U. of Iowa 71-16
C00-2059-7

MASTER

Nonlinear Evolution of the
Dory-Guest-Harris Instability*

Glenn Joyce

and

Georg Knorr

Department of Physics and Astronomy
The University of Iowa
Iowa City, Iowa 52240

May 1971

*This work was supported in part by Atomic Energy Commission
Grant No. AT(11-1)-2059.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

The nonlinear evolution of the Dory-Guest-Harris type instability is investigated by numerical integration of the Vlasov equation in one spatial and two velocity dimensions. Two methods are used: a particle simulation and the Fourier-Hermite transform. The two methods are compared for $f_0(v_\perp) = (2\pi 5!)^{-1} (v_\perp/\sqrt{2})^{10} \exp(-v_\perp^2/2)$ for k values which are unstable. A method of predicting the limiting amplitude of the electric field is presented for the case in which one mode dominates the system.

I. INTRODUCTION

The linear behavior of the Dory-Guest-Harris (DGH) instability is well known.^{1,2} Recently a numerical study of the nonlinear behavior was made by Byers and Grewal³ using a particle simulation model of the plasma. They found that the electric field saturates after reaching a certain level as is expected and thereafter the field rises and falls with a period of roughly the cyclotron period.

We concentrate on a rather special type of DGH instability, that of zero frequency. We model this instability in two ways; by Fourier-Hermite transformations of the Vlasov equation and by particle simulation. The simplifying assumption is made that it suffices to consider only one spatial dimension (x) and two velocity dimensions (v_x, v_y) while the magnetic field is assumed to be constant and directed along the z axis.

Sections II and III present a general outline of the two methods, Section IV compares the results, and Section V attempts an explanation of the electric field saturation.

II. THE NONLINEAR VLASOV EQUATION

If we consider a constant magnetic field directed along the z axis and all quantities to be functions only of the spatial dimension x , and the velocity dimensions v_x, v_y , the Vlasov-Poisson system can be written in dimensionless variables as

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + E_x \frac{\partial f}{\partial v_x} + R \left(v_x \frac{\partial f}{\partial v_x} - v_y \frac{\partial f}{\partial v_y} \right) = 0$$

$$\frac{\partial E_x}{\partial x} = 1 - \int f dv_x dv_y \quad (1)$$

where t is in units of ω_p , v is in units of the thermal velocity, x is in units of the Debye length, and $R = \omega_c / \omega_p$, the ratio of the cyclotron frequency to the plasma frequency.

The Vlasov equation may be Fourier transformed in x, v_x, v_y by the transformation (compare Ref. 4)

$$F_n(y_1, y_2, t) = \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_0^L dx e^{ink_0 x}$$

$$e^{i(v_x y_1 + v_y y_2)} f(x, v_x, v_y, t) \quad (2)$$

which yields

$$\frac{\partial F_n}{\partial t} + nk_0 \frac{\partial F_n}{\partial y_1} + R \left(y_2 \frac{\partial F_n}{\partial y_1} - y_1 \frac{\partial F_n}{\partial y_2} \right) - i \sum_{m=-\infty}^{\infty} E_m y_1 F_{n-m} = 0, \quad (3)$$

$$ik_0 m E_m = F_m(0, 0, t).$$

We then expand F_n in the series

$$F_n(y_1, y_2, t) = \sum_{v_1=0}^{\infty} \sum_{v_2=0}^{\infty} g_{v_1} g_{v_2} a_{n, v_1, v_2}(t) y_1^{v_1} y_2^{v_2} e^{-\frac{1}{2}(y_1^2 + y_2^2)},$$

$$g_v = 2^{v/2} \frac{\Gamma(\frac{v+2}{2})}{\Gamma(v+1)} \quad (4)$$

which gives a coupled system of ordinary differential equations

$$\begin{aligned} \dot{a}_{n, v_1, v_2} = & -k_0 n \gamma_{v_1} \left((v_1+1) a_{n, v_1+1, v_2} - v_1 a_{n, v_1-1, v_2} \right) \\ & + i \gamma_{v_1} v_1 \sum_m E_m a_{n-m, v_1-1, v_2} \\ & - R \gamma_{v_1} \gamma_{v_2} \left(v_2 (v_1+1) a_{n, v_1+1, v_2-1} - v_1 (v_2+1) a_{n, v_1-1, v_2+1} \right) \end{aligned} \quad (5)$$

