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A FORTRAN PROGRAM TO ANALYZE
MASS SPECTROMETER ION OPTICS

by

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A FORTRAN PROGRAM TO ANALYZE MASS SPECTROMETER ION OPTICS

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ABSTRACT

A Fortran program is described for the computation of ion-optical aberration coefficients through second order for any number of tandem electric and magnetic sector fields, such as those used in multistaged mass spectrometers. Also computed are the image positions and their magnifications, the system mass dispersion, and the inclinations of the direction- and energy-focusing planes with respect to the optic axis.

INTRODUCTION

Since 1950 extensive studies have been made of the ion-optical properties of tandem electric and magnetic sector fields by Johnson and Nier,⁽¹⁾ Voorhies,⁽²⁾ Hintenberger and König,⁽³⁾ Ewald and Liebl,⁽⁴⁾ Tasman, Boerboom, and Wachsmuth,⁽⁵⁾ and others. These studies formed the basis for the computer program described herein, although other results were derived when needed.

This program considers ions having the following initial conditions different from those of ions which traverse the systems optic axis (see Fig. 1):

- (1) an initial angle to the optic axis
 α in the radial direction;
 α_z in the axial direction;
- (2) a different velocity or energy
 $V = V_0(1 + \beta)$ or
 $E = E_0(1 + \eta)$;
- (3) a different mass
 $M = M_0(1 + \gamma)$;
- (4) an initial displacement to the optic axis
 δ in the radial direction;
 δ_z in the axial direction.

Fig. 1. Figure Showing the Trajectories of Two Ions through an Electromagnetic Sector Field. The initial displacement of ion A to the optic axis is resolved into the radial component δ and an axial component δ_z . Also, its initial direction to the optic axis is resolved into radial and axial components α and α_z , respectively.

The program calculates ion trajectories as functions of α , β , γ , δ , α_z , and δ_z . In the radial plane the trajectories are given as power series to second-order terms, and in the axial direction to first-order terms. In field-free regions the trajectories are straight lines given directly by the program output. Points along the trajectories in the sector fields can be found by subdividing the field into a number of tandem sector fields, zero distance apart, and reading out computed data for each edge of the subfields.

The electric sectors may have curved boundaries and toroidal shape, which includes cylindrical and spherical analyzers as special cases. The magnetic fields are assumed homogeneous, but can have non-normal entrance and exit, as well as circularly curved boundaries.

INPUT DATA

The input data to this program are punched on cards, one card for each field in the system. For convenience these data are divided into three types, each of which is discussed below.

TYPE I DATA: A set of parameters for each field specifying its configuration:

(1) For a homogeneous magnetic field, as shown in Fig. 2, the relevant parameters are the field radius r_m , the beam-deflection angle ϕ_m , the

$$c = r_e/R_e,$$

and

$$R'_e = \left(\frac{\partial R}{\partial r} \right)_{r=r_e, z=0},$$

where R is the radius of curvature of an equipotential surface near the optic axis and the other symbols are as defined in Fig. 3. The values of c and R'_e for a general toroidal sector are discussed by Tasman⁽⁶⁾ and Albrecht.⁽⁷⁾ To first order,

$$c = \frac{r_e(a+b)}{bR_a + aR_b},$$

and

$$R'_e = \frac{R_a - R_b}{a + b}.$$

Two special types of toroidal electric fields are frequently used: the cylindrical analyzer which has $c = 0$ and $R'_e = 1$, and the spherical analyzer which has $c = 1$ and $R'_e = 1$.

TYPE II DATA: A set of parameters describing the relation of each field to its neighbors (see Fig. 4):

(1) On the card for the first field, one enters the distance from the source slit to the first field entrance boundary (ℓ').

(2) On the rest of the cards for the system, one enters the distance from the exit boundary of the previous field to the entrance boundary of the field considered (d), and also whether the deflection is in the same or opposite sense as that of the previous field.

(3) On the card for the last field in the system, one enters the distance (ℓ'') from the exit boundary to the point where the program is to calculate the aberration coefficients. Henceforth, this point will be referred to as the collector point and the plane perpendicular to the optic axis at this point as the collector plane.

(4) On each field card enter the sequence number of each field along with the total number of fields in the system.

Fig. 4. A Tandem Sector Field System

TYPE III DATA: Digits for controlling the following program options:

- (1) For a single symmetric field, the program can calculate both the entrance and the exit distances (l' and l''), so the point of first-order angle (α) focus is the collector point.
- (2) For any system of fields, the program can calculate the distance (l'') from the last field to the point of first-order angle (α) focus and use it as the collector point.
- (3) The aberration coefficients can be calculated in terms of energy differences η rather than velocity differences β .
- (4) The coefficient and ray trace arrays, described under output, may be read out if desired.

A detailed listing of the data card format is given in Table I.

Table I
INPUT DATA CARD FORMAT

Card Columns	Format	Fortran Symbol	Explanation	Punched on
1-6	F6.0	Q	Field deflection angle in degrees (θ_e or θ_m)	all cards.
7-12	F6.0	R	Field radius (r_e or r_m)	all cards.
13-18	F6.0	DA	The distance from the source slit to the first field (l_e' or l_m')	card for the first field only. Not necessary at all if 1 is punched in CC77.
19-24	F6.0	DB	The distance from the previous field to the present field (d)	all cards except the one for the first field.
25-30	F6.0	DC	The distance from the last field to the point where an aberration coefficient summary is desired (collector point) (l_e'' or l_m'')	card for the last field only. Not necessary at all if 1 is punched in CC76.
31-36	F6.0	R1	The radius of curvature of the entrance field boundary (R' or R_I)	all cards that do not have a 2 or 3 punched in CC74.
37-42	F6.0	R2	The radius of curvature of the exit field boundary (R'' or R_{II})	all cards that do not have a 2 or 4 punched in CC74.

