41, 219

Neutrino Charge Radius in a Model of Weak and Electromagnetic Interactions*

Gordon J. Aubrecht II[†] and K. TanakaDepartment of Physics
The Ohio State University
Columbus, Ohio 43210

ABSTRACT

We introduce a modified $SO(3)$ model of weak and electromagnetic interaction symmetry. The model is renormalizable and reproduces the usual theory of weak interactions. Four additional heavy leptons and five additional quarks are necessary. The electromagnetic charge radius of the neutrino is calculated in the ξ -limiting procedure and a finite result is found.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

*Research supported in part by the U.S. Atomic Energy Commission under Contract AT(11-1)-1545.

[†]Present address: Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403

MASTER

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

We introduce a Weinberg type model of leptons and hadrons based on the spontaneously broken gauge symmetry $SO(3)$ so that the only gauge fields are the intermediate charged bosons $W_\mu^\pm$ and the photon A_μ .¹

The model is similar to those proposed by Georgi and Glashow,² Lee,³ and Prentki and Zumino,⁴ but differs in the number of additional heavy particles required and on their multiplet structure. The model satisfies the conditions of renormalizability,⁵ (in particular the cancellation of anomalies),⁶ and yields the conventional Cabibbo theory. The electromagnetic current is the only neutral current; and the model emphasizes the symmetry between the lepton and hadron currents.

Previously,⁷ we expressed the leptons in the Lagrangian in terms of Konopinski-Mahmoud type triplets $\psi_L = (L\nu, Le^-, L\mu^+)$ and $\psi_R = (R\mu^+, R\nu, Re^-)$ where $L = \frac{1}{2}(1 + \gamma_5)$ and $R = \frac{1}{2}(1 - \gamma_5)$. Such a model based on known leptons has anomalies.² To remedy this, a heavy muon M^- , a heavy positron E^+ , and neutral leptons X^0 and Y^0 are introduced. These form the triplets and singlets,⁸

$$\Psi_{eL} = (E_L^+, X_L^0 \cos \beta + \nu_L \sin \beta, e_L^-) \quad \Psi_{eR} = (E_R^+, X_R^0, e_R^-),$$

$$S_{eL} = -X_L^0 \sin \beta + \nu_L \cos \beta,$$

$$\Psi_{\mu L} = (\mu_L^+, Y_L^0, M_L^-) \quad \Psi_{\mu R} = (\mu_R^+, Y_R^0 \cos \beta + \nu_R \sin \beta, M_R^-),$$

$$S_{\mu R} = -Y_R^0 \sin \beta + \nu_R \cos \beta. \quad (1)$$

The coupling of the vector bosons to the leptons $\psi_e = \psi_{eL} + \psi_{eR}$, etc., is:

$$e A_{i\mu} J_{i\mu}^l, \quad (2)$$

where

$$W_\mu^\pm = (A_{1\mu} \mp i A_{2\mu}) / \sqrt{2}, \quad (3)$$

$$A_\mu = A_{3\mu}, \quad (4)$$

$$J_{i\lambda}^l = \bar{\Psi}_e \gamma_\lambda t_i \Psi_e + \bar{\Psi}_\mu \gamma_\lambda t_i \Psi_\mu, \quad (5)$$

$$[t_i, t_j] = i \epsilon_{ijk} t_k. \quad (6)$$

In the limit in which M^- , X^0 , Y^0 , and E^+ are regarded as heavy, Eq.(2) becomes the conventional weak and electromagnetic interaction,⁹

$$e \sin \beta \left[W_\mu^+ (\bar{\nu}_L \gamma_\mu e^- - \bar{\mu}^+ \gamma_\mu \nu_R) + W_\mu^- (\bar{e} \gamma_\mu \nu_L - \bar{\nu}_R \gamma_\mu \mu^+) \right] \\ + e A_\mu (\bar{\mu}^+ \gamma_\mu \mu^+ - \bar{e} \gamma_\mu e^-) \quad (7)$$

The weak dimensionless coupling constant g_W is, from Eq.(7), equal to $\frac{1}{2} e \sin \beta$,

and so yields an intermediate boson mass of $m_W = [\sqrt{2} g_W^2 / G]^{\frac{1}{2}} \sin \beta = 53 \sin \beta \text{ GeV}/c^2$

