGURE B-1

MODEL FOR HEAT TRANSFER, PERFECTLY INSULATED CORE TUBE

Since no heat flows from the region $T_i(X)$ to the region $T_o(X)$, a heat balance on the outer tube between X and $X+\Delta X$ is sufficient to obtain the differential equation describing the temperature, $T_o(X)$, as a function of tube length. Thus,

$$\underbrace{Wh/X+\Delta X}_{\text{Input by flow @ } X+\Delta X} - \underbrace{Wh/X}_{\text{Output by flow @ } X} - \underbrace{\int_X^{X+\Delta X} Ua_o [T_o(X) - T_B] dX}_{\text{Output by heat transfer to fluidized bed}} = 0 \quad (1b)$$

where W = flow of hot fluid, lb/hr

h = enthalpy of fluid, Btu/lb

U = overall heat transfer coefficient between tube surface and the fluidized bed, Btu/hr-ft²-°F

a_o = heat transfer area per foot of tube length, ft.

$T_i(X)$ = temperature within core tube at X , °F

$T_o(X)$ = temperature within outer tube at X , °F

T_B = perfect mixing temperature of fluidized bed, °F

Application of the Theorem of the Mean of Differential Calculus and the Mean Value Theorem of Integral Calculus results in the following differential equation

$$\frac{dT_o(X)}{dX} - \left(\frac{Ua_o}{WC_p} \right) [T_o(X) - T_B] = 0 \quad (2b)$$

with the boundary condition, $T_o(L) = T_{in}$.

Longitudinal conduction within the liquid metal was neglected and the heat capacity and heat transfer coefficient were assumed constant.

Equation (2b) can be simplified by definition of a new dependent variable, $\theta(X) = T_o(X) - T_B$. The boundary value problem for temperature, $T_o(X)$, as a function of distance along the tube is then

$$\frac{d\theta(X)}{dX} + \left(\frac{Ua_o}{WC_p} \right) \theta(X) = 0 ; \quad \theta(L) = T_{in} - T_B \quad (3b)$$

The solution to Equation (3b) can be written as

$$\theta(X) = (T_{in} - T_B) e^{-U_o(L - X)}$$

2. Heat Transfer from Core Tube to Annulus by Conduction

Heat can be transferred from the core tube to the annulus by conduction through metal walls and through any gas gap which might be present. The model for heat transfer in this case is shown in Figure B-2.

The system of differential equations describing the temperature as a function of distance along the core and outer tubes can be derived as follows. A heat balance on the region, $T_i(X)$, between X and $X+\Delta X$ results in the following equation

$$Wh/X - Wh/X+\Delta X - \int_X^{X+\Delta X} Q_c(X) dX = 0 \quad (4b)$$

FIGURE B-2

HEAT TRANSFER BY CONDUCTION FROM CORE TUBE TO ANNULUS

$$\text{where } Q_c(X) = \frac{[T_i(X) - T_o(X)]}{\left[\frac{d_1}{k_m a_{m1}} + \frac{d_g}{k_g a_{mg}} + \frac{d_2}{k_m a_{m2}} \right]}$$

d_1, d_g, d_2 = thickness of core metal wall, gas gap, and outer tube metal wall, respectively, ft.

k_m, k_g = thermal conductivity of metal and gas respectively, Btu/hr-ft-°F

a_{m1}, a_{mg}, a_{m2} = log mean area of core tube wall, gas gap, and outer tube wall, respectively, per foot of tube, ft.

In the limit as $\Delta X \rightarrow 0$, the differential equation for core fluid temperature as a function of distance along the tube becomes

$$\frac{dT_i}{dX} + U_c [T_i(X) - T_o(X)] = 0 \quad (5b)$$

A heat balance on the region, $T_o(X)$, between X and $X+\Delta X$ yields the following equation

$$Wh/X+\Delta X - Wh/X + \int_X^{X+\Delta X} Q_c(X) dX - \int_X^{X+\Delta X} U_{a_o} [T_o(X) - T_B] dX = 0 \quad (6b)$$

