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RF SEPARATION TECHNIQUES AT A SUPER-HIGH-ENERGY ACCELERATOR - II

J. Sandweiss *

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I. General

In a previous report¹ (hereafter referred to as RF-I), certain aspects of rf beams with design momentum up to 500 BeV/c were considered. Particular attention was given to the ultimate limitations imposed by achievable beam transmission. In the present report we develop further the considerations of RF-I and in particular we consider the effects of various sources of background and image broadening.

As in RF-I we make use of the scaled beam design. That is, we consider an ultra-high-energy rf beam designed by scaling up the reference beam as described in RF-I. The scaling proceeds by scaling the lengths in the pre- and post-deflector sections linearly with beam momentum and scaling intercavity lengths quadratically with the beam momentum. We also consider the fractional momentum bite and the angular acceptances in both planes to (each) scale in inverse proportion to the beam design momentum. We observe that for this type of scaling the quadrupole magnets in the pre- and post-deflector sections need be no stronger (gradient $\times$ length) than the corresponding quadrupoles in the reference beam. In an actual beam design one might wish to use more focusing power and make the pre- and post-deflector sections of the beam shorter. The quadrupoles in the scaled-intercavity section actually require less focusing power than the

* Yale University.

1. J. Sandweiss, "Rf Separation Techniques at a Super-High-Energy Accelerator - I", BNL Accelerator Dept. Int. Rpt. AADD-74, March 25, 1965.

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corresponding quadrupoles in the reference beam. However, the length of the intercavity section is set by the required drift space and cannot be made smaller without increasing the rf frequency.

The transmission estimates in RF-I were made assuming that the target dimensions would be about the same as for the reference beam (~ 1 cm). This assumption is clearly overconservative. The studies for the proposed 200-BeV proton synchrotron indicate that targets as small as 0.02 cm vertically by 0.1 cm horizontally are feasible.² With a 600 to 1000-BeV accelerator even smaller targets will be feasible.³

With the transmission enhancements made possible by these small targets it is clear that 500-BeV/c design momentum can be reached before the beam is flux limited. In order to utilize the small target size much higher magnification in the pre-deflector section would be used than is the case for the reference beam design. The overall length of the beam will not be very different from the length of the scaled beam because most of the length is in the intercavity section and the need for higher magnification in the pre-deflector stage and hence for a longer pre-deflector section will be compensated in part by the use of relatively stronger magnets.

Despite the expected deviations from the scaled beam design we shall frequently use the scaled design in the following analysis. Generally we can indicate qualitatively the effect of the design changes discussed above and we can establish the results for the scaled system as upper or lower bounds. Also the scaled system, although certainly not optimum, is a system which would work (from a flux standpoint) up to ~ 300 BeV/c and as such represents an interesting "yardstick" to compare with more refined designs. In this respect we note that the transmission given in RF-I for the 100-BeV/c scaled beam agrees to within a factor of two with the detailed design prepared for the 200-BeV design study² provided the correction is applied for the different target sizes used.

II. Deflector and Rf Considerations

The basic requirement on the stability of the relative phase and amplitude of the deflecting forces exerted by the two (or more) deflectors is independent

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2. J. Lach, Yale University, private communication, and "100-BeV/c Rf Beam for the 200-BeV Proton Synchrotron", to be published as UCRL Report.
 3. E.D. Courant, Brookhaven National Laboratory, private communication.

of the beam design momentum. As can readily be seen from the phasor diagrams, the relative phase must be kept stable within $\sim 10^\circ$ and the amplitudes equal (or in the correct ratio for 3 deflector systems) within $\sim 5\%$ if the effective deflection of the unwanted components is to be less than $\sim 10\%$ of the deflection of the wanted components.

As noted in RF-I we assume that by the time the beams under discussion here are constructed, superconducting deflectors will be feasible. The techniques for operating these deflectors are yet to be developed, and can certainly be expected to be substantially different from current practice.⁴ However, it is clear that the relative phase of the two deflectors must be measured and the phase error controlled. The very difficult problem of transmitting the required phase information with adequate fidelity over the long interdeflector distance will be a major factor in the design of the rf system. It is very likely that the high-energy particle beam itself will be useful. For example, the spread in the unwanted image at the stopper is a direct measure of the phase error. An array of particle detectors at the stopper can sample the distribution of the unwanted particle image and transmit this information to an on-line computer. The computer could also receive information about the relative amplitudes and the natural (deflectors off) image width and could thus analyze the distribution for phase errors. The resultant error signal could be used to complete the phase servo loop. This system is schematically illustrated in Fig. 1.

