SOME TECHNIQUES FOR DETERMINING TENSOR PRESSURE EQUILIBRIA

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SOME TECHNIQUES FOR DETERMINING TENSOR PRESSURE EQUILIBRIA

C. L. Hedrick, G. E. Guest, and D. B. Nelson

I. INTRODUCTION

In this report we discuss several iterative techniques for computing finite-beta, tensor-pressure equilibria of guiding center plasmas. These techniques have evolved in the course of investigations of the equilibrium and stability of plasmas confined in several existing and proposed experimental devices at ORNL: ELMO, The Canted-Mirror Facility, and the Elmo Bumpy Torus;¹ and INTEREM, IMP and the proposed Long Field Experiment.² These devices encompass magnetic configurations ranging from axisymmetric simple mirrors through various forms of magnetic "wells", to a closed-line torus. For this reason the present review must deal with several distinct plasma models of varying complexity required to describe the experimentally observed equilibria in these different devices.

While the computation of equilibria in these various configurations requires distinctly different analytical techniques, there is an underlying unity which is apparent in the basic equilibrium equations. In Section II we cast these equations into a form suitable for the iteration schemes discussed in the following Sections and develop a number of relations needed for studies of equilibrium and stability.

The organization of the report is around various functional models for the pressure tensor, $\underline{P}$, the simplest being $\underline{P}(\underline{B})$; that is, a pressure tensor depending only on the magnitude of the magnetic field strength.³ Such a plasma is confined only in magnetic traps with nested closed

surfaces of constant $|\vec{B}|$, such as INTEREM and IMP. Despite the limited applicability of $P(B)$, the methods described in Section III for treating such a plasma model form the basis and pattern for the more general models considered in subsequent Sections. Of particular importance is the discussion of boundary conditions needed for numerical solution of the equations.

For devices in which surfaces of constant $|\vec{B}|$ are not closed so that $P(B)$ is not adequate to describe a confined plasma, it is necessary that P depend on one or more additional coordinates. It is often convenient to use coordinates based on the magnetic field lines or related surface quantities such as the elements of a Clebsch representation, $\vec{B} = \vec{\nabla}\alpha \times \vec{\nabla}\beta$, or, for axisymmetric geometries, the flux function, $\psi = \int \vec{B} \cdot d\vec{S}$. Section IV considers the latter case, exemplified by simple mirror devices such as ELMO, and discussed at length in ORNL-TM-4077.⁴ The analysis of $P(\alpha, \beta, B)$ models can often be simplified without serious loss of generality by assuming P to be a separable function of B and the remaining coordinates. In ORNL-TM-4077 this assumption permitted useful analytic estimates of the various equilibrium parameters entering the interchange stability criterion. The consequences of separable pressure tensors are discussed more generally in Section V, with particular stress on the complications resulting from the non-axisymmetry of both the magnetic-well devices and the Canted Mirror and EBT configurations.

The most immediate of such complications is the fact that the surfaces (of constant α , say) entering into the pressure model are not uniquely determined without recourse to a subsidiary condition. For open-ended devices magnetic field lines pass from the plasma into vacuum

regions; and a reasonable subsidiary condition is that $j_{\parallel} = 0$ in the vacuum. In Section V we demonstrate how this constraint acts to determine surfaces of constant α , just as the requirement of single-valuedness determines that scalar pressure isobars (in closed-line torii) are the surfaces of constant $\int d\ell/B$. Other choices for α -surfaces may be dictated by the experimentally observed properties of the particular plasma. For example, we consider here the choice in which α -surfaces coincide with $|\vec{B}|$ -contours on the midplane of the Canted-Mirror Facility.

For toroidal closed line devices such as EBT a similar restriction arises and must be observed. However the separable model is probably not adequate for EBT. Thus in Section VI we discuss the general case in which the pressure tensor depends upon B , α and β . We then specialize to a model which is probably adequate for our present purposes. Again there are restrictions imposed similar to that encountered for scalar pressure closed line systems, associated with questions of topology.

The initial sections of this report constitute an annotated review, using a uniform notation, of the work of a number of authors. Sections II, III and IV are based in part on a previous review by H. Grad.⁵ The discussion of $P(B)$ is also based on the original papers by Taylor³ and Northrup and Whiteman⁶ as well as a subsequent paper by Hall.⁷ A number of interesting numerical techniques have been discussed by Anderson and Killeen⁸ and Anderson.⁹

The iteration scheme for the general case discussed in Section VI is new. Its origins are in the work of L. Hall¹⁰ and H. Grad.¹¹ Its principal advantage over the scheme proposed by Hall is that it does not

employ singular differential equations, and hence the restrictions arising from $j = 0$ in vacuum for open-ended devices are more transparent.

II. BASIC EQUATIONS AND USEFUL RELATIONS

The fundamental equilibrium equations are

$$\vec{j} \times \vec{B} = \nabla \cdot \vec{P}, \quad (2.1)$$

where (see ORNL-TM-3943,¹² eq. 3.2)

$$\begin{aligned} \nabla \cdot \vec{P} &= \overline{\nabla p_{\perp}} + \hat{x}_i \frac{\partial}{\partial x_i} \left[(p_{\parallel} - p_{\perp}) b_i b_j \right] \\ &= \overline{\nabla p_{\perp}} + \hat{b} \frac{\partial}{\partial \ell} (p_{\parallel} - p_{\perp}) + (p_{\parallel} - p_{\perp}) \left(-\frac{1}{B} \frac{\partial B}{\partial \ell} \hat{b} + \vec{\kappa} \right). \end{aligned} \quad (2.2)$$

and ℓ is arc length along a field line. From Maxwell's equations we have

$$\nabla \times \vec{B} = \mu_0 \vec{j} \quad \text{and} \quad \nabla \cdot \vec{B} = 0. \quad (2.3)$$

