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OF SINGLET-TRIPLET PARTICLES

M. J. Mandell

Laboratory of Atomic and Solid State Physics
Cornell University
Ithaca, New York 14850

October 1972

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Dynamic Effects in the Collective Excitations
of Singlet-Triplet Particles*

M. J. Mandell

Laboratory of Atomic and Solid State Physics
Cornell University
Ithaca, New York 14850

ABSTRACT

We consider the collective excitations of a lattice of particles having singlet ground state and triplet excited state. Beginning with a representation of single particle operators as the direct product of two spin one-half operators, we derive explicit expressions for the polarization and approximate normal-mode operators. We show that the Green's functions of these operators satisfy a Dyson-like equation and illustrate dynamic effects contained therein by a crude calculation for a one-dimensional dielectric. We comment on the implications of this calculation for relating theory to experiment.

1. Introduction

The dynamics of many-particle problems is made more complex when the individual particles have an intrinsic excitation spectrum which exhibits anharmonic properties. A well-known example of this is the non-linear spin-wave problem.¹ Another which has recently attracted some attention is that of singlet ground-state particles coupled through their excitonic states. This has been of particular interest with respect to magnetic behavior in rare earths²⁻⁴, where the single particle levels are due to crystal field splittings. It is also relevant to various problems in molecular crystals⁶, ferroelectrics, and interacting impurity systems⁷.

The case where the excited state is a singlet has been treated extensively by Cooper and Wang⁸, and by Pfeuty⁹. Triplet excited states have been treated by Cooper¹⁰, and by Pink¹¹, and Cooper¹² has recently done some calculations for more complex "real" systems. When modes of such systems become unstable (soft) phase transitions occur. It has been a feature of all models of this sort that no transition occurs unless the interaction is stronger than some threshold value. In the disordered (paramagnetic) regime one should see a temperature-dependent spectrum of elementary excitations, with a mode becoming soft as the temperature is lowered toward the transition (or toward zero). This soft-mode behavior seems to be absent in some systems⁴, while there are indications that it may exist in others⁵.

In this work we focus on the excitation spectrum in the disordered phase for singlet-triplet systems. The single-particle states are represented using the direct product of two spin one-half operators

in a manner similar to that of Pink¹¹. However, we derive explicit expressions for the polarization operator, the bare-exciton operators, and the approximate normal-mode operators whose frequencies agree with those found by Pink¹¹. In addition, we derive a Dyson-like equation for the Green's functions of the approximate normal-mode operators, and proceed in higher approximation to consider dynamics of the interacting modes. We illustrate the dynamic effects by a crude perturbative calculation on a one-dimensional dielectric. We are led thereby to the conclusion that non-linear interactions, i.e., scattering of quasiparticles, can produce substantial effects in these systems.

II. Formalism

A. Single-Site Operators

We associate with each site two spin one-half Heisenberg operators $\underline{S}_i$ and $\underline{T}_i$, with the commutation relations

$$[\underline{S}_i, \underline{T}_j] = 0$$

$$[\underline{S}_i, \underline{S}_j] = 0 \text{ for } i \neq j$$

$$[S_{i\alpha}, S_{i\beta}] = iS_{i\gamma} \text{ for } \alpha\beta\gamma \text{ cyclic} \quad (2.1)$$

and the unperturbed hamiltonian

$$\begin{aligned} H_0 &= \Delta \sum_i \underline{S}_i \cdot \underline{T}_i \\ &= \Delta \sum_i (1/2 J_i^2 - 3/4) \end{aligned} \quad (2.2)$$

For the moment we consider a single site, suppressing the site index. Define for each site an operator

$$\underline{P}_i = \frac{2}{\sqrt{3}} \underline{S}_i \times \underline{T}_i$$

so that

$$P_\alpha = \frac{2}{\sqrt{3}} (S_\beta T_\gamma - S_\gamma T_\beta) \text{ } (\alpha\beta\gamma \text{ cyclic}) \quad (2.3)$$

