A THEORETICAL TREATMENT OF THE SCHWARZ-HORA
EFFECT

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A theoretical treatment of
the Schwarz-Hora effect

by

George Ronald Hadley

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TABLE OF CONTENTS

	Page
FOREWORD	v
PARTIAL LIST OF SYMBOLS USED	vi
ABSTRACT	vii
I. INTRODUCTION	1
A. Schwarz's Original Experiment	1
B. Experimental Attempts by Others	3
C. Theoretical Contributions	6
D. Present Work	8
II. THEORY OF THE MODULATION	10
A. General Discussion	10
B. Interaction Potential	11
C. Modulation of an Incoming Plane Wave	15
D. Wave Packet Treatment	26
III. PRODUCTION OF LIGHT AT THE TARGET	35
A. Introductory Remarks	35
B. Quantum-Mechanical Viewpoint	35
C. Classical Viewpoint	38
IV. EVALUATION AND DISCUSSION	44
V. REFERENCES CITED	47
VI. ACKNOWLEDGMENT	49
VII. APPENDIX	50

FOREWORD

This thesis represents a part of the culmination of two years research by the author in collaboration with Mr. E. J. Stanek, Professor R. H. Good, Jr., and Professor D. W. Lynch. Professor Good provided the initial enthusiasm for work on this problem. Later, when necessary data was found lacking, the experiment itself was performed (1) by the group with the helpful assistance of Dr. Elmer Rosauer, Director of the Electron Microscope Laboratory of the Ceramic Engineering Department at Iowa State University. Parts of the material contained herein were published jointly by the author, Dr. Good, and Mr. Stanek (2).

PARTIAL LIST OF SYMBOLS USED

d	crystal (film) thickness
$\bar{A}(\bar{x}, t)$	vector potential
$\varphi(\bar{x}, t)$	incident electron wave function
$\Psi_{SC}(\bar{x}, t)$	scattered wave function
$U(\bar{x})$	crystal potential
$U(\bar{p})$	Fourier transform of $U(\bar{x})$
$u(x)$	step function (defined in text)
m	electron mass
e	electron charge
$\bar{E}(\bar{x}, t)$	electric field amplitude
$G_0(\bar{x}, t; \bar{x}', t')$	free particle electron propagator
c	speed of light
ω	laser angular frequency
ϵ	dielectric constant of crystal
$V(\bar{x}, t)$	interaction potential
p	electron momentum
E	electron energy
$\hbar$	$(\frac{1}{2\pi}) \times$ Planck's constant
∇	gradient operator
λ	electron wavelength $= \frac{2\pi\hbar}{p}$

A theoretical treatment of
the Schwarz-Hora effect

George Ronald Hadley

Under the supervision of Roland H. Good, Jr.
From the Department of Physics
Iowa State University

In the Schwarz-Hora effect, an electron beam passing through a crystal is modulated by laser light and then reproduces the light when it strikes a nonluminescent target. In this paper the theory for the modulation process is developed for incident plane wave electrons using the Born approximation to second order. A wave packet treatment of the incident electrons is also included to study interparticle phasing. Finally, light production at a non-resonant target is discussed classically and quantum mechanically, and an upper limit for the amount of light emitted at the laser frequency is calculated using a semi-classical model.

I. INTRODUCTION

A. Schwarz's Original Experiment

In December of 1969 a paper (3) was published in Applied Physics Letters describing a very remarkable experiment in which a beam of electrons was modulated at an optical frequency. The authors of the paper were Drs. Helmut Schwarz and Heinrich Hora, both of the Rensselaer Polytechnic Institute. (Since then Dr. Hora has returned to the Institut Fur Plasmaphysik, Max Planck Gesellschaft, Garching, Germany). This experiment has since captured the interest of both theoreticians and experimentalists and has sparked several attempts to reproduce it.

According to Reference 3, the experiment was performed in the following manner:

- 1) Ordinary electron diffraction patterns of thin monocrystalline films were produced and made visible on a luminescent target such as a zinc sulfide screen. Films used were SiO_2 and Al_2O_3 with thicknesses varying from 600 to 2000 Å. The electron beam was operated at an energy of 50 keV and a current of approximately one microampere.

- 2) An argon-ion laser supplying 10 watts of power at 4880 Å wavelength was then focused edgewise on the film as shown in Figure 1. Supporting grid wires were moved out of the way so that the laser beam could pass through the film unobstructed. The polarization vector of the laser was aligned parallel to the electron beam as shown.

- 3) The fluorescent screen normally used to view the diffraction pattern was replaced by an alumina sheet (polycrystalline Al_2O_3). With

Figure 1. Basic arrangement for the Schwarz-Hora experiment

the laser off, this new target gave no pattern whatsoever. When the laser was turned on, however, the original diffraction pattern reappeared in its previous position. Now the pattern appeared the color of the laser rather than the yellow-green characteristic of zinc sulfide. It was also somewhat broken - see the photograph, page 349 of Reference 3.

In order to insure that the blue-green diffraction spots were due to the presence of modulated electrons rather than scattered laser light, a hand magnet was brought near the beam, and was shown capable of displacing the spots or making them disappear altogether. Another important experimental feature reported was that the pattern would disappear when the laser polarization was rotated by 90° . Although the spots appeared to be identical in color to the laser light, the emitted light's frequency was not measured.

B. Experimental Attempts by Others

It is known that at least three groups have seriously attempted to reproduce the Schwarz-Hora Effect: one under the supervision of Dr. Donald Scarl at the Brooklyn Polytechnic Institute; one at Bell Laboratories, Murray Hill, New Jersey; and one at Iowa State University. Only the experiment done at Iowa State has been published (1), but all three have failed to observe the effect. The effort at Bell Laboratories was extremely well financed and of almost two years duration. The author knows from private communication with the experimenters that this experiment was carefully performed and that many different setups were tried. Not much is known about the effort at the Brooklyn Polytechnic Institute

except that a heavy emphasis was being placed on electronic detection of emitted light so that intensities far below visible levels could be detected. In the Iowa State experiment, an electron microscope was used as the electron beam source, and a laser similar to the one used by Schwarz was rented¹. Several different films and target materials were tried, but no laser-produced diffraction pattern was ever observed, even with a dark-adapted eye. In a private communication with the author, however, Dr. Schwarz stated that the pattern was not on the threshold of visibility, but rather was easily visible from across the room.

As in the case with all new experiments, the results obtained by Schwarz are doubted by many, particularly in view of the difficulty experienced by those trying to reproduce the effect. Other factors have led to even stronger doubts, however. First of all, no outside groups have been allowed to witness the effect even though many (including the one from Iowa State) have asked to see it. In fact even Dr. Hora himself had no part in performing the experiment and has not seen it performed². Secondly, ambiguities and confusion exist as to exactly what the original experimental setup was like. As an example,

¹The statement made by the representative of Carson Labs Inc. to the experimenters was that the laser used at Iowa State was identical to the one Schwarz used although it carried only a 1 watt rating instead of 10 as claimed in Reference 3.

²This information was obtained directly from Dr. Hora by private communication. He claimed only to have seen the apparatus set up and never actually observed the modulation itself.

the description in Reference 3 of the electron beam apparatus reads

" ... usual electron optics were applied to obtain an electron beam of 50 keV energy and a beam current of the order of a microampere in a spot of a few microns diameter." This implies a simple setup which would yield a beam with an energy uncertainty of roughly 1 eV. In a later publication (4), however, Schwarz claims to have a very elaborate electron beam apparatus consisting of bending magnets and a specially designed monochromator system which supposedly cuts the beam uncertainty to approximately 0.01 eV. A more detailed description of this system given recently (April 1972) reads

The apparent incompatibility of two requirements, namely, extremely narrow energy spread of the 50 keV beams of less than 10 meV and relatively high electron currents of $0.5 \mu\text{a}$, is overcome by shaping the electron beams into a "filamentary" structure while passing through a magnetic monochromator. This is achieved with diaphragms having many equally spaced holes of diameters as small as 5μ . Proper adjustments of the spacings and of the characteristics of the monochromator provide for zero phase difference between the single electron beam filaments (5).