Application of the Theorem of the Mean and Mean Value Theorem results in the desired differential equation

$$\frac{dT_o(X)}{dX} + U_c [T_i(X) - T_o(X)] - U_o [T_o(X) - T_B] = 0 \quad (7b)$$

$$\text{where } U_c = \frac{1}{WC_p \left[\frac{d_1}{k_m a_{m1}} + \frac{dg}{k_g a_{mg}} + \frac{d_2}{k_m a_{m2}} \right]}$$

$$U_o = \frac{U_{a_o}}{WC_p}$$

Combining Equations (5b) and (7b) results in the following boundary value problem

$$\frac{dT_i(X)}{dX} + U_c [T_i(X) - T_o(X)] = 0 \quad (8b)$$

$$\frac{dT_o(X)}{dX} + U_c [T_i(X) - T_o(X)] - U_o [T_o(X) - T_B] = 0 \quad (9b)$$

Boundary conditions:

(1) $T_i(o) = T_{in}$	(3) $T_i(L) = T_o(L)$
(2) $T_o(o) = T_o(\text{unknown})$	(coupling equation)

This system of differential equations can be readily solved by use of the Laplace transformation⁽²⁾. Laplace transforming Equations (8b) and (9b) and using the boundary conditions (1) and (2) yields:

$$SU_i(S) + U_c [U_1(S) - U_2(S)] = T_{in} \quad (10b)$$

$$U_1(S) = \frac{T_{in}}{S+U_c} + \frac{U_c U_2(S)}{S+U_c}$$

$$\begin{aligned}
SU_2(S) - T_o + U_c [U_1(S) - U_2(S)] - U_o U_2(S) + \frac{U_o T_B}{S} &= 0 \\
U_2(S) &= \frac{T_o}{S-C} - \frac{U_o T_B}{S(S-C)} - \frac{U_c U_1(S)}{S-C}
\end{aligned} \tag{11b}$$

$$\text{where } C = U_o + U_c$$

Substituting Equation (10b) into Equation (11b) yields an equation solvable for $U_2(S)$, thus

$$U_2(S) = \frac{T_o(S+U_c)}{[(S-C)(S+U_c)+U_c^2]} - \frac{U_o T_B(S+U_c)}{S[(S-C)(S+U_c)+U_c^2]} - \frac{U_c T_{in}}{[(S-C)(S+U_c)+U_c^2]}$$

Simplifying for ease of inversion gives

$$U_2(S) = \frac{U_c(T_o - T_{in}) - U_o T_B}{(S-S_1)(S-S_2)} + \frac{T_o S}{(S-S_1)(S-S_2)} - \frac{U_o U_c T_B}{S(S-S_1)(S-S_2)} \tag{12b}$$

$$\text{where } S_1 = \frac{U_o - (U_o^2 + 4U_o U_c)^{1/2}}{2}$$

$$S_2 = \frac{U_o + (U_o^2 + 4U_o U_c)^{1/2}}{2}$$

Inversion of Equation (11b) yields the following function for the temperature, $T_o(X)$

$$\begin{aligned}
T_o(X) &= [U_c(T_o - T_{in}) - U_o T_B] \left[\frac{e^{S_1 X} - e^{S_2 X}}{(S_1 - S_2)} \right] \\
&+ T_o \left[\frac{S_1 e^{S_1 X} - S_2 e^{S_2 X}}{(S_1 - S_2)} \right] \\
&- U_o U_c T_B \left[\frac{1}{S_1 S_2} + \frac{e^{S_1 X}}{S_1 (S_1 - S_2)} + \frac{e^{S_2 X}}{S_2 (S_2 - S_1)} \right]
\end{aligned} \tag{13b}$$

The final form of $U_1(S)$ can be written as follows

$$\begin{aligned}
U_1(S) &= \frac{T_o}{S-C} + \frac{U_c}{S+U_c} \left[\frac{U_c(T_o - T_{in}) - U_o T_B}{(S-S_1)(S-S_2)} + \frac{T_o S}{(S-S_1)(S-S_2)} \right. \\
&\quad \left. - \frac{U_o U_c T_B}{S(S-S_1)(S-S_2)} \right]
\end{aligned} \tag{14b}$$

