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CHANNEL-WALL LIMITATIONS IN
THE MHD INDUCTION GENERATOR

by

Edward S. Pierson and
William D. Jackson

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Reactor Engineering Division

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March 1966

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TABLE OF CONTENTS

	<u>Page</u>
NOMENCLATURE	6
ABSTRACT	7
I. INTRODUCTION.	7
1.1 Liquid-metal MHD Power-generation Systems.	7
1.2 Scope and Objectives.	9
1.3 The MHD Induction Machine.	9
1.4 Previous Work.	11
1.5 Material Constraints.	13
II. THE INFINITE-LENGTH MACHINE WITHOUT CHANNEL WALLS AND INSULATION.	17
2.1 The Model.	17
2.2 The Equations	18
2.3 The Exact Solution	21
2.4 The Slit-channel Solution.	25
III. THE INFINITE-LENGTH MACHINE WITH CHANNEL WALLS AND THERMAL INSULATION.	27
3.1 Introduction.	27
3.2 The Electromagnetic Fields	27
3.3 The Powers.	31
3.4 Discussion	34
IV. THE FINITE-LENGTH MACHINE WITH CONSTANT FLUID VELOCITY	38
4.1 Introduction.	38
4.2 Performance without Walls and Insulation.	38
4.3 Problems with Exact Solution with Walls and Insulation. . .	42
4.4 Slit-channel Solution with Walls and Insulation.	44
V. CONCLUDING REMARKS.	49
ACKNOWLEDGMENT	50
REFERENCES	51

LIST OF FIGURES.

<u>No.</u>	<u>Title</u>	<u>Page</u>
1.	The Flat, Linear, MHD Induction Machine.	10
2.	Model of Infinite-length Machine.	17
3.	Generator Efficiency versus α_f as a Function of s for $\alpha_w = 0.08$	35
4.	Generator Efficiency versus α_f as a Function of s for $\alpha_w = 0.02$	35
5.	Generator Efficiency versus α_w as a Function of s and α_f	36
6.	Generator Efficiency versus s as a Function of α_f	36
7.	Model of Finite-length Machine.	38
8.	Generator Efficiency versus Length of $\alpha_w = 0.1$, $\alpha_i = 0.1$, $R_{Mf} = 10$, $R_{Mw} = 10$, and $\kappa = 0$	47

LIST OF TABLES

<u>No.</u>	<u>Title</u>	<u>Page</u>
I.	Refractory-metal Properties and Wall Thicknesses for an Annular Duct One meter in Diameter	14
II.	The Six Boundary Conditions	28
III.	Equations Used to Calculate Constants for $\mu_f = \mu_w = \mu_i = \mu_o$	29
IV.	Constants for Exact and Slit-channel Solutions for Vector Potential for $R_{Mf} = 10$, $R_{Mw} = 10$, $s = -0.1$, and $\kappa = 0$	30
V.	Powers per Wavelength, with Walls and Insulation	31
VI.	Powers per Wavelength for Exact and Slit-channel Solutions for $R_{Mf} = 10$, $R_{Mw} = 10$, $s = -0.1$, and $\kappa = 0$	33
VII.	Powers per Wavelength for Exact and Slit-channel Solutions for $R_{Mf} = 10$, $R_{Mw} = 10$, $s = -0.4$, and $\kappa = 0$	33
VIII.	Exact and Slit-channel Generator Efficiencies for $R_{Mf} = 11$, $R_{Mw} = 20$, $\alpha_i = 0.04$, and $\kappa = 0$	36
IX.	The Slit-channel Powers for a Finite-length Machine	46
X.	Powers and Efficiencies for a Finite-length Slit-channel Generator for $R_{Mf} = 10$, $R_{Mw} = 10$, and $\kappa = 0$	47

NOMENCLATURE

Rationalized mks units are used. An arrow above a symbol indicates a vector, underlining indicates the complex amplitude, * the complex conjugate, and $\|$ the magnitude or absolute value. The common subscripts are f, c, e, i, and w for fluid, core, exciting winding, insulation, and channel-wall variables, respectively; and x, y, and z for the components of a vector. Variables used in a single section may not be included.

Symbol	Definition	Symbol	Definition
A	Vector potential (Eq. 2.2.9)	j	$\sqrt{-1}$
A ₁	Slit-channel vector potential in fluid, wall, and insulation	k	Wave number
B	Magnetic flux density	k	Thermal conductivity (Section 3.4)
B _{sat}	Saturation magnetic flux density of the core	n	Length of machine in direction of fluid flow in wavelengths
E	Electric field intensity	p	Pressure
E	Joint efficiency (Eq. 1.5.1)	p	Transform variable (Chapter IV)
I	Exciting-current amplitude	P _{em}	Electromagnetic part of the pressure
J	Current density	q	Electric charge density
K	Surface current density of excitation	r	Radius of pressure vessel (Eq. 1.5.1)
N	Turns-density amplitude of excitation	s	Slip based on phase velocity, $s = (v_s - v)/v_s$
P _{in}	Net electrical power supplied to machine by exciting winding	t	Time
P _m	Mechanical power output (Eq. 2.2.18)	t	Thickness of pressure vessel (Eq. 1.5.1)
P ₀	Normalization constant for powers, $P_0 = \mu_f v_s N^2 I^2 c n \lambda$	v	Fluid velocity
P _r	Power dissipated in volume due to finite conductivity (Eq. 2.2.20)	v _s	Field phase velocity, $v_s = \omega/k$
P _s	Power supplied to volume (Eq. 2.2.16)	x	Axis in direction of fluid flow
P ₁	Normalization constant for powers, $P_1 = \lambda c v_s k^2 / \mu_0$	y	Axis transverse to flow
R _M	Magnetic Reynolds number, $R_M = \mu \sigma v_s / k$	z	Axis in direction of current flow
R _{MF}	Magnetic Reynolds number based on fluid velocity, $R_{MF} = \mu_f \sigma_f v / k$	α	$\alpha = ak$
R _{Ma}	$R_{Ma} = \frac{1}{1 + \kappa / \alpha_f} R_{Mf}$	γ_c	Spatial constant in core, $\gamma_c^2 = 1 + jR_{Mc}$
S	Poynting vector (Eq. 2.2.17)	γ_f	Spatial constant in fluid, $\gamma_f^2 = 1 + jsR_{Mf}$
S	Maximum allowable stress (Eq. 1.5.1)	γ_w	Spatial constant in wall, $\gamma_w^2 = 1 + jR_{Mw}$
a _f	Channel half-height	ϵ	Permittivity
a _i	Insulation thickness	η	Absolute viscosity
a _w	Wall thickness	κ	Ratio of fluid to core permeability, $\kappa = \mu_f / \mu_c$
b	Thickness of exciting plate	λ	Wavelength, $\lambda = 2\pi/k$
c	Machine width	μ	Permeability
e	Efficiency	μ_0	Permeability of free space
e _g	Generator efficiency	ρ	Fluid mass density
e _p	Pump efficiency	σ	Electrical conductivity
i	Order of pole for finite-length solution	σ_s	Surface conductivity
$\vec{i}$	Unit vector	ϕ	Scalar potential (Eq. 2.2.10)
		ω	Frequency in rad/sec
		∇	Vector differential operator
		Δ_i	$\Delta_i = 2$ for $i = 0$, and 1 for $i \neq 0$

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ABSTRACT

The materials problems influencing the design and performance of the MHD induction generator are discussed. The previous analysis of the machine is reviewed and extended to include electrically-conducting channel walls and thermal insulation. The infinite-length machine is examined in detail, including both an approximate analysis to show the general behavior, and an exact solution to provide numerical data. The power flow between the field structure and the fluid is used to determine the effect of the conducting wall with practical material properties and machine dimensions. It is shown that the wall thickness has to be very small for acceptable generator performance, and that this requirement raises severe structural problems. The effect of the insulation is determined to be small. By means of an approximate solution, the previous analysis of the finite-length machine is also extended to include walls and insulation.

I. INTRODUCTION

1.1. Liquid-metal MHD Power-generation Systems

The basic attraction of thermal to electric power-generation systems utilizing an MHD generator is that a fluid can serve both as the thermodynamic working substance and as the moving component of the generator. The consequent elimination of turbines is potentially advantageous, particularly for high-temperature systems, and MHD conversion schemes have accordingly been studied in detail during the past 10 years.

Initial interest centered on the use of an MHD generator in a topping cycle for fossil-fuel central power stations to increase the overall thermal efficiency.¹ In this case, the obvious procedure is operation of the generator directly on combustion gases in an open cycle, and this approach led directly to extensive studies^{2,3} of combustion-gas plasma MHD generators.

Recent progress with nuclear power for central-station installations and the prospects for developing high-temperature reactors have extended interest in large-scale MHD generators to include their use in topping

cycles with nuclear heat sources.⁴⁻⁶ In contrast to combustion-driven MHD generators, a closed cycle is clearly required, and accordingly there is some freedom in the selection of a working fluid.

One approach is to employ a gas-cooled reactor with helium as the working fluid in a noncondensing or Brayton cycle. Since the temperature limitations imposed by nuclear heat sources prevent an adequate equilibrium electrical conductivity from being obtained in this type of MHD generator, where cesium-seeded helium usually provides the plasma flow, a conductivity-enhancement scheme is required. The most favored approach^{7,8} is nonequilibrium ionization, in which the electric field produced by motional induction is used to heat electrons preferentially. A Brayton cycle, using helium or neon seeded with cesium, requires an efficient compressor and high magnetic fields.⁹ When the factors involved in matching a nonequilibrium plasma generator to a gas-cooled nuclear reactor are taken into account, the performance of a binary cycle including a steam plant shows little gain over established technology with pressurized-water reactors, and superconducting magnetic systems would certainly be required.⁹

In the temperature range of interest (1500-3000°F), alkali metals, particularly potassium and cesium, have thermodynamic properties that make them well-suited to nuclear-driven condensing or Rankine-type topping cycles. A condensing cycle implies that electrical power may be extracted from either the liquid or vapor phases. With nonequilibrium ionization, the electrical conductivity¹⁰ of the vapor flow is probably adequate for an MHD generator essentially similar to the cesium-seeded helium type.

An alternate approach with alkali metals is to generate electric power in an MHD generator operated on the liquid phase. Several liquid-metal cycles have been proposed for the conversion system,¹¹⁻¹⁴ and cycle analysis¹⁵ has shown that the potential efficiency of binary cycles including a liquid-metal topping stage is superior to the corresponding plasma cycles. This performance is achieved at lower maximum temperatures than with plasmas, and in addition the electrical conductivity of the liquid phase yields a magnetic Reynolds number sufficient for direct a.c. generation by means of an MHD induction machine.¹⁶

Estimates of electrical-power requirements for space-vehicle missions to the far planets indicate that a nuclear heat source coupled in a closed-cycle system to an electrical generator offers potentially attractive specific weights.¹⁷ Since heat is rejected by radiation, a high-temperature system is required to keep the radiator area (and weight) to an acceptable level. Use of MHD generators as the conversion device again enables rotating components to be dispensed with and promises greater reliability for long-duration operation.

Space-power systems may also be of either the Brayton or Rankine types. In the former case, the compressor must be driven from the MHD generator and highly efficient components are required to enable the cycle to operate.⁹ The development of these components is a formidable task, which is not duplicated in the case of liquid-metal cycles. The liquid-metal cycles can yield competitive specific weights at substantially lower heat-source temperatures than any cycle in which a plasma MHD generator is employed, and they also allow for the possibility of direct a.c. generation.

1.2. Scope and Objectives

This report deals with liquid-metal MHD induction generators intended for central-station installations, although many of the results and conclusions are applicable to the smaller machines required in space-vehicle power systems. As the basic analysis of this type of induction generator is essentially complete, the report is concerned with performance limitations imposed by materials considerations. The major objective is to emphasize the design problems and performance limitations of large induction generators.

The major factors that have to be taken into account, in addition to the electrical loss in the fluid inherent in induction machine operation, are:

1. Friction loss in the generator channel and end regions.
2. Electrical and thermal loss due to the channel walls.
3. Electrical loss in the end regions.
4. Winding loss.
5. Core loss.

The remainder of this chapter is directed first to brief reviews of the principle of operation of the MHD induction machine and of prior work on the device, and secondly to a discussion of the implications of the losses and their relation to materials limitations. This discussion shows that the wall losses are a primary factor demanding first attention. Thus, Chapter II presents the basic analytical material required for the analysis of this loss, which is considered in detail in Chapter III. Finite machine length, including walls, is discussed in Chapter IV. The results are summarized in Chapter V, and the next steps to be taken in preliminary design studies are outlined.

1.3. The MHD Induction Machine

The MHD induction machine, shown schematically in Fig. 1, consists of a channel containing a flowing, electrically-conducting fluid and a set of exciting windings, which produce a traveling magnetic field. Currents

Fig. 1. The Flat, Linear, MHD Induction Machine

are induced in the fluid because of the relative motion between the field and the fluid. The force of electrical origin, due to the interaction of the magnetic field and the fluid current, is always in a direction such that the fluid tends to travel at the same velocity as the field. The slip s , defined as

$$s = \frac{v_s - v}{v_s}, \quad (1.3.1)$$

is the difference between the field velocity v_s and the fluid velocity v normalized with respect to v_s . If v is less than v_s (s positive), the force accelerates the fluid, energy is transferred to the fluid, and the device acts as a pump or accelerator. If v is greater than v_s , the force opposes the motion and the machine is a generator, converting mechanical energy to electrical energy. If v and v_s are in opposite directions, the machine is a flow damper, absorbing both mechanical and electrical energy. At synchronous speed ($v = v_s$), there is no relative motion between the fluid and the field, and consequently there is no interaction.

There is no direct electrical connection to the fluid, so that all fluid currents are due to a combination of transformer and motional induction. The effect of the fluid is detected from the electrical terminals only by the change in the electrical impedance, or equivalently, the electromagnetic fields. If there is to be an appreciable energy transfer relative to the stored magnetic energy, as is required for high efficiency, the induced magnetic field due to the fluid currents must be comparable with the applied magnetic field. Approximate solutions for the magnetic field or complete neglect of the induced field are not valid. The real part of the terminal impedance, representing in part the energy transfer to or from mechanical form, is negative for operation as a generator. The imaginary part is always inductive, requiring external capacitive-energy storage.