Resolving (2.2) into components parallel and perpendicular to $\vec{B}$ and noting (2.1), we obtain the useful identity

$$(\nabla \cdot \vec{P})_{\parallel} = 0 = \frac{\partial p_{\parallel}}{\partial \ell} - \left(\frac{p_{\parallel} - p_{\perp}}{B} \right) \frac{\partial B}{\partial \ell} = \left[\frac{\partial p_{\parallel}}{\partial B} - \frac{(p_{\parallel} - p_{\perp})}{B} \right] \frac{\partial B}{\partial \ell}$$

or

$$\frac{\partial p_{\parallel}}{\partial B} - \frac{(p_{\parallel} - p_{\perp})}{B} = 0 \quad (2.4)$$

The perpendicular component of $\nabla \cdot \vec{P}$,

$$(\nabla \cdot \vec{P})_{\perp} = \overline{\nabla_{\perp} p_{\perp}} + (p_{\parallel} - p_{\perp}) \vec{\kappa}, \quad (2.5)$$

can be rewritten by using the standard vector identity,

$$\frac{1}{2} \nabla \vec{B}^2 = (\vec{B} \cdot \nabla) \vec{B} + \vec{B} \times (\nabla \times \vec{B}) , \quad (2.6)$$

to express the curvature in the form

$$\vec{\kappa} = \frac{1}{B} \nabla_{\perp} \vec{B} + \frac{(\nabla \times \vec{B}) \times \vec{B}}{B^2} . \quad (2.7)$$

Using this form for the curvature in 2.5, we can write 2.1 as

$$\vec{j} \times \vec{B} = \nabla_{\perp} p_{\perp} + \frac{(p_{\parallel} - p_{\perp})}{B} \left[\nabla_{\perp} \vec{B} + \frac{(\nabla \times \vec{B}) \times \vec{B}}{B} \right] , \quad (2.8)$$

which, together with 2.3 becomes

$$\begin{aligned} \left[\frac{1}{\mu_0} - \frac{(p_{\parallel} - p_{\perp})}{B^2} \right] (\nabla \times \vec{B}) \times \vec{B} &\equiv \sigma (\nabla \times \vec{B}) \times \vec{B} \\ &= \nabla_{\perp} p + \frac{(p_{\parallel} - p_{\perp})}{B} \nabla_{\perp} \vec{B} . \end{aligned} \quad (2.9)$$

Note that

$$\begin{aligned} \vec{B} \times [\nabla \times (\sigma \vec{B})] &= \sigma \vec{B} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \sigma \times \vec{B}) \\ &= \sigma \vec{B} \times (\nabla \times \vec{B}) + B^2 \nabla_{\perp} \sigma \\ &= \sigma \vec{B} \times (\nabla \times \vec{B}) - B^2 \nabla_{\perp} \left(\frac{p_{\parallel} - p_{\perp}}{B^2} \right) , \end{aligned} \quad (2.10)$$

whence

$$\sigma (\nabla \times \vec{B}) \times \vec{B} = [\nabla \times (\sigma \vec{B})] \times \vec{B} - B^2 \nabla_{\perp} \left(\frac{p_{\parallel} - p_{\perp}}{B^2} \right) .$$

Substituting this result into 2.9 gives

$$[\nabla \times (\sigma \vec{B})] \times \vec{B} = \nabla_{\perp} p_{\parallel} - \left(\frac{p_{\parallel} - p_{\perp}}{B} \right) \nabla_{\perp} \vec{B} . \quad (2.11)$$

When the right-hand side is resolved into components, a striking simplification appears. For the moment we consider a general coordinate system in which field lines are labeled by (α, β) where $\vec{B} = \vec{\nabla}\alpha \times \vec{\nabla}\beta$. For $p_{\parallel} = p_{\parallel}(\alpha, \beta, B)$, (2.11) becomes

$$\begin{aligned} [\vec{\nabla} \times (\sigma \vec{B})] \times \vec{B} &= \frac{\partial p_{\parallel}}{\partial B} \vec{\nabla}_{\perp} B + \frac{\partial p_{\parallel}}{\partial \alpha} \vec{\nabla}_{\perp} \alpha + \frac{\partial p_{\parallel}}{\partial \beta} \vec{\nabla}_{\perp} \beta - \left(\frac{p_{\parallel} - p_{\perp}}{B} \right) \vec{\nabla}_{\perp} B \\ &= \frac{\partial p_{\parallel}}{\partial \alpha} \vec{\nabla} \alpha + \frac{\partial p_{\parallel}}{\partial \beta} \vec{\nabla} \beta, \end{aligned} \quad (2.12)$$

where we have used 2.4 and the fact that α and β are constant along field lines.

Now define a vector $\vec{K}$ such that $\vec{K}_{\perp} \times \vec{B} = \text{Right-Hand Side of eq. (2.12)}$.

$$\vec{K}_{\perp} = \frac{\hat{b}}{B} \times \left(\frac{\partial p_{\parallel}}{\partial \alpha} \vec{\nabla} \alpha + \frac{\partial p_{\parallel}}{\partial \beta} \vec{\nabla} \beta \right), \quad (2.13)$$

so that (2.12) can be re-written as

$$[\vec{\nabla} \times (\sigma \vec{B}) - \vec{K}_{\perp}] \times \vec{B} = 0, \quad (2.14)$$

that is,

$$\vec{\nabla} \times (\sigma \vec{B}) = \vec{K} = \vec{K}_{\perp} + \lambda_1 \vec{B} \quad (2.15)$$

Expanding $\vec{\nabla} \times (\sigma \vec{B}) = \vec{\nabla} \sigma \times \vec{B} + \sigma \vec{\nabla} \times \vec{B}$ and replacing $\vec{\nabla} \times \vec{B}$ from 2.3, we can identify the constant λ_1 in (2.15):

$$\vec{\nabla} \sigma \times \vec{B} + \sigma \mu_0 \vec{j} = \vec{K}_{\perp} + \lambda_1 \vec{B} \quad (2.16)$$

$\vec{K}_\perp$ is clearly perpendicular to $\hat{b}$ from (2.13) so that the scalar product of (2.16) by $\hat{b}$ gives

$$\sigma \mu_0 j_\parallel = \lambda_\perp B. \quad (2.17)$$

If we use the constitutive relation

$$\vec{B} = \mu \vec{H} = \frac{1}{\sigma} \vec{H}$$

then (2.15) reduces to

$$\nabla \times \vec{H} = \vec{K}$$

and $\vec{K}$ is then interpreted as the "macroscopic" current (total current minus magnetization current). In the next section we examine the case where the pressure tensor is only a function of B. As we shall see, this causes the "macroscopic" current to be zero. More general forms for the pressure tensor lead to non-zero "macroscopic" currents of increasing complexity.

