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ISABELLE PROJECT

BROOKHAVEN NATIONAL LABORATORY
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ACCELERATOR DEPARTMENT
Informal Report

POSSIBLE EFFECTS OF WEAKLY COUPLED NEUTRAL CURRENTS in $pp \rightarrow \ell^+\ell^- + X$

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CERN - Geneva
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The possibility of observing weak neutral currents at ISABELLE has been pointed out by Lederman [1]. Here we probe this possibility further. We shall discuss some inherent difficulties of the proposed experiment and present a (model dependent) estimate of one type of effect (charge asymmetry).

Generally, there are two effects which can signal the presence of weak neutral currents interfering with single photon exchange :

a) parity violation :

$$\langle (\vec{p}_+ \times \vec{p}_-) \cdot \vec{p}_{beam} \rangle \neq 0$$

and b) charge asymmetries :

$$\langle (\vec{p}_+ - \vec{p}_-) \cdot \vec{p}_{beam} \rangle \neq 0$$

$$\langle (E_+ - E_-) \rangle \neq 0$$

However, the beam direction is not defined due to the identity of the two initial state particles ; the first two asymmetries vanish identically. In an inclusive pair production experiment, the only non-vanishing momentum-dependent effects are of the form :

$$\langle [(\vec{p}_+ \times \vec{p}_-) \cdot \vec{p}_{beam}] \cdot (\vec{Q} \cdot \vec{p}_{beam}) \rangle$$

and

$$\langle [(\vec{p}_+ - \vec{p}_-) \cdot \vec{p}_{beam}] \cdot (\vec{Q} \cdot \vec{p}_{beam}) \rangle$$

where $\vec{Q} = \vec{p}_+ + \vec{p}_-$ is the three-momentum transfer to the lepton pair. Thus the effects depend on the virtual photon not being produced at rest. (This is of course also true for the energy asymmetry which vanishes identically for $\vec{Q} = 0$.) In the parton models [2] generally used to describe these processes (Fig. 1), $\vec{Q}$ is parallel to the beam direction and peaked toward zero. This is one factor which could suppress observable effects.

The observation of parity violation is really the only clear signature for the presence of weak interactions, since charge asymmetries can also arise from the interference of two-photon exchange with one-photon exchange. The dominant process for two-photon exchange (Fig. 2) is expected to be comparable with one-photon exchange at ISABELLE energies [3]. On the other hand, if no polarizations are measured, the only evidence for parity violation appears in the non-vanishing expectation value of a triple momentum product, which is odd under time reversal. So the presence of such a term depends on the presence of strong interaction effects. In an inclusive reaction, where we sum over all final hadron states, only strong interactions in the initial pp state can give rise to parity violating effects of this type. In the parton model, where strong interactions are ignored, the effect is identically zero.

This leaves us with charge asymmetries. However, in this case the parton model works in our favour. Since the inclusive pp cross-section is just an incoherent sum (weighted with probability distributions) over cross-sections for $\bar{q}q \to \ell^+\ell^-$, there is no interference with the dominant two-photon contribution, which arises from quark bremsstrahlung rather than annihilation. The only two-photon process (Fig. 3) which can interfere with the one-photon process is expected to be small [3]. We might try to calculate this contribution using point-like quarks ; however, in the presence of an off-mass shell quark the theoretical grounds for such an assumption are weak and appear to be contradicted by the data [4].

In the following we shall consider one photon and one Z^0 (weakly coupled massive neutral boson) exchange in the parton model, and neglect two-photon effects. It may be possible to test our assumptions a posteriori. Single vector meson exchange implies a quadratic distribution in the lepton four-momentum difference $q = p_- - p_+$. For two-photon exchange the dependence in q_μ is more complicated. Although the size of the effects we shall estimate depends critically on the model we choose for parton distribution functions in the proton, there are other predictions of the parton model which are independent of the distribution functions. Verification of these would lend further confidence to the one-particle exchange assumption. In particular the energy asymmetry and momentum asymmetry are not independent in the parton model. The contribution of Fig. 2 alone can of course be corrected for.

KINEMATICS

If p_1 and p_2 are the incoming proton momenta and p_+, p_- the lepton momenta, we define the follwing variables :

$$P = p_1 + p_2 = (\sqrt{s}, 0)$$
$$K = p_1 - p_2 = (0, \vec{K})$$
$$Q = p_+ + p_-, \qquad q = p_- - p_+$$

(in the c.m. system).