The charges $Q_{\bar{W}}$, Q_W , and Q_E corresponding to currents associated with W_μ^+ , W_μ^- and A_μ , respectively, generate the $SO(3)$ algebra²

$$[Q_{\bar{W}}, Q_W] = Q_E, \quad [Q_E, Q_{\bar{W}}] = Q_{\bar{W}}, \quad [Q_W, Q_E] = Q_W. \quad (8)$$

The hadron triplets, represented by integrally charged quarks, are introduced in a manner which enhances the symmetry with the leptons, incorporates the Cabibbo angle θ , and takes into account the suppression of $K_L^0 \rightarrow \bar{\mu} \mu$ and $K^+ \rightarrow \pi^+ e^+ e^-$ decays.^{10,11}

The coupling of $[\phi_i] = [(\phi^+ + \phi^-)/\sqrt{2}, i(\phi^+ - \phi^-)/\sqrt{2}, \phi^0]$ to the leptons is similar to Eq.(2),

$$\phi_i \left[g_1 (\bar{\psi}_e t_i \psi_e + \bar{\psi}_\mu t_i \psi_\mu) + g_2 (\bar{\psi}_{eL} S_{eL} + \bar{\psi}_{\mu L} S_{\mu L} + h.c.) \right] \quad (9)$$

where $\bar{\psi}_{ei}$, etc. are expressed in the form of $[\phi_i]$ rather than of (1). Muon and electron number is conserved in Eq.(9). The $[\phi_i]$ couple in a similar manner to hadrons. Due to the fact that there are lepton singlets, the neutral leptons X^0 and Y^0 acquire an additional mass.¹²

The constraints imposed on the present $SO(3)$ model from muon magnetic moment have been obtained by various authors.¹³ We study here the neutrino charge radius.¹⁴

The neutrino has only one form factor $F(q^2)$ (due to CP and γ_5 invariance) where q^2 is the square of the momentum transfer. We have calculated the neutrino electromagnetic radius $\langle r^2 \rangle = -6 [d F(q^2)/d q^2]_{q^2=0}$ in the ξ -limiting procedure¹⁵ in order to extract a finite value. The entire calculation is carried out for a lepton of arbitrary mass m (but with the condition $m^2/m_W^2 \ll 1$). The divergent contributions from the muon and heavy muon to the charge radius cancel in taking the appropriate combinations because of their opposite charges. The leading terms in the

expansion of $\langle r^2 \rangle$ agrees with previous results.¹⁶ The contribution for a lepton of mass m are given in Fig.1(a) and Fig.1(b).

The matrix element for Fig.1(a) is $\bar{u}(k + \frac{1}{2}q) J^\sigma(q^2) u(k - \frac{1}{2}q)$, where $J^\sigma(q^2)$ is given by

$$\begin{aligned} [J^\sigma]_a &= F_a(q^2) \gamma^\sigma \\ &= \frac{g_w^2}{16\pi^4} \int \frac{d^4 l}{i} \frac{[\gamma^\mu (1+\gamma_5)(k-l+\frac{1}{2}q+m) \gamma^\sigma (k-l-\frac{1}{2}q+m) \gamma^\nu (1+\gamma_5) P_{\mu\nu}(l)]}{[(k-l+\frac{1}{2}q)^2 - m^2][(k-l-\frac{1}{2}q)^2 - m^2]}, \end{aligned} \quad (10)$$

where

$$P_{\mu\nu}(l) = [g_{\mu\nu} - l_\mu l_\nu (1 - \xi^{-1}) / l^2 - M^2] / (l^2 - m_w^2),$$

$$\xi^{-1} = M^2 / m_w^2 \quad g_w = \frac{1}{2} e \sin \beta. \quad (11)$$

For Fig.1(b), J^σ is

$$\begin{aligned} [J^\sigma]_b &= \frac{g_w^2}{16\pi^4} \int \frac{d^4 l}{i} \gamma^\mu (1+\gamma_5)(l+m) \gamma^\sigma (1+\gamma_5) P_{\mu\lambda}(k-l+\frac{1}{2}q) \\ &\quad \times P_{\rho\nu}(k-l-\frac{1}{2}q) \Lambda^{\lambda\rho\sigma}(k, l, q) / (l^2 - m^2), \end{aligned} \quad (12)$$

where

$$\Lambda^{\lambda\rho\sigma}(k, l, q) = 2g^{\lambda\rho}(k-l)^{\sigma} - (1-\xi) [g^{\lambda\sigma}(k-l)^{\rho} + g^{\rho\sigma}(k-l)^{\lambda}] + \frac{1}{2}(2\chi+1+\xi)(g^{\rho\sigma}q^{\lambda} - g^{\lambda\sigma}q^{\rho}). \quad (13)$$