The above description corresponds exactly to a conventional rotating induction machine,^{18,19} and previous knowledge of the rotating version can be applied to the MHD version with appropriate modifications for the fluid and geometry. The fluid is more complex because the velocity is not constant, and there are additional losses due to viscosity. The geometry is similar to linear induction motors²⁰ or induction motors with arc-shaped stators (a stator that occupies only a fraction of the rotor periphery),²¹ and shares with them the distortion of the electromagnetic fields at the entrance and exit regions to the traveling field structure. While the channel may be of constant cross-sectional area, variable-area geometry enables the dynamic head to be utilized for energy conversion

and reduces the static pressure at the generator entrance. The machine may have either a flat linear or an annular configuration.

Previous studies have aimed at establishing the performance and operating conditions of idealized models of the device. The basic design problem, however, is to determine the dimensions of the fluid channel and field structure that permit operation at maximum efficiency when the generator is operated between the fluid inlet and outlet conditions determined by analysis of the appropriate MHD power cycle. Limitations arise from the properties of materials, and the design procedure must be carried through with the parameters that express these. From the electromagnetic viewpoint, the basic limitations are imposed by saturation of the teeth of the field structure and by the maximum current density in the exciting winding as determined by heat-transfer considerations. The mechanical, thermal, and electrical properties of the duct wall must also be included.

1.4. Previous Work

The earliest consideration of MHD induction machines was in connection with electromagnetic (or MHD) pumps for liquid metals since these pumps avoid the leakage and seal problems associated with conventional pumps.

An induction pump was first proposed by Chubb²² in 1915 to pump a liquid metal by means of a rotating magnetic field in a spiral-channel induction pump. Einstein and Szilard²³ in 1928 suggested an annular traveling-field pump for use with alkali metals, and its application to a refrigeration cycle.

The development of the MHD pump proceeded slowly until the need arose for pumps to handle liquid metals for cooling nuclear reactors.^{24, 25} The theory of the conduction pump (and generator) is now well understood,^{26, 27} and efforts have been devoted to consideration of end effects, boundary layers, and turbulence. The theory of the induction pump is not as well developed as that of the conduction pump since exact solutions are not possible. Design procedures based on a combination of theory, experiment, and experience from conventional rotating induction machines have been used by Baker,²⁸ Barnes *et al.*,²⁹ and Watt.³⁰ This basic analysis has been extended by several authors³¹⁻³³ to include approximate velocity profiles and more detailed experimental and theoretical results. The efficiencies obtained with actual pumps are less than 50%, and this is too low to be acceptable if the pump is to be operated in the generator mode. Both conduction and induction pumps are commercially available.^{34, 35}

The present investigation is the outgrowth of several previous studies. Harris³⁶ studied induction-driven flows and obtained the approximate

one-dimensional velocity profile for the slit-channel machine.* He also investigated the power flow in the induction machine and extended his work on turbulence to induction-driven flows. Penhune³⁷ examined the internal behavior of laminar induction-driven flows in more detail, but did not attempt to study machine operation. Jackson and Pierson,¹⁶ utilizing a highly idealized model, obtained the operating characteristics of the induction generator. Pierson³⁸ studied the power flow and terminal properties in more detail, obtaining the parameter regions of acceptable generator operation. Fluid viscosity and finite length were included in this analysis.

Several significant experimental studies have been undertaken, but more work is required. Reid and Jackson³⁹ first operated a small generator, although the machine was not self-excited. Porter⁴⁰ designed a coil system and tested it in the pump mode to study the interaction. Later experiments with this coil system will extend the tests to generator operation. Elliott⁴¹ started an experimental program on electrically-short induction generators, but no results are as yet available.

Considerable attention has been devoted to induction generators operated on plasma flows. Most of the analyses are valid only for $sR_{Mf}^{**} \ll 1$ (which is an impractical operating regime for energy conversion) and are restricted to an inviscid and incompressible fluid. Also, these analyses have not determined the machine properties and the conditions for best operation, which clearly show that a plasma is not practical in an induction generator unless higher conductivities are obtainable.¹⁶ Fanucci *et al.*⁴² made a comprehensive study of the induction generator, including an experiment.⁴³ The theory considers compressibility and finite length, but not the basic properties necessary to design a practical machine. The experimental power flow was small.

Sudan⁴⁴ treated the machine with a finite-length current sheet, but did not interpret his results in terms of a practical winding. Peschka, Kelm, and Engeln⁴⁵ considered the induction machine with a finite-length winding, but their results differ significantly from those of Pierson,⁴⁶ who made a detailed study for the case of a finite-length winding and infinite-length core considering both the electrical impedance and power flow. Elliott⁴⁷ and Weh and Waltke⁴⁸ considered the finite-length problem with an iron core of the same length as the winding, which is the condition occurring in actual machines if the core does not appreciably overhang the winding.

*A split-channel machine has a channel height that is much less than the pole spacing of the excitation that produces the traveling field.

** sR_{Mf} , the magnetic Reynolds number based on the velocity difference $v_s - v$ and the wavelength, indicates the magnitude of the field-fluid interaction. The case $sR_{Mf} \ll 1$ means that the field induced in the fluid is negligible compared to the applied field.

1.5. Material Constraints

Previous studies of the induction generator have shown that between the fluid and core there should be no material that will either decrease the magnetic-flux density or shield it out of the fluid. It has also been established that the core must possess a high permeability, a high-saturation magnetic-flux density (for high power density), and low loss. In all induction-generator configurations thus far considered, a channel is required to contain the liquid metal at the high temperatures ($\geq 1600^\circ\text{F}$) envisioned for a practical high-power system. Thermal insulation is also required to restrict heat loss from the fluid and keep the temperature of the iron core sufficiently below the Curie point to maintain the required magnetic properties.

The fabrication of the channel walls is a primary problem because they must withstand corrosion and erosion by the hot, high-velocity liquid metal, and must be able to contain pressures that may be on the order of hundreds of pounds per square inch. These considerations imply a thick wall at these temperatures, but all presently available materials are good electrical and thermal conductors, so that electrical dissipation and heat loss must be taken into account. The electrical conductivities of the refractory metals are in the same range as those for the liquid metals of interest in MHD power systems. As the walls experience a slip s of unity, or a higher time-rate-of-change of the magnetic field than the fluid, the electrical-power loss density in the walls may substantially exceed the corresponding loss density in the fluid, and the total loss may even exceed the power generated by the fluid.

The materials presently available to handle liquid metals at these temperatures are the refractory metals--columbium (niobium), molybdenum, rhenium, tantalum, tungsten, and their alloys. They are hard to work with and very brittle, but they do retain structural strength at high temperatures and are not seriously corroded by liquid metals, so that systems can be built from them with reasonable lifetimes. For a large power-generation system, the channel should have a lifetime at least as great as the elapsed time between shutdowns of the nuclear heat source.

To evaluate the various wall materials, the required wall thickness is calculated by using the ASME Boiler and Pressure Vessel Code for Unfired Pressure Vessels.⁴⁹ Attention is confined to the annular geometry because the wall thickness and structural requirements are an order of magnitude less than for the rectangular geometry at these dimensions. Also, with the annular geometry there is no reduction in power level due to edge effects where the current loops in the fluid must close. For a cylindrical shell under internal pressure, the wall thickness is⁵⁰

$$t = \frac{pr}{SE - 0.6p} \quad (1.5.1)$$

for

$$t \leq \frac{r}{2} \text{ or } p \leq 0.385SE,$$

where t is the thickness (in inches), p is the design or maximum pressure (in psi), r is the inside radius of the cylinder (in inches), S is the maximum allowable stress (in psi), and E is the joint efficiency, which can be taken as unity for completely radiologically tested welds.⁵¹ The restriction, $t \leq r/2$, is imposed because the stress is no longer uniformly distributed if the wall becomes too thick. Formulas are available for this case, which is not of interest for a practical MHD induction machine. The above formula has two safety factors: the $-0.6p$ in the denominator, which requires thick shells if SE is not much larger than p , and the choice of allowable values of S . The values of S tabulated in the code are usually one-fourth of the minimum tensile strength of the material at the operating temperature.⁵²

The properties of several refractory metals at 1600 and 2200°F are specified in Table I. The high-temperature properties are more complex because tensile strength alone is not sufficient; materials tend to creep under sustained stress and eventually to rupture. For pure columbium at 1600°F, the tensile strength is 16,000 psi, but the stress for rupture is 9000 psi in 1 hr, 7900 psi in 10 hr, and 6900 psi in 100 hr.⁵³ For columbium alloy B-33 (developed by Westinghouse), at 2000°F the tensile strength is 35,000 psi and the 0.2% yield strength is 31,000 psi, but the stress for rupture in 1 hr is 22,000 psi.⁵⁴ There is a lack of uniformity in specifying data for the various materials, and in addition, the measured properties

TABLE I. Refractory-metal Properties and Wall Thicknesses for an Annular Duct One meter in Diameter

Material	Temp, °F	Tensile Strength, 1000 psi	Rupture Stress		Electrical Conductivity, 10 ⁶ mhos/m	S, 1000 psi	Thickness, in.	
			Time, hr	Stress, 1000 psi			p = 600 psi	p = 200 psi
Columbium ⁵³	1600	16	100	6.9	2	4	3.3	1.0
	2200	6	100	4		1.5	a	2.9
Molybdenum ⁵⁵	1600	30	100	32	2	7.5	1.7	0.53
	2200	18			1.5	4.5	2.9	0.90
Tantalum ⁵⁶	1600	23			2	5.8	2.2	0.70
	2200	15			1.6	3.8	3.5	1.1
Columbium alloy B-33 ⁵⁴ (4% vanadium)	1600	65				16	0.76	0.25
	2200	22				5.5	2.3	0.73
Columbium alloy D-31 ⁵⁷ (10% titanium, 10% molybdenum)	1600	56				14	0.87	0.28
	2200	20				5	2.6	0.81
Tantalum alloy T-111 ⁵⁸ (7-9% tungsten, 2-2.8% hafnium)	2200	67 ^b			1.6	17	0.71	0.23
Stainless steel ⁵⁹	1500					1.5	a	2.9

^aFormula not valid.

^bRanges from 49 to 85 depending on treatment.

are very dependent on the method used. Other important factors are small amounts of impurities in the metal, the atmosphere in which the tests are conducted, and the previous treatment of the metal. When available, both tensile and rupture stresses are supplied. Where several figures are given for different tests or experimental conditions, an average is used in the table. The electrical conductivity is also given when available.

The wall thicknesses required to contain pressures of 600 and 200 psi, calculated by using Eq. 1.5.1, are also given in Table I. The diameter of the annular duct is taken to be one meter, a reasonable size for a large generator. One-quarter of the tensile strength is used for S , but this value should be checked, probably experimentally, for both rupture and creep over long periods of time at the operating temperature. The joint efficiency E is taken as unity. All the thicknesses, although small compared to the diameter, are shown in Chapter III to be too large for acceptable operation because of the induced electrical loss.

The allowable operating temperature of the field structure so that it retains its ferromagnetic properties controls the amount of thermal insulation required between it and the fluid, and also the method chosen for cooling. The desired properties of the thermal insulation are high-temperature operation, low thermal conductivity, and high compressive strength. Ideally, the insulation should be able to withstand contact with the hot, high-velocity liquid metal, eliminating the separate channel wall; but no suitable material has yet been identified. With high compressive strength, thin channel walls can be used and the stress transmitted through the insulation to the core, the walls serving only to protect the rest of the system from the liquid metal.

Important constraints imposed by the field structure are:

1. The peak flux density in the structure (at the roots of the teeth) must be less than the saturation flux density.
2. The temperature must be sufficiently below the Curie point to avoid the loss of saturation flux density and relative permeability.
3. The temperature must be low enough to allow the laminations and exciting winding to be electrically insulated.

When allowance is made for the slots in the core and the stacking factor due to electrical insulation between the laminations, the peak field in the gap will be about half of the saturation value. The lamination thickness and insulation must be chosen to reduce the core loss to an acceptable value at the operating temperature of the field structure.

The large air gap of the MHD induction machine introduces additional difficulties relative to the rotating version. The large air gap requires a much greater number of ampere-turns and (since the current density is limited by heat-transfer considerations) relatively wide and deep slots. Wide slots result in less average magnetic-flux density, and deep slots require tapering of the slots to avoid premature saturation at the roots of the teeth. The primary difficulty of deep slots, though, is the large leakage reactance.

The field structure may be cooled by the development of conventional methods, but alternatives are worth serious consideration. Conventional cooling of the exciting winding is probably not sufficient because the current density is too low to allow for slots of acceptable dimensions. The use of cryogenic coils with a separate refrigerator may be feasible, although superconducting windings cannot be employed for a.c.⁶⁰ Hollow conductors cooled by sodium circulation, where the sodium carries part or all of the current, offer the possibility of higher current densities but introduce difficulties in the electrical insulation between turns. A separate cooling system may be required for the core since the electrical insulation will also isolate it thermally from the exciting winding.

In this review of limitations imposed by materials considerations on the MHD induction generator, clearly the ultimate design optimization is a complex procedure that must take friction loss, wall loss, end loss, and loss in the field structure into account. Previous work has shown that the friction loss can be reduced to an acceptable value, at least in large machines, but insufficient information is available to deal satisfactorily with end and field-structure losses. Since the fabrication of walls of sufficient strength and acceptable erosion and corrosion performance is a first requirement in generator construction, the wall losses were selected for detailed treatment and are dealt with in this report.

II. THE INFINITE-LENGTH MACHINE WITHOUT CHANNEL WALLS AND INSULATION

2.1. The Model

The basic model to be analyzed is shown in Fig. 2. The average fluid flow is in the x-direction between two parallel channel walls of infinite extent in the x- and z-directions

Fig. 2. Model of Infinite-length Machine

spaced a distance $2a_f$ apart. The channel walls of thickness a_w , permeability μ_w , and electrical conductivity σ_w ; and the thermal insulation of thickness a_i , permeability μ_i , and zero conductivity; are required to contain the fluid and protect the rest of the machine from corrosion and heat. The region outside the excitation sheets is filled with a core of permeability μ_c and conductivity σ_c . The exciting plates are assumed thin so that they can be replaced by current sheets with a surface conductivity $\sigma_s = \sigma_e b$, where b is the plate thickness and σ_e the material conductivity. The details of the excitation system are used only in calculating the power loss and the leakage reactance. Insulators

of infinitesimal thickness are required between the different regions to prevent current flow in the y-direction.*⁶¹

The exciting plates are driven by a current source that gives an even or symmetric surface current density

$$\vec{K} = \vec{i}_z NI \cos(\omega t - kx). \quad (2.1.1)$$

This represents a traveling-current wave of amplitude NI , frequency ω , and wavelength and velocity

$$\lambda = \frac{2\pi}{k} \quad (2.1.2)$$

*These insulators will actually exist, except between the fluid and the conducting channel wall. In a rectangular machine, current may flow between the fluid and channel wall. For an annular machine, there will be no current flow between the fluid and the wall.

and

$$v_s = \frac{\omega}{k}. \quad (2.1.3)$$

Excitation with odd symmetry was previously shown to be impractical.⁶² Unbalanced excitation, where the currents in the multiphase winding producing the traveling current are not equal, can be treated as a direct extension of the balanced case.⁶³ The machine of finite length in the x-direction is discussed in Chapter IV.