These ambiguities have proven frustrating to experimentalists who, after designing and building an apparatus to duplicate Schwarz's experiment, find out they haven't duplicated his electron optics. Further doubts have arisen as a result of calculations done by several authors (6,2) who predict much lower output intensities than Schwarz claims to have produced. These comments all typify the air of mystery which has surrounded the Schwarz-Hora Effect from the time the first paper was published right up until the present (April 1972).

C. Theoretical Contributions

The first attempts at a theoretical explanation of the Schwarz-Hora Effect were classical in nature and modelled after the classical theory of the klystron, a "tube" of sorts used as an oscillator and power amplifier at wavelengths of the order of centimeters. The analogy is close, as can be seen in Figure 2. In the klystron the accelerated electrons enter the bunching region or "buncher" and are either accelerated or decelerated by an electric field at the operating frequency. During their flight down the "drift region" the electrons bunch together so that the "catcher" will see a modulated beam. The accelerating anode voltage and drift region length are adjusted for maximum bunching. The bunched beam then reradiates when stopped in the catcher at the same frequency at which it was bunched. This radiation is fed back through wave guides to the buncher, producing an oscillation. In the Schwarz-Hora Effect the crystal or film is the buncher, the space between crystal and target the drift region, and the target the catcher. In this case, of course, there is no feedback or resulting oscillation.

Classical explanations modelled after the klystron were published early in 1970 by Rubin (7) and by Oliver and Cutler (8). Several authors were not satisfied by classical calculations so quantum-mechanical theories soon appeared. Some authors directed their efforts toward an understanding of the modulation process (Hutson (9), Van Zandt and Meyer (10), and Salat (11)) while others took a modulated wave function as given and aimed at treating the emitted radiation at the target (Varshalovich and D'yakonov (12) and Van Zandt (13)).

Figure 2. Schematic drawing of a klystron

During the next year (1971) the classical approach was abandoned in favor of quantum mechanics. Some authors took new directions entirely. A group of theorists at Wayne State University espousing a many-particle approach to a demodulation mechanism published several papers (14-18). In this same direction Lipkin and Peshkin (19), Marcuse (20, 21), Kondo (22), and Becchi and Morpurgo (6) showed conclusively that a many-electron theory was necessary. Other authors concentrated on extending previous quantum-mechanical calculations to include diffracted spots or to make quantitative assessments of certain models. Included in this group are Becchi and Morpurgo (6) and Hadley, Stanek, and Good (2,23).

Recent (early 1972) publications have been largely concerned with more innovative suggestions for producing and detecting the modulation. Schmieder (24) has proposed using electron interferometry techniques and two separate modulating crystals in order to detect the beat frequency between two closely-tuned lasers instead of a single optical frequency. Bergmann (25) has even suggested a new class of experiments which utilize two electron beams and operate without laser or modulating crystal. These publications are evidence that a new idea has taken root and will eventually become a new area of technology.

D. Present Work

This thesis will cover the non-relativistic theory of the modulation process, including a wave packet treatment of the incoming electron, and a brief semi-classical treatment of the radiation from a non-resonant

target. Part of this material has been published by Hadley, Stanek and Good in the Journal of Applied Physics (2).

II. THEORY OF THE MODULATION

A. General Discussion

The theory of this process may be divided into three general areas:

1) The resultant potential a single electron sees as it approaches the interaction region where laser and crystal intersect. This involves first a treatment of the resultant electric and magnetic fields due to the presence of the crystal, and then the incorporation of the crystal scattering centers into one overall scattering potential.

2) The electron wave function, including any effects due to the presence of neighboring electrons, after leaving the crystal. This wave function should show some modulation properties at the laser frequency which disappear when the laser is turned off.

3) The production of light at the target showing a visible diffraction pattern the same color as the laser light. This is the most challenging part theoretically since most estimates, including the one to be presented here, give a total light output much too small to be seen with the naked eye.

The theoretical treatment which follows will be divided according to this format. Electrons will be treated non-relativistically in this paper, although a relativistic generalization is straightforward and follows this treatment very closely (23). For ease in following the notation, a list of the major symbols used is given on page iv. Other symbols will be defined in the text.

B. Interaction Potential

The interaction region is that region in space where laser and crystal intersect. The crystal scatters electrons even without the laser into a diffraction pattern and so one term in $V(\vec{x},t)$ will be $U(\vec{x})$, the crystal potential. This term carries information about the crystal structure, and, if put into the expression for the scattered wave function by itself, will yield the appropriate diffraction pattern only. This pattern would of course be visible only on a target which luminesces when struck by electrons, and not on the alumina sheet.

The effect of radiation fields in non-relativistic quantum theory is treated by inserting $\vec{p} - \frac{e}{c} \vec{A}$ in place of $\vec{p}$ in the Schrödinger equation, and this results in the term

$$-\frac{e}{2mc} [\vec{A}(\vec{x},t) \cdot \vec{p} + \vec{p} \cdot \vec{A}(\vec{x},t)]$$

appearing as a potential term along side other such terms not due to the radiation field. Thus

$$V(\vec{x},t) = U(\vec{x}) - \frac{e}{2mc} [\vec{A}(\vec{x},t) \cdot \vec{p} + \vec{p} \cdot \vec{A}(\vec{x},t)]. \quad (1)$$

This expression assumes Gaussian units, as will the entire paper.

The next step is then to determine $\vec{A}(\vec{x},t)$. Looking at the crystal on edge as in Figure 3, one can visualize the focused laser beam as being a sum of plane waves coming in at a continuum of angles, entering at the edge. This is a very complicated situation, and determining the fields exactly would be an arduous task indeed. One is saved from

Figure 3. Close-up of the crystal, showing the component waves of the focused laser

doing this because, as will be shown later, only spatial variations of $\bar{A}(\bar{x}, t)$ with wavelengths less than half the light wavelength contribute to the modulation process. Such wavelengths enter only because of the crystal since both $\bar{E}$ and $\bar{A}$ have discontinuities at its surfaces. The resultant notch in $\bar{A}$, which is a fraction of a wavelength in width, will then possess the correct Fourier components when the optimum combination of beam energy and crystal thickness is used. Therefore the most exacting calculations of the fields around and through the crystal are ineffectual because the addition of plane wave terms with wavelengths greater than or equal to the light wavelength can never result in a spatial variation any sharper than this. Guided modes also fall into this category.

This being the case, only the values of the fields at the crystal boundaries enter into the problem and one might as well use spatially constant values elsewhere. This is an important simplification and will be carefully justified in the next section.