III. PRESSURE A FUNCTION OF ONLY B

Here $p_\parallel = \hat{p}_\parallel(B)$ so that (2.13) reduces to $\vec{K}_\perp = 0$. In order to demonstrate that $K_\parallel$ is also zero we note that (2.16) reduces to

$$\mu_0 \vec{j} = \frac{1}{\sigma} \lambda_\perp \vec{B} + \vec{B} \times \nabla \ln \sigma$$

Since $\nabla \cdot \vec{j} = 0$, we have

$$\begin{aligned} 0 &= \frac{B}{\sigma} \frac{\partial \lambda_\perp}{\partial \ell} - \frac{\lambda_\perp B}{\sigma^2} \frac{\partial \sigma}{\partial \ell} + \nabla(\ln \sigma) \cdot (\nabla \times \vec{B}) \\ &= \frac{B}{\sigma} \frac{\partial \lambda_\perp}{\partial \ell} - \frac{\lambda_\perp B}{\sigma^2} \frac{\partial \sigma}{\partial \ell} + \frac{\nabla(\ln \sigma)}{\sigma} \cdot [\lambda_\perp \vec{B} + \vec{B} \times \nabla \ln \sigma] \\ &= \frac{B}{\sigma} \frac{\partial \lambda_\perp}{\partial \ell} \end{aligned}$$

Thus $\partial\lambda_{\perp}/\partial\ell = 0$ and $\lambda_{\perp}$ is constant along a field line. The natural choice for a mirror device is that $\lambda_{\perp} = 0$ so that there is no current in the vacuum regions.

Thus $\vec{K} = 0$ and (2.15) reduces to

$$\nabla \times (\sigma \vec{B}) = 0$$

with solution

$$\vec{H} \equiv \sigma \vec{B} = \nabla \varphi \quad (3.1)$$

Substituting (3.1) into $\nabla \cdot \vec{B} = 0$ yields

$$\nabla^2 \varphi - \frac{1}{\sigma} \nabla \sigma \cdot \nabla \varphi = 0 \quad (3.2)$$

Because the permeability, σ^{-1} , is a function of B and hence a function of φ through (3.1) equation (3.2) is non-linear. One can demonstrate that (3.2) is an elliptic differential equation if $\tau = \mu_0^{-1} + B^{-1} \partial p_{\perp} / \partial B \geq 0$. This condition is often cited as the criterion for stability of "mirror" modes, just as $\sigma \geq 0$ is a stability criterion for "firehose" modes.

In order to make some of the concepts more concrete, we present a specific model for the plasma pressure.

$$\begin{aligned} \mu_0 p_{\perp} &= \frac{1}{2} S (B_C^2 - B^2) \quad , \text{ if } B \leq B_C \\ \mu_0 p_{\parallel} &= \frac{1}{2} S (B_C^2 - B^2) \quad , \text{ if } B \leq B_C \\ \mu_0 \sigma &= 1 + S \left(\frac{B_C}{B} - 1 \right) \quad , \text{ if } B \leq B_C \\ \mu_0 \tau &= 1 - S \quad , \text{ if } B \leq B_C \end{aligned} \quad (3.3)$$

For $B \geq B_C$, $p_{\perp} = p_{\parallel} = 0$ and $\mu_0 \sigma = \mu_0 \tau = 1$. Here S and B_C are constants.

Notice that for $S < 1$, σ and τ are positive and the "mirror" and "firehose" stability criteria are guaranteed at every point in the plasma.

Because of the nonlinearity of the problem an iteration technique is indicated. We shall first give an overview of the iteration scheme and then sketch some of the more detailed considerations necessary for numerical solution. One begins the iteration scheme by using the $\vec{B}$ produced by the coils alone to compute σ from (3.3) and hence compute the coefficients in (3.2), $\vec{\nabla}\sigma/\sigma$. One then solves (3.2) for φ . From (3.1) one then computes a new $\vec{B}$. From this new value of $\vec{B}$ one recalculates $\vec{\nabla}\sigma/\sigma$ and solves (3.2) again for φ . This procedure is repeated until a desired degree of convergence is obtained.

In actual practice it is best to write

$$\varphi = \varphi_0 + \varphi_1$$

where φ_0 is the contribution from the coils alone and φ_1 is the contribution from the plasma. Since φ_0 is known, (3.2) is then cast in the form

$$\nabla^2 \varphi_1 - \frac{1}{\sigma} \vec{\nabla}\sigma \cdot \vec{\nabla}\varphi_1 = \frac{1}{\sigma} \vec{\nabla}\sigma \cdot \vec{\nabla}\varphi_0 \quad (3.4)$$

where we have noted that $\nabla^2 \varphi_0 = 0$. The initial guess for φ_1 is usually zero, so that φ is initially due to the coils alone.

Since (3.4) is an elliptic differential equation we must specify appropriate boundary conditions to determine the solution. For the steady state plasmas which we have in mind there are two convenient sets of boundary conditions and a third realistic one. The first (convenient) boundary condition is that φ_1 is constant on the boundary. This boundary condition is often termed ferromagnetic since it corresponds to the boundary being surrounded by a high (infinite) permeability material. A second convenient boundary condition is that the normal derivative of

φ_1 vanish at the boundary. This is the so-called flux conserving boundary condition. It is often used for pulsed experiments which are surrounded by a highly conducting wall so that the magnetic flux cannot soak into the wall in the duration of the experiment.

