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MASTER

The Design of a Medium-Energy Pion Beam (4-16 Gev/c) for the
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I. Introduction

We describe here a medium-energy (4-16 GeV) pion beam transport system for the AGS. The design of the beam was determined by the requirements of a π -p large-angle elastic scattering experiment recently carried out by the BNL-Cornell collaboration group.¹ At the liquid H₂ target in which the π -p scattering took place, it was desired to obtain positive and negative pion beams of as high an intensity as possible, with the conditions that the beam must be small in size ($\sim 1''$ in diameter) and must have small angular divergence ($\sim \pm 2.5$ mr in both horizontal and vertical planes). A momentum spread of up to $\pm 2\%$ would be acceptable.

The pion beam is produced, at an internal Be wire target in the G-10 straight section, at an angle of 4.5° to the circulating proton beam. The choice of this target location and beam production angle is discussed in Section II. A discussion of the optics and detailed layout of the beam transport system is given in Section III. In Section IV we describe the computational methods used for optimizing the beam parameters and for estimating the intensity, size and divergence of beams of various momenta; results of these computations are presented in tabular and graphical form. We describe in Section V the procedures for optimizing the beam, and some of the results obtained in its use.

II. Beam Origin and Production Angle

Two locations for the AGS internal target were considered--in the G-9 and G-10 straight sections. See Figure I. Note that a straight section has the same name as the preceding (to the left, in Fig. I) magnet unit. Negative beams with 0° production could be obtained from a G-9 target. However, due to the deflection of the secondary particles in the field of the G-10 magnet, it is difficult to use a G-9 target for positive beams; also, because of the focusing effects of the G-10 magnet field, orbits of negative beams $\lesssim 12$ GeV/c are distorted such that the object for a subsequent beam transport system

appears to be large and thus gives undesirable properties to the final image. Since we required both positive and negative beams, mostly below 12 GeV/c, and also wished to enable other experimenters to work simultaneously from the same target, a G-10 target position was most suitable. A particle production angle of 4.5° was chosen as this enabled the particles to emerge through a thin window in the G-10 straight section vacuum box, and consequently minimized multiple scattering. For this production angle, the particle trajectories are not distorted too greatly by the fringe field of the G-11 magnet.

The target is a 30-mil diameter, L-shaped Be wire. Most of the p-Be interactions take place in the horizontal arm of the "L", which is 1/2" in length and is pointed along the axis of the secondary beam in order to minimize the apparent size of the source viewed by the beam transport system. This type of target has been found to be satisfactory in many experiments at the AGS.

III. Layout of the Beam Transport System

A. Beam optics

A schematic drawing of the beam optics is shown in Figure II. The lens L_1 focuses the beam from the AGS target O at a slit S ("intermediate focus"). The beam is dispersed by the deflection magnet D_1 ; the band of desired momenta passes through the slit S. The lens L_2 focuses the beam, emerging through S, at the image point I, where the liquid H_2 target for the BNL-Cornell π -p scattering experiment was placed. The momentum band accepted by the slit S is recombined by the deflection magnet D_2 into an achromatic image at I.

The lengths a, b, c, and d are determined by the following considerations:

(i) a should be small, in order to maximize the solid angle of emission from the target O, which is accepted by the aperture of lens L_1 .

(ii) b should be small, so that the "intermediate foci", for different particle momenta, are not too widely spaced along the

beam line (chromatic aberration). A wide spread in foci would produce a poorly defined "object" for the second half of the transport system.

(iii) c should be small, so that all of the beam (which, since b is small, is diverging rapidly in the region $S-L_2$) can pass through the aperture of lens L_2 .

(iv) d should be large, to give the desired small angular divergence at I .

If θ^0 is the half-angle of beam acceptance at O in either the horizontal or vertical planes, then the beam divergence (in the same plane) at I is

$$\theta^1 \sim a/b \times c/d \times \theta^0.$$

The beam transport system is designed for an angular magnification $\sim x 1/3$; there is therefore a corresponding linear magnification $\sim x 3$.