When these integrals are evaluated, we find the divergent terms to be

$$\chi \frac{M^2}{m_w^2} + (2\chi + \frac{10}{3}) \ln \frac{M^2}{m_w^2}. \quad (14)$$

Keeping terms of order $\ln x$ (except for the lepton-to- m_w mass ratio), we find the result

$$\langle r_{\nu}^2 \rangle_a = \frac{g_w^2}{8\pi^2 m_w^2} \left\{ 9 \frac{m^2}{m_w^2} + 4 \ln \frac{m_w^2}{m^2} + \frac{25}{24} + \frac{1}{5} \frac{m_w^2}{M^2} \right\}, \quad (15)$$

$$\begin{aligned} \langle r_{\nu}^2 \rangle_b = \frac{g_w^2}{8\pi^2 m_w^2} \left\{ -\frac{3}{2} \chi \frac{M^2}{m_w^2} - (3\chi + 5) \ln \frac{M^2}{m_w^2} + \left(\frac{21}{2} \chi + 5 - \frac{1}{8} - \frac{1}{240} \right) \right. \\ \left. - 9\chi \frac{m_w^2}{M^2} \ln \frac{M^2}{m_w^2} + 3 \frac{m^2}{m_w^2} + \left(6\chi - 6 - \frac{1}{10} + \frac{3}{400} \right) \frac{m_w^2}{M^2} \right\}. \quad (16) \end{aligned}$$

We combine the contributions for μ^+ and M^- from Eqs. (15) and (16)

and obtain

$$\langle r_e^2 \rangle = \frac{g_w^2}{2\pi^2 m_W^2} \left\{ \ln \frac{m_M^2}{m^2} + 3 \frac{(m^2 - m_M^2)}{m_W^2} \right\}. \quad (17)$$

Note that the dependence of $\langle r_\nu^2 \rangle$ on M^2 , m_W^2 , and κ dropped out to yield a finite result. The constraint on m_M from requiring that $\langle r_\nu^2 \rangle$ be positive definite ($m_M < 2m_W$) is satisfied here.

The present model based on $SO(3)$ is free of anomalies and neutral currents, and is renormalizable. It has the usual features of weak interactions such as universality and the Cabibbo current. The leptons and hadrons are introduced in a symmetric way.¹⁷ The decays $K_L^0 \rightarrow \mu^+ \mu^-$ and $K^+ \rightarrow \pi^+ e^+ e^-$ are suppressed by the use of additional quarks.¹⁾ The price which must be paid here is the proliferation of unobserved heavy leptons M , X^0 , Y^0 , E and quarks q^0 , q^- , p' , $q^{0'}$, and $q^{-'}$.

ACKNOWLEDGMENT

The authors would like to thank R. Torgerson for comments on the manuscript, and one of us (G.A.) acknowledges an enlightening discussion with R. Zimmerman.

References

1. S. Weinberg, Phys. Rev. Letters 19, 1264 (1967); 27, 1688 (1971);
Phys. Rev. D 5, 1412 (1972).
A. Salam, in Elementary Particle Theory, edited by N. Svartholm
(Almquist and Forlag AB, Stockholm, 1968).
2. H. Georgi and S. L. Glashow, Phys. Rev. Letters 28, 1494 (1972).
3. B. W. Lee, Phys. Rev. D 6, 1188 (1972).
4. J. Prentki and B. Zumino, Nucl. Phys., B47, 99 (1972).
5. G. 't Hooft, Nucl. Phys. B33, 173 (1971); B35, 167 (1971);
B. W. Lee, Phys. Rev. D 5, 823 (1972);
B. W. Lee and J. Zinn-Justin, Phys. Rev. D 5, 3121, 3137, 3155 (1972).
6. H. Georgi and S. L. Glashow, Phys. Rev. D 6, 429 (1972);
D. J. Gross and R. Jackiw, Phys. Rev. D 6, 477 (1972).
7. K. Tanaka and R. Torgerson, Nucl. Phys. B45, 637 (1972);
See also A. Salam and J. C. Ward, Phys. Letters 13, 168 (1964).
8. The ψ_e and ψ_μ are distinguished by electron and muon number. The scalar meson triplet ϕ is invariantly coupled to the leptons so as to conserve muon and electron numbers.