Inversion of Equation (14b) results in the expression for the temperature in the core tube as a function of distance along the tube

$$\begin{aligned}
 T_i(X) = & T_{in} e^{-U_c X} + U_c T_o \left[\frac{-U_c e^{-U_c X}}{(S_1 + U_c)(S_2 + U_c)} + \frac{S_1 e^{S_1 X}}{(S_1 + U_c)(S_1 - S_2)} \right. \\
 & \left. + \frac{S_2 e^{S_2 X}}{(S_2 + U_c)(S_2 - S_1)} \right] + \left[U_c^2 (T_o - T_{in}) - U_c U_o T_B \right] \left[\frac{e^{-U_c X}}{(S_1 + U_c)(S_2 + U_c)} \right. \\
 & \left. + \frac{e^{S_1 X}}{(S_1 + U_c)(S_1 - S_2)} + \frac{e^{S_2 X}}{(S_2 + U_c)(S_2 - S_1)} \right] - U_c^2 U_o T_B \\
 & \left[\frac{1}{U_c S_1 S_2} + \frac{e^{-U_c X}}{(-U_c)(S_1 + U_c)(S_2 + U_c)} + \frac{e^{S_1 X}}{S_1 (S_1 + U_c)(S_1 - S_2)} \right. \\
 & \left. + \frac{e^{S_2 X}}{S_2 (S_2 + U_c)(S_2 - S_1)} \right] \quad (15b)
 \end{aligned}$$

The unknown temperature, T_o , is found by evaluating Equations (13b) and (15b) at $X = L$.

3. Heat Transfer from Core Tube to Annulus by Both Conduction and Radiation

Inclusion of radiative transport across the gas gap necessitates the use of a numerical procedure for solving the simultaneous differential equations. The model is identical to that shown in Figure B-2, with the exception that radiation across the gas gap is included. The differential equations describing the temperature variation along the tube are derived as follows.

A steady-state heat balance on a section of the core tube between X and $X + \Delta X$ results in

$$W_h/X - W_h/X + \Delta X - \int_X^{X + \Delta X} Q_c(X) dX - \int_X^{X + \Delta X} Q_R(X) dX = 0 \quad (16b)$$

$$\text{where } Q_R(X) = \bar{F} a_{mi} [T_1^4(X) - T_o^4(X)]$$

$$\bar{F} = \frac{0.173}{\frac{1}{\epsilon_m} + \left(\frac{a_{mi}}{a_{mi}} \right) \left[\frac{1}{\epsilon_m} - 1 \right]}$$

After taking the limit $\Delta X \rightarrow 0$ and rearranging, the differential equation for the temperature within the core tube can be written in the form

$$\frac{dT_i(X)}{dX} + U_c [T_i(X) - T_o(X)] + U_R [T_i^4(X) - T_o^4(X)] = 0 \quad (17b)$$

$$\text{where } U_R = \frac{\bar{F}a_{mi}}{WC_p}$$

Similarly, a heat balance in the annulus between X and $X+\Delta X$ yields the following equation

$$Wh_{X+\Delta X} - Wh_X + \int_X^{X+\Delta X} Q_c(X) dX + \int_X^{X+\Delta X} Q_R(X) dX - \int_X^{X+\Delta X} U_o a_o [T_o(X) - T_B] dX = 0 \quad (18b)$$

Applying the Theorem of the Mean and the Mean Value Theorem to Equation (18b) and letting $\Delta X \rightarrow 0$ gives the following differential equation describing temperature as a function of distance in the annulus

$$\begin{aligned} \frac{dT_o(X)}{dX} - U_o [T_o(X) - T_B] + U_c [T_i(X) - T_o(X)] + U_R [T_i^4(X) \\ - T_o^4(X)] = 0 \end{aligned} \quad (19b)$$

Equations (17b) and (19b) are non-linear and are solved by approximating the differential equations by a standard finite difference procedure. The following finite difference equations approximate Equations (17b) and (19b)⁽⁴⁾.

$$\begin{aligned} \frac{T_{i,m+1} - T_{i,m}}{\Delta X} + U_c [T_{i,m} - T_{o,n}] + U_R [T_{i,m}^4 - T_{o,n}^4] = 0 \\ \frac{T_{o,n+1} - T_{o,n}}{\Delta X} + U_c [T_{i,m} - T_{o,n}] - U_o [T_{o,n} - T_B] + U_R [T_{i,m}^4 - T_{o,n}^4] = 0 \end{aligned} \quad (20b)$$

A pictorial representation of Equation (20b) is shown in Figure B-3.

