31
10-20-69
DRAGON

Gulf General Atomic
Incorporated

P O Box 608, San Diego, California 92112

C-80
MASTER

INFORMAL AEC RESEARCH
AND DEVELOPMENT REPORT

GAMD-9404

**DYNAMIC ANALYSIS OF AN ELASTOMERIC
PRESTRESSED CONCRETE REACTOR VESSEL SUPPORT**

by

P. D. Smith

This document, which was prepared primarily for internal use at Gulf General Atomic Incorporated, may contain preliminary or incomplete data. It is informal and is subject to revision or correction; it does not, therefore, represent a final report.

Gulf General Atomic Project 317
U. S. Atomic Energy Commission
Contract AT(04-3)-167
Project Agreement No. 17

REPRODUCTION OF THIS DOCUMENT IS UNLIMITED

June 3, 1969

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

TABLE OF CONTENTS

	PAGE
1. INTRODUCTION	1
2. PCRV SUPPORT DESIGN	1
3. FLEXIBILITY CALCULATIONS	3
3.1 STATIC Model	7
3.2 Column Flexibilities	8
3.3 Bearing Flexibility	8
3.4 Ground Flexibility	11
4. STATIC ANALYSIS	13
5. PCRV MASS	15
6. DYNAMIC ANALYSIS	16
7. STRESS ANALYSIS	19
8. CONCLUSIONS	24
9. LIST OF SYMBOLS	25
10. REFERENCES	27

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission

A. Makes any warranty or representation expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights, or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor

1. INTRODUCTION

It was proposed in Ref. 1 that elastomeric bearing pads may be utilized for supporting a large PCRV and protecting it against damaging earthquake loads. The most promising design for such a bearing appears to be a laminated neoprene and steel bearing of the type commonly used for bridge supports.

A preliminary design of an elastomeric support for a large PCRV has been made, and a dynamic response analysis of this design has been performed using the SHAKE computer code (Ref. 2). The purpose of this report is to present the results and to demonstrate that the elastomeric bearing pad is, in fact, a feasible support concept.

Four cases, each with different support flexibility assumptions, are analyzed. The first case uses the relatively crude flexibility assumptions which were made in the preliminary sizing calculations. Each succeeding case represents an improvement in the model, with the fourth case being the best approximation. The reason for doing this is to check the validity of the simplifying assumptions and to determine the effect that each succeeding improvement will have on the results.

2. PCRV SUPPORT DESIGN

The PCRV support design evaluated in this study consists of six variable diameter columns of prestressed concrete construction. The elastomeric bearings are located at the top of the columns and are doweled to both the columns and the PCRV. The basic configuration and bearing details are shown on Figs. 1 through 3.

The function of the six prestressed concrete columns is to elevate the PCR/V and to allow access to the bottom for piping and for installation of the steam generator modules. The column length is dictated by access requirements. The column diameter is governed by strength and by access requirements. A relatively large diameter at the bottom is needed for strength, and a relatively small diameter at the top is required for access to PCR/V prestressing tendons. The stepped column design shown on page satisfies these requirements. A further advantage of the stepped design is that it facilitates the prestressing of the columns themselves.

The elastomeric bearings were designed to limit the maximum PCR/V response by ensuring a definite natural period of vibration. After preliminary investigation of the dynamic response criteria (Ref. 1), the natural period of the system in the horizontal direction was fixed at two seconds. It was assumed that the foundation and the concrete columns were rigid when compared to the elastomeric bearings and that the bearings had flexibility only in the horizontal direction. These assumptions permit one to calculate the natural period of the PCR/V as a function of the rubber bearing dimensions alone. There are many possible combinations of bearing geometry and rubber hardness which will result in a natural period of two seconds, and the final design choice depends on strength, economy, and size limitations.

The most useful strength basis for the design of an elastomeric bearing is not stress but strain level. Experience has shown that a shear strain of 50% is a safe allowable design value (Ref. 4) and that the ultimate strain capability of the bearing is considerably greater.