This system would certainly serve to eliminate the phase drifts with long time constants relative to the time between beam pulses.

In order for the two deflectors to operate at the same frequency it will probably be necessary to transmit some rf power from one deflector station to another. This transmission must not introduce phase jitter with time constants faster than a few seconds if the beam-referenced phase servo is to be used. Several possibilities for transmitting this rf power have been suggested. One technique might be the use of a subharmonic, with consequent frequency multiplication at the deflector station. A subharmonic has the advantage of lower attenuation relative to the (higher) operating frequency. It is interesting to note that the Stanford Two-Mile Electron Linear Accelerator uses a 476-Mc/sec drive line to operate the string of 2856-Mc/sec accelerating stations with a beam-referenced phase servo for long-term stability.

4. The author is indebted to H. Hahn and H. Halama for useful discussions on this topic and on the possible approaches to the phasing problem.

Alternately, rf power might be transmitted between deflectors with a very precisely thermostated waveguide, perhaps at liquid nitrogen temperature. If suitable modulators and demodulators can be developed, laser beams (in vacuum) might be useful for transmission.

Another possibility might be the use of two separate rf sources, each "free running" but of such stability and low noise that the required coherence is maintained. That is, the two oscillators must remain within a few degrees of each other for a period of several seconds. As noted above, slower drifts could be compensated by the beam-referenced phase servo. The hydrogen maser is capable of the required stability ($1^0/\text{sec}$ at 10 kMc is equivalent to ~ 3 parts in 10^{13}) but the phase noise characteristics are not sufficiently well known (at least to the author) at present to evaluate the maser scheme. The hydrogen maser operates at ~ 1.4 kMc so that only relatively modest frequency multiplication would be necessary to provide X-band power.

It is perhaps useful to point out that for the beam designs considered here, which would provide adequate separated flux of K-mesons up to ~ 500 BeV/c (with small targets), the peak transverse impulse required from each deflector is independent of beam design momentum. As noted in RF-I the angular acceptance in each plane (and the fractional momentum bite) must scale inversely as the beam design momentum. Thus the required deflection angle decreases linearly with design momentum. We are thus considering deflectors of the same deflecting power ($E_0\theta$) as are in current use (i.e., capable of imparting ~ 15 -MeV/c transverse momentum). As noted in RF-I the angular acceptance must decrease in order to keep the chromatic aberration from increasing as the design momentum is scaled up, hence the constancy of the required transverse impulse. There is another fundamental requirement for the angular acceptance to decrease linearly with design momentum, namely the necessary isochronism of all rays between deflectors. It is easy to see that

$$\Delta L = \frac{\theta^2}{2} L \quad (1)$$

where ΔL is the path length difference between the central ray and the ray inclined (on the average) by an angle θ to the central ray and L is the inter-deflector separation. Since L scales as p^2 , θ must scale as p^{-1} if ΔL is to stay constant.

Since the required impulse is ~ 15 MeV/c it is quite clear that short pulse deflectors (few μsec), essentially scaled versions of the present BNL deflector, could be built and would be useful for bubble chamber beams. Such short pulse beams would be useful over a significant fraction of the potential K-meson energy range. This of course assumes the deflector phasing problem can be solved. However, as noted above, many approaches are possible and the development of an adequate system seems very likely. The availability of superconducting deflectors will allow long pulse operation and provide beams useful for counter, spark chamber experiments.

We conclude that although somewhat novel developments will be necessary, the rf problems present no fundamental barriers to the realization of rf separated beams up to ~ 500 BeV/c.

III. Survey and Alignment Tolerances

We consider first the scaled system discussed in RF-I. That is, a scaled version [of the present (1966) rf separated beam at the AGS] in which the fractional momentum bite decreases linearly with increasing beam design momentum. We consider here only 10,000-Mc/sec systems. As noted in RF-I, all beam dimensions, except intercavity dimensions, increase linearly with beam design momentum whereas intercavity dimensions increase quadratically.