The fluid flow will in general be turbulent, so that details of the flow are of limited value. The fluid velocity is assumed to be constant, which uncouples the electromagnetic and fluid equations and allows an analytical solution to be obtained. This emphasizes the important features of the field-fluid interaction without obscuring it in mathematical detail. Approximate solutions for laminar and turbulent fluid flows have been obtained elsewhere.⁶⁴

2.2. The Equations

The equations to be solved are Maxwell's equations for the electromagnetic field with the usual MHD approximation that neglects displacement currents, and the Navier-Stokes or conservation of momentum equation for the incompressible fluid:

$$\nabla \times \vec{B} = \mu \vec{J}, \quad (2.2.1)$$

$$\nabla \cdot \vec{B} = 0, \quad (2.2.2)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (2.2.3)$$

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}), \quad (2.2.4)$$

$$\nabla \cdot \vec{J} = 0, \quad (2.2.5)$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}, \quad (2.2.6)$$

$$\rho \left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \vec{v} = -\nabla p + \eta \nabla^2 \vec{v} + \vec{J} \times \vec{B}, \quad (2.2.7)$$

and

$$\nabla \cdot \vec{v} = 0; \quad (2.2.8)$$

where the symbols are defined in the nomenclature. Equation 2.2.5 is a consequence of neglecting displacement currents in Eq. 2.2.1. Equation 2.2.6 serves solely to determine the required charge density since the other equations are sufficient to determine the fields. Equations 2.2.4 and 2.2.8 are Ohm's law and conservation of mass, respectively, for the moving, incompressible fluid.

The analysis is simplified by the use of a vector potential $\vec{A}$ defined by

$$\vec{B} = \nabla \times \vec{A}, \quad (2.2.9)$$

which satisfies Eq. 2.2.2. Equation 2.2.3 then becomes

$$\vec{E} = -\nabla\phi - \frac{\partial\vec{A}}{\partial t}, \quad (2.2.10)$$

defining the scalar potential ϕ . To complete the specification of the vector potential, its divergence is chosen to satisfy

$$\nabla \cdot \vec{A} + \mu\sigma\phi = 0, \quad (2.2.11)$$

which uncouples the equations for the two potentials. Substituting Eqs. 2.2.4, 2.2.9, and 2.2.10 into Eq. 2.2.1 and using condition 2.2.11 gives

$$\nabla^2 \vec{A} - \mu\sigma \frac{\partial\vec{A}}{\partial t} + \mu\sigma(\vec{v} \times \nabla \times \vec{A}) = 0. \quad (2.2.12)$$

Equations 2.2.11 and 2.2.12 are sufficient because the vector potential, due solely to currents, will be assumed to be z-directed and independent of z. Consequently,

$$\phi = 0 \quad (2.2.13)$$

from Eq. 2.2.11 for any material of nonzero conductivity.

The resulting two coupled, nonlinear, vector partial-differential equations are Eq. 2.2.12 and

$$\rho\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right) \vec{v} = -\nabla p + \eta \nabla^2 \vec{v} - \frac{1}{\mu} (\nabla^2 \vec{A}) \times (\nabla \times \vec{A}), \quad (2.2.14)$$

obtained by writing $\vec{J} \times \vec{B}$ in terms of $\vec{A}$. A general solution cannot be obtained because of the nonlinear terms and the two-way coupling, plus the difficulty in describing the fluid behavior, although approximate solutions have been obtained for laminar and turbulent fluid flows.⁶⁴

The formulas required to calculate the powers are summarized below. Consideration is confined to the time-average powers because of the field products involved. The analysis is restricted to a time-independent fluid velocity, so that the vector potential and field components will have the same sinusoidal time dependence as the excitation since Eq. 2.2.12 is linear with coefficients independent of time. In complex notation, the field components are written as

$$\vec{A}(x, y, z, t) = \text{Re}\{\underline{\vec{A}}(x, y, z) e^{j\omega t}\}, \quad (2.2.15)$$

where underlining indicates the complex amplitude.

The time-average power supplied to a volume is

$$P_s = \text{Re}\left\{-\oint_{\text{surf}} (\underline{\vec{S}} \cdot \underline{\vec{i}}_n) da\right\}, \quad (2.2.16)$$

where

$$\underline{\vec{S}} = \frac{1}{2\mu} (\underline{\vec{E}} \times \underline{\vec{B}}^*) \quad (2.2.17)$$

is the time-averaged complex Poynting vector, the * denotes the complex conjugate, $\underline{\vec{i}}_n$ is a unit vector normal to the surface and outwardly directed, and the surface encloses the volume.

The mechanical power output (the power supplied to the external fluid system by the machine) is the product of the time-average electromagnetic pressure gradient and the velocity integrated over the fluid volume,

$$P_m = \int_{\text{vol}} \langle \nabla P_{em} \rangle \cdot \vec{v} dxdydz, \quad (2.2.18)$$

where

$$\langle \nabla P_{em} \rangle = \text{Re}\left\{\frac{\underline{\vec{J}} \times \underline{\vec{B}}^*}{2}\right\}. \quad (2.2.19)$$

The power dissipated in a material due to its finite conductivity, always positive, is

$$P_r = \int_{\text{vol}} \frac{\underline{\vec{J}} \cdot \underline{\vec{J}}^*}{2\sigma} dxdydz. \quad (2.2.20)$$

This is used for the fluid loss, but the wall and core losses may be found from Eq. 2.2.16 or 2.2.20.

2.3. The Exact Solution

The previous methods and results⁶⁵ are summarized here for reference and comparison before conducting channel walls and thermal insulation are added in Chapter III. The model is as shown in Fig. 2 without the walls and insulation. The constant fluid velocity eliminates the need for the fluid equations. Since the system is linear, the vector potential must possess the same x - and t -dependence as the excitation. In complex notation it becomes

$$\vec{A}(x, y, t) = \text{Re} \left\{ \vec{i}_z \underline{A}(y) e^{j(\omega t - kx)} \right\}, \quad (2.3.1)$$

where $\vec{A}$ is in the z -direction and independent of z for a machine of infinite width.

The equations for the complex vector potential in the fluid and core, denoted by the subscripts f and c , respectively, from Eq. 2.2.12, are

$$\frac{d^2 \underline{A}_f}{dy^2} - \gamma_f^2 k^2 \underline{A}_f = 0, \quad (2.3.2)$$

and

$$\frac{d^2 \underline{A}_c}{dy^2} - \gamma_c^2 k^2 \underline{A}_c = 0; \quad (2.3.3)$$

where

$$\gamma_f^2 = 1 + jsR_{Mf}, \quad (2.3.4)$$

$$\gamma_c^2 = 1 + jR_{Mc}, \quad (2.3.5)$$

$$R_{Mf} = \frac{\mu_f \sigma_f v_s}{k}, \quad (2.3.6)$$

$$R_{Mc} = \frac{\mu_c \sigma_c v_s}{k}, \quad (2.3.7)$$

and

$$s = \frac{v_s - v}{v_s}. \quad (2.3.8)$$

Here, R_{Mf} and R_{Mc} are the fluid and core magnetic Reynolds numbers, respectively, based on synchronous speed and wavelength. It is appropriate to use a magnetic Reynolds number for the core because it moves relative to the field with a velocity v_s . The slip based on synchronous (field) speed s represents the motion of the field relative to the fluid. The magnitude of the field-field interaction is indicated by sR_{Mf} , which is based on the velocity difference $v_s - v$, rather than by R_{Mf} alone.

The forms of the potentials, from the above equations, are

$$\underline{A}_f = \underline{A}_{f1} \cosh \gamma_f k y + \underline{A}_{f2} \sinh \gamma_f k y, \quad (2.3.9)$$

and

$$\underline{A}_c = \underline{A}_{c1} e^{-\gamma_c k |y|}, \quad (2.3.10)$$

since $\underline{A}_c$ must approach zero far from the core. The vector potential is symmetric, from the symmetric excitation, so that $\underline{A}_{f2} = 0$. The remaining constants come from the boundary conditions on the magnetic field; the electric field conditions do not enter because displacement currents are neglected. At the fluid-core interface, the normal magnetic field is continuous, and the discontinuity in the tangential magnetic field is proportional to the surface current. In terms of the vector potential, the conditions at $y = \pm a_f$ are

$$\underline{A}_f = \underline{A}_c, \quad (2.3.11)$$

and

$$\frac{1}{\mu_f} \frac{\partial \underline{A}_f}{\partial y} - \frac{1}{\mu_c} \frac{\partial \underline{A}_c}{\partial y} = \pm \underline{K}(x), \quad (2.3.12)$$

where $\underline{K}(x)$ is the complex amplitude of the exciting current (NI for this case).

The resulting vector potentials are

$$\underline{A}_f = \frac{\mu_f NI \cosh \gamma_f k y}{k(\gamma_f \sinh \gamma_f a_f + \kappa \gamma_c \cosh \gamma_f a_f)} \text{ for } |y| \leq a_f, \quad (2.3.13)$$

and

$$\underline{A}_c = \frac{\mu_f NI e^{-\gamma_c k(|y| - a_f)}}{k(\gamma_f \tanh \gamma_f a_f + \kappa \gamma_c)} \text{ for } |y| \geq a_f; \quad (2.3.14)$$

in which

$$\alpha_f = a_f k \quad (2.3.15)$$

and

$$\kappa = \frac{\mu_f}{\mu_c} \quad (2.3.16)$$

are the ratios, respectively, of the channel width to the excitation wavelength and the fluid to core permeability. The electromagnetic fields are determined from the potentials through Eqs. 2.2.9 and 2.2.10.

The impedance seen at the terminals of an exciting coil can be found from the vector potential, or alternatively, from the powers.⁶⁶ As the power flow determines the design and evaluation of a machine, and the impedance is used only for interconnecting with other devices, only the powers are considered here.

The time-average powers are calculated from the equations of Section 2.2. The complex Poynting vector in the fluid is

$$\underline{\vec{S}}_f = \frac{\mu_f v_s N^2 I^2 (\vec{i}_x \cosh \gamma_f k y \cosh \gamma_f^* k y - \vec{i}_y j \gamma_f^* \cosh \gamma_f k y \sinh \gamma_f^* k y)}{2(\gamma_f \sinh \gamma_f \alpha_f + \kappa \gamma_c \cosh \gamma_f \alpha_f)(\gamma_f^* \sinh \gamma_f^* \alpha_f + \kappa \gamma_c^* \cosh \gamma_f^* \alpha_f)} \quad (2.3.17)$$

The x-component represents a constant power flow; it makes no contribution to the power supplied to the fluid. The powers for a segment out of an infinite length and width machine n wavelengths long in the x-direction and of depth c in the z-direction are

$$P_{sf} = \frac{P_0 \text{Im}\{\gamma_f \tanh \gamma_f \alpha_f\}}{(\gamma_f \tanh \gamma_f \alpha_f + \kappa \gamma_c)(\gamma_f^* \tanh \gamma_f^* \alpha_f + \kappa \gamma_c^*)}, \quad (2.3.18)$$

$$P_m = (1 - s) P_{sf}, \quad (2.3.19)$$

and

$$P_{rf} = s P_{sf}; \quad (2.3.20)$$

where

$$P_0 = \mu_f v_s N^2 I^2 c n \lambda. \quad (2.3.21)$$

The above results for P_{sf} , P_m , and P_{rf} are identical to the power-flow relations for a rotating induction machine.^{18,19} The output power for a generator for NI fixed, from Eq. 2.3.18, is highest if $\gamma_f \alpha_f$ is small, and best performance is expected as α_f approaches zero. Note that P_{sf} is the electrical power supplied to the fluid, and differs from the electrical input power to the machine by the core and excitation losses. Similarly, the mechanical power output is decreased by the viscous loss. The core loss is

$$P_{rc} = \frac{P_0 \kappa \text{Im}\{\gamma_c\}}{(\gamma_f \tanh \gamma_f \alpha_f + \kappa \gamma_c)(\gamma_f^* \tanh \gamma_f^* \alpha_f + \kappa \gamma_c^*)} \quad (2.3.22)$$

The electrical efficiency, including only the power loss in the fluid conductivity, is

$$e_g = \frac{1}{1 - s} \quad (2.3.23)$$

for generator operation ($s < 0$), or, for pump operation ($s > 0$),

$$e_p = 1 - s. \quad (2.3.24)$$

This represents the upper limit to the attainable efficiency inherent in the induction machine.

To achieve optimum utilization of the generator volume, the machine will normally be operated at the highest magnetic-flux density consistent with saturation of the teeth of the iron core. Instead of specifying NI, B_{sat} would be set and NI chosen to operate at this point. The magnetic flux density in the core, from Eq. 2.3.14, is

$$\vec{B}_c = (-\vec{i}_x \gamma_c + \vec{i}_y j) \frac{\mu_f NI e^{-\gamma_c k(|y| - \alpha_f)}}{\gamma_f \tanh \gamma_f \alpha_f + \kappa \gamma_c} \quad (2.3.25)$$

For a core with small loss, so that $\gamma_c \approx 1$, this represents a circularly polarized field, so that

$$NI = \frac{B_{sat}}{\mu_f} (\gamma_f \tanh \gamma_f \alpha_f + \kappa \gamma_c) \quad (2.3.26)$$

in terms of the field in the core corresponding to saturation of the teeth. Allowance has to be made in choosing B_{sat} for the stacking factor of the laminations in addition to the slots. In terms of B_{sat} , the powers become

$$P_{sf} = \frac{cn\lambda v_s}{\mu_f} (B_{sat})^2 \text{Im}(\gamma_f \tanh \gamma_f \alpha_f), \quad (2.3.27)$$

with Eqs. 2.3.19 and 2.3.20 unchanged. In this case, as opposed to NI constant, the highest power is obtained for α_f large, but if $\gamma_f \alpha_f$ is not small compared to unity, the volume and fluid flow are not fully utilized because the field does not completely penetrate the fluid. If the field does not completely penetrate the fluid, the velocity profile is not flat, as assumed here, and generator operation is not possible.⁶⁷ The equations in terms of B_{sat} will normally be used in generator design.