where $\gamma_v = g_{v+1}/g_v$, and

$$iE_m = \frac{a_{m, 0, 0}(t)}{mk_0} \quad m \neq 0 \quad (6)$$

The transform-expansion scheme is equivalent to a Fourier transformation in space and Hermite transformation in velocity except that here all the a_{n, v_1, v_2} 's are real. The infinite set is truncated and the coefficients $a_{n, v_{\max}+1, v_2}$ are estimated by a polynomial fit to the coefficients $a_{n, v_{\max}, v_2} \dots a_{n, v_{\max}-3, v_2}$ at each time step. The details of this truncation procedure are presented elsewhere⁴ for the one dimensional case of which this is a simple generalization.

Because only one spatial variable is included, it turns out that if the a_{n, v_1, v_2} 's are confined to a triangle in terms of v_1, v_2 (see Fig. 1) we need only extrapolate the terms $a_{n, v_{\max}+1, v_2}$.

For the present calculation, five Fourier modes are included and $v_{1 \max} = v_{2 \max} = 40$, which seem to be sufficient to represent the growth of the instability up to its saturation.

III. PARTICLE SIMULATION

The mechanics of particle simulation codes are well known.⁶

The model we use is a particle-in-cell model with two velocity dimensions and one spatial dimension. The electric field is solved via fast Fourier transforms. Thirty-two Fourier modes are allowed of which the unstable mode of interest is mode 1 or 2. The particles are loaded uniformly in space by a quiet start technique. The perturbation of the uniform state is accomplished with an "external" electric field. The simulation particles attempt to shield the external field and when the field is switched off, a density perturbation is produced. This allows good control of the initial conditions.

IV. NUMERICAL RESULTS

The distribution function used in the Hermite model is

$$f_0(v_1, t=0) = \frac{1}{2\pi 5!} \left(\frac{1}{2} v_1^2 \right)^5 \exp \left(-\frac{1}{2} v_1^2 \right) \quad (7)$$

which has a reasonably simple Hermite representation. This distribution function exhibits Dory-Guest-Harris instabilities for $R \leq \frac{1}{6}$.² The function used for the particle simulation matches this for v_1 around the maximum ($v_1 = \sqrt{10}$) but is truncated when $f_0(v_1)$ falls to 1/2 its maximum value. The distribution functions at $t = 0$ are exhibited in Fig. 2.

The maximum growth of the $\omega_r = 0$ mode occurs for $k \cong R \left(= \frac{\omega_c}{\omega_p} \right)$ where $R = .125$. These are the values of k_0 and R for the electric fields shown in Fig. 3 and 4. The behavior of the two systems is similar, but not exactly the same. This is due, in part, to the differences in initial conditions. For both cases, for a single wave number, two modes are excited one with $\omega_r = 0$ and unstable, and the other a stable mode with $\omega_r \simeq \omega_p$. The effect of the stable mode is diminished as the electric field grows becoming only a ripple when the field is large enough to reach its maximum value.

Figure 5 shows the space averaged distribution function $f_0(v_1, t)$ (from the particle simulation) for various times during the course of electric field growth and saturation. The flattening of the distribution function is reminiscent of numerical calculations with no magnetic field present.

V. A MODEL OF THE INSTABILITY

The features of saturation of the electric field appear to be the following. The change of electric field from its linear growth appears quite abruptly. This change occurs over less than half a cyclotron period (2π on the time scales of Fig. 3 and 4). The field falls after saturation, but then recovers in about a cyclotron period. The distribution function during this time flattens out and then tends to regain some of its hump corresponding to the decline and recovery of the electric field.