In discussing tolerances it is convenient to refer all misalignments to equivalent target displacements. This can be easily carried out via the relation:

$$\begin{pmatrix} Z \\ Z' \end{pmatrix}_s = M(s_0, s) \left[\begin{pmatrix} Z \\ Z' \end{pmatrix}_{s_0} + M^{-1}(s_0, s_1) \begin{pmatrix} 0 \\ k\Delta \end{pmatrix} \right]. \quad (2)$$

The vector $(Z, Z')_s$ represents the position Z and slope Z' at position s in the beam for a ray which originated at the target with $(Z, Z')_{s_0}$. The position s is downstream of the quadrupole located at s_1 which is assumed to be displaced by an amount Δ . The principal effect of the displacement is an angular deflection of $k\Delta$ relative to the correct direction of the ray. For beams scaled as discussed above the quantity k decreases linearly with increasing beam design momentum for pre- and post-deflector sections and quadratically for intercavity sections. We write

$$M(s_0, s_1) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

whence (recalling $\det m = 1$),

$$M^{-1}(s_0, s_1) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

and

$$M^{-1}(s_0, s_1) \begin{bmatrix} 0 \\ k\Delta \end{bmatrix} = \begin{bmatrix} -b k \Delta \\ a k \Delta \end{bmatrix} \quad (3)$$

The matrix element a is the magnification which is independent of scale factor and the matrix element b is essentially the displacement of a ray originating at the target center with unit slope. The element b is clearly proportional to the scale factor for pre- and post-deflector optics and proportional to the scale factor squared for intercavity optics. The quantity bk is thus independent of scale factor and we conclude that so far as effective target displacements are concerned the quadrupole transverse location error Δ must be kept within the same limits for both scaled and unscaled systems. The quantity ak decreases linearly with beam design momentum as does the angular divergence accepted from the target. Thus, to keep the fractional spread in effective target emission angle constant, we again see that Δ must be the same for scaled and unscaled systems.

The unscaled reference system (15 BeV/c) required location tolerances of 0.01" for some of the quadrupoles. For the scaled systems we will, (as shown above), again require transverse location errors to be $\sim 0.01''$. However, this tolerance must now be maintained over lengths ranging from ~ 1.3 kilometers (100-BeV/c design momentum) to ~ 21.2 kilometers (500-BeV/c design momentum)! This appears to be a rather formidable feat. It is not unlike the alignment required for the Stanford Two-Mile Linear Electron Accelerator.

If, as is almost certain, the possibilities for reducing the target size are exploited in the beam optical design, larger magnification between the target and first deflector will result. The required survey tolerance will become substantially more stringent because larger values of k (relative to the system scaled as above with constant target size) will result, at least for some

magnets. It appears to the author that it will not be possible, or rather economically feasible, to physically locate the magnets to the desired precision. Rather, small dipoles, both vertical and horizontal, should be provided for each quadrupole and used to correct the displacements of the quadrupoles. The beam itself can then be used for the final "alignment". It will, of course, be desirable to come as close as possible physically and the laser alignment techniques developed at Stanford will be very useful.

The location of uniform field regions (bending magnets) is obviously less critical. Here the major difficulty will be that of achieving the correct orientation. Again the technique of small corrector magnets will be useful in a final beam alignment.

IV. Magnet Aberrations and Stability

We consider now the allowable errors in the magnetic fields of the beam transport magnets. Bending magnet fields must be stable against fractional variations comparable to the fractional momentum bite. We somewhat arbitrarily set the allowed variation at one-third the fractional momentum bite. For the scaled beam design, we list in Table I the required stability of bending magnet fields as a function of beam design momentum. The 15-BeV/c reference beam had a $\Delta p/p$ of 1% (full width at base).

TABLE I

Required Stability of Bending Magnet Fields

<u>Design Momentum (BeV/c)</u>	<u>Required Stability (%)</u>
50	0.1
100	0.05
150	0.033
200	0.025
250	0.020
300	0.017
350	0.014
400	0.013
450	0.011
500	0.010

We now consider errors in focusing fields. We shall derive an expression for the effect of a focusing field error at position s_1 in the beam line, on the sharpness of the image at a point in the beam (beyond s_1) where in the ideal case an image would be formed. This could be an image of the target or it could be an image of some other point in the beam. For example, an image of the mass slit is formed at the stopper. Let:

- s_0 = position of the object
- s_1 = position of the aberrated element
- s = position of the image
- $M(s)$ = unaberrated transfer matrix from s_0 to s
- $M^\delta(s)$ = aberrated transfer matrix (with error at s_1)
from s_0 to s
- A = unaberrated transfer matrix from s_0 to s_1
(including element at s_1)
- B = unaberrated transfer matrix from s_1 to s
- ℓ_1 = length of the aberrated field at s_1
- m_0 = unaberrated transfer matrix through the element
of length ℓ_1 at s_1
- m = aberrated transfer matrix through the element of
length ℓ_1 at s_1

We can write:

$$M(s) = B A \quad (4)$$

$$m_0 = \begin{bmatrix} 1 & \ell_1 \\ k \ell_1 & 1 \end{bmatrix} \quad (5)$$

Equation (5) is valid for ℓ_1 small compared to the focal length of the magnet at s_1 . This will certainly be a good approximation. The quantity k is the

correct value of the inverse focal length of the magnet at s_1 . We now consider the aberrated magnet, $k \rightarrow k + \delta k$.

$$m = \begin{bmatrix} 1 & & l_1 \\ (k + \delta k) & l_1 & 1 \end{bmatrix} \quad (6)$$

We observe that we can write the aberrated transfer matrix $M^\delta(s)$ in the form:

$$M^\delta(s) = B m (m_0)^{-1} A \quad (7)$$

From Eqs. (5) and (6) we see that to first order in l_1 :

$$m m_0^{-1} = \begin{bmatrix} 1 & 0 \\ (\delta k) & l_1 & 1 \end{bmatrix} \quad (8)$$

whence

$$M^\delta(s) = B \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ (\delta k) & l_1 & 0 \end{pmatrix} \right] A$$

and

$$M^\delta(s) = M(s) + \begin{bmatrix} B_{11} A_{11} (\delta k) l_1 & B_{12} A_{12} (\delta k) l_1 \\ B_{12} A_{11} (\delta k) l_1 & B_{22} A_{12} (\delta k) l_1 \end{bmatrix} \quad (9)$$

Because s_0 and s are conjugate points the matrix $M(s)$ has the form

$$M(s) = \begin{bmatrix} \eta & 0 \\ m_{21} & \frac{1}{\eta} \end{bmatrix} \quad (10)$$

where η is the magnification of the system from s_0 to s . The effect of the error at s_1 is to spread the image of a point at s into a "blur" with extent Δz given by:

$$\Delta z = M_{12}^{\delta}(s) \alpha \quad (11)$$

where α is the angular spread (at s_0) of the transmitted beam. From Eqs. (9) and (11) we see that:

$$\Delta z = B_{12} A_{12} (\delta k) \ell_1 \alpha \quad (12)$$

We can express $B_{12} A_{12}$ in terms of the unaberrated transfer matrix from s_0 to s_1 , $[M(s_1)]$, and η , the magnification from s_0 to s . From the definition of A and B we write:

$$A = M(s_1) \quad (13)$$

$$B = M(s) M^{-1}(s_1)$$

and

$$B = \begin{bmatrix} \eta & 0 \\ M_{12}(s) & \frac{1}{\eta} \end{bmatrix} \begin{bmatrix} M_{22}(s_1) & -M_{12}(s_1) \\ -M_{21}(s_1) & M_{11}(s_1) \end{bmatrix} \quad (14)$$

From Eqs. (13) and (14) we see:

$$A_{12} = M_{12}(s_1)$$

$$B_{12} = -\eta M_{12}(s_1)$$

So that we have for Δz

$$\Delta z = - \eta \left[M_{12}(s_1) \right]^2 (\delta k) l_1 \alpha \quad . \quad (15)$$

We apply Eq. (15) to estimate the required magnet quality in three separate cases.

(a) Pre- and Post-Deflector Quadrupoles

The matrix element M_{12} is just the position, z , of a ray leaving s_0 with $z_0 = 0$ and $dz/ds|_0 = 1$. Thus

$$M_{12}(s_1) \propto \frac{p}{p_0}$$

for the scaled beam design. As mentioned previously

$$\alpha = \alpha_0 \frac{p_0}{p}$$

and

$$\eta = \eta_0 \quad .$$

Thus to keep $\Delta z = \Delta z_0$ (i.e., no worse than in the reference beam) we require

$$\delta k l_1 \propto \frac{p_0}{p} \quad .$$

However, for the scaled beam design we recall $k l_1 \propto (p_0/p)$ so that the fractional tolerance on field linearity for pre- and post-deflector quadrupoles is the same for the scaled beam and the reference beam.