FIGURE B-3

MODEL FOR HEAT TRANSFER FROM CORE TUBE BY BOTH CONDUCTION AND RADIATION

The core tube has been divided into a number of perfectly mixed "cells", $m=1$ to $m=M$, each ΔX units long. Similarly, the outer annulus can be divided into a series of perfectly mixed cells $n=M+1$ to N . Using the concept of cells in conjunction with Equation (20b) results in the following system of finite difference equations.

$$T_1 + \frac{U_R \Delta X}{(1+U_c \Delta X)} T_1^4 = \frac{T_{in}}{(1+U_c \Delta X)} + \frac{U_c \Delta X}{(1+U_c \Delta X)} T_N + \frac{U_R \Delta X T_N^4}{(1+U_c \Delta X)} \quad (21b)$$

$$T_m + \left(\frac{U_R \Delta X}{1+U_c \Delta X} \right) T_m^4 = \frac{T_{m-1}}{(1+U_c \Delta X)} + \left(\frac{U_c \Delta X}{1+U_c \Delta X} \right) T_n + \left(\frac{U_R \Delta X}{1+U_c \Delta X} \right) T_n^4 \quad (22b)$$

$$(m = 2, 3, \dots, M)$$

$$T_{M+1} + \left(\frac{U_R \Delta X}{1+U_R \Delta X + U_c \Delta X} \right) T_{M+1}^4 = \left(\frac{1+U_c \Delta X}{1+U_c \Delta X + U_R \Delta X} \right) T_M + \frac{U_R \Delta X}{(1+U_c \Delta X + U_R \Delta X)} T_M^4 + \frac{U_o \Delta X}{(1+U_c \Delta X + U_R \Delta X)} T_B \quad (23b)$$

$$T_n + \left(\frac{U_R \Delta X}{1+U_c \Delta X + U_R \Delta X} \right) T_n^4 = \frac{T_{n-1}}{(1+U_c \Delta X + U_R \Delta X)} + \frac{U_c \Delta X}{(1+U_c \Delta X + U_R \Delta X)} T_m + \left(\frac{U_R \Delta X}{1+U_c \Delta X + U_R \Delta X} \right) T_m^4 + \left(\frac{U_o \Delta X}{1+U_c \Delta X + U_R \Delta X} \right) T_B \quad (24b)$$

$$(n = M+2, \dots N-1)$$

$$(m = 2, 3, \dots M-1)$$

$$\begin{aligned} T_N + \frac{U_R \Delta X}{(1+U_R \Delta X+U_c \Delta X)} T_N^4 &= \frac{T_{N-1}}{(1+U_c \Delta X+U_R \Delta X)} + \frac{U_c \Delta X}{(1+U_c \Delta X+U_R \Delta X)} T_1 \\ &+ \frac{U_R \Delta X}{(1+U_c \Delta X+U_R \Delta X)} T_1^4 + \frac{U_o \Delta X}{(1+U_c \Delta X+U_R \Delta X)} T_B \end{aligned} \quad (25b)$$

The system of Equations (21b) through (25b) were solved using the method of "regula falsi" (False position) in conjunction with an iterative procedure. A digital computer program, written in FORTRAN IV for solving the set of equations, is described in Appendix C.

APPENDIX C - COMPUTER PROGRAM

The flow diagram of the computer program used to solve Equations (21b) through (25b) is shown in Figure C-1. For the general case in which radiative transfer across the gas gap is considered, the following logic was employed:

1. The steady-state temperature of each hypothetical cell was assumed.
2. Constant coefficients were calculated.
3. New temperatures for each cell were calculated using the method of "regula falsi."
4. Iterations were repeated throughout the cell matrix until the change in all temperatures between iterations was less than a specified quantity.