Table I (Contd)

Card Columns	Format	Fortran Symbol	Explanation	Punched on
43-48	F6 0	E1	ϵ' indegrees for a magnetic field 0 for a cylindrical electrostatic field 1 for a spherical electrostatic field c parameter for a toroidal electrostatic field	all cards except those magnetic field cards that have $\epsilon' = 0$
49-54	F6 0	E2	ϵ'' in degrees for a magnetic field 1 for a cylindrical or spherical electrostatic field R_0^1 parameter for a toroidal electrostatic field	all cards except those magnetic field cards that have $\epsilon'' = 0$
55-59	5X		Not used	
60	I1	I9	1 in this column if the printer is to bring up a new page before doing any more printing	those cards where this option is desired
61-66	I6	K1	The field number (fields to be numbered sequentially beginning at the source end)	all cards
67-72	I6	K2	The total number of fields in the particular system considered	all cards
73	I1	I1	1 for a magnetic field, blank for an electric field	all cards
74	I1	I2	1 if R1 and R2 (CC 31-42) are both <u>not</u> equal to infinity 2 if R1 and R2 (CC 31-42) are both equal to infinity 3 if R1 (CC 31-36) is equal to infinity and R2 (CC 37-42) is <u>not</u> equal to infinity 4 if R1 (CC 31-36) is <u>not</u> equal to infinity and R2 (CC 37-42) is equal to infinity	all cards
75	I1	I3	1 if the deflection is in the opposite sense as the previous field, blank if it is in the same sense	all cards except the one for the first field.
76	I1	I4	1 if the program is to calculate the distance from the last field to the image plane which gives first-order angle focusing - this distance will be printed out (labeled DC) and used in subsequent calculations	card for the last field when this option is desired
77	I1	I5	1 if the program is to calculate the distance from the source to the first field - this feature can be used for a <u>single</u> symmetric field only	field card on which this option is wanted
78	I1	I6	1 if the aberrations are to be obtained in the η (energy) representation, blank for aberrations in the β (velocity) representation	all cards blanks or ones must be used consistently throughout a system
79	I1	I7	1 for the coefficient print feature	card for field for which this option is desired
80	I1	I8	1 for the ray trace print feature	card for field for which this option is desired.

Notes

- 1 CC abbreviation for card column(s)
- 2 The distances required above may be measured in any consistent unit

PROGRAM OUTPUT

For each field-free space this program calculates the radial displacement y to second order and the axial displacement z to first order as a function of α , β , γ , δ , α_z , and δ_z . This is accomplished by calculating certain constants for each field, then altering the aberration coefficients by use of these constants with a subprogram.

After each field is calculated the coefficients may be read out through a program option. This feature is called the coefficient print.

If this option is elected, thirty-six coefficients are read out for each field, in the order C_1 to C_{18} , then D_1 to D_{18} . This output is labeled by C followed by the field number. Specifically, the C's and D's are defined by the following equations, where the superscript

1 refers to the first field of a system,

ℓ refers to the last field of a system, and

N refers to the Nth field of a system:

$$\begin{aligned}
 Y^N = & r^N \left[C_1^N \alpha + C_2^N \beta + C_3^N \gamma + C_4^N \left(\frac{\delta}{r^1} \right) + C_7^N \alpha^2 + C_8^N \alpha \beta + C_9^N \beta^2 + C_{10}^N \alpha \gamma + C_{11}^N \beta \gamma + C_{12}^N \gamma^2 \right. \\
 & + C_{13}^N \alpha \left(\frac{\delta}{r^1} \right) + C_{14}^N \beta \left(\frac{\delta}{r^1} \right) + C_{15}^N \left(\frac{\delta}{r^1} \right)^2 + C_{16}^N \alpha_z^2 + C_{17}^N \alpha_z \left(\frac{\delta_z}{r^1} \right) + C_{18}^N \left(\frac{\delta_z}{r^1} \right)^2 \left. \right] \\
 & + X^N \left[D_1^N \alpha + D_2^N \beta + D_3^N \gamma + D_4^N \left(\frac{\delta}{r^1} \right) + D_7^N \alpha^2 + D_8^N \alpha \beta + D_9^N \beta^2 + D_{10}^N \alpha \gamma + D_{11}^N \beta \gamma \right. \\
 & + D_{12}^N \gamma^2 + D_{13}^N \alpha \left(\frac{\delta}{r^1} \right) + D_{14}^N \beta \left(\frac{\delta}{r^1} \right) + D_{15}^N \left(\frac{\delta}{r^1} \right)^2 + D_{16}^N \alpha_z^2 + D_{17}^N \alpha_z \left(\frac{\delta_z}{r^1} \right) + D_{18}^N \left(\frac{\delta_z}{r^1} \right)^2 \left. \right], \\
 Z^N = & r^N \left[C_5^N \alpha_z + C_6^N \left(\frac{\delta_z}{r^1} \right) \right] + X^N \left[D_5^N \alpha_z + D_6^N \left(\frac{\delta_z}{r^1} \right) \right];
 \end{aligned}$$

In these equations r is a field radius, X^N is the distance along the optic axis measured from the exit boundary of the Nth field, and Y^N and Z^N are the displacements measured in a plane located at X^N perpendicular to the optic axis. Note that Y^N is positive on the opposite side of the optic axis from the field center of curvature, Z^N is positive if above the optic axis, and (X^ℓ, Y^ℓ, Z^ℓ) are the coordinates in image space.

At this time twelve first-order coefficients: W_1 to W_6 for the entrance boundary, and W_1 to W_6 for the exit boundary, may be printed out. These twelve are labeled by R followed by the field number. This option is called the ray trace. The W's are defined by the equations

$$Y^N = W_1^N \alpha + W_2^N \beta + W_3^N \gamma + W_4^N \delta$$

and

$$Z^N = W_5^N \alpha_z + W_6^N \delta_z.$$

Just before the program summary a line labeled RS is read out, which contains W_1 to W_6 calculated at the collector point.