The most realistic boundary condition for steady state plasmas is that $\vec{B}_1$ be regular at infinity. Because one must use finite size grids to solve the problem numerically, one specifies the normal derivative of φ_1 on the grid boundary. To specify the normal derivative of φ_1 on the boundary one calculates $\vec{B}_1'$ on the boundary which can, in principle be obtained from the law of Biot and Savart after the plasma currents are determined. In practice one uses a multipole expansion (expansion in vector spherical harmonics) to calculate $\vec{B}_1$ rather than the full integral appearing in the law of Biot and Savart. Because the plasma currents change in the course of iteration, they must be recalculated repeatedly. This procedure obviously involves considerable computation but is necessary if one requires the far field of the plasma.

Fortunately, if one is only interested in the region near the plasma, any one of these sets of boundary conditions can be used provided the boundary is sufficiently far from the plasma. In any given problem the size of the grid relative to the plasma must be determined by experimentation. A useful ruse for determining the effect of boundary conditions upon the near fields is to use both ferromagnetic and flux conserving boundary conditions and compare the results.

For fixed σ and φ_0 equation (3.4) looks remarkably like Poisson's equation. An iteration scheme which is adequate under these circumstances is successive over relaxation (SOR). However in the overall scheme σ is

not fixed and numerical instability can often occur. This can be avoided by introducing an artificial viscosity. That is, instead of using the newly computed σ in equation (3.4) one uses a linear combination of the newly computed sigma and the previous sigma [eg. $\bar{\sigma} = \frac{1}{2} (\sigma_{\text{new}} + \sigma_{\text{old}})$]. More sophisticated and hence more efficient procedures can also be utilized (see Anderson and Killeen).⁸

IV. AXISYMMETRIC SYSTEMS

Here we restrict ourselves to axisymmetric systems such as ELMO with $j_{\parallel} = 0 = \lambda_1$. Then all the currents are in the azimuthal direction and the vector potential has a single component so that (in cylindrical coordinates)

$$\vec{B} = \nabla \times \vec{A} = -\frac{\hat{r}}{r} \frac{\partial}{\partial z} (rA) + \frac{\hat{z}}{r} \frac{\partial}{\partial r} (rA)$$

Thus

$$\hat{\theta} \times \vec{B} = \frac{1}{r} \left[\hat{r} \frac{\partial}{\partial r} (rA) + \hat{z} \frac{\partial}{\partial z} (rA) \right] \equiv \frac{1}{r} \nabla \psi \quad (4.1)$$

or

$$\vec{B} = \frac{\nabla \psi \times \hat{\theta}}{r} = \nabla \psi \times \nabla \theta \quad (4.2)$$

Notice that $\vec{B}$ is written in a Clebsch representation ($\vec{B} = \nabla \alpha \times \nabla \beta$) with $\psi = \alpha$ and $\theta = \beta$. Because of the axisymmetry θ is an ignorable coordinate so that $p_{\parallel}(\alpha, \beta, B) = p_{\parallel}(\psi, B)$ and (2.12) reduces to

$$(\nabla \times \sigma \vec{B}) \times \vec{B} = \frac{\partial p_{\parallel}}{\partial \psi} \nabla \psi$$

Inserting (4.2) into the left hand side and expanding in cylindrical coordinates yields

$$-\left(\frac{\partial}{\partial z} \frac{\sigma}{r} \frac{\partial \psi}{\partial z} + \frac{\partial}{\partial r} \frac{\sigma}{r} \frac{\partial \psi}{\partial r} \right) \frac{1}{r} \left(\hat{r} \frac{\partial \psi}{\partial r} + \hat{z} \frac{\partial \psi}{\partial z} \right) = \frac{\partial p_{\parallel}}{\partial \psi} \nabla \psi$$

or

$$\Delta^* \psi + \frac{1}{\sigma} \vec{\nabla} \sigma \cdot \vec{\nabla} \psi = - \frac{r^2}{\sigma} \frac{\partial p_{\parallel}}{\partial \psi} \quad (4.3)$$

where

$$\Delta^* \psi \equiv \left(\frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) \psi$$

Equation (4.3) is analogous to (3.2) and is also elliptic provided $\tau > 0$.

An obvious choice for $p_{\parallel}(B, \psi)$ is a separable function of B and ψ .

That is

$$p_{\parallel} = \hat{p}_{\parallel}(B) g(\psi) \quad (4.4)$$

$$p_{\perp} = \hat{p}_{\perp}(B) g(\psi)$$

where $g(\psi)$ is a form factor such that $0 \leq g \leq 1$, and the functional form for $\hat{p}_{\parallel}(B)$ and $\hat{p}_{\perp}(B)$ is given for example by (3.3) so that

$$\mu_0 p_{\parallel} = \frac{1}{2} S g (B_C - B)^2 \quad \text{if } B < B_C \quad (4.5)$$

$$\mu_0 p_{\perp} = \frac{1}{2} S g (B_C^2 - B^2) \quad \text{if } B < B_C$$

Again, since $0 \leq g \leq 1$, "mirror" and "firehose" stability are guaranteed for $S < 1$.

The choice for the form factor $g(\psi)$ is usually guided by the experimental device. For example it may be possible to measure the pressure in the midplane ($z = 0$) as a function of radius. Consequently one would write

$$g(\psi) = \hat{g} [r_M(\psi)]$$

where r_M is the radius in the midplane and is regarded as a function of ψ . This procedure is closely related to one which is discussed in the next section for a non-axisymmetric case.

An alternate procedure which works well in practice is to treat $g(\psi)$ as a polynomial in ψ and periodically adjust the coefficient so that reasonable agreement between calculated and experimental pressure profiles is achieved. (If this adjustment is not done one can easily compute nonsense for sufficiently high beta. For example one can find that the plasma pressure is nonzero outside the container of the actual device.)

The iteration procedure used in this instance is similar to that discussed in the previous section. The initial estimate for the flux function ψ is taken to be that produced by the coils alone. From (4.2) one calculates B . Using equations (4.5) the coefficients and driving term in (4.3) can be calculated. One then solves (4.3) for a new ψ . From the new ψ one recalculates the coefficients and driving term in (4.3) and solves for ψ again. This process is continued until convergence occurs.

