SUMMARY OF ANALYSES OF PLATE VS. PIN MEASUREMENTS
EMPHASIZING REACTIVITY AND SODIUM VOID EFFECTS *

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I. Introduction

This paper presents analyses of measurements performed in pin and plate environments of nearly identical composition in critical assemblies ZPPR-2 and ZPR-6-7 conducted by Argonne National Laboratory. Nearly all fast reactor critical assembly measurements, prior to the ZPPR-2 and ZPR-6-7 assemblies, have been made in plate environments using typically 1/16 to 1/4 inch wide plates to simulate fast reactor compositions. The plate environment of critical assemblies introduces heterogeneity effects which differ significantly from that of an LMFBR which is comprised of fuel pins typically about 1/4 inch in diameter. Consequently, the recent pin versus plate measurements are an important contribution to removing uncertainties in the extrapolation from plate critical measurements and analyses to the LMFBR pin environment. An improved capability for confident calculations of plate versus pin effects will significantly improve the applicability of future plate criticals toward reducing LMFBR physics design uncertainties.

The pin versus plate measurements in ZPPR-2 and ZPR-6-7 were performed by replacing small regions of the normal plate core with pin calandria containing 16 fuel rods (0.410 and 0.348-inch diameter clad and mixed oxide fuel pellet diameters, respectively) across the approximately 2x2 inch cross sectional area. Figure 1 shows a cross section view of the ZPPR-2 assembly which outlines the central pin region (69 matrix drawers) and the radial pin sector. Initial pin versus plate measurements were performed in the central 69 drawers and the pin calandria were then rearranged into the radial pin sector for later measurements. Figure 1 also shows cross sections of the ZPPR-2 inner core zone plate arrangement and of the pin calandria. Assembly ZPR-6-7 was a large single zone assembly having the same plate arrangement and composition as the inner core zone of ZPPR-2.

In ZPR-6-7, the pin sector was comprised of 25 drawers. Axially, the ZPPR-2 pin region included the full 36 inch core height and 6 inches into the axial blankets while the ZPR-6-7 pin region included only the central 24 inches of the 60 inch core height.

The principal sources for differences in measurements or calculations are due to changes in heterogeneity affecting resonance self-shielding and the detailed fine neutron flux structure between fissile or fertile regions and the diluent (sodium, stainless steel, or iron) regions. An unresolved potential problem is whether neutron leakage is affected differently in plate or pin environment. The flux and resonance heterogeneity effects are dominated by changes affecting Pu-239 and U-238 with contributions from other isotopes comparatively small although not completely negligible. Heterogeneity effects lead to slightly softer neutron flux spectrum in the plate environment than in the pin or hypothetical truly homogeneous environment. The effects on eigenvalues (criticality) and on sodium voiding are the most difficult to calculate as the net effects on these measurements are comprised of compensating positive and negative reactivity effects. Heterogeneity effects on other measurements such as the Doppler effect, central cavity reaction rate ratios, etc., are due primarily to effects on the gross flux spectra and are generally easier to calculate.

This paper presents detailed results of analyses by the present authors on the principal plate versus pin measurements includes reactivity (eigenvalue), sodium void, reaction rate ratio and Doppler (heated sample measurements) effects. Principal emphasis is placed on eigenvalue and sodium void effects. Summary results of the authors are compared with currently available calculations from ANL-Idaho (Refs. 1, 2, 3) and ANL-Chicago (Refs. 4, 5). As more details are currently available to the authors for the WARD calculations, the author's results are initially presented and then compared with the ANL calculations.

Current uncertainties in delayed neutron yields lead to uncertainties on the order of 5% in the inhours/ Δk conversion factor which is necessary to compare experimentally measured inhours with calculated Δk . The WARD results in this paper have been converted using 1000 inhours for 1% Δk , as this value is comparable with previously published values for ZPPR-2. However, a recent evaluation of delayed neutron yields by the present authors^[7] leads to a calculated value of 953 ih/% Δk for ZPPR-2. Consequently, the ratios of calculation/experiment given in this paper could be reduced by about 5% for consistency with current evaluations of delayed neutron data.

II. Plate vs. Pin Reactivity Effect

The reactivity effect on replacing pins by plates is an important consideration in comparing calculated eigenvalues with experiment and corresponding estimates of eigenvalue uncertainties or adjustments for LMFBRs. Prior to the recent pin measurements, the only experimental data for testing heterogeneity estimates involved plate bunching experiments wherein reactivity effects of varying plate drawer arrangements were varied. The pin replacement experiments provide a more direct test of heterogeneity effects even though the experimental pin cells are somewhat more heterogeneous than an LMFBR fuel assembly.

Methods of Analysis

Based on current calculational methods, the principal eigenvalue effects are expected to be resonance self-shielding and cell structure of the neutron flux which also lead to effects on the neutron spectrum. Methods used for the plate calculations involved separate resonance self-shielding for each heavy element in each plate of the assembly. The WARD codes XSRES and WIDX (modifications of the 1DX^[6] code) are used for resonance self-shielding and one-dimensional diffusion theory spatial flux averaging of the cross sections. The methods for resonance self-shielding use resonance equivalence principles^[7] for assymmetric slab cells and cylindrical lattices. The cylindrical cell was based on the actual fuel pin diameter and matrix tube average volume ratios and material densities. Comparison of resonance self-shielding for the average cell with individual self-shielding for corner, edge and center pins indicates no significant differences for the matrix averaged cross sections. However, the differences could be significant for individual pin reaction rates. The calculations have been performed using 30 energy groups with modified ENDF/B-II data^[8]. Comparative calculations^[7] using this data and the ENDF/B-III data indicate the only significant differences between these data is that the ENDF/B-III data yield central sodium worth about 10% higher than the modified data.

Spatial cell homogenization^[7] is performed using the one-dimensional transport theory code, ANISN^[9], modified to perform the required cell flux weighting. Periodic boundary conditions are used for the plate cell calculations with input, plate independent values of $D_g B_g^2$ (based on two-dimensional diffusion theory calculations) for cell leakage representation.

The cylindrical cell calculations for the pin studies are particularly sensitive to the cell boundary conditions. Based on thermal reactor experience, "White" boundary conditions (isotropic reflection) are typically used for cylindrical cell calculations. However, the optical thickness of the moderation region in a thermal reactor is on the order of a neutron mean free path. Similarly, the fast reactor fuel pin is not as "black" to neutrons as a thermal reactor fuel pin. Using isotropic reflection for a fast reactor cell tends to force the angular flux in the cell to be almost isotropic everywhere in the cell. Some recent work by Choong^[10] indicates that, even for thermal reactor cells, the angular distribution for the returned neutrons should be slightly more peaked than the isotropic reflection assumption. Consequently, the white boundary condition can lead to an underestimate of the flux variations in the cell. For this reason, parallel calculations were conducted using both white and reflective boundary conditions. A four region cell model was used based on fuel, cladding, sodium and calandria can plus matrix volume fractions. Reflective boundary conditions overestimate the flux variations in the cell and the calculated fuel to cell average flux ratios do not converge as the angular order is increased from S6 to S16. The S6 calculations yield a net flux heterogeneity reactivity effect similar to the plate cell while S16 effects are considerably greater than for the plate cell. Cross sections from S6 calculations were used in ZPPR-2 analyses to provide results for an upper bound (S6 reflective boundary conditions) and lower bound (white boundary conditions) on the fuel pin to cell average flux variations. Peaking factors using the white boundary conditions are independent of angular orders greater than S6.

The ANL calculations used the MC² code^[11] for cross section averaging and resonance self-shielding with the CALHET code^[12] for spatial flux weighting. CALHET uses collision probability methods with the collision probabilities based on boundary conditions similar to the white condition.