Choose the gauge in which the scalar potential is zero. Then $\bar{E}(\bar{x}, t)$ is related to $\bar{A}(\bar{x}, t)$ by

$$\bar{E}(\bar{x}, t) = - \frac{1}{c} \frac{\partial \bar{A}(\bar{x}, t)}{\partial t}.$$

Since time dependence is uniformly of frequency ω , $\bar{E}(\bar{x})$ may be related to $\bar{A}(\bar{x})$ by

$$\bar{A}(\bar{x}) = \frac{c}{\omega} \bar{E}(\bar{x}),$$

where $\bar{A}(\bar{x}, t) = \bar{A}(\bar{x}) \cos \omega t$ and $\bar{E}(\bar{x}, t) = \bar{E}(\bar{x}) \sin \omega t$. Then

$$\bar{A}(\bar{x}, t) = \frac{c}{\omega} \bar{E}(\bar{x}) \cos \omega t \quad (2)$$

Referring again to Figure 3, it is convenient to think of $\bar{E}(\bar{x})$ as a sum of two polarizations, one along the z-axis (referred to as the normal component) and another perpendicular to the z-axis (referred to as the longitudinal component). Requiring continuity of E_{long} and D_{normal} allows one to write $\bar{E}(\bar{x})$, and thus $\bar{A}(\bar{x})$, in the form

$$\bar{A}(\bar{x}) = \bar{A}_\ell(\bar{x}) + \bar{A}_n(\bar{x}),$$

where

$$\bar{A}_\ell(\bar{x}) = \frac{c\bar{E}_0}{\omega} \cos \omega t, \quad (3)$$

and $\bar{A}_n$ is in the z-direction with

$$\bar{A}_{nz}(\bar{x}) = \frac{c}{\omega} E_{0z} \frac{1-\epsilon}{\epsilon} u(z) \cos \omega t. \quad (4)$$

Here ϵ is the dielectric constant of the material, $\bar{E}_0 \sin \omega t$ is the field produced by the laser at the crystal, and the function $u(z)$ is unity for $-\frac{d}{2} \leq z \leq \frac{d}{2}$ and zero otherwise. This breaks the total potential $\bar{A}$ into a part $\bar{A}_\ell$ that gives the contribution $\bar{E}_0 \sin \omega t$ to the field as if the crystal were not present and into a part $\bar{A}_n$ that makes a notch in the z-component. The part $\bar{A}_n$ affects only E_z inside the crystal, making it be $\frac{E_{0z}}{\epsilon} \sin \omega t$.

In summary, the interaction potential is composed of a term which gives a diffraction pattern plus laser field terms which are considerably simplified and reflect only the magnitude and frequency of the

field plus two discontinuities at the crystal boundaries.

C. Modulation of an Incoming Plane Wave

Let the incident electron wave function propagate in the z direction and be of the form

$$\varphi(\vec{x}, t) = e^{i/\hbar (pz - Et)}, \quad (5)$$

where E is $p^2/2m$. The scattered wave function $\Psi(\vec{x}, t)$ must satisfy the Schrödinger equation

$$(i\hbar \frac{\partial}{\partial t} - \frac{p^2}{2m}) \Psi(\vec{x}, t) = V(\vec{x}, t) \Psi(\vec{x}, t), \quad (6)$$

where $V(\vec{x}, t)$ is the interaction potential discussed in the previous section. Since the modulation under consideration is small, perturbation theory applies, and the final wave function may be developed as a series. The techniques for doing this are well known, and the expressions in the notation of Bjorken and Drell (26) are, through second order,

$$\Psi(\vec{x}, t) = \varphi(\vec{x}, t) + \Psi_{SC}^{(1)}(\vec{x}, t) + \Psi_{SC}^{(2)}(\vec{x}, t)$$

where

$$\Psi_{SC}^{(1)}(\vec{x}, t) = \int d^3\vec{x}' dt' G_0(\vec{x}, t; \vec{x}', t') V(\vec{x}', t') \varphi(\vec{x}', t') \quad (7)$$

$$\Psi_{SC}^{(2)}(\vec{x}, t) = \int d^3\vec{x}' dt' G_0(\vec{x}, t; \vec{x}', t') V(\vec{x}', t')$$

$$\times \int d^3\vec{x}'' dt'' G_0(\vec{x}', t'; \vec{x}'', t'') V(\vec{x}'', t'') \varphi(\vec{x}'', t''), \quad (8)$$

and where the free particle propagator is

$$\begin{aligned}
 G_0(\bar{x}, t; \bar{x}', t') &= \frac{1}{(2\pi\hbar)^4} \int d^3 p' e^{\frac{i}{\hbar} p' \cdot (\bar{x} - \bar{x}')} \\
 &\times \int d\omega \frac{e^{-\frac{i}{\hbar} \omega(t-t')}}{\omega - \left(\frac{p'^2}{2m}\right) + i\epsilon} \\
 &= -\frac{i}{(2\pi)^3} \frac{\theta(t-t')}{\hbar^4} \int d^3 p' e^{\frac{i}{\hbar} (p' \cdot (\bar{x} - \bar{x}') - (E' - i\epsilon)(t-t'))} \quad (9)
 \end{aligned}$$

Here E' is $\frac{p'^2}{2m}$; $\theta(t-t')$ is the step function, unity for $t > t'$ and zero otherwise; and ϵ is to be set equal to zero later.

We shall examine $\Psi_{SC}^{(1)}(\bar{x}, t)$ first. Since $V(\bar{x}, t)$ is the sum of a laser term and a crystal term, and since $\Psi_{SC}^{(1)}$ is linear in V , two terms are produced. One is laser independent and gives Bragg scattering only. This term is unmodulated and will become the "background" for the modulated $\Psi_{SC}^{(2)}$ wave. For the time being we shall ignore it. The laser term is modulated but undeflected and corresponds to the central spot of a diffraction pattern. Putting together Equations (1), (2), (5), (7), and (9) and doing the integration on t' gives

$$\begin{aligned}
 \Psi_{SC}^{(1)}(x, t) &= \frac{iep}{2(2\pi\hbar)^3 m\omega} \int d^3 p' [\delta(E - E' + \hbar\omega) \\
 &+ \delta(E - E' - \hbar\omega)] \int d^3 x' e^{\frac{i}{\hbar} (\bar{p} - \bar{p}') \cdot x'} E_z(\bar{x}') e^{\frac{i}{\hbar} (\bar{p}' \cdot x - E' t)} \quad (10)
 \end{aligned}$$

where $\bar{p} = p\hat{z}$. Here the $\nabla \cdot \bar{A}$ term in Equation (1) has been omitted because it gives a contribution smaller than the $\bar{A} \cdot \bar{p}$ term kept by a factor of $\frac{\hbar\omega}{E} \simeq 5 \times 10^{-5}$.

Equation (10) displays $\psi_{SC}^{(1)}(\bar{x}, t)$ as a sum of plane waves which represent either emission or absorption of a photon at the laser frequency. For even further clarity one may write this as

$$\int d^3 p' a(\bar{p}') e^{\frac{i}{\hbar}(\bar{p}' \cdot \bar{x} - E' t)}.$$

The amplitude $a(\bar{p}')$ is essentially the Fourier transform of $E_z(\bar{x}')$ evaluated at the momentum transfer $\bar{p} - \bar{p}'$. In this case E_z depends hardly at all on x' or y' , giving the delta functions $\delta(p_x - p'_x)$ $\delta(p_y - p'_y)$. To allow for such dependence only adds transverse momentum components and does not add to the effect under study. The remaining transform is

$$\int dz' e^{\frac{i}{\hbar}(p_z - p'_z)z'} E_z(z')$$

and is evaluated at a wavelength of $\frac{2\pi\hbar}{p_z - p'_z}$. The delta functions require

$$|p_z - p'_z| \simeq \frac{m\hbar\omega}{p}$$

so that this wavelength is $\frac{v}{c}\lambda$. For electron energies in the vicinity of 50 keV (the energy used by Schwarz), $\frac{v}{c} \simeq .4$. Thus as promised in the last section, the modulation amplitude is only appreciable when E_z has Fourier components at less than half a light wavelength. The presence of the crystal provides such components, and it is for this

reason that the crystal is an essential part of the modulation process. Its presence is not required to conserve momentum as thought by some previous authors (9,12). In fact, even a laser beam by itself will lead to some modulation, although the amplitude is vanishingly small since spatial dependence is too slowly varying.