V. SEPARABLE PRESSURE

Here we assume that the components of the pressure tensor can be expressed as separable functions of B and α , say, similar to those given by (4.5):

$$\begin{aligned} p_{\parallel} &= \hat{p}_{\parallel}(B) g(\alpha) \\ p_{\perp} &= \hat{p}_{\perp}(B) g(\alpha) \end{aligned} \tag{5.1}$$

The restriction that g not be a function of β is a notational convenience. As contrasted to the axisymmetric case (where $\alpha = \psi$) the surfaces of constant α are not uniquely determined in an obvious way without an auxiliary condition. If one demands that the parallel current go to zero where the

pressure goes to zero in a mirror device (or at a symmetry plane in EBT) then the topology of the constant α surfaces is determined. Note however that in many experimental configurations the high pressure plasma is imbedded in a cold plasma which can provide good electrical connection to conducting end walls. In this instance it is not clear that the parallel current must go to zero where the pressure is negligibly small and the topology of the α -surfaces can be chosen in some other way. (For example one might ask that they agree with single particle drift surfaces for a certain class of particles.)

Because of the experimental observations we chose the contours of constant α to match the vacuum $|\vec{B}|$ contours in the midplane in calculating equilibria for the Canted Mirror Facility. For low plasma pressure this is equivalent to matching the α -surfaces to the drift surfaces for particles with $v_{||} = 0$ since they drift along $|\vec{B}|$ contours in the midplane. Note that for short annular plasmas (i.e., $v_{||}$ not too large) all the drift surfaces are approximately the same so that in addition to agreeing with experimental observations the choice for the α -surfaces is a reasonable assumption from a theoretical standpoint.

We begin by developing the basic iteration scheme (independent of how the α contours are chosen or restricted), and return later to the choice of isobaric surfaces. From (2.15), $\nabla \times (\sigma \vec{B}) = \vec{K}$, it follows that $\nabla \cdot \vec{K} = 0$. Now any vector field with vanishing divergence can be described by a Clebsch representation:

$$\vec{K} = \nabla \varphi_1 \times \nabla \varphi_2 = \nabla \times (\varphi_1 \nabla \varphi_2) \quad (5.2)$$

Then (2.15) becomes

$$\nabla \times (\sigma \vec{B} - \varphi_1 \vec{\nabla} \varphi_2) = 0 \quad (5.3)$$

with solutions of the form

$$\sigma \vec{B} = \varphi_1 \vec{\nabla} \varphi_2 + \vec{\nabla} \varphi_3 \quad (5.4)$$

The choice of the functions φ_1 and φ_2 is somewhat arbitrary. To motivate our choice we use (5.2) to form:

$$\vec{K} \times \vec{B} = \vec{\nabla} \varphi_2 (\vec{B} \cdot \vec{\nabla} \varphi_1) - \vec{\nabla} \varphi_1 (\vec{B} \cdot \vec{\nabla} \varphi_2) \quad (5.5)$$

Alternatively if we substitute (5.1) into (2.13) we find that

$$\vec{K} \times \vec{B} = \hat{p}_{\parallel} \vec{\nabla} g$$

Comparing these last two equations suggests that we choose

$$\varphi_2 = g(\alpha) \quad (5.6)$$

for then (5.5) reduces to

$$\vec{K} \times \vec{B} = \vec{\nabla} g (\vec{B} \cdot \vec{\nabla} \varphi_1) = \hat{p}_{\parallel} \vec{\nabla} g \quad (5.7)$$

provided

$$\vec{B} \cdot \vec{\nabla} \varphi_1 = \hat{p}_{\parallel}$$

or

$$\varphi_1 = \int_0^{\ell} \frac{d\ell'}{B} \hat{p}_{\parallel} \quad (5.8)$$

To complete our system of equations we substitute (5.4) into $\nabla \cdot \vec{B} = 0$

$$\nabla^2 \varphi_3 - \frac{1}{\sigma} \vec{\nabla} \sigma \cdot \vec{\nabla} \varphi_3 = - \nabla \cdot (\varphi_1 \vec{\nabla} g) + \frac{\vec{\nabla} \sigma}{\sigma} \cdot (\varphi_1 \vec{\nabla} g) \quad (5.9)$$

Notice that for the special case $g = 1$, (5.9) reduces to (3.2).

We now obtain the restriction on α for mirror like plasmas for which the pressure vanishes at some point on the field line. From (5.2) and (5.3) we have

$$\mu_0 \vec{j} = \nabla \times \vec{B} = \frac{1}{\sigma} (\nabla \varphi_1 \times \nabla \varphi_2 + \vec{B} \times \nabla \sigma) \quad (5.10)$$

Regarding φ_1 as a function of α , β , ℓ and using $\varphi_2 = g(\alpha)$, we find that (5.10) reduces to.

$$\begin{aligned} \mu_0 \vec{j} = \frac{1}{\sigma} \left\{ -g'(\alpha) \frac{\partial \varphi_1}{\partial \beta} (\nabla \alpha \times \nabla \beta) - g'(\alpha) \frac{\partial \varphi_1}{\partial \ell} (\nabla \alpha \times \nabla \ell) \right. \\ \left. + \vec{B} \times \nabla \sigma \right\} \end{aligned} \quad (5.11)$$

Now for $\hat{p}_{\parallel}$ such as indicated in (4.5), the pressure is zero and σ is constant for $\ell > \ell_C$ (ℓ_C corresponding to B_C). Thus for $\ell > \ell_C$, is equal to

$$\bar{\varphi}_1 \equiv \int_0^{\ell_C} \frac{d\ell}{B} \hat{p}_{\parallel} \quad (5.12)$$

and $\partial \varphi_1 / \partial \ell = \nabla \sigma = 0$ so that (5.11) reduces to

$$\mu_0 \vec{j} = -g'(\alpha) \frac{\partial \bar{\varphi}_1}{\partial \beta} \vec{B} \quad (5.13)$$

If we demand that the current go to zero where the pressure is zero, then we obtain the constraint

$$\partial \bar{\varphi}_1 / \partial \beta = 0 ; \quad (5.14)$$

i.e, $\bar{\varphi}_1$ is independent of β and surfaces of constant α are surfaces of constant $\bar{\varphi}_1$. Consequently $g(\alpha)$ is a function of $\bar{\varphi}_1$.