Figure 6 shows the velocity coordinates of a single charged particle in a constant magnetic field and an electric field of the form $E(\vec{x}, t) = E_0 \sin(kx(t) - \phi)$ where E_0 is chosen to be the limiting amplitude of the field in Fig. 3. At $t = 0$, $\vec{v} = \sqrt{10} \hat{x}$. No self consistency is attempted. One can see that although the basic motion is circular, rather wide excursions from the $E = 0$ path occur. It is not too surprising, then, that the distribution function becomes distorted. Another interesting observation is that the particle returns almost to its initial coordinates after one revolution. We have run particles several times (~ 10) around this path and the deviation from the first transit coordinates is negligible. From this, we expect that any behavior of the system subsequent to the saturation time will have periodicity on the order of a cyclotron period.

We have made a model based on our observations of the numerical calculations which has been quite successful in predicting the limiting amplitudes of the electric field. The ingredients consist of a perturbation model for discrete spatial modes and a constant magnetic field. Mode coupling effects have been neglected. In the appendix, a perturbation equation is derived for the space averaged distribution function $f_o(v_\perp, t)$ in terms of its initial value. This equation is

$$\frac{\partial f_o(v_\perp, t)}{\partial t} = \frac{1}{R^2} \sum_n \gamma_n \left(\frac{R}{nk_o} \right)^2 |E_n(t)|^2 \left(\frac{1}{v_\perp} + \frac{\partial}{\partial v_\perp} \right) \frac{1}{v_\perp^2} \frac{\partial f_o(v_\perp, 0)}{\partial v_\perp} \quad (8)$$

where

$$E_n(t) = E_n(0) \exp \left\{ \int_0^t (i\omega_n + \gamma_n) dt \right\} .$$

We can simplify this if we assume that one mode (the n th) dominates the solution. Then

$$\frac{\partial f_o(v_1, t)}{\partial t} = \frac{\gamma_n}{R^2} |E_n(t)|^2 \left(\frac{R}{nk_o} \right)^2 \left(\frac{\partial}{\partial v_1} + \frac{1}{v_1} \right) \left(\frac{1}{v_1^2} \frac{\partial f_o(v_1, 0)}{\partial v_1} \right) \quad (9)$$

Since the only time dependence on the R.H.S. is

$$\gamma_n(t) |E_n(0)|^2 e^{2 \int_0^t dt'} \gamma_n(t'),$$

the equation can be integrated to give

$$f_o(v_1, t) - f_o(v_1, 0) = \frac{1}{2R^2} |E_n(t)|^2 \left(\frac{R}{nk_o} \right)^2 \left(\frac{\partial}{\partial v_1} + \frac{1}{v_1} \right) \frac{1}{v_1^2} \frac{\partial f_o(v_1, 0)}{\partial v_1} \quad (10)$$

If we have some way to characterize $f_o(v_1, t)$, we can obtain an estimate of $|E_n(t)|$. This is provided by the linearized dispersion relation, which in terms of dimensionless quantities is

$$1 = - \frac{1}{k^2} \sum_m \frac{mR}{\omega - mR} \int d^2v J_m^2 \left(\frac{nk_o v_{\perp}}{R} \right) \frac{1}{v_{\perp}} \frac{\partial f_o(v_{\perp})}{\partial v_{\perp}} . \quad (11)$$

One can easily demonstrate that for a zero frequency mode, a distribution function which exhibits marginal stability must satisfy the following integral condition

$$k^2 = \int d^2v J_o^2 \left(\frac{nk_o v_{\perp}}{R} \right) \frac{1}{v_{\perp}} \frac{\partial f_o(v_{\perp})}{\partial v_{\perp}} . \quad (12)$$

If the picture is correct that the instability is turned off simply by a flattening of the distribution function so much that the stability boundary is reached, the maximum electric field can be obtained by differentiating Eq. 10 with respect to $v_{\perp}$, multiplying by $J_o^2 \left(\frac{nk_o v_{\perp}}{R} \right)$, and integrating over $v_{\perp}$.