B. Layout of beam components

The layout of the beam is shown in detail in Fig. 1. For the quadrupole and deflection magnets, we follow the nomenclature used by Danby.² All of the magnets (except for Q_1 --see the following paragraph) are standard items of BNL-AGS beam transport equipment. We shall discuss now some significant features of this layout.

(i) The distance a (desired to be small) is determined by the physical size of the first quadrupole magnet, Q_1 , which is placed as far upstream as spatial limitations allow. Q_1 is a Berkeley 8Q32 (Mark I)--of 24" overall horizontal width.

(ii) The distance b (also desired to be small) is determined by the finite length and maximum focusing power, at 16 GeV/c, of the quadrupole lens system Q_{1-6} .

(iii) The six quadrupole magnets Q_{1-6} (lens L_1) are run in effect as a triplet lens. The order of focusing is $V_1 V_2 H_3 H_4 V_5 V_6$ (H =horizontal, V =vertical).

(iv) Since the beam passes through a focus between D_1 and

D_2 , these two magnets must deflect the beam in the same sense in order to obtain an achromatic image at I. The values chosen for the deflection angles at D_1 and D_2 were determined with the help of a beam-optimization computer program (see Section IV).

(v) The "intermediate focus" is at the center of collimator C. The size of the aperture of this collimator determines the magnitude of the accepted momentum bite.

(vi) D_3 can be used to steer the beam to different final image locations. In the BNL-Cornell experiment its field was small (angle of deflection 0.69°) and it was not a significant factor in the optics of the transport system.

(vii) The amount of multiple Coulomb scattering of the beam particles, along the length of the transport system, should be minimal because of the stringent requirements on beam spot size and angular divergence. In order to minimize this scattering, the beam passes, wherever practicable, through evacuated aluminum pipes of $7\frac{3}{4}$ " I.D., with 10 mil mylar end windows.

(viii) The main AGS shielding wall lies between D_1 and collimator C; it is made as thick as practicable ($\sim 20'$) to stop unwanted particles. The shielding immediately following collimator C stops most of the "spray" produced at the walls of this collimator.

(ix) An iron absorber (~ 3 -6 ft. long) is placed ~ 50 ft. downstream from the H_2 target; pions are stopped by this absorber, while muons produced by pion decay in flight pass through it. By counting the muons, one can determine the muon contamination of a given pion beam. The fraction of pions which decay in flight over the whole beam path ranges from $\sim 8\%$ at 16 GeV/c to $\sim 30\%$ at 4 GeV/c; many of the decay muons are removed from the beam by the deflection magnets.

(x) In the BNL-Cornell experiment, a threshold gas Cerenkov counter was used to discriminate against $K^\pm$ and $p^\pm$ in the $\pi^\pm$ beams.

IV. Calculational Methods

A. Some useful computer programs

The beam passes through the fringe field of the G-11 magnet. The trajectories of particles passing through this field were calculated with the help of the IBM 7094 programs, BEAM and BEAMFINDER.³ Two 7094 programs were used for the optimization of beam parameters:

(i) AUTOBEAM.⁴ Given a set of initial conditions (so-called "starting values") for the magnetic fields, collimator apertures, distances between components, etc., which are fairly close to the optimum design conditions, this program will maximize a "merit function" F , a figure-of-merit for the performance of the system. F can be any desired function of the beam parameters. Each of n beam parameters, $V_1, V_2, \dots, V_n$, is varied, in turn, until a relative maximum for F , in V -space, is found. The size, divergence, solid angle of acceptance and dispersive properties of the beam are then calculated by the program. The principal difficulty in using the program is that, for $n > 2$, there may be many relative maxima for F in V -space; the program does not necessarily find the desired (i.e., the largest) relative maximum for F .