For closed line devices such as EBT where the pressure need not vanish anywhere along a field line, Grad has used a different argument to show that

$$\bar{\varphi}_1 \equiv \oint \frac{d\ell}{B} \hat{p}_{\parallel}(B) \quad (5.15)$$

is again a function of α only. These results are the tensor-pressure analogs of the fact that for scalar pressure the pressure must be a function of $\oint d\ell/B$ for closed line systems.

We shall first discuss the iteration scheme associated with these equations for a device where $j_{\parallel}$ need not vanish and (5.14) can be neglected. (We have in mind the Canted Mirror Facility with conducting end plates.) We will then consider the modifications necessary to meet the restriction imposed by (5.14).

For the Canted Mirror Facility with conducting end plates, $g(\alpha)$ is regarded as a fixed function of position in the midplane (namely contours of constant α are contours of constant vacuum B .) As before the field produced by the coils alone is used for the starting point in the iteration scheme. In order to determine g and φ_1 for points not in the midplane we simply follow field lines back from the point in question to the midplane. Since g is constant along a field line and φ_1 is given by a line integral along the field line (5.8), they are readily determined.

Having determined g and φ_1 for all points of interest, we then evaluate the coefficients and driving term in (5.9). We then solve (5.9) for φ_3 .

In the next pass through the iteration scheme we use (5.4) to compute a new $\vec{B}$. Using the new field, g and φ_1 can be recalculated for points which are not in the midplane (again by following field lines back to the midplane). The coefficients and driving term in (5.4) can then be recalculated and (5.9) solved for φ_3 again. This procedure is repeated until a satisfactory degree of convergence is obtained.

We now consider the necessary modifications to this scheme when the restriction (5.14) is imposed (as required for EBT). Here $g(\alpha)$ cannot be regarded as a fixed function of position in the midplane, but must be recalculated at each pass through the iteration scheme, since g must be a function of $\bar{\phi}_1$, itself a function $p_{||}$ whose value at any point in space changes from one iteration to the next. Thus at the start of each pass through the iteration process we compute $\bar{\phi}_1$ for a number of points in the midplane so that contours of constant $\bar{\phi}_1$ can be determined. For low plasma pressure these contours are topologically equivalent to nested circles. In this case we need only specify g along some ray in the midplane (say $x \geq 0, y = 0$); g is then specified for the remainder of the midplane by mapping along contours of constant $\bar{\phi}_1$.

At higher values of plasma pressure Grad¹¹ has pointed out that the $\bar{\phi}_1$ contours need not be topologically equivalent to nested circles (i.e. "islands" can occur). As an example of how this can happen suppose that the plasma suppresses B in the midplane more for positive x than for negative x , so that in the midplane B has a saddle point. If the plasma is concentrated near the midplane (a short annulus) then one would expect $\bar{\phi}_1$ to be dominated by the behavior of B in the midplane and hence $\bar{\phi}_1$ would also have a saddle point. (See figure 1.) Since g must be a function of $\bar{\phi}_1$, care must now be taken in specifying g along the line $y = 0$ for $x_1 \leq x \leq x_2$. Traditionally (for scalar pressure) one arranges that g be constant for $x_1 \leq x \leq x_2$ so that it has the form shown in Figure 2, which also illustrates that the g chosen for low plasma pressure can be significantly modified by this procedure. Thus as the plasma pressure is increased the shape of g as a function of x changes; hence our control of g is somewhat indirect. This may present difficulties in matching calculation and experiment.

Fig. 1. Complicated topology associated with $\bar{\phi}_1$. The topology of $\bar{\phi}_1$ is due to the fact that the well depth of the magnetic field varies the short way around the torus.

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Fig. 2. g as a function of x for $x \geq 0$. Solid curve shows g for the topology of $\bar{\phi}_1$ indicated in Figure 1. Dashed curve shows g for a lower value of plasma pressure for which $\bar{\phi}_1$ is topologically equivalent to nested circles.

In the example of Fig. 1 only a single saddle point occurred. More generally, several saddle points can occur and the topology becomes more complicated. However once the topology for a given situation is known, specific numerical methods for determining the contours for g can be developed.

VI. PRESSURE A FUNCTION OF α , β AND B

Finally we consider the general case where $p_{\parallel}$ is an arbitrary function of α , β and B (subject to certain constraints). While we do not foresee an immediate need for the β dependence, we do anticipate that for EBT the separability assumed in the previous section will not be appropriate; since, in addition to the anisotropic, high-pressure annulus characteristic of ELMO, there will be a component of circulating plasma which can be described in terms of a scalar pressure, $f(\alpha)$, so that (in contrast to 5.1)

$$p_{\parallel}(\alpha, B) = \hat{p}_{\parallel}(B) g(\alpha) + f(\alpha) \quad (6.1)$$

We begin by making the ansatz:

$$\sigma \vec{B} = \psi_1 \vec{\nabla} \alpha + \psi_2 \vec{\nabla} \beta + \vec{\nabla} \varphi_3 \quad (6.2)$$

Substitution of (6.2) into (2.15) yields

$$\vec{\nabla} \psi_1 \times \nabla \alpha + \vec{\nabla} \psi_2 \times \nabla \beta = \vec{K} \quad (6.3)$$

or

$$\vec{K} \times \vec{B} = (\vec{B} \cdot \vec{\nabla} \psi_1) \vec{\nabla} \alpha + (\vec{B} \cdot \vec{\nabla} \psi_2) \vec{\nabla} \beta \quad (6.4)$$

But from (2.13) we have

$$\vec{K} \times \vec{B} = \frac{\partial p_{\parallel}}{\partial \alpha} \vec{\nabla} \alpha + \frac{\partial p_{\parallel}}{\partial \beta} \vec{\nabla} \beta \quad (6.5)$$