Denoting the ground state by $|0\rangle$, and defining the angular momentum $\underline{J} = \underline{S} + \underline{T}$, and the operator $\underline{L} = \underline{S} - \underline{T}$, it is straightforward to show that

$$P_{\alpha} |0\rangle = \frac{1}{\sqrt{3}} |\alpha\rangle \quad (\text{an excited state}) \quad (2.4a)$$

$$P^2 = 1/2 + 2/3 \underline{S} \cdot \underline{T} \quad (2.4b)$$

$$P_{\alpha} P_{\beta} |0\rangle = 0 \quad (\alpha \neq \beta) \quad (2.4c)$$

$$P_{\alpha} P_{\beta} |\beta\rangle = 1/3 |\alpha\rangle \quad (2.4d)$$

$$[P_{\alpha}, P_{\beta}] = \frac{i}{3} J_{\gamma} \quad (\alpha\beta\gamma \text{ cyclic}) \quad (2.4e)$$

$$[P_i, H_0] = \frac{i\Delta}{\sqrt{3}} L_i \quad (2.4f)$$

$$[M_0, L_i] = i\Delta/\sqrt{3} P_i \quad (2.4g)$$

$$[J_{\alpha}, P_{\beta}] = i P_{\gamma} \quad (2.4h)$$

$$[P_{\alpha}, L_{\beta}] = \begin{cases} -4i/\sqrt{3} (\underline{S} \cdot \underline{T} - S_{\alpha} T_{\alpha}) & \text{for } \alpha = \beta \\ \frac{2i}{\sqrt{3}} (S_{\alpha} T_{\beta} + S_{\beta} T_{\alpha}) & \text{for } \alpha \neq \beta \end{cases} \quad (2.4i)$$

etc.

We are now in a position to construct the bare exciton operators c_{α}, c_{α}^+ , corresponding to the "boson" operators of Cooper. We require that $[H_0, c_{\alpha}^+] = \Delta c_{\alpha}^+$, which is satisfied by

$$\begin{aligned} c_{\alpha}^+ &= \frac{\sqrt{3}}{2} P_{\alpha} - \frac{i}{2} L_{\alpha} \\ c_{\alpha} &= \frac{\sqrt{3}}{2} P_{\alpha} + \frac{i}{2} L_{\alpha} \end{aligned} \quad (2.5)$$

The commutation relations of these operators are

$$[c_{\alpha}, c_{\alpha}^+] = -\frac{i\sqrt{3}}{2} \quad [P_{\alpha}, L_{\alpha}] = -2 (\underline{S} \cdot \underline{T} - S_{\alpha} T_{\alpha})$$

$$[c_{\alpha}, c_{\beta}] = [c_{\alpha}^+, c_{\beta}^+] = 0$$

$$[c_{\alpha}, c_{\beta}^+] = -c_{\beta}^+ c_{\alpha}$$

Note that these operators act as boson operators on the single particle ground state, suggesting a boson treatment valid at low temperatures, i.e., the treatment of Cooper.

B. Interacting particles; Bogoliubov Transformation

We will consider an interaction of the general form

$$\begin{aligned} H_1 &= 1/2 \sum_{ij} \sum_{\alpha\beta} P_{i\alpha} J_{\alpha\beta} \\ &= 1/6 \sum_{\underline{k}\alpha\beta} (c_{\alpha}^{+}(\underline{k}) + c_{\alpha}(-\underline{k})) J_{\alpha\beta}(\underline{k}) (c_{\beta}^{+}(-\underline{k}) + c_{\beta}(\underline{k})) \end{aligned} \quad (2.7)$$

where

$$\begin{aligned} c_{\alpha}^{+}(\underline{k}) &= N^{-1} \sum_{\underline{i}} e^{i\underline{k} \cdot \underline{r}_{\underline{i}}} c_{i\alpha}^{+} \\ J_{\alpha\beta}(\underline{k}) &= \sum_{\underline{j}} e^{i\underline{k} \cdot (\underline{i}-\underline{j})} J_{\alpha\beta}(\underline{i}-\underline{j}) \end{aligned} \quad (2.8)$$

We may suppose $J_{\alpha\beta}(\underline{k})$ is diagonal for each $\underline{k}$. (The eigenvalue transformation will, in general, be $\underline{k}$ - dependent.) We wish to make a transformation to new operators

$$d_{\alpha}^{+}(\underline{k}) = \cosh \phi_{\underline{k}\alpha} c_{\alpha}^{+}(\underline{k}) + \sinh \phi_{\underline{k}\alpha} c_{\alpha}(-\underline{k}) \quad (2.9)$$

such that

$$[H, d_{\alpha}^{+}(\underline{k})] = \epsilon_{\underline{k}\alpha}^{(0)} d_{\alpha}^{+}(\underline{k})$$