Given the natural period of the PCR/V, the maximum displacement response to an earthquake excitation can be found from the response spectrum (Ref. 1). Assuming a standard El Centro earthquake with an input ground acceleration of 0.16g (seismic zone 2), and also assuming a period of 2 seconds and a damping factor of 2%, the maximum displacement response was found to be 4.0 inches. In order for the operating shear

strain to be less than the 50% allowable, the thickness of the rubber must be at least 8.0-in.

Making the plausible assumption that the smallest bearing is the least costly, the "optimum design" from a cost standpoint is one that uses the hardest possible rubber and which has the minimum bearing area and rubber thickness. In our case, however, there are many uncertainties that make it advisable to overdesign the bearing. The magnitude of the earthquake, the ability of the rubber to withstand deformations larger than 50%, and the aging characteristics of the rubber are some of the unknown factors.

The final bearing design shown on Fig. 3 has a rubber thickness of 13 inches. It therefore satisfies the strain criterion by an ample margin. This thickness was obtained by assuming the bearings to have the largest practical area, i.e., slightly less than the total area on top of the columns. Smaller, cheaper bearings could be designed, but they would not have as great a margin of safety.

3. FLEXIBILITY CALCULATIONS

The flexibilities of the various structural components are calculated separately and combined by either the STATIC or the SHAKE computer programs. Although the SHAKE code itself is theoretically capable of assembling all of the flexibilities, a peculiarity of this dynamic problem renders solution by SHAKE alone impossible. The difficulty is that the structure is geometrically three dimensional, while, due to symmetry, the stiffnesses in the two horizontal directions are identical. This results in repeated eigenvalues. Unfortunately, the iterative solution scheme employed by SHAKE cannot handle repeated eigenvalues. The way to circumvent this difficulty is to first reduce the problem to two dimensions. The two-dimensional problem will not have repeated eigenvalues and can be solved successfully by SHAKE. Two dimensional flexibility matrices for the structure were therefore calculated by an auxiliary code. The STATIC code was used to calculate a three dimensional flexibility, and the two-dimensional flexibility was derived by simply deleting rows and columns.

Basic Configuration

FIGURE 1

Bearing and Column Detail

Columns : prestressed concrete (six total)

Bearing : details shown on Figure 3

Figure 2

Neoprene Bearing Details

35 rubber layers, 3/8" thick

34 steel layers, .10" thick

FIGURE 3

3.1 STATIC Model

The structural model used by STATIC to calculate the three dimensional flexibility matrices is shown below. Note that the ground flexibility is not included in this model. This is because the ground flexibility is readily obtainable in two-dimensional form and can be put into SHAKE directly. Also, to include the ground in the STATIC model would necessitate adding a number of additional structural members.

FIGURE 4

3.2 Column Flexibilities

The column flexibilities are calculated internally by the STATIC code, using beam formulas that account for shear deformation. A summary of the section properties for the three segments of the stepped column is given below.

Column #	Diameter (in.)	Length (in.)	Area (in. ²)	Ix, Iz in. ⁴	J in. ⁴
1	168.	141.48	2.22x10 ⁴	3.92x10 ⁷	7.84x10 ⁷
2	204.	120.	3.27x10 ⁴	8.48x10 ⁷	16.96x10 ⁷
3	240.	120.	4.50x10 ⁴	1.625x10 ⁸	3.250x10 ⁸

3.3 Bearing Flexibility

The flexibility of the laminated neoprene bearing is calculated by means of the method given in Ref. 3. A brief summary of the method and of the numerical results is given here.

In matrix notation, the force vs displacement relationship is

$$\{X\} = [A] \{F\}$$

The flexibility matrix [A] is the inverse of the stiffness matrix [K]. Since the stiffness matrix for the laminated elastomeric bearing is a diagonal matrix (i.e., there is no coupling), the inverse is found by simply taking the reciprocal of the diagonal elements. Furthermore, due to structural symmetry, both the bending and the shear stiffnesses are identical in the two horizontal directions. Only four independent flexibilities need be calculated. These are

$$A_v = \frac{1}{K_v} \quad A_s = \frac{1}{K_s}$$

$$A_\theta = \frac{1}{K_\theta} \quad A_t = \frac{1}{K_t}$$

Formulas for the stiffnesses (K) are given on the following page. The complete 6x6 flexibility matrix required by the STATIC code is shown on page 10.