2.4. The Slit-channel Solution

The case in which $\gamma_f \alpha_f$ is small compared to unity is of particular interest because the magnetic field in the fluid is primarily transverse to the flow or y -directed and independent of y . The field-fluid interaction is accordingly constant throughout the volume. This case corresponds to maximum utilization of the fluid and is normally preferred. If $\gamma_f \alpha_f$ is large, the magnetic field does not penetrate the fluid, the interaction is confined to the boundaries, and the rest of the fluid does not contribute to the electromagnetic interaction. For this reason it is expected that a practical machine will be designed with $\gamma_f \alpha_f \ll 1$, referred to as a slit-channel machine. This may not be true for a generator with wall loss, as shown later in Section 3.3, because the power generated in the fluid must be substantially greater than the loss in the walls. The operation must be close to the slit-channel case, however, for the generator mode to be attained.

It is significantly easier to analyze the slit-channel machine and to visualize its dependence on machine parameters, although the performance predictions may be slightly optimistic. The approach used in this report, particularly in Chapter III where the equations are quite complex, is to determine trends from the slit-channel theory and numerical results from the exact theory. The two solutions are compared numerically in Chapter III both with and without channel walls.

The equations for the slit-channel machine without walls can be obtained from the general case by taking the limit as $\gamma_f \alpha_f$ approaches zero. However, it is just as simple here to repeat the derivation starting from the initial assumption that the magnetic field in the fluid is y -directed and independent of y , or the vector potential is independent of y . This approach is necessary with the channel wall where the general equations cannot be easily simplified. The differential form of Maxwell's equations cannot be used because of the neglected y -dependence. Instead, $\vec{B}$ or $\vec{A}$ is determined from the integral form of Eq. 2.2.1, which is

$$\oint_{\text{cont}} \frac{\vec{B}}{\mu} \cdot d\vec{\ell} = \int_{\text{surf}} (\vec{J} \cdot \vec{i}_n) da. \quad (2.4.1)$$

This equation is used with the dashed contour of Fig. 2, so that $\vec{J}$ is the sum of the fluid current (from Eq. 2.2.4) and the exciting current. The form of the vector potential in the core is unchanged (Eq. 2.3.10), and the single boundary condition required is that the normal magnetic flux density is continuous. The result is

$$\underline{A_f} = \frac{\mu_f NI}{k(\gamma_f^2 \alpha_f + \kappa \gamma_c)}, \quad (2.4.2)$$

which is identical to Eq. 2.3.13 for $\gamma_f \alpha_f \ll 1$.

Poynting's theorem cannot be used for the slit-channel machine because of the neglect of the x-component of the magnetic field. Instead, P_m and P_{rf} are calculated from their formulas; then, from conservation of energy,

$$P_{sf} = P_m + P_{rf}. \quad (2.4.3)$$

The result, for a lossless core, is

$$P_{sf} = P_0 \left(\frac{1}{\kappa + \alpha_f} \right) \frac{s R_{M\alpha}}{1 + s^2 R_{M\alpha}^2}, \quad (2.4.4)$$

where

$$R_{M\alpha} = \left(\frac{1}{1 + \frac{\kappa}{\alpha_f}} \right) R_{Mf} \quad (2.4.5)$$

is the pertinent magnetic Reynolds number. The power relations, Eqs. 2.3.19 and 2.3.20, are unchanged. The slit-channel formulas are accurate for α_f less than a few tenths. For operation at saturation of the magnetic core,

$$NI = \frac{B_{sat} |\gamma_f^2 \alpha_f + \kappa \gamma_c|}{\mu_f}, \quad (2.4.6)$$

and

$$P_{sf} = \frac{cn \lambda \alpha_f v_s}{\mu_f} (B_{sat})^2 s R_{Mf}. \quad (2.4.7)$$

III. THE INFINITE-LENGTH MACHINE WITH CHANNEL WALLS AND THERMAL INSULATION

3.1. Introduction

From Chapter I, it is evident that a channel wall and thermal insulation are required to contain the fluid and protect the field structure from corrosion, erosion, and excessive temperatures. Since with presently available materials at least part of the wall has to be a good electrical conductor, it is important to include its effect in the theory. The analysis of Chapter II is extended here to include a conducting channel wall and insulation using the model shown in Fig. 2 and described in Section 2.1. Both exact and slit-channel theories are presented and compared. The slit-channel equations are easier to interpret, but the general numerical results are preferred for design purposes.

3.2. The Electromagnetic Fields

The addition of walls and insulation results in a much more complicated system algebraically. There are now four regions with their characteristic solutions and, after using symmetry, six boundary conditions. Using the same notation as before, with the addition of w and i subscripts for the wall and insulation, respectively, the potentials from Eq. 2.3.2 are

$$\underline{A}_f = \underline{A}_{f1} \cosh \gamma_f k y, \quad (3.2.1)$$

$$\underline{A}_c = \underline{A}_{c1} e^{-\gamma_c k |y|}, \quad (3.2.2)$$

$$\underline{A}_w = \underline{A}_{w1} \cosh \gamma_w k y + \underline{A}_{w2} \sinh \gamma_w k y, \quad (3.2.3)$$

and

$$\underline{A}_i = \underline{A}_{i1} \cosh k y + \underline{A}_{i2} \sinh k y; \quad (3.2.4)$$

where

$$\gamma_w^2 = 1 + j R_{Mw}, \quad (3.2.5)$$

and

$$R_{Mw} = \frac{\mu_w \sigma_w v_s}{k}. \quad (3.2.6)$$

Applying the boundary conditions on the normal and tangential magnetic fields (Eqs. 2.3.11 and 2.3.12) to the fluid-wall, wall-insulation, and insulation-core interfaces gives the six equations of Table II, where

$$\alpha_w = a_w k, \quad (3.2.7)$$

and

$$\alpha_i = a_i k. \quad (3.2.8)$$

An analytical solution of the six simultaneous equations for the constants is of limited value because of the complexity of the terms. A numerical solution runs into difficulty for large values of α_f , α_w , and α_i because of the exponentials, but this approach is satisfactory for values of practical interest. The limit is set by the number of digits carried by the computer. The solution is checked by substituting the constants back into the original equations.

TABLE II. The Six Boundary Conditions

<u>Equations</u>			
$H_{11} \frac{A_{f1}}{\mu_f}$	$+ H_{13} \frac{A_{w1}}{\mu_w} + H_{14} \frac{A_{w2}}{\mu_w}$		$= 0$
$H_{21} \frac{A_{f1}}{\mu_f}$	$+ H_{23} \frac{A_{w1}}{\mu_w} + H_{24} \frac{A_{w2}}{\mu_w}$		$= 0$
	$H_{33} \frac{A_{w1}}{\mu_w} + H_{34} \frac{A_{w2}}{\mu_w} + H_{35} \frac{A_{i1}}{\mu_i} + H_{36} \frac{A_{i2}}{\mu_i}$		$= 0$
	$H_{43} \frac{A_{w1}}{\mu_w} + H_{44} \frac{A_{w2}}{\mu_w} + H_{45} \frac{A_{i1}}{\mu_i} + H_{46} \frac{A_{i2}}{\mu_i}$		$= 0$
$H_{52} \frac{A_{c1}}{\mu_c}$	$+ H_{55} \frac{A_{i1}}{\mu_i} + H_{56} \frac{A_{i2}}{\mu_i}$		$= 0$
$H_{62} \frac{A_{c1}}{\mu_c}$	$+ H_{65} \frac{A_{i1}}{\mu_i} + H_{66} \frac{A_{i2}}{\mu_i}$		$= NI$
<u>Coefficients</u>			
$H_{11} = \cosh \gamma_f \alpha_f$	$H_{21} = \frac{\gamma_f}{\mu_f} \sinh \gamma_f \alpha_f$		
$H_{13} = -\cosh \gamma_w \alpha_f$	$H_{23} = -\frac{\gamma_w}{\mu_w} \sinh \gamma_w \alpha_f$		
$H_{14} = -\sinh \gamma_w \alpha_f$	$H_{24} = -\frac{\gamma_w}{\mu_w} \cosh \gamma_w \alpha_f$		
$H_{33} = \cosh \gamma_w (\alpha_f + \alpha_w)$	$H_{43} = \frac{\gamma_w}{\mu_w} \sinh \gamma_w (\alpha_f + \alpha_w)$		
$H_{34} = \sinh \gamma_w (\alpha_f + \alpha_w)$	$H_{44} = \frac{\gamma_w}{\mu_w} \cosh \gamma_w (\alpha_f + \alpha_w)$		
$H_{35} = -\cosh (\alpha_f + \alpha_w)$	$H_{45} = -\frac{1}{\mu_i} \sinh (\alpha_f + \alpha_w)$		
$H_{36} = -\sinh (\alpha_f + \alpha_w)$	$H_{46} = -\frac{1}{\mu_i} \cosh (\alpha_f + \alpha_w)$		
$H_{52} = e^{-\gamma_c (\alpha_f + \alpha_w + \alpha_i)}$	$H_{62} = \frac{k \gamma_c}{\mu_c} e^{-\gamma_c (\alpha_f + \alpha_w + \alpha_i)}$		
$H_{55} = -\cosh (\alpha_f + \alpha_w + \alpha_i)$	$H_{65} = \frac{k}{\mu_i} \sinh (\alpha_f + \alpha_w + \alpha_i)$		
$H_{56} = -\sinh (\alpha_f + \alpha_w + \alpha_i)$	$H_{66} = \frac{k}{\mu_i} \cosh (\alpha_f + \alpha_w + \alpha_i)$		

The range over which the constants can be calculated is improved by obtaining a partial analytical solution and simplifying before the numerical computation. The procedure is to eliminate $\underline{A}_{f1}$ and $\underline{A}_{w2}$ in terms of $\underline{A}_{w1}$, and $\underline{A}_{c1}$ in terms of $\underline{A}_{i1}$ and $\underline{A}_{i2}$. The remaining three equations are solved for $\underline{A}_{i1}$, and the other constants are then determined from $\underline{A}_{i1}$. The odd-numbered constants, corresponding to the solution for a slit-channel machine, $\alpha_f + \alpha_w + \alpha_i \ll 1$, are determined first because they are larger for that case than the even-numbered terms. The equations used on the computer, except for combining terms as necessary to avoid large exponentials, are shown in Table III for the special case $\mu_f = \mu_w = \mu_i = \mu_0$, so that $\kappa = \mu_0/\mu_c$.

TABLE III. Equations Used to Calculate Constants
for $\mu_f = \mu_w = \mu_i = \mu_0$

$$\begin{aligned}\underline{A}_{i1} &= \frac{\cosh(\alpha_f + \alpha_w) - K_1 \gamma_w \sinh(\alpha_f + \alpha_w)}{(1 + K_1 \gamma_w \kappa \gamma_c) \sinh \alpha_i + (\kappa \gamma_c + K_1 \gamma_w) \cosh \alpha_i} \frac{\mu_0 N I}{k} \\ \underline{A}_{i2} &= \frac{[K_1 \gamma_w - \tanh(\alpha_f + \alpha_w)] \underline{A}_{i1}}{1 - K_1 \gamma_w \tanh(\alpha_f + \alpha_w)} \\ \underline{A}_{c1} &= [\underline{A}_{i1} \cosh(\alpha_f + \alpha_w + \alpha_i) + \underline{A}_{i2} \sinh(\alpha_f + \alpha_w + \alpha_i)] e^{\gamma_c(\alpha_f + \alpha_w + \alpha_i)} \\ \underline{A}_{w1} &= [\underline{A}_{i1} \cosh(\alpha_f + \alpha_w) + \underline{A}_{i2} \sinh(\alpha_f + \alpha_w)] K_2 \\ \underline{A}_{w2} &= \frac{(\gamma_f \tanh \gamma_f \alpha_f - \gamma_w \tanh \gamma_w \alpha_f) \underline{A}_{w1}}{\gamma_w - \gamma_f \tanh \gamma_f \alpha_f \tanh \gamma_w \alpha_f} \\ \underline{A}_{f1} &= \frac{\gamma_w \underline{A}_{w1}}{(\gamma_w - \gamma_f \tanh \gamma_f \alpha_f \tanh \gamma_w \alpha_f) \cosh \gamma_f \alpha_f \cosh \gamma_w \alpha_f} \\ K_1 &= \frac{\gamma_f \tanh \gamma_f \alpha_f + \gamma_w \tanh \gamma_w \alpha_w}{\gamma_w + \gamma_f \tanh \gamma_f \alpha_f \tanh \gamma_w \alpha_w} \\ K_2 &= \frac{\gamma_w - \gamma_f \tanh \gamma_f \alpha_f \tanh \gamma_w \alpha_f}{\gamma_w + \gamma_f \tanh \gamma_f \alpha_f \tanh \gamma_w \alpha_w} \frac{\cosh \gamma_w \alpha_f}{\cosh \gamma_w \alpha_w}\end{aligned}$$

The equations can be simplified for a slit-channel machine, but it is easier to use an integral approach, as described in Section 2.4. The magnetic field, completely transverse or y-directed and independent at y, is the same in the fluid, wall, and insulation. Solving for the vector potential, as before, gives

$$\underline{A}_1 = \frac{\mu_0 N I}{k(\gamma_f^2 \alpha_f + \gamma_w^2 \alpha_w + \alpha_i + \kappa \gamma_c)} \quad (3.2.9)$$

for $\mu_f = \mu_w = \mu_i = \mu_0$. The slit-channel formulas are simpler to work with, but of limited validity because R_{Mw} is generally large, and its effect is not diminished, as is that of R_{Mf} , by the slip s , which is small.

The coefficients for the exact and approximate solutions are compared in Table IV for $R_{Mf} = R_{Mw} = 10$, $s = -0.1$, and $\kappa = 0$. In practice, R_{Mf} and R_{Mw} are of the same order of magnitude, but R_{Mw} will be slightly larger because $\sigma_w > \sigma_f$. The slit-channel results are good up to reasonably wide channels, an error of less than 5% in A_{f1} for $\alpha_f = \alpha_w = \alpha_i = 0.1$, but the discrepancy increases above this. The addition of the wall markedly decreases A_{f1} , but the insulator, unless quite thick, has relatively little effect.

