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MEASUREMENT OF THE POLARIZATION PARAMETER
IN π^+p SCATTERING FROM 356 TO 519 MeV/c

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(Ph. D. thesis)

January 1973

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-iii-

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<u>Contents</u>	<u>Page</u>
I. Introduction	1
II. Experimental Method and Equipment	
A. Beam	9
B. Target	16
C. Detector Layout and Terminology	19
D. Electronics	27
III. Data Analysis	
A. Background Suppression	34
B. Separation of Pion Up from Pion Down Events	42
C. Determination of the Scattering Angle	47
IV. Results	
A. Calculation of the Polarization Parameter	49
B. Errors	
1. Statistical Errors	51
2. Systematic Errors	52
V. Discussion of the Results	
A. Comparison with Other Experiments	77
B. Application to Phase Shift Analysis	
1. Effect on Previous Analyses	79
2. Ambiguities in Single Energy Phase Shift Analysis	87
Acknowledgments	94
References	96

MEASUREMENT OF THE POLARIZATION PARAMETER IN $\pi^{\pm}p$ SCATTERING
FROM 356 TO 519 MeV/c^{*}

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Abstract

Using a polarized proton target, the polarization parameter for both π^+p and π^-p scattering was measured at four pion laboratory momenta: 356 MeV/c, 417 MeV/c, 468 MeV/c, and 519 MeV/c. The results are compared with those of other experiments, and the effects of the discrepancies between this and other experiments on the established phase shift solutions at these energies is examined.

* Work done under the auspices of the U.S. Atomic Energy Commission.

I. INTRODUCTION

Because the scattering of a spin-zero particle from a spin-1/2 target is the simplest measurement of the strong interactions of the elementary particles that one can make, pion-nucleon scattering has been intensively studied by experimental particle physicists. There are four observables which completely describe 0-1/2 scattering: the differential cross-section $I_0(\theta)$, the polarization parameter $P(\theta)$, and the two spin rotation parameters $A(\theta)$ and $R(\theta)$. The polarization parameter is defined to be the polarization of the outgoing spin-1/2 particle when the initial state spin-1/2 particle is unpolarized. (The A and R parameters are defined to be the transverse polarization of the outgoing spin-1/2 particle when the initial state spin-1/2 particle is polarized, in the plane of scattering, parallel to and perpendicular to the beam momentum, respectively.)

The definition of the polarization parameter suggests that a double scattering experiment is required to measure $P(\theta)$ for πp scattering. In such an experiment, the first interaction produces the polarized proton, while the second interaction is necessary to analyze its polarization. While $P(\theta)$ was indeed first measured this way, it has become possible, through the advent of the polarized proton target, to measure it in a single scattering experiment. This second method, which is much simpler

and freer from systematic error, is the method used in the present experiment.

In the double scattering experiment, pions are scattered from a liquid hydrogen target, and the recoil protons are scattered in turn from a second target, typically made of carbon. If the asymmetry in the scattering of protons of known polarization from carbon has been previously determined, then the polarization of these recoil protons may be found by measuring the asymmetry of their scattering from the second target. One way to measure this asymmetry is with two scintillation counter telescopes pointing at the carbon target at equal angles to either side of the recoil protons' incident direction.⁽¹⁾ Another method is to use a carbon plate spark chamber as both the second target and the detector of the asymmetry.⁽²⁾ In the counter telescope experiment, only one scattering angle is measured at a time, making it very tedious to cover a large angular range. Furthermore, very few of the protons scattered from the target reach the telescopes, so the data acquisition rate is very slow. In the carbon plate spark chamber experiment the counting rate is again slow, due to the intrinsic slowness of pulsing and photographing optical spark chambers. The polarized proton target experiment entails only a single scattering and the scattered particles may be detected with simple scintillation counter hodoscopes. This facilitates both a high counting rate and a large range of

scattering angles, although very small pion scattering angles are inaccessible because the recoil proton has too little energy to escape from the polarized target cryostat.

(Measurements of the spin rotation parameters in πp scattering necessarily involve initially polarized protons, and therefore such measurements were impossible before the development of the polarized proton target. Very few such measurements have been made to date because the geometry of most existing polarized targets is such that the plane of scattering is constrained to be perpendicular to the direction of polarization by the polarizing magnets' pole pieces. The use of superconducting coils instead of iron magnets in the future will open the way to extensive measurements of these parameters.)

The differential cross-section for the scattering of pions from a polarized proton target is given by the expression:

$$I(\theta) = I_0(\theta) [1 + P(\theta) \vec{P}_t \cdot \hat{n}] \quad , \quad \text{Eq. 1}$$

where:

$I_0(\theta)$ = differential cross-section for scattering
from an unpolarized target at center-of-mass
angle θ ,

$\vec{P}_t$ = target polarization,

$\hat{n}$ = unit normal to the scattering plane

$\hat{n} = \frac{\vec{k}_i \times \vec{k}_f}{|\vec{k}_i \times \vec{k}_f|}$, and $\vec{k}_i$, $\vec{k}_f$ are the initial and

final state momenta of the pion in the center-of-mass system.

It can be shown that the parameter $P(\theta)$ in Eq. 1 is in fact the same as the polarization parameter defined above, under the assumption that the πp interaction conserves parity or is time-reversal invariant (or both).⁽³⁾

In this experiment the target was polarized horizontally and the plane of scattering was vertical. Hence $\vec{P}_t \cdot \hat{n} = \pm |\vec{P}_t|$, which maximizes the effect on the cross-section of a given magnitude of target polarization. Furthermore, we see from Eq. 1 that the differential cross-section is a linear function of the magnitude of the target polarization, with slope proportional to the polarization parameter. Our method of measuring the polarization parameter, then, consists in measuring the scattering intensity for different values of the target polarization, fitting the results to a straight line by the method of least squares, and extracting $P(\theta)$ from the slope of the fit.

The systematic errors introduced by the positioning of counters in the two-counter-telescope double scattering experiment are absent. Instead, all scintillation counters remain fixed and the target polarization is changed by electrical rather than mechanical means. (The magnitude of the target polarization is typically kept maximized, while its direction is varied between parallel and antiparallel to $\hat{n}$.) Also absent are systematic errors due to inelastic proton-carbon scattering in the analyzer. On the other hand, the polarized target experi-

ment suffers from a high background due to scattering from bound protons in the non-hydrogenous elements which make up the bulk of the target material.

The ultimate aim of experiments such as this is to reconstruct the strong interaction scattering amplitudes involved, which are a complete phenomenological description of the interaction. The most general form for the amplitude describing parity conserving 0-1/2 scattering is:

$$M(\theta) = f(\theta) + ig(\theta)\vec{\sigma} \cdot \hat{n} ,$$

where:

$f(\theta)$ is the spin non-flip part of the amplitude,

$g(\theta)$ is the spin flip part of the amplitude,

$\vec{\sigma}$ is the vector operator constructed from the

three Pauli spin matrices.

Taking into account isotopic spin, the πp system can have total isospin equal to 3/2 or 1/2. Hence, there are four complex scattering amplitudes to be determined:

$$f_{3/2}(\theta); f_{1/2}(\theta); g_{3/2}(\theta); g_{1/2}(\theta) .$$

In $\pi^+ p$ elastic scattering, the initial and final states are pure $I = 3/2$ and the scattering amplitudes are:

$$f^+(\theta) = f_{3/2}(\theta); g^+(\theta) = g_{3/2}(\theta) .$$

In $\pi^- p$ elastic scattering, the initial and final states are a linear superposition of $I = 3/2$ and $I = 1/2$ states and the scattering amplitudes are:

$$f^-(\theta) = \frac{1}{3}f_{3/2}(\theta) + \frac{2}{3}f_{1/2}(\theta);$$

$$g^-(\theta) = \frac{1}{3}g_{3/2}(\theta) + \frac{2}{3}g_{1/2}(\theta) \quad .$$

In charge exchange scattering, $\pi^-p \rightarrow \pi^0n$, the initial and final states are again superpositions of the two isospin eigenstates and the scattering amplitudes are:

$$f^0(\theta) = \frac{\sqrt{2}}{3} [f_{3/2}(\theta) - f_{1/2}(\theta)];$$

$$g^0(\theta) = \frac{\sqrt{2}}{3} [g_{3/2}(\theta) - g_{1/2}(\theta)] \quad .$$

In terms of these amplitudes, the various observables for π^+p elastic scattering are:

$$I_0(\theta) = |f^+(\theta)|^2 + |g^+(\theta)|^2$$

$$P(\theta) = \frac{2\text{Im}[\{f^+(\theta)\}^* g^+(\theta)]}{|f^+(\theta)|^2 + |g^+(\theta)|^2}$$

$$\begin{pmatrix} A(\theta) \\ R(\theta) \end{pmatrix} = \begin{pmatrix} -\cos(\theta) & -\sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \beta(\theta) \\ \gamma(\theta) \end{pmatrix}$$

where

$$\beta(\theta) = \frac{2\text{Re}[\{f^+(\theta)\}^* g^+(\theta)]}{|f^+(\theta)|^2 + |g^+(\theta)|^2}$$

$$\gamma(\theta) = \frac{|f^+(\theta)|^2 - |g^+(\theta)|^2}{|f^+(\theta)|^2 + |g^+(\theta)|^2}$$

and similarly for the reactions $\pi^-p \rightarrow \pi^-p$ and $\pi^-p \rightarrow \pi^0n$.

It has been shown⁽⁴⁾ that in order to solve unambiguously for the four complex scattering amplitudes at a given energy, at least eight of the 12 different observables must be known. To date, only the three differential cross-sections and the three polarization parameters have been extensively measured. Consequently, it has been necessary to determine the amplitudes at each energy by a fitting procedure. This in general does not give a unique result, but rather gives several solutions at each energy. Uniqueness is then obtained by imposing energy continuity on the various fit solutions, starting from a low energy where the solution is uniquely known.

The usual method of fitting starts with a parameterization of the four isospin amplitudes. They are expanded in partial wave angular momentum states:

$$f = \frac{1}{k} \sum_{\ell=0}^L \left((\ell+1) T_{\ell+} + \ell T_{\ell-} \right) P_{\ell}(\cos(\theta)),$$

$$g = \frac{1}{k} \sum_{\ell=1}^L (T_{\ell+} - T_{\ell-}) P'_{\ell}(\cos(\theta)) ,$$

where:

$P_{\ell}(\cos(\theta))$ is the Legendre polynomial of order ℓ ,

$P'_{\ell}(\cos(\theta))$ is the associated Legendre polynomial of order ℓ ,

L is a cut-off value for the angular momentum,

k is the center-of-mass momentum,

$\ell+$ means $j = \ell + 1/2$,

l - means $j = l - 1/2$,

$T = \frac{ne^{2i\delta} - 1}{2i}$ is a partial wave amplitude.