$$\begin{aligned} & 2\pi \int_0^{\infty} dv_{\perp} J_o^2 \left(\frac{nk_o v_{\perp}}{R} \right) \frac{\partial}{\partial v_{\perp}} \left\{ f_o(v_{\perp}, t) - f_o(v_{\perp}, 0) \right\} \\ &= \frac{|E_{\max}|^2}{2R^2} \left(\frac{R}{nk_o} \right)^2 \left(\frac{\partial}{\partial v_{\perp}} + \frac{1}{v_{\perp}} \right) \left(\frac{1}{v_{\perp}} \frac{\partial f_o(v_{\perp}, 0)}{\partial v_{\perp}} \right) \Bigg\} \end{aligned} \quad (13)$$

where we must replace the first term by k^2 from Eq. 12. Finally we obtain

$$E_{\max} = \frac{R}{\sqrt{\pi}} \left(\frac{nk_o}{R} \right) \frac{\left(k^2 - 2\pi \int_0^\infty dv_1 J_o^2 \left(\frac{nk_o v_1}{R} \right) \frac{\partial f_o}{\partial v_1} (v_1, 0) \right)^{\frac{1}{2}}}{\left(\int_0^\infty dv_1 J_o^2 \left(\frac{nk_o v_1}{R} \right) \left(\frac{\partial^2}{\partial v_1^2} \frac{1}{v_1^2} \frac{\partial f_o}{\partial v_1} + \frac{\partial}{\partial v_1} \frac{1}{v_1^3} \frac{\partial f_o}{\partial v_1} \right) \right)^{\frac{1}{2}}}, \quad (14)$$

We have used Eq. 14 to predict the limiting amplitudes of the electric field for a number of different sets of parameters and have compared these predictions with the maximum fields determined from our computer models. The results are summarized in Table I. The agreement of the predicted and calculated values of the maximum electric field indicate that the essential features of the mechanism which causes the electric field to cease growing are included in our calculation.

APPENDIX: PERTURBATION EQUATION

We consider the Vlasov equation

$$\frac{\partial \tilde{f}_n}{\partial t} + i n k_0 v \tilde{f}_n + \sum_{n'} E_{n-n'} \frac{\partial \tilde{f}_n}{\partial v_x} + R \left(v_y \frac{\partial \tilde{f}_n}{\partial v_x} - v_x \frac{\partial \tilde{f}_n}{\partial v_y} \right) = 0 \quad (A.1)$$

$$i n k_0 E_n = \int \tilde{f}_n dv_x dv_y \quad (A.2)$$

Let us assume at $t = 0$ $\tilde{f}_0(v_x, v_y) = \tilde{f}_0(v_\perp)$ and define

$$f_0(v_\perp, t) = \frac{1}{2\pi} \int_0^{2\pi} d\phi \tilde{f}_0 \quad (A.3)$$

then

$$\frac{\partial f_0}{\partial t} = - \frac{1}{2\pi} \int_0^{2\pi} d\phi \sum_n E_{-n} \frac{\partial \tilde{f}_n}{\partial v_x} \quad (A.4)$$

The $\tilde{f}_n$ on the right-hand side of A.4, we obtain from the linearized theory

$$\tilde{f}_n(\vec{v}, t) = \frac{E_n(t)}{R} \frac{\partial f_o(v_{\perp})}{\partial v_{\perp}} \frac{i}{2} \sum_{\alpha=-\infty}^{\infty} J_{\alpha}^2 \left(\frac{nk_o v_{\perp}}{R} \right) e^{\frac{ink_o v_{\perp}}{R} \sin \varphi}$$

$$\left[\frac{e^{i(1-\alpha)\varphi}}{\left(1 + \frac{\omega_n + i\gamma_n}{R} - \alpha\right)} + \frac{e^{-i(1+\alpha)\varphi}}{\left(-1 + \frac{\omega_n + i\gamma_n}{R} - \alpha\right)} \right]$$