ZPPR-2 Plate vs. Pin Reactivity Calculations

Table 1 presents results of ZPPR-2 plate versus pin heterogeneity calculations including perturbation theory isotopic contributions to the net effect. Also given are the corresponding components for the plate versus homogeneous calculations. Pin versus homogeneous effects can be obtained by subtracting the plate versus pin from the plate versus homogeneous effects.

The ZPPR-2 experimentally measured plate versus pin reactivity effect of 76.1 ± 1.6 inhours includes composition differences between the matched plate and pin calandria for which ANL has determined a correction of 8.43 ± 6.65 inhours based on experimental worth traverses. Then the composition corrected experimental value is 67.7 ± 6.8 inhours for the plate minus pin reactivity effect. The calculated composition correction is 16.2 inhours which included calculational errors for material worths. For ZPPR-2, the present ratio of calculation/experiment (C/E) for the central fissile worth is 1.15. About 5% of this C/E value is believed to be due to the use of 1000 ih/% Δk as noted in the introduction. Based on an argument (a probable but unproven argument) that the fissile central worth error is primarily due to errors in calculated flux and adjoint distributions, about half of the remaining 10% C/E error could be assigned to the axial distributions and half to the radial distributions. Since the rod zone in ZPPR-2 has full axial core length, only the radial 5% integrated worth errors would apply to the rod zone. Acceptance of these arguments leads to an expected C/E of 1.1 for axially integrated worths using 1000 ih/% Δk . Table 1 includes in parentheses the composition contributions adjusted by multiplying the calculated isotopic contributions by $1.1/(\text{Central Worth C/E})$. The adjusted composition correction

of 10.5 inhours is in acceptable agreement with the experimentally based correction of 8.43 ± 6.65 inhours. It should be noted that the above assumptions on the central worth error have been made, to further speculation, while seeking a solution to the problem and are not technically verified arguments.

The calculated plate versus pin reactivity effect is 137.8 inhours (88.4 ih from cell flux effects plus 49.4 ih from resonance self-shielding effects) compared to the experimental value of 67.7 ± 6.8 ih. Using reflective cylindrical cell boundary conditions (expected to overestimate the flux heterogeneity), the calculated reactivity effect is only 38.5 ih. The pin-homogeneous calculated reactivity effect is 87.0 ih ($224.8 - 137.8$) and 186.3 ih for white and reflective boundary conditions, respectively, compared to 224.8 ih for the plate-homogeneous reactivity effect. The results using reflective boundary conditions can be expected to overestimate the flux heterogeneity for the pin cell and consequently underestimate the plate-pin flux contribution.

The discrepancy between calculation and measurement for the plate-pin reactivity effect could be due to an overestimate of the plate flux or resonance heterogeneity or corresponding underestimates for the pin geometry. The strong sensitivity to the cylindrical cell boundary condition indicates this problem to be a potentially significant source of error since the adequacy of white boundary conditions for a fast reactor cell does not appear to be established. Sensitivity calculations performed for the plate cell by the present authors indicate variations of only about 15% based on acceptable combinations of models, current methods and data. Even assuming a 15% overestimate of the plate heterogeneity, the experimental plate-pin heterogeneity would be overestimated by 50% for the white boundary condition calculation. The cylindrical resonance self-shielding appears to be least sensitive to methods and data errors. It appears that unexpectedly large modifications of the plate heterogeneity calculations combined with an underestimate of the pin flux heterogeneity (when using white boundary conditions) would be required to obtain agreement between calculation and experiment. Alternately, the calculations may not include a reactivity

effect present experimentally but not included in the calculations. This possibility is further examined below. The present calculations have not included either flux or resonance self-shielding heterogeneity contributions from the axial blanket. Qualitatively, ~~these~~ contributions are expected to be small and likely to lead to a more positive plate-pin reactivity effect which would further ~~increase~~ the discrepancy with experiment. However, a calculation of the blanket contribution has not yet been performed.

Since plate versus pin reactivity measurements have been performed in both ZPPR-2 and ZPR-6-7, it is desirable to quantitatively check the relative consistency of the measurements. If the pin zones for either assembly introduce no large perturbations or unexpected reactivity effects, the measurements should be approximately the same when adjusted for differences in pin region volumes, perturbation denominators (integrated fission worth) and peak to average worth ratio of the Δk plate versus pin reactivity effect. The resulting ratio would then be given by:

$$\frac{\Delta k(\text{ZPPR-2})}{\Delta k(\text{ZPR-6-7})} \approx (\text{Pin volume ratio}) \div (\text{perturbation denominator ratio})$$

$$\div (\text{pin zone peak/average worth ratio})$$

$$\frac{\Delta k(\text{ZPPR-2})}{\Delta k(\text{ZPR-6-7})} \approx \left(\frac{69 \times 36''}{25 \times 24''} \right) \div \left(\frac{161.3}{118.2} \right) \div \left(\frac{1.7}{1.2} \right)$$

$$\frac{\Delta k(\text{ZPPR-2})}{\Delta k(\text{ZPR-6-7})} \approx 2.1 \quad \text{estimated ratio.}$$

The first term in the above ratio corrects for the number and length of the pin calandria used in each experiment. The second term uses the experimental central Pu-239 worths as an indicator of the perturbation denominator ratio. The third term is based on calculated peak/average worth distributions for the plate versus pin reactivity change in ZPPR-2. Neglecting unexpected contributions to this ratio, the above ratio of 2.1 should be accurate to better than 25%. The same ratio based on calculated central worths might be 10% lower due to the higher C/E ratio for the Pu-239 worth normally calculated for ZPR-6-7.

The experimental plate versus pin reactivity effect is:

$$\frac{\Delta k(\text{ZPPR-2})}{\Delta k(\text{ZPR-6-7})} = \frac{67.7 \pm 6.8}{52.3 \pm 3.4} = 1.29 \pm 0.23$$

The difference between the expected ratio of 2.1 and the experimental ratio of 1.29 may be an indication of an unexpected reactivity effect present in ZPPR-2 but not in ZPR-6-7. If the ZPPR-2 calculated value of 137.8 is adjusted by the expected 2.1 ratio, the resulting value of 65.6 ih would be in much better agreement with the ZPR-6-7 measurements than for the ZPPR-2 measurement. Some possible sources for the "apparent" inconsistency between the ZPPR-2 and ZPR-6-7 measurements could be: blanket effects in ZPPR-2 which would not occur in ZPR-6-7; local perturbation greater in ZPR-6-7; unknown experimental errors in one or both assemblies; and an unexpected reactivity effect more dominant in one of the two assemblies.

The possibility of an unexpected reactivity effect appears to be worth continued speculation in view of possible relation to the well known central worth discrepancy. A potential and conceivable explanation which could have significant effects on the use of critical experiments for design applications is the following: The plate criticals could have more leakage parallel to the plates due to streaming considerations than allowed for in present calculations and more leakage than in a rod environment. Then substitutions of pins with plates would lead to a less positive reactivity effect than currently calculated due to the enhanced leakage effect in a plate environment. This effect would be substantially larger in ZPPR-2 than ZPR-6-7 due to the greater axial flux gradients over the pin region of ZPPR-2 and thus reduce the expected reactivity ratio in the direction of the experimental ratio. Also the effect would tend to yield spatial flux distributions somewhat flatter in the core regions than currently calculated. This could lead to integrated fission worths (perturbation denominator) higher than presently calculated and tend to reduce the central worth discrepancy.

This possible explanation appears to be worth more careful study due to the potential impact of such a problem. Colin Durston of WARD has suggested (private communication) the possibility that different diffusion coefficients (or transport cross sections) may be required parallel and transverse to the plates such as frequently used for thermal reactor applications. The current theory on this problem seems to be qualitatively consistent with the above leakage speculation.