The η 's and δ 's are called phase shift parameters. It is these parameters which are varied in the fitting procedure until a minimum χ^2 for the fit is found:

$$\chi^2 = \sum_{\substack{\text{all} \\ \text{measured} \\ \text{points}}} \left[\frac{\text{measured value} - \text{fit value}}{\text{error in measured value}} \right]^2$$

The results of this experiment, in preliminary form, have been used in an extensive phase shift analysis which has been reported elsewhere.⁽⁵⁾ Using some of the techniques developed for that effort, we will examine some of the consequences of this experiment for phase shift analyses in our energy range.

The experiment was performed at the 184" Cyclotron of the Lawrence Berkeley Laboratory, Berkeley, California, during 1966-67. The polarization parameter was measured for both π^+p and π^-p scattering at four pion laboratory momenta: 356 MeV/c, 417 MeV/c, 468 MeV/c, and 519 MeV/c.

II. EXPERIMENTAL METHOD AND EQUIPMENT

A. Beam

The pion beams were generated as follows. The 740 MeV external proton beam of the cyclotron was focused onto a production target by means of a quadrupole triplet magnet. Overall EPB intensity was monitored by a secondary emission monitor (SEM) positioned as far upstream as possible, at the port where the beam emerged from the primary cyclotron shielding. Production targets for positive pion beams were made of polyethylene (CH_2), while beryllium was used for negative pions. For each momentum, the production target length was varied to maximize the pion flux at the polarized target.

The π^+ flux, when plotted against CH_2 production target length for a given beam momentum, exhibits two maxima, as shown in Fig. 1. The broad maximum is due chiefly to three processes: $pp \rightarrow \pi^+ np$ where the target proton is either free or bound in carbon, and $pn \rightarrow \pi^+ nn$ where the target neutron is bound in carbon. Since the final state has three particles in it, the momentum of the forward-going pions is not unique. Furthermore, the Fermi momenta of the target nucleons in carbon spreads the pion momenta even more. Hence the broadness of this peak. The sharp peak is due to the reaction $pp \rightarrow \pi^+ d$. Here the final state is two-body and the forward-going pions have a well-defined momentum. In addition, the pion momentum turns out to be nearly independent of the thickness of production target

Fig. 1. π^+ Beam Intensity vs. CH₂ Production Target Length, 417 MeV/c.

traversed by the incident proton before the interaction occurs. This is so because the energy losses of 740 MeV protons and 200-500 MeV pions in CH_2 are roughly equal: $-2.6 \text{ MeV cm}^2/\text{g}$ and $-2.2 \text{ MeV cm}^2/\text{g}$ respectively. Hence the sum of the energy losses of the incident proton and the outgoing pion are roughly constant, and the pion always emerges with approximately the same energy. The $pp \rightarrow \pi d$ reaction was the more copious, except at the highest momentum.

The pion beam transport system is shown in Figs. 2 and 3. The beams typically had a horizontal full width at half maximum of $1\text{-}1/2''$, with a vertical FWHM of $1\text{-}1/4''$, as measured with a magnetostrictive wire spark chamber with $1/4''$ resolution, situated at the eventual position of the polarized target. The major properties of the beams are summarized in Tables 1 and 2. Beam momenta were measured first by wire-orbiting of the downstream leg, from the intermediate focus to the final focus. This was checked by measuring the ranges of the beam particles. Figure 4 is an example of a range curve obtained with positive pions, plus some muon contaminant, in copper absorber. The peak for the pions gives a momentum of $424 \pm 1.5 \text{ MeV}/c$, while the muon peak gives $423 \text{ MeV}/c$. The range measurements agreed with the wire-orbiting within experimental error. Final beam momenta were then determined by calculating and subtracting off the energy losses in the polarized target cryostat and crystals.

Fig. 2. Beam Layout

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↑↓ Converging quadrupole
 ↓↑ Diverging quadrupole

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Fig. 3. Beam Optics.

TABLE 1. Properties of π^+ Beams

Pion Momentum, MeV/c	$\Delta p/p$, %	Length of CH ₂ production target	Pions per second on target	Events per second	Pions/ all particles
356 ± 8	± 1.0	14"	4×10^5	20	.56
417 ± 10	± 1.7	18"	6×10^5	35	.53
468 ± 13	± 2.3	9.5"	2×10^5	15	.37
519 ± 17	± 2.9	2.5"	3×10^5	15	.12

TABLE 2. Properties of π^- Beams

Pion Momentum, MeV/c	$\Delta p/p$, %	Length of Be production target	Pions per second on target	Events per second	Pions/ all particles
356 ± 8	± 1.0	6"	1.5×10^5	2.0	.91
417 ± 10	± 1.7	4"	1.5×10^5	1.5	.88
468 ± 13	± 2.3	4"	1.0×10^5	1.0	.90
519 ± 17	± 2.9	4"	0.4×10^5	0.4	.86

Fig. 4. π^+ Differential Range Curve in Copper,
424 MeV/c.

(The spread in each beam momentum quoted includes, in quadrature, ± 7 MeV due to the difference in energy losses of pions scattering at different points within the target crystals.)

The length of the beam line, from production target to polarized target, was 48'. A relativistic particle requires about 50 ns to traverse this length, or about twice the lifetime of charged pions at rest. Consequently, an appreciable percentage of the pions produced decayed in flight. At the lowest beam energy, 243 MeV, this amounted to a 53% attrition rate. At the highest energy, 398 MeV, the rate was 40%.

The proton contamination of the π^+ beams was reduced below the levels shown in the table by the insertion of polyethylene at the intermediate focus. Since the protons lose more momentum in this absorber than do the pions, they are bent more than the pions by the bending magnet in the downstream leg of the beam. The result is that the final proton focus is displaced laterally from the final pion focus, and most of the protons miss the polarized target.

Helium bags were placed everywhere within the beam transport system (except at the intermediate focus), to cut down multiple scattering.

B. Target

The polarized target used in this experiment consisted of six crystals of $\text{La}_2\text{Mg}_3(\text{NO}_3)_{12} : 24\text{H}_2\text{O}$ (LMN) doped with Nd.

The crystals were each 1/4" thick, stacked to make a total thickness of 1-1/2". The density of these crystals is 2.0 g/cm^3 . As they are 3% by weight hydrogen, the amount of hydrogen in the pion beam is 0.25 g/cm^2 . The transverse dimension of the crystals is roughly 1-1/4".

The free protons in the water of hydration of these crystals can be polarized by dynamic nuclear orientation⁽⁵⁾, a method requiring low temperature, high magnetic field, and irradiation by microwaves of a suitable frequency. A temperature of 1°K was achieved by cooling the crystals in a vertical He^4 cryostat. The magnetic field was 18.8 Kilogauss, with a corresponding microwave frequency of 70 GHz. Target polarizations thus achieved ranged from 45% to 55%.

By making a 0.2% change in the microwave frequency, one can reverse the direction of the polarization of the protons from parallel to the magnetic field (plus polarization) to antiparallel to it (minus polarization). Such reversals were effected every 1-1/2 to 2 hours during data-taking, in order to minimize systematic errors due to long-term changes in experimental conditions.

The polarization was measured by the proton magnetic resonance technique,⁽⁶⁾ using a constant-current Q-meter at 78.5 MHz. The rf sampling field was made fairly uniform over the whole sample by generating it with two figure-eight coils separated by a plane septum (see Fig. 5). There were three crystals on each side of the septum.

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Fig. 5. NMR Pickup Design.

A read-out of the target polarization consisted of sweeping the rf frequency through the proton resonance line and digitizing the resultant Q-meter voltage output for later analysis. Each read-out took seven minutes, with read-outs being taken every 15-20 minutes (once per run). Between read-outs the peak of the resonance signal was monitored continuously, and the results of the analyses of the digitized read-outs were corrected for any variation in the target polarization thus observed.

The read-out system was calibrated by measuring the polarization when the target was at thermal equilibrium with the surrounding helium bath and magnetic field (i.e., brute force polarization, rather than dynamic nuclear polarization using microwaves.) At thermal equilibrium, the target polarization is calculable from Boltzmann statistics. Its NMR signal is large enough to measure to $\pm 5\%$ of itself with our system. Calibration of the read-out system was carried out once every 24 hours or less.

C. Detector Layout and Terminology

The polarized target and associated particle detectors are shown in Fig. 6. (Scintillation counters were the only type of detector used in this experiment, apart from the previously mentioned SEM and the wire chamber used in beam tuning.) Particles

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Fig. 6. Detector Layout.

incident on the target were defined by coincidences between hodoscope H and counter F. H was a non-overlapping array of thin scintillators, two wide by three high, which determined the angle of incidence of the particle onto the target. The primary function of counter F was to provide a coincidence with the H counters, and therefore it was made large enough to intercept the whole beam.

A second output signal from F was pulse-height analyzed into two channels to provide a trigger veto of many of the events caused by beam protons, when the beam was positive, and by two nearly-simultaneous pions. Actually, pp events did not greatly affect our measurement, in spite of the substantial proton contamination in the beam. This is because it typically required at least 300 MeV/c for a proton to escape from the polarized target and its magnetic field, so that one of the protons usually did not escape. Similarly, simultaneous events from two beam pions would generally be rejected when too many particles were detected in the final state. Nevertheless it was felt useful to resort to this simple expedient to help suppress background. The stability of the discriminator level of the two-channel pulse-height analyzer, then, was not critical. To protect against large changes in this level, the rate at which FH coincidences were vetoed was monitored throughout the experiment.

Since the focused beam spot was kept appreciably larger

than the target crystals' transverse dimensions, for reasons which will be explained later, some of it scattered from the polarizing magnet and cryostat parts. Most of this background was easily eliminated with appropriately-positioned veto counters. Events originating in the iron of the magnet were vetoed by four counters P which covered the pole-tips both upstream and downstream of the cryostat. Similarly, pions striking the cryostat above or to the sides of the region containing the crystals were vetoed by three counters placed just upstream of it, top, left, and right ($\bar{T}$, $\bar{L}$, $\bar{R}$). (Although a bottom veto counter was contemplated, it had to be omitted because it would have interfered with the detection of some of the down-going scattered particles.) The ratio of counts in L and R provided a monitor of the horizontal beam steering.

Data were taken both with the field of the polarizing magnet set to bend positive beam particles up (or negative particles down), and with it set to bend in the opposite direction. This was done to enable different center-of-mass angular regions to be investigated with the given detector geometry, and had nothing to do with reversing the direction of the target polarization, which, as noted above, was accomplished by changing the frequency of the polarizing microwaves. When the beam was bent down, it struck the magnet yoke. To eliminate the background due to this, a counter $\bar{B}$ was positioned on the yoke to intercept the transmitted beam and veto any

event occurring simultaneously with a beam particle hitting the yoke.