J^{γ}_{μ} is the hadronic current and we take the weak neutral coupling to be :

$$\left(M_Z^2 \frac{G}{\sqrt{2}}\right)^{1/2} Z_\mu \left(J_\mu^Z + j_\mu^Z \right)$$

where G is the Fermi constant, J^Z_μ is the hadronic neutral current and

$$j_\mu^Z = \bar{l}\, \gamma_\mu (a + b\gamma_5)\, l$$

is the lepton current. Then the inclusive cross-section in the centre-of-mass frame is :

$$d\sigma = \left(d^3p_+\, d^3p_- / 32(2\pi)^6 S E_+ E_-\right) \times$$

$$\Big\{ \left(\frac{4\pi\alpha}{Q^2}\right)^2 \langle p_1 p_2 | J^\gamma_\mu(-Q)\, J^\gamma_\nu(Q) | p_1 p_2 \rangle \, \mathrm{Tr}\, \not{p}_+ \gamma^\mu \not{p}_- \gamma^\nu$$

$$- \frac{4\pi\alpha}{Q^2} \frac{G}{\sqrt{2}} \langle p_1 p_2 | J^\gamma_\mu(-Q)\, J^Z_\nu(Q) + J^Z_\mu(-Q)\, J^\gamma_\nu(Q) | p_1 p_2 \rangle \times \qquad (1)$$

$$\times\, \mathrm{Tr}\, \not{p}_+ \gamma^\mu (a + b\gamma_5) \not{p}_- \gamma^\nu / (1 - Q^2/M_Z^2)$$

$$+ \frac{G^2/2}{(1 - Q^2/M_Z^2)^2} \langle p_1 p_2 | J^Z_\mu(-Q)\, J^Z_\nu(Q) | p_1 p_2 \rangle \, \mathrm{Tr}\, \not{p}_+ \gamma^\mu (A + B\gamma_5) \not{p}_- \gamma^\nu \Big]$$

where $\quad A = a^2 + b^2, \qquad B = 2ab,$

$$\langle p_1 p_2 | J^1_\mu(-Q)\, J^2_\nu(Q) | p_1 p_2 \rangle \equiv \mathcal{W}^{12}_{\mu\nu}(P, K, Q)$$

$$\equiv \int d^4x\, e^{-iQx} \langle p_1 p_2 | J^1_\mu(x)\, J^2_\nu(0) | p_1 p_2 \rangle$$

and we have neglected the lepton masses. The requirements of hermiticity and Fermi statistics restrict the hadronic matrix elements to the form :

$$
\begin{aligned}
\mathcal{M}_{\mu\nu} \equiv \tfrac{1}{2}(\mathcal{M}^{12}_{\mu\nu} + \mathcal{M}^{21}_{\mu\nu}) &= -g_{\mu\nu} F_1 + P_\mu P_\nu F_2 \\
&- K_\mu K_\nu F_3 + (P_\mu K_\nu + P_\nu K_\mu)(Q\cdot K) F_4 \\
&+ i\,\varepsilon_{\mu\nu\rho\sigma}\{(Q\cdot K)P^\rho K^\sigma G_1 + Q^\sigma P^\rho G_2 + (Q\cdot K) K^\rho Q^\sigma G_3\} \\
&+ i(P_\mu K_\nu - K_\mu P_\nu)(Q\cdot K) F_5 \\
&+ [(P_\mu \varepsilon_{\nu\sigma\tau} + P_\nu \varepsilon_{\mu\rho\sigma\tau})(Q\cdot K) G_4 + (K_\mu \varepsilon_{\nu\sigma\tau} + K_\nu \varepsilon_{\mu\rho\sigma\tau}) G_5] P^\rho Q^\sigma K^\tau
\end{aligned} \tag{2}
$$

plus terms in Q_μ , Q_ν which we neglect since their contributions vanish with vanishing lepton mass. If we can also neglect pp interactions in the initial state, time reversal requires :

$$F_5 = G_4 = G_5 = 0 \tag{3}$$

The structure functions F_i and G_i depend on the invariants s, Q^2 and $(Q\cdot K)^2$. The sum over lepton spins gives the general expression :

$$
\begin{aligned}
\tfrac{1}{4} \ell_{\mu\nu} &\equiv \tfrac{1}{4} \mathrm{Tr}\, \not{p}_+ \gamma_\mu (\alpha + \beta \gamma_5) \not{p}_- \gamma_\nu = \\
&= \frac{\alpha}{2}\left[Q_\mu Q_\nu - q_\mu q_\nu - Q^2 g_{\mu\nu}\right] \\
&\quad - i\frac{\beta}{2}\, \varepsilon_{\mu\nu\rho\sigma} Q^\rho q^\sigma
\end{aligned} \tag{4}
$$