Comparisons of Independent Plate versus Pin Heterogeneity Calculations

In addition to the present calculations, the plate versus pin reactivity effects have been calculated for ZPPR-2 by ANL-Idaho^[1] and for ZPR-6-7 by ANL-Chicago^[4]. Results of these calculations are given in Table II and include the results of total core plate heterogeneity (plate versus homogeneous) calculations. The ANL calculations of 82.6 ih for the ZPPR-2 plate vs. pin reactivity effect differs significantly from the present authors' result of 137.8 ih. About half of this difference may be attributable to the plate heterogeneity differences of 1.2% Δk by ANL and 1.63% Δk by WARD. ANL's latest calculation of 1.6% Δk for the ZPPR-2 plate heterogeneity is more consistent with the WARD value. Recalculation of the plate vs. pin effect is currently underway at ANL and more detailed comparisons will be possible when these calculations are completed.

The ANL-Chicago calculation of 50.9 ih for ZPR-6-7 is in significantly better agreement with experiment than the ZPPR-2 calculations. This improved agreement with experiment for ZPR-6-7 calculations is consistent with expectations based on the previous discussions although the ANL C/E value for ZPPR-2 is notably better than for the WARD calculations.

III. Reaction Rate Ratio and Doppler Effect Calculations

In Table III, comparisons of calculation and experiment are made for pin and plate environment central reaction rate ratios, central worths and the ^{238}U Doppler effect. Calculations are included in the table using both white and reflective boundary conditions for the cylindrical cell calculations. The white boundary condition calculations yield a slightly harder neutron flux spectrum than obtained with reflective conditions. This result is indicated by the lower ^{238}U Doppler effect, ^{10}B central worth and $\sigma_Y^{28}/\sigma_f^{49}$ ratio obtained with the white boundary condition. Overall the calculated pin/plate ratios are in better agreement with experiment using the white boundary condition.

The ZPPR-2 experimental reaction rates given in the table are fission counter measurements while the ZPR-6-7 values are foil measurements averaged over the cell. Since the counter measurements tend to be an average measurement over the central drawer spectrum, they can be best compared with a calculation using homogeneous cross sections as given for the non-asterisked calculations in Table III. The ZPR-6-7 cell-average measurements are best represented by the calculations using heterogeneous cell-homogenized cross sections as indicated with an asterisk in Table III. For both types of calculations, it appears that the pin/plate increase in the $\sigma_f^{28}/\sigma_f^{49}$ and $\sigma_f^{40}/\sigma_f^{49}$ ratios are underestimated while the $\sigma_f^{25}/\sigma_f^{49}$ and $\sigma_Y^{28}/\sigma_f^{49}$ ratios are well-calculated. For the plate core, the sodium voided reaction rate ratios are calculated with essentially the same accuracy as for the normal core.

The pin/plate ratio of 0.96 for the ^{239}Pu central worth is in good agreement with the experimental ratio. The pin/plate calculated ratio of 0.96 for the sodium central worth is in apparent disagreement with the experimental ratio of 1.16 ± 0.10 . However, local geometry effects on these measurements appear to be large and the large region sodium void results of the next section of this paper are more amenable to calculation.

For the ^{238}U Doppler effect, the pin/plate calculated ratio is a few percent higher than the experimental ratio although within error bounds of the experimental ratios. The Doppler effect for the sodium voided core is underpredicted relative to experiment by about 12% compared to about 3% for the normal core. This discrepancy would indicate an over prediction of the spectral hardening upon sodium voiding.

IV. Sodium Void Calculations

The basic method used for sodium void calculations was first order perturbation theory. Cross section differences between the sodium-in and sodium-out conditions were obtained by performing separate resonance self-shielding and cell flux calculations. Checks on the accuracy of first order perturbation theory were performed using direct eigenvalue and "exact" (perturbed flux, unperturbed adjoint) perturbation theory calculations.

Central Sodium Void Calculations

Table IV compares calculation and experiment for ZPPR-2 sodium voiding of the central regions up to 69 matrices. The first order perturbation theory calculations for the plate core indicate C/E values slightly greater than unity near the core center and slightly less than unity for the 69 matrix, 12 inch axial height (12 inches above and below core midplane) void region. A direct eigenvalue calculation for 18 inch (full core height) axial voiding gave a C/E of 0.98 compared to 1.10 for perturbation theory. As noted later, this difference is consistent with an underestimation of the leakage component for sodium voiding. As the voided region increases in size up to 69 matrices, the flux spectrum near the core center tends to become progressively harder leading to more positive sodium voiding than predicted by perturbation theory. Direct eigenvalue calculations, which can account for the spectral hardening, predict more constant C/E values with progressive voiding than perturbation theory.

The pin voiding calculations using white boundary conditions yield about 5% less positive void effects and lower pin/plate ratios than the calculations using reflective boundary conditions. The white boundary condition results tend to yield a somewhat flatter C/E value as the void size increases although no clear preference on boundary conditions can be made based on the C/E values. As the void height increases, the calculated pin/plate ratios increase by 7-10%.

However, the experimental accuracy for the central 12 and 18 inch void regions is insufficient to judge the accuracy of this increase.

Overall, the voiding for the central regions is calculated within about 5% accuracy for both the pin and plate environments with no unexpected difficulties in calculating either the pin or plate measurements.

Components of Sodium Voiding

Table V presents the components of sodium voiding including the principal isotopic contributions from Na, ^{238}U , and ^{239}Pu . The contributions from ^{238}U and ^{239}Pu are given separately for cross section changes due to cell flux and resonance self-shielding differences between sodium-in and sodium-out.

From the table, it is seen that contributions from isotopes other than sodium contribute 15 to 20% to the net (given by $\delta\sigma^{\text{core}}$) void effect. As expected, the principal resonance self-shielding contribution occurs in ^{238}U which is a large positive effect while ^{239}Pu self-shielding contributes a small negative effect. The principal effects from the cell flux variations for the plate core is a negative fission component dominated by ^{239}Pu . Upon sodium voiding, the cell flux variations are reduced (the cell becomes more homogeneous). In the plate case, the largest effect is a reduction of the high energy flux peaking effects in the fissile plates so that the voided fission cross section is lower than the sodium-in cross section. Calculations of this flux effect for sodium voiding are sensitive to representation of axial leakage in the cell model so that the calculated flux terms are probably uncertain to about 25%.

The dominant calculated differences between pin and plate voiding are due to a smaller ^{238}U resonance self-shielding contribution and a large reduction in the contributions from cell flux variations. Resonance self-shielding contributions from ^{238}U for the plate case are reduced

by about 20% (0.85 ih to 0.64 ih for central 6 inch voiding) for the same for both pin and plate. The flux effect contributions for the pin calculations are significantly different between white and reflective boundary conditions. Using white conditions, cell flux variations are small and flux effect contributions to sodium voiding are very small. With reflective conditions, the principle effect is a reduction in the low energy flux depressions in the fuel pin so that voided fission cross sections are higher than for sodium-in. This leads to a positive ^{239}Pu fission contribution to sodium voiding in contrast to the negative effect for plate voiding.

It is seen that the differences between pin and plate voided are almost completely dominated by differences in the heterogeneity contributions with cell flux effects being the largest difference. Moderation and leakage components are very similar for both pin and plate calculations although the white boundary condition pin calculations have 5% lower moderation and leakage components. The fact that the moderation components are similar for both pin and plate environment is largely due to a cancellation of effects. The adjoint spectrum for the plate case has a greater slope than the pin case, particularly at high energies. The positive void effect of this increased adjoint slope is compensated by a harder neutron spectrum for the pin case.

The increase in the pin/plate ratio upon progressive axial sodium voiding is largely due to the smaller leakage components in the pin environment compared to the plate environment. This effect may be due to differences in composition between the pin and plate environments rather than the true plate versus pin effects.