A subroutine (TRAN) is present within the program for computing the analytical solution where possible.

Figure C-1

FLOW DIAGRAM FOR COMPUTER PROGRAM

1. Fortran Source Program For Computing The Temperature Along A Bayonet Tube

```

PROGRAM HEAT
COMMON UO, UC, TL, TI, TB, TO, CL, CAL, MP1, XX, TC, TOR
1, TCO, MP, TCA, TAA
DIMENSION T(200), T1(200), F1(200), CON2(200), X(10), XX(100)
1, TC(200), TCO(100), TCA(100), TAA(100)
READ 2, CA, W, CP, TL, TB, TI, PI, CO, D1, D2, D3, D4, D5
READ 1, NP
KK = 0
300 READ 1, M, N, NP1, MP1, NM1, ND
MP = M
II = 0
KK = KK + 1
IF(KK=NP) 80, 80, 100
80 READ 2, EM, CM, DEL, B, EV, ES, CAG, DEL1, CAL
II = II + 1
CL = TL/CM
A1 = PI * D1
A2 = PI * D2
A3 = PI * D3
A4 = PI * D4
A5 = PI * D5
A1 = (A2 - A1)/(LOGF(A2/A1))
A2 = (A3 - A2)/(LOGF(A3/A2))
A3 = (A4 - A3)/(LOGF(A4/A3))
UC = 1./((W*CP*((D2-D1)/(CA*A1)+(D3-D2)/(CAG*A2)+(D4-D3)/(CA*A5)))
UR = (.173*A2)/(W*CP*((1./EM)*(A2/A3)*((1./EM)+1.)))
UO = CO*A0/(W*CP)
C4 = UC/UR
C3 = 1./(1. + UC*CL)
C2 = UC*CL*C3
C4 = 1./(1. + CL*(UC + UO))
C5 = UO*CL*C4
C6 = UC*CL*C4
C7 = (1. + UC*CL)*C4
C8 = UR*CL*C3
C9 = UR*C4*CL
T1(1) = TI - DEL
T1(N+1) = 0.0
IF(EM) 81, 900, 900
900 DO 5 K=2, N
5 T1(K) = T1(K-1) - DEL
90 DO 3 K=1, N
T(K) = T1(K)
T(NP1-K) = T1(NP1-K)
T(K-1) = T1(K-1)
IF(K=1) 4, 4, 6
4 CON2(K) = T1*C3*C8*(4.6+T(NP1-K)*1.E-2)**4. + C2*T(NP1-K)
GO TO 3
6 IF(K=MP1) 7, 7, 9
7 CON2(K) = C2*T(NP1-K) + C8*(T(NP1-K)*1.E-2 + 4.6)**4.0
1 *C3*T(K-1)
GO TO 3
9 IF(K=MP1) 10, 10, 11
10 CON2(K) = C7*T(NP1-K) + C9*(4.6+T(NP1-K)*1.E-2)**4. + C5*TB
GO TO 3
11 CON2(K) = C4*T(K-1) + C9*(4.6+T(NP1-K)*1.E-2)**4. + C6*T(NP1-K)*