After the last field calculation has been completed, the program begins a summary calculation. This summary includes the aberration coefficients for the collector plane and other constants of the system. Eighteen coefficients B_1 to B_{18} , labeled S1, are read out. They are

defined by the equations

$$\begin{aligned} \frac{Y^N}{r^\ell} = & B_1\alpha + B_2\beta + B_3\gamma + B_4\left(\frac{\delta}{r^1}\right) + B_7\alpha^2 + B_8\alpha\beta + B_9\beta^2 + B_{10}\alpha\gamma + B_{11}\beta\gamma + B_{12}\gamma^2 \\ & + B_{13}\alpha\left(\frac{\delta}{r^1}\right) + B_{14}\beta\left(\frac{\delta}{r^1}\right) + B_{15}\left(\frac{\delta}{r^1}\right)^2 + B_{16}\alpha_z^2 + B_{17}\alpha_z\left(\frac{\delta_z}{r^1}\right) + B_{18}\left(\frac{\delta_z}{r^1}\right)^2 \end{aligned}$$

and

$$\frac{Z^N}{r^\ell} = B_5\alpha_z + B_6\left(\frac{\delta_z}{r^1}\right).$$

Following the S1 readout is the last output record, labeled S2, which contains the following six constants in order;

- (1) the position of the axial focusing plane, i.e., the distance X^ℓ to the point where $B_5 = 0$;
- (2) the system mass dispersion per $\Delta M/M$;
- (3) the radial image magnification (measured at the point of α focus, i.e., where $B_1 = 0$);
- (4) the axial image magnification (measured at the point of α_z focus, i.e., where $B_5 = 0$);
- (5) the angle ψ_α that the direction-focusing curve makes with the optic axis;
- (6) the angle ψ_β (or ψ_η) that the velocity (energy)-focusing curve makes with the optic axis.

The angles ψ_α and ψ_β (or ψ_η) (see Fig. 5) are defined as follows: the locus of points of direction or α focus described as an ion mass or γ is varied will be a surface in image space. The intersection of this surface with the system median plane is the direction-focusing curve χ_α . The slope, in degrees, of the tangent to χ_α at its intersection with the optic axis is ψ_α . The X^ℓ coordinate of this point is the one for which $B_1 = 0$. In a completely analogous manner, the velocity or energy parameters χ_β and ψ_β (or χ_η and ψ_η) may be defined. The X^ℓ coordinate for the intersection of χ_β or χ_η with the optic axis is that value which makes $B_2 = 0$. The angle ψ_β (or ψ_η) is of most interest for systems which are "double focusing," that is, where B_1 and B_2 both equal zero for the same value of X^ℓ .

Fig. 5

Figure Illustrating the Direction and Energy Focusing Curves and the Angles These Curves Make with the Optic Axis. In this figure ψ_α is positive and ψ_β is negative.

SAMPLE PROBLEM

As an example of the use of this program consider the Argonne 100-in. mass spectrometer, shown pictorially in Fig. 6. This instrument consists of a 100-in.-radius, 75° spherical electrostatic analyzer, followed by a 100-in.-radius, 110° magnetic analyzer.

Fig. 6. 100-in. Radius Double Focusing Mass Spectrometer

The parameters to be used in this example were read off Fig. 6 and punched on data cards as in Table II. The program output listing is shown in Table III, in which the second S1 and S2 outputs are for the energy (η) rather than the velocity (β) representation used in the preceding output records.

The ray trace feature was used to calculate the curves plotted in Figs. 7A and 7B. For this calculation the electric and magnetic sectors were each divided into ten subfields zero distance apart. The ray trace output for these twenty fields gave enough points to plot the curves. This output takes into account first-order* terms only, i.e., rays an infinitesimal distance away from the optic axis. In the figures the dimensions measured perpendicular to the optic axis can be considered greatly magnified. For this reason the magnetic sector boundaries are shown at right angles to the optic axis rather than inclined as they look to rays at a finite distance away. However, the calculation does take into account the focal properties of non-normal entrance and exit from the magnetic field.

*Second-order terms could easily be taken into account by calculating the curves directly from the coefficient print feature.

INPUT DATA FOR SAMPLE PROBLEM

[illegible]

Table III

SAMPLE PROBLEM OUTPUT

C 1	1.03528933 1.03528933 -0.02614593 -0.96602072 0.76733677 -0.00004911 -0.00004911 -0.03407404 -1.25884443 0.99997631	1.48236198 0.25881901 0.00000000 3.73205088 0.09591587 1.93185167 -0.96592584 0.00000000 2.93185155 -0.23296295	0.00000000 -0.00004912 0.00000000 -0.93301272 0.00000000 -1.03534018 0.00000000 -0.24999997	0.25881901 3.86389310 0.00000000 -0.26774917 -0.96592584 3.03542511 0.00000000 0.03472601
R 1	26.80000000 26.80000000 0.00000000	0.00000000 1.00000000 0.25881901	0.00000000 103.52893304 103.52893304	1.00000000 148.23619794 0.25881901
C 2	-0.36573166 -0.43555374 -10.52564577 1.82385587 -2.26343107 1.49091663 -1.07108789 0.81198440 -3.41156817 2.62351919	-1.24278774 -1.81704510 -0.70047860 -2.17157112 -0.32902210 5.06829209 -2.17244144 -0.74548292 9.73563290 -0.21561004	0.67101010 0.19775253 -2.61798160 0.57892024 -0.00000004 -0.60531209 -1.56822031 -1.72109583	0.93908818 -9.78742093 -0.16775250 -0.33628085 -1.09397747 12.63628107 0.00000003 -0.33287384
R 2	-103.52716495 103.52716495 67.10100965	-217.78285810 -0.08891429 0.93908818	-0.00000000 -36.57316597 -43.55537433	0.08891429 -124.27877381 -1.81704510
RS	-0.04570846 -69.79702763	-0.10561756 -2.34929325	67.10100859	0.67106371
S1	-0.00045708 -0.69797028 -10.32670959 0.98802167 -1.62066887	-0.00105618 -2.34929325 -0.88312192 0.21365894 -0.38184657	0.67101009 0.04945107 -3.00219557 0.15725176	0.67106371 -6.69153207 -0.16775250 -0.41783494
S2	-40.66461282 48.56350975	67.10100859 48.56285248	0.67072831	-0.93363020
S1	-0.00045708 -0.69797028 -2.66548765 0.98802167 -1.62066887	-0.00052809 -2.34929325 -0.88312192 0.10682947 -0.38184657	0.67101009 0.04945107 -1.50109779 0.15725176	0.67106371 -3.34576604 -0.16775250 -0.41783494
S2	-40.66461282 48.56350975	67.10100859 48.56285248	0.67072831	-0.93363020