The first order perturbation theory calculations given in this paper used the conventional assumption that:

$$\delta D = \frac{-\delta \Sigma_{tr}}{\Sigma_{tr} \Sigma_{tr}^1}$$

can be approximated by:

$$\delta D \approx \frac{-\delta \Sigma_{tr}}{\Sigma_{tr}^2}$$

where Σ_{tr}^1 is the perturbed transport cross section. For sodium voiding in ZPPR-2, the first, more exact expression yields leakage terms about 20% larger than the second approximation. The choice of terms is related to changes in the flux gradients upon sodium voiding. The present calculations indicate that agreement between first order perturbation theory and more exact calculations is improved if leakage terms calculated using the second expression are increased by 10%. Table V includes (in parenthesis) net calculations have the leakage term increased by 10%. These adjusted results are in better agreement with the direct eigenvalue calculations for 18 inch axial voiding.

Off-Center Plate Sodium Void Calculations

At the present time, no significant analyses have been reported for measurements performed in the pin sector of ZPPR-2, shown in Figure 1. Both R-Z and X-Y geometry calculations of plate core sodium voiding for regions A and B of Figure 1 have been performed and the results are given in Table VI.

Based on the comparisons of R-Z and R calculations, reference methods for the X-Y calculation included: region dependent, group independent B^2 averaged over a 12-inch core height; axial leakage calculations based on R-Z calculations; and a normalization correction of 0.81 for 12-inch axial voiding relative to 18-inch core height. Sensitivity calculations were also made using group dependent B^2 . The X-Y calculations were performed using quarter-core symmetry with four mesh points per ZPPR-2 matrix position (drawer). The inner and outer core radii for the R-Z calculation are 64.67 and 91.45 cm., respectively, (based on equal areas of actual core zones) while the X-Y radial distances to the outer

core region B are 63.53 and 91.15 cm., for the inner and outer edge, respectively. Due to the shorter radial distance to region B in the actual X-Y geometry, the flux levels in region B will be slightly higher (particularly near the inner core) than obtained in the R-Z calculations.

Table VI compares calculation and experiment for the measurements reported in regions A and B. For the R-Z void calculations, the calculations for region B are given as direct R-Z results in inh/kg and as a linear average over the five column subdivisions of region B (indicated by footnote e in the Table). The first procedure is representative of voiding a truly radial sector while the linear average better represents the experimental square region voiding. Components of the calculated worths including leakage terms in the radial and transverse directions from the X-Y calculation are given in the Table. The fission, absorption and moderation terms from first order perturbation theory are about 3% higher in the X-Y calculation than the R-Z calculation (footnote e in the table). Radial leakage (sum of radial and transverse terms in X-Y) is about 3% lower in the X-Y calculation. These differences are due primarily to the shift of 1.14 cm. in the radial inner-outer core boundary as noted above.

The measurements shown in Table VI for each column of region B provides a qualitative check on the predictions for the spatial dependence of the void coefficient across the outer core zone. The results given in Table VI shows that experiment predicts the void coefficient to become negative for Column 3 while calculation predicts a negative contribution starting in Column 2. This discrepancy is consistent with an underprediction of the positive void contributions (as also indicated for the 69 matrix central voidings) if one assumes that the leakage terms are accurately predicted. For example, if the moderation component is increased by 10%, the X-Y calculations agree with experiment to better than

5% for all measurements regions A and B except Column 2 of B which has a large experimental uncertainty. However, the results from the central region voiding and the exact perturbation theory results of Table VI indicate that the first order perturbation theory leakage terms should be increased by 10%. Tests of diffusion theory against transport theory may be necessary to check the accuracy of diffusion theory for outer core zone voiding. For the regions A and B of ZPPR-2, it appears that differences in the radial location of and B of ZPPR-2, it appears that differences in the radial location of the voided regions between R-Z and X-Y geometry are more important than representation of the detailed geometry of the voided region (square in X-Y versus ring in R-Z). However, for the 5x5 drawer voiding of region B as contrasted to whole outer core voiding, the R-Z exact perturbation theory results tend to overpredict the spectral hardening upon voiding (due to the necessity to void the whole outer core region in R-Z geometry) compared to the X-Y calculations. This trend is indicated by the differences between first order and exact perturbation theory of -0.09 and -0.27 ih/kg for R-Z and X-Y geometry, respectively.

The experimental values indicate that pin voiding for regions A and B is less positive than plate voiding which is the opposite effect than found for the central void regions. Since the pin sectors is smaller than the central pin zone, local boundary effects could be more influential for the pin sector. Pin sector calculations should be performed and are particularly significant in view of the relatively large C/E value of 1.20 found for region B compared to about 0.9 for region A.

In this study, comparisons were made of R-Z geometry void calculations with one-dimensional R calculations. These studies indicated that the use of group dependent bucklings (obtained from R-Z geometry neutron flux solutions) gave excellent agreement for the neutron flux between the R calculations and R-Z values averaged over the height used to calculate the bucklings. However, the adjoint spectrum obtained using the group dependent bucklings gave poorer agreement with the R-Z adjoint spectrum than obtained with group independent bucklings. These studies

also indicated that the use of transverse δDB^2 terms to calculate axial leakage for partial core height voiding is subject to very large inaccuracies. The buckling changes in core regions adjacent to the voided region (particularly axially adjacent regions) can contribute as much to the leakage term as the voided region itself. It would be difficult to incorporate these effects into X-Y calculations.

The methods used for the X-Y void calculations (combining X-Y and R-Z results) appears to provide a practical calculational scheme with adequate accuracy for voiding of irregular core geometries. Direct three-dimensional calculations would probably be required to obtain significantly improved accuracy. For full core height voiding, the use of axial leakage based on DB^2 can be expected to provide accuracy comparable to an R-Z leakage calculation although it might be necessary to include the DB^2 calculation in regions adjacent to the voided region for highly accurate calculations. The use of X-Y methods rather than R-Z methods for sodium voiding can be expected to be of greater importance for cores containing heterogeneities such as control rods (a reactor design, ZPPR-Assembly 3, FFTF mockup critical assemblies). In these cases, the spatial variation of the void coefficient is significantly greater than the nearly radial dependence in ZPPR-2.

V. Comparisons of ANL and WARD Calculations

Table VII compares currently available analyses for pin and plate measurements in ZPPR-2 and ZPR-6-7. As noted previously, the calculation/experiment ratios for the pin versus plate reactivity effect differ significantly between the ANL results for ZPPR-2 and ZPR-6-7. The largest difference, however, is the ZPPR-2 C/E value of 2.04 by the present authors compared to 1.22 by ANL. These experiments are currently being recalculated by ANL using the SDX code and ENDF/B-III data.

Preliminary ANL values for the total plate heterogeneity (see Table II) indicate better agreement with the WARD results than the previous values using MC² and ENDF/B-I data.

Calculated pin/plate ratios for both ANL calculations and the WARD calculations are in good agreement for sodium voiding, ²³⁸U Doppler effect, central reaction rate ratios and the central ²³⁹Pu worth. There is a consistent trend to underpredict the pin/plate reaction ratios for the threshold fission of ²³⁸U and ²⁴⁰Pu although the calculated ratios are close to the error bounds of the experimental data. The experimental pin/plate ratios for sodium voiding in ZPR-6-7 show equal or smaller pin voiding effects than plate voiding in contrast to the larger pin voiding effects in ZPPR-2. This experimental difference could be due to the small pin region and larger uncertainties for the ZPR-6-7 measurements.

VI. Conclusions

With the exception of the pin versus plate reactivity effects, calculations at both ANL and WARD reproduce quite well the measured values for pin/plate ratios including sodium void, Doppler effect, reaction rate ratio, and central ^{239}Pu worth measurements. Differences in absolute values calculated for the latter measurements are largely due to differences in the cross section data used at ANL and WARD.

