Conf-720705--3

PARAMETRIC INSTABILITY AND ITS
INDUCED HEATING

by

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ABSTRACT

Laboratory experiments having high-frequency excitation electric fields near ω_{pe} are reviewed. Effects of the inhomogeneity of plasmas are discussed and the potential use of parametric instability in plasma heating is assessed.

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MASTER

Parametric instabilities may occur in systems having more than one resonance frequency (or degree of freedom). Consider a system with two degrees of freedom, characterized by one high- and one low-frequency resonance. Under static conditions both have stable equilibrium. When an external alternating force field is applied at a frequency near the high-frequency resonance, the two degrees of freedom may become coupled through the excitation field, resulting in the destabilization of the low-frequency degree of freedom.¹ An example is the harmonic oscillator, represented by a pendulum with a stiff rod in a gravitational field. The equilibrium $\theta = 0$ is stable and $\omega^2 = g/l$. This is the low-frequency resonance, and its associated degree of freedom is θ . If the stiff rod is replaced by a spring of spring constant K , the system now has a new degree of freedom, namely, r , and its associated resonance frequency is $\Omega_0 = (K/m)^{1/2}$. In the static gravitational field the θ degree of freedom remains stable at equilibrium. If an alternating gravitational field $\tilde{g} \cos \Omega t$ is applied to the system in the $\theta = 0$ direction, the r and θ degrees of freedom become coupled through this applied alternating field: the high-frequency potential, from which the high-frequency forces are derived, is

$$m (\tilde{g} \cos \Omega t) (r \cos \theta)$$

This coupling, basically, may destabilize the low-frequency θ degree of freedom.

The response of the r degree of freedom has a resonance denominator $r \propto 1/(\Omega^2 - \Omega_0^2)$. This has two important consequences: 1) r undergoes an abrupt phase change of π when Ω passes through Ω_0 , and 2) the coupling can be made very effective by placing Ω near Ω_0 . The first specifies the resonance frequency as stability boundary (in the

frequency domain), and the second indicates efficient destabilization of the equilibrium. When damping is included, the phase change becomes gradual and the coupling effectiveness more limited.

The threshold condition depends on: 1) linear damping of the system, and 2) the proximity of driving frequency to resonance frequency.

Plasmas have numerous resonance frequencies. One analogous 0-degree of freedom is the ion fluctuation $\tilde{n}_i$; its low frequency resonance is $\omega^2 \approx \omega_{pi}^2$ and it has a stable equilibrium at $\tilde{n}_i = 0$. The analogous r degree of freedom is $\tilde{n}_e$ and its resonance frequency is ω_{pe} . The spring is automatically provided by Poisson's equation. When an alternating electric field (pump) E is applied, $\tilde{n}_i$ and $\tilde{n}_e$ become coupled. The high-frequency potential obtained from Poisson's equation is proportional to

$$E \cos \Omega t \tilde{n}_e \Delta \tilde{n}, \quad \Delta \tilde{n} \equiv \tilde{n}_e - \tilde{n}_i$$

and the resonance denominator for $\tilde{n}_e$ is $\tilde{n}_e \propto 1/(\Omega^2 - \omega_{pe}^2)$. Again, we can destabilize the ω_{pi} wave by driving ω_{pe} ; and threshold conditions depend on: 1) collisional or Landau damping, and 2) closeness of pump to resonance frequency. The frequency spectrum of the parametric instability therefore consists of the pump frequency Ω , destabilized low-frequency instability at ω , and a high-frequency sideband $\Omega \pm \omega$. In general the frequency ω may be zero or finite, respectively labelled as purely growing mode or decay instability. When momentum considerations are included, wave number selection rule must also be satisfied. Incident and decay waves can be electrostatic or electromagnetic modes satisfying the boundary requirements.