In terms of these operators

$$P_{\alpha}(\underline{k}) = \frac{1}{\sqrt{3}} e^{-\phi_{\underline{k}\alpha}} (d_{\alpha}(-\underline{k}) + d_{\alpha}^{+}(\underline{k})) \quad (2.10)$$

(For the moment, we suppress the indices $\underline{k}, \alpha$.) Equation (2.9)

becomes

$$\begin{aligned} & \Delta (\cosh \phi c^+ - \sinh \phi c) \\ & + \frac{J}{3} C_\alpha (\cosh \phi - \sinh \phi) (c^+ + c) \\ & = \epsilon^{(0)} (\cosh \phi c^+ + \sinh \phi c) \end{aligned} \quad (2.11)$$

where

$$C_\alpha = - \frac{2}{N} \sum_i (\underline{S}_i \cdot \underline{T}_i - S_{i\alpha} T_{i\alpha}) \quad (2.12)$$

and we have neglected terms involving other spectral branches which we will shortly resurrect. To achieve an RPA solution we replace C_α by $\langle C_\alpha \rangle$, giving

$$\epsilon^{(0)^2} = \Delta^2 + \frac{2}{3} \Delta J \langle C_\alpha \rangle \quad (2.13a)$$

$$\phi = \frac{1}{2} \log (\epsilon^{(0)} / \Delta) \quad (2.13b)$$

The transformed operators thus satisfy

$$\begin{aligned} & [H - \epsilon_{q\gamma}^{(0)}, d_{\gamma(q)}^+] = 1/6 J_{\gamma\gamma}(q) e^{-\phi_{q\gamma}} \{ (c_\gamma(q) + c_\gamma(-q)), (C_\gamma - \langle C_\gamma \rangle) \}_+ \\ & + \sum'_{\underline{k}, \alpha, \beta} \frac{1}{6} J_{\alpha\beta}(-k) \{ (c_{k\alpha}^+ + c_{-k\alpha}) (\sinh \phi_{q\gamma} [c_\beta^+(-k), c_\gamma(-q)] \\ & + \cosh \phi_{q\gamma} [c_\beta(k), c_\gamma^+(q)]) + (\sinh \phi_{q\gamma} [c_\alpha^+(k), c_\gamma(-q)] \\ & - \cosh \phi_{q\gamma} [c_\gamma^+(q), c_\alpha(-k)]) (c_\beta^+(-k) + c_\beta(k)) \} \end{aligned} \quad (2.14)$$

The prime on the sum indicates the omission of commutators between operators with the same polarization index. We define the right-hand side of (2.14) as $R_{q\gamma}$.

The commutators appearing in R take the form of a hard-core interaction

$$[c_{\beta}^{\dagger}(-k), c_{\gamma}(-q)] = \frac{1}{N} \sum_p c_{\beta}^{\dagger}(-k-p) c_{\gamma}(-q-p) \quad (2.15)$$

leading to

$$\begin{aligned} R_{\delta\gamma} = & \frac{1}{6N} \sum_{kp} \sum_{\alpha\beta} J_{\alpha\beta}(\underline{k}) e^{-\phi_{k\alpha}} \{ (d_{\alpha}^{\dagger}(\underline{k}) + d_{\alpha}(-\underline{k})), \\ & [\sinh \phi_{q\gamma} (\cosh \phi_{k+p,\beta} d_{\beta}^{\dagger}(-\underline{k}-\underline{p}) - \sinh \phi_{k+p,\beta} d_{\beta}(\underline{k}+p)) \\ & \times (\cosh \phi_{q+p,\gamma} d_{\gamma}^{\dagger}(-q-p) - \sinh \phi_{q+p,\gamma} d_{\gamma}^{\dagger}(q+p)) \\ & - \cosh \phi_{q\gamma} (\cosh \phi_{q+p,\gamma} d_{\gamma}^{\dagger}(q+p) - \sinh \phi_{-q-p,\gamma} d_{\gamma}(-q-p)) \\ & \times (\cosh \phi_{k+p,\beta} d_{\beta}(\underline{k}+p) - \sinh \phi_{k+p,\beta} d_{\beta}^{\dagger}(-\underline{k}-\underline{p})) \} + \end{aligned} \quad (2.16)$$

if we assume $J_{\alpha\beta}(\underline{k}) = J_{\alpha\beta}(-\underline{k})$, and neglect $(C_{\alpha} - \langle C_{\alpha} \rangle)$.