(b) Interdeflector Quadrupoles

Here,

$$M_{12}(s_1) \propto \left(\frac{p}{p_0} \right)^2$$

whence for $\Delta z = \Delta z_0$ we require

$$\delta k \ell_1 \propto \left(\frac{p_0}{p} \right)^3 .$$

However, the scaled design has $k \ell_1 \propto (p_0/p)^2$ for quadrupoles in the interdeflector section. Thus the fractional tolerance on field linearity for interdeflector quadrupoles becomes more stringent in direct proportion to the scale factor.

This result is quite general. Design modifications (from the scaled beam) to utilize small targets will not change the interdeflector optics.

(c) Field Uniformity of Bending Magnets

Ideally $k = 0$ in bending magnets. However, nonuniformities give rise to aberrative focusing effects described here by $\delta k \ell_1$. As before

$$M_{12}(s_1) \propto \frac{p_0}{p}$$

$$\alpha = \alpha_0 \frac{p_0}{p} .$$

Thus for $\Delta z = \Delta z_0$ we have

$$\delta k \ell_1 \propto \frac{p_0}{p} .$$

For the bending magnets in the scaled beam design $\ell_1 \propto (p/p_0)$ and the central field H_0 was the same as for the reference beam (without superconducting magnets). Thus we require

$$\frac{\delta k}{H_0} \propto \left(\frac{p_0}{p} \right)^2 . \tag{16}$$

Equation (16) represents a formidable requirement. Extensive use of multi-pole windings would certainly be necessary together with very sensitive and sophisticated magnetic measuring techniques.

The use of a smaller target than that assumed in the scaled design will substantially relax the uniformity requirement. If all of the "gain" from the small target is used to alleviate the uniformity requirement, ℓ_1 will stay constant. This assumes that the target size can decrease in proportion to p_0/p . Target heating effects may prevent this above a certain energy. In any case if ℓ_1 is constant Eq. (16) should read:

$$\frac{\delta k}{H_0} \propto \frac{p_0}{p} \quad (17)$$

If however, some of the small target "gain" is used to increase transmission higher magnification will be needed and hence more bending to achieve the desired momentum analysis. We thus conclude that Eqs. (16) and (17) represent upper and lower bounds respectively for the required field uniformity.

The problem of magnet "quality" may be a limiting factor in the extension of the rf technique to high energies. Present methods would not produce magnets of the required tolerance for 500-BeV/c beams. There is no fundamental barrier to developing the required techniques and in fact they will not be appreciably different from the techniques necessary to build the accelerator magnets themselves.

V. Slits and Collimators

The thickness of beam-defining slits and collimators is chosen to satisfy two criteria. First, the jaws must present sufficiently many nuclear mean free paths to adequately attenuate the strongly interacting particle component which strikes them. Secondly, traversal of the slit jaws by muons must cause significant momentum loss so that the transmitted muons will be rejected by the subsequent magnetic optics.

In the projected super-high-energy rf separated beams under discussion here, the fraction of wanted particles (K^+ , K^- , $\bar{p}$) at the target is expected to be approximately independent of energy and about the same as that observed in present day (1966) rf separated beams at the AGS and the CERN PS. These fractions are typically 0.10 for K^-/π^- and 0.05 for $\bar{p}/\pi^-$. Thus from the standpoint of attenuating strongly interacting background the slits do not have to be longer than those in current use.

As noted in RF-I and above, the super-high-energy beams will probably be designed with momentum transmission inversely proportional to beam momentum. This has the result that the slits need not be lengthened in order to cause significant momentum loss relative to the transmitted momentum bite as the beam momentum is increased. Thus we anticipate slits and collimators of approximately the current size. Typically they are one meter in the beam direction, made of brass, possibly with heavimet jaws.

Because the angular spread of the beam decreases with increasing beam momentum, the natural acceptance of the slit is relatively greater. Thus the slit transmission factors will be better for the super-high-energy beams than for the current (1966) beams.