Upon taking the transform of $u(z')$, Equation (10) may be simplified to the form

$$\begin{aligned} \psi_{SC}^{(1)}(\bar{x}, t) = & i \frac{cE_0 z^d}{2\hbar\omega} \frac{\epsilon-1}{\epsilon} \left[\frac{p}{p_+} \frac{\sin \frac{1}{2} d \hbar^{-1} (p-p_+)}{\frac{1}{2} d \hbar^{-1} (p-p_+)} e^{\frac{i}{\hbar} (p_+ z - E_+ t)} \right. \\ & \left. + \frac{p}{p_-} \frac{\sin \frac{1}{2} d \hbar^{-1} (p-p_-)}{\frac{1}{2} d \hbar^{-1} (p-p_-)} e^{\frac{i}{\hbar} (p_- z - E_- t)} \right], \end{aligned} \quad (11)$$

where $E_{\pm}$ is $\frac{p_{\pm}^2}{2m}$ and $p_{\pm}$ are the positive momenta defined by

$$\frac{p_{\pm}^2}{2m} = \frac{p^2}{2m} \pm \hbar\omega.$$

For optical photons and 50 keV electrons we expand $p_{\pm}$ as

$$p_{\pm} = p \left(1 \pm \frac{\hbar\omega}{E} \right)^{1/2} = p \pm \frac{1}{2} \left(\frac{\hbar\omega}{E} \right) p - \frac{1}{8} \left(\frac{\hbar\omega}{E} \right)^2 p. \quad (12)$$

Different orders of this expansion are appropriate at different places in Equation (11). The factors $\frac{p}{p_{\pm}}$ may be replaced by unity to get the main contribution. The first two terms must be retained to evaluate $p - p_{\pm}$ in the sine functions, and the first three kept in the exponential. Equation (11) then simplifies to

$$\begin{aligned} \psi_{SC}^{(1)}(\vec{x}, t) = & -i \frac{eE_0 z d}{\hbar \omega} \frac{\epsilon - 1}{\epsilon} \frac{\sin \frac{m\omega d}{2p}}{\frac{m\omega d}{2p}} e^{\frac{i}{\hbar}(pz - Et)} \\ & \times e^{-\frac{i}{8} \left(\frac{\hbar \omega}{E}\right)^2 \frac{pz}{\hbar}} \cos \omega \left(\frac{z}{p/m} - t\right). \end{aligned} \quad (13)$$

One combines this with Equation (5) to get the probability density, including the ripple, for the straight-through beam:

$$\begin{aligned} \rho = & (\varphi + \psi_{SC}^{(1)})^* (\varphi + \psi_{SC}^{(1)}) \\ \rho = & 1 - 2 \frac{eE_0 z d}{\hbar \omega} \frac{\epsilon - 1}{\epsilon} \frac{\sin \frac{m\omega d}{2p}}{\frac{m\omega d}{2p}} \sin \frac{1}{8} \left(\frac{\hbar \omega}{E}\right)^2 \frac{pz}{\hbar} \\ & \times \cos \omega \left(\frac{z}{p/m} - t\right). \end{aligned} \quad (14)$$

The probability current

$$\vec{J} = \frac{\hbar}{2im} (\Psi^* \nabla \Psi - (\nabla \Psi)^* \Psi)$$

is, in this case, in the z direction and simply $\rho \frac{p}{m}$. In taking the gradient of $\psi_{SC}^{(1)}$, the z -dependence in the factor $e^{i p \frac{z}{\hbar}}$ is the important effect, the other z -dependences contributing terms that are powers of $\left(\frac{\hbar \omega}{E}\right)$ smaller.

In a typical experiment, E_0 is 15 esu (for a laser power of 1.5 watt focused down to a beam 50μ in diameter), $d = 10^{-5}$ cm, $\hbar \omega = 2.5$ eV, and $E = 50$ keV. In that case, $\frac{eE_0 d}{\hbar \omega}$ is 0.02, $\frac{m\omega d}{2p}$ is 1.5, and

$\frac{1}{8} \left(\frac{\hbar\omega}{E} \right)^2 \frac{p}{\hbar}$ is $\frac{2\pi}{1.6} \text{ cm}^{-1}$. Altogether, with $\epsilon \simeq 3$, the second term in Equation (14) is about 0.02 times the trigonometric functions. In summary, with these operating conditions, there is a 2% modulation of the electron probability density. The ripple in the probability density has the laser frequency ω and moves with the particle velocity p/m .

The dependence on crystal thickness and momentum shows up in the factor $p \sin \frac{m\omega d}{2p}$, which is graphed in Figure 4 as a function of beam energy for selected values of crystal thickness. It is seen that the modulation disappears completely at thicknesses for which $u(z)$ does not have Fourier components at a wavelength $\frac{v}{c} \lambda$.

Modulation of the diffracted beams is examined by calculating $\psi_{SC}^{(2)}$ and comparing it with the first order Bragg scattering term mentioned previously. The calculation of $\psi_{SC}^{(2)}$ proceeds in much the same way as for $\psi_{SC}^{(1)}$ except that now the expression is quadratic in $V(\bar{x}, t)$ and one discards expressions which do not contain both a laser and a crystal potential term. This leaves two expressions in which $U(\bar{x})$ and the laser interaction $-\frac{e}{mc} \bar{A} \cdot \bar{p}$ are used for V in Equation (8) in the two possible ways.

The expression for $\psi_{SC}^{(2)}$ is easily divided into contributions from $\bar{A}_n$ and $\bar{A}_\ell$, as was $\psi_{SC}^{(1)}$ (though there the $\bar{A}_\ell$ part was zero). The contribution from $\bar{A}_n$ is similar to the first order case already done. In fact, if a certain direction toward a diffraction spot is chosen and $\psi_{SC}^{(2)}$ normalized to the Bragg-scattered wave function there, the resulting probability density is identical to that found for $\psi_{SC}^{(1)}$, Equation (14).

Figure 4. Graph of the crystal dependence factor as a function of beam energy for different crystal thicknesses

In a general direction, the result is

$$\psi_{SC}^{(2n)}(\vec{x}, t) = -im(2\pi)^{-2} \int d\Omega' p U(\vec{p}-\vec{p}') e^{\frac{i}{\hbar}(\vec{p}' \cdot \vec{x} - Et)}$$

$$\times \left[1 - i \frac{eE_{oz}d}{\hbar\omega} \frac{\epsilon-1}{\epsilon} \frac{\sin \frac{m\omega d}{2p}}{\frac{m\omega d}{2p}} e^{-\frac{i}{8} \left(\frac{\hbar\omega}{E} \right)^2 \frac{\vec{p}' \cdot \vec{x}}{\hbar}} \cos \left[\frac{\omega}{2E} \vec{p}' \cdot \vec{x} - \omega t \right] \right].$$

The Bragg-scattered wave function is

$$-im(2\pi)^{-2} \int d\Omega' U(\vec{p}-\vec{p}') p e^{\frac{i}{\hbar}(\vec{p}' \cdot \vec{x} - E't)} \quad (15)$$

where in both cases $U(\vec{p}-\vec{p}')$ stands for the transform of the lattice potential $U(\vec{x})$. The normalized probability density is thus

$$\rho^{(n)} = 1 - 2 \frac{eE_{oz}d}{\hbar\omega} \frac{\epsilon-1}{\epsilon} \frac{\sin \frac{m\omega d}{2p}}{\frac{m\omega d}{2p}} \sin \frac{1}{8} \left(\frac{\hbar\omega}{E} \right)^2 \frac{\vec{p}' \cdot \vec{x}}{\hbar} \cos \left[\frac{\omega}{2E} \vec{p}' \cdot \vec{x} - \omega t \right]. \quad (16)$$