There are other resonance frequencies in plasmas. Here, we

restrict the discussion to laboratory experiments in which the frequency of the excitation field is near ω_{pe} and the destabilized low-frequency mode is an ion acoustic wave near ω_{pi} ; we exclude ionspheric-plasma experiments² and laboratory experiments concerning other resonances such as cyclotron harmonics³ or hybrid resonances.⁴

Experiments in which the excitation field is electrostatic, launched by probes or grids, are summarized in Fig. 1. The experiments by Franklin, Hamburger, Lampis, and Smith⁵ determined threshold and selection rules and compared wave damping in the stable and parametric regimes. In addition, since the experiment is performed in a single-ended Q device at very low density, the ions are flowing, allowing both fast and slow modes and two accompanying high-frequency side bands, as observed in the experiment. The experimental arrangement of Ref. 6 is similar to that of the previous one. The density was varied over more than three decades, and wave numbers were measured at low plasma density. No threshold or wave damping was measured. The experiment of Stenzel and Wong⁷ is unique in that the plasma can be considered "infinite" and without boundary effects on parametric instability. Threshold conditions agree with linear theory based on an infinite-plasma model. It is characteristic of all these experiments that the plasmas are low-density; hence Debye length and acoustic wavelength are long when compared to probe size so that wave numbers could be measured with probes.

Experiments in which the excitation field is electromagnetic, as in wave guides and resonant cavities, are summarized in Fig. 2. Gekker and Sizukhin⁸ did not relate that the monotonic decrease of reflection coefficient with increasing power is due to parametric

instability. The experimental arrangement is simple, and only incident and reflected power need be concerned. A similar experiment was reported by Sergeichev⁹ who observed that, above a threshold, the particle flux to collectors increases. In the experiment of Eubank,¹⁰ since the microwave horn is placed outside the plasma column whose boundary has a large curvature, not all the microwave power can be accounted for -- an unknown amount is neither absorbed by the plasma nor reflected into the wave guide. This experiment demonstrated the increase of ion temperature and low-frequency spectrum near the ion plasma frequency. Parametric coupling was demonstrated in different approach in an experiment by Phelps, Rynn, and Van Hoven¹¹. When a strong pump, a weak interacting signal were launched, a third wave, satisfying both frequency and wave number selection rules was observed.

The first quantitative threshold measurement on anomalous absorption came from an experiment by Dreicer, Henderson, and Ingraham,¹² although no instability measurements were reported. By frequency-sweeping the cavity resonance and monitoring its half width from transmitted power, the dissipation of the cavity-plasma system, $1/Q$, was found to increase above a threshold in the transmitted power, demonstrating anomalous absorption. Similar results, obtained by measuring incident, transmitted, and reflected power, along with measurements of instability amplitude, frequency selection rule, increase of electron temperature, effective collision frequency determined from the gross power dissipation, and field reduction due to the instability were reported in Ref. 13.

Caution must be exercised in interpreting the cavity resonance curve from monitored transmitted power in the frequency-modulation

method. When parametric instabilities set in during the sweep, the plasma absorbs more power; hence the ratio of transmitted to absorbed power is no longer constant at different points of the resonance curve and the Q value varies. In addition, for frequency sweep, the residence time of the incident electromagnetic wave should be longer than the instability growth time.

What are the advantages of using electrostatic excitation by grids versus electromagnetic excitation by cavities? Grids can be used to launch a wide range of frequencies. In a grid experiment, the frequency spectrum can easily be measured, and the wave number spectrum can be obtained at low densities with probes. Electromagnetic waves launched by cavities have no such advantages: it is difficult to find out what is going on inside the cavity without disturbing the cavity-plasma system. But precise power measurements (incident, reflected, and transmitted) can be made so that the gross properties are known while power-balance measurements in a grid experiment are difficult. In both cases, measurement of wavenumbers are more difficult, especially at high densities. From the viewpoint of rf plasma heating, excitation fields provided by electromagnetic fields are perhaps of more practical importance.

One important plasma parameter which affects the experiment is the inhomogeneity of plasma. Table 1 shows the measured threshold and driving frequency. Also shown are theoretical threshold values based on models of a) infinite plasma¹⁴, and b) inhomogeneous plasma having a uni-direction density decrease¹⁵. A plasma can be considered inhomogeneous if the mean free path is longer than the density-gradient scale length¹⁵. It should be noted that the theory based

on infinite-plasma models requires that the decay instability can be excited only when $\Omega_{\text{pump}} > \omega_{pe}$.