(ii) BEAM II.⁵ This program has been used to solve two distinct types of problems. Firstly, for a magnetic lens system in which only two parameters are allowed to vary, these parameters are varied in order to satisfy both of two "focus" conditions. For either "focus" requirement, an incident particle of any given momentum, position and direction, is specified. A "focus" condition is a requirement that the displacement from the beam axis or the direction of the particle, at a given distance along the beam line from the source, shall lie within some desired specified limits. The focusing routine was used for calculating the optimal magnetic fields for some of the quadrupole lens elements. Secondly, another routine of BEAM II was used to trace the paths of various rays, of given initial momentum, position and direction, through the beam transport system. This ray-trace routine was used to complement the evaluation made by AUTOBEAM of the properties of the beam.

B. The radial position of the Be target

The trajectory of a particle passing through the fringe field of the G-11 magnet is a function of $p_{12} = p_1/p_2$, where p_1 is the momentum of the circulating proton beam and p_2 is the momentum of the secondary particle, with the same sign as the charge of the particle. It would be necessary to adjust the radial position of the target, R_T , for different values of p_{12} , in order that each of the corresponding secondary beams should have its virtual source on the axis of lens L_1 . The relation between R_T and p_{12} necessary for this has been calculated with the help of the BEAM and BEAMFINDER programs; a plot of R_T versus p_{12} is shown in Fig. III. $R_T = 0$ corresponds to the mean equilibrium orbit of the synchrotron, as defined by Courant⁶ (Section (1)). Note that the target position is positive (i.e., outwards from the AGS equilibrium orbit) for π^- beams and negative for π^+ beams.

C. Optimization of the beam, up to collimator C

The beam optimization calculations were done in two stages--firstly, up to and including collimator C, and secondly, the remainder of the system.

The first quantities to be calculated were the magnetic fields in Q_{1-6} which produce a focus at the center of collimator C. The beam properties at the exit of the fringe field of the AGS, calculated by the BEAM program, were used as input data for this focusing problem. No program apparently exists which can optimize six quadrupole magnet strengths simultaneously in a reasonable amount of computer time, if good "guess" values for the field strengths are not available. The most reasonable procedure for solving this problem was found to be to guess some "reasonable" starting values for the AUTOBEAM program. For our first guess, we scaled the fields in a similar six-element lens by Crittenden,⁷ using thin lens approximations and taking into account the different momenta and the different distances a and b in the two beams. These starting-values were fed into the focusing routine of BEAM II, allowing two of the six fields to vary until the desired focus conditions at collimator C were

satisfied. The output of BEAM II was then fed into AUTOBEAM, which adjusted the fields by small amounts to give the maximum beam flux through the aperture of the collimator. It was found that satisfactory solutions could be obtained with $B(Q_3) = B(Q_4)$, and $B(Q_5) = B(Q_6)$ (where $B(X)$ is the field strength in element X); these two conditions were retained in all subsequent calculations.

AUTOBEAM was next used to determine the magnitude of the angle of deflection in M_1 . For this computation, the following merit function F was maximized:

$$F = \sum_{p=p_0}^{p=p_0 + \Delta p_1} (\Delta\Omega)_p - 5 \sum_{p=p_0 + \Delta p_2}^{p=p_0} (\Delta\Omega)_p$$

$(\Delta\Omega)_p$ is the solid angle subtended at the Be target by the accepted beam particles of momentum p , p_0 is the nominal beam momentum, $(p_0 + \Delta p_1)$ and $(p_0 - \Delta p_1)$ are momenta which it is desired to accept in the beam, and $(p_0 + \Delta p_2)$ and $(p_0 - \Delta p_2)$ are momenta which it is desired to reject. In our case, $\Delta p_1/p_0 = 1\%$, and $\Delta p_2/p_0 = 2\%$. The results of these calculations indicated that an accepted momentum bite of $\pm 1\%$, a collimator C aperture of $1/2" \times 1/2"$, and a deflection of 2.9° in M_1 , gave a satisfactory solution. A larger momentum bite was found to produce too much chromatic aberration in the second half of the system, giving too large a spot size and angular divergence at the H_2 target.