If we note that

$$\int_0^{2\pi} d\varphi \frac{\partial}{\partial v_x} \tilde{f}_n = \int_0^{2\pi} d\varphi \left(\frac{\partial}{\partial v_{\perp}} + \frac{1}{v_{\perp}} \right) \cos \varphi \tilde{f}_n, \quad (A.5)$$

we can write

$$\frac{1}{2\pi} \int_0^{2\pi} d\varphi \frac{\partial}{\partial v_x} \tilde{f}_n(\vec{v}, t) = \frac{i}{4} \frac{E_n(t)}{R} \left(\frac{\partial}{\partial v_{\perp}} + \frac{1}{v_{\perp}} \right) \left\{ \frac{\partial f_o}{\partial v_{\perp}} \left(\frac{2R}{nk_o v_{\perp}} \right)^2 \sum_{\alpha} \frac{\alpha^2 J_{\alpha}^2 \left(\frac{nk_o v_{\perp}}{R} \right)}{-\alpha + \frac{\omega_n + \gamma_n}{R}} \right\} \quad (A.6)$$

or

$$\begin{aligned} \frac{\partial f_o(v_{\perp}, t)}{\partial t} = \frac{i}{R} \sum_n \left(\frac{R}{nk_o} \right)^2 |E_n|^2 \left(\frac{\partial}{\partial v_{\perp}} + \frac{1}{v_{\perp}} \right) \frac{1}{v_{\perp}^2} \frac{\partial f_o(v_{\perp}, t)}{\partial v_{\perp}} \\ \sum_{\alpha} \frac{\alpha^2 J_{\alpha}^2 \left(\frac{nk_o v_{\perp}}{R} \right)}{-\alpha + \frac{\omega_n + i\gamma_n}{R}} \end{aligned} \quad (A.7)$$

This is a quasi-linear like equation.

If we assume E_n is a first order quantity, we can write⁸

$$f_o(v_{\perp}, t) = f_o^{(0)}(v_{\perp}, t) + f_o^{(2)}(v_{\perp}, t) + \dots \quad (A.8)$$

$$\frac{\partial f_o^{(0)}(v_{\perp}, t)}{\partial t} = 0 ; f_o^{(0)}(v_{\perp}, t) = f_o^{(0)}(v_{\perp}, 0) \quad (A.9)$$

$$\begin{aligned} \frac{\partial f_o^{(2)}(v_{\perp}, t)}{\partial t} = \frac{i}{R} \sum_n \left(\frac{R}{nk_o} \right)^2 |E_n|^2 \left(\frac{\partial}{\partial v_{\perp}} + \frac{1}{v_{\perp}} \right) \frac{1}{v_{\perp}^2} \frac{\partial f_o^{(0)}(v_{\perp}, 0)}{\partial v_{\perp}} \\ \sum_{\alpha} \frac{\alpha^2 J_{\alpha}^2 \left(\frac{nk_o v_{\perp}}{R} \right)}{-\alpha + \frac{\omega_n + i\gamma_n}{R}} \end{aligned} \quad (A.10)$$

Then to 2nd order

$$\begin{aligned} \frac{\partial f_o(v_1, t)}{\partial t} &= \frac{i}{R} \sum_n \left(\frac{R}{nk_o} \right)^2 |E_n|^2 \left(\frac{\partial}{\partial v_1} + \frac{1}{v_1} \right) \frac{1}{v_1^2} \frac{\partial f_o(v_1, 0)}{\partial v_1} \\ &\quad \sum_{\alpha} \frac{\alpha^2 J_{\alpha}^2 \left(\frac{nk_o v_1}{R} \right)}{-\alpha + \frac{\omega_n + i\gamma_n}{R}} \end{aligned} \quad (A.11)$$

This can be written as

$$\begin{aligned} \frac{\partial f_o(v_1, t)}{\partial t} &= \sum_n \left(\frac{1}{nk_o} \right)^2 \gamma_n |E_n|^2 \left(\frac{\partial}{\partial v_1} + \frac{1}{v_1} \right) \frac{1}{v_1^2} \frac{\partial f_o(v_1, 0)}{\partial v_1} \\ &\quad \sum_{\alpha} \frac{\alpha^2 J_{\alpha}^2 \left(\frac{nk_o v_1}{R} \right)}{\left(\frac{\omega_n}{R} - \alpha \right)^2 + \left(\frac{\gamma_n}{R} \right)^2} \end{aligned} \quad (A.12)$$

if we notice that the imaginary part of the R.H.S. sums to zero.