In the case of $\psi_{SC}^{(1)}$, the contribution from $\bar{A}_\ell$ was zero because in first order energy conservation forbids a free electron from emitting or absorbing a photon from a spatially constant electromagnetic field. In second order this restriction no longer applies and both terms contribute to $\psi_{SC}^{(2)}$. The result for $\bar{A}_\ell$ is

$$\psi_{SC}^{(2\ell)}(\vec{x}, t) = -im(2\pi)^{-2} \int d\Omega' p U(\vec{p}-\vec{p}') e^{\frac{i}{\hbar}(\vec{p}' \cdot \vec{x} - Et)}$$

$$\times \frac{-ie}{m\hbar\omega} \vec{E}_0 \cdot (\vec{p}-\vec{p}') e^{-\frac{i}{8} \left(\frac{\hbar\omega}{E} \right)^2 \frac{\vec{p}' \cdot \vec{x}}{\hbar}} \sin \left[\frac{\omega}{2E} \vec{p}' \cdot \vec{x} - \omega t \right]. \quad (17)$$

In a definite direction $\vec{p}'$, this combines with Equation (15) to make

$$\rho^{(1)} = 1 - \left(\frac{2e}{m\hbar\omega}\right) \vec{E}_0 \cdot (\vec{p} - \vec{p}') \sin \frac{1}{8} \left(\frac{\hbar\omega}{E}\right)^2 \frac{\vec{p}' \cdot \vec{x}}{\hbar} \\ \times \sin \left(\frac{\omega}{2E} \vec{p}' \cdot \vec{x} - \omega t\right). \quad (18)$$

Under similar conditions as before the factor in front of the trigonometric functions has a value of 0.007. For large angle scattering, however, it may be larger and may be of comparable size to the other term. In that case the two ripples are to be added and one has

$$\rho^{(2)} = 1 + A \cos \left[\frac{\omega}{2E} \vec{p}' \cdot \vec{x} - \omega t + \text{phase}\right], \quad (19)$$

where

$$A^2 = \sin^2 \left(\frac{1}{8} \left(\frac{\hbar\omega}{E}\right)^2 \frac{\vec{p}' \cdot \vec{x}}{\hbar}\right) \left[\left(\frac{2e}{m\hbar\omega}\right) \vec{E}_0 \cdot (\vec{p} - \vec{p}')\right]^2 \\ + \left(\frac{4eE_0 z^p}{m\hbar\omega} \frac{\epsilon-1}{\epsilon} \sin \frac{m\omega d}{2p}\right)^2. \quad (20)$$

This completes the treatment of plane wave modulation both in the forward direction and in the direction of a diffracted spot. Before going on to a wave packet treatment some observations may be made concerning the scattered wave functions:

1) There exists an interesting relationship between the crystal dependence factor graphed in Figure 4 and the length of time the electron spends inside the crystal. This time is $\frac{d}{p/m}$ and during this time the laser field changes phase an amount given by

$$\omega t = \frac{m\omega d}{p}.$$

The crystal dependence factor $\sin \frac{m\omega d}{2p}$ is at maximum when

$$\frac{m\omega d}{2p} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

or when this phase change is

$$\pi, 3\pi, 5\pi \dots$$

So the amplitude for photon pickup is maximized when the electron spends a half-odd number of laser periods in the crystal. This makes some sense classically, because the electron should have a net amount of work done on it by the field when it sees that field for odd multiples of a half period (provided the phase is correct). On the other hand the crystal factor is seen to be zero when the electron spends an even number of periods inside the crystal. In this case the classical work is always zero. This idea neglects the field outside the crystal but nonetheless seems to make some sense.

2) A useful comparison with the klystron may be made at this point by letting ω become orders of magnitude smaller in Equation (14). This raises the amplitude like ω^{-2} so that soon the Born approximation would fail to apply unless many terms in the series were computed. Since higher order terms involve many photon exchange (a process which often may be described classically), we then see the Schwarz-Hora Effect as the quantum-mechanical counterpart to the klystron. As one increases the frequency past klystron limits, the amplitude for many photon

exchange gradually goes down and the process becomes entirely quantum-mechanical.

3) The amplitude for the absorption of one photon is seen to be the same as that for the emission of one photon (to lowest order) so that the average momentum of any electron is the same before and after it enters the crystal. This nullifies the early classical attempts (7,8) to describe the Schwarz-Hora Effect since they assumed that bunching of the electron beam takes place. In the klystron, bunching does occur because the appreciable multi-photon absorption terms mentioned above need not have the same amplitude as their emission counterparts. Again as one goes to the high frequency limit, bunching gradually disappears until one is left with only one-photon processes which have equal absorption and emission amplitudes.

D. Wave Packet Treatment

There are certain questions concerning the behavior of the modulated beam which cannot be answered by using a plane wave description. Is, for example, the amplitude for an electron to absorb a photon dependent on when the electron enters the crystal? This is certainly the case in the classical limit. Is the phase of the "ripples" on one electron locked to that of a neighboring electron irregardless of their relative separation? In order to answer these questions, an electron must be described as a localized particle, and this demands a wave packet construction.

We shall work with solutions for the undeflected beam, since they

are less complicated, and form packets with respect to the z-direction. This can be extended, if desired, to the diffracted beams, but many of the questions at hand may be answered by considering only the central beam. The initial packet is constructed out of plane waves in the following manner:

$$\varphi = \int dp_z e^{-\frac{(p_z - p)^2}{\Delta p^2}} e^{\frac{i}{\hbar}(p_z z - \frac{p_z^2}{2m} t)} \quad (21)$$

This gives, upon doing the integral and normalizing in the z direction,

$$\varphi = (2\pi)^{1/4} \sqrt{\frac{\Delta p}{\hbar \alpha}} e^{-\frac{(z - \frac{p}{m} t)^2 \Delta p^2}{4\alpha \hbar^2}} e^{\frac{i}{\hbar}(p z - \frac{p^2}{2m} t)} \quad (22)$$

where

$$\alpha = 1 + \frac{it\Delta p^2}{2m\hbar}.$$

Here the wave function is left independent of and unnormalized with respect to the x and y directions. Equation (22) describes a packet which moves with average momentum $p \hat{z}$ and has a momentum uncertainty Δp . The spreading parameter is α and becomes different from one when

$$\frac{\Delta p^2 t}{2m\hbar} \simeq 1,$$

or, in terms of distance traveled D,

$$\frac{D \Delta p^2}{p\hbar} \simeq 1.$$

For a 50 keV electron beam with an energy spread of 0.1 eV this factor is 0.72 at $D = 30$ cm. Thus for this crystal-to-screen distance spreading should be considered unless ΔE is much less than 0.1 eV.

The final packets are found from Equation (7) just as before except that now φ is no longer a plane wave.

$$\begin{aligned} \Psi_{SC}^{(1)}(\bar{x}, t) = & \int d^3x' \int dt' G_0(\bar{x}, t; \bar{x}', t') V(\bar{x}', t' - t_0) \\ & \times \int dp_z e^{-\frac{(p_z - p_0)^2}{\Delta p^2}} e^{\frac{i}{\hbar}(p_z z' - \frac{p_z^2}{2m} t')} \sqrt{\frac{\alpha}{\hbar}} \frac{(2\pi)^{-1/4}}{\pi} \Delta p^{-3/2}, \end{aligned} \quad (23)$$

where $\alpha = 1 + \frac{it' \Delta p^2}{2m\hbar}$.