```

```

1  C5*TB
3  CONTINUE                                0072
   DO 12 K=1, N                            0072
   X(1) = T(K)                            0072
   X(2) = T(K) + DEL1                     0072
84  DO 13 I=1, 2                          0072
15  IF( K= M) 16, 16, 19                 0072
16  F1(I) = X(I) + C8* (X(I)+1.E=2 +4,6)**4, = CON2(K)
   GO TO 13                                0072
19  F1(I) = X(I)+ C9*( X(I)+1.E=2 +4,6)**4, = CON2(K)
13  CONTINUE
   X(3) = X(2) -((X(2)-X(1))* F1(2)/ (F1(2)- F1(1))) 0072
   IF( ABSF( X(3)- X(2))= B) 85, 88, 88 0072
88  X(1) = X(2)                            0072
   X(2) = X(3)                            0072
   GO TO 84                              0072
85  T1(K) = X(3)                          0072
12  CONTINUE                              0072
   DO 20 K= 1, N                          0072
   IF( ABSF( T1(K) - T(K)) = B) 20, 20, 90 0072
20  CONTINUE                              0072
81  CALL TRAN
21  PRINT 60                              0072
   IF(ES) 200,200,201
200  PRINT 70,CA
   70  FORMAT(/ 2X, 17HMETAL WALL , K = F8,2)
   GO TO 501
201  IF(EV) 204,204,205
204  PRINT 71, EM
   71  FORMAT(/ 2X, 26HAIR GAP WITH VAC, EMISS = F8,4)
   GO TO 203
205  PRINT 72,CAG,EM,CA
   72  FORMAT(/ 2X, 21HAIR GAP , K OF GAS = F8,4,4X, 17HEMISS OF METAL
1 = F8,4,4X, 13HK OF METAL = F8,2)
203  PRINT 61                              0072
   PRINT 62 , ( K, T1(K), K=1, N)         0072
   PRINT 48, TO, TOR
   IF(CAL)501,500,501
501  PRINT 502
502  FORMAT(/ 4X, 18HANALYTICAL RESULTS)
   PRINT 504
504  FORMAT(/ 4X,8H XX(I) ,4X, 8H TC(I) , 4X, 8H TCO(I),20X,
1 4HTCA , 8X, 3HTAA)
   DO 505 I = 1, MP1
505  PRINT 506, XX(I), TC(I) , TCO(I) , TCA(I) , TAA(I)
506  FORMAT(/ 3(4X, F10,3))
   48  FORMAT(/ 4X, 29HOUTLET TEMP IF FED TO CORE = F10,2, 4X,
1 32HOUTLET TEMP IF FED TO ANNULUS = F10,2)
500  IF (II= ND) 80, 300,300
   1  FORMAT( 10I4)                        0072
   2  FORMAT( 8F10,4)
   60  FORMAT( // 2X, 26HBAYONET TUBE HEAT TRANSFER) 0072
   61  FORMAT(/ 5( 4X, 21HCELL NO, TEMP, F )) 0072
   62  FORMAT( / 5( 6X, 13, 6X, F8,1))
   91  FORMAT(/ 8E14,4)
100  RETURN                                0072