Plasma inhomogeneity may play two different, unrelated roles:

1) it increases threshold values above the uniform plasma theory by convecting energy away from the interaction region,¹⁵ and 2) it produces those decayed modes which satisfy the dispersion relation of a finite plasma (such as the Gould-Trivelpiece mode of an electron plasma wave). The first explains why all measured threshold values, except those of Eubank's, agree with the "infinite" plasma model; the second explains why the driving frequency, except that in the experiment of Stenzel and Wong, in which the plasma can be considered as infinite both from the viewpoint of parametric instability and the dispersion of plasma waves, is less than the maximum plasma frequency.

The use of the parametric instability for the purpose of plasma heating hinges critically on the low onset threshold (typically $v_{\text{drift}}/v_{\text{thermal}}$ is a few percent, $v_{\text{drift}} \equiv eE/m_e\Omega$); so that the efficient absorption regime can be easily reached. According to theory based on inhomogeneous plasmas such advantage appears to be lost if the plasma has a strong density gradient. Laboratory plasmas, however, are unlikely to satisfy the requirements of the presently published theory¹⁵ of "inhomogeneous" plasmas for the following reason.

The theory of inhomogeneous plasma¹⁵ is based on a single plasma slab having uni-direction density increase in x and extending to infinity according to $e^{-x/H}$, H being the density-gradient scale length. Such a model simulates situations in which only a single turning point of the wave is needed (as in ionospheres) and assumes an infinite

internal region for convection (when the excitation frequency is less than the cyclotron frequency, the case for most laboratory plasmas). An implicit requirement is that the plasma size is much larger than all other scale lengths in the model. Laboratory plasmas, however, are finite in size. Cylindrical geometry permits the setting up of standing wave patterns inside (between turning points) and results in no convection. Since the time required for these waves to traverse the distance between turning points is usually shorter than the wave damping time, waves are essentially undamped between turning points and their threshold value should not increase substantially. It would be useful to repeat an experiment in the regime of long mean free path like that of Eubank's and with the entire power balance accounted for, so that the precise threshold value is known.

How much is known about the parametric instability experimentally?

Very little. Measurements include threshold, frequency selection rule, and some values on wave-number selection rule and growth rate. The lack of experimental results is particularly acute for the case of electromagnetic excitation. Nothing is known beyond threshold, frequency selection rule and gross dissipation at low $v_{\text{drift}}/v_{\text{thermal}}$ values. There has been no measurement of instability in real time and its spatial variation, and of plasma heating showing power balance including heat loss. Are (low-frequency) transport coefficients affected? what happens in the regime of large $v_{\text{drift}}/v_{\text{thermal}}$ values? does the incident wave interact with the plasma? and what is the change of the microwave field due to the instability? There is also no direct knowledge on the saturation mechanism. Finally, purely growing

modes have not been observed.

A compelling question must be asked: can the parametric instability be used in plasma heating for reactor purposes? The answer, in the case of electromagnetic excitation field near ω_{pe} , and to various degrees apply to other resonances, is not overly optimistic as judged by the limited experimental results. One practical limitation is the low power level available at these high frequencies within the present technology. Another problem is detuning. The instability has a growth time $\lesssim 1/\omega_{pi}$, much shorter than the confinement time in most devices. Heating results in local plasma expansion, and therefore a reduction of plasma density. Such change of plasma parameters results in the plasma resonance being tuned out after initial parametric heating. This is reflected in the fact that all electromagnetic field excitation experiments were either pulsed or unsteady (frequency modulation or amplitude modulation in the cavities). Attempts to achieve steady state in the time scale of the plasma confinement time (in the regime that the skin depth is larger than the plasma size) results in relaxation oscillation of the entire plasma volume at an acoustic mode¹⁶. Perhaps plasma inhomogeneity can be advantageously used to overcome this, so that there is always one location at which an incident electromagnetic wave may interact with the plasma to destabilize the instability, especially in the high-density regime where the required incident wavelength becomes much shorter than the density-gradient scale length. A second possibility is to operate the devices in a parameter regime that fusion reaction can be completed before detuning of the resonance due to expansion. From this viewpoint, it is worthwhile to reiterate the importance of knowing the instability behavior in real

time and its spatial variation.