A. Radial Plane

B. Axial Plane

Fig. 7. First-order Approximation to the Ion Trajectories for the Argonne 100-in. Mass Spectrometer. These plots were found by use of the ray trace feature of the program. Note that the distance in the radial and axial directions are shown greatly magnified.

REMARKS

The program described in this report is primarily for system analysis. It would be considerably more difficult to write a general computer program for system synthesis since each designer may wish to optimize different parameters. A few special syntheses of general interest have been intensively studied by groups in Germany and the Netherlands.⁽⁸⁾

No account is taken of the fringing fields, except for the Z or axial focusing effect of non-normal entrance into magnetic fields. In this case the fringing fields act as thin lenses with focal length equal to the field radius divided by the tangent of ϵ (defined in Fig. 2). The fringing field of electrostatic analyzers is usually eliminated by the use of Herzog end shields.⁽⁹⁾ The effects of the magnetic fringing fields can be taken into account by finding a field without fringing effects equivalent to an actual field and using this equivalent field in the computer calculations. Some help in finding this equivalent field can be found in the paper by Coggeshall and Muskat.⁽¹⁰⁾

From the discussions given in the papers listed at the beginning of this report one can see that the second-order radial coefficients depend on the radii of curvature of the field boundaries, but that the first-order coefficients and the general system properties do not. In fact, the second-order coefficients are linear functions of the reciprocals of these radii. With curved field boundaries, the ions traveling off the optic axis see slightly more or less field than those moving along the optic axis. Balestrini and White⁽¹¹⁾ have suggested the use of magnetic shims to give the same effect as the curved field boundaries. An alternative suggestion made by C. Stevens⁽¹²⁾ is to use small coils, similar to Helmholtz Coils, at various places along the ion path in the magnetic field. These coils have the advantage that the current through them can be varied, thus allowing some "tuning" of the instrument. The following example illustrates the use of this program in planning these methods of aberration correction.

Example: Consider three coils placed at the entrance, the middle, and the exit of the 100-in. mass spectrometer magnet used in the sample problem (p. 12). We wish to find the coil field strengths which will make $B_{11} = 0$ and $|B_{12}| + |B_{22}|$ a minimum. It is convenient to measure the coil field strengths in units of effective boundary radii of curvature, which can be related to the coil currents for a given coil construction. A practical upper limit of coil field strength or minimum radius of curvature may be set at 10 in.

Solution: Since B_{11} , B_{12} , and B_{22} are linear functions of the reciprocals of the coil strengths, they can be written as

$$B_{11} = \frac{C_1}{R_1} + \frac{C_2}{R_2} + \frac{C_3}{R_3} + C_4;$$

$$B_{12} = \frac{C_5}{R_1} + \frac{C_6}{R_2} + \frac{C_7}{R_3} + C_8;$$

$$B_{22} = \frac{C_9}{R_1} + \frac{C_{10}}{R_2} + \frac{C_{11}}{R_3} + C_{12},$$

where the C's are constants, and R_1 , R_2 , and R_3 are the coil field strengths. To find the C's, set all three R's equal to infinity and obtain C_4 , C_8 , and C_{12} by use of this program. Next, take any three linearly independent sets of (R_1, R_2, R_3) and calculate the B coefficients for each set. This procedure gives three sets of three linear simultaneous equations which can be solved for the nine remaining C's. At this point we have,

$$B_{11} = \frac{71.67}{R_1} + \frac{61.09}{R_2} + \frac{2.981}{R_3} + 0.1805;$$

$$B_{12} = \frac{301.5}{R_1} + \frac{372.7}{R_2} + \frac{20.26}{R_3} - 6.140;$$

$$B_{22} = \frac{317.2}{R_1} + \frac{568.4}{R_2} + \frac{34.42}{R_3} - 9.747.$$

The magnetic field is broken into two separate fields, zero distance apart, for calculating the effects of the middle coil. Setting the B's equal to zero and solving for the R's yields a physically unrealizable result. However, the stated problem of making $B_{11} = 0$ and $|B_{12}| + |B_{22}|$ a minimum, subject to the conditions $|R_i| \geq 10$, can be solved by straightforward linear programming methods.⁽¹³⁾ The solution is $R_1 = -28.912$, $R_2 = 23.528$, $R_3 = -10$, and minimum of $|B_{12}| + |B_{22}| = 2.755$. Other simple synthesis problems can be handled by similar methods.

THE FORTRAN PROGRAM

The program was written in "basic" Fortran through use of the most elementary forms of Fortran statements. All input-output statements are written for cards and are enclosed by lines of asterisks. This program should be easy to adapt to any computer installation. The program running time is negligible, the CDC 3600 machine at Argonne compiled the program in 35 sec and calculated about 120 fields per minute. A program flowchart (Fig. 8) and listing (Table IV) follow.