```



```

SUBROUTINE TRAN
COMMON UO, UC, TL, TI, TB, TO, CL, CAL, MP1, XX, TC, TOR
1, TCO, MP, TCA, TAA
DIMENSION T(200), T1(200), F1(200), CON2(200), X(10), XX(100)
1, TC(200), TCO(100), TCA(100), TAA(100)
1 SR = SQRTF(UO**2, + 4, *UC*UO)
S1 = (UO+SR)/2.
S2 = (UO+SR)/2.
G1 = EXPF(-UC*TL)/((S1+UC)*(S2+UC))
G2 = EXPF(S1*TL)/((S1+UC)*(S1-S2))
G3 = EXPF(S2*TL)/((S2+UC)*(S2-S1))
A1 = UC**2, * (G1 + G2 + G3) + UC*(UC*G1 + S1*G2 + S2*G3)
B1 = TI*EXPF(-UC*TL) = (UC**2, *TI + UC*UO*TB) * (G1 + G2 + G3)
1 -UC**2, *UO*TB*((1./(UC*S1*S2)) + (G1/(-UC)) + (G2/S1) + (G3/S2))
Q1 = EXPF(S1*TL)
Q2 = EXPF(S2*TL)
C1 = UC*((Q1-Q2)/(S1-S2)) + ((S1*Q1-S2*Q2)/(S1-S2))
R1 = (UC*TI + UO*TB)*((Q1-Q2)/(S1-S2)) - UO*TB*UC*
1 ((1./(S1*S2)) + (Q1/(S1*(S1-S2))) + (Q2/(S2*(S2-S1))))
TO = (R1 - B1)/ (A1 - C1)
S1 = (UO+SR)/2.
S2 = (UO+SR)/2.
G1 = EXPF(UC*TL)
G2 = EXPF(S1*TL)
G3 = EXPF(S2*TL)
Y1 = G1 - UC**2, *(G1/((UC-S1)*(UC-S2)) + G2/((S1-UC)*(S1-S2)) +
1 G3/((S2-UC)*(S2-S1))) - UC*(G2/(S1-S2) + G3/(S2-S1))
Y2 = (S1-UC)*G2/(S1-S2) + (S2-UC)*G3/(S2-S1) + UC*(G2/(S1-S2) +
1 G3/(S2-S1))
Y3 = UC*(1./ (S1*S2) + G2/(S1*(S1-S2)) + G3/(S2*(S2-S1))) +
1 (-UC/(S1*S2) + (S1-UC)*G2/(S1*(S1-S2)) + (S2-UC)*G3/(S2*(S2-S1)))
1 )
TOR = ((Y2*TI) + (UO*TB*Y3))/Y1
PRINT 50, Y1, Y2, Y3, TOR, TO
50 FORMAT(/ / 10X, 5(F10.2, 4X))
X1 = UC*TI
X2 = UC*UO*TB
X3 = UC**2, *TOR
X4 = UO*TB
X5 = UC*TOR
Y = 0.0
XX(1) = 0.0
DO 400 I = 1, MP
TCA(I) = TOR*EXPF(UC*Y) + X1*((EXPF(S1*Y)/(S1-S2)) + (EXPF(S2*Y)/(S2
1 -S1))) + X2*((1./ (S1*S2)) + (EXPF(S1*Y)/(S1*(S1-S2))) + (EXPF(S2*Y)/(S
2 2*(S2-S1)))) + X3*((EXPF(UC*Y)/((UC-S1)*(UC-S2))) + (EXPF(S1*Y)/((
3 S1-UC)*(S1-S2))) + (EXPF(S2*Y)/((S2-UC)*(S2-S1))) )
TAA(I) = TI*((S1-UC)/(S1-S2))*EXPF(S1*Y) + ((S2-UC)/(S2-S1))*EXPF
1 (S2*Y) + X4*((UC/(S1*S2)) + ((S1-UC)/(S1*(S1-S2))))*EXPF(S1*Y)
2 + ((S2-UC)/(S2*(S2-S1)))*EXPF(S2*Y) + X5*((EXPF(S1*Y)/(S1-S2)) +
3 (EXPF(S2*Y)/(S2-S1)))
Y = Y + CL
KK = KK + 1
XX(KK) = Y
400 CONTINUE
IF(CAL)101,100,101

```

```

101 X1 = UC*(TO-TI) - UO*TB
    X2 = - UO*UC*TB
    Y = 0.0
    XX(1) = 0.0
    S1 = (UO+SR)/2.
    S2 = (UO+SR)/2.
    G1 = 1./((S1 + UC)*(S2+UC))
    G2 = 1./((S1 + UC)*(S1+S2))
    G3 = 1./((S2 + UC)*(S2 + S1))
    X3 = UC*TO
    X4 = UC**2.*(TO-TI) - UC*UO*TB
    X5 = -UC**2. *UO*TB
    DO 21 = 1, MP
    TC(1) = X1 * ( EXPF(S1*Y) - EXPF(S2*Y))/(S1 - S2)
1  * TO*(S1* EXPF(S1*Y) + S2*EXPF(S2*Y))/(S1 - S2)
2  *X2*( 1./((S1+S2) + EXPF(S1*Y)/(S1*(S1 - S2)) + EXPF(S2*Y)/(S2*(
3  S2-S1)))
    TCO(1) = TI*EXPF(-UC*Y) + X3*(UC*EXPF(-UC*Y)*G1 + S1*G2*EXPF(S1*Y
1) + S2*G3*EXPF(S2*Y)) + X4*(G1*EXPF(-UC*Y) + G2*EXPF(S1*Y) + G3*EXP
2F(S2*Y) + X5*(1./((UC*S1*S2) + G1*EXPF(-UC*Y)*(1./(-UC)) + G2*EXPF
3(S1*Y)*(1./S1) + G3*EXPF(S2*Y)*(1./S2))
    Y = Y + CL
    KK = 1 + 1
    XX(KK) = Y
2  CONTINUE
100  RETURN
    END

```

2. Program Nomenclature

A1, A2, A3, A4 - Area per foot of tube corresponding to radii shown in Figure 3.