An experiment which imposes less stringent requirements on beam size and divergence at I, could perhaps accept a larger momentum bite than the $\pm 1\%$ momentum acceptance of the beam used by the BNL-Cornell group.

D. Optimization of the second half of the beam transport system

The optimum values for the distances c and d were calculated according to criteria discussed previously, thus determining the approximate locations of the lens Q_{7-9} and the H_2 target. The separation distances Q_7-Q_8 and Q_8-Q_9 were arbitrarily set equal to the magnetic length (51") of an 8Q48. The next calculation was to determine

the magnetic fields in Q_7 , Q_8 and Q_9 , which give foci in both the horizontal and vertical planes at the H_2 target, and which give angular magnifications in the two planes such that the beam divergence was ± 2.5 mr. in the vertical plane and ± 3 mr. in the horizontal plane. Various solutions to the focusing requirements were found by setting the Q_9 field equal to each of several arbitrary fixed values and using the BEAM II in the "focusing" mode to determine the corresponding Q_7 and Q_8 fields. One of these solutions was found to give image properties which satisfied the design requirements.

The value of the field in D_3 was arbitrarily fixed at 2.5 Kgauss (for $p_0 = 12$ GeV/c), giving an angle of deflection in D_3 of 0.69° . The angle of deflection in D_2 (2.0°) was now determined by using AUTOBEAM, maximizing the same merit function F which was used to determine the angle of deflection in D_1 . By calculating the angle of deflection in D_2 in this way, the desired recombination of momenta, at the H_2 target, was obtained.

E. Focus of the beam at different image points I

The H_2 target was located at different points along the beam line, at different stages of the BNL-Cornell experiment. The solutions for the fields in Q_{7-9} having been calculated for one position of I, the value for the Q_9 field for a second position was scaled according to the thin lens formula. The fields in Q_7 and Q_8 were then calculated, as before, by using the BEAM II focusing routine. The beam was then optimized and its properties evaluated by AUTOBEAM.

F. Aperture stops of the system

The BEAM II program, used in the "ray-trace" mode, was used to determine what are the aperture stops of the system. Trajectories through the system were calculated for particles with several different values of momentum and angle from the AGS target. A typical

set of such ray-traces is shown in Figs. IV and V. The vertical stops are the apertures of D_1 and collimator C, the horizontal stops are Q_3 , Q_4 and collimator C. The fact that collimator C was the final aperture stop helped to minimize the number of particles arriving at the H_2 target which had been scattered from the wall of beam transport elements.

V. Results of Calculations

The relation between R_T and p_{12} , discussed earlier, is plotted in Fig. 3. The optimum Q_1 field is a function of p_{12} and of the beam sign and momentum, whereas the Q_{2-9} and D_{1-3} fields are dependent only upon the beam sign and momentum. Figure VI shows the dependence of Q_1 field upon p_{12} .

Table I gives a list of the beam elements, the separation distances between their centers, and the values of other relevant parameters (fields, apertures, etc.) for beams of various momenta. The values for Q_1 field, which are given in the table, correspond to $p_{12} = 0$. The actual value used for Q_1 field, in a given beam, depends of course upon p_{12} , and is determined by reference to the curve of Fig. VI.

Table II gives a list of calculated values of the currents to be used in the magnetic elements. The magnet calibration data of Danby² were used in calculating the data given in this table. A typical momentum acceptance spectrum, spatial distribution and angular distribution, of a beam, as calculated by AUTOBEAM, are shown in Fig. VII. Some estimated beam intensities, calculated using the AGS beam-survey data of Baker, et al.⁸, are shown in Table III.