Comparing these last two equations we see that it is natural to choose

$$\vec{B} \cdot \vec{\nabla} \psi_1 = \partial p_{\parallel} / \partial \alpha \quad (6.6)$$

$$\vec{B} \cdot \vec{\nabla} \psi_2 = \partial p_{\parallel} / \partial \beta$$

or

$$\begin{aligned} \psi_1 &= \int_0^{\ell} \frac{d\ell'}{B} \frac{\partial p_{\parallel}}{\partial \alpha} \\ \psi_2 &= \int_0^{\ell} \frac{d\ell'}{B} \frac{\partial p_{\parallel}}{\partial \beta} \end{aligned} \quad (6.7)$$

Because of (2.15), $\nabla \times (\sigma \vec{B}) = \vec{K}$, it must be true that $\nabla \cdot \vec{K} = 0$. Since $\vec{\nabla} \psi_1 \times \vec{\nabla} \alpha = \nabla \times (\psi_1 \vec{\nabla} \alpha)$ it follows that

$$\vec{K} = \nabla \times (\psi_1 \vec{\nabla} \alpha) + \nabla \times (\psi_2 \vec{\nabla} \beta)$$

verifying that $\nabla \cdot \vec{K} = 0$. Using (2.16) we find after some manipulation that

$$\mu_0 \nabla \cdot \vec{j} = \frac{1}{\sigma} \nabla \cdot \vec{K}$$

so that $\nabla \cdot \vec{j} = 0$ as well.

The system of equations is completed as in section V by substituting (6.2) into $\nabla \cdot \vec{B} = 0$. Thus

$$\begin{aligned} \nabla^2 \varphi_3 - \frac{1}{\sigma} \vec{\nabla} \sigma \cdot \vec{\nabla} \varphi_3 &= - \nabla \cdot (\psi_1 \vec{\nabla} \alpha + \psi_2 \vec{\nabla} \beta) \\ &\quad + \frac{\vec{\nabla} \sigma}{\sigma} \cdot (\psi_1 \vec{\nabla} \alpha + \psi_2 \vec{\nabla} \beta) \end{aligned} \quad (6.8)$$

For mirror plasmas we can find constraints on (6.7) in essentially the same way that we used (5.8) to obtain (5.14). Thus for the regions

along a field line where the pressure is zero we find $\psi_1 = \bar{\psi}_1$ and $\psi_2 = \bar{\psi}_2$ where

$$\begin{aligned}\bar{\psi}_1 &= \int_0^{\ell_C} \frac{d\ell}{B} \frac{\partial p_{\parallel}}{\partial \alpha} \\ \bar{\psi}_2 &= \int_0^{\ell_C} \frac{d\ell}{B} \frac{\partial p_{\parallel}}{\partial \beta}\end{aligned}\tag{6.9}$$

and in these regions

$$\mu_0 \vec{j} = \left(\frac{\partial \bar{\psi}_2}{\partial \alpha} - \frac{\partial \bar{\psi}_1}{\partial \beta} \right) (\nabla \alpha \times \nabla \beta)\tag{6.10}$$

so that if we demand that $\vec{j} = 0$ in these regions we obtain

$$\frac{\partial \bar{\psi}_1}{\partial \beta} = \frac{\partial \bar{\psi}_2}{\partial \alpha}\tag{6.11}$$

For devices such as EBT we must use a slightly different argument. We begin by assuming that ψ_1 and ψ_2 are functions of α , β and η where η is the angle the long way around the torus. Then from (6.3) and (2.16) we obtain

$$\begin{aligned}\sigma \mu_0 j_{\parallel} &= \hat{b} \cdot \left\{ (\nabla \alpha \times \nabla \beta) \left[\frac{\partial \psi_2}{\partial \alpha} - \frac{\partial \psi_1}{\partial \beta} \right] \right. \\ &\quad \left. + (\nabla \eta \times \nabla \alpha) \frac{\partial \psi_1}{\partial \eta} + (\nabla \eta \times \nabla \beta) \frac{\partial \psi_2}{\partial \eta} \right\}\end{aligned}\tag{6.12}$$

Because of the assumed symmetry of EBT, $\nabla \eta$ is parallel to $\hat{b}$ at all mid-planes and coil-planes. Consequently in any of these planes

$$\sigma \mu_0 j_{\parallel} = \hat{b} \cdot (\nabla \alpha \times \nabla \beta) \left[\frac{\partial \psi_2}{\partial \alpha} - \frac{\partial \psi_1}{\partial \beta} \right]$$

But in these planes the assumed symmetry of $\vec{j}$ requires that $j_{\parallel} = 0$. Consequently in these planes we require

$$\frac{\partial \psi_1}{\partial \beta} = \frac{\partial \psi_2}{\partial \alpha} \quad (6.13)$$

We now assume for concreteness that $\eta = \ell = 0$ corresponds to a midplane, and $\eta = \eta_1$ and $\ell = \ell_1$ corresponds to an adjacent coil-plane. Then for $\eta = \eta_1$

$$\begin{aligned} \psi_1 &= \bar{\psi}_1 \equiv \int_0^{\ell_1} \frac{d\ell}{B} \frac{\partial p_{\parallel}}{\partial \alpha} \\ \psi_2 &= \bar{\psi}_2 \equiv \int_0^{\ell_1} \frac{d\ell}{B} \frac{\partial p_{\parallel}}{\partial \beta} \end{aligned} \quad (6.14)$$

and we again obtain (6.11) as a condition on $\bar{\psi}_1$ and $\bar{\psi}_2$.

The iteration scheme to be used in this instance is nearly identical to that discussed in Section V. Here ψ_1 and ψ_2 play an analogous role to that of φ_1 while α and β play the role of φ_2 ($\varphi_2 = g$ in section V). Thus the relevant equations for the present iteration scheme are (6.2), (6.7) and (6.8). If we can avoid the constraint condition (6.11), then the present case is little more difficult than that discussed in section V. However for EBT we must observe (6.11) and this constraint is considerably more complicated than that obtained in section V ($\partial \varphi_1 / \partial \beta = 0$).