The trend to overpredict the pin versus plate reactivity effect in ZPPR-2 compared to ZPR-6-7, particularly in the WARD calculations, indicates the possibility of a reactivity effect which is not accounted for in the calculations. The possibility of a reactivity effect due to greater neutron streaming parallel to the plates than occurs in the rod environment has been postulated in this paper. C. Durston of WARD has suggested an examination of methods for using different diffusion coefficients parallel and perpendicular to the plates in order to account for neutron diffusion asymmetry. If additional studies should verify an asymmetry effect, significant effects on eigenvalues, worth calculations and reaction rate distributions could be expected. Sodium void calculating could be particularly affected if directional leakage effects were important. The most immediate effect would be on design applications where bias factors or uncertainties are established from plate critical experiments and applied to a rodged reactor where asymmetry effects would be considerably smaller.

The strong sensitivity of cylindrical cell flux calculations to the cell boundary conditions indicates a need for further study on fast reactor applications. Although white boundary conditions are clearly superior to reflective conditions, it appears likely that the white conditions underestimate the cell flux variations. The use of collision probabilities for a pin in a square lattice, such as those developed by Brissendon^[13], could offer an improved solution for the critical experiment analyses while triangular pitch solutions might be required for fast reactor lattices.

- ☒ OUTER CORE DOUBLE COLUMN DRAWERS
- ☐ CONTROL RODS - C₁ - HALF 1, C₂ - HALF 2
- ☐ POISON SAFETY (OR SHIM) RODS

HALF 2 IS A MIRROR IMAGE
A & B Na VOIDING ZONES

ZPPR HALF 1

Central Pin Zone and Rodded Pin Sector

Fig. 1. Positions of Large Off-center Sodium voiding zones

Central Plate

Central Pin Plate

- ☐ OUTER CORE DOUBLE COLUMN DRAWERS
 - ☐ CONTROL RODS C₁ - HALF 1, C₂ - HALF 2
 - ☐ POISON SAFETY (OR SHIM) RODS
- HALF 2 IS A MIRROR IMAGE
A & B NO VOIDING ZONES

ZPPR HALF 1

Central Pin Zone and Axial Pin Sector

Fig. 2. Positions of Large Off-center Sodium-voiding Zones

TABLE I. CONTRIBUTIONS TO PIN AND PLATE HETEROGENEITY IN ZPPR-2 BY PERTURBATION THEORY

Isotope	Plate Het. - Homogeneous			Plate Het. - Pin Het. (White B.C.)			
	Flux	Resonance	Net	Flux	Resonance	Composition	Net
²³⁸ U oxide	-15.5	63.2	47.7	18.6(-24.1)*	73.0	4.6(3.8) ⁺	96.2
²³⁸ U metal	46.3	72.6	118.9				
²³⁹ Pu	62.3	-29.5	32.8	51.3(5.2)	-28.0	0.7(0.7)	24.0
²⁴⁰ Pu	14.0	5.5	19.5	13.8(6.9)	4.4	-0.5(-0.5)	17.7
²⁴¹ Pu	-1.1	-0.1	-1.2	-1.2(-1.4)		-22.1(-20.7)	-23.3
²⁴¹ Am	-	-	-	-		16.5(15.3)	16.5
²⁴² Pu	0.3		0.3	0.2(0.2)		0.1(0.1)	0.3
²³⁵ U	-0.2		-0.2	-0.1(-0.2)		-1.8(-1.7)	-1.9
Na	2.7		2.7	1.1(-2.2)		-0.3(-0.3)	0.8
G	2.5		2.5	4.9(15.2)		-0.8(-0.9)	4.1
Fe	-0.7		-0.7	-0.9(-8.0)			
Cr	0.8		0.8	-0.2(-1.9)		29.4(23.5)	29.2
Ni	0.6		0.6	0.3(-1.2)		15.1(15.5)	15.4
Mo	1.1		1.1	1.4(1.5)		-1.5(-1.3)	-0.1
Mn				-0.7(-0.8)		4.4(2.7)	3.7
Al, Si, Cu				-0.1(-0.1)		0.1(0.1)	0.0
²³⁸ U Blkt.				-5.1(-4.3)			-5.1
²³⁵ U Blkt.				1.1(1.0)			1.1
Na + O Blkt.				-1.4(-1.5)			-1.4
Fe, Ni, Cr Blkt.				-3.5(-3.3)			-3.5
Net Calculation (Perturbation Theory)	113.1	111.7	224.8	88.4(-10.9)*	49.4	16.2(10.5) ⁺	153.0
Calculation (Direct Eigenvalue)							140(36)*
Experiment				67.7±6.8		8.43±6.65	76.1±1.6

* Values based on reflective boundary conditions for transport theory cell calculations.

	Flux	Resonance	Net	Flux	Resonance	Composition	Net
²³⁸ U oxide	-15.5	63.2	47.7	18.6(-24.1)*	73.0	4.6(3.8) ⁺	96.2
²³⁸ U metal	46.3	72.6	118.9				
²³⁹ Pu	62.3	-29.5	32.8	51.3(5.2)	-28.0	0.7(0.7)	24.0
²⁴⁰ Pu	14.0	5.5	19.5	13.8(6.9)	4.4	-0.5(-0.5)	17.7
²⁴¹ Pu	-1.1	-0.1	-1.2	-1.2(-1.4)		-22.1(-20.7)	-23.3
²⁴¹ Am	-	-	-	-		16.5(15.3)	16.5
²⁴² Pu	0.3		0.3	0.2(0.2)		0.1(0.1)	0.3
²³⁵ U	-0.2		-0.2	-0.1(-0.2)		-1.8(-1.7)	-1.9
Na	2.7		2.7	1.1(-2.2)		-0.3(-0.3)	0.8
O	2.5		2.5	4.9(15.2)		-0.8(-0.9)	4.1
Fe	-0.7		-0.7	-0.9(-8.0)			
Cr	0.8		0.8	-0.2(-1.9)		29.4(23.5)	29.2
Ni	0.6		0.6	0.3(-1.2)		15.1(15.5)	15.4
Mo	1.1		1.1	1.4(1.5)		-1.5(-1.3)	-0.1
Mn				-0.7(-0.8)		4.4(2.7)	3.7
Al, Si, Cu				-0.1(-0.1)		0.1(0.1)	0.0
²³⁸ U Blkt.				-5.1(-4.3)			-5.1
²³⁵ U Blkt.				1.1(1.0)			1.1
Na + O Blkt.				-1.4(-1.5)			-1.4
Fe, Ni, Cr Blkt.				-3.5(-3.3)			-3.5
Net Calculation (Perturbation Theory)	113.1	111.7	224.8	88.4(-10.9)*	49.4	16.2(10.5) ⁺	153.0
Calculation (Direct Eigenvalue)							140(36)*
Experiment				67.7±6.8		8.4±6.65	76.1±1.6

* Values based on reflective boundary conditions for transport theory cell calculations.

+ Normalized by 1.1/Central Worth C/E.

TABLE II. PLATE AND PIN HETEROGENEITY CALCULATIONS

Quantity	Experiment	C A L C U L A T I O N		
		W-ARD	ANL Idaho	ANL Chicago
Calculated Total Core Plate				
Heterogeneity (%Δk)				
<u>ZPPR-2</u>				
Flux		1.00	0.9	
Resonance		0.63	0.3	
Net		1.63	1.2(1.6)**	
<u>ZPR-6-7</u>				
Flux				1.11
Resonance				0.24
Net				1.35(1.66)**
Plate-Pin Δk(1h)				
<u>ZPPR-2</u>				
Flux		88.4(-10.9)+		
Resonance		49.4		
Net	67.7±6.8*	137.8(38.5)+	82.6	
<u>ZPR-6-7</u>				
Net	52.3±3.4			50.9

* Expected ratio for Δk (ZPPR-2)/ Δk (ZPR-6-7) is approximately 2.1.