TABLE 1
SUMMARY OF STIFFNESS FORMULAS
FOR ELASTOMERIC BEARINGS*

Single Bearing			
Load		Stiffness	Page
Compression		$K_v = \frac{1}{F_1 + F_2}$ see below	14
	No-bulge flexibility†	$F_1 = F_s + F_r$, where	12
		$F_s = \frac{n_s t_s}{E_s A} \left(1 + \frac{2v_s t_r}{t_s} \right)$	11
		$F_r = \frac{n_r t_r}{E_B A} \left\{ 1 + \frac{2E_B}{E_s} \left[(1-v_s) \frac{t_r}{t_s} + v_s \right] \right\}$	12
	Bulge Flexibility	$F_2 = \frac{n_r t_r}{E_r A} \left(\frac{1}{1 + 2 k S^2} \right)$	13
Shear		$K_s = \frac{AG}{n_r t_r}$	15
Bending		$K_\theta = K_v \left(\frac{I}{A} \right)$	15
Torsion		$K_T = K_s \left(\frac{J}{A} \right)$	17

* Obtained from Ref. 3

† F_s and F_r are simplified by assuming $E_s \gg E_r$ and $v_r = 0.5$. For more accurate values of F_1 , use Eqs. 21 and 24 on page 11 of Ref. 3.

The total six degree of freedom flexibility for the bearing is given below. The matrix is arranged according to the displacement components given in the parentheses beneath the matrix. The displacement sign convention for a typical beam-like structural element (see Ref. 2) is shown to the right.

$$[A] = \begin{bmatrix} A_s & 0 & 0 & 0 & 0 & 0 \\ 0 & A_v & 0 & 0 & 0 & 0 \\ 0 & 0 & A_s & 0 & 0 & 0 \\ 0 & 0 & 0 & A_\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & A_T & 0 \\ 0 & 0 & 0 & 0 & 0 & A_\theta \end{bmatrix}$$

(x y z θ_x θ_y θ_z)

From the geometrical information shown on page 6 we have

$$n_r = 35$$

(see p. 25 for list of symbols)

$$n_s = 34$$

$$t_r = 3/8 \text{ in.}$$

$$t_s = .10 \text{ in.}$$

$$A = \frac{\pi(13)^2}{4} (144) = 19150 \text{ in}^2$$

Assuming rubber with a shore hardness of 70, we obtain the following material properties from Table 1 of Ref. 4.

$$G = 245 \text{ psi}$$

$$E_r = 1040 \text{ psi}$$

$$E_B = 180,000 \text{ psi}$$

$$k = .53$$

In addition, we assume for the steel plates

$$E_s = 30 \times 10^6, \quad \nu_s = .30$$

and for the rubber

$$\nu_r = .5$$

Using the formulas given in Table 1 (p. 9), the following numerical values were obtained for the flexibility of a single bearing:

$$A_v = 4.016 \times 10^{-9} \text{ in/lb}$$

$$A_s = 2.79 \times 10^{-6} \text{ in/lb}$$

$$A_\theta = 2.565 \times 10^{-12} \text{ radian/in.-lb}$$

$$A_T = .918 \times 10^{-9} \text{ radian/in.-lb}$$

These values, along with the column section properties shown on page 8, are used as input for the STATIC code. A six degree of freedom flexibility matrix for the support structure is calculated for two cases: (1) columns rigid and bearings flexible, and (2) both columns and bearings flexible. The flexibility is given as a 6x6 matrix of influence coefficients for space point number 1 of the STATIC model (see p. 7). The 6x6 matrix is then reduced to a 3x3 (to describe motion in the x-y plane alone) by deleting the rows and columns for all out-of-plane components.