To illustrate the complexities associated with the constraint condition (6.11) let us return to the simpler case indicated by (6.1). Here $p_{\parallel}$ has no β dependence and we may take $\psi_2 = \bar{\psi}_2 = 0$. Then (6.11) reduces to

$$\partial \bar{\psi}_1 / \partial \beta = 0 \quad (6.15)$$

with

$$\bar{\psi}_1 = g'(\alpha) u_1 + f'(\alpha) u_2 \quad (6.16)$$

and

$$u_1 = \oint \frac{d\ell}{B} \hat{p}_{\parallel}(B) \quad (6.17)$$

$$u_2 = \oint \frac{d\ell}{B}$$

We notice that u_1 is the same as $\bar{\phi}_1$ in section V. As we saw in section V, $\bar{\phi}_1$ (or u_1) can have saddle points. Here u_2 can also possess saddle points and even if neither u_1 nor u_2 have saddle points, a linear combination of them, such as (6.16) can have saddle points. As we saw before the occurrence of saddle points makes one adjust the functional form of $g(\alpha)$ to conform to the topology. Here we must also adjust $f(\alpha)$ to conform to the topology. Notice however that adjusting g and f can change the topology. This interdependence is of course possible to deal with in a variety of ways. The disconcerting point is that these various methods almost surely lead to different equilibria. The choice of methods is probably governed by a combination of convenience and one's concept of the time dependent build up of pressure in the actual device.

Even after a decision is made on the precise form for f and g , assigning f and g to the required grid points is more difficult than for the separable case. Here one must solve an equation of the form

$$\bar{\psi}_1 = u_1 \hat{g}(\bar{\psi}_1) + u_2 \hat{f}(\bar{\psi}_1)$$

to determine the contours of constant $\bar{\psi}_1$ so that f and g can be assigned over the entire midplane.

VII. SUMMARY AND CONCLUSIONS

We have presented a number of iteration schemes for various specializations of the general case considered in Section VI. (The equations of Section IV are not readily derived from those of Section VI because of explicit use of axisymmetry.) Because the equations of Sections III and V can be obtained from those of Section VI, computer codes for calculating equilibria for more general forms of the pressure tensor can be developed simply by adding new subroutines rather than by complete revision. To illustrate this point we now collect and compare the basic equations from these three sections.

For the initial case considered in Section III, $p_{\parallel} = p_{\parallel}(B)$, and we obtained

$$\sigma \vec{B} = \vec{\nabla} \varphi \quad (3.1)$$

For the separable case discussed in Section V, $p_{\parallel} = \hat{p}_{\parallel}(B) g(\alpha)$, the analogous equation is

$$\begin{aligned} \sigma \vec{B} &= \vec{\nabla} \varphi + \varphi_1 \vec{\nabla} \varphi_2 = \vec{\nabla} \varphi + \varphi_1 \vec{\nabla} g \\ &= \vec{\nabla} \varphi + \varphi_1 g'(\alpha) \vec{\nabla} \alpha \end{aligned} \quad (5.4)$$

where

$$\varphi_1 = \int_0^{\ell} \frac{d\ell'}{B} \hat{p}_{\parallel}(B) \quad (5.8)$$

while for the general case of Section VI

$$\sigma \vec{B} = \vec{\nabla} \varphi + \psi_1 \vec{\nabla} \alpha + \psi_2 \vec{\nabla} \beta \quad (6.2)$$

where

$$\begin{aligned}\psi_1 &= \int_0^L \frac{d\ell}{B} \frac{\partial p_{\parallel}}{\partial \alpha} \\ \psi_2 &= \int_0^L \frac{d\ell}{B} \frac{\partial p_{\parallel}}{\partial \beta}\end{aligned}\tag{6.7}$$

If we substitute $p_{\parallel} = \hat{p}_{\parallel}(B)g(\alpha)$ into (6.7) we find that $\psi_2 = 0$ and $\psi_1 = g'(\alpha)\varphi_1$, so that (6.2) reduces to (5.4). By taking $g(\alpha)$ equal to unity so that $g'(\alpha) = 0$, we obtain (3.1) as the expression for $\sigma\vec{B}$.

In each instance substitution into $\nabla \cdot \vec{B} = 0$ gives a second order differential equation for φ (3.4, 5.9, 6.8) of the form

$$\nabla^2 \varphi - \frac{\nabla \sigma}{\sigma} \cdot \nabla \varphi = D\tag{7.1}$$

Thus a basic ingredient of all these iteration schemes is a technique for solving differential equations of the form (7.1).

Various potentials (φ_1 , ψ_1 , ψ_2) required to evaluate the pressure components necessitate numerical evaluation of line integrals along magnetic lines of force. For the general case one must be able to map both α and β along field lines and compute the two line integrals for ψ_1 and ψ_2 . This does not represent a substantial change in the algorithms and storage requirements of the separable case.

The specification of α and β contours in the midplane (or some other convenient surface) is relatively simple if one can neglect questions associated with $j_{\parallel}$. The requirement that $j_{\parallel} = 0$ in vacuum for mirror-like devices or at symmetry planes for closed line torii complicates matters somewhat. Actually the differences between the two cases is minor unless saddle points occur. Dealing with the complicated topology

associated with saddle points is non-trivial for the separable case and even more difficult for non-separable cases.

It is tempting to regard these questions of the topology of $\bar{\psi}_1$ ($\bar{\psi}_1$ and $\bar{\psi}_2$ in the general case) as a baroque worry brought about by an esoteric mathematical formulation of the problem. That this is not the case is illustrated by the example considered in figure 1, in which "islands" appear in the $|\vec{B}|$ -contours as well as in the $\bar{\psi}_1$ -contours. Since the particles with $v_{||} = 0$ drift along these $|\vec{B}|$ -contours, such a magnetic field causes some peculiar single particle motions. Whether high beta equilibria can be found which do not have a complicated topology remains an interesting question.

In conclusion, while there are still unresolved questions of topology and its numerical treatment, they can no doubt be resolved as warranted. The basic iteration techniques now exist for handling the most general form of the pressure tensor for open ended and closed line devices.

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