Fig. 8. Fortran Program Flowchart

Table IV

FORTTRAN PROGRAM LISTING

C	ION OPTICS OF TANDEM ELECTRIC AND MAGNETIC SECTOR FIELDS	IOP	001
C	WRITTEN BY KURT KAISER	IOP	002
C	9-63	IOP	003
C	DIMENSION F(2,20),C(2,18),S(18),T(2,18),W(2,6)	IOP	004
C	PI=.017453293	IOP	005
C	*****	IOP	006
C	READ INPUT DATA CARD	IOP	007
C	1 READ 2,Q,R,DA,DB,DC,R1,R2,E1,E2,I9,K1,K2,I1,I2,I3,I4,I5,I6,I7,I8	IOP	008
C	2 FORMAT (9F6.0,5X11,2I6,8I1)	IOP	009
C	RESTORE PRINTER IF I9=1	IOP	010
C	71 IF (I9) 50,69,70	IOP	011
C	70 PUNCH 3	IOP	012
C	3 FORMAT(78X2H -)	IOP	013
C	*****	IOP	014
C	69 GO TO(4,5,6,7),I2	IOP	015
C	4 P1=R/R1	IOP	016
C	P2=R/R2	IOP	017
C	GO TO 8	IOP	018
C	5 P1=0.	IOP	019
C	P2=0.	IOP	020
C	GO TO 8	IOP	021
C	6 P1=0.	IOP	022
C	P2=R/R2	IOP	023
C	GO TO 8	IOP	024
C	7 P1=R/R1	IOP	025
C	P2=0.	IOP	026
C	8 IF(I1) 50,39,40	IOP	027
C	MAGNETIC SECTOR CALCULATION	IOP	028
C	40 Q=PI*Q	IOP	029
C	E1=PI*E1	IOP	030
C	E2=PI*E2	IOP	031
C	G=COSF(E1)	IOP	032
C	D=COSF(E2)	IOP	033
C	T1=SINF(E1)/G	IOP	034
C	T2=SINF(E2)/D	IOP	035
C	P1=.5*P1/(G*G*G)	IOP	036
C	P2=.5*P2/(D*D*D)	IOP	037
C	ZEEs MAGNETIC	IOP	038
C	Z1=SINF(Q)	IOP	039
C	Z2=COSF(Q)	IOP	040
C	Z3=Z1*Z1	IOP	041
C	Z4=Z2*Z2	IOP	042
C	Z5=1.-Z2	IOP	043
C	Z6=Z1*Z2	IOP	044
C	Z7=-.5*Z3	IOP	045
C	Z8=Z1*Z5	IOP	046
C	Z9=-.5*Z5	IOP	047
C	Z10=-.5*Z1	IOP	048
C	Z11=Z2-Z4	IOP	049
C	Z12=.5*Z11	IOP	050
C	Z13=Z4-Z3	IOP	051
C	Z14=Z2-Z13	IOP	052
C	Z15=T1*T1	IOP	053
C	Z16=T2*T2	IOP	054
C	Z17=T2*Z16	IOP	055
C	Z18=T1*T2	IOP	056
C	Z19=T1*Z16	IOP	057
C	Z20=Z15*Z16	IOP	058
C	Z21=T2*Z15	IOP	059
C	Z22=T1*Z17	IOP	060
C	F(I,J) MAGNETIC	IOP	061
C	F(1,1)=Z1	IOP	062
C	F(1,2)=Z2+Z1*T1	IOP	063
C	F(1,3)=-Z9	IOP	064
C	F(1,4)=F(1,2)	IOP	065
C	F(1,5)=Z5	IOP	066
C	F(1,6)=Z12-Z7*Z16	IOP	067
C	F(1,7)=Z6+2.*Z12*T1+Z1*Z15+Z6*Z16+Z3*Z19	IOP	068
C	F(1,8)=Z7+Z6*(T1+Z19)+Z15*(Z12-.5*Z2)+.5*(Z4*Z16+Z3*Z20)+P1*Z1	IOP	069
C	F(1,9)=Z8*(1.+Z16)	IOP	070
C	F(1,10)=Z3-Z6*T1+Z11*Z16+Z8*Z19	IOP	071
C	F(1,11)=Z7+.5*Z5*Z5*Z16	IOP	072
C	F(1,12)=Z9	IOP	073
C	F(1,13)=Z5*T1	IOP	074
C	F(1,14)=Z9*Z15	IOP	075
C	F(1,15)=Q	IOP	076
C	F(1,16)=1.-Q*T1	IOP	077
C		IOP	078
C		IOP	079
C		IOP	080

Table IV (Contd.)