Here for simplicity we shall ignore spreading, and so set $\alpha = 1$. Exchanging orders of integration,

$$\begin{aligned} \Psi_{SC}^{(1)}(\bar{x}, t) = & \frac{(2\pi)^{-1/4}}{\pi} \frac{\Delta p^{-3/2}}{\sqrt{\hbar}} \int dp_z e^{-\frac{(p_z - p_0)^2}{\Delta p^2}} \int d^3x' \int dt' \\ & \times G_0(\bar{x}, t; \bar{x}', t') V(\bar{x}', t' - t_0) e^{\frac{i}{\hbar}(p_z z' - \frac{p_z^2}{2m} t')}. \end{aligned} \quad (24)$$

The integrations over $\bar{x}'$ and t' have already been evaluated except for the presence of $t' - t_0$ in place of t' in the potential V . This substitution was made so that the laser may be adjusted to have any arbitrary phase when the wave packet passes through the interaction region. In this case it passes through at $t' = 0$, at which time the laser vector potential has the form $\cos \omega t_0$. This will allow an investigation of laser phase influence on the scattering of the packet. Upon doing the integrals again, Equation (24) becomes

$$\psi_{SC}^{(1)}(\vec{x}, t) = \frac{(2\pi)^{-1/4} \Delta p^{-3/2}}{\pi \sqrt{\hbar}} \int dp_z e^{-\frac{(p_z - p_0)^2}{\Delta p^2}} \left[A_+ e^{i\omega t_0} e^{\frac{i}{\hbar}(p_+ z - E_+ t)} + A_- e^{-i\omega t_0} e^{\frac{i}{\hbar}(p_- z - E_- t)} \right], \quad (25)$$

where

$$A_{\pm} = -i \frac{e E_0 d}{2\hbar \omega} \frac{\epsilon - 1}{\epsilon} \frac{p_z}{p_{\pm}} \frac{\sin \frac{d}{2\hbar}(p_z - p_{\pm})}{\frac{d}{2\hbar}(p_z - p_{\pm})}$$

$$p_{\pm} = \sqrt{p_z^2 \pm 2m\hbar\omega}.$$

Now $A_{\pm}$ is a slowly varying function of p_z compared with $e^{-\frac{(p_z - p_0)^2}{\Delta p^2}}$ so it may be extracted from the integral. In addition, A_+ and A_- are equal to lowest order, so

$$\psi_{SC}^{(1)}(\vec{x}, t) = \frac{(2\pi)^{-1/4} \Delta p^{-3/2}}{\pi \sqrt{\hbar}} A \int dp_z e^{-\frac{(p_z - p_0)^2}{\Delta p^2}} \left[e^{i\omega t_0} e^{\frac{i}{\hbar}(p_+ z - E_+ t)} + e^{-i\omega t_0} e^{\frac{i}{\hbar}(p_- z - E_- t)} \right], \quad (26)$$

where A is just $A_{\pm}$ with $p_{\pm}$ replaced by p_z in front of the sine function.

It is convenient to write $p_{\pm}$ in the form

$$p_{\pm} = \xi_{\pm} p_z,$$

where

$$\xi_{\pm} = 1 \pm \frac{1}{2} \left(\frac{\hbar\omega}{E} \right) - \frac{1}{8} \left(\frac{\hbar\omega}{E} \right)^2 \pm \dots$$

Then the integral on p_z in Equation (26) can be rewritten (after a change of integration variable)

$$\begin{aligned} & e^{i\omega t_0} e^{\frac{i}{\hbar}(\xi_+ p_0 z - \frac{\xi_+^2 p_0^2 t}{2m})} \int dp e^{-\frac{p^2}{\Delta p^2}} e^{\frac{i}{\hbar}(p(\xi_+ z - \frac{p_0}{m} \xi_+^2 t) - \frac{p^2 \xi_+^2 t}{2m})} \\ & + e^{-i\omega t_0} e^{\frac{i}{\hbar}(\xi_- p_0 z - \frac{\xi_-^2 p_0^2 t}{2m})} \\ & \int dp e^{-\frac{p^2}{\Delta p^2}} e^{\frac{i}{\hbar}(p(\xi_- z - \frac{p_0}{m} \xi_-^2 t) - \frac{p^2 \xi_-^2 t}{2m})}. \end{aligned}$$

The two integrals left can be easily evaluated and $\xi_{\pm} p_0$ recombined to make $p_{\pm}$. The scattered wave function is then seen to be the sum of two discrete packets:

$$\begin{aligned} \psi_{SC}^{(1)}(\vec{x}, t) = & \frac{(2\pi)^{-1/4} \Delta p^{1/2}}{\sqrt{\hbar}} A \left[e^{\frac{i}{\hbar}(p_+ z - E_+ t)} e^{-\frac{(z - \frac{p_+}{m} t)^2 \xi_+^2 \Delta p^2}{4\hbar^2}} e^{i\omega t_0} \right. \\ & \left. + e^{\frac{i}{\hbar}(p_- z - E_- t)} e^{-\frac{(z - \frac{p_-}{m} t)^2 \xi_-^2 \Delta p^2}{4\hbar^2}} e^{-i\omega t_0} \right]. \end{aligned} \quad (27)$$

Since one packet is traveling with velocity $\frac{p_+}{m}$ and the other $\frac{p_-}{m}$, they will gradually separate until they no longer overlap and the laser frequency modulation disappears. It is instructive to watch the

modulation at the position of the central (unscattered) packet with all three packets added in. This may be done by evaluating the slowly varying packet envelopes at $z = \frac{p_0}{m} t$ while leaving the z -dependence free in the traveling-wave exponentials. This is legitimate since these vary much more quickly than the packet envelopes, and it is this variation we would like to examine. In this form the wave function is

$$\begin{aligned} \psi_{SC}^{(1)}(\bar{x}, t) = & (2\pi)^{-1/4} \sqrt{\frac{\Delta p}{\hbar}} e^{\frac{i}{\hbar}(p_0 z - E_0 t)} \left[1 - i \frac{e E_0 z d}{\hbar \omega} \right. \\ & \times \frac{\epsilon - 1}{\epsilon} \frac{\sin \frac{m \omega d}{2p}}{\frac{m \omega d}{2p}} e^{-\frac{1}{8} \left(\frac{\hbar \omega}{E} \right)^2 \frac{p z}{\hbar}} e^{-\frac{1}{16} \left(\frac{\hbar \omega}{E} \right)^2 \frac{\Delta p^2}{\hbar^2} \left(\frac{p_0 t}{m} \right)^2} \\ & \left. \cos \omega \left(\frac{z}{p/m} - (t - t_0) \right) \right]. \end{aligned} \quad (28)$$

Here we may make two important observations:

- 1) The amplitude of the ripple term decreases exponentially with the square of the distance of the packet from the crystal¹:

$$e^{-\frac{1}{16} \left(\frac{\hbar \omega}{E} \right)^2 \frac{\Delta p^2}{\hbar^2} D^2}.$$

¹This dependence was first found by Favro, Fradkin, and Kuo (15). Their expression in the present notation is

$$e^{-\frac{1}{4\pi} \left(\frac{\hbar \omega}{E} \right)^2 \frac{\Delta p^2}{\hbar^2} D^2}.$$

This term begins to deviate from one when

$$D \simeq \frac{4\hbar}{\Delta p} \left(\frac{E}{\hbar\omega} \right),$$

or

$$D \simeq \frac{8\hbar}{p} \left(\frac{E}{\Delta E} \right) \left(\frac{E}{\hbar\omega} \right). \quad (29)$$

This in effect says that in order for the modulation to exist 30 cm from the crystal with other parameters the same as before requires $\Delta E \simeq 0.01$ eV. This is a difficult energy resolution to obtain in the laboratory and is in fact responsible for most of the controversy surrounding the Schwarz-Hora Effect. That is, there exists some doubt as to whether Schwarz actually obtained that figure in his experiment as he now claims to have done.