+ Reflective boundary conditions for pin cell.

** ANL results using ENDF/B-III data and SDX cross section averaging.

TABLE III. PLATE AND PIN CALCULATIONS FOR REACTOR RATE RATIOS, CENTRAL WIRING, AND TUBULAR SUPPLER

Item	270-2				270-3				270-4				270-5				270-6				270-7				270-8				270-9				270-10				270-11				270-12				270-13				270-14				270-15				270-16				270-17				270-18				270-19				270-20				270-21				270-22				270-23				270-24				270-25				270-26				270-27				270-28				270-29				270-30				270-31				270-32				270-33				270-34				270-35				270-36				270-37				270-38				270-39				270-40				270-41				270-42				270-43				270-44				270-45				270-46				270-47				270-48				270-49				270-50				270-51				270-52				270-53				270-54				270-55				270-56				270-57				270-58				270-59				270-60				270-61				270-62				270-63				270-64				270-65				270-66				270-67				270-68				270-69				270-70				270-71				270-72				270-73				270-74				270-75				270-76				270-77				270-78				270-79				270-80				270-81				270-82				270-83				270-84				270-85				270-86				270-87				270-88				270-89				270-90				270-91				270-92				270-93				270-94				270-95				270-96				270-97				270-98				270-99				270-100				270-101				270-102				270-103				270-104				270-105				270-106				270-107				270-108				270-109				270-110				270-111				270-112				270-113				270-114				270-115				270-116				270-117				270-118				270-119				270-120				270-121				270-122				270-123				270-124				270-125				270-126				270-127				270-128				270-129				270-130				270-131				270-132				270-133				270-134				270-135				270-136				270-137				270-138				270-139				270-140				270-141				270-142				270-143				270-144				270-145				270-146				270-147				270-148				270-149				270-150				270-151				270-152				270-153				270-154				270-155				270-156				270-157				270-158				270-159				270-160				270-161				270-162				270-163				270-164				270-165				270-166				270-167				270-168				270-169				270-170				270-171				270-172				270-173				270-174				270-175				270-176				270-177				270-178				270-179				270-180				270-181				270-182				270-183				270-184				270-185				270-186				270-187				270-188				270-189				270-190				270-191				270-192				270-193				270-194				270-195				270-196				270-197				270-198				270-199				270-200				270-201				270-202				270-203				270-204				270-205				270-206				270-207				270-208				270-209				270-210				270-211				270-212				270-213				270-214				270-215				270-216				270-217				270-218				270-219				270-220				270-221				270-222				270-223				270-224				270-225				270-226				270-227				270-228				270-229				270-230				270-231				270-232				270-233				270-234				270-235				270-236				270-237				270-238				270-239				270-240				270-241				270-242				270-243				270-244				270-245				270-246				270-247				270-248				270-249				270-250				270-251				270-252				270-253				270-254				270-255				270-256				270-257				270-258				270-259				270-260				270-261				270-262				270-263				270-264				270-265				270-266				270-267				270-268				270-269				270-270				270-271				270-272				270-273				270-274				270-275				270-276				270-277				270-278				270-279				270-280				270-281				270-282				270-283				270-284				270-285				270-286				270-287				270-288				270-289				270-290				270-291				270-292				270-293				270-294				270-295				270-296				270-297				270-298				270-299				270-300				270-301				270-302				270-303				270-304				270-305				270-306				270-307				270-308				270-309				270-310				270-311				270-312				270-313				270-314				270-315				270-316				270-317				270-318				270-319				270-320				270-321				270-322				270-323				270-324				270-325				270-326				270-327				270-328				270-329				270-330				270-331				270-332				270-333				270-334				270-335				270-336				270-337				270-338				270-339				270-340				270-341				270-342				270-343				270-344				270-345				270-346				270-347				270-348				270-349				270-350				270-351				270-352				270-353				270-354				270-355				270-356				270-357				270-358				270-359				270-360				270-361				270-362				270-363				270-364				270-365				270-366				270-367				270-368				270-369				270-370				270-371				270-372				270-373				270-374				270-375				270-376				270-377				270-378				270-379				270-380				270-381				270-382				270-383				270-384				270-385				270-386				270-387				270-388				270-389				270-390				270-391				270-392				270-393				270-394				270-395				270-396				270-397				270-398				270-399				270-400				270-401				270-402				270-403				270-404				270-405				270-406				270-407				270-408				270-409				270-410				270-411				270-412				270-413				270-414				270-415				270-416				270-417				270-418				270-419				270-420				270-421				270-422				270-423				270-424				270-425				270-426				270-427				270-428				270-429				270-430				270-431				270-432				270-433				270-434				270-435				270-436				270-437				270-438				270-439				270-440				270-441				270-442				270-443				270-444				270-445				270-446				270-447				270-448				270-449				270-450				270-451				270-452				270-453				270-454				270-455				270-456				270-457				270-458				270-459				270-460				270-461				270-462				270-463				270-464				270-465	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Calculation using heterogeneous cell averaged cross sections while other calculations use homogeneous cross sections.

Table IV. Calculations for ZPPR-2 Sodium Void Measurements

Zone	Axial Void Length(in.)	Experiment (lh/kg)			Calculation			
		Plate	Pin	Ratio	Plate		Pin	Ratio
				$\frac{\text{Pin}}{\text{Plate}}$	ih/kg	C/E	C/E	$\frac{\text{Pin}}{\text{Plate}}$
Central Sample		4.57 $\pm$ 0.12	5.31 $\pm$ 0.31	1.16 $\pm$ 0.10	4.55			1.02
		5.12 $\pm$ 0.20						0.96*
Central Regions (Average of 1, 4, and 9 matrix voiding)	6	4.44 $\pm$ 0.05	4.87 $\pm$ 0.05	1.10 $\pm$ 0.02	4.60	1.04	1.02	1.08
							0.97*	1.03*
	12	3.00 $\pm$ 0.12	3.25 $\pm$ 0.10	1.08 $\pm$ 0.06	3.23	1.08	1.09	1.10
	18	1.36 $\pm$ 0.20	1.51 $\pm$ 0.07	1.12 $\pm$ 0.16	1.52	1.10	1.19	1.18
					1.33+	0.98+	1.03+	1.17+
45 Matrices ($\approx$ 7x7)							1.11*	1.10*
	12	3.20 $\pm$ 0.03	3.31 $\pm$ 0.03	1.03 $\pm$ 0.02	3.14	0.98	1.04	1.10
69 Matrices ($\approx$ 9x9)							1.00*	1.06*
	12	3.23 $\pm$ 0.02	3.38 $\pm$ 0.02	1.05 $\pm$ 0.01	3.08	0.95	1.00	1.10
							0.97*	1.08*
					3.05+	0.94+	0.96+	1.07+
							0.95+*	1.05+*

* Calculations using "white" boundary condition in transport theory cell calculations.

+ Direct eigenvalue difference calculation.