VI. Testing and Performance

The following procedure was found to be the most convenient for setting the magnet element currents: 3 scintillation counters were used which we denote by S_1 , S_2 and S_3 . S_1 ($\sim 3'' \times 3''$) was placed a few yards upstream of the image point I (Fig. 1); S_2 ($\sim 3'' \times 3''$) and S_3 (of the desired final spot size $1''$ hor. $\times 1\frac{1}{2}''$ vert.) were placed

at I. For monitoring the AGS beam striking the Be target, we used a scintillator telescope $M = M_1 M_2 M_3$ (see Fig. 1).

All magnet currents were first set at their calculated values. In the adjustment procedure, D_2 was kept fixed and used as the definer of momentum; Q_2 was also generally kept fixed in order to reduce the number of variables, since any variation needed could be achieved by a change of Q_1 current.

The ratio (coincidence $S_1 S_2$ /Monitor M) was maximized for each of the following in turn: D_1 , D_3 , Q_1 , Q_{34} , Q_{56} (this ensured that the beam travelled the correct path through the magnets, and that the maximum intensity through collimator C was obtained.) Next, the ratio $S_1 S_3/M$ was maximized in sequence for the remaining magnets Q_7 , Q_8 , Q_9 ; this focused the beam to the required final spot size. Sometimes a final tuning of D_1 and D_3 , maximizing $S_1 S_3/M$, obtained a slight increase in intensity.

The experimental values for the magnet fields were always within a few percent of those calculated. There was some indication of a slight misalignment in quadrupoles Q_1 to Q_6 , necessitating small changes in some fields. In Table IV are given a set of experimental currents, together with the calculated values from Table II, for $p = -8$ GeV/c, $p_{12} = -0.35$, $R_T = +0.6$, $\ell = 951''$.

During almost all of the BNL-Cornell experiment, R_T remained at $+0.6''$, since this enabled other experimenters to use the G10 target with least inconvenience, and it was found that this did not affect the properties of this beam significantly. The main effect, as expected, was that the optimum values for the fields of Q_1 and D_1 were changed.

Two profiles of the beam are shown in Fig. 8. These were measured using a $1/2'' \times 1/2''$ scintillation counter at the final image point. The resolution of this counter has not been unfolded.

The yield of pions was generally smaller than that calculated by up to a factor of 1.5 to 2. The cause of this is not known, though some of it may be due to uncertainties in the data of Baker et al.⁸, and in the normalization of our Monitor $M_1 M_2 M_3$ to the circulating AGS beam striking the Be target.

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TABLE I

Components for Beams

a) Positions and apertures of components

Length of a magnet, given in the Table, is the effective magnetic length.

<u>Component type</u>	<u>Length</u>	<u>Distance of downstream end of component from beam origin</u>	<u>Aperture</u>	<u>Remarks</u>
1. Fringe field	199"	199"		Particle trajectories calculated with BEAM program
2. Free space	97"	296"		
3. Q1 8Q32 Mk.I	34"	330"	8" diameter	Vertically focusing
4. Free space	27"	357"		
5. Q2 8Q48	52"	409"	8" diameter	Vertically focusing
6. Free space	8"	417"		
7. Q3 8Q48	52"	469"	8" diameter	Horizontally focusing
8. Free space	8"	477"		
9. Q4 8Q48	52"	529"	8" diameter	Horizontally focusing
10. Free space	8"	537"		
11. Q5 8Q48	52"	589"	8" diameter	Vertically focusing
12. Free space	8"	597"		
13. Q6 8Q48	52"	649"	8" diameter	Vertically focusing
14. Free space	12"	661"		
15. D1 18D72	79.5"	740.5"	18" Hor.x6" Vert.	Deflection angle: 2.89° c.w.
16. Free space	372"	1,112.5"		
17. Collimator C	40"	1,152.5"	$\frac{1}{2}$ " Hor.x $\frac{1}{2}$ " Vert.	"Intermediate focus"