ACKNOWLEDGEMENT

It is a pleasure to acknowledge valuable discussions with Drs. H. W. Hendel, Y. C. Lee, R. Motley, and E. Valeo, and Mr. J. Flick.

This work was jointly supported by U. S. Atomic Energy Commission Contract AT(11-1)-3073 and by the Advanced Research Projects Agency under Contract No. F30602-72-C-0053.

REFERENCES

- ¹ T. K. Chu and Y. C. Lee, Bull. Amer. Phys. Soc. 16, 1245 (1971)
- ² W. F. Utlaut and R. Cohen, Science 174, 245 (1971);
H. C. Carlson, J. of Geophys. Res. 77, 1242 (1972)
- ³ R. P. H. Chang, M. Porkolab, and B. Grek, Phys. Rev. Letters 28, 206 (1972)
- ⁴ W. M. Hooke and S. Bernabei, Phys. Rev. Lett. 28, 407 (1972);
S. Bernabei and W. M. Hooke, Bull. Amer. Phys. Soc. 17, 772 (1972)
- ⁵ R. N. Franklin, S. M. Hamberger, G. Lampis, and G. J. Smith, Phys. Rev. Lett. 27, 1119 (1971)
- ⁶ T. K. Chu and H. W. Hendel, Bull. Amer. Phys. Soc. 16, 1297 (1971)
- ⁷ R. Stenzel and A. Y. Wong, Phys. Rev. Lett. 28, 274 (1972)
- ⁸ I. R. Gekker and O. V. Sizukhin, Zh.ETF Pis. Red. 9, 408 (1969)
(Soviet Physics JETP 9, 243 (1969))
- ⁹ K. F. Sergeichev, Zh. Eksp. Teor. Fiz. 58, 1157 (1970)
(Soviet Physics JETP 31, 620 (1971))
- ¹⁰ H. P. Eubank, Phys. Fluids 14, 2551 (1971)
- ¹¹ D. Phelps, N. Rynn, and G. Van Hoven, Phys. Rev. Lett. 26, 688 (1971)
- ¹² H. Dreicer, D. B. Henderson, and J. C. Ingraham, Phys. Rev. Lett. 26, 1618 (1971)
- ¹³ T. K. Chu and H. W. Hendel, Bull. Amer. Phys. Soc. 17, 685 (1972)
- ¹⁴ D. F. Du Bois and M. V. Goldman, Phys. Rev. Lett. 14, 544 (1965) and Phys. Rev. 164, 207 (1967); V. P. Silin, Zh. Eksp. Teor. Fiz. 48, 1679 (1965) (Sov. Phys.--JETP 21, 1127 (1965)); and Zh. Eksp. Teor. Fiz. 51, 1842 (1966) (Sov. Phys.--JETP 24, 1242 (1967)); K. Nishikawa, J. Phys. Soc. Japan 24, 916 (1968); and 24, 1152 (1968); J. R. Sanmartin, Phys. Fluids 13, 1533 (1970); J. M. Dawson and W. L. Kruer, Phys. Fluids 12, 2586 (1969)
- ¹⁵ F. W. Perkins and J. Flick, Phys. Fluids 14, 2012 (1971)
- ¹⁶ H. W. Hendel and T. K. Chu, Bull. Amer. Phys. Soc. 16, 1294 (1971)

FIGURE CAPTIONS

Figure 1. Principal results of experiments with electrostatic excitation

Figure 2. Principal results of experiments with electromagnetic excitation

Table 1. Comparison of measured threshold and pump frequency with calculated values for various experiments. Also listed are $\lambda_{\text{mfp}}/(n/n)$.