2) In the preceding calculation, the packet was constructed so as to pass through the crystal at $t=0$ when the laser phase was $-\omega t_0$. Since these are the only two phases of physical interest, it is just as valid to have the laser with phase 0 at $t=0$ and have the packet pass the crystal at $t=-t_0$. In either case the laser has a phase $-\omega t_0$ when the packet goes through, but the latter way of looking at the situation enables one to more easily handle more than one electron with a fixed laser phase. Equation (28) may be interpreted in this new way by first making the time transformation $t - t_0 \rightarrow t'$ and then changing t_0 to $-t_1$. This gives an incoming wave packet which passes the crystal at $t=t_1$. The ripple on this packet now has the form

$$\cos \omega \left(\frac{z}{p/m} - t' \right)$$

and is clearly independent of t_1 . This means that every electron in the beam has the same ripple phase as every other electron, irregardless of their relative position. The target will then see a coherent pulsation of charge density in which all electrons are locked in phase with one another, rather than a series of random phase wave trains.

(See Figure 5).

Figure 5. Two adjacent modulated wave packets

III. PRODUCTION OF LIGHT AT THE TARGET

A. Introductory Remarks

Having modulated the beam by a few percent, the next question is whether or not the modulation can be detected and if so, how. The detection method reported by Schwarz and Hora (3,27) was the observation of emitted light at the laser frequency from a non-resonant target. At the time of this writing there is no generally accepted theory for this process. Other methods can be imagined, including the use of targets with resonances at the laser frequency.

In this section the method of detection which involves emitted light from a non-resonant target will be examined from different points of view. For one particular model, namely treating the wave function modulation as a classical current, an upper limit will be obtained for the radiated light intensity.

The author does not pretend to present an acceptable theory, but simply to examine the demodulation process Schwarz used and comment on it from a theoretical point of view.

B. Quantum-Mechanical Viewpoint

Here we view the target as a quantum-mechanical system with a ground state plus a discrete set of excited states. The notation here will be $|E\rangle$ where $E = E_0$ for the ground state. Initially the electrons and the target do not interact so that if $|p\rangle$ is an electron plane wave eigenstate we may write the combined state as $|E, p\rangle$.

One can easily show that one modulated electron cannot give off very much more light than an unmodulated electron¹. To see this, let the interaction operator for one electron with target be H_1 . Then the amplitude for exciting a transition in the target to some excited state $\hbar\omega$ in energy above the ground state is

$$\sum_{p'} \langle E_0 + \hbar\omega, p' | H_1 | E_0, p \rangle, \quad (30)$$

where plane electron waves have been used for convenience and the sum is over final electron states. For a non-luminescent target, this amplitude is small and a smooth function of p . Now do the same with a modulated electron whose wave function is

$$\varphi = |p\rangle + A_+ |p_+\rangle + A_- |p_-\rangle.$$

The amplitude for exciting a transition is now

$$\begin{aligned} \sum_{p'} [& \langle E_0 + \hbar\omega, p' | H_1 | E_0, p \rangle + A_+ \langle E_0 + \hbar\omega, p' | H_1 | E_0, p_+ \rangle \\ & + A_- \langle E_0 + \hbar\omega, p' | H_1 | E_0, p_- \rangle]. \end{aligned}$$

Since Equation (30) is a smooth function of p , we may replace $p_{\pm}$ by p to get

$$(1 + A_+ + A_-) \sum_{p'} \langle E_0 + \hbar\omega, p' | H_1 | E_0, p \rangle.$$

¹This proof has been given before by Lipkin and Peshkin (19).

This is hardly different from the amplitude for excitation by an unmodulated electron whenever A_+ and A_- are small. The point of view in which one calculates the number of photons emitted per electron and multiplies by the number of electrons incident per second is then seen to fail. This viewpoint would give an answer linear in the electron beam current. Adding a resonance to the target spectrum does not actually improve the situation, because it implies only a different dependence of Equation (30) on ω , not on p .

One might suspect that one reason for the failure of a one-particle model is that the electron-electron phase correlation demonstrated in a previous section is not utilized. To do so necessitates a many-particle calculation and several authors have attacked the problem from this point of view¹. Here the idea is that N electrons excite the target simultaneously, and in phase, giving N times the transition amplitude but N^2 times the transition probability. N is determined to be the number of electrons which pass the target within one lifetime of the excited target state. If N turns out to be one, then this simply reduces to the one-electron model and no enhancement is expected. Using the parameters of the original Schwarz-Hora experiment and ignoring transverse separations, one finds that there were about 500 electrons per cm in Schwarz's beam. This gives for $N = 10$ a time of 8×10^{-13} sec, which should then be compared with lifetimes of possible solid state

¹For specific references, see the discussion presented in the introduction.

emission processes.

At least one many-particle theory (21) has been applied to both transition radiation and brehmstrahlung and has failed to produce enhancement large enough to see with the unaided eye when the parameters used by Schwarz were inserted in the theory.

C. Classical Viewpoint

The word "classical" here must not be taken too literally, because if the electrons in the beam are viewed as true point particles, then no modulation is present. This is not true of the klystron since there beam bunching occurs, but is true of the Schwarz-Hora Effect where the modulation is confined to individual particle wavefunctions.

One may proceed in a "semi-classical" manner, though, by viewing the ripple on the electron's wave function as giving rise to an alternating current as seen by the target. From this viewpoint, the target sees the electron not as a point charge, but as an elongated washboard. Upon seeing this kind of charge density pass by, it will oscillate at the modulation frequency and subsequently radiate at that frequency.

A semi-classical calculation of this nature may be criticized, and the questions raised thereby are worthy of examination. Here are the the chief arguments against its validity:

1) To treat $\frac{e\mathbf{p}}{m} \Psi^* \Psi$ as a current density in the classical sense is somewhat subject to doubt. This is where the bridge between classical and quantum mechanics lies, because Ψ is a quantum-mechanical entity whereas once a classical current density is found the problem from there

on is entirely classical.

2) The alternating component of the current itself is treated as the source of radiation. This is probably not what takes place.

3) The treatment is basically a one particle calculation and is therefore subject to the proof given earlier against the workability of such models.

In support are the following points:

1) First and foremost, this way of looking at the problem permits obtaining a numerical answer for the amount of light radiated with a minimum of information required. The output power radiated depends only on the modulation amplitude and the electron beam current. The answer obtained is reasonable and believable.

2) The calculation is intended to be an upper limit rather than an accurate determination. Such a limit is very desirable to experimentalists trying to reproduce the effect, since it provides an order of magnitude intensity estimate. This interpretation partially compensates for some of the uncertainties mentioned above regarding principle. Such rough order of magnitude calculations are invaluable when an exact theory is unknown, and yet can seldom be thoroughly substantiated step by step.

3) This semi-classical viewpoint is essentially a many-particle rather than a one-particle calculation, and incorporates particle-to-particle phasing. The first point should be evident since the light output depends on the square of the beam current. A true one-particle

picture involves multiplying the power radiated by one electron by the number of electrons per second striking the target, giving a wattage linear in the beam current. Electron-to-electron phasing is incorporated very inconspicuously when the alternating current incident upon the target is assumed to be the beam current multiplied by the modulation amplitude. If each electron were out of phase with its nearest neighbor, the net alternating current seen by the target would be a small fraction of J_{AC} used below (see the appendix for a proof). Correct phasing, a many-electron feature, is thus used implicitly.