3.4 Ground Flexibility

The flexibility of the ground is calculated by formulas given by Barkan (Ref. 5). These formulas are based on the assumption that the foundation can be treated as a rigid plate on a homogeneous, isotropic, elastic half-space. An inspection of Fig. 1 (p. 4) will show that this assumption is clearly not true for our foundation. Exact determination of the flexibility of our foundation by analytical means is beyond the present state of the art. Furthermore, material properties of the soil

and bedrock are completely unknown. Any analytical flexibility value, regardless of the method used, is therefore little more than an educated guess. The Barkan formulas are used here because they are simple and representative of average conditions. The material properties assumed are those of a soft rock. The resulting flexibility values are therefore an order-of-magnitude estimate. The purpose of using these values in the dynamic analysis is to determine whether or not the ground flexibility might have an important influence on the dynamic response. The results are not intended to be an accurate analysis of the ground flexibility effect.

A brief summary of the soil stiffness formulas is given below.

Normal Force

$$K_v = \frac{P}{\delta_p} = \frac{1.13 E \sqrt{A}}{1-\nu^2} \left(\frac{lb}{in.} \right)$$

Shear

$$K_s = \frac{S}{\delta_s} = \frac{k_\tau E \sqrt{A}}{1-\nu^2} \left(\frac{lb}{in.} \right)$$

Bending

$$K_\theta = \frac{M}{\theta_m} = \frac{k_\phi}{(1-\nu^2) \sqrt{A}} \left(\frac{in.-lb}{radian} \right)$$

where \$k_\tau\$ and \$k_\phi\$ are factors given in Ref. 5 which depend on the shape of the plate.

Assuming a circular plate of radius r on a soil with $\nu = .3$, the above formulas reduce to

$$K_v = 2.20 Er$$

$$K_s = 1.69 Er$$

$$K_\theta = .968 Er^3$$

Assuming a plate with a diameter of 100 ft. ($r = 600$ in.) and a soil with $E = 3.0 \times 10^5$ (a soft rock), we obtain the following stiffness matrix.

$$K = \begin{bmatrix} K_s & 0 & 0 \\ 0 & K_v & 0 \\ 0 & 0 & K_\theta \end{bmatrix} = \begin{bmatrix} 3.04 \times 10^8 & 0 & 0 \\ 0 & 3.96 \times 10^8 & 0 \\ 0 & 0 & 6.28 \times 10^{13} \end{bmatrix}$$

The flexibility is simply the inverse of the stiffness, i.e.

$$A = \begin{bmatrix} A_s & 0 & 0 \\ 0 & A_v & 0 \\ 0 & 0 & A_\theta \end{bmatrix} = \begin{bmatrix} .338 \times 10^{-8} & 0 & 0 \\ 0 & .252 \times 10^{-8} & 0 \\ 0 & 0 & .159 \times 10^{-13} \end{bmatrix}$$

4. STATIC ANALYSIS

Two cases are analyzed by the STATIC code in order to calculate composite structural flexibilities. The stiffness are calculated for six degrees of freedom. Three of these degrees of freedom are unnecessary because of symmetry, however, and they are eliminated before using these flexibilities in the SHAKE analysis. The two cases both make use of the same model (see p. 7), except that in case A the columns are assumed to be rigid. The results of the STATIC code analysis for these two cases are given below.

Case A - Bearings Flexible and Columns Rigid

The total six degree of freedom flexibility, which consists of influence coefficients for space point 1 of the model, is shown below

$$A = \begin{bmatrix} 4.65 \times 10^{-7} & 0 & 0 & 0 & 0 & 1.03 \times 10^{-13} \\ 0 & 6.69 \times 10^{-10} & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.65 \times 10^{-7} & -1.03 \times 10^{-13} & 0 & 0 \\ 0 & 0 & -1.03 \times 10^{-13} & 6.03 \times 10^{-13} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.09 \times 10^{-12} & 0 \\ 1.03 \times 10^{-13} & 0 & 0 & 0 & 0 & 6.03 \times 10^{-13} \end{bmatrix}$$