C. Green's Functions; Dyson Equation

We define retarded and advanced Green's functions by

$$G_{\alpha\beta}^R(\underline{k}_1, \underline{k}_2, t) = -i \langle e^{iHt} d_{\alpha}(\underline{k}_1) e^{-iHt} d_{\beta}^{\dagger}(\underline{k}_2) \rangle \theta(t)$$

$$G_{\alpha\beta}^A(\underline{k}_1, \underline{k}_2, t) = -i \langle e^{iHt} d_{\alpha}^{\dagger}(\underline{k}_1) e^{-iHt} d_{\beta}(\underline{k}_2) \rangle \theta(t)$$

In what follows we restrict ourselves to $\alpha = \beta$, $k_1 = k_2$

(We suppose $J_{\alpha\beta}$ to have been diagonalized as mentioned earlier. Off-diagonal Green's functions would be important in higher approximations than we shall consider in this paper.) We have

$$G_{\alpha}^R(\underline{k}, 0^+) - G_{\alpha}^A(\underline{k}, 0^+) = -i \langle C_{\alpha} \rangle. \quad (2.18)$$

In addition we will suppose

$$G_{\alpha}^R(\underline{k}, 0^+) = e^{\beta \epsilon_{k\alpha}} G_{\alpha}^A(\underline{k}, 0^+)$$

where $\epsilon_{k\alpha}$ is not necessarily equal to $\epsilon_{k\alpha}^{(0)}$.

To derive a Dyson-like equation, we write (see Appendix B)

$$\begin{aligned}
 i \frac{\partial}{\partial t} G_{\alpha}^R(\underline{k}, t) &= i \delta(t) G_{\alpha}^R(\underline{k}, 0^+) + i \langle e^{iHt} [H, d_{\alpha}(\underline{k})] e^{-iHt} d_{\alpha}^+(\underline{k}) \rangle \theta(t) \\
 &= \frac{\delta(t) \langle C_{\alpha} \rangle}{1 - e^{-\beta \epsilon_{k\alpha}}} + \epsilon_{k\alpha}^{(0)} G_{\alpha}^R(\underline{k}, t) \\
 &\quad - i \langle e^{iHt} R_{k\alpha}^+ e^{-iHt} d_{\alpha}^+(\underline{k}) \rangle \theta(t)
 \end{aligned} \tag{2.20}$$

or, fourier transforming,

$$(\omega - \epsilon_{k\alpha}^{(0)}) G_{\alpha}^R(\underline{k}, \omega) = \frac{\langle C_{\alpha} \rangle}{1 - e^{-\beta \epsilon_{k\alpha}}} - \left\{ i \theta(t) \langle e^{iHt} R_{k\alpha}^+ e^{-iHt} d_{\alpha}^+(\underline{k}) \rangle \right\}_{\omega}$$

We may further treat the last term by writing (2.21)

$$\begin{aligned}
 i \frac{\partial}{\partial t} \left\{ \theta(t) \langle R_{k\alpha}^+ e^{-iHt} d_{k\alpha}^+ e^{iHt} \rangle \right\} &= i \delta(t) \langle R_{k\alpha}^+ d_{k\alpha}^+ \rangle \\
 &\quad + \langle R_{k\alpha}^+ e^{-iHt} [H, d_{k\alpha}^+] e^{iHt} \rangle \theta(t)
 \end{aligned} \tag{2.22}$$

Taking as vanishing the first term on the right, we have

$$\begin{aligned}
 (\omega - \epsilon_{k\alpha}^{(0)}) \left\{ \langle R_{k\alpha}^+ e^{-iHt} d_{k\alpha}^+ e^{iHt} \rangle \theta(t) \right\}_{\omega} \\
 = \langle e^{iHt} R_{k\alpha}^+ e^{-iHt} R_{k\alpha} \theta(t) \rangle_{\omega}
 \end{aligned} \tag{2.23}$$

Equation (2.21) then becomes

$$(\omega - \epsilon_{k\alpha}^{(0)}) G_{\alpha}^R(\underline{k}, \omega) = \frac{\langle C_{\alpha} \rangle}{1 - e^{-\beta \epsilon_{k\alpha}}} - \frac{i}{\omega - \epsilon_{k\alpha}^{(0)}} \left\{ \langle e^{iHt} R_{k\alpha}^+ e^{-iHt} R_{k\alpha} \rangle \theta(t) \right\}_{\omega} \quad (2.24)$$