We can write

$$\sum_{\alpha} \frac{\alpha^2 J_{\alpha}^2 \left(\frac{nk_o v_1}{R} \right)}{\left(\frac{\omega_n}{R} - \alpha \right)^2 + \left(\frac{\gamma_n}{R} \right)^2} = 1 - \left(\frac{\gamma_n}{R} \right)^2 \sum_{\alpha} \frac{J_{\alpha}^2 \left(\frac{nk_o v_1}{R} \right)}{\left(\frac{\omega_n}{R} - \alpha \right)^2 + \left(\frac{\gamma_n}{R} \right)^2} \quad (A.13)$$

Eq. (A.12) contains a distribution function which is peaked at fairly large values of v_1 . Since the Bessel functions are small in this region, we keep only the first term of this series.

Therefore, Eq. (A.12) can be written

$$\frac{\partial f_o(v_1, t)}{\partial t} = \sum_n \gamma_n \left(\frac{1}{nk_o} \right)^2 |E_n|^2 \left(\frac{\partial}{\partial v_1} + \frac{1}{v_1} \right) \frac{1}{v_1^2} \frac{\partial f_o(v_1, 0)}{\partial v_1} .$$

(A.14)

Table I. The comparison of $E_{\max}$ predicted by Eq. (14) and $E_{\max}$ calculated from computer models of the Dory-Guest-Harris instability. The first entry is from the Hermite model because the initial value for that model is given by $(v_1)^{10} e^{-\frac{1}{2}v_1^2}$.

Distribution Function	nk_0	$\frac{\omega_c}{\omega_p}$	Predicted $E_{\max}$	Calculated $E_{\max}$
$(v_1)^{10} e^{-\frac{1}{2}v_1^2}$	.125	.125	.15	.20 (Hermite)
$\delta(v_1 - 3.16)$	.125	.125	.25	.27 (Particle)
$\delta(v_1 - 3.16)$	.0625	.0625	.14	.14 (Particle)
$\delta(v_1 - 6.32)$	.0625	.125	.60	.63 (Particle)
$\delta(v_1 - 6.32)$	.03125	.0625	.29	.22 (Particle)

REFERENCES

- ¹ E. G. Harris, J. Nucl. Energy C2, 138 (1961).
- ² R. A. Dory, G. E. Guest, and E. G. Harris, Phys. Rev. Letters
14, 131 (1965).
- ³ J. A. Byers and M. Grewal, Phys. Fluids 13, 1819 (1970).
- ⁴ G. Joyce, G. Knorr, and H. Meier, J. Comp. Phys., to be published.
- ⁵ See for example R. L. Morse and C. Nielson, Methods in Computational
Physics (Academic Press, New York, 1970), Vol. 9, pp. 213-239.
- ⁶ S. P. Gary, Phys. Fluids 10, 570 (1967).

ACKNOWLEDGMENTS

We are grateful for the use of the facilities of the Thermonuclear Division, Oak Ridge National Laboratories where the initial stages of this research were carried out.

One of us (G. J.) would like to thank Dr. David Montgomery for a number of helpful discussions and suggestions.

FIGURE CAPTIONS

Figure 1. The a_n, v_1, v_2 's which are used in the calculation of the Fourier-Hermite expansion of the Vlasov equation.

Figure 2. The spatially uniform distribution function $F_0(v_1, 0)$. The cross-hatched area is the representation used in the particle representation.

Figure 3. The electric fields E_1, E_2, E_3 of the Fourier-Hermite representation as a function of time for $k_0 = .125$,

$$R = \left(\frac{\omega_c}{\omega_p} \right) = .125.$$

Figure 4. The electric fields E_1, E_2, E_3 of the particle representation as a function of time for $k_0 = .125, R = .125$.

Figures 5 a and b. The spatially averaged distribution function $F_0(v_1, t)$ determined from the particle representation.