A two-counter telescope, T_1 - T_2 , looked at particles backscattered from the cryostat and was one of several pion beam monitors.

The scattered pions and the recoil protons were detected in two scintillator hodoscopes, one positioned above the beam line and called the up array, and the other below the beam line and called the down array. Each of these two arrays consisted of two orthogonal sub-arrays. Since the plane of scattering was constrained to be roughly vertical by the 4" vertical gap between the pole pieces of the polarizing magnet, those counters with their long direction oriented horizontally defined the scattering angle of the event and were called θ counters. The orthogonal counters, which defined the azimuthal angle, over the small angular region permitted by the pole-piece geometry, were called ϕ counters.

Measuring angles from the direction of the beam center line shown in Fig. 6, the θ_{up} array extended from $\theta_{lab} = 0^\circ$ to $\theta_{lab} = 53^\circ$. The θ_{down} array went from $\theta_{lab} = -50^\circ$ to $\theta_{lab} = -144^\circ$. The opening angle of the polarizing magnet was $\pm 7^\circ$ from the magnet's vertical plane of symmetry, and the ϕ_{up} and ϕ_{down} arrays were wide enough to cover that whole region.

There were 30 θ_{down} counters, overlapped to provide 59 distinguishable bins, and 24 θ_{up} counters, overlapped (except

in two cases) to provide 45 bins. Since each θ array consisted of two intersecting planes of counters, as seen in Fig. 6, we chose to use separate plane ϕ arrays behind each of these planes, rather than single arrays of extremely long, bent counters. Hence there were 20, rather than 10, ϕ_{up} counters, overlapped to create 19 bins, and 12 ϕ_{down} counters, overlapped for 11 bins.

The θ and ϕ arrays existed before this experiment was planned. Consequently we did not have complete freedom in the choice of bin sizes and array positions. Our primary aim in locating these arrays was to subtend the largest center-of-mass angular region possible. This criterion determined the distance from the target to one of the arrays. In addition, we wanted to have each bin in the up array subtend roughly the same center-of-mass angular region as the conjugate bin in the lower array. This criterion determined the distance to the other array.

However, because of the polarizing magnetic field, the second criterion could only be met during part of the data-taking. In Fig. 7 we show the four basic configurations used in this experiment, where we scattered both positive and negative pions in a polarizing field pointed in either of two directions. For each configuration, scattering at two different center-of-mass angles is depicted, with the solid trajectories corresponding to the events with the more forward-going, high momentum pions. We see that in some situations the outgoing particles

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Fig. 7. Focusing Effects of the Polarized Target Magnet.
(Dashed trajectories correspond to events with higher momentum transfer.)

tend to be focused by the polarizing field, while in others they are defocused. Consequently, it is clearly impossible to arrange the bin sizes in the arrays such that conjugate bins always subtend the same center-of-mass angular region. We chose to satisfy this criterion in the first configuration shown. As a result, in the other configurations the ratio between conjugate up and down bins was as high as three to one. Distances to the up array counters ranged from 70" to 80", while distances to the down array counters ranged from 35" to 60".

Two counters, up-defining and down-defining (U_d and D_d), were positioned adjacent to the cryostat such that any particle coming from the crystals and reaching the up array had to pass through U_d , and similarly for D_d . One output from each of these counters went into the event trigger. A second output was fed into a six-channel pulse-height analyzer consisting of a linear amplifier, passive splitter, five discriminators, and five flip-flops. The information thus obtained proved to be very useful in distinguishing pion-up proton-down events from pion-down proton-up events, where kinematics and geometry allowed for the detection of both.

A specially difficult case of this latter problem occurred where the kinematics for the two cases were nearly the same, i.e., where the down proton was detected in the same bin as the down pion

(or very near by), while the up pion was detected in the same bin as the up proton. To aid in resolving this kinematic ambiguity, a copper slab 1-1/4" thick, followed by counter R, was placed behind the up array in the ambiguous region. Only pions triggered R. The protons were stopped in the copper.

D. Electronics

Data was acquired according to the scheme depicted in Figs. 8 and 9. Signals from each of the counters in the five hodoscope arrays H, θ_{up} , θ_{down} , ϕ_{up} , and ϕ_{down} were fed through threshold discriminators and then split in two. One signal went into a delay line, while the other was OR'ed, with all of the counters in each array being added to form $\sum H$, $\sum \theta_{up}$, $\sum \theta_{down}$, $\sum \phi_{up}$, and $\sum \phi_{down}$. These summed signals, together with the signals from the beam defining counters described above, were required to satisfy pre-imposed logic requirements in fast logic circuitry in order to produce a trigger pulse. If the master trigger condition was met, gates were opened at the end of each delay line, allowing short-memory flip-flops to be set by pulses coming from any of the array counters. A small computer then interrogated each of these flip-flops to find out which counters fired.

The first pre-condition for a master trigger was a coincidence condition called BD, the coincidence of a beam pion with counts in the up-defining and down-defining counters. This

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Fig. 8. Beam-Defining Fast Logic.

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Fig. 9. Trigger-Defining Fast Logic.

indicated an interaction in the target. (Actually, when the polarizing magnet was set to bend the beam up, the beam would have passed through the Ud counter. Since there is a good deal of hydrogen in scintillator material, this would have made the Ud scintillator a particularly serious source of background events. For this reason, Ud was physically removed from the experiment when the beam was bent up.) The presence of a beam pion incident on the target was signalled by the coincidence $FH = \sum H \cdot F$ along with no counts in the pole-tip antis or the top, left, and right antis: $BEAM = FH \cdot \sum \bar{P} \cdot \bar{T} \cdot \bar{L} \cdot \bar{R} \cdot \bar{M}$. (M was the second signal from F which was pulse-height analyzed. A pulse on M meant either a beam proton or two simultaneous beam pions.) The indication that the beam pion interacted in the target was a coincidence between BEAM and counts in Ud and Dd, along with no count in the beam back-up counter:

$$BD = BEAM \cdot Ud \cdot Dd \cdot \bar{B}.$$

The second pre-condition for a master trigger was the detection of scattered particles in each of the arrays θ_{up} , ϕ_{up} , θ_{down} , ϕ_{down} , with the separate coincidences $\theta - \phi_{up} = \sum \theta_{up} \cdot \sum \phi_{up}$ and $\theta - \phi_{down} = \sum \theta_{down} \cdot \sum \phi_{down}$ being formed.

The master trigger, then, was the coincidence

$$\theta\theta = BD \cdot (\theta - \phi_{up}) \cdot (\theta - \phi_{down}).$$

BD , $\theta\theta$, and several of the various sub-coincidences from

which BD was formed were counted on 10 MHz decade scalers to provide monitors of experimental conditions.

Because of the lengths of the array counters, and because of the greatly varying times of flight to the different counters (particularly in the situation where the pion could go either to the up or the down array), the $\theta\theta$ coincidence timing had to be left fairly loose. Once a master trigger coincidence had been achieved, however, it was important that the timing at the coincidence gates at the ends of the delay lines be made tight so that only the flip-flops corresponding to the counters which actually fired would be set. Otherwise, large noise pulses or counts from other events might set the flip-flops, resulting in events with two non-adjacent counts recorded in one of the arrays, which we called NG2's. This tight timing was accomplished by making coincidences between a long $\theta\theta$ pulse and a shortened array pulse such that the coincidence occurred a fixed time after the array counter fired. These coincidences are marked in Fig. 9 as $\theta_{up} TOG$, etc.; in the toggle mode the circuits cannot be triggered a second time until they receive a reset pulse. The toggle circuit outputs were fed into pulse generators whose 15 ns wide outputs were sent to the coincidence gates just in time to meet the original pulses coming out of the delay lines, thus setting the appropriate flip-flops.

With this timing scheme, the rate of NG2's per master trigger never exceeded 15%. The rate of NGØ's (no counts recorded in one array) per master trigger was always negligible. Both of these rates were counted on the 10 MHz scalars.

Besides permitting the flip-flops to be set, the master trigger signal served both to alert the small on-line computer, a PDP-5, that an event had occurred, and to gate off the beam-defining logic circuitry and the 10 MHz scalars so that the various monitors being scaled would reflect conditions only during the times when the data acquisition circuitry was enabled. On receiving a master trigger, the computer stopped whatever it was doing and read out the status of all the flip-flops. Since 12 bits is the size of a PDP-5 word, batches of 12 flip-flops were simply stored as binary words in the computer's memory. Each event consisted of eight such words. Once all eight had been transferred to core, the PDP-5 reset the flip-flops and the toggle circuits, gated on the logic and the scalars, and returned to its interrupted task. The system was then ready to accept another event.

The time to acquire a single event in the above manner was about 60 μ sec, with essentially all of that time being taken by the computer reading in the data.

Due to the limited amount of core space available in the computer for data storage (necessitated by a lengthy data-

acquisition program), the accumulated data had to be written out onto magnetic tape when the number of events stored reached 128. It took .75 sec to perform this write-out operation.

During those times when the computer was not occupied either in reading in or writing out data, it coded the events in into a more compact form (three PDP-5 words per event), checked for obviously invalid events (NG1's and NG2's) and made various useful on-line displays of experimental conditions, such as the distribution of counts in each of the arrays, the pulse-height spectra in Ud and Dd, etc.

III. DATA ANALYSIS

In order to calculate the polarization parameter, we first had to determine the counting rate of elastic events as a function of center-of-mass scattering angle and target polarization. Then the least squares method outlined in the introduction could be applied. Hence the three basic tasks of the data analysis were the separation of πp elastic events from background, the determination of the center-of-mass scattering angle of the elastic events, and the calculation of the target polarization.

A. Background Suppression

The output tapes from the PDP-5 on-line computer consisted of a chronological sequence of events, each of which was specified by eight numbers. The first five numbers gave the location of the event in the five arrays H , θ_{up} , θ_{down} , ϕ_{up} , and ϕ_{down} , in terms of which bin registered a particle. The sixth and seventh numbers specified the pulse height of the defining counters Ud and Dd . The eighth number was the output of the counter R ; one if R fired (indicating a pion-up event in the kinematically ambiguous region), zero otherwise. One could thus think of each event as initially lying somewhere in an eight-dimensional matrix of event possibilities. The

number of sites in this matrix, hence the number of different events conceivable, is on the order of 10^8 .