then, neglecting strong interactions in the initial state, the three terms appearing in the centre-of-mass cross-section, Eq. (1), are each expressions of the form :

$$\frac{1}{4} M_{\mu\nu} \ell^{\mu\nu} = \alpha \Big\{ Q^2 F_1 + s F_2 \left(2E_+ E_- - Q^2/2\right) - s F_3 \left(2 p_\parallel^+ p_\parallel^- + Q^2/2\right) + \frac{s^{3/2}}{2} Q_\parallel F_4 \left(Q_0 Q_\parallel - q_0 q_\parallel\right) \Big\} + \beta \Big\{ s^{3/2} Q_\parallel G_1 \left(Q_0 q_\parallel - Q_\parallel q_0\right) + s^{1/2} Q^2 q_0 G_2 - s Q_\parallel q_\parallel Q^2 G_3 \Big\} \tag{5}$$

where $p_\parallel$ is the component of the vector $\vec{p}$ along the beam direction and we have neglected the proton mass : $P^2 = -K^2 = s$.

QUARK PARTON MODEL

If $p_i(x)$ is the probability for finding the i^{th} type of quark $(i = 1 \ldots 6)$ in the proton with fraction x of longitudinal momentum, the inclusive cross-section for pair production is :

$$d\sigma = \sum_{i=1}^{6} \int dx_1\, dx_2\, p_i(x_1)\, p_{\bar{i}}(x_2)\, d\sigma_i(x_1, x_2) \tag{6}$$

where $d\sigma_i$ is the differential cross-section for $q_i q_{\bar{i}} \to \ell^+ \ell^-$. One finds for the structure functions defined in Eq. (2) :

$$F_1 = s F_2 = s F_3 = (2\pi)^4 \delta^2(\vec{Q}_T)\, 4 \sum_{i=1}^{6} \alpha_i\, p_i(x_+)\, p_{\bar{i}}(x_-) \tag{7}$$

$$s(Q\cdot K)\, G_1 = (2\pi)^4 \delta^2(\vec{Q}_T)\, 4 \sum_{i=1}^{6} \beta_i\, p_i(x_+)\, p_{\bar{i}}(x_-) \tag{8}$$

$$F_4 = G_2 = G_3 = 0 \tag{9}$$

where $\vec{Q}_T$ is the component of $\vec{Q}$ perpendicular to the beam,

$$x_\pm = Q_\pm/\sqrt{s} = (Q_0 \pm Q_\parallel)/\sqrt{s} \ , \qquad x_+ x_- = Q^2/s \tag{10}$$

and

$$\begin{array}{lll} \alpha_i = Q_i^2 \ , & \beta_i = 0 & \text{for } \gamma \text{ exchange,} \\ \alpha_i = a_i^2 + b_i^2 \ , & \beta_i = 2a_i b_i & \text{for } Z \text{ exchange,} \\ \alpha_i = Q_i a_i \ , & \beta_i = Q_i b_i & \text{for the} \\ & & \text{interference term.} \end{array} \tag{11}$$

Q_i is the quark charge and the Z quark coupling is

$$\left(M_Z^2 \frac{G}{\sqrt{2}}\right)^{1/2} Z_\mu \bar{q}_i \gamma_\mu (a_i + b_i \gamma_5) q_i \tag{12}$$

The predictions of : (a) no transverse momentum for the lepton pair, and (b) only two independent structure functions are general features of the parton model, which in principle allow us to determine a posteriori whether the numerical estimates we may make have any meaning.

The charge conjugation properties of the currents imply

$$\alpha_i = \alpha_{\bar{i}} \ , \qquad \beta_i = -\beta_{\bar{i}} \tag{13}$$

so that the charge asymmetry [Eq. (8)] will clearly vanish if quark and anti-quark distribution functions in the proton are the same. This is still another possible suppression factor.