A0 - Area per foot at outer surface of tube.

A1, Ag, A5 - Log mean areas of core tube, gas gap, and outer tube respectively.

B - Error criteria used in "regula falsi" calculation.

CA, CAG - Thermal conductivity of metal walls and gas gap respectively.

CO - Heat transfer coefficient at outer tube surface.

CM - Number of cells within core tube.

CL - Length of a single cell.

C2 - The quantity $\left(\frac{U_c \Delta X}{1 + U_c \Delta X} \right)$, (Equation 21b).

C3 - The quantity $\left(\frac{1}{1 + UC \Delta X} \right)$, (Equation 21b).

C4 - The quantity $\left(\frac{1}{1 + \Delta X (UC + UR)} \right)$, (Equation 23b).

C5 - The quantity $\left(\frac{UO \Delta X}{1 + \Delta X (UC + UR)} \right)$, (Equation 23b).

C6 - The quantity $\left(\frac{UC \Delta X}{1 + \Delta X (UC + UR)} \right)$, (Equation 23b).

C7 - The quantity $\left(\frac{1 + UC \Delta X}{1 + \Delta X (UC + UR)} \right)$, (Equation 23b).

C8 - The quantity $\left(\frac{UR \Delta X}{1 + UC \Delta X} \right)$, (Equation 21b).

C9 - The quantity $\left(\frac{UR \Delta X}{1 + \Delta X (UC + UR)} \right)$, (Equation 23b).

CON2(1) - The right hand side of Equation (21b).

CON2(K), K=2, ...M - The right hand side of Equation (22b).

CON2(K), K=M+2, ...N - The right hand side of Equation (24b).

D1, D2, D3, D4 - Diameters shown in Figure 3.

D5 - Outside diameter of bayonet tube.

DEL - Temperature increment used to establish the initial cell temperatures.

DEL1 - Temperature increment used in "regula falsi" subroutine.

ES - Control word, +1 corresponds to metal wall only, 0 corresponds to metal wall with a gas gap.

EV - Control word, +1 corresponds to no radiative transport, 0 corresponds to radiation.

EM - Emissivity of metal walls in gas gap.

F1(I) - A function used in calculating the roots of Equations (21b), (22b), 23b), 24b), and (25b).

M - The last cell within the core tube.

MP1 - The first cell in the outer annulus.

N - The last cell in the outer annulus.

NML - The next to last cell in the outer annulus.
 NP1 - A hypothetical cell at the outlet of the outer annulus.
 ND - Control word specifying number of problems to be run.
 TL - Length of bayonet tube.
 TO - Temperature of NaK leaving annulus.
 TI - Inlet temperature of NaK.
 T(K), K=1, N - Temperature of Cell K.
 Tl(K), K=1, N - Calculated temperature of Cell K during an iteration.
 UC - The quantity defined in Equation (7b).
 UO - The quantity defined in Equation (7b).
 UR - The quantity defined in Equation (17b).
 X(1), X(2) - Dummy variables used in the calculation of the roots of
 Equations (21b), (22b), (23b), (24b), and (25b).

SUBROUTINE TRAN

A1 - Quantity on the right hand side of Equation (15b).
 B1 - Quantity on the right hand side of Equation (15b).
 C1 - Quantity on the right hand side of Equation (15b).
 G1, G2, G3 - The exponential quantities on the right hand side of
 Equation (15b).
 R1 - The quantity on the right hand side of Equation (13b).
 Q1, Q2 - The quantities $\text{EXP}(S1*TL)$ and $\text{EXP}(S2*TL)$ respectively.
 SR - The square root quantity $[UO^2 + 4UC*UO]^{1/2}$.
 S1, S2 - The quantities $\left(\frac{UO - SR}{2}\right)$ and $\left(\frac{UO + SR}{2}\right)$ respectively.