This matrix contains both πp elastic events and background events. We want to identify those matrix regions with the highest concentration of elastic events. We do this by imposing cuts on the eight parameters. These cuts are of two kinds: dynamical and kinematical. As an example of the first, consider the pulse height information from the up- and down-defining counters. If the elastic events which we want to isolate have an up-going proton and a down-going pion, then we require a large pulse in Ud and a small pulse in Dd. An example of a kinematic constraint would be the requirement that the incident pion and the two outgoing particles be coplanar. By imposing conditions of these kinds, we not only will be able to eliminate from consideration the vast majority of background events, but we will also be able to reduce the number of parameters needed to specify an event from eight to two; namely θ_{up} and θ_{down} .

Reduction of the matrix proceeded in three stages. The first stage was accomplished by means of a computer program called AUGUSTUS, named for a Roman emperor noted for his census-taking. Here, the events were examined one by one and cuts were applied to divide them into three categories: good, bad, and so-so. A good event was one which fell at or near a site in the matrix predicted by πp elastic kinematics, with pulse

heights in Ud and Dd consistent with πp elastic scattering. Bad events were those far from such sites. So-so events were those falling in questionable matrix regions. All three categories contained both elastic events and background, in varying degrees.

The usual way to determine the dividing line between these categories was by visual inspection. In a plot of the number of events in each θ_{down} bin vs. θ_{down} bin number, for a given θ_{up} bin, there is generally a peaking of elastic events above the background about the bin conjugate to that θ_{up} for elastic scattering. A typical example of such a hydrogen peak is shown in Fig. 10. By examining the effect upon these peaks of a cut on the data we may determine into which category the discarded data belongs.

Where visual inspection was uncertain, we examined the effect of a given cut upon the final values of the polarization parameter and its statistical error. The cut was then made such that the error in the polarization parameter was a minimum.

The first cut made was on the basis of the pulse height information from the defining counters Ud and Dd. There were $6 \times 6 = 36$ combinations of Ud and Dd bins possible. The events were all processed a first time, with weak constraints on the other six parameters, to see which pairs of Ud and Dd bins exhibited hydrogen peaks. The events were then processed a

Fig. 10. Hydrogen Peak.

second time and all events in those pairs of bins which did not show hydrogen peaks were rejected as so-so's.

AUGUSTUS next tested the $\phi_{\text{up}}-\phi_{\text{down}}$ correlation of the surviving events for coplanarity. In doing so, it first eliminated the horizontal degree of freedom in the incident particle hodoscope array H. The coordinates of each event were rotated about a vertical axis so that the incident pion approached the target in the vertical plane of symmetry of the experiment. This reduced the number of H bins from six to three, called H_y , which specified the incident angle of the incoming particle in the vertical plane of symmetry.

The coplanarity constraint was treated as follows. From the incident pion angle and the position of the event in the up array the expected ϕ_{down} bin for perfect coplanarity was calculated. This calculation used information about πp elastic kinematics in the presence of the polarizing magnet's field supplied by a separate kinematics program called POSTPIAKIN. The ϕ_{down} bins were then classified into five regions or stripes for the event. The central region, stripe 3, was centered about the expected ϕ_{down} bin with a certain width W. Stripes 2 and 4, to either side of stripe 3, were also of width W. The remainder of the ϕ_{down} array was in stripes 1 and 5. Events with the down-going particle going into stripe 3 were considered coplanar. Events falling into stripes 2 or 4 were mildly

non-coplanar. Stripes 1 and 5 events were very non-coplanar and were discarded as bad.

The width parameter W was chosen by first processing all of the events several times, each with a different value for this parameter. The size of the hydrogen peaks in stripes 2, 3, and 4 as a function of W was then examined by eye and a compromise made between including all of the elastic events in stripe 3 and including too much of the background.

Since the laboratory momenta of the outgoing particles varied with scattering angle one can expect that the optimal constraints on U_d and D_d and on the size of W would also vary with that angle. We allowed for these variations by dividing the θ_{up} array into four regions, each with its own cuts.

The final job that AUGUSTUS performed was to crudely check the $\theta_{up}-\theta_{down}$ correlation of each event. POSTPIAKIN predicted the expected θ_{down} bin for each θ_{up} bin. If the actual θ_{down} position of the event was further than 15 bins from the expected position the event was discarded as bad.

The surviving events were scaled in arrays and then written out onto magnetic tape, run by run, for subsequent analysis. These arrays consisted of both the stripe 3 coplanar events and the stripes 2 and 4 near-coplanar events. Of the eight parameters which specified the original events only three remained: θ_{up} , θ_{down} , and H_y .

The second stage of data reduction was accomplished by a computer program called PTHETA. Here, the $\theta_{\text{up}} - \theta_{\text{down}}$ correlations were carefully examined to optimize the number of hydrogen events and the hydrogen-to-background ratio of the final sample of events. This program used as input the AUGUSTUS output tapes, the π p elastic scattering information from POSTPIAKIN, and the values of the target polarization for each run, calculated from the digitized NMR readouts by a program called PIAPOL.

PTHETA could be run in either of two modes. In the first, it plotted the hydrogen peaks for each θ_{up} , summed over all runs. In the second, it calculated the polarization parameter.

In either mode, the program's first task was to eliminate the three H_y bins. Using the kinematics information from POSTPIAKIN, it rotated the coordinates of all events about a horizontal axis until the incident pion direction was along the beam center line.

Then the plots of the hydrogen peaks were made. A typical stripe 3 plot included good, bad, and so-so events, and appropriate cuts had to be made to separate these three categories. The good events, of course, were those under the hydrogen peak, while the bad ones were those far out on the tails. The bad events in stripes 2, 3, and 4, were summed up and used as a monitor with which to weight each run. There were plenty

of obviously bad events for this purpose, so it was easy to choose the limits of the bad event region on the tails.

The limits of the peak itself, however, were not so easily determined. The procedure was to choose one of the momenta, for a given sign of incident pion and sign of polarizing field, and compute the polarization parameter and its uncertainty as a function of peak width, for each θ_{up} bin, over a wide range of widths. The peak width which minimized the error in the polarization parameter was chosen. The value of the polarization parameter was not very sensitive to the width used. The peak widths for the other three momenta were then picked by eye, on the basis of the experience gained with the first momentum. Only the stripe 3 coplanar events were considered here.

The third and final stage of data reduction was carried out by PTHETA in its second mode of operation. The background under the hydrogen peaks was subtracted off, each run was appropriately weighted, and the least squares calculation of the polarization parameter was made.

Determination of the amount of background under the peaks was made experimentally. "Target empty" data was taken for all data-taking configurations, where "target empty" meant running with what we called dummy crystals in the cryostat. The dummy crystals were of roughly the same size, shape, and density as the LMN crystals, and the percentages of elements, other than

hydrogen, were about the same. (Actually, the elemental make-up of the dummy crystals was somewhat different from the LMN, with such changes as Ba⁵⁶ for La⁵⁷ being made for convenience.) Of course, the dummy crystals contained no hydrogen.

This dummy data underwent the same processing by AUGUSTUS as the LMN data, resulting in analysis into hydrogen peaks which consisted wholly of background events. Using the bad events in the tails of both the crystal and dummy data as weighting factors. PTHETA then made the correct run-by-run subtraction of background events under the crystal hydrogen peaks. This resulted in the purest sample of elastic πp events that we could isolate.

B. Separation of Pion-up from Pion-down Events

As noted previously, there were data-taking configurations where elastically scattered pions could go into either the up or the down array. Specifically this occurred at the three highest momenta, when incident positive pions were being bent down by the polarizing magnet. Additional analysis was necessary to cleanly separate these two types of events.

In Fig. 11 we have plotted the θ_{up} angle against the θ_{down} angle for elastic scattering at one of the momenta, for both pion-up and pion-down events. At the point of intersection of these two curves, the center-of-mass angle for pion-up scattering is equal to the c.m. angle for pion-down scattering, and the

Fig. 11. $\theta_{up} = \theta_{down}$ Correlation for
Elastic π^+p Scattering.

laboratory momenta of the up pion, down pion, up proton, and down proton are all equal in magnitude. This may be seen as follows:

Fig. 12. Pion-Up and Pion-Down Scattering

Figure 12 shows π^+ up and π^+ down scattering in the laboratory system for the same c.m. angle, in the absence of the polarizing magnetic field. When $\alpha = \beta$, conservation of the transverse component of momentum implies that the pion and proton laboratory momenta are equal. If we now switch on the magnetic field, both up particles will be bent the same amount, as will both down particles.

Due to the momentum spread in the incident beam and the multiple scattering of the recoil particles in the target, the curves in Fig. 11 actually have finite width. Hence, as we move away from the intersection point, the pion-up and pion-down events remain ambiguous over a significant c.m. angular region of 10° - 15° ; their hydrogen peaks are not resolved.

The systematic error which would be introduced by poor separation of the pion-up from the pion-down events may easily be estimated. Consider the idealized experiment of πp scattering from a very well-behaved polarized target whose polarization is either $+P_t$ or $-P_t$. Then, from Eq. 1, the asymmetry

$$A = \frac{I_+ - I_-}{I_+ + I_-}$$

(where I_+ is the scattering intensity from a positively polarized target, etc.) is equal to $P(\theta) \cdot P_t$. Hence,

$$P(\theta) = \frac{A}{P_t}.$$

In the region of ambiguity the polarization parameter is roughly constant. Hence at a given c.m. angle in this region the asymmetry for pion-up scattering will be equal in magnitude and opposite in sign to that for pion-down scattering. (The sign is opposite because where the pion-up event will have $\vec{P}_t \cdot \hat{n}$ positive, the pion-down event will have it negative.) Hence, a 10% impurity of one sort of event mixed in with the other sort will reduce the observed asymmetry, and hence the polarization parameter, by 10% of itself. Then, when the pion-up results are averaged with the pion-down results, the final numbers will be 10% low (assuming an equal degree of contamination in each case).

Since kinematic analysis will clearly not suffice to separate the two kinds of events, we had to depend upon the

other information we had available, namely the pulse heights from the defining counters Ud and Dd, and the output of the range counter R. (The latter covered too narrow an angular region to be of much use, but, as we shall see, it was crucial in determining how good a separation was achieved.) The first step in the AUGUSTUS analysis of this data, then, was to determine where an event lay in the Ud-Dd two-dimensional pulse-height defining sub-matrix. By looking at the hydrogen peaks for each Ud-Dd combination separately we were easily able to determine which region of the sub-matrix contained primarily pion-up events, which contained primarily pion-down events, and which contained primarily inelastic background events. The pion-up and pion-down regions proved to be well resolved. After labelling an event as pion-up or pion-down or bad according to this criterion, its R signal was checked. If R had fired, that information overrode the defining information and the event was labelled pion-up. The rest of the analysis then proceeded as described above.