(x y z θ_x θ_y θ_z) *

Considering motions in the x-y plane only, this matrix is reduced to a (3x3) by eliminating rows and columns for all out-of-plane motions.

$$A = \begin{bmatrix} 4.65 \times 10^{-7} & 0 & 1.03 \times 10^{-13} \\ 0 & 6.69 \times 10^{-10} & 0 \\ 1.03 \times 10^{-13} & 0 & 6.03 \times 10^{-15} \end{bmatrix}$$

(x y θ_z) *

Case B - Both Bearings and Columns Flexible

The total six degree of freedom flexibility is

$$A = \begin{bmatrix} 4.71 \times 10^{-7} & 0 & 0 & 0 & 0 & 5.89 \times 10^{-13} \\ 0 & 1.09 \times 10^{-9} & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.71 \times 10^{-7} & -5.89 \times 10^{-13} & 0 & 0 \\ 0 & 0 & -5.89 \times 10^{-13} & 9.82 \times 10^{-15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.12 \times 10^{-12} & 0 \\ 5.89 \times 10^{-13} & 0 & 0 & 0 & 0 & 9.82 \times 10^{-15} \end{bmatrix}$$

(x y z θ_x θ_y θ_z) *

* These symbols list the order of the components (see page 10).

The corresponding three degree of freedom flexibility is

$$A = \begin{bmatrix} 4.71 \times 10^{-7} & 0 & 5.89 \times 10^{-13} \\ 0 & 1.09 \times 10^{-9} & 0 \\ 5.89 \times 10^{-13} & 0 & 9.82 \times 10^{-15} \end{bmatrix}$$

(x y θ_z)

5. PCRV MASS

The total weight of the PCRV and its internals is assumed to be 85×10^6 lb. It is assumed that the mass is uniformly distributed throughout the PCRV volume and that the center of gravity is at the geometric center. The dimensions of the PCRV are as shown below.

The volume of the PCRV is

$$V = \frac{\pi D^2 h}{4} = \frac{\pi (92.75)^2 (84.667)}{4}$$

$$= 572,000 \text{ ft}^3$$

The effective uniform density is

$$V_{\text{eff}} = \frac{W}{V} = \frac{85 \times 10^6 \text{ lb}}{57.2 \times 10^4 \text{ ft}^3} = 148.4 \frac{\text{lb}}{\text{ft}^3}$$

The mass moment of inertia about the center of gravity is

$$\begin{aligned} I_{cg} &= \frac{W}{12g} (h^2 + 3r^2) \\ &= \frac{82 \times 10^6}{12(32.2)} \left[84.667^2 + \frac{3(92.75)^2}{4} \right] \\ &= 2.99 \times 10^9 \text{ lb sec}^2 \text{ ft} \end{aligned}$$

or

$$I_{cg} = 2.99 \times 10^9 (12) = 3.59 \times 10^{10} \text{ lb sec}^2 \text{ in}$$

The above weight plus the mass moment of inertia are both used as SHAKE code input.

6. DYNAMIC ANALYSIS

The dynamic response is determined for four cases, each using different support flexibility assumptions. The first of these is a simplified single degree of freedom model, and the calculations are performed by hand using the response spectrum directly. The others, each having an increasingly flexible and more accurate model, are all three degree of freedom systems, and the SHAKE code is employed for their analysis. The characteristics of the four cases are summarized below.

Case 1 - Columns and ground assumed rigid. Bearings flexible in horizontal direction only.

Case 2 - Same as Case 1 except complete bearing flexibility used.

Case 3 - Same as Case 2 except column flexibility is added.

Case 4 - Same as Case 3 except ground flexibility is included.

The model analyzed by SHAKE is shown below

In Case 2, member 0-3 is assumed rigid and member 2-3 is given the flexibility of Case A on page 14. In Case 3, member 0-3 is still rigid, but member 2-3 is given the flexibility of Case B on page 15. In Case 4, member 2-3 is unchanged but member 0-3 is given the ground flexibility shown on page 13.