<u>Component type</u>	<u>Length</u>	<u>Distance of downstream end of com- ponent from beam origin</u>	<u>Aperture</u>	<u>Remarks</u>
18. Free space	257.5"	1,410"		
19. D2 18D72	80"	1,490"	18" Hor.x6" Vert.	Deflection angle: 1.99° c.w.
20. Free space	12"	1,502"		
21. Q7 8Q48	52"	1,544"	8" diameter	Horizontally focusing
22. Free space	51"	1,605"		
23. Q8 8Q48	52"	1,657"	8" diameter	Vertically focusing
24. Free space	51"	1,708"		
25. Q9 8Q48	52"	1,760"	8" diameter	Horizontally focusing
26. Free space	78.5"	1,838.5"		
27. D3 18D72	80"	1,918.5"	18"Hor. x 6"Vert.	Deflection angle: 0.69° c.w.
28. Free space	?	?		Distance from D3 to experiment: varies from exper- iment to experi- ment.

b) Field strengths for deflection magnets

Momentum (GeV/c)	$\int B \cdot dL$		
	D1	D2	D3
4	265 Kg-in.	182 Kg-in.	63 Kg-in.
8	531 "	364 "	126 "
12	796 "	546 "	189 "
16	1,061 "	728 "	252 "

c) Field strengths for quadrupoles

Field strength for Q_1 corresponds to $p_{12} = 0$. Dependence of Q_1 field strength upon p_{12} is shown in Fig. 3. Field strengths for Q_7, Q_8, Q_9 , depend upon the distance ℓ from D3 to final beam focus.

$\int G \cdot dL$								
Quadrupole	(p = 4 GeV/c)		(p = 8 GeV/c)		(p = 12 GeV/c)		(p = 16 GeV/c)	
Q_1 ($p_{12} = 0$)	13.5	Kg/in x in	27.0	Kg/in x in	40.5	Kg/in x in	50.0	Kg/in x in
Q_2	29.2	"	58.5	"	87.7	"	116.9	"
Q_3	31.3	"	62.6	"	93.9	"	125.2	"
Q_4	31.3	"	62.6	"	93.9	"	125.2	"
Q_5	17.0	"	33.9	"	50.9	"	67.9	"
Q_6	17.0	"	33.9	"	50.9	"	67.9	"
Q_7 ($\ell = 951''$)	13.7	"	27.3	"	41.0	"	54.7	"
Q_8 ($\ell = 951''$)	41.5	"	83.0	"	124.5	"	166.0	"
Q_9 ($\ell = 951''$)	29.7	"	59.4	"	89.1	"	188.8	"
Q_7 ($\ell = 1111''$)	15.7	"	31.3	"	47.0	"	62.7	"
Q_8 ($\ell = 1111''$)	41.7	"	83.4	"	125.1	"	166.8	"
Q_9 ($\ell = 1111''$)	28.5	"	57.0	"	85.5	"	144.0	"
Q_7 ($\ell = 1287''$)	17.9	"	35.8	"	53.7	"	71.6	"
Q_8 ($\ell = 1287''$)	40.6	"	81.3	"	121.9	"	162.5	"
Q_9 ($\ell = 1287''$)	27.3	"	54.7	"	82.0	"	109.3	"