We estimated the degree of contamination in the two regions of the Ud-Dd sub-matrix by looking at the distribution of all highly coplanar events for which R fired. (High coplanarity was required to minimize the number of inelastic events in the sample.) These events all had to be pion-up, but some of them fell into the pion-down matrix region. This gave us a ratio of mis-assigned pion-ups to correctly assigned pion-ups. From

the total sample of events we knew the ratio of events in the pion-up region to those in the pion-down region. Hence we knew the ratio of mis-assigned pion-ups to total events assigned to the pion-down region. This number was about 1.5%. The contamination of pion-down events in the pion-up region could not be measured but is assumed to be about the same.

An independent check on the systematic errors due to the poor separation of events in the ambiguous region could be made because the data exhibiting the ambiguity was collected at two different times during the course of the experiment, in roughly equal amounts. In the first instance, the Ud and Dd signals were pulse-height analyzed into only two channels each, whereas the second had the full benefit of six channels each. The method of separation outlined above was used for both sets of data. Clearly, the resultant separation will not be as clean in the two channel case as in the six channel case. Any significant degree of contamination should therefore result in the polarization parameter calculated from the former being systematically lower than the results from the latter, in the ambiguous region. This is not observed. Hence, any such systematic error must be small compared with the statistical errors.

C. Determination of the Scattering Angle

The center-of-mass scattering angle corresponding to each θ_{up} bin is calculated by the program POSTPIAKIN. Input includes

the masses and charges of the initial state particles, the direction of the polarizing magnetic field, and the momentum and incident angle of the pion. The program contains a field map of the polarizing magnet, consisting of field values on 14 annular regions out to a radius of 21.5", where the field is down to 1% of its central value. The geometry of the counter hodoscopes is read in and the positions of each θ bin calculated. Finally, information is furnished on the energy lost by the incident pion in getting into the cryostat, and the energy losses of the final state particles getting out of the cryostat, as a function of laboratory momenta.

The program's output, punched on cards for use by AUGUSTUS and PTHETA, is a tabulation of the center-of-mass scattering angles corresponding to each θ_{up} bin, plus the conjugate θ_{down} bin.

IV. RESULTS

A. Calculation of the Polarization Parameter

A complete derivation of the formula used to compute the polarization parameter may be found elsewhere.⁽⁵⁾ The basic ideas of the least squares fitting procedure follow.

Writing $\vec{P}_T \cdot \hat{n} = T$, Equation 1 may be rewritten for the i^{th} data taking run:

$$I_i(\theta) = I_0(\theta) + [I_0(\theta) \cdot P(\theta)] T_i \quad (\text{Eq. 2})$$

In a plot of $I_i(\theta)$ versus T_i this equation represents a straight line of intercept $I_0(\theta)$ and slope $I_0(\theta) \cdot P(\theta)$. Here, $I_i(\theta)$ is the number of elastic scattering events per unit monitor in the hydrogen peak corresponding to the center-of-mass angle θ . It is obtained from the total number of counts in the hydrogen peak, $N_i(\theta)$, the total number of counts in the equivalent region of the dummy target data, $D(\theta)$, and the number of monitor counts associated with each, M_i and C respectively:

$$I_i(\theta) = [N_i(\theta) - \frac{M_i}{C} D(\theta)] / M_i \quad (\text{Eq. 3})$$

The discrepancy between the measured and fitted counting rates is:

$$d_i(\theta) \equiv I_i(\theta) - [I_0(\theta) + \{I_0(\theta) \cdot P(\theta)\} T_i]$$

Writing $\delta I_i(\theta)$ as the uncertainty in $I_i(\theta)$, the appropriate quantity to be minimized is:

$$J(\theta) \equiv \sum_i \frac{d_i(\theta)^2}{\delta I_i(\theta)^2}$$

We may evaluate $\delta I_i(\theta)$ from Eq. 3. Noting that $M_i \gg N_i$, $C \gg D$, and $C \gg M_i$, we find:

$$[\delta I_i(\theta)]^2 \sim \frac{1}{M_i} \left(\frac{N_i(\theta)}{M_i} \right).$$

From Eqs. 2 and 3, this expression is a function of the polarization parameter $P(\theta)$. We now make the simplifying approximation of "equal weighting". We take

$$\frac{N_i(\theta)}{M_i} \sim \text{const.}$$

for all runs, so that the only dependence of $J(\theta)$ on $P(\theta)$ is in the numerators of the several terms. This gives:

$$J(\theta) = \sum_i M_i d_i(\theta)^2.$$

The conditions for a minimum are:

$$\frac{\partial J(\theta)}{\partial [I_o(\theta)]} = 0, \quad \frac{\partial J(\theta)}{\partial [I_o(\theta) \cdot P(\theta)]} = 0.$$

Straightforward calculation yields the result:

$$P = \frac{\sum_i M_i \cdot S_i \cdot T_i - \langle T \rangle \sum_i M_i S_i}{\langle T^2 \rangle \sum_i M_i S_i - \langle T \rangle \sum_i M_i S_i T_i} \quad (\text{Eq. 4})$$

where:

$$\langle T \rangle \equiv \frac{\sum_i A_i T_i}{\sum_i M_i}, \quad \langle T^2 \rangle \equiv \frac{\sum_i M_i T_i}{\sum_i M_i}.$$

(We have suppressed the c.m. angular dependence for simplicity.)

The formula for the uncertainty in the polarization parameter $P(\theta)$ may be obtained by straightforward partial differentiation of Eq. 4 with respect to all of the independent variables. The result is not quoted.

The results of this experiment are presented in Tables 3 through 10, and in Figs. 13 through 20. Also plotted are measurements of the polarization parameter by other groups, at nearby energies. The errors quoted are statistical only.

B. Errors

1. Statistical Errors

There are two principal contributions to the statistical errors quoted. These are counting statistics and the target polarization measurement.

The number of counts in the hydrogen peaks, the number of counts in the tails of those peaks (which made up the monitor) and the number of counts in the dummy target data used in the background subtraction all added to the statistical error. The former was by far the dominant term in nearly all cases.

Measurement of the target polarization when the polarization was dynamically enhanced to $\sim \pm 50\%$ was fairly precise and introduced negligible statistical error. Calibration measurements of the polarization when the target was at thermal equilibrium (TE) were much less precise, since here the polarization was only $\sim 15\%$ and the signal-to-noise difficulties were appreciable. Typically, each calibration consisted of about six readouts of the TE polarization, with a standard deviation of $\lesssim \pm 1.5\%$ of the average. At times when the readout system was particularly noisy that error went as high as $\pm 2.5\%$. In calculating the value of the polarization from readouts of the enhanced target during data taking, an average of the TE calibration factors measured before and after the data taking period was used. The resultant statistical error was $\leq 1.0\%$ typically, with 1.5% as the worst case. This error was small compared with the counting statistical error.

2. Systematic Errors

There were two principal contributions to the systematic error. These were the errors associated with the target polarization measurement, and the errors in the determination of the center-of-mass scattering angle.

As mentioned above, the calibration factor used to calculate the target polarization was an average of an initial calibration and a final calibration. Often, these two calibrations differed

by an amount greater than the standard deviation of either one of them, indicating a drift in the calibration during data taking. The typical amount of drift was different at different times during the experiment, ranging from a 1% shift to 2.5%. The worst case was 9.5%. Since we average the initial and final calibrations, the systematic errors in the calibration factors, and hence in the target polarizations, will be half of these numbers, or generally under 1.5%, with the worst case being under 5%. The resultant systematic error in the polarization parameter, then, will also be under 1.5% of itself in general.

Additional systematic error contributions to the target polarization are due to approximations made in calculating the polarization from the recorded NMR signal. A major one comes about due to the fact that NMR measures the effects of both the absorptive and dispersive parts of the magnetic susceptibility of the LMN crystals, while the target polarization depends only upon the absorptive part.⁽⁷⁾ A crude estimate showed that this unwanted term led to calculated target polarizations in error by less than 4% of themselves. However, unlike the systematic error due to calibration shifts, which essentially scales the target polarizations, and hence the polarization parameter, up or down, this error has the same sign for both positive and negative target polarizations. That is, the zero of target polarization is shifted. This is equivalent to a change in the

intercept of Eq. 1. Since the polarization parameter is proportional to the slope of Eq. 1, it is unaffected to first order. More careful analysis shows that, if both signs of target polarization are shifted by 4%, the change in the polarization parameter is

$$\frac{\Delta P(\theta)}{P(\theta)} = .04 \cdot P(\theta) \quad .$$

As mentioned previously, during data taking the sign of the target polarization was reversed every 1-1/2 to 2 hours to minimize the errors introduced by slow drifting of experimental conditions. If instead we had first taken all the positively-polarized data (of a given momentum, pion sign, etc.), then taken all the negatively-polarized data, such problems as slow changes in counter efficiencies would have led to false asymmetries. The higher the frequency of target polarization reversals the more such systematic errors due to drift cancel out. (The time required to change the sign of polarization was 15-20 minutes. Since no data could be taken during that interval, reversals could not be made too often without cutting into the data acquisition rate.)

Any appreciable drift in running conditions over a time interval less than two hours, then, could result in false asymmetries. We protected ourselves against such fast drifts and sudden changes in three ways. First, various counting rates and monitors of experimental conditions were recorded on-line by the PDP-5 computer, and the displays and print-outs were

continually examined for anomalies. Second, an off-line study was made of the counting rates per unit monitor as a function of run number for all of the scintillation counters. Finally, an analysis of all the data was performed which quantitatively measured the amount of false asymmetry present.

In order to measure the false asymmetry, it must be separated from the true asymmetry. This may be done as follows. The data taking during the time the target was polarized in one direction usually consisted of four distinct runs, each containing roughly the same number of events. There would be four runs of positively-polarized data, followed by four of negatively-polarized data, then back to four of positively-polarized data, etc. We can abbreviate this sequence as ++++-----+... If we tell the analysis programs that the sequence was ++-----++... and ask what is the polarization parameter, we should get the answer zero, since each batch of "plus" runs actually contains an equal amount of + and - data, and similarly for the "minus" runs. However, this procedure should not affect any false asymmetry present. Our quantitative measure of systematic error, then, is the goodness of fit of the polarization parameter, calculated in this way, to zero.

The data in this experiment were analyzed in 31 separate batches, corresponding to different experimental parameters. Besides the various combinations of different beam momenta,

different pion sign, different sign of the polarizing magnetic field, and whether the pion went up or down, data were sometimes taken at different beam intensities, and some data were taken in more than one block, separated in time. For each of these 31 batches the zero asymmetry test was made and a χ^2 obtained. The number of degrees of freedom equalled the number of center-of-mass angle bins. For the experiment as a whole, the χ^2 was 694 for 748 degrees of freedom, corresponding to a confidence level of 92%. Batch by batch, there were only five cases with confidence levels less than 20%. In four of these, one or two points alone accounted for the bad χ^2 . Since, in a sample of 748 normally distributed quantities, we can expect about 35 fluctuations of two standard deviations or more, these bad points are not cause to conclude that a systematic error was present.