The effects of single and multiple coulomb scattering decrease at high energies as is evident from the well known $(p\beta)^{-1}$ dependence of the Rutherford scattering cross section. Although the total cross section for nuclear diffraction scattering is expected to be independent of energy, the characteristic angular range of the diffraction scattering varies as p^{-1} . Thus the diffraction contribution to slit edge scattering also decreases with increasing beam momentum.

We note the perhaps curious result that in all respects the design of beam defining slits and collimators becomes easier as the beam momentum is increased.

VI. $\pi - \mu$ Decay

Past experience indicates that the most serious source of muon contamination is the decay of beam pions after the first momentum analysis section (and before the stopper). Beam-like muons arising from pion decay before the first momentum analysis are deflected exactly as pions and will strike the stopper. The stopper must be sufficiently thick so that on-momentum muons striking the stopper lose enough energy to be rejected by the post-analysis section.

To a good approximation then we may consider the intercavity length as the effective length for pion decay. This neglects the section from the last cavity to the stopper which will certainly be short in comparison with the intercavity separation. We consider a two-deflector, two-contaminant rejection, 10,000-Mc/sec system (see Table III, RF-I). Table II below gives the fraction of pions which decay in the intercavity section and the muon-to-kaon ratio which would result if all the muons within the post-analysis momentum bite were transmitted to the chamber. For this purpose we have assumed the fractional momentum

transmission of the beam to decrease linearly with beam momentum from a value of 1% at 15 BeV/c. As is well known, the muon momentum spectrum is essentially flat from the beam momentum to 57% of the beam momentum. The K/ π ratio at production has been taken at 10% and the K-meson survival fraction (target to chamber) has been taken from Table VI, RF-I. As noted there, it may be desirable to decrease the momentum bite quadratically with increasing beam momentum. However, for the present analysis we are interested in the most unfavorable situation for muon background so we consider only the linear case.

TABLE II

Muon Contamination Neglecting Stopper Energy Loss for 10,000-Mc/sec.
Two Deflector, Double Rejection System

p (BeV/c)	M.F.P. Pion Decay (meters)	Intercavity Length (meters)	Pion Decay Fraction	K-Meson Survival Fraction (target to chamber)	μ/K for $\Delta p/p = 0.01$ (15/p) Neglecting Muons Energy Loss in Stopper
50	2.740×10^3	1.83×10^2	0.0667	0.276	1.70×10^{-2}
100	5.480×10^3	7.32×10^2	0.1250	0.169	2.60×10^{-2}
150	8.22×10^3	16.47×10^2	0.1817	0.102	4.17×10^{-2}
200	10.96×10^3	29.28×10^2	0.2345	0.063	6.54×10^{-2}
250	13.70×10^3	45.75×10^2	0.2840	0.038	0.1051
300	16.44×10^3	65.88×10^2	0.331	0.023	0.1686
350	19.18×10^3	89.67×10^2	0.3740	0.0142	0.2641
400	21.92×10^3	117.12×10^2	0.4140	0.0086	0.4230
450	24.66×10^3	148.23×10^2	0.4524	0.00525	0.6723
500	27.40×10^3	183.0×10^2	0.488	0.0032	1.07

The calculations of Table II show that muon contamination will not be a serious problem for rf separated beams even up to 500-BeV/c beam momentum. It will be necessary, of course, to keep the off-momentum muons from striking the chamber. This, however, will present no particular difficulty. For example, magnetized slit jaws would be particularly effective at high energies.

It is interesting to note that the muon problem for these projected super-high-energy beams is less severe than it is for present day (1966) rf separated

beams. There are several factors which affect the comparison. First, the projected beams are "designed" for a higher frequency (10,000 Mc/sec vs. 3000 Mc/sec) with correspondingly shorter decay paths. Secondly, the projected beams are "designed" with a proportionately smaller momentum bite whereas the decay muons are spread out over a constant fraction (43%) of the initial beam momentum.

VII. Costs

Although the author has spent considerable time in attempts to estimate the possible costs, and wishes to thank his colleagues⁵ for their assistance, the subject is sufficiently complex that quantitative estimates at the present level of study would be more misleading than informative.

JS/ke

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5. J. Grisoli and J. Schuchman of AGS Division and H. Hahn and H. Halama of AADD, Accelerator Department, Brookhaven National Laboratory, private communication.

Fig. 1 - Schematic illustration of a possible beam reference phase servo system utilizing an on-line computer.