The ground excitation used in the analysis is the El Centro standard earthquake with a .33g ground acceleration input. Vertical and horizontal excitations are treated in separate runs so that their effects can be observed separately and later superimposed. The actual design earthquake for the PCR in a zone 2 application has a .16g horizontal and a .11g vertical ground acceleration. Since the dynamic response is proportional to the earthquake ground acceleration, this condition is analyzed by multiplying the horizontal and vertical SHAKE results by appropriate scaling factors and superimposing them.

Tabular summaries of the analysis results are given in Tables 2 through 4.

TABLE 2
FREQUENCIES AND MODE SHAPES

Case	Mode	Frequency f (cps)	Period T (sec)	Eigenvector		
				x (in)	y (in)	θ (radian)
1	1	.498	2.010	1.000	.000	.000
2	1	.496	2.014	1.000	.000	small
	2	10.84	.092	1.000	.000	-.903
	3	13.11	.076	.000	1.000	.000
3	1	.493	2.029	1.000	.000	small
	2	8.50	.118	1.000	.000	-.519
	3	10.26	.097	.000	1.000	.000
4	1	.484	2.065	1.000	.000	small
	2	5.32	.187	1.000	.000	-.149
	3	5.64	.177	.000	1.000	.000

TABLE 3
MAXIMUM DYNAMIC LOADS* AT BOTTOM OF PCRV[†]

Case	Axial Load (lb)	Shear (lb)	Bending Moment (in.-lb)
1	$.915 \times 10^7$	$.860 \times 10^7$	4.37×10^9
2	1.19×10^7	$.859 \times 10^7$	4.37×10^9
3	1.31×10^7	$.853 \times 10^7$	4.35×10^9
4	1.91×10^7	$.840 \times 10^7$	4.32×10^9

* Root sum square of all contributing modes. Assumes .16g horizontal and .11g vertical ground acceleration input.

[†] The bottom of the PCRV is space point 2 in the SHAKE model (see p. 17).

TABLE 4
MAXIMUM DYNAMIC DEFLECTION AT PCRV C.G.

Case	x (in)	y (in)	θ (radian)
1	4.00	small	0.00
2	4.01	.00797	2.72×10^{-5}
3	4.04	.0143	4.77×10^{-5}
4	4.12	.069	1.70×10^{-4}

- 1) Root sum square of all contributing modes. Assumes
.16g horizontal and .11g vertical ground acceleration input.

The results of this dynamic analysis (see Table 2) indicate that the increasingly flexible support flexibility assumptions have very little effect on the horizontal oscillations but may have a significant effect on the vertical response. This is because the bearing is already so flexible horizontally that the increased flexibility due to the ground, etc., is insignificant in comparison. Since the bearing alone is quite stiff in the vertical direction, however, the vertical response is very sensitive to changes in the vertical foundation flexibility. One must be careful, therefore, that in an effort to design a horizontally flexible support, one does not indiscriminately add flexibility in such a way that the vertical response criteria are jeopardized. It is seen that, in this case, the best results (i.e. the lowest vertical response) are obtained if the elastomeric bearings are mounted on a foundation that is as stiff as possible.

7. STRESS ANALYSIS

The dynamic loads calculated and presented in Section 6 have been applied to the support structure to determine whether the design is structurally adequate. These results are given briefly on page 20.

Bearing Array Section Properties

$$r = 6\frac{1}{2} \text{ ft} = 78 \text{ in}$$

$$R = 39 \text{ ft} = 468 \text{ in}$$

Individual Bearing:

$$A_o = \pi r^2 = \pi (78)^2 = 1.915 \times 10^4 \text{ in}^2$$

$$I_o = \frac{\pi r^4}{4} = \frac{\pi (78)^4}{4} = 2.915 \times 10^7 \text{ in}^4$$

Bearing Array:

$$A = 6A_o = 6(1.915 \times 10^4) = 1.150 \times 10^5 \text{ in}^2$$

$$\begin{aligned} I &= A_o R^2 \sum \cos^2 \theta + 6I_o \\ &= (1.915 \times 10^4)(468)^2 + 6(2.915 \times 10^7) \\ &= 1.274 \times 10^{10} \text{ in}^4 \end{aligned}$$