In the fifth case there were two c.m. regions where the false asymmetry was systematically greater than zero. Fortunately, this was one of those cases where data had been taken at two separate times during the experiment. Comparison of the real polarization parameter computed from the two batches of data showed excellent agreement ($\chi^2 = 9.5$ for 19 degrees of freedom). Hence, if there is a systematic error here it is small compared to the statistical errors.

Determination of the c.m. angle associated with each hydrogen peak is done by the program POSTPIAKIN. Systematic errors may arise due to such factors as errors in the measurement of the counter positions, estimation of the incident angle of the pion beam, and calculation of the energy losses of the incident and recoil particles in the target material. The best way to obtain a quantitative estimate of this error is to compare the predicted positions of the hydrogen peaks (see Fig. 10) with the observed positions. The discrepancy is typically zero or one bin, corresponding to $\leq 2^\circ$ - 4° in the center-of-mass. At the ends of the accessible measuring range, however, it gets to be twice that amount.

In order to determine the magnitude of systematic effects due to fluctuations in pion beam intensity, we took data at 417 MeV/c, with positive pions, at two different beam intensities; $6 \times 10^5/\text{sec}$ and $12 \times 10^5/\text{sec}$. (Actual fluctuations during the experiment were never allowed to exceed $\pm 10\%$ or so.) Comparison of the results gave a χ^2 of 25 for 17 degrees of freedom, or an 8% confidence level. This indicates the possible onset of beam-dependence at these beam intensities. As a consequence, the final results quoted do not include the data at the higher beam intensity. Since beam intensities during the rest of the experiment were always less than $6 \times 10^5/\text{sec}$ (see Tables 1 and 2), and since fluctuations in beam intensity never approached a factor of two, we believe that such fluctua-

tions caused negligible systematic error in our final results.

Another possible source of systematic error is what we might call incorrect averaging over the finite volume of the LMN crystals. Referring to Eq. 1, we see that the essence of this experiment is the simultaneous measurement of a scattering intensity and a target polarization. Because of the finite target size, both of these measurements will be volume averages over the target. In the case of the scattering intensity, the volume average will be weighted by the transverse intensity distribution of the incident beam. In the case of the target polarization, the volume average will be weighted by the square of the transverse component of the RF magnetic field, H_T . It is essential that these two weighting factors be as close to equal as possible.

This necessity may be seen by considering an extreme case. Suppose $H_T = H_0$ in the upper half of the target, while it equals $2H_0$ in the lower half. Suppose further that the beam intensity over the upper half is twice the intensity over the lower half. Then, the resultant value for the target polarization will predominantly reflect the polarization of the lower half of the target, while the measured scattering intensity will come predominantly from the upper half of the target. Clearly, if the target polarization is different in the upper and lower halves, we will get the wrong answer for the polarization parameter.

The remedy is two-fold. First, we try to keep the target polarization constant throughout the LMN crystals. Second, we aim to have the distribution of H_T^2 over the target equal to the transverse distribution of beam intensity. In practice, of course, this second condition is most easily met by having both as uniform as possible.

The chief factor which would cause the crystals to be polarized non-uniformly is non-uniformity of crystal temperature. The crystals are heated somewhat by the polarizing microwaves. The temperature of the interior of the crystals may become higher than that of the surface which is in direct contact with the liquid helium refrigerant. This problem arose in previous experiments when the crystals were stacked tightly together. It was avoided in the present experiment by keeping the crystals slightly separated by means of thin Teflon spacers which allowed the liquid helium in. Nevertheless, because of the finite thermal conductivity of LMN, the interiors of the crystals will still be slightly warmer, and hence slightly less polarized, than the surfaces.

The transverse dimension of the crystals, which are non-regular hexagons, is roughly 1-1/4". The full widths at half maximum of the horizontal and vertical beam profiles were typically 1-1/2" and 1-1/4", respectively. Hence, all of the crystals were irradiated reasonably uniformly, without allowing too much of the beam to miss them and produce background counts.

In order to have H_T as uniform as possible over the volume of the crystals, we used the brass septum illustrated in Fig. 5. The presence of this septum constrains the RF magnetic field to be parallel to it at its surface, as do the brass walls of the cylindrical cavity.

Because of the beam size, spacing of the crystals, and the use of a septum, we expect the systematic errors due to incorrect volume averaging to be negligible.

Fig. 13. $P(\theta)$, π^+p Elastic Scattering,
356 MeV/c.

XBL729-4105

XBL 729-4106

Fig. 14. $P(\theta)$, π^+p Elastic Scattering,
41⁻ MeV/c.

XBL 729-4107

Fig. 15. $P(\theta)$, π^+p Elastic Scattering, 468 MeV/c.

XBL 729-4108

Fig. 16. $P(\theta)$, π^+p Elastic Scattering, 519 MeV/c.

XBL 729-4109

Fig. 17. $P(\theta)$, π^-p Elastic Scattering,
356 MeV/c.

Fig. 18. $P(\theta)$, π^-p Elastic Scattering,
417 MeV/c.

XBL729 - 4110

Fig. 19. $P(\theta)$, π^-p Elastic Scattering,
468 MeV/c.

XBL729-4111

Fig. 20. $P(\theta)$, π^-p Elastic Scattering,
519 MeV/c.

XBL729-4112

TABLE 3. $P(\theta)$, π^+p Elastic Scattering
 356 ± 8 MeV/c = 243 ± 8 MeV = 1260 MeV c.m.

$\cos(\theta_{\text{c.m.}})$	$P(\theta)$	$\delta P(\theta)$
.11	.673	.100
.03	.436	.048
-.07	.501	.035
-.16	.383	.028
-.25	.316	.025
-.33	.214	.022
-.40	.141	.022
-.46	.037	.021
-.51	.067	.024
-.58	-.038	.024
-.65	.007	.031
-.71	.044	.060
-.79	.000	.020
-.88	-.002	.013
-.95	.005	.013
-.99	.039	.014
-1.00	.007	.021

Errors quoted are statistical only.

TABLE 4. $P(\theta)$, π^+p Elastic Scattering
 $417 \pm 10 \text{ MeV}/c = 300 \pm 10 \text{ MeV} = 1300 \text{ MeV c.m.}$

$\cos(\theta_{\text{c.m.}})$	$P(\theta)$	$\delta P(\theta)$
.13	.427	.021
.03	.477	.021
-.05	.470	.019
-.16	.421	.025
-.24	.362	.029
-.30	.268	.030
-.36	.180	.030
-.42	.160	.032
-.51	-.005	.032
-.58	-.028	.051
-.64	-.160	.049
-.72	-.160	.020
-.83	-.142	.016
-.92	-.126	.017
-.97	-.070	.018
-1.00	-.092	.021

Errors quoted are statistical only.

TABLE 5. $P(\theta)$, π^+p Elastic Scattering
 $468 \pm 13 \text{ MeV}/c = 349 \pm 13 \text{ MeV} = 1330 \text{ MeV c.m.}$

$\cos(\theta_{\text{c.m.}})$	$P(\theta)$	$\delta P(\theta)$
.55	.138	.302
.46	.138	.094
.40	.141	.098
.25	.261	.051
.18	.317	.028
.09	.327	.027
-.01	.369	.034
-.08	.370	.025
-.18	.310	.050
-.25	.165	.053
-.31	.162	.060
-.38	.026	.060
-.47	-.153	.056
-.57	-.185	.059
-.67	-.170	.035
-.79	-.186	.026
-.89	-.159	.028
-.96	-.027	.029
-.99	-.018	.031

Errors quoted are statistical only.

TABLE 6. $P(\theta)$, π^+_p Elastic Scattering
 519 ± 17 MeV/c = 398 ± 17 MeV = 1370 MeV c.m.

$\cos(\theta_{c.m.})$	$P(\theta)$	$\delta P(\theta)$
.53	.148	.076
.44	.145	.038
.15	.204	.025
.03	.259	.024
-.04	.242	.036
-.13	.208	.026
-.19	.230	.053
-.25	.054	.064
-.32	-.045	.068
-.42	-.229	.077
-.52	-.444	.084
-.64	-.306	.049
-.78	-.316	.039
-.88	-.188	.041
-.95	-.118	.042
-.98	-.116	.041

Errors quoted are statistical only.

TABLE 7. $P(\theta)$, $\pi^- p$ Elastic Scattering
 $356 \pm 8 \text{ MeV}/c = 243 \pm 8 \text{ MeV} = 1265 \text{ MeV c.m.}$

$\cos(\theta_{\text{c.m.}})$	$P(\theta)$	$\delta P(\theta)$
.09	-.167	.101
- .03	-.212	.111
- .10	-.089	.097
- .18	.076	.105
- .25	-.029	.125
- .77	.062	.064
- .84	-.054	.048
- .92	.103	.044
- .97	.027	.044
-1.00	.060	.059

Errors quoted are statistical only.

TABLE 8. $P(\theta)$, π^-p Elastic Scattering
 417 ± 10 MeV/c = 300 ± 10 MeV = 1300 MeV c.m.

$\cos(\theta_{c.m.})$	$P(\theta)$	$\delta P(\theta)$
.37	-.537	.202
.30	-.612	.086
.18	-.493	.069
.06	-.236	.067
-.06	-.140	.078
-.13	.097	.081
-.21	.213	.079
-.29	.253	.079
-.39	.195	.122
-.67	.311	.055
-.76	.221	.037
-.86	.137	.033
-.93	.055	.036
-.98	.099	.039

Errors quoted are statistical only.

TABLE 9. $P(\theta)$, π^-p Elastic Scattering
 468 ± 13 MeV/c = 349 ± 13 MeV = 1330 MeV c.m.

$\cos(\theta_{c.m.})$	$P(\theta)$	$\delta P(\theta)$
.37	-.842	.099
.27	-.779	.064
.15	-.673	.065
.01	-.515	.072
-.09	-.273	.088
-.16	.064	.089
-.25	.107	.087
-.33	.701	.100
-.44	.682	.110
-.61	.843	.083
-.72	.525	.054
-.84	.494	.046
-.92	.331	.053
-.97	.174	.055

Errors quoted are statistical only.

TABLE 10. $P(\theta)$, π^-p Elastic Scattering
 519 ± 17 MeV/c = 398 ± 17 MeV = 1370 MeV c.m.