Introducing the "scaling" variables

$$x = x_+, \quad y = \tfrac{1}{2}(1 + q_\parallel/Q_+), \quad \lambda = Q^2/s \tag{14}$$

with $q_+ = q_0 + q_\parallel$, we may write the contribution of each term as

$$\frac{d\sigma^{\gamma, Z, (Z\gamma)}}{d\lambda\, dx\, dy} = \frac{g^{\gamma, Z, (Z\gamma)}(Q^2)}{8\pi} \frac{s\lambda}{x} \sum_{i=1}^{6} p_i(x) \bar{p}_i\!\left(\tfrac{\lambda}{x}\right)\left[\alpha\alpha_i (1 - 2y + 2y^2) + \beta\beta_i (1 - 2y)\right] \tag{15}$$

where

$$g^{\gamma}(Q^2) = (4\pi\alpha/Q^2)^2 \tag{16}$$

$$R(Q^2) \equiv g^{Z\gamma}/g^{\gamma} = g^{Z}/g^{Z\gamma} = \frac{G\, Q^2/\sqrt{2}}{4\pi\alpha(1 - Q^2/M_Z^2)} \tag{17}$$

The variable y is related to the lepton angle in the di-lepton rest frame by

$$2y - 1 = \cos\theta^* \tag{18}$$

In the pp centre-of-mass frame we have :

$$\left.\begin{aligned} q_{\parallel} &= \sqrt{s}\, x(2y-1)(1+\tau/x^2) \\ q_0 &= \sqrt{s}\, x(2y-1)(1-\tau/x^2) \\ Q_{\parallel} &= \sqrt{s}\, x(1-\tau/x^2) \end{aligned}\right\} \text{with} \left\{\begin{aligned} & 0 \leq y \leq 1 \\ & \tau \leq x \leq 1 \\ & 0 \leq \tau \leq 1 \end{aligned}\right. \tag{19}$$

Then

$$\left.\begin{aligned} & q_0 > 0 \\ & q_{\parallel}/Q_{\parallel} > 0 \end{aligned}\right\} \text{ if } \left\{\begin{aligned} & y > 1/2,\ x > \sqrt{\tau} \\ & \qquad \text{or} \\ & y < 1/2,\ x < \sqrt{\tau} \end{aligned}\right. \tag{20}$$

We define the charge asymmetry for fixed Q^2 by :

$$A = \frac{d\sigma(q_0>0) - d\sigma(q_0<0)}{d\sigma} = \frac{d\sigma(q_{\parallel}/Q_{\parallel}>0) - d\sigma(q_{\parallel}/Q_{\parallel}<0)}{d\sigma} \tag{21}$$

We find

$$A(Q^2, s) = \frac{2R(Q^2)\sum_{i=1}^{3}\left[-2bQ_i b_i + R(Q^2)4ab\, a_i b_i\right]\left(I_1^i(\tau) - I_2^i(\tau)\right)}{\sum_{i=1}^{3}\left[Q_i^2 - 2R(Q^2)a\, a_i Q_i + R^2(Q^2)(a^2+b^2)(a_i^2+b_i^2)\right] I_3^i(\tau)} \tag{22}$$

where the sum is over quark states only, $\bar{R}(Q^2)$ is defined in Eq. (17) and

$$I_1^i(\tau) = \int_{\sqrt{\tau}}^{1} \frac{dx}{x}\, p_i(x)\, p_{\bar{i}}(\tau/x) \int_{1/2}^{1-y_0(x)} dy\,(1-2y) \tag{23a}$$

$$I_2^i(r) = \int_{\sqrt{r}}^{1} \frac{dx}{x}\, p_{\bar{i}}(x)\, p_i(r/x) \int_{1/2}^{1-y_0(x)} dy\,(1-2y) \tag{23b}$$

$$I_3^i(r) = \int_{r}^{1} \frac{dx}{x}\, p_i(x)\, p_{\bar{i}}(r/x) \int_{y_0(x)}^{1-y_0(x)} dy\,(1-2y+2y^2) \tag{23c}$$

$y_0(x) = y_0(r/x)$ represents a cut-off on lepton angle or transverse momentum. The lepton momenta in the pp centre-of-mass frame are :

$$p_{\parallel}^{-} = \sqrt{s}\,[\,xy - \frac{r}{x}(1-y)\,]\,, \qquad p_{\parallel}^{+} = \sqrt{s}\,[(1-y)x - \frac{r}{x}y\,]\,,$$
$$p_T^2 = r s y(1-y) \tag{24}$$