which in the usual way can be written

$$\frac{\langle C_{\alpha} \rangle}{1 - e^{-\beta \epsilon_{k\alpha}}} (G_{\alpha}^R(\underline{k}, \omega))^{-1} = (\omega - \epsilon_{k\alpha}^{(0)} + \Sigma_{k\alpha}(\omega)) \quad (2.25)$$

where $\Sigma_{k\alpha}(\omega) = (\omega - \epsilon_{k\alpha}^{(0)})^{-1}$

$$\times \left\{ -i \langle e^{iHt} R_{k\alpha}^+ e^{-iHt} R_{k\alpha} \rangle \theta(t) \right\}_{\omega} \quad (2.26)$$

$$\times (G_{\alpha}^R(\underline{k}, \omega))^{-1}$$

III. Illustrative Calculation

In this section we apply the preceding formalism to a one-dimensional dielectric having dipolar interactions between nearest neighbors. Thus (with unit lattice constant)

$$\begin{aligned} J_{xx}(q) &= J_{yy}(q) = 2 J_0 \cos q \\ J_{zz}(q) &= -4 J_0 \cos q \\ J_{xy}(q) &= J_{yz}(q) = J_{yx}(q) = \dots = 0 \end{aligned} \quad (3.1)$$

for any q . The spectrum will have a longitudinal mode which softens to zero at $q = 0$ for a sufficiently strong interaction, and two transverse modes with minima at $q = \pi$. We restrict this calculation to the framework of perturbation theory in the parameter J_0/Δ , and henceforth set $\Delta = 1$ and measure T in units of Δ/k_B .

The value of $\langle C_\alpha \rangle$ may be reasonably well calculated by solving the two particle problem for $\langle C_\alpha^{(2)} \rangle$ and putting $(\langle C_\alpha \rangle - \langle C_\alpha^{(1)} \rangle)$ equal to $2(\langle C_\alpha^{(2)} \rangle - \langle C_\alpha^{(1)} \rangle)$. (This amounts to saying that each bond contributes independently to $(\langle C_\alpha \rangle - \langle C_\alpha^{(1)} \rangle)$). We then use this value to compute the approximate Green's functions

$$G^{R(0)}(k, \omega) = \langle C_\alpha \rangle (1 - e^{-\beta \epsilon_{k\alpha}^{(0)}})^{-1} (\omega - \epsilon_{k\alpha}^{(0)} + i\delta)^{-1} \quad (3.2)$$

The largest term in $\Sigma_{q\gamma}$ is now

$$\begin{aligned}
 & \frac{\langle C_Y \rangle}{1 - \exp(-\beta \epsilon_{qY}^{(0)})} (6N)^{-2} \sum_{\substack{k, p \\ \alpha}} \sum_{\substack{k', p' \\ \alpha'}} J_{\alpha\alpha}(k) J_{\alpha'\alpha'}(k') \exp(-(\phi_{k\alpha} + \phi_{k'\alpha'})) \\
 & \times \cosh^2 \phi_{qY} \cosh \phi_{q+p, Y} \cosh \phi_{k+p, \alpha} \cosh \phi_{q+p, Y} \cosh \phi_{k'+p', \alpha'} \\
 & \times (-i) < \theta(t) e^{iHt} \{ (d_{k\alpha} + d_{-k\alpha}^+), d_{k+p, \alpha}^+ d_{q+p, Y} \}_+ e^{-iHt} \\
 & \times \{ (d_{k'\alpha'}^+ + d_{-k'\alpha'}), d_{q+p', Y}^+ d_{k'+p', \alpha'} \}_+ >_w
 \end{aligned} \tag{3.3}$$

The last factor may be decoupled to give

$$\begin{aligned}
 & - \frac{1}{\pi^2} \int d\omega_1 \int d\omega_2 G_{\alpha}^R(k, \omega_1) G_Y^R(q+p, \omega_2) G_{\alpha}^A(k+p, \omega - \omega_1 - \omega_2) \\
 & \times \delta_{\alpha\alpha'} \delta(k-k') \delta(p-p') + \text{smaller terms}
 \end{aligned} \tag{3.4}$$

This term corresponds to scattering from a thermally excited particle, and may be diagrammatically represented as Figure 1. This and other terms may be evaluated (using the approximate normal mode Green's functions) as described in Appendix A. The term we have represented is proportional to J^2 and vanishes exponentially at low temperatures. The expansion of Σ includes as well terms finite at low temperatures but proportional to higher powers of J .