TABLE IV. Constants for Exact and Slit-channel Solutions for Vector Potential for $R_{Mf} = 10$, $R_{Mw} = 10$, $s = -0.1$, and $\kappa = 0$

α_f	α_w	α_i	A_{f1}^*	A_{w1}^*	A_{w2}^*	A_{i1}^*	A_{i2}^*	A_{c1}^*	A_1^*
0.01	0	0	50.0					50.5	50.0
			50.0					50.5	50.0
0.1	0	0	4.98					5.56	5.00
			5.00					5.53	5.00
1.0	0	0	0.352					2.21	0.500
			0.484					1.41	0.500
0.01	0.01	0.01	3.33	3.34	-0.335	3.32	0.900	3.45	3.33
			-10.0	-10.0	0.147	-10.0	0.300	-10.3	-10.0
0.1	0.1	0.1	0.318	0.372	-0.330	0.173	0.906	0.617	0.333
			-0.984	-0.966	0.143	-1.03	0.301	-1.33	-1.00
1.0	1.0	1.0	-0.00857	0.146	-0.146	-1.43	1.53	17.1	0.0333
			-0.0157	0.00408	-0.00100	-0.621	0.618	-1.24	-0.100
0.01	0.001	0.001	83.3	83.3	-1.94	83.3	3.06×10^{-5}	84.3	83.3
			-0.000764	0.0451	-2.14	-0.00535	6.42×10^{-5}	-0.00541	0
0.1	0.01	0.01	8.29	8.29	-1.91	8.29	0.00307	9.41	8.33
			-0.00754	-0.450	-2.17	-0.0536	0.00640	-0.0600	0
1.0	0.1	0.1	0.515	-1.96	1.87	0.160	0.419	3.06	0.833
			-0.0217	3.72	-3.75	-0.664	0.554	-1.22	0
0.2	0.1	0.1	0.484	0.694	-0.738	0.253	0.829	0.916	0.500
			-0.954	-0.841	0.154	-1.07	0.406	-1.47	-1.00
0.5	0.1	0.1	0.787	1.40	-1.53	0.449	0.525	-1.94	0.946
			-0.528	0.764	-1.08	-0.909	0.549	-1.46	-0.676
0.1	0	0.1	3.96			3.95	0.200	4.98	4.00
			2.00			2.02	-0.398	2.42	2.00
0.1	0	1.0	0.738			0.738	0.00879	3.73	0.902
			0.0888			0.0925	-0.0741	0.167	0.0820
0.1	0.1	0	0.224	0.281	-0.326			0.324	0.235
			-1.05	-1.04	0.182			-1.30	-1.06

*The constants are divided by $\mu_0 NI/k$. The upper number of each pair is the real part; the lower number is the imaginary part.

For the three cases in which $\alpha_f = 10\alpha_w$ and $s = -0.1$, the vector potential, and thus the powers, are increased over that for the same α_f

with no wall and insulation. This is because the induced currents in the fluid and wall cancel, as is shown by writing A_1 as

$$\underline{A}_1 = \frac{\mu_0 NI}{k[(\alpha_f + \alpha_w + \alpha_i) + j(\alpha_f s R_{Mf} + \alpha_w R_{Mw}) + \kappa \gamma_c]}, \quad (3.2.10)$$

and noting that the induced currents are proportional to the product of R_{Mf} and the transverse magnetic field. This represents a sort of "resonance" condition between the fluid and wall currents, but has no practical application because the wall loss is too large, as shown in the next section.

Although only one value of s is considered in Table IV, Eq.(3.2.10) shows that making s more negative effectively decreases the wall effect for a slit-channel machine because the terms to compare are $\alpha_f s R_{Mf}$ and $\alpha_w R_{Mw}$.

3.3. The Powers

The procedure for calculating the powers is as in Sections 2.3 and 2.4 for the general and slit-channel machines, with the addition of the wall loss P_{rw} . The resulting equations for the general case are given in Table V in terms of the constants of Table III for a segment one wavelength long and of the width c out of an infinite-length machine. The electrical power supplied to the machine, P_{in} , is determined from Poynting's theorem applied to the exciting winding. Alternatively,

$$P_{in} = P_{sf} + P_{rw} + P_{rc} \quad (3.3.1)$$

from conservation of energy. The calculation of P_{rw} breaks down for large values of $\alpha_f + \alpha_w + \alpha_i$ because of the exponentials, although it works for all cases of practical interest, so that Poynting's theorem is preferred for P_{in} .

TABLE V. Powers per Wavelength, with Walls and Insulation

$\frac{P_{sf}}{P_1} = \text{Re} \{ j \underline{A}_f \underline{A}_w^* \cosh \gamma_f a_f \sinh \gamma_f^* a_f \}$
$P_m = (1 - s) P_{sf}$
$P_{rf} = s P_{sf}$
$\frac{P_{rw}}{P_1} = \frac{\mu_f}{2\mu_w} \text{Im} \left\{ \underline{A}_w \underline{A}_w^* \left[\gamma_w \sinh \gamma_w (a_f + a_w) \cosh \gamma_w^* (a_f + a_w) - \gamma_w^* \sinh \gamma_w^* (a_f + a_w) \cosh \gamma_w (a_f + a_w) - \gamma_w \sinh \gamma_w a_f \cosh \gamma_w^* a_f + \gamma_w^* \sinh \gamma_w^* a_f \cosh \gamma_w a_f \right] \right.$
$\quad + \underline{A}_w \underline{A}_w^* \left[\gamma_w \sinh \gamma_w^* (a_f + a_w) \cosh \gamma_w (a_f + a_w) - \gamma_w^* \sinh \gamma_w (a_f + a_w) \cosh \gamma_w^* (a_f + a_w) - \gamma_w \sinh \gamma_w^* a_f \cosh \gamma_w a_f + \gamma_w^* \sinh \gamma_w a_f \cosh \gamma_w^* a_f \right]$
$\quad + \underline{A}_w \underline{A}_w^* \left[\gamma_w \sinh \gamma_w (a_f + a_w) \sinh \gamma_w^* (a_f + a_w) - \gamma_w^* \cosh \gamma_w (a_f + a_w) \cosh \gamma_w^* (a_f + a_w) - \gamma_w \sinh \gamma_w a_f \sinh \gamma_w^* a_f + \gamma_w^* \cosh \gamma_w a_f \cosh \gamma_w^* a_f \right]$
$\quad \left. + \underline{A}_w \underline{A}_w^* \left[\gamma_w \cosh \gamma_w (a_f + a_w) \cosh \gamma_w^* (a_f + a_w) - \gamma_w^* \sinh \gamma_w (a_f + a_w) \sinh \gamma_w^* (a_f + a_w) - \gamma_w \cosh \gamma_w a_f \cosh \gamma_w^* a_f + \gamma_w^* \sinh \gamma_w a_f \sinh \gamma_w^* a_f \right] \right\}$
$\frac{P_{rc}}{P_1} = \frac{\mu_f}{\mu_c} R_{Mc} \underline{A}_c \underline{A}_c^* e^{-\frac{(\gamma_c + \gamma_c^*) (a_f + a_w + a_i)}{\gamma_c + \gamma_c^*}}$
$\frac{P_{in}}{P_1} = \frac{\mu_f}{\mu_i} \text{Re} \left\{ j \left[\underline{A}_{i1} \cosh (a_f + a_w + a_i) + \underline{A}_{i2} \sinh (a_f + a_w + a_i) \right] \left[\underline{A}_1 \sinh (a_f + a_w + a_i) + \underline{A}_2^* \cosh (a_f + a_w + a_i) \right] + j \frac{\mu_i}{\mu_c} \underline{A}_c \underline{A}_c^* \gamma_c^* e^{-\frac{(\gamma_c + \gamma_c^*) (a_f + a_w + a_i)}{\gamma_c + \gamma_c^*}} \right\}$
$e_g = \frac{P_{in}}{P_m}$
$P_1 = \frac{\lambda c v_s k^2}{\mu_f}$

The powers per wavelength for a slit-channel machine, in terms of the vector potential (Eq. 3.2.9), are

$$\frac{P_{sf}}{P_1} = \alpha_f s R_{Mf} \underline{A_1 A_1^*}, \quad (3.3.2)$$

$$P_m = (1 - s)P_{sf}, \quad (3.3.3)$$

$$P_{rf} = sP_{sf}, \quad (3.3.4)$$

$$\frac{P_{rw}}{P_1} = \alpha_w R_{Mw} \underline{A_1 A_1^*}, \quad (3.3.5)$$

and

$$\frac{P_{in}}{P_1} = (\alpha_f s R_{Mf} + \alpha_w R_{Mw}) \underline{A_1 A_1^*}; \quad (3.3.6)$$

where

$$P_1 = \frac{\lambda c v_s k^2}{\mu_0}, \quad (3.3.7)$$

and $\mu_f = \mu_w = \mu_i = \mu_0$. Core loss has been neglected in P_{in} , but can be added by using the appropriate equation from Table V.

The slit-channel equations, although of limited validity, show the conditions under which the induction machine will operate as a generator. First, $\gamma_f \alpha_f$ cannot be too large or the field will not penetrate into the fluid, and the velocity profile will destroy any chance of generator operation. Thus the slit-channel equations must be at least approximately correct. Then, $\alpha_f s R_{Mf} + \alpha_w R_{Mw} < 0$ for generator operation, and

$$\left| \frac{\alpha_w R_{Mw}}{\alpha_f s R_{Mf}} \right| = \left| \frac{a_w \sigma_w}{a_f s \sigma_f} \right| \ll 1 \quad (3.3.8)$$

for the wall loss to be small compared to the power generated in the fluid. For a given fluid and wall material, with a wall thickness set by mechanical considerations, this gives a lower limit to $|a_f s|$, which will generally be large enough so that the slit-channel predictions are optimistic. The efficiency for the slit-channel generator is

$$e_g = \left(\frac{1}{1 - s} \right) \left(1 + \frac{\alpha_w R_{Mw}}{\alpha_f s R_{Mf}} \right). \quad (3.3.9)$$

Note that for the "resonance" condition, where the vector potential is increased by the addition of the wall, the loss in the wall just equals the power generated in the fluid, or the net output is zero.

The powers for both the general and slit-channel cases are given in Tables VI and VII for the same parameters as used in Table IV, except that slips of -0.1 and -0.4 are used. Note that for $s = -0.1$ none of the cases that include a wall will produce a net electrical power output, although several will for $s = -0.4$. As s becomes more negative, the efficiency initially improves as the power generated in the fluid increases relative to the wall loss, but eventually decreases because the electrical loss in the fluid increases, as shown later in Figs. 3-6. With no wall, the $s = -0.1$ case is better. However, with a wall, the $s = -0.4$ case will always be better unless the wall is very thin or Eq. 3.3.8 is satisfied.

TABLE VI. Powers per Wavelength for Exact and Slit-channel Solutions for $R_{Mf} = 10$, $R_{Mw} = 10$, $s = -0.1$, and $\kappa = 0$

α_f	α_w	α_i	Exact					Slit-channel		
			$\frac{P_{sf}}{P_0}$	$\frac{P_{rw}}{P_0}$	$\frac{P_{in}}{P_0}$	$\frac{P_m}{P_0}$	$e_g, \%$	$\frac{P_{sf}}{P_0}$	$\frac{P_{rw}}{P_0}$	$\frac{P_m}{P_0}$
0.01	0	0	-50.0		-50.0	-55.0	90.9	-50.0		-55.0
0.1	0	0	-5.00		-5.00	-5.50	90.9	-5.00		-5.50
1.0	0	0	-0.518		-0.518	-0.570	90.9	-0.500		-0.550
0.01	0.01	0.01	-1.11	11.1	10.0	-1.22		-1.11	11.1	-1.22
0.1	0.1	0.1	-0.107	1.09	0.987	-0.118		-0.111	1.11	-0.122
1.0	1.0	1.0	-0.000464	0.0621	0.0617	-0.000511		-0.0111	0.111	-0.0122
0.01	0.001	0.001	-69.4	69.4	0.00535	-76.4		-69.4	69.4	-76.4
0.1	0.01	0.01	-6.89	6.95	0.0532	-7.58		-6.94	6.94	-7.64
1.0	0.1	0.1	-0.383	0.750	0.367	-0.422		-0.694	0.694	-0.764
0.2	0.1	0.1	-0.232	1.22	0.988	-0.255		-0.250	1.25	-0.275
0.5	0.1	0.1	-0.489	1.21	0.724	-0.538		-0.676	1.35	-0.743
0.1	0	0.1	-1.98		-1.98	-2.18	90.9	-2.00		-2.20
0.1	0	1.0	-0.0555		-0.0555	-0.0610	90.9	-0.0820		-0.0902
0.1	0.1	0	-0.115	1.18	1.06	-0.127		-0.118	1.18	-0.129

$$P_0 = \mu_0 v_s (NI)^2 c \lambda$$

TABLE VII. Powers per Wavelength for Exact and Slit-channel Solutions for $R_{Mf} = 10$, $R_{Mw} = 10$, $s = -0.4$, and $\kappa = 0$

α_f	α_w	α_i	Exact					Slit-channel		
			$\frac{P_{sf}}{P_0}$	$\frac{P_{rw}}{P_0}$	$\frac{P_{in}}{P_0}$	$\frac{P_m}{P_0}$	$e_g, \%$	$\frac{P_{sf}}{P_0}$	$\frac{P_{rw}}{P_0}$	$\frac{P_m}{P_0}$
0.01	0	0	-23.5		-23.5	-32.9	71.4	-23.5		-32.9
0.1	0	0	-2.35		-2.35	-3.29	71.4	-2.35		-3.29
1.0	0	0	-0.301		-0.301	-0.422	71.4	-0.235		-0.329
0.01	0.01	0.01	-8.88	22.2	13.3	-12.4		-8.89	22.2	-12.4
0.1	0.1	0.1	-0.839	2.14	1.30	-1.17		-0.889	2.22	-1.24
1.0	1.0	1.0	-0.000920	0.0632	0.0623	-0.00129		-0.0889	0.222	-0.124
0.01	0.001	0.001	-38.3	9.58	-28.7	-53.6	53.6	-38.3	9.58	-53.6
0.1	0.01	0.01	-3.82	0.962	-2.86	-5.35	53.4	-3.83	0.958	-5.36
1.0	0.1	0.1	-0.368	0.307	-0.0609	-0.516	11.8	-0.383	0.0958	-0.536
0.2	0.1	0.1	-2.86	3.78	0.926	-4.00		-4.00	5.00	-5.60
0.5	0.1	0.1	-0.924	0.657	-0.267	-1.29	20.6	-1.34	0.671	-1.88
0.1	0	0.1	-1.97		-1.97	-2.76	71.4	-2.00		-2.80
0.1	0	1.0	-0.184		-0.184	-0.257	71.4	-0.292		-0.409
0.1	0.1	0	-0.957	2.44	1.49	-1.34		-1.00	2.50	-1.40

$$P_0 = \mu_0 v_s (NI)^2 c \lambda$$

Two important points are illustrated by the tabulated numbers. First, the slit-channel analysis is valid for any α up to 0.1, but the powers and efficiencies are too large if $\alpha_f + \alpha_w + \alpha_i$ is much larger than 0.1. Second, even with α_w an order of magnitude less than α_f , the efficiency is too low, with about a 55% maximum. This is not acceptable for a power system (especially since viscous and other losses still have to be considered) and throws a sobering light on the wall problem.