Table V. Perturbation Theory Components for Sodium Voiding

Components of Sodium Voiding (1h/kg)								
Cross Section Change	Flux Solution	Fission	Abs.	Mod.	Radial Leakage	Axial Leakage	Net Calc. (1h/kg)	Experiment
Central Matrix -6" Axial Void Length								
$\delta \Sigma_{\text{core}}$	Plate	-0.80	1.85	3.88	0.00	-0.31	4.62(4.59)*	4.44 $\pm$ 0.05
$\delta \Sigma_{\text{Na}}$	"		0.78	3.52		-0.30	4.00	
$\delta \Sigma^{28}_{\text{(Flux)}}$	"	-0.13	-0.04	0.05			-0.17	
$\delta \Sigma^{49}_{\text{(Flux)}}$	"	-0.40	0.02	0.02			-0.36	
$\delta \Sigma^{28}_{\text{(Res.)}}$	"		0.85	0.01			0.86	
$\delta \Sigma^{49}_{\text{(Res.)}}$	"	-0.19	0.14				-0.05	
$\delta \Sigma_{\text{core}}$	Pin-Refl.	-0.04	1.40	3.90	0.00	-0.30	4.96(4.93)*	4.87 $\pm$ 0.05
$\delta \Sigma_{\text{Na}}$	"		0.77	3.54		-0.29	4.02	
$\delta \Sigma^{28}_{\text{(Flux)}}$	"	-0.08	-0.11	0.04			-0.15	
$\delta \Sigma^{49}_{\text{(Flux)}}$	"	0.23	-0.13				0.10	
$\delta \Sigma^{28}_{\text{(Res.)}}$	"		0.64	0.01			0.65	
$\delta \Sigma^{49}_{\text{(Res.)}}$	"	-0.18	0.13				-0.05	
$\delta \Sigma_{\text{core}}$	Pin-White	-0.21	1.52	3.74	0.00	-0.30	4.75(4.72)*	4.87 $\pm$ 0.05
$\delta \Sigma_{\text{Na}}$	"		0.75	3.40		-0.29	3.86	
Central Matrix -18" Axial Void Length								
$\delta \Sigma_{\text{core}}$	Plate	-0.55	1.34	2.95	0.00	-2.20	1.54(1.32)*	1.36 $\pm$ 0.20
$\delta \Sigma_{\text{core}}$	Pin-Refl.	-0.02	1.03	2.92		-2.13	1.80(1.59)*	1.51 $\pm$ 0.07
$\delta \Sigma_{\text{core}}$	Pin-White	-0.16	1.12	2.82		-2.09	1.69(1.48)*	1.51 $\pm$ 0.07
Central 69 Matrices -12" Axial Void Length								
$\delta \Sigma_{\text{core}}$	Plate	-0.67	1.57	3.32	-0.04	-1.10	3.08(2.97)*	3.23 $\pm$ 0.02
$\delta \Sigma_{\text{Na}}$	"		0.66	3.01	-0.04	-1.05	2.58	
$\delta \Sigma_{\text{core}}$	Pin-Refl.	-0.03	1.20	3.32	-0.04	-1.06	3.39(3.28)*	3.38 $\pm$ 0.02
$\delta \Sigma_{\text{Na}}$	"		0.65	3.02	-0.04	-1.02	2.61	
$\delta \Sigma_{\text{core}}$	Pin-White	-0.18	1.31	3.23	-0.03	-1.05	3.28(3.17)*	3.38 $\pm$ 0.02
$\delta \Sigma_{\text{Na}}$	"		0.64	2.96	-0.03	-1.02	2.53	

Central Matrix -6" Axial Void Length

$\delta \epsilon_{\text{core}}$	Plate	-0.80	1.85	3.68	0.00	-0.31	4.62(4.59)*	4.44±0.05
$\delta \epsilon_{\text{Na}}$	"		0.78	3.52		-0.30	4.00	
$\delta \epsilon^{26}_{\text{Na}}$ (Flux)	"	-0.13	-0.04	0.05			-0.17	
$\delta \epsilon^{49}_{\text{Na}}$ (Flux)	"	-0.40	0.02	0.02			-0.36	
$\delta \epsilon^{28}_{\text{Na}}$ (Res.)	"		0.65	0.01			0.86	
$\delta \epsilon^{43}_{\text{Na}}$ (Res.)	"	-0.19	0.14				-0.05	
$\delta \epsilon_{\text{core}}$	Pin-Ref.	-0.04	1.40	3.90	0.00	-0.30	4.96(4.93)*	4.87±0.05
$\delta \epsilon_{\text{Na}}$	"		0.77	3.54		-0.29	4.02	
$\delta \epsilon^{23}_{\text{Na}}$ (Flux)	"	-0.08	-0.11	0.04			-0.15	
$\delta \epsilon^{49}_{\text{Na}}$ (Flux)	"	0.23	-0.13				0.10	
$\delta \epsilon^{28}_{\text{Na}}$ (Res.)	"		0.64	0.01			0.65	
$\delta \epsilon^{43}_{\text{Na}}$ (Res.)	"	-0.18	0.13				-0.05	
$\delta \epsilon_{\text{core}}$	Pin-White	-0.21	1.52	3.74	0.00	-0.30	4.75(4.72)*	4.87±0.05
$\delta \epsilon_{\text{Na}}$	"		0.75	3.40		-0.29	3.86	

Central Matrix -18" Axial Void Length

$\delta \epsilon_{\text{core}}$	Plate	-0.55	1.34	2.95	0.00	-2.20	1.54(1.32)*	1.36±0.20
$\delta \epsilon_{\text{core}}$	Pin-Ref.	-0.02	1.03	2.92		-2.13	1.80(1.59)*	1.51±0.07
$\delta \epsilon_{\text{core}}$	Pin-White	-0.16	1.12	2.82		-2.09	1.69(1.48)*	1.51±0.07

Central 69 Matrices -12" Axial Void Length

$\delta \epsilon_{\text{core}}$	Plate	-0.67	1.57	3.32	-0.04	-1.10	3.08(2.97)*	3.23±0.02
$\delta \epsilon_{\text{Na}}$	"		0.66	3.01	-0.04	-1.05	2.58	
$\delta \epsilon_{\text{core}}$	Pin-Ref.	-0.03	1.20	3.32	-0.04	-1.06	3.39(3.28)*	3.38±0.02
$\delta \epsilon_{\text{Na}}$	"		0.65	3.02	-0.04	-1.02	2.61	
$\delta \epsilon_{\text{core}}$	Pin-White	-0.18	1.31	3.23	-0.03	-1.05	3.28(3.17)*	3.38±0.02
$\delta \epsilon_{\text{Na}}$	"		0.64	2.94	-0.03	-1.02	2.53	

* Values in parenthesis are obtained by increasing leakage component by 10%.

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Table VI. Comparison of Calculation and Experiment for Off-Center Sodium Voiding