$\cos(\theta_{\text{c.m.}})$	$P(\theta)$	$\delta P(\theta)$
.40	-1.270	.279
.32	-1.015	.108
.20	-.942	.098
.06	-.556	.098
-.06	-.428	.102
-.14	-.037	.142
-.22	.043	.146
-.31	.931	.202
-.41	1.001	.135
-.52	.986	.195
-.57	1.025	.123
-.68	.881	.076
-.81	.689	.062
-.90	.385	.067
-.96	.300	.068

Errors quoted are statistical only.

V. DISCUSSION OF THE RESULTS

A. Comparison with Other Experiments

The graphs of the experimental results indicate some systematic disagreements between this experiment and other experiments at nearby momenta. Specifically, for π^+p there appears to be a discrepancy with Bareyre et al⁽²⁾ at 532 MeV/c. For π^-p the present experimental results seem to differ from those of Rugge and Vik⁽⁸⁾ at 427 MeV/c, Dickenson et al⁽⁹⁾ at 484 MeV/c, and Bareyre et al⁽¹⁰⁾ at 532 MeV/c. In order to decide whether or not these discrepancies are significant, however, we must take into account the small differences in beam momenta in these various experiments.

In the case of the 519 MeV/c π^+p results the discrepancy is marked for $\cos(\theta_{c.m.}) > -.3$, while agreement is good for $\cos(\theta_{c.m.}) < -.3$. It is difficult to see how the small difference in beam momentum could account for the large separations of the points in the forward direction while not significantly affecting the points in the backwards direction. The two sets of results seem to be fundamentally inconsistent.

In the case of the 417 MeV/c π^-p results, there are two other experiments with which to compare. This experiment agrees very well with Vasilevsky et al⁽¹¹⁾ at 415 MeV/c, while disagreeing strongly with Rugge and Vik at 427 MeV/c. Only very rapid change

of the polarization parameter with beam momentum could reconcile this seeming discrepancy. A glance at the π^-p results from this experiment at all four momenta shows that such rapid variation is not present. This disagreement is real.

In the remaining two cases of π^-p results it does appear that the differences in beam momenta could account for the disagreements, in as much as the pairs of experiments seem to differ by scale factors. In order to test this hypothesis quantitatively we make the simplest assumption about the momentum dependence of the polarization parameter, namely that the dependence is linear. We can get the slope of this dependence with the help of a third set of results at the next highest or lowest momentum.

Actually, since the various sets of experimental points are measured at different center-of-mass angles, we cannot make such a simple comparison. Instead, we fit each experiment with associated Legendre polynomials and require that each Legendre coefficient vary linearly with momentum. The χ^2 of the fit is then a quantitative measure of the disagreement between the experiments, under the hypothesis of linear variation.

To test the discrepancy with Dickenson we use as the third experiment our own results at 417 MeV/c. The confidence level of this fit is less than 1%. So, the difference in momenta between the two experiments doesn't seem to be able to

account for the difference in experimental results.

To test the discrepancy with Bareyre we use as the third experiment his results at 616 MeV/c.⁽¹⁰⁾ The confidence level of this fit is 35%, indicating that the simple hypothesis of linear momentum dependence is sufficient to account for the apparent discrepancy between the two experiments. However, there still remain a few individual points which disagree.

Of course it must be noted that even when it is possible to reconcile two experiments by attributing the differences to a momentum dependence of the polarization parameter, the apparent discrepancy might nevertheless be a real one due to some systematic error.

B. Application to Phase Shift Analysis

1. Effect on Previous Analyses

Having seen that some of our results are not in agreement with the results of other experiments, we would now like to determine the implication of these discrepancies for the existing phase shift solutions for πp scattering. In order to do that, a discussion of some of the details of making a phase shift analysis is necessary.

The aim of any phase shift analysis is the determination of the scattering amplitudes as a function of energy over some

energy interval. In so-called energy-independent phase shift analyses, the ultimate solution is arrived at in two steps, as indicated in the introduction. In the first step, one finds many solutions at each energy, each of which fits the data at that energy. In the second step, one solution from each energy is selected to form a phase shift path. In energy-dependent phase shift analyses, on the other hand, fits are made simultaneously at more than one energy.

The standard procedure by which energy-independent analyses find solutions at each energy is the χ^2 minimization procedure described in the introduction. The standard procedure by which energy-independent analyses select a single solution at each energy to form their phase shift path is the method of shortest path length, which we will now describe. This method, which is the general basis for the specific procedures used in the various extant analyses, depends upon the defining of a distance between two phase shift solutions. This may be done as follows.

For a given solution, each of the complex partial wave amplitudes $T_{l\pm}$ can be thought of as a two-dimensional vector. There are $2(2L + 1)$ such vectors (where L is the cut-off value of the angular momentum in the partial wave expansions of the amplitudes). Each of these vectors may be considered to be the projection of a single vector, in a $4(2L + 1)$ -dimensional Euclidean space, onto one of $2(2L + 1)$ orthogonal two-dimensional

subspaces. This single vector completely characterizes the solution. The Euclidean distance between the vectors representing two different solutions is a measure of the closeness of the solutions:

$$d_{12} \equiv \left\{ \sum_{\substack{\text{all} \\ \text{partial} \\ \text{amplitudes}}} [(\text{Re } T_{\ell\pm}(2) - \text{Re } T_{\ell\pm}(1))^2 + (\text{Im } T_{\ell\pm}(2) - \text{Im } T_{\ell\pm}(1))^2] \right\}^{1/2}.$$

We normalize this metric so that two maximally dissimilar solutions (all η 's equal to 1, all δ 's different by 90°) are separated by 100 distance units. This metric gives us a measure of the distance between two solutions at a given energy, or between two solutions at different energies.

Suppose now that we have found several acceptable phase shift solutions at each of several energies, except at the lowest energy where we know the solutions uniquely. We now arbitrarily choose one of the solutions at each energy. The resultant set of solutions is some arbitrary energy-dependent phase shift path. We define the path length as the sum of the Euclidean distances between each pair of solutions at adjacent energies, starting from the lowest energy unique solution. We may similarly calculate the path lengths for all other possible paths. From general considerations of continuity we expect that the path with the shortest length may be the one with the correct energy dependence.

Some of the more prominent phase shift analyses that have been done are those of Lovelace,⁽¹²⁾ Bareyre,⁽¹³⁾ Davies,⁽¹⁴⁾

and Johnson.⁽⁵⁾ The latter effort, resulting in the Berkeley path, was performed essentially as described above. Bareyre's analysis, which resulted in the Saclay path, is very similar to Johnson's, differing principally in the use of more elaborate metrics than the simple Euclidean one which we have defined. The work of Lovelace, which produced the CERN path, is similar to the other two analyses. It differs chiefly in the use of partial wave dispersion relations to restrict the region at each energy in which the random searches were made, and in the use of techniques which resulted in a smoother energy dependence of the path. Davies' work was an energy-dependent analysis in which the amplitudes were parametrized as resonances plus background rather than expanded in terms of n 's and δ 's. The result was the Glasgow A path. The data from the present experiment, in preliminary form, was used in the work of Johnson and Lovelace.

The gross features of these four paths are in general agreement. In the energy range of the present experiment (1265 MeV to 1370 MeV in the center-of-mass, which is above the $\Delta(1238)$ resonance but below all of the other πN resonances) there are no significant differences. In Table 11 are listed for comparison the phase shift parameters of the solutions of each of these analyses at our highest energy. Table 12 is a matrix showing the Euclidean distances between each pair of these solutions.

TABLE 11. Comparison of Phase Shift Solutions at 532 MeV/c.

		CERN	SACLAY	GLAS- GOW A	BERK- ELEY	AB	AG	BERK- ELEY 490 MeV/c			CERN	SACLAY	GLAS- GOW A	BERK- ELEY	AB	AG	BERK- ELEY 490 MeV/c
S ₃₁	n	1.00	.96	.93	1.00	.96	.92	1.00	S ₁₁	n	.97	1.00	.88	1.00	1.00	1.00	1.00
	δ	-23.8°	-23.5°	-26.2°	-25.5°	-25.5°	-25.7°	-23.8°		δ	16.1°	16.3°	17.6°	11.7°	22.1°	16.8°	13.0°
	n	1.00	.98	.99	1.00	.90	1.00	1.00		n	.64	.74	.61	.78	.66	.71	.86
F ₃₁	δ	-13.1°	-9.2°	-12.0°	-10.6°	-9.7°	-9.8°	-8.9°	F ₁₁	δ	43.2°	36.6°	37.7°	38.2°	39.1°	43.1°	29.2°
	n	1.00	1.00	.99	1.00	1.00	1.00	1.00		n	1.00	1.00	1.02	.92	.99	.90	.93
F ₃₃	δ	-30.7°	-32.0°	-30.2°	-31.7°	-32.0°	-32.3°	-35.0°	F ₁₃	δ	-5.2°	-7.3°	-6.2°	-7.4°	-5.1°	-4.4°	-6.9°
	n	.99	1.00	.98	1.00	.98	1.00	1.00		n	.96	.96	.88	.96	.99	1.00	.99
D ₃₃	δ	.2°	1.2°	.1°	.8°	1.7°	1.5°	.8°	D ₁₃	δ	9.6°	10.4°	8.0°	9.9°	8.9°	8.7°	7.9°
	n	1.00	1.00	.99	.98	.99	.93	1.00		n	1.00	.98	.99	.99	.96	1.00	1.00
D ₃₅	δ	-1.4°	-2.8°	-1.9°	-2.4°	-3.2°	-2.5°	-1.4°	D ₁₅	δ	2.3°	1.8°	2.9°	3.0°	2.3°	2.3°	1.5°
	n	1.00	1.00	1.00	1.00	.98	1.00	1.00		n	1.00	1.00	.97	1.00	1.00	1.00	1.00
F ₃₅	δ	-.1°	-.7°	.3°	.0°	.0°	.0°	.1°	F ₁₅	δ	2.3°	1.8°	1.9°	2.4°	1.3°	1.3°	1.8°
	n	1.00	1.00	1.00	1.00	1.00	1.00	1.00		n	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F ₃₇	δ	1.0°	1.0°	.7°	.6°	.9°	.5°	.6°	F ₁₇	δ	-.2°	-1.6°	.3°	-1.3°	-.9°	-.5°	-.7°
	n	1.11	1.00	1.00				1.00		n	1.00	.98	1.00				1.00
C ₃₇	δ	.1°	.3°	.0°				.0°	C ₁₇	δ	.2°	.1°	.4°				.3°
	n	1.00	1.00	1.00				1.00		n	1.00	1.00	1.00				1.00
C ₃₉	δ	.0°	.3°	.3°				.3°	C ₁₉	δ	.2°	-.9°	.2°				-.5°
	n		1.00	1.00						n		1.00	1.00				
E ₃₉	δ		.0°	.0°					E ₁₉	δ		.0°	.0°				
	n		1.00	1.00						n		1.00	1.00				
E _{3,11}	δ		.0°	.0°					E _{1,11}	δ		.0°	.2°				
	n									n							

TABLE 12. Euclidean Distances Between Phase Shift Solutions at 532 MeV/c.