Electrostatic Excitation

Experiments

Principal Results

1. Franklin, Hamburger, Lampis, and Smith (1971) in single-ended Q device
 $(\omega_{ce} \gg \omega_{pe})$

1. threshold
2. $\omega_1 = \omega_2 + \omega_3^*$
3. $\vec{k}_1 = \vec{k}_2 + \vec{k}_3^*$
4. damping

$$\left. \begin{array}{l} n = 3 \times 10^7 \text{ cm}^{-3} \\ T = 0.2 \text{ eV} \\ \Omega = 30 \text{ to } 60 \text{ MHz} \end{array} \right\} \begin{array}{l} 1. e\phi/kT \text{ up to } 30\% \\ 2. \Omega \lesssim \omega_{pe} \end{array}$$

2. Stenzel and Wong (1972) in DP device
 $(\omega_{ce} = 0)$

1. threshold
2. $\omega_1 = \omega_2 + \omega_3$
3. $\vec{k}_1 = \vec{k}_2 + \vec{k}_3$
4. growth rate

launching grids detector probe

$$\left. \begin{array}{l} n \approx 10^9 \text{ cm}^{-3} \\ T_i = 0.2 \text{ eV}, T_e = 2 \text{ eV} \\ \Omega = 400 \text{ MHz} \end{array} \right\} \begin{array}{l} 1. \frac{v_d}{v_{th}} \text{ up to } 2.1\% \\ 2. \Omega \gtrsim \omega_{pe} \end{array}$$

3. Chu and Hendel (1971) in double-ended Q device $(\omega_{ce} \gg \omega_{pe})$

1. $\omega_1 = \omega_2 + \omega_3$
2. $\vec{k}_1 = \vec{k}_2 + \vec{k}_3$
3. frequency dependence on density

$$\left. \begin{array}{l} n = 10^7 \text{ to } 5 \times 10^{10} \text{ cm}^{-3} \\ T = 0.25 \text{ eV} \end{array} \right\} 1. \Omega \lesssim \omega_{pe}$$

Electromagnetic Excitation

Experiments

Principal Results

1. Gekker and Sizukhin (1969) in wave guide

1. change of reflection coefficient R

2. Sergeichev (1970) in wave guide

1. increase of particle flux to collector when power exceeds a threshold

3. Eubank (1970) ($\omega_{ce} \gg \omega_{pe}$)

1. threshold
2. change of R
3. ω_{pi} oscillations
4. heating

4. Phelps, Rynn, and VanHoven (1971) in single-ended Q device with cavity excitation

1. launch strong pump and weak interacting 2nd signal, observe the 3rd wave satisfying ω and k relations.

5. Dreicer, Henderson, and Ingraham (1971) in single-ended Q device with high-Q cavity ($\omega_{ce} \gg \omega_{pe}$)

cavity ($Q \approx 20,000$)
(~100 mW)

1. threshold of anomalous absorption

6. Chu and Hendel (1972) in double-ended Q device with high-Q cavity ($\omega_{ce} \gg \omega_{pe}$)

cavity ($Q \approx 10,000$)
(~100 mW)

1. threshold
2. $\omega_i = \omega_s + \omega_j$
3. heating
4. effective collision freq.
5. field depletion

Effects of Plasma Inhomogeneity

Experiments	$\frac{\lambda_{mfp}}{(n/\nabla n)}$	Threshold Field, V/cm			Pump and Plasma Frequency
		Experiment	Theory		
			Homogeneous	Inhomogeneous	
(Electrostatic Excitation Field)					
1. Franklin, HLS	--	$(e\phi/kT = 10\%)$	$(e\phi/kT = 10\%)$	--	$\Omega \leq \omega_{pe}$
2. Stenzel & Wong	--	2.5	1.9	--	$\Omega > \omega_{pe}$
3. Chu & Hendel	--	--	--	--	$\Omega < \omega_{pe}$
(Electromagnetic Excitation Field)					
1. Eubank	~8	500	~40	400 - 600	$\Omega < \omega_{pe}$
2. Dreicer, H & I	~1	15	~8	100	$\Omega < \omega_{pe}$
3. Chu & Hendel	~1	12	~8	100	$\Omega < \omega_{pe}$