We have performed the calculation for the longitudinal mode of the one-dimensional dielectric previously described. The integrals were done using Simpson's Rule on the Brillouin zone $-\pi < k < \pi$ divided into 40 subintervals, with the help of the Cornell University IBM 360/65. All terms were retained involving single scattering from a thermal particle. The results are shown in Figures 2-3. This constitutes a rather crude calculation so that, while we believe the qualitative features be correct, the numerical values should not be taken seriously.

IV. Discussion and Conclusions

One feature which is readily understood is that the soft mode is somewhat depressed and relatively highly damped by the scattering. These excitations scatter strongly with other low-energy excitations, and particularly with excitations in low-energy portions of other spectral branches. The intermediate states of such scatterings will be of equal or higher energy. The lowering of the group velocity and small damping near $k = \pi/2$ is probably peculiar to this model, and related to the large group velocity in this region.

These dynamical effects will be particularly large in systems which interact strongly enough to exhibit phase transitions. Such a system can be disordered only if there are enough thermal excitations to prevent ordering, and furthermore scattering matrix elements will be large. Our results suggest the possibility that mode softening effects in some systems may be concentrated in a small region near $q = 0$. For weak interactions the energies of small q modes due to thermal population at finite q , but this relationship is reversed when the interaction is strong. One must be wary of any statements concerning finite wavevector behavior in the regime where it is governed primarily by the poorly understood soft modes.

The presence of higher single-particle levels will drastically alter the scattering effects treated here. In that case, two excitations are no longer excluded from occupying the same site, but can combine to form a (virtual) heavy excitation in a higher band. This effect should tend to counteract the mode-softening effect for temperatures $T < \Delta$.

We conclude that the mean-field-like theories thus far published cannot be expected to fit very well the excitations of interacting singlet-triplet systems unless the interaction is weak. There is room for a good deal more work along this direction.

Appendix A

To evaluate $-(1/\pi^2) \int d\omega_1 \int d\omega_2 G^{R(0)}(\omega_1, k_1) G^{R(0)}(\omega_2, k_2) G^{R(0)}(\omega - \omega_1 - \omega_2, k_3)$

The quantity desired may be written

$$Q(\omega) = \frac{1}{\pi^2} \int d\omega_1 \int d\omega_2 \frac{A_1}{\omega_1 - \epsilon_1 + i\delta} \frac{A_2}{\omega_2 - \epsilon_2 + i\delta} \frac{A_3}{\omega - \omega_1 - \omega_2 - \epsilon_3 + i\delta} \quad (\text{A.1})$$

We close the contours for ω_1 and ω_2 in the upper half-plane, giving

$$Q(\omega) = 4 A_1 A_2 A_3 / (\omega - \epsilon_3 + \epsilon_2 - \epsilon_1 + i\delta) \quad (\text{A.2})$$

$$= 4 A_1 A_2 A_3 \left[P \frac{1}{\omega + \epsilon_1 - \epsilon_1 - \epsilon_3} - i\pi\delta(\omega + \epsilon_2 - \epsilon_1 - \epsilon_3) \right] \quad (\text{A.3})$$

This may then be integrated over k_1 , k_2 , and k_3 by usual methods to find $\Sigma_{q\alpha}(\omega = \epsilon_{q\alpha}^{(0)})$, the real part giving an energy shift and the imaginary part a damping.

Appendix B

This derivation uses the standard equation of motion technique. To make this method work, we need not write the hamiltonian in terms of the operators, provided we can write the commutators of the hamiltonian with the operators.

If the quasi-particles were bosons, we would have

$$\langle C_{\alpha} \rangle = 1$$

$$H = \sum_{k\alpha} \epsilon_{k\alpha} d_{k\alpha}^{\dagger} d_{k\alpha} + V$$

$$R_{\delta\alpha}^{\dagger} = [d_{\delta\alpha}, V]$$

and the calculation would lead to the standard result.

Figure 1

Figure 2

Figure 3

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FIGURE CAPTIONS

Fig. 1: Diagrammatic representation of the main dynamic correction to $G_Y(q)$. The wavy line represents a thermal excitation.

Fig. 2: Energy (solid line) and damping (dot-dash line) for various temperatures and $J_0 = 0.4$. The dashed line is the approximate energy of equation (2.13a).

Fig. 3: Same as Figure 2, but for various values of J_0 at $k_B T/\Delta = 0.5$.

ACKNOWLEDGEMENTS

The author would like to acknowledge fruitful interactions with J. A. Krumhansl throughout the course of this work, as well as conversations with P. Pfeuty, and a communication from R. J. Birgenau.