CERN	0						
SACLAY	3	0					
GLAS- GOW A	3	3	0				
BERK- ELEY	3	3	4	0			
AB	4	3	3	5	0		
AG	3	3	4	3	3	0	
BERK- ELEY 490 MeV/c.	6	4	4	4	6	5	0
	CERN	SACLAY	GLAS- GOW A	BERK- ELEY	AB	AG	BERK- ELEY 490 MeV/c.

Previous experience has shown that two solutions should not be considered distinct if they are closer together than five distance units.

It is clear from these two tables that the results of these four phase shift analyses are in agreement at 532 MeV/c. Some of these analyses used data from the present experiment, while others used the data of Bareyre, so preliminary indications are that discrepancies on the order of those between these two experiments have no serious consequences for phase shift analyses. We can test this conclusion more carefully. First, we will see what the effect of the discrepancies is on the known solution at 532 MeV/c. Then, we will see whether there is any effect on the results of the shortest path length procedure.

Since, as we have seen, the solution at 532 MeV/c is well known, we expect that any of the solutions in Table 11 will be a fairly good fit to either the data of this experiment or that of Bareyre. Some small adjustments of the phase shift parameters will be necessary. To determine the sizes of these adjustments, we reminimize one of these solutions, using as the polarization data first the results of Bareyre and then our own results. We choose as our initial solution the Berkeley solution at this energy. The differential cross-section data used in the fits is: $\pi^{\pm}p$ differential cross-sections from Ogden et al;⁽¹⁵⁾ $\pi^{-}p$ charge exchange differential cross-section of Hauser et al.⁽¹⁶⁾ Various total cross-section measurements made before 1967 were

also included as four more data points to be fit, in the form of the real and imaginary parts of the forward scattering amplitudes.⁽¹⁷⁾ The real parts were calculated by means of dispersion relations, while the imaginary parts were obtained from the optical theorem. The energy difference between the two polarization measurements is ignored.

The resulting two solutions are called AB and AG, with the former being generated by fitting the Bareyre results and the latter fitting this experiment. The η 's and δ 's are listed in Table 11, with the distances to other solutions given in Table 12. Also listed is the Berkeley solution at 490 MeV/c, the next lowest energy, for comparison. We see that the parameters for all of the solutions at 532 MeV/c are generally closer to each other than to the parameters for the solution at 490 MeV/c, indicating no serious disagreements.

We now look at the effect of these discrepancies on the energy continuation procedure. We perform three shortest path continuations as follows. In the first continuation, the set of solutions from which the path is selected is the set which gave the Berkeley path. In the second, the set is the same at all energies except 532 MeV/c. There, we include only solution AB, thus constraining the resulting path to go through this solution. In the third, we similarly constrain the path to go through solution AG. The differences between the Berkeley solution and solutions AB and AG would be significant if they

resulted in different paths. This, however, is not the case. The shortest paths are identical in all three cases (except, of course, at 532 MeV/c).

In conclusion, then, we have found that, in the energy range of this experiment, discrepancies in polarization parameter results of the magnitude of those between the present experiment and the experiments of Bareyre et al. have little effect on a single-energy phase shift analysis, and no effect on determining the energy dependence of the solutions. The major effect, as we shall see below, is on the number of distinct solutions found at a single energy.

2. Ambiguities in Single-Energy Phase Shift Analysis

One of the reasons for performing the present experiment was a hope that more definitive data in this energy region would reduce the number of phase shift solutions found at each energy. First is the quality of the data. If the angular regions examined are incomplete, or the errors on the measured points large, it may be possible for several solutions to give comparably good fits to the data. Second, there are intrinsic ambiguities in going from the observables, which are bilinear in the scattering amplitudes, to the amplitudes themselves. As mentioned in the introduction, at least eight different experiments must be performed to unambiguously determine the πp scattering amplitudes at a single energy. In the energy range of this experiment only

five different observables have been measured. These are the differential cross-sections for π^+p and π^-p elastic and π^-p charge exchange scattering and the polarization in π^+p and π^-p elastic scattering. Hence, multiple solutions are possible no matter how good the data are.

To get some idea of how the multiplicity of solutions depends upon the quality of the data, we made an intensive single-energy search for solutions at 532 MeV/c. The method used was the straightforward χ^2 minimization outlined in the introduction. The differential cross-sections which were fit were the π^+p elastic data of Ogden et al. and the π^-p charge exchange cross-section data of Kurz and Lind.⁽¹⁸⁾ The real and imaginary parts of the forward amplitudes were also fit. For polarization data the results of this experiment were used. In addition, an equal amount of searching was done using the polarization data of Bareyre et al.

Minimization of χ^2 proceeded from a trial set of phase shift parameters. The only constraints on these initial sets of parameters were that no initial partial wave amplitude alone was allowed to contribute more to the total cross-section, total elastic cross-section, or total inelastic cross-section than the known values of those cross-sections. Within these bounds, the initial sets of parameters were chosen randomly. Five hundred such random starts were made using the polarization data from this experiment, and another 500 using the data of Bareyre. The energy difference between the two experiments was ignored.

The χ^2 's of the resultant minimized solutions ranged from roughly equal to the number of degrees of freedom up to several times that number. We retained for further consideration only those solutions whose confidence level was greater than about 10%-15%. Using the Euclidean measure of distance described above we found that there were 10 distinct solutions generated from the data of the present experiment and 13 distinct solutions generated from Bareyre's data. A few solutions showed up only once in the 500 searches while the others were found several times. The most ubiquitous solution was found 12 times.

Surprisingly enough, the canonical solutions AB and AG described in the preceding section were not among the solutions found. This fact, plus the fact that some of the 23 solutions were found only once, indicates that there are probably other solutions which were never found. Nevertheless, the solution set obtained is sufficient to determine how much of this redundancy is due to inadequacies in the data, and how much is due to inherent ambiguities.

One way of answering this question is to follow the analysis of Gersten,⁽¹⁹⁾ the method of Barrelet zeros. The analysis, in brief, goes as follows. As shown in the introduction, the differential cross-section and polarization parameter for either π^+p or π^-p elastic scattering may be written in terms of two amplitudes:

$$I_0^\pm(\theta) = |f^\pm(\theta)|^2 + |g^\pm(\theta)|^2$$

$$I_0^\pm(\theta)P^\pm(\theta) = 2 \operatorname{Im} f^\pm(\theta)^* g^\pm(\theta) \quad .$$

If we now define:

$$h^\pm(\theta) \equiv f^\pm(\theta) + ig^\pm(\theta) \quad ,$$

we may rewrite the observables as:

$$I_0^\pm(\theta) = \frac{1}{2} \left[|h^\pm(\theta)|^2 + |h^\pm(-\theta)|^2 \right] \quad \text{Eq. 5}$$

$$I_0^\pm(\theta)P^\pm(\theta) = \frac{1}{2} \left[|h^\pm(\theta)|^2 - |h^\pm(-\theta)|^2 \right] \quad \text{Eq. 6}$$

If the f and g amplitudes are expanded in Legendre polynomials to order L , the h amplitudes may be similarly expanded. By straightforward trigonometry we may rewrite these expansions in terms of the variable $z = \tan(\frac{1}{2}\theta)$, in which variable they are polynomials of order $2L$. If we can find the complex roots $\{z_i\}$ of these polynomials, we may write them as products:

$$h^\pm = A^\pm \prod_{i=1}^{2L} (z - z_i^\pm) \quad . \quad \text{Eq. 7}$$

And now we see from Eqs. 5 and 6 that by replacing any combination of the z_i in Eq. 7 by their complex conjugates we get back the same observables. Since there are 2^{2L} ways of choosing subsets of all the Barrelet zeros, independently for h^+ and h^- , we have found 2^{4L} different pairs of amplitudes, all of which predict exactly the same observables. In the searches

we performed, $L = 3$, so the number of Barrelet equivalent amplitudes is 4096. However, the amplitudes obtained by making such Barrelet transformations will often exceed the unitarity limit, so the actual number of acceptable ambiguous amplitudes thus generated may be far less.

The above result depends upon the fact that the observables we were considering in Eqs. 5 and 6 each depended upon either h^+ or h^- . If in addition we consider charge exchange scattering, all of this ambiguity should disappear. This is because the charge exchange amplitude h^0 is a linear combination of h^+ and h^- . This constitutes a constraint: we cannot make a Barrelet transformation on either of the latter two without changing the former in some arbitrary way which is certain to change the predictions for the charge exchange observables. So, multiple solutions at a single energy, all of which fit charge exchange data, should not be equivalent under Barrelet transformations.

The data set used to produce the 23 solutions included the charge exchange differential cross-section measurements of Kurz and Lind. This data, however, is limited both in precision and in angular region covered. Hence it should not constitute much of a constraint on the fitting and we might expect Barrelet ambiguities to be present. We checked this as follows. We found the set of zeros for each of the 23 solutions. For each set we then generated a new solution by making the Barrelet

transformation which resulted in a set of zeros all of whose imaginary parts were positive. Hence, if two of the initial solutions had been Barrelet equivalents, the two resultant solutions would be identical. Because of the charge exchange data these new solutions were no longer minima in χ^2 , so we reminimized them. However, this resulted in only slight changes in the phase shift parameters. We then calculated the Euclidean distances between all pairs of solutions.

The results, for the solutions originally generated using the data from this experiment, were that all 10 solutions were Barrelet equivalent to each other and to solution AG, the canonical solution. The only ambiguity in these solutions, then, is the Barrelet ambiguity. The 13 Bareyre solutions reduced to four inequivalent subsets, one of which was equivalent to canonical solution AB. Hence, these solutions exhibit some further ambiguity, possibly one related to the quality of the data.

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This is a time of hundred-member collaborations, hundred billion electron volt accelerators, and million dollar experiments. And yet, in spite of the impersonality of big physics, the metamorphosis from green graduate student to certified physicist remains an intensely personal thing. In fact, while to the general reader this manuscript must seem a tedious stringing out of picayune details leading up to a resoundingly anticlimactic heap of numbers, to me the story of each scintillation counter, beam magnet, or computer program is a nostalgic anecdote about the associates and friends who for these past few years have been my life. And so, while these people are thoroughly deserving of scientific acknowledgment for their help in my work, I prefer to offer them my heartfelt thanks.

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