Figure 6. The orbit of a particle under the influence of a magnetic field and electric field of the form $E = .25 \sin(k_0 x - \phi)$ for $k_0 = .125$ and $\frac{\omega_c}{\omega_p} = .125$, where the dimensionless time is ω_p^{-1} . The dashed line is the orbit for $E = 0$.

A - G71 - 181

Figure 1.

Figure 2.

A-G70-632

Figure 3.

A-670-630-1

Figure 4.

Figure 5a.

Figure 5b.

A - G71-180-1

Figure 6.

INSTRUCTIONS

Who uses this form: AEC contract administrators will designate the AEC contractors who are to use this form. Generally speaking, it will be used by educational institutions and other "not for profit" institutions. AEC National Laboratories and other major contractors will generally use the longer form, AEC-426.

When to use this form: AEC contractors are required under their contracts to transmit specified types of documents to the AEC. Some, but not all, of these are transmitted by AEC contract administrators to AEC's Division of Technical Information (DTI) and may be incorporated into AEC's technical information documentation system. Types of documents which will be transmitted to DTI are identified in instructions which the contractor receives from his contract administrator. Each such document is to be accompanied by one copy of this transmittal form in order to instruct DTI regarding announcement and distribution of the document. (Exception: Reprints of journal articles may be transmitted to DTI, but no transmittal form should be used with such reprints.) Documents which the contractor may be required to submit to the AEC under his contract but which are not of the type to be transmitted to DTI should not be accompanied by a copy of this transmittal form. Examples of types of documents in this latter category include such items as: contract proposals, manuscripts and preprints of journal articles, and manuscripts of oral presentations which are to be published in a journal in substantially the same form and detail.

Where to send this form: Send the document and the attached form AEC-427 to the AEC contract administrator for transmittal to DTI unless the AEC contract administrator specifies otherwise.

Item instructions:

- Item 1.** The first element in the number shall be an AEC-approved code. This may be a code which is unique to the contractor, e.g., MIT, or it may be the code of the AEC Operations Office, i.e., NYO, COO, ORO, IDO, SRO, SAN, ALO, RLO, NVO. The contract administrator will specify the code which is to be used.

The code shall be followed by a sequential number, or by a contract number plus a sequential number, as follows: (a) Contractors with unique codes may complete the report number by adding a sequential number to the code, e.g., MIT-101, MIT-102, etc.; or they

may add the identifying portion of the contract number and a sequential number, e.g., ABC-2105-1, ABC-2105-2, etc.; (b) Contractors using the operations office code shall complete the report number by adding the identifying portion of the contract number and a sequential number, e.g., NYO-2200-1, NYO-2200-2, etc.

- Item 2.** Give title exactly as on the document itself.
- Item 3.** If box c is checked, indicate type of item being sent, e.g., thesis, translation, computer program, etc.
- Item 4.** The "normal announcement and distribution procedures" for unclassified documents may include listing in DTI's "Weekly Accessions List of Unlimited Reports" (TID-4401); abstracting in "Nuclear Science Abstracts" (NSA); and distribution to appropriate TID-4500 ("Standard Distribution for Unclassified Scientific and Technical Reports") addressees, to AEC depository libraries, to the Clearinghouse for Federal Scientific and Technical Information for sale to the public, and to authorized foreign recipients. Check 4b or 4c if there is need for limiting announcement and distribution procedures described above. The normal expectation is that there should seldom be a necessity to check 4c.
- Item 5.** If 4b or 4c is checked, give reason for recommending announcement or distribution restrictions, e.g., "preliminary information", "prepared primarily for internal use", etc.
- Item 6.** Enter name of person to whom inquiries concerning the recommendations on this form may be addressed.
- Item 7.** AEC contract administrators may use this space to show concurrence or nonconcurrence with the recommendation in item 4 and to make other recommendations.
- Item 8.** AEC contract administrator or patent group representative should check a, b, or c, and forward this form and the document to:

USAEC - Technical Information
P. O. Box 62
Oak Ridge, Tennessee 37830