Distance to extreme fiber,

$$c = R+r = 468+78 = 546 \text{ in.}$$

Max Compressive Stress (Design Earthquake)

$$\sigma_c = \frac{P_{\max}}{A} + \frac{M_c}{I}$$

$$P_{\max} = 85 \times 10^6 + 19.1 \times 10^6 = 104.1 \times 10^6 \text{ lb}$$

$$M = 4.32 \times 10^9 \text{ in-lb} \quad (\text{Ref. p. 18})$$

$$\therefore \sigma_c = \frac{1.041 \times 10^8}{1.15 \times 10^5} + \frac{4.32 \times 10^9 (.546 \times 10^3)}{1.274 \times 10^{10}}$$

$$= 905 + 184 = 1089 \text{ psi}$$

Max Tensile Stress (Design Earthquake)

$$\sigma_T = \frac{-P_{\min}}{A} + \frac{M_c}{I}$$

$$P_{\min} = -85 \times 10^6 + 19.1 \times 10^6 = -65.9 \times 10^6 \text{ lb}$$

$$M = 4.32 \times 10^9 \text{ in-lb} \quad (\text{Ref. p. 18})$$

$$\sigma_T = \frac{-65.9 \times 10^6}{1.15 \times 10^5} + \frac{4.32 \times 10^9 (.546 \times 10^3)}{1.274 \times 10^{10}}$$

$$= -573 + 184 = -389 \text{ psi} \quad (\text{compression})$$

There is no tensile stress for the design earthquake.

Vertical Strain in Rubber

Assuming maximum compressive stress exists over the entire bearing area, the effective pressure load is

$$p = 1089 \text{ psi}$$

The vertical strain in the rubber is

$$\begin{aligned}\epsilon_y^r &= \frac{pA_o}{K_v n_r t_r} = \frac{1089(1.915 \times 10^4)}{.249 \times 10^9 (13.12)} \\ &= .0064\end{aligned}$$

Since the allowable vertical strain, according to Ref. 4, is 15%, it is clear that there is no problem here. The steel offers almost complete restraint.

Shear Strain in Rubber

From Table 4, p. 19, the maximum horizontal deformation of the bearing is

$$\delta = 4.12 \text{ inches}$$

The shear strain is

$$\gamma = \frac{\delta}{n_r t_r} = \frac{4.12}{13.12} = .313$$

This compares favorably with an allowable of 50% (Ref. 4).

Steel Tensile Stress

$$\sigma_s \approx p \left(\frac{t_r}{t_s} \right) = 1089 \left(\frac{3/8}{.1} \right) = 4080 \text{ psi}$$

Steel stress is no problem either.

Ultimate Earthquake

In addition to operating safely at the "operating earthquake," which is an input acceleration of

.16g horizontal and
.11g vertical,

the structure must also be able to withstand an "ultimate earthquake," which is an input acceleration of

.25g horizontal and
.17g vertical

without a catastrophic collapse. The only two possible critical areas are tension and/or shear strain in the rubber.

Using the same approach as above, the minimum axial load is

$$\begin{aligned} P_{\min} &= 85 \times 10^6 - 19.1 \times 10^6 \left(\frac{.17}{.11} \right) \\ &= 85 \times 10^6 - 29.6 \times 10^6 = 55.4 \times 10^6 \text{ lb} \end{aligned}$$

The new bending moment is

$$M = 4.32 \times 10^9 \left(\frac{.25}{.16} \right) = 6.75 \times 10^9.$$

The maximum tensile stress is then

$$\begin{aligned} \sigma_T &= \frac{-55.4 \times 10^6}{1.15 \times 10^5} + \frac{6.75 \times 10^9 (.546 \times 10^3)}{1.274 \times 10^{10}} \\ &= -482 + 289 = -193 \text{ psi (compression)}. \end{aligned}$$

There is no tensile stress for the ultimate earthquake.