	F(2,1)=Z2+Z1*T2	IOP	081
	F(2,2)=-Z1+Z2*(T1+T2)+Z1*Z18	IOP	082
	F(2,5)=Z1+Z5*T2	IOP	083
	F(2,4)=F(2,2)	IOP	084
	F(2,3)=.5*F(2,5)	IOP	085
	F(2,6)=Z10+Z12*T2-Z6*Z16+Z3*(P2-.5*Z17)	IOP	086
	F(2,7)=2.*Z10*T1+Z6*(T2-Z17-2.*Z19)+Z2*Z15+2.*Z12*Z18-Z13*Z16	IOP	087
	1+Z1*Z21-Z3*Z22+2.*P2*(Z6+Z3*T1)	IOP	088
	F(2,8)=Z7*(T2+Z15*Z17)-.5*Z4*(Z17+Z21)+Z6*(Z16+Z18-Z20-Z22)	IOP	089
	1-Z13*Z19+P2*(Z4+2.*Z6*T1+Z3*Z15)+P1*(Z2+Z1*T2)	IOP	090
	F(2,9)=-Z6*T2-Z14*Z16+Z8*(2.*P2-Z17)	IOP	091
	F(2,10)=Z1-Z2*T1+(Z3-Z2)*T2-Z16*(2.*Z6-Z1)-Z18*(Z1+Z6)-Z11*Z17	IOP	092
	1-Z14*Z19-Z8*Z22+2.*P2*(Z11+Z8*T1)	IOP	093
	F(2,11)=-Z1+(Z7-Z5)*T2-Z8*Z16+Z5*Z5*(P2-.5*Z17)	IOP	094
	F(2,12)=Z10+Z9*T2	IOP	095
	F(2,13)=-2.*(Z10*T1+Z9*Z18)	IOP	096
	F(2,14)=Z10*Z15+Z9*Z21	IOP	097
	F(2,15)=1.-Q*T2	IOP	098
	F(2,16)=Q*Z18-T1-T2	IOP	099
	DO 55 I=1,2	IOP	100
	F(1,17)=.5*F(1,9)	IOP	101
	F(1,18)=.5*F(1,10)	IOP	102
	F(1,19)=.5*F(1,5)+F(1,11)	IOP	103
	F(1,20)=.25*F(1,11)-.125*F(1,5)	IOP	104
	IF(I6) 50,55,58	IOP	105
58	F(1,5)=.5*F(1,5)	IOP	106
	F(1,9)=F(1,17)	IOP	107
	F(1,10)=F(1,18)	IOP	108
	F(1,11)=F(1,20)	IOP	109
	F(1,19)=.5*F(1,19)	IOP	110
55	CONTINUE	IOP	111
	GO TO 9	IOP	112
C	ELECTRIC SECTOR CALCULATION	IOP	113
39	A=SQRTF(2.-E1)	IOP	114
	Q=PI*Q	IOP	115
	G=SQRTF(E1)	IOP	116
	F1=Q*G	IOP	117
	E2=E1*(1.+E2)	IOP	118
	RE=E1*E2	IOP	119
	E2=1.+E2	IOP	120
	B=3.*(E1-1.)-.5*RE	IOP	121
	D=E1+RE	IOP	122
	F2=Q*A	IOP	123
C	ZEES ELECTRIC	IOP	124
	Z1=SINF(F2)	IOP	125
	Z2=COSF(F2)	IOP	126
	Z3=SINF(F1)	IOP	127
	Z4=COSF(F1)	IOP	128
	Z5=Z2*Z2	IOP	129
	Z6=Z4*Z4	IOP	130
	Z7=Z1*Z2	IOP	131
	Z8=Z3*Z4	IOP	132
	Z9=A*A	IOP	133
	Z10=3.*Z9	IOP	134
	Z11=A*Z10	IOP	135
	Z12=Z9*Z9	IOP	136
	Z13=3.*Z12	IOP	137
	Z14=A*Z13	IOP	138
	Z15=B+Z9	IOP	139
	Z16=2.*B-Z9	IOP	140
	Z17=B-2.*Z9	IOP	141
	Z18=2.-3.*E1	IOP	142
	Z19=Z18-2.*E1	IOP	143
	Z20=Z5*Z17	IOP	144
	Z21=3.*F2*(2.*B+Z9+Z12)	IOP	145
	Z22=Z1*Z21	IOP	146
	Z21=Z2*Z21	IOP	147
C	F(I,J) ELECTRIC	IOP	148
	F(1,1)=Z1/A	IOP	149
	F(1,2)=Z2	IOP	150
	F(1,3)=0.	IOP	151
	F(1,4)=Z2	IOP	152
	F(1,5)=2.*(1.-Z2)/Z9	IOP	153
	F(1,6)=(Z15-Z2*Z16+Z20)/Z13	IOP	154
	F(1,7)=(Z1*Z16-2.*Z7*Z17)/Z11	IOP	155
	F(1,8)=(Z16-Z2*Z15-Z5*Z17)/Z10	IOP	156
	F(1,9)=((Z13+4.*Z10+Z16)*Z1+4.*Z7*Z17-Z21)/Z14	IOP	157
	F(1,10)=(4.*(Z20+Z2*Z15-Z16)+Z22)/Z13	IOP	158
	F(1,11)=(20.*Z15+Z13-(16.*Z15+4.*Z10+Z13)*Z2-4.*Z20-2.*Z22)	IOP	159
	1/(A*Z14)	IOP	160
	F(1,12)=(E2*(2.*E1*Z2+Z18-Z6*Z9)/(2.*Z19)+Z2-1.)/Z9	IOP	161
	F(1,13)=G*E2*Z8/Z19-Z1*D/(A*Z19)	IOP	162
	F(1,14)=(D*(Z6*Z9-Z2*Z18-2.*E1)/(2.*Z19)+E1*(Z2-1.))/Z9	IOP	163
	1+.5*F(1,1)*P1	IOP	164
	IF(E1) 10,11,10	IOP	165

Table IV (Contd.)