TABLE II
Currents in magnetic transport elements

Element	C u r r e n t (k Amps)			
	p=4 GeV/c	p=8 GeV/c	p=12 GeV/c	p=16 GeV/c
Q1 ($p_{12} = 0$)	0.172*	0.344*	0.514*	0.688*
Q2	0.434	0.875	1.314	1.765
Q3	0.465	0.938	1.410	1.899
Q4	0.465	0.938	1.410	1.899
Q5	0.249	0.505	0.760	1.015
Q6	0.249	0.505	0.760	1.015
D1	0.361	0.722	1.084	1.451
D2	0.247	0.494	0.741	0.988
Q7($\ell = 951''$)	0.200	0.406	0.611	0.818
Q8($\ell = 951''$)	0.637	1.289	1.943	2.668
Q9($\ell = 951''$)	0.440	0.887	1.334	1.783
Q7($\ell = 1111''$)	0.230	0.465	0.700	0.937
Q8($\ell = 1111''$)	0.640	1.295	1.952	2.681
Q9($\ell = 1111''$)	0.422	0.851	1.280	1.711
Q7($\ell = 1287''$)	0.262	0.531	0.801	1.071
Q8($\ell = 1287''$)	0.624	1.262	1.902	2.612
Q9($\ell = 1287''$)	0.405	0.816	1.228	1.641
D3	0.087	0.170	0.255	0.340

* Current for Q1 corresponds to $p_{12} = 0$. Dependence of Q1 field strength upon p_{12} is shown in Fig. 6. Field strength $\propto$ current, for Q1. Currents for Q7, Q8, Q9, depend upon the distance ℓ from D3 to final beam focus.

TABLE III

Estimated π^+ beam intensitiesAssumed circulating proton intensity: $I=3.10^{11}$ /burstValues for $\frac{\Delta^2 N}{\Delta p \cdot \Delta \omega}$ are taken from Baker, et al⁸

Pion momentum (GeV/c)	4	8	12
Incident proton energy (GeV)	25	30	30
Momentum bite: Δp_{FWHM} (GeV/c)	0.08	0.16	0.24
Solid angle: $\Delta \omega$ (Sr)	1.6×10^{-4}	2.10^{-4}	2×10^{-4}
Momentum phase space: $\Delta p \cdot \Delta \omega$ (GeV/c.Sr)	1.3×10^{-5}	3.2×10^{-5}	4.8×10^{-5}
Pion yield factor: $\frac{\Delta^2 N}{\Delta p \cdot \Delta \omega}$ per circulating proton (GeV/c.Sr) ⁻¹	2×10^{-1}	9×10^{-2}	1.7×10^{-2}
No. of π^+ /burst = $I \cdot \frac{\Delta^2 N}{\Delta p \cdot \Delta \omega} \cdot \Delta p \cdot \Delta \omega$	8×10^5	9×10^5	3×10^5

TABLE IV

Magnet currents for a negative beam, -8 GeV/c, with $p_{12} = -0.35$

$R_T = +0.6$, and $\ell = 951''$.

<u>Magnet</u>	<u>Experimental current (k Amps)</u>	<u>Calculated current (k Amps)</u>
Q1	0.374	0.387
Q2	0.858	0.875
Q3,4	0.930	0.938
Q5,6	0.503	0.505
D1	0.741	0.722
D2	0.493	0.494
Q7	0.398	0.406
Q8	1.225	1.289
Q9	0.863	0.889
D3	0.169	0.170

Figure Captions

- Fig. 1 Layout of the beam transport system
- Fig. 2 Schematic drawing of the beam optics.
- Fig. 3 Radial target position, R_T , as a function of p_{12} .
 p_{12} = ratio of circulating to secondary momentum;
 $p_{12} > 0$ for π^+ and $p_{12} < 0$ for π^- .
- Fig. 4 Some ray-traces: horizontal plane.
- Fig. 5 Some ray-traces: vertical plane.
- Fig. 6 Variation of Q_1 , as a function of p_{12} . p_{12} = ratio of circulating to secondary momentum; $p_{12} > 0$ for π^+ and $p_{12} < 0$ for π^- .
- Fig. 7 Estimated beam properties for 12 GeV/c π^- at image point I; (a) momentum spectrum, (b) spatial distribution, and (c) angular distribution.
- Fig. 8 Measured spatial distributions at image point I, for $p = -8$ GeV/c, $\ell = 951''$.

FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 7

FIGURE 8