The preceding calculations were performed with constant current excitation, NI fixed, although a practical machine will be operated at the highest magnetic-flux density consistent with saturation of the iron core. The magnetic-flux density in the core is

$$\vec{B}_c = (-\vec{i}_x \gamma_c + \vec{i}_y j) k \underline{A}_{c1} e^{-\gamma_c k y}, \quad (3.3.10)$$

which for a low-loss core, $\gamma_c = 1$, is circularly polarized. The saturation field is

$$B_{sat} = k \left| \underline{A}_{c1} e^{-\gamma_c (\alpha_f + \alpha_w + \alpha_i)} \right|, \quad (3.3.11)$$

which can be used to find NI in terms of B_{sat} by using the numerical results for $\underline{A}_{c1}$. This will change the absolute values of the powers, but not the relative values or the efficiencies.

3.4. Discussion

At this point it is appropriate to put realistic numbers into the equations and observe the results. For operation at 1600°F, potassium, with $\sigma_f = 1.1 \times 10^6$ mhos/m,⁶⁸ is preferred from thermodynamic considerations.⁶⁹ For the wall at that temperature, $\sigma_w = 2 \times 10^6$ mhos/m for any of the refractory metals or their alloys (see Table I). Assuming a wavelength of one meter and 50-cps operation gives $R_{Mf} = 11$ and $R_{Mw} = 20$. The ratio of R_{Mf} to R_{Mw} and the slit-channel generator efficiencies shown in the accompanying figures and tables are independent of the values chosen for λ and ω . The ratio is smaller than for the cases treated in the previous section, and consequently the performance is worse.

The thickness of the thermal insulation is chosen to restrict the heat loss from the fluid to a small fraction of the generator output power, and it is assumed that the cooling of the field structure is adequate to handle this quantity of heat. Values of the thermal conductivity k in Btu-in./hr-ft²-°F range from 1.3 to 3.5 for firebricks, up to 14.4 for zirconia and 38.5 for alumina.⁷⁰ For Johns-Manville castable hydraulic-setting refractory cement Type 3X, which might be suitable for the induction generator since it has a high crushing strength, $k = 7.5$ at a mean temperature of 1400°F.⁷¹ In terms of k and the insulation thickness a_i , the

heat loss in megawatts for an annular channel one meter in diameter and three meters (three wavelengths) long is $(k/a_i)(3.57 \times 10^{-2})$, where the fluid and field structure temperatures are taken as 1600 and 1000°F, respectively, the area is doubled because the annular channel has inner and outer surfaces, and the dependence of k on the temperature is neglected. Assuming a 200-MW generator and an acceptable heat loss of 1% or 2 MW gives $a_i = 0.18$ in. for $k = 10$. Since k , the core temperature, and the allowable heat loss from the field are not known, an insulation thickness of 1/4 in. is used for the remaining calculations, which corresponds to $a_i = 0.04$. The results are essentially independent of a_i , provided a_i is small compared to $a_f + a_w$.

The generator efficiency for $R_{Mf} = 11$, $R_{Mw} = 20$, $a_i = 0.04$, and an ideal core is plotted in Figs. 3-6 for a variety of values of s , a_f , and a_w .

Fig. 3. Generator Efficiency versus α_f as a Function of s for $\alpha_w = 0.08$

Fig. 4. Generator Efficiency versus α_f as a Function of s for $\alpha_w = 0.02$

Fig. 5. Generator Efficiency versus α_w as a Function of s and α_f

Fig. 6. Generator Efficiency versus s as a Function of α_f

For $\alpha_w = 0.08$ ($a_w = 1/2$ in.) in Fig. 3, the efficiency for a given slip goes through a peak when plotted against α_f . This is not expected from the slit-channel theory, which predicts a steady rise and eventual leveling off at $1/(1-s)$ when the fluid completely dominates. The exact and slit-channel solutions are both plotted for $s = -1.0$ to show the difference. For this case there is never a good correspondence, or the slit-channel condition is never satisfied for generator operation. The cause of the peak, although not clear, is probably due to the interaction between the excitations on the two sides of the fluid. For moderate or large values of α_f , the field and the power loss in the walls are essentially independent of α_f . However, the field in the fluid (the sum of the two exponentials from the two excitation windings) drops off rapidly as α_f increases, and the power density (since it is the square of the field) drops off faster than the volume increases.⁷² The initial increase in the fluid volume as α increases causes the performance to improve, but the eventual decrease in the fluid power density causes the efficiency to drop off. For thinner walls, where the wall loss is less important, the drop should not be as marked, as shown in Fig. 4 and Table VIII, but it is observed even for $\alpha_w = 0.0025$ (Table VIII).

The efficiency curves in the figures cross for different sets of parameters. This does not mean that a degeneracy occurs, but only that the ratio of output power to input power is the same for both parameter sets. The actual values and, more important, the distribution of losses between the fluid and wall, will be different.

For $\alpha_w = 0.08$, the maximum efficiency of about 20% is not acceptable for a generator. The maximum efficiency has improved to about 55% for $\alpha_w = 0.02$, but this is still too low since viscous and other losses are not included. Since $\alpha_w = 0.02$ corresponds to a $1/8$ -in. wall, which is shown

in Section 1.5 and Table I to be much too thin from structural considerations, unconventional construction techniques are required. Either the conducting wall and its loss must be eliminated completely, or only a thin coating of a refractory metal or alloy must be used to protect the rest of the system from the liquid metal. The former requires the identification or development of new materials, or else new construction techniques such as a laminated wall. The latter requires the field structure to supply the structural strength and the thermal insulation to transmit the stress from the fluid to the field structure. The exact and slit-channel efficiencies for 1/64- and 1/16-in. coatings ($\alpha_w = 0.0025$ and 0.01) are listed in Table VIII and compared with the efficiency with no wall. Since large values of α_f are not acceptable, because of the effect on the fluid velocity profile, only a wall thickness less than 1/64 in. will yield an acceptable performance. This places severe restrictions on even a coating process, since the coating must be thick enough to withstand corrosion and erosion.

TABLE VIII. Exact and Slit-channel Generator Efficiencies
for $R_{Mf} = 11$, $R_{Mw} = 20$, $\alpha_f = 0.04$, and $\kappa = 0$

		eg, %, for $\alpha_w = 0.0025$		eg, %, for $\alpha_w = 0.01$		eg, % No Wall			eg, %, for $\alpha_w = 0.0025$		eg, %, for $\alpha_w = 0.01$		eg, % No Wall
α_f	s	Exact	Slit-channel	Exact	Slit-channel		α_f	s	Exact	Slit-channel	Exact	Slit-channel	
0.05	-0.1	8.1	8.3	-	-	90.9	0.4	-0.1	79.4	80.6	44.8	49.6	90.9
	-0.3	53.6	53.6	-	-	76.9	-0.3	73.6	74.0	63.5	65.3	76.9	
	-0.5	54.5	54.5	18.1	18.2	66.7	-0.5	64.8	65.2	59.3	60.6	66.7	
	-0.7	51.2	51.2	28.2	28.3	58.8	-0.7	57.6	57.9	53.8	55.0	58.8	
	-1.0	45.4	45.5	31.8	31.8	50.0	-1.0	49.2	49.4	46.6	47.7	50.0	
0.1	-0.1	49.3	49.6	-	-	90.9	0.5	-0.1	81.2	82.6	51.8	57.9	90.9
	-0.3	65.2	65.3	29.9	30.3	76.9	-0.3	74.0	74.6	65.2	67.6	76.9	
	-0.5	60.6	60.6	42.2	42.4	66.7	-0.5	65.0	65.5	59.8	61.8	66.7	
	-0.7	55.0	55.0	43.4	43.5	58.8	-0.7	57.6	58.1	53.9	55.8	58.8	
	-1.0	47.7	47.7	40.8	40.9	50.0	-1.0	49.1	49.5	46.4	48.2	50.0	
0.2	-0.1	69.7	70.2	5.9	8.3	90.9	0.6	-0.1	82.2	84.0	55.9	63.4	90.9
	-0.3	70.9	71.1	52.9	53.6	76.9	-0.3	74.2	75.0	65.9	69.2	76.9	
	-0.5	63.5	63.6	54.1	54.5	66.7	-0.5	65.0	65.7	59.8	62.6	66.7	
	-0.7	56.8	56.9	50.9	51.2	58.8	-0.7	57.5	58.2	53.6	56.3	58.8	
	-1.0	48.8	48.9	45.2	45.5	50.0	-1.0	49.0	49.6	46.0	48.5	50.0	
0.3	-0.1	76.3	77.1	32.2	35.8	90.9							
	-0.3	72.8	73.0	60.2	61.4	76.9							
	-0.5	64.5	64.6	57.8	58.6	66.7							
	-0.7	57.4	57.6	53.1	53.7	58.8							
	-1.0	49.1	49.2	46.4	47.0	50.0							

The dependence of efficiency on α_w is shown in Fig. 5. As expected, e_g decreases with increasing α_w until eventually wall loss dominates. The low- and high-slip curves cross because for small α_w the wall loss is small and the efficiency is best for small $|s|$, but for large α_w the relative wall effect is less for larger $|s|$. Note that the two pairs of curves are crossed with respect to α_f , indicating that the slit-channel equations are not valid. This correlates with the peak in Figs. 3 and 4.

The previous comments on the dependence of generator performance on the slip are illustrated in Fig. 6 where e_g is plotted against s . For no wall, e_g drops off with increasing $|s|$. With a wall, the generated power is zero at $s = 0$ but the wall loss is not, so that the generator mode is not attained until $|s|$ becomes somewhat negative. A peak in the curves occurs with increasing $|s|$ when the $1/(1-s)$ limit or the fluid loss becomes important. This is also shown in Fig. 4 and Table VIII.

IV. THE FINITE-LENGTH MACHINE WITH CONSTANT FLUID VELOCITY

4.1. Introduction

A finite distance is required to establish electromagnetic fields in the fluid close to those that would exist in an infinite-length machine. If the machine length is not long compared to this distance, severe discrepancies will exist between the predictions based on a segment out of an infinite-length machine and the actual operating characteristics. The main effect of the ends for a machine more than a few wavelengths long is to reduce the power level, with only a small change in the electrical efficiency.⁷³

Two models have been proposed for treating finite length, by using either an infinite-length^{44,74} or a finite-length⁴⁷ core with the finite-length exciting winding. The infinite-length-core model is applicable if the iron overhangs the winding by more than one wavelength, and is readily analyzed by transform methods. Only approximate analyses are available for the finite-length core, and the problems associated with it are not fully understood. More theoretical and experimental work is needed on this case.

Fig. 7. Model of Finite-length Machine

rent, where n is in fractions of a wavelength. The electromagnetic field equations are unchanged by the finite length. The vector potential is still in the z -direction and independent of z , and must have the same $e^{j\omega t}$ dependence as the exciting current, but now the e^{-jkx} dependence of the excitation is preserved only in the driven part of the solution. Writing Eq. 2.2.12 for the complex amplitude $\underline{A}_f(x, y)$ gives

$$\nabla^2 \underline{A}_f - jR_{Mf}k^2 \underline{A}_f - R_{MF}k \frac{\partial \underline{A}_f}{\partial x} = 0 \quad (4.2.1)$$

In this chapter the previous analysis for a machine without walls and insulation is reviewed, and then extended by approximations to cover the case with walls and insulation. Only the infinite-length core is considered.

4.2. Performance without Walls and Insulation

The model to be analyzed (Fig. 7 without the walls), differs from that of the infinite-length machine only in the finite length $n\lambda$ of the exciting surface cur-

and a similar equation for $\underline{A}_c(x, y)$. Here

$$R_{MF} = R_{Mf}(1 - s) = \frac{\mu_f \sigma_f v}{k}, \quad (4.2.2)$$

the magnetic Reynolds number based on the fluid velocity, is the important parameter because it controls the ratio of convection to diffusion for the magnetic field.

The normal method of solving boundary-value problems is to determine the natural modes in each region from the geometry, and then use the boundary conditions to find the constants. Here the modes are not evident except for the special case of an ideal core.⁷⁵ Instead the double-ended Laplace transform of the x-variation is taken, and then the boundary conditions are used. This gives the total solution at once, provided that the inverse transform can be evaluated.

The transform used is

$$\underline{A}_f(p, y) = \int_{-\infty}^{\infty} \underline{A}_f(x, y) e^{-pkx} dx, \quad (4.2.3)$$

where p is the complex transform variable, normalized with respect to the wave number k of the excitation. The transformed equations are

$$\frac{\partial^2 \underline{A}_f(p, y)}{\partial y^2} + k^2(p^2 - jR_{Mf} - R_{MFP}) \underline{A}_f(p, y) = 0 \quad (4.2.4)$$

and a similar equation for $\underline{A}_c(p, y)$. This is identical in form with Eq. 2.3.2, so that the y -dependence will again be an exponential. Using symmetry and the boundary conditions at the fluid-core interface, the solution is

$$\underline{A}_f(p, y) = \frac{\mu_f \underline{K}(p) \cosh \gamma_f(p)ky}{k[\gamma_f(p) \sinh \gamma_f(p)\alpha_f + \kappa \gamma_c(p) \cosh \gamma_f(p)\alpha_f]} \text{ for } |y| \leq a_f, \quad (4.2.5)$$

where

$$[\gamma_f(p)]^2 = -p^2 + jR_{Mf} + R_{MFP} \quad (4.2.6)$$

and

$$[\gamma_c(p)]^2 = -p^2 + jR_{Mc} \quad (4.2.7)$$

have the same form as before but now depend on p , and $\underline{K}(p)$ is the transform of the exciting current.