The maximum horizontal deformation is

$$\delta = 4.12 \left(\frac{.25}{.16} \right) = 6.42 \text{ in.},$$

and the maximum shear strain is

$$\gamma = \frac{6.42}{13.12} = .490$$

This is still less than the operating allowable, so there is clearly no problem. Tests show ultimate shear strain capabilities of well over 100% (Ref. 6).

8. CONCLUSIONS

The results given in this report demonstrate three significant facts:

1. The original assumptions used to design the elastomeric bearing are quite good, i.e., most of the flexibility in the horizontal direction is due to shear in the bearings. Accurate assessment of the vertical response, however, can be made only after refining the model to include the effects of vertical bearing flexibility, column flexibility, and ground flexibility.
2. Vertical response is kept to a minimum if the elastomeric bearings are mounted on a foundation which is as stiff as possible.
3. The preliminary bearing design analyzed in this report is proven to be structurally sound.

The elastomeric bearing concept is clearly a feasible support concept. It can be readily designed, using simplifying assumptions, to satisfy a great variety of design conditions.

The maximum horizontal deformation is

$$\delta = 4.12 \left(\frac{.25}{.16} \right) = 6.42 \text{ in.},$$

and the maximum shear strain is

$$\gamma = \frac{6.42}{13.12} = .490$$

This is still less than the operating allowable, so there is clearly no problem. Tests show ultimate shear strain capabilities of well over 100% (Ref. 6).

8. CONCLUSIONS

The results given in this report demonstrate three significant facts:

1. The original assumptions used to design the elastomeric bearing are quite good, i.e., most of the flexibility in the horizontal direction is due to shear in the bearings. Accurate assessment of the vertical response, however, can be made only after refining the model to include the effects of vertical bearing flexibility, column flexibility, and ground flexibility.
2. Vertical response is kept to a minimum if the elastomeric bearings are mounted on a foundation which is as stiff as possible.
3. The preliminary bearing design analyzed in this report is proven to be structurally sound.

The elastomeric bearing concept is clearly a feasible support concept. It can be readily designed, using simplifying assumptions, to satisfy a great variety of design conditions.

9. LIST OF SYMBOLS

A	Flexibility or area
c	Distance to extreme fiber
D	Diameter
E	Young's modulus
E_B	Bulk modulus of rubber
G	Shear modulus
h	Height
I	Moment of inertia
J	Polar moment of inertia
k	Constant
K	Stiffness
M	Bending moment
n	Number of layers
p	Pressure
P	Axial compression load
r,R	Radius
S	Shear force or shape factor
t	Thickness
T	Torque
V	Volume
W	Weight of PCRV
x,y,z	Coordinate directions
σ	Normal stress
τ	Shear stress
ϵ	Normal strain
γ	Shear strain
ν	Poisson's ratio
θ	Angle (bending rotation)

Subscripts

s	Steel or Shear
r	Rubber
t	Torque or Tension
c	Compression
v	Vertical
θ	Bending

10. REFERENCES

1. Esztergar, E. P., and P. D. Smith, "Seismic Study of Support Concepts for Prestressed Concrete Reactor Vessels," USAEC Report GAMD-9282, Gulf General Atomic Incorporated, May 1969.
2. McLennan, G. A., W. P. Henschel, and E. P. Esztergar, Gulf General Atomic Incorporated unpublished data.
3. Smith, P. D., "Stiffness of Laminated Elastomeric Bearings," USAEC Report GAMD-9375, Gulf General Atomic Incorporated, May 14, 1969.
4. Lindley, P. B., "Design of Natural Rubber Bridge Bearings," N.R.P.R.A. Technical Bulletin No. 7, Natural Rubber Bureau, London, September 1962.
5. Barkan, D., Dynamics of Bases and Foundations, translated from the Russian by L. Drashevskaya, McGraw-Hill, New York, 1962.
6. "Design of Neoprene Bridge Bearing Pads," Elastomer Chemicals Department, E. I. du Pont de Nemours and Co., Inc., April 1959.