9. In the representation $t_1 = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} / \sqrt{2}$ $t_2 = i \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} / \sqrt{2}$

$t_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, Eq. (2) can be rewritten as

$$e \left[\bar{\psi}_\mu \gamma_\mu \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \psi_e + \bar{\psi}_e \gamma_\mu \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \psi_e + A_\mu \bar{\psi}_e \gamma_\mu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \psi_e \right]$$

$$+ (\psi_e \rightarrow \psi_\mu) = -i A_\mu \bar{\psi}_e \gamma_\mu \times \psi_e + (\psi_e \rightarrow \psi_\mu).$$

10. B. W. Lee, J. R. Primack, and S. B. Treiman, NAL - THY preprint, June 1972; S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).
11. For the hadrons, we have the triplets $\psi = \psi_L + \psi_R$, $\psi' = \psi'_L + \psi'_R$ and singlets s and s'

$$\psi_L = (p_L, g_L^0 \cos \beta + n_L' \sin \beta, g_L^-)$$

$$\psi_R = (p_R, g_R^0, g_R^-),$$

$$\psi'_L = (p'_L, g'_L, g'^-_L)$$

$$\psi'_R = (p'_R, g'^0_R \cos \beta + \lambda'_R \sin \beta, g'^-_R),$$

$$s = -g_L^0 \sin \beta + n_L' \cos \beta,$$

$$s' = -g'^0_R \sin \beta + \lambda'_R \cos \beta,$$

where

$$n' = n \cos \theta + \lambda \sin \theta$$

$$\lambda' = -n \sin \theta + \lambda \cos \theta.$$

These are a variant of the 8-quark version of Georgi-Glashow $SO(3)$ model, proposed by Bjorken. The mesons are constructed from quark-antiquark pairs while to form the baryons one must add q^0 (regarded as a singlet) to the quark-antiquark pair.

The coupling of the vector bosons to the hadrons $\psi = \psi_L + \psi_R$, etc., is

$$e A_{i\mu} J_{i\mu}^h,$$

where

$$J_{i\mu}^h = \bar{\Psi} \gamma_\mu t_i \psi + \bar{\Psi}' \gamma_\mu t_i \psi'.$$

In the limit in which quarks p' , q^0 , q'^0 , q^- , and q'^- are regarded as heavy, the above reproduces the Cabibbo coupling.

$$e \sin \beta \left[-W_\mu^+ (\bar{p} \gamma_\mu n'_1) - W_\mu^- (\bar{n}'_1 \gamma_\mu p) \right] + e A_\mu (\bar{p} \gamma_\mu p) .$$

In addition, the triplets of leptons and triplets and singlets of hadrons are coupled invariantly to a Higgs scalar meson triplet ϕ which is introduced in order to facilitate spontaneous breaking of the gauge symmetry. The neutral member of ϕ develops a vacuum expectation value, the charged members disappear, and the $W^\pm$ bosons become massive when the gauge symmetry is broken.

12. Because the model is 'vector like' the leptons and hadrons may have masses before the gauge symmetry is broken. See Ref.2.
13. J. R. Primack and H. R. Quinn, Phys. Rev. D 6, 3171 (1972);
K. Fujikawa, B. W. Lee and A. I. Sanda, Phys. Rev. D 6, 2923 (1972).
14. W. A. Bardeen, R. Gastmans, and B. Lautrup, Nucl. Phys. B46, 319 (1972).
15. T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962).
16. J. Bernstein and T. D. Lee, Phys. Rev. Letters 11, 512 (1963);
P. H. Meyer and D. Schiff, Phys. Letters 8, 217 (1964).
17. See for example, J. Schechter and Y. Ueda, Phys. Rev. D 2, 736 (1970) and
K. Tanaka, Phys. Rev. D 5, 3243 (1972).

Figure Caption.

Figure 1. Feynman diagram contributing to the charge radius of the neutrino.

(a.)

(b.)