11	F(1,15)=Q	IOP	166
	GO TO 12	IOP	167
10	F(1,15)=Z3/G	IOP	168
12	F(1,16)=Z4	IOP	169
	F(2,1)=Z2	IOP	170
	F(2,2)=-A*Z1	IOP	171
	F(2,3)=0.	IOP	172
	F(2,4)=F(2,2)	IOP	173
	F(2,5)=2.*Z1/A	IOP	174
	F(2,6)=Z16*(Z1-Z7)/Z11	IOP	175
	F(2,7)=Z16*(1.+Z2-2.*Z5)/Z10	IOP	176
	Z7=Z7*Z16	IOP	177
	F(2,8)=(Z1*Z15+Z7)/(3.*A)	IOP	178
	F(2,9)=(2.*Z16*(2.*Z5-Z2-1.)+Z22)/Z13	IOP	179
	F(2,10)=(2.*Z15+Z10+Z13)*Z1-4.*Z7+Z21)/Z11	IOP	180
	F(2,11)=(4.*Z15+2.*Z10-Z13)*Z1+4.*Z7-2.*Z21)/Z14	IOP	181
	F(2,12)=F(1,13)-F(1,1)+.5*F(1,15)*F(1,15)*P2	IOP	182
	F(2,13)=D*(2.*Z6-Z2-1.)/Z19+F(1,15)*F(1,16)*P2	IOP	183
	F(2,14)=D*(.5*Z1*Z18/A-G*Z8)/Z19-E1*Z1/A+.5*(F(2,1)*P1	IOP	184
	1+F(1,16)*F(1,16)*P2)	IOP	185
	F(2,15)=Z4	IOP	186
	F(2,16)=-G*Z3	IOP	187
	DO 56 I=1,2	IOP	188
	IF(16) 50,59,60	IOP	189
60	F(1,5)=.5*F(1,5)	IOP	190
	F(1,9)=.5*F(1,9)	IOP	191
	F(1,10)=.5*F(1,10)	IOP	192
	F(1,11)=.25*F(1,11)-.125*F(1,5)	IOP	193
59	F(1,17)=0.	IOP	194
	F(1,18)=0.	IOP	195
	F(1,19)=0.	IOP	196
56	F(1,20)=0.	IOP	197
9	IF(K1-1) 50,13,14	IOP	198
C		IOP	199
	13 IF(K2-1) 50,15,16	IOP	200
	15 IF(15) 50,16,17	IOP	201
	17 A=F(1,2)+F(2,1)	IOP	202
	DA=R*(-A-SQRTF(A*A-4.*F(1,1)*F(2,2)))/(2.*F(2,2))	IOP	203
16	DA=DA/R	IOP	204
	DS=DA*DA	IOP	205
	DO 18 I=1,2	IOP	206
	C(1,1)=F(1,1)+F(1,2)*DA	IOP	207
	C(1,2)=F(1,5)	IOP	208
	C(1,3)=F(1,3)	IOP	209
	C(1,4)=F(1,4)	IOP	210
	C(1,5)=F(1,15)+F(1,16)*DA	IOP	211
	C(1,6)=F(1,16)	IOP	212
	C(1,7)=F(1,6)+F(1,7)*DA+F(1,8)*DS	IOP	213
	C(1,8)=F(1,9)+F(1,10)*DA	IOP	214
	C(1,9)=F(1,11)	IOP	215
	C(1,10)=F(1,17)+DA*F(1,18)	IOP	216
	C(1,11)=F(1,19)	IOP	217
	C(1,12)=F(1,20)	IOP	218
	C(1,13)=F(1,7)+2.*F(1,8)*DA	IOP	219
	C(1,14)=F(1,10)	IOP	220
	C(1,15)=F(1,8)	IOP	221
	C(1,16)=F(1,12)+F(1,13)*DA+F(1,14)*DS	IOP	222
	C(1,17)=F(1,13)+2.*F(1,14)*DA	IOP	223
18	C(1,18)=F(1,14)	IOP	224
	RB=R	IOP	225
	W(1,1)=DA*R	IOP	226
	W(1,2)=0.	IOP	227
	W(1,3)=0.	IOP	228
	W(1,4)=1.	IOP	229
	W(1,5)=DA*R	IOP	230
	W(1,6)=1.	IOP	231
	DO 61 J=1,6	IOP	232
61	W(2,J)=R*C(1,J)	IOP	233
	W(2,4)=W(2,4)/R	IOP	234
	W(2,6)=W(2,6)/R	IOP	235
C		IOP	236
C	*****	IOP	237
C	PUNCH COEFFICIENT PRINT OR RAY TRACE	IOP	238
	30 FORMAT(//)	IOP	239
	19 FORMAT(/4X2H RI2,4F18.8/(8X4F18.8))	IOP	240
	65 PUNCH 30	IOP	241
	20 IF(17) 50,62,37	IOP	242
	37 PUNCH 52,K1,((C(1,J),J=1,18),I=1,2)	IOP	243
	52 FORMAT(/4X2H CI2,4F18.8,3(/8X4F18.8)/8X2F18.8,/(8X4F18.8))	IOP	244
	62 IF(18) 50,21,41	IOP	245
	41 PUNCH 19,K1,((W(1,J),J=1,6),I=1,2)	IOP	246
C	*****	IOP	247
C		IOP	248
	21 RA=R	IOP	249
	IF(K2-K1) 50,23,1	IOP	250

Table IV (Contd.)