For a uniform traveling current of length $n\lambda$,

$$\underline{K}(p) = \frac{NI [1 - e^{-(p+j)2\pi n}]}{k(p+j)}. \quad (4.2.8)$$

This is the sum of a positive step at $x = 0$ and a negative step at $x = n\lambda$, so that superposition can be used to find the total field solution.

The transformed vector potential in the fluid is

$$\underline{\Lambda}_f(p, y) = \frac{\mu_f NI [1 - e^{-(p+j)2\pi n}] \cosh \gamma_f(p)ky}{k^2(p+j)[\gamma_f(p) \sinh \gamma_f(p)\alpha_f + \kappa \gamma_c(p) \cosh \gamma_f(p)\alpha_f]}. \quad (4.2.9)$$

The inverse transform of this is evaluated as a contour integral, which is the sum of the residues at the poles enclosed within the contour. The poles are the values of p at which the denominator is zero. One pole is

$$p_e = -j \quad (4.2.10)$$

due to the excitation, and this gives the field with no ends. The other poles are determined by

$$\gamma_f(p) \sinh \gamma_f(p)\alpha_f + \kappa \gamma_c(p) \cosh \gamma_f(p)\alpha_f = 0. \quad (4.2.11)$$

There is no general solution for Eq. 4.2.11, but an analytical solution for the poles is known for an ideal iron core,^{44,74,46} as outlined below. The poles for a lossless core of arbitrary permeability, where $\gamma_f(p)$ must be pure imaginary, can be determined numerically by iteration,⁷⁶ but even the numerical approach becomes difficult for a lossy core. Otherwise the procedure is as sketched below for an ideal core, except that the calculation is more difficult because there are more terms and no orthogonalities in the power products.

For an ideal core, $\kappa = 0$, the poles occur when $\gamma_f(p)\alpha_f = ji\pi$, where i is an integer, or when

$$p_i^\pm = \frac{R_{MF}}{2} \left[1 \pm \sqrt{1 + \frac{j^4}{R_{MF}(1-s)} + \left(\frac{2i\pi}{\alpha_f R_{MF}} \right)^2} \right]. \quad (4.2.12)$$

The residues at the poles are given by

$$\text{Residue} \{ \underline{A}_f(p, y) \} \Big|_{p=p_i^\pm} = k(p - p_i^\pm) \underline{A}_f(p, y) \Big|_{p=p_i^\pm}. \quad (4.2.13)$$

To evaluate this, the denominator is expanded in a Taylor series about the pole or, alternatively, small-angle formulas are used for $\sinh \gamma_f \alpha_f$. Care must be taken with the residues for $i = 0$.

Examination of the square-root term in Eq. 4.2.12 shows that the real part is always greater than one, and that both the real and imaginary parts are positive. The $+$ poles, with positive real and imaginary parts, are the poles for negative x since the natural response must decay away from the entrance. Similarly, the $-$ poles have negative real and imaginary parts and apply to positive x . This can also be seen from the contour integral. The inverse transform, including the validity of the transform with a doubly-infinite series of poles, is discussed in Ref. 77.

The vector potential, the sum of the residues of the transformed potential, including the contributions from both the beginning and end of the machine, is

$$\underline{A_f(x, y)} = - \sum_{i=0}^{\infty} \frac{\mu_f N I (-1)^i \cos \frac{i\pi y}{a_f} e^{p_i^+ kx} \left[1 - e^{-(p_i^+ + j)2\pi n} \right]}{\alpha_f k \Delta_i (p_i^+ + j) \left(\frac{R_{MF}}{2} - p_i^+ \right)}$$

for $x \leq 0$, (4.2.14)

$$\underline{A_f(x, y)} = \frac{\mu_f N I \cosh \gamma_f k y e^{-jkx}}{k \gamma_f \sinh \gamma_f \alpha_f} + \sum_{i=0}^{\infty} \frac{\mu_f N I (-1)^i \cos \frac{i\pi y}{a_f} e^{p_i^- kx}}{\alpha_f k \Delta_i (p_i^- + j) \left(\frac{R_{MF}}{2} - p_i^- \right)}$$

$$+ \sum_{i=0}^{\infty} \frac{\mu_f N I (-1)^i \cos \frac{i\pi y}{a_f} e^{p_i^+ kx} e^{-(p_i^+ + j)2\pi n}}{\alpha_f k \Delta_i (p_i^+ + j) \left(\frac{R_{MF}}{2} - p_i^+ \right)}$$

for $0 \leq x \leq n\lambda$, (4.2.15)

and

$$\underline{A_f(x, y)} = \sum_{i=0}^{\infty} \frac{\mu_f N I (-1)^i \cos \frac{i\pi y}{a_f} e^{p_i^- kx} \left[1 - e^{-(p_i^- + j)2\pi n} \right]}{\alpha_f k \Delta_i (p_i^- + j) \left(\frac{R_{MF}}{2} - p_i^- \right)}$$

for $n\lambda \leq x$; (4.2.16)

where γ_f without a p corresponds to the previous definition (Eq. 2.3.4) and

$$\Delta_i = \begin{cases} 2 & \text{for } i = 0 \\ 1 & \text{for } i \neq 0 \end{cases} \quad (4.2.17)$$

The negative signs arise because the contour encircles the $+$ poles in a clockwise direction.

The powers are calculated by using Eqs. 2.2.16 to 2.2.20 with the above vector potential. This is complicated because of the large number of products (although many integrate to zero for an ideal core) and is not reproduced here.⁷⁸ The resulting equations are identical in form to those shown later in Table IX, except that most of the single terms are replaced by sums over all the poles and the poles are given by Eq. 4.2.12. Fortunately, the zero-order poles are the dominant terms in the powers.⁷⁸

Much useful information is obtained by examining the magnitudes of the poles and residues for optimum machine parameters: $|s| \ll 1$ for high efficiency, $\alpha_f \ll 1$ for a uniform power density, and $|sR_{Mf}| \gg 1$ for the reactive power to be small compared to the real power. The poles for $i = 0$ are small compared to the others, so that their residues are larger and they decay slower and thus dominate the fields and powers. This is to be expected for a slit-channel machine because the vector potential is approximately independent at y , so that the natural response required to match the boundary conditions at $x = 0$ and $x = n\lambda$ should also be independent at y , i.e., only the terms for $i = 0$ in Eqs. 4.2.14 to 4.2.16. For α_f less than a few tenths, good accuracy is obtained by using only the slit-channel powers.

4.3. Problems with Exact Solution with Walls and Insulation

Attempting to apply the procedure of Section 4.2 with walls and insulation runs into severe analytical problems, although conceptually it is no harder. There are now four equations like Eq. 4.2.4 instead of two, and six simultaneous equations to determine the constants. The potentials in the four regions are

$$\underline{A}_f(p, y) = \underline{A}_{f1} \cosh \gamma_f(p)ky, \quad (4.3.1)$$

$$\underline{A}_c(p, y) = \underline{A}_{c1} e^{-\gamma_c(p)k|y|}, \quad (4.3.2)$$

$$\underline{A}_w(p, y) = \underline{A}_{w1} \cosh \gamma_w(p)ky + \underline{A}_{w2} \sinh \gamma_w(p)ky, \quad (4.3.3)$$

and

$$\underline{A}_i(p, y) = \underline{A}_{i1} \cosh \gamma_i(p)ky + \underline{A}_{i2} \sinh \gamma_i(p)ky; \quad (4.3.4)$$

where

$$[\gamma_w(p)]^2 = -p^2 + jR_{Mw}; \quad (4.3.5)$$

and

$$[\gamma_i(p)]^2 = -p^2. \quad (4.3.6)$$

These equations are identical to those of Section 3.2 except for the p -dependence and the addition of $\gamma_i(p)$. Thus, these are the only changes required in the equations of Tables II and III, except that NI is replaced by $\underline{K}(p)$.

The natural response and the poles will be the same for all regions, so that only one of the constants has to be considered. Adding $\gamma_i(p)$ to the equation for $\underline{A}_{i1}$ in Table III and expanding the denominator gives

$$\begin{aligned} & \tanh \gamma_i(p) \alpha_i \{ [\gamma_i(p)]^2 [\gamma_w(p) + \gamma_f(p) \tanh \gamma_f(p) \alpha_f \tanh \gamma_w(p) \alpha_w] \\ & + \kappa \gamma_c(p) \gamma_w(p) [\gamma_f(p) \tanh \gamma_f(p) \alpha_f + \gamma_w(p) \tanh \gamma_w(p) \alpha_w] \} \\ & + \kappa \gamma_c(p) \gamma_i(p) [\gamma_w(p) + \gamma_f(p) \tanh \gamma_f(p) \alpha_f \tanh \gamma_w(p) \alpha_w] \\ & + \gamma_i(p) \gamma_w(p) [\gamma_f(p) \tanh \gamma_f(p) \alpha_f + \gamma_w(p) \tanh \gamma_w(p) \alpha_w] = 0 \end{aligned} \quad (4.3.7)$$

to determine the poles, except for the excitation pole, for the case $\mu_f = \mu_w = \mu_i = \mu_0$. Determining the roots of this equation, even for $\kappa = 0$, is not practical. The only alternative is to go to the slit-channel case, replacing all sinh's by their arguments and all cosh's by unity. The result, for $\kappa = 0$, is

$$\begin{aligned} & p^4(\alpha_f \alpha_w \alpha_i) + p^3(-\alpha_f \alpha_w \alpha_i R_{MF}) + p^2(-\alpha_f - \alpha_w - \alpha_i - j\alpha_f \alpha_w \alpha_i R_{Mf}) \\ & + p(\alpha_f R_{MF}) + j(\alpha_f R_{Mf} + \alpha_w R_{Mw}) = 0, \end{aligned} \quad (4.3.8)$$

which is still inconvenient to solve. One special case can be treated, i.e., $\alpha_i = 0$, no insulation. Then,

$$p^2(-\alpha_f - \alpha_w) + p(\alpha_f R_{MF}) + j(\alpha_f R_{Mf} + \alpha_w R_{Mw}) = 0, \quad (4.3.9)$$

which can be solved for approximations to the two lowest-order poles.

This still gives no information on the powers. Several alternatives are available. If the poles from Eq. 4.3.9 are used, the fields and powers can be determined, but it is not clear from this analysis how valid the

results are or how to include the insulation. Another possibility is to use the power formulas without walls and insulation with the poles determined from Eq. 4.3.9. Comparing Eq. 4.3.9 with Eq. 4.2.11 for $\gamma_f \alpha_f \ll 1$ and $\kappa = 0$, which becomes

$$p^2(-\alpha_f) + p(\alpha_f R_{MF}) + (j\alpha_f R_{MF}) = 0, \quad (4.3.10)$$

allows the definition of the effective parameters

$$\alpha_e = \alpha_f + \alpha_w, \quad (4.3.11)$$

$$R_{Me} = \frac{(\alpha_f R_{Mf} + \alpha_w R_{Mw})}{\alpha_e}, \quad (4.3.12)$$

and

$$s_e = \frac{(\alpha_f s R_{Mf} + \alpha_w R_{Mw})}{\alpha_e R_{Me}}. \quad (4.3.13)$$

If insulation is included, it is expected that the only change required is that

$$\alpha_e = \alpha_f + \alpha_w + \alpha_i. \quad (4.3.14)$$

Unfortunately, the use of the equivalent parameters in the power formulas without walls and insulation is invalid for three reasons:

1. There is no reason to expect any correlation between the effective parameters and the higher-order poles.
2. The effective parameters are only used in some parts of the formulas.
3. There is no equivalent to handle the change in wall loss, which is highly important.

Thus, this approach is not pursued further. It is more satisfying to repeat the solution for a slit-channel machine, which will answer the above objections at the expense of only including the lowest-order poles. Using the slit-channel solution from the beginning also makes the limitations on the validity of the results more evident.

4.4. Slit-channel Solution with Walls and Insulation

Since the exact solution is prohibitive, some information can be obtained from the slit-channel solution. The integral approach is used with

the dashed contour of Fig. 7, which is closed for $x \ll 0$ so that all field quantities are zero at the left edge. The form of the transformed vector potential in the core is unchanged (Eq. 4.3.2), but now the transformed vector potential in the fluid, wall, and insulation is a constant, $A_1(p)$. The boundary condition at the core, $y = a_f + a_w + a_i$, is

$$\underline{A}_1(p) = \underline{A}_{c1} e^{-\gamma_c(p)(\alpha_f + \alpha_w + \alpha_i)} \quad (4.4.1)$$

Determining the currents in terms of the potentials and substituting into the transform of Eq. 2.4.1, as in Section 2.4, gives

$$\underline{A}_1(p) = \frac{\mu_0 \underline{K}(p)}{k\{[\gamma_f(p)]^2 \alpha_f + [\gamma_w(p)]^2 \alpha_w + [\gamma_i(p)]^2 \alpha_i + \kappa \gamma_c(p)\}} \quad (4.4.2)$$

for the contour of Fig. 7, where $\mu_f = \mu_w = \mu_i = \mu_0$, and $\underline{K}(p)$ is given by Eq. 4.2.8.

The poles for the inverse transform, which are the roots of the denominator of Eq. 4.4.2, are $p_e = -j$ from the excitation, and

$$p^\pm = \frac{\alpha_f R_{MF} + j\kappa}{2(\alpha_f + \alpha_w + \alpha_i)} \left[1 \pm \sqrt{1 + \frac{4j(\alpha_f R_{Mf} + \alpha_w R_{Mw})(\alpha_f + \alpha_w + \alpha_i)}{(\alpha_f R_{MF} + j\kappa)^2}} \right] \quad (4.4.3)$$

for the natural response with a lossless core. For $\kappa = 0$ this reduces to the poles for $i = 0$ with no wall (Eq. 4.2.12) when the equivalences of Section 4.3 are used. The vector potential in terms of the poles is

$$\underline{A}_1(x) = \frac{\mu_0 NI e^{p^+ kx} [1 - e^{-(p^+ + j)2\pi n}]}{k(\alpha_f + \alpha_w + \alpha_i)(p^+ + j)(p^+ - p^-)} \quad \text{for } x \leq 0, \quad (4.4.4)$$

$$\begin{aligned} \underline{A}_1(x) = & \frac{\mu_0 NI e^{-jkx}}{k(\gamma_f^2 \alpha_f + \gamma_w^2 \alpha_w + \alpha_i + \kappa)} - \frac{\mu_0 NI e^{p^- kx}}{k(\alpha_f + \alpha_w + \alpha_i)(p^- + j)(p^- - p^+)} \\ & - \frac{\mu_0 NI e^{p^+ kx} e^{-(p^+ + j)2\pi n}}{k(\alpha_f + \alpha_w + \alpha_i)(p^+ + j)(p^+ - p^-)} \quad \text{for } 0 \leq x \leq n\lambda, \end{aligned} \quad (4.4.5)$$

and

$$\underline{A}_1(x) = \frac{-\mu_0 NI e^{p^- kx} [1 - e^{-(p^- + j)2\pi n}]}{k(\alpha_f + \alpha_w + \alpha_i)(p^- + j)(p^- - p^+)} \quad \text{for } n\lambda \leq x; \quad (4.4.6)$$

where γ_f and γ_w without a p correspond to the infinite-length definitions, and the core is lossless. This is the same as the solution without the wall and insulation, except written in a slightly different form.