Region Voided	Type of Calculation	Perturbation Theory Components of Na Voiding							Experiment	
		Fission	Abs.	Mod.	Radial Leakage	Axial Leakage	Transverse Leakage	Net Calc. (C/E)	Plate	Pin
A	R-Z Pert.- $\delta\epsilon^{\text{core}}$	-0.54	1.14	2.46	-0.24	-0.86	-	1.96(0.86) ^a	2.28 $\pm$ 0.05	2.15 $\pm$ 0.05
	R-Z Pert.- $\delta\epsilon^{\text{Na}}$		0.48	2.26	-0.24	-0.83	-	1.67	2.24 $\pm$ 0.05 ^b	2.17 $\pm$ 0.97 ^b
	R-Z Direct k- $\delta\epsilon^{\text{core}}$							2.05(0.90)		
	X-Y Pert.- $\delta\epsilon^{\text{core}}$	-0.56	1.18	2.55	-0.22	-0.86	-0.01	2.08(0.91)		
	X-Y Pert.- $\delta\epsilon^{\text{Na}}$		0.50	2.34	-0.21	-0.83	-0.01	1.79		
	(X-Y Pert.- $\delta\epsilon^{\text{core}}$) ^d	-0.55	1.19	2.08	-0.22		-0.01			
B	R-Z Pert.- $\delta\epsilon^{\text{core}}$	-0.16	0.45	1.28	72.50	-0.47	-	-1.40(1.39)	-1.01 $\pm$ 0.05	-1.01 $\pm$ 0.05
	R-Z Pert.- $\delta\epsilon^{\text{Na}}$		0.23	1.17	-2.41	-0.45	-	-1.46	-0.97 $\pm$ 0.05 ^c	-1.02 $\pm$ 0.05 ^c
	(R-Z Pert.- $\delta\epsilon^{\text{core}}$) ^a	-0.17	0.46	1.30	-2.39	-0.48		-1.28(1.27)		
	X-Y Pert.- $\delta\epsilon^{\text{core}}$	-0.18	0.48	1.34	-2.34	-0.48	-0.03	-1.21(1.20)		
	X-Y Pert.- $\delta\epsilon^{\text{Na}}$		0.25	1.22	-2.26	-0.46	-0.03	-1.27		
	(X-Y Pert.- $\delta\epsilon^{\text{core}}$) ^d	-0.17	0.48	1.10	-2.32		-0.03			
	X-Y Exact Pert.- $\delta\epsilon^{\text{core}}$	-0.29	0.59	1.39	-2.60	-0.54	-0.03	-1.48(1.47)		
A + B	R-Z Exact Pert.- $\delta\epsilon^{\text{core}}$	-0.29	0.57	1.41	-2.64	-0.54		-1.49(1.48)		
	R-Z Pert.- $\delta\epsilon^{\text{core}}$	-0.32	0.73	1.77	-1.59	-0.63	-	-0.04(-)	0.68 $\pm$ 0.03	0.59 $\pm$ 0.03
	X-Y Pert.- $\delta\epsilon^{\text{core}}$	-0.37	0.83	1.95	-1.27	-0.67	-0.03	0.44(0.65)		
Col. 1 of B	(R-Z Pert.- $\delta\epsilon^{\text{core}}$) ^e	-0.36	0.80	1.88	-1.31	-0.67	-	0.34(0.50)		
	R-Z Pert.- $\delta\epsilon^{\text{core}}$	-0.25	0.69	1.71	-0.69	-0.71	-	0.75(0.74)	1.02 $\pm$ 0.33 ^c	
	X-Y Pert.- $\delta\epsilon^{\text{core}}$	-0.26	0.71	1.76	-0.63	-0.71	-0.20	0.85(0.83)		
Col. 2 of B	R-Z Pert.- $\delta\epsilon^{\text{core}}$	-0.23	0.58	1.45	-1.54	-0.61	-	-0.35(-)	0.21 $\pm$ 0.33 ^c	
Col. 3 of B	R-Z Pert.- $\delta\epsilon^{\text{core}}$	-0.18	0.46	1.24	-2.46	-0.49		-1.43(1.10)	-1.30 $\pm$ 0.33 ^c	
	X-Y Pert.- $\delta\epsilon^{\text{core}}$	-0.19	0.48	1.27	-2.39	-0.49	-0.034	-2.33(1.10)	-2.11 $\pm$ 0.20 ^c	
Col. 4 of B	R-Z Pert.- $\delta\epsilon^{\text{core}}$	-0.13	0.35	1.09	-3.31	-0.36		-2.36(1.12)	-2.11 $\pm$ 0.20 ^c	
	X-Y Pert.- $\delta\epsilon^{\text{core}}$	-0.14	0.36	1.11	-3.27	-0.36	-0.034	-2.33(1.10)		
Col. 5 of B	R-Z Pert.- $\delta\epsilon^{\text{core}}$	-0.07	0.24	1.02	-3.95	-0.24		-3.00(1.02)	-2.93 $\pm$ 0.33 ^c	
	X-Y Pert.- $\delta\epsilon^{\text{core}}$	-0.07	0.24	1.03	-3.96	-0.24	-0.031	-3.03(1.03)		

a) Ratio of calculation/experiment given in parenthesis.

b) Value for voiding A with B previously voided.

c) Value for voiding B (or drawers of B) with A previously voided.

d) Calculations using group and region dependent B^2 . All other calculations used group independent B^2 .

e) Calculated as a linear average of Columns 1 to 5 of B.

Table VII. Comparisons of ANL and WARD Calculations for Pin and Plate Measurements

Measurement	Experiment		Calculation					
	Plate	Pin Plate	(W)ARD		ANL-Idaho		ANL-Chicago	
			Plate C/E	Pin Plate	Plate C/E	Pin Plate	Plate C/E	Pin Plate
Plate vs. Pin $\Delta k(\text{ih})$	67.7 $\pm$ 6.8 52.3 $\pm$ 3.4*		2.04		1.22		0.97*	
Sodium Voiding (ih/kg)								
69 matrices	3.23 $\pm$ 0.02	1.05 $\pm$ 0.01	0.94	1.05	0.78	1.06		
1 matrix*	6.70 $\pm$ 0.2	1.01 $\pm$ 0.04					1.03*	1.05*
9 matrices*	6.94 $\pm$ 0.6	0.92 $\pm$ 0.11					0.95*	1.06*
^{238}U Doppler (ih/kg)								
300-1100°K	-0.621 $\pm$ 0.009	0.887 $\pm$ 0.03	0.96	0.91	1.02	0.89		
Central Reaction Rate Ratios								
$\sigma_f^{25}/\sigma_f^{49}$	1.067 $\pm$ 0.016	1.001 $\pm$ 0.03	1.03	1.00	1.09	0.99		
$\sigma_f^{28}/\sigma_f^{49}$	0.0215 $\pm$ 0.0005	1.057 $\pm$ 0.03	1.00	1.02	1.02	1.03		
$\sigma_f^{33}/\sigma_f^{49}$	0.1918 $\pm$ 0.003	1.046 $\pm$ 0.03	1.01	1.02	1.02	1.03		
$\sigma_f^{25}/\sigma_f^{49}$ *	1.065 $\pm$ 0.021	0.990 $\pm$ 0.03					1.08*	1.00*
$\sigma_f^{25}/\sigma_f^{49}$ *	0.0234 $\pm$ 0.0005	1.056 $\pm$ 0.03					0.99*	1.02*
$\sigma_f^{28}/\sigma_f^{49}$ *	0.143 $\pm$ 0.003	1.002 $\pm$ 0.03					1.15*	1.00*
Central Worths (ih/kg)								
^{239}Pu	120.2 $\pm$ 0.1	0.974 $\pm$ 0.01	1.15	0.96	1.14	0.98		
^{239}Pu *	157.9 $\pm$ 0.9	0.975 $\pm$ 0.01					1.33*	0.96*

* ZPR-6-7 results. Other results are for ZPR-2.

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This possible explanation appears to be worth more careful study due to the potential impact of such a problem. Colin Durston of WARD has suggested (private communication) the possibility that different diffusion coefficients (or transport cross sections) may be required parallel and transverse to the plates such as frequently used for thermal reactor applications. The current theory on this problem seems to be qualitatively consistent with the above leakage speculation.

Comparisons of Independent Plate versus Pin Heterogeneity Calculations

In addition to the present calculations, the plate versus pin reactivity effects have been calculated for ZPPR-2 by ANL-Idaho^[1] and for ZPR-6-7 by ANL-Chicago^[4]. Results of these calculations are given in Table II and include the results of total core plate heterogeneity (plate versus homogeneous) calculations. The ANL calculations of 82.6 ih for the ZPPR-2 plate vs. pin reactivity effect differs significantly from the present authors' result of 137.8 ih. About half of this difference may be attributable to the plate heterogeneity differences of 1.2% Δk by ANL and 1.63% Δk by WARD. ANL's latest calculation of 1.6% Δk for the ZPPR-2 plate heterogeneity is more consistent with the WARD value. Recalculation of the plate vs. pin effect is currently underway at ANL and more detailed comparisons will be possible when these calculations are completed.

The ANL-Chicago calculation of 50.9 ih for ZPR-6-7 is in significantly better agreement with experiment than the ZPPR-2 calculations. This improved agreement with experiment for ZPR-6-7 calculations is consistent with expectations based on the previous discussions although the ANL C/E value for ZPPR-2 is notably better than for the WARD calculations.