		MORE THAN 1 FIELD COMBINATION SUBPROGRAM	
C	14	DO 44 J=1,6 W(1,J)=R*C(1,J)+DB*C(2,J) IF(13) 50,44,82 82 IF(J-5) 83,44,44 83 W(1,J)=-W(1,J) 44 CONTINUE W(1,4)=W(1,4)/RB W(1,6)=W(1,6)/RB U=RA/R V=DB/R DO 24 J=1,18 24 S(J)=U*C(1,J)+V*C(2,J) IF(13) 50,25,26 26 DO 27 I=1,2 F(1,1)=-F(1,1) F(1,2)=-F(1,2) F(1,9)=-F(1,9) F(1,10)=-F(1,10) F(1,17)=-F(1,17) 27 F(1,18)=-F(1,18) 25 D1=C(2,1) D2=C(2,2) D3=C(2,3) D4=C(2,4) H3=C(2,5) H4=C(2,6) DO 28 I=1,2 DO 28 J=1,18 28 T(I,J)=F(1,1)*C(2,J)+F(1,2)*S(J) DO 29 I=1,2 C(1,1)=T(1,1) C(1,2)=T(1,2)+F(1,5) C(1,3)=T(1,3)+F(1,3) C(1,4)=T(1,4) C(1,5)=F(1,15)*H3+F(1,16)*S(5) C(1,6)=F(1,15)*H4+F(1,16)*S(6) C(1,7)=T(1,7)+F(1,6)*D1*D1+F(1,7)*D1*S(1)+F(1,8)*S(1)*S(1) C(1,8)=T(1,8)+2.*F(1,6)*D1*D2+F(1,7)*(D1*S(2)+D2*S(1)) 1+2.*F(1,8)*S(1)*S(2)+F(1,9)*D1+F(1,10)*S(1) C(1,9)=T(1,9)+F(1,6)*D2*D2+F(1,7)*D2*S(2)+F(1,8)*S(2)*S(2) 1+F(1,9)*D2+F(1,10)*S(2)+F(1,11) C(1,10)=T(1,10)+2.*F(1,6)*D1*D3+F(1,7)*(D1*S(3)+D3*S(1)) 1+2.*F(1,8)*S(1)*S(3)+F(1,17)*D1+F(1,18)*S(1) C(1,11)=T(1,11)+2.*F(1,6)*D2*D3+F(1,7)*(D2*S(3)+D3*S(2)) 1+2.*F(1,8)*S(2)*S(3)+F(1,9)*D3+F(1,10)*S(3)+F(1,19) 2+F(1,17)*D2+F(1,18)*S(2) C(1,12)=T(1,12)+F(1,6)*D3*D3+F(1,7)*D3*S(3)+F(1,8)*S(3)*S(3) 1+F(1,17)*D3+F(1,18)*S(3)+F(1,20) C(1,13)=T(1,13)+2.*F(1,6)*D1*D4+F(1,7)*(D1*S(4)+D4*S(1)) 1+2.*F(1,8)*S(1)*S(4) C(1,14)=T(1,14)+2.*F(1,6)*D2*D4+F(1,7)*(D2*S(4)+D4*S(2)) 1+2.*F(1,8)*S(2)*S(4)+F(1,9)*D4+F(1,10)*S(4) C(1,15)=T(1,15)+F(1,6)*D4*D4+F(1,7)*D4*S(4)+F(1,8)*S(4)*S(4) C(1,16)=T(1,16)+F(1,12)*H3*H3+F(1,13)*H3*S(5)+F(1,14)*S(5)*S(5) C(1,17)=T(1,17)+2.*F(1,12)*H3*H4+F(1,13)*(H3*S(6)+H4*S(5)) 1+2.*F(1,14)*S(5)*S(6) 29 C(1,18)=T(1,18)+F(1,12)*H4*H4+F(1,13)*H4*S(6)+F(1,14)*S(6)*S(6) DO 47 J=1,6 47 W(2,J)=R*C(1,J) W(2,4)=W(2,4)/RB W(2,6)=W(2,6)/RB GO TO 20	IOP 251 IOP 252 IOP 253 IOP 254 IOP 255 IOP 256 IOP 257 IOP 258 IOP 259 IOP 260 IOP 261 IOP 262 IOP 263 IOP 264 IOP 265 IOP 266 IOP 267 IOP 268 IOP 269 IOP 270 IOP 271 IOP 272 IOP 273 IOP 274 IOP 275 IOP 276 IOP 277 IOP 278 IOP 279 IOP 280 IOP 281 IOP 282 IOP 283 IOP 284 IOP 285 IOP 286 IOP 287 IOP 288 IOP 289 IOP 290 IOP 291 IOP 292 IOP 293 IOP 294 IOP 295 IOP 296 IOP 297 IOP 298 IOP 299 IOP 300 IOP 301 IOP 302 IOP 303 IOP 304 IOP 305 IOP 306 IOP 307 IOP 308 IOP 309 IOP 310 IOP 311 IOP 312 IOP 313 IOP 314 IOP 315
C	23	S(1)=-RA*C(1,5)/C(2,5) S(3)=1./C(2,1) S(4)=1./C(2,5) IF(15) 50,32,31 32 IF(14) 50,33,31 31 DC=-RA*C(1,1)/C(2,1)	IOP 316 IOP 317 IOP 318 IOP 319 IOP 320
C	SUMMARY		IOP 321
C	*****		IOP 322
C	PUNCH DC		IOP 323
C	PUNCH 34,DC		IOP 324
C	34 FORMAT(/4X4H DC F18.8)		IOP 325
C	*****		IOP 326
C	33	DO 64 J=1,6 64 T(1,J)=R*C(1,J)+DC*C(2,J) T(1,4)=T(1,4)/RB T(1,6)=T(1,6)/RB DC=DC/RA S(7)=C(1,3)+DC*C(2,3) S(8)=C(1,10)+DC*C(2,10) S(9)=C(1,11)+DC*C(2,11)	IOP 327 IOP 328 IOP 329 IOP 330 IOP 331 IOP 332 IOP 333 IOP 334 IOP 335

Table IV (Contd.)

	IF(S(8)) 53,54,53	IOP	336
54	S(5)=90.	IOP	337
	S(6)=90.	IOP	338
	GO TO 57	IOP	339
53	S(5)=ATANF(-C(2,1)*S(7)/S(8))/PI	IOP	340
	S(6)=ATANF(-C(2,2)*S(7)/S(9))/PI	IOP	341
57	DO 35 J=1,18	IOP	342
35	C(1,J)=C(1,J)+DC*C(2,J)	IOP	343
	S(2)=RA*C(1,3)	IOP	344
C		IOP	345
C	*****	IOP	346
	PUNCH SUMMARY	IOP	347
66	FORMAT(/4X4H RS 4F18.8/8X2F18.8)	IOP	348
67	FORMAT(/4X4H S1 4F18.8/(8X4F18.8))	IOP	349
68	FORMAT(/4X4H S2 4F18.8/8X2F18.8)	IOP	350
	IF(I8) 50,51,49	IOP	351
49	PUNCH 66,(T(1,J),J=1,6)	IOP	352
51	PUNCH 67,(C(1,J),J=1,18)	IOP	353
	PUNCH 68,(S(J),J=1,6)	IOP	354
C	*****	IOP	355
		IOP	356
	GO TO 1	IOP	357
50	STOP	IOP	358
	END	IOP	359

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