The powers, calculated from the equations of Section 2.2, are given in Table IX. Each region must be evaluated separately and then the terms added together. Since this is a slit-channel machine, the input power can only be evaluated as

$$P_{in} = P_{rf} + P_{rw} + P_m + P_{rc}, \quad (4.4.7)$$

using the conservation of energy. Comparison with the powers without the wall⁷⁹ shows that these powers represent the terms due to the zero-order poles if the effective parameters of Section 4.3 are used for the poles but not elsewhere in the equations. The equations for P_{rf} and P_{rw} are the same, in that setting $v = 0$ or $s = 1$ in P_{rf} and using the appropriate parameters gives P_{rw} .

TABLE IX. The Slit-channel Powers for a Finite-length Machine

$$\begin{aligned} \frac{P_{rf}}{P_0} &= \frac{a_f R_{Mf}}{2\pi n} \left\{ \frac{2\pi n s^2}{CC^*} - \frac{js[-j+(1-s)p^-] [e^{(p^--j)2\pi n} - 1]}{Ca_T(p^--j)^2(p^--p^+)} + \frac{js[j+(1-s)p^-] [e^{(p^++j)2\pi n} - 1]}{C^*a_T(p^++j)^2(p^--p^+)} - \frac{js[-j+(1-s)p^+] [1-e^{-(p^++j)2\pi n}]}{Ca_T(p^++j)^2(p^+-p^-)} + \frac{js[j+(1-s)p^+] [1-e^{-(p^++j)2\pi n}]}{C^*a_T(p^++j)^2(p^+-p^-)} \right. \\ &\quad + \frac{[j+(1-s)p^+] [-j+(1-s)p^+] [2-e^{-(p^++j)2\pi n} - e^{-(p^+-j)2\pi n}]}{a_f^2(p^++j)(p^+-j)(p^+-p^+)(p^+-p^-)(p^++p^+)} - \frac{[j+(1-s)p^-] [-j+(1-s)p^-] [2-e^{(p^--j)2\pi n} - e^{(p^--j)2\pi n}]}{a_f^2(p^--j)(p^--j)(p^--p^+)(p^--p^-)(p^--p^+)} \\ &\quad + \left. \frac{[j+(1-s)p^-] [-j+(1-s)p^+] [e^{p^-2\pi n} - e^{-p^+2\pi n}] e^{j2\pi n}}{a_f^2(p^--j)(p^+-j)(p^+-p^+)(p^+-p^-)(p^++p^+)} + \frac{[j+(1-s)p^+] [-j+(1-s)p^-] [e^{p^+2\pi n} - e^{-p^+2\pi n}] e^{-j2\pi n}}{a_f^2(p^++j)(p^--j)(p^--p^+)(p^--p^-)(p^++p^+)} \right\} \\ \frac{P_{rw}}{P_0} &= \frac{a_w R_{Mw}}{2\pi n} \left\{ \frac{2\pi n}{CC^*} - \frac{e^{(p^--j)2\pi n} - 1}{Ca_T(p^--j)^2(p^--p^+)} - \frac{e^{(p^++j)2\pi n} - 1}{C^*a_T(p^++j)^2(p^--p^+)} - \frac{1-e^{-(p^++j)2\pi n}}{Ca_T(p^++j)^2(p^+-p^-)} - \frac{1-e^{-(p^++j)2\pi n}}{C^*a_T(p^++j)^2(p^+-p^-)} + \frac{2-e^{-(p^++j)2\pi n} - e^{-(p^+-j)2\pi n}}{a_f^2(p^++j)(p^+-j)(p^+-p^+)(p^+-p^-)(p^++p^+)} \right. \\ &\quad - \frac{2-e^{(p^--j)2\pi n} - e^{(p^--j)2\pi n}}{a_f^2(p^--j)(p^--j)(p^--p^+)(p^--p^-)(p^--p^+)} + \frac{(e^{-p^-2\pi n} - e^{-p^+2\pi n}) e^{j2\pi n}}{a_f^2(p^--j)(p^+-j)(p^+-p^+)(p^+-p^-)(p^++p^+)} + \frac{(e^{p^+2\pi n} - e^{-p^+2\pi n}) e^{-j2\pi n}}{a_f^2(p^++j)(p^--j)(p^--p^+)(p^--p^-)(p^++p^+)} \left. \right\} \\ \frac{P_m}{P_0} &= \frac{a_f R_{Mf}}{2\pi n} \operatorname{Re} \left\{ \frac{2\pi n s}{CC^*} + \frac{jsp^- [e^{(p^--j)2\pi n} - 1]}{Ca_T(p^--j)^2(p^--p^+)} + \frac{j[j+(1-s)p^-] [e^{(p^++j)2\pi n} - 1]}{C^*a_T(p^++j)^2(p^--p^+)} + \frac{jsp^+ [1-e^{-(p^++j)2\pi n}]}{Ca_T(p^++j)^2(p^+-p^-)} + \frac{j[j+(1-s)p^+] [1-e^{-(p^++j)2\pi n}]}{C^*a_T(p^++j)^2(p^+-p^-)} \right. \\ &\quad - \frac{[j+(1-s)p^+] p^+ [2-e^{-(p^++j)2\pi n} - e^{-(p^+-j)2\pi n}]}{a_f^2(p^++j)(p^+-j)(p^+-p^+)(p^+-p^-)(p^++p^+)} + \frac{[j+(1-s)p^-] p^- [2-e^{(p^--j)2\pi n} - e^{(p^--j)2\pi n}]}{a_f^2(p^--j)(p^--j)(p^--p^+)(p^--p^-)(p^--p^+)} \\ &\quad - \left. \frac{[j+(1-s)p^-] p^+ [e^{p^-2\pi n} - e^{-p^+2\pi n}] e^{j2\pi n}}{a_f^2(p^--j)(p^+-j)(p^+-p^+)(p^+-p^-)(p^++p^+)} - \frac{[j+(1-s)p^+] p^- [e^{p^+2\pi n} - e^{-p^+2\pi n}] e^{-j2\pi n}}{a_f^2(p^++j)(p^--j)(p^--p^+)(p^--p^-)(p^++p^+)} \right\} \\ C &= \gamma_f^2 a_f + \gamma_w^2 a_w + a_i + \kappa \\ a_T &= a_f + a_w + a_i \end{aligned}$$

To illustrate the dependence of generator performance on length, the normalized powers and efficiencies for several sets of parameters are plotted in Fig. 8 and listed in Table X. The efficiencies are calculated by

Fig. 8. Generator Efficiency versus Length for $\alpha_w = 0.1$, $\alpha_i = 0.1$, $R_{Mf} = 10$, $R_{Mw} = 10$, and $\kappa = 0$

using the ratios of the slit-channel powers for finite-to-infinite-length machines with the infinite-length exact powers calculated as in Section 3.3. Figure 8 shows the same behavior observed previously without the wall and insulation,⁷⁸ i.e., a minimum length for the generator mode to be attained, then a sharp rise in the efficiency, followed by a gradual improvement to the infinite-length case, and oscillations for some parameter sets. Only integral-wavelength-long machines are considered, so that the curves are made up of straight-line segments. The $\alpha_f = 0.7$, $s = -0.4$ case is peculiar in that a three-wavelength machine is a generator, but then it again becomes a damper until $n = 6$.

TABLE X. Powers and Efficiencies for a Finite-length Slit-channel Generator for $R_{Mf} = 10$, $R_{Mw} = 10$, and $\kappa = 0$

α_f	α_w	α_i	s	$n = 3$				$n = 6$				$n = 10$				$n = \infty$
				P_{rf}^a	P_{rw}^a	P_m^a	$e_g, \%$	P_{rf}^a	P_{rw}^a	P_m^a	$e_g, \%$	P_{rf}^a	P_{rw}^a	P_m^a	$e_g, \%$	
0.1	0	0	-0.1	1.10		0.407	75.5	0.981		0.722	87.7	0.995		0.842	89.3	90.9
0.1	0.1	0.1	-0.1	6.68	0.976	1.87		3.84	0.988	1.44		2.70	0.993	1.26		
0.1	0.01	0.01	-0.1	1.02	0.580	0.550		0.989	0.767	0.734		0.989	0.859	0.837		
0.2	0.1	0.1	-0.1	4.93	0.976	1.79		2.96	0.987	1.39		2.17	0.992	1.24		90.9
0.5	0.1	0.1	-0.1	2.03	0.711	0.990		1.58	0.857	1.02		1.35	0.915	1.01		
0.1	0	0.1	-0.1	1.58		0.486	70.5	1.25		0.753	84.9	1.15		0.852	87.7	
0.1	0	1.0	-0.1	6.19		0.421		3.59		0.709	53.9	2.55		0.825	71.9	90.9
0.1	0.1	0	-0.1	9.40	1.03	2.44		5.20	1.01	1.72		3.52	1.01	1.43		
0.1	0	0	-0.4	0.978		1.01	72.4	0.986		0.989	71.5	0.993		0.988	71.3	71.4
0.1	0.1	0.1	-0.4	1.55	0.959	1.23		1.27	0.979	1.11		1.16	0.988	1.07		
0.1	0.01	0.01	-0.4	1.00	2.25	1.17	40.9	0.982	1.46	1.03	47.2	0.992	1.27	1.02	49.9	
0.2	0.1	0.1	-0.4	0.823	0.679	0.738		0.914	0.833	0.867		0.949	0.900	0.921		20.6
0.5	0.1	0.1	-0.4	0.916	1.59	1.04		0.976	1.24	1.03	11.4	0.983	1.15	1.01	14.7	
0.1	0	0.1	-0.4	0.962		0.942	70.8	0.979		0.957	70.8	0.988		0.972	71.0	
0.1	0	1.0	-0.4	1.15		0.816	59.6	1.07		0.909	66.3	1.04		0.945	68.4	71.4
0.1	0.1	0	-0.4	1.97	1.06	1.49		1.49	1.03	1.25		1.29	1.02	1.15		
0.1	0	0	-0.7	0.992		0.933	56.2	0.994		0.986	58.5	0.998		0.991	58.6	58.8
0.1	0.1	0.1	-0.7	1.02	0.828	0.938		1.01	0.917	0.973		1.01	0.950	0.984		
0.1	0.01	0.01	-0.7	0.997	1.78	1.04	45.9	1.00	1.66	1.04	46.8	1.01	1.63	1.05	47.4	
0.2	0.1	0.1	-0.7	0.904	1.24	0.971	3.85	0.978	1.14	1.01	9.25	0.983	1.08	1.00	10.9	13.5
0.5	0.1	0.1	-0.7	1.06	1.90	1.18	14.3	1.04	1.64	1.12	17.5	1.02	1.35	1.06	21.9	
0.1	0	0.1	-0.7	0.985		0.934	56.6	0.990		0.981	58.4	0.995		0.989	58.6	
0.1	0	1.0	-0.7	0.994		0.893	54.2	0.997		0.950	56.8	0.998		0.970	57.6	58.8
0.1	0.1	0	-0.7	1.28	0.952	1.15		1.14	0.983	1.08		1.08	0.990	1.05		

^aNormalized to be 1 if there is no end effect.

The tabulated values show the same gradual improvement with length. It is hard to make generalizations from the data, but the entry length

(the decay length for the fields due to the entrance and exit) appears to be longer with the wall than without it. In most cases, the wall loss is increased for a generator over the infinite-length machine and decreased for a damper. Note that this is a slit-channel theory; scaling α_f , α_w , and α_i together has no effect on the power ratios, so that fewer values are required in Table X than in the Tables V-VII. Caution has to be used in applying the slit-channel results to the larger values of α_f , α_w , and/or α_i contained in Table X and Fig. 8 as they may not be valid.

V. CONCLUDING REMARKS

Structural considerations have shown that a refractory-metal channel wall at least $1/4$ in. thick is required to contain the pressure of the liquid metal and provide protection from corrosion and erosion. However, a wall of these materials more than $1/64$ in. thick cannot be tolerated if an acceptable generator efficiency is to be obtained. These two conflicting requirements must be reconciled, either by finding suitable nonconducting wall materials, or by determining methods of using coatings of a refractory metal only a few mills thick on an insulator that will transmit the stress to the field structure. Another possibility is a laminated wall structure made of sheets of refractory metal with interposed sheets of insulation. This is the aspect of MHD induction-generator development requiring most urgent attention, since a successful resolution of the wall problem is essential to the ultimate success of this type of MHD generator.

The other major requirement is to perform a design study of an MHD induction generator matched to a complete liquid power cycle, including the results of a further investigation of wall construction. This should include the design of a field structure and its associated cooling for operation at a temperature determined by system and materials considerations. The electrical winding design must consider tooth saturation and leakage reactance. Both are important because of the large air gap and the resulting large number of ampere-turns required. The outcome of this study would be to show the overall performance to be expected from the generator and cycle, the machine dimensions and weight, and the required auxiliary capacitance.

Two further investigations are required to complete the information required for comprehensive design studies. First, a solution for a variable-area MHD induction machine has not yet been obtained. A practical generator will have a variable area to enable dynamic head to be converted directly to electrical energy, and consequently to reduce the static pressure levels at the generator inlet.

The second problem area is concerned with the fundamental character of turbulent MHD flows in both d.c. and traveling magnetic fields. The meager experimental data and analytical work currently available mean that the viscous loss and velocity profile, both of which play a major part in the calculation of the machine performance,⁶⁴ are unknown. Previous analytical study using a boundary-layer analysis⁸⁰ is of limited value until experimental verification is available. The simultaneous experimental measurement of both the pressure gradient and the velocity profile is required. This will add to basic knowledge in addition to aiding the design study.

Combined with the design study should be an experimental program of testing small-scale MHD induction generators to verify the theoretical analysis. Since the models analyzed are crude approximations to an actual machine, it is not known how well the predictions will correlate with actual performance.

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