It is the author's conclusion that a semi-classical treatment at this point is valuable and informative as an upper limit to the amount of radiated light from the target at the laser frequency. The calculation accordingly proceeds as follows:

The electron current $\bar{J}$ is written as

$$\bar{J} = \bar{J}_{dc} + \bar{J}_{ac}$$

where

$$\bar{J}_{AC} = \bar{J}_{DC} A \cos \left[\frac{\omega}{2E} pz - \omega t \right] \quad (31)$$

and

$$A = - \frac{2eE_{oz}d}{\hbar\omega} \frac{\epsilon-1}{\epsilon} \frac{\sin \frac{m\omega d}{2p}}{\frac{m\omega d}{2p}} \sin \frac{1}{8} \left(\frac{\hbar\omega}{E} \right)^2 \frac{pz}{\hbar} .$$

Although the central beam only is considered here, the same type of treatment applies to the diffracted beam.

One now thinks of this current as being transformed with perfect efficiency into a surface current (charge per unit time per unit length in the surface perpendicular to the current) $K = JL$ for a spot on the target of dimension L . This sort of a surface current, originating from an area large compared to a wavelength, produces waves as illustrated in Figure 6. The boundary conditions $\Delta \bar{E}_{\text{tan}} = 0$ and $\hat{n} \times \Delta \bar{B}_{\text{tan}} = \frac{4\pi}{c} \bar{K}$ (Gaussian units) imply that

$$E = B = \frac{2\pi}{c} J_{AC} L$$

for one of the output beams. In time average, the energy flux emitted is

$$\begin{aligned} \bar{S} &= \frac{c}{4\pi} \bar{E}^2 \\ &= \frac{\pi}{2c} J_{DC}^2 L^2 A^2 \end{aligned}$$

and the total power radiated is

$$P = \bar{S} L^2 = \frac{\pi}{2c} I^2 A^2, \quad (32)$$

where I is the total current incident on the spot. Under the conditions described in section II, the parameter A is about 0.02. For a total current of 10^{-6} amperes incident on a spot, Equation (32) gives a radiated power of 10^{-14} watt. This is four orders of magnitude lower than the estimated power output from Schwarz's experiment (27).

As mentioned before, this estimate is designed as an upper limit

Figure 6. Illustration of the semi-classical target radiation process. The radiation shown is from a single diffraction spot

and has therefore been made generously. Its application is restricted, however, to non-resonant targets such as the one used by Schwarz. A treatment of resonant targets would probably be more meaningful if done quantum-mechanically.

IV. EVALUATION AND DISCUSSION

The principle result of this thesis is the calculation through the second-order Born approximation of the wave function of an electron which has been "modulated" at an optical frequency. This has been done using both plane waves and wave packets to describe the electron. The resultant wave function shows, within a certain range of parameters, the conditions under which an electron beam may be optically modulated. The following is a summary of these conditions:

A free electron may absorb or emit a photon (become modulated) in the presence of a laser field only provided the laser's polarization component parallel to the electrons momentum has a spatial variation in that same direction with appreciable Fourier components at a wavelength of $\frac{v}{c}\lambda$. Although this applies for all electron energies large compared to the photon energy, the amplitude is proportional to $\sqrt{E}$ and is already rather small (2%) at 50 keV.

An electron scattered from a crystal (as part of a diffraction pattern, for example) may become modulated if the laser field intersects the crystal and has a large polarization component in the direction of the electron's momentum or momentum transfer. One term in the modulation corresponds identically to the process just mentioned and depends critically on the spatial variation of the laser field produced by the presence of the crystal. Another term requires only the presence of the laser field inside the crystal and is a second-order induced emission or absorption process.

Since the modulation is small under the best of circumstances, the biggest problem one has to face is that of detecting and measuring the modulation. This effect has been proposed as being useful in areas ranging from the study of organic molecules (15) to color television (28). Before any of these ideas can be realized, however, much more must be understood about the detection process. In particular a target must be devised which is sensitive to Fourier components in the electron's wave function at the laser frequency. It's response must be highly peaked in order to show much enhancement from a 2% modulation.

Not of small importance is the problem of keeping the electron wave packet components together until they reach the target. From Equation (29) it is seen that this places a rather severe restriction experimentally on either target distance, or electron energy uncertainty, or both. At present it is a major feat in itself to prepare a 50 keV beam with an uncertainty of only 0.01 eV, the necessary value for a target distance of 30 cm.

One possible solution to this dilemma is to go to relativistic energies. Calculations at 1 MeV (23) indicate that an energy spread of a few eV suffices for $D = 30$ cm. Since the energy spreads easily obtainable from accelerator devices are mostly independent of beam energy, this might be a practical idea. It would hardly benefit the color television industry but is encouraging as regards other uses, since there are at present electron microscopes operating in this energy range. But even here the problem of an efficient demodulator looms large enough to be discouraging.

In any event, Drs. Schwarz and Hora have introduced to the scientific community a new class of problems in which tremendous interest has been shown. Even the fact that the effect has not been successfully reproduced has not dampened the enthusiasm of those working in this area. With laser technology booming it is certain that the future will bring an exhaustive study of electron-photon interaction problems of this type and that the results will be extremely enlightening.

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VII. APPENDIX

It was contended in section III that the semi-classical reradiation treatment given there was in essence a many-electron calculation. The reader may recall that in this treatment the electron beam was said to have an alternating current component J_{AC} which was J_{DC} times the modulation amplitude (roughly 2%). The following argument is given to show that the phase correlation between the ripples of each successive electron is implicitly taken into account here, and that this model must necessarily be counted as a many-electron model.

Consider two successive electrons as shown in Figure 5. Let their wave functions be written as

$$f(x - v_0 t - x_1) e^{i(kx - \omega t)}$$

for the first electron and

$$f(x - v_0 t - x_2) e^{i(kx - \omega t + \varphi)}$$

for the second. Here $f(x)$ is a slowly varying function representing the packet envelope which is centered at $x = 0$, and φ represents a possible phase difference between the two electrons. The packets are shown 100% modulated for simplicity. When these electrons pass the target, it will see a time-varying current whose Fourier transform is given by

$$\int e^{-i\omega' t} \left[f(x_t - v_0 t - x_1) e^{i(kx_t - \omega t)} + f(x_t - v_0 t - x_2) e^{i(kx_t - \omega t + \varphi)} \right] dt.$$

In order to see if this Fourier transform has a sizeable component at the laser frequency, one may proceed to do the integration and obtain

$$-e^{ikx_t} \left[e^{-i(\omega' - \omega) \left(\frac{x_t}{v_0} - \frac{x_1}{v_0} \right)} \int e^{i(\omega' - \omega)t'} f(v_0 t') dt' \right. \\ \left. + e^{i\varphi} e^{-i(\omega' - \omega) \left(\frac{x_t}{v_0} - \frac{x_2}{v_0} \right)} \int e^{i(\omega' - \omega)t'} f(v_0 t') dt' \right].$$

Since f is a slowly varying function, its transform $f_T(\omega' - \omega)$ will have a large peak at $\omega' = \omega$. So the original transform is approximately (at $\omega' = \omega$)

$$-e^{ikx_t} f_T(0) [1 + e^{i\varphi}].$$

This quantity is large when $\varphi = 0$ or 2π and is zero when $\varphi = \pi$.

Surely the statement

$$J_{AC} = (\text{Amplitude}) \times J_{DC}$$

assumes the electron current to have an appreciable Fourier component at the laser frequency. But it has just been shown that for two electrons this component will nearly vanish unless the ripples are in phase. This argument applies for more than two electrons in a similar manner if $e^{i\varphi}$ is replaced in the answer by

$$e^{i\varphi_1} + e^{i\varphi_2} + \dots + e^{i\varphi_N}.$$