A MODEL DEPENDENT ESTIMATE

Numerical calculations require both a model for the parton distribution functions and a model for the z coupling constants. To get an idea of the magnitude of the effect one might expect, we use the parton model used by Berman, Bjorken and Kogut [2], where two independent distributions are assumed, one for valence quarks and one for the quark-anti-quark sea which is assumed to be SU_3 symmetric. Specifically :

$$\begin{aligned} x p_p(x) &= 2V(x) + 3/4\, S(x) \\ x p_n(x) &= V(x) + 3/4\, S(x) \\ p_{\bar{p}}(x) = p_{\bar{n}}(x) &= p_{\bar{\lambda}}(x) = p_{\lambda}(x) = 3/4\, S(x)/x \end{aligned} \tag{25}$$

with

$$V(x) = 1.1\sqrt{x}\,(1-x)^3$$
$$S(x) = 0.3 \quad (1-x)^{7/2} \tag{26}$$

To simplify the integrals, we do not consider x dependent cut-offs. Then the asymmetry is proportional to :

$$F(y_0) \quad \frac{I_-(\lambda)}{I_+(\lambda) + \delta\, I_S(\lambda)} \tag{27}$$

where

$$I_\pm(\lambda) = \int_{\sqrt{\lambda}}^{1} \frac{dx}{x} \left\{ V(x)\,S(\lambda/x) \pm S(x)\,V(\lambda/x) \right\} \tag{28}$$

$$I_S(\lambda) = \int_{\lambda}^{1} \frac{dx}{x}\, S(x)\, S(\lambda/x) \tag{29}$$

$$F(y_0) = -\frac{3}{8}(1-2y_0)\left[1 - y_0(1-y_0)\right]^{-1} \tag{30}$$

and δ depends on the quark coupling. In fact I_S does not contribute significantly. Figure 4 shows the function

$$\left(F(y_0)/F(0)\right)\left(I_-(\lambda)/I_+(\lambda)\right)$$

for $y_0 = 0$ (whole solid angle) and for $p_T^2/s = ry(1-y) > 0.1$. For a cut off $y_0 = 1/4$ ($|\cos\theta^*| < 0.5$), we have

$$F(.25)/F(0) = 2/3$$

This gives an idea how angular cuts will suppress the effect.

To estimate the asymmetries, we use the four-quark version of the Weinberg model [5)], and assume that there are no charmed quarks in the proton. Then the weak couplings are :

$$
\begin{aligned}
a &= -(1-4\sin^2\theta), \qquad b = -1 \\
a_p &= \tfrac{1}{2}(1-\tfrac{8}{3}\sin^2\theta) \\
a_n = a_\lambda &= -\tfrac{1}{2}(1-\tfrac{4}{3}\sin^2\theta) \\
b_p = -b_n &= -b_\lambda = \tfrac{1}{2}
\end{aligned} \tag{31}
$$

and the charges are

$$
Q_p = -2Q_n = -2Q_\lambda = 2/3 \tag{32}
$$

The angle θ is the γ, Z mixing angle [5]; empirically [6] $\sin^2\theta < 0.4$. We take $\theta = 0$ (V-A theory) and $\theta = 30^\circ$, $\sin^2\theta = 0.25$ (pure axial coupling for the lepton).

The asymmetries obtained using these values are shown in Figs. 5 and 6 for $s = (54\ \mathrm{GeV})^2$ (ISR) and $s = (400\ \mathrm{GeV})^2$ (ISABELLE) for different values of M_Z. For small values of the relative weak electromagnetic coupling $R(Q^2)$ [Eq. (18)], the asymmetry is proportional to R and independent of the Weinberg angle. At ISR energies the asymmetry exceeds 5% only for low M_Z and high Q^2. Moreover, the cross-section drops rapidly with increasing Q^2 [$d\sigma(r=0.1)/ /(d\sigma(r=0.9) \sim 10^{-9}$ in this model]. On the other hand if $Q^2 \gg M_Z^2$ (ISABELLE, M_Z small) the relative strength $R(Q^2)$ again becomes weak as the boson propagator falls like $1/Q^2$. Finally for $R(Q^2) \gg 1$ (ISABELLE, M_Z large), the weak interaction dominates and the asymmetry remains appreciable for $\theta = 0$ (V-A theory) and vanishes for $\theta = 30^\circ$, since in this case only the axial current couples to the leptons.

The asymmetries shown in Figs. 5 and 6 are of course limited by the asymmetry in the parton distributions (Fig. 4) which is between 15 and 25 per cent in the model used. Since this model is probably too simple to account for all the data, it may be that a better model will predict a higher asymmetry [7].

We have benefited from discussions with L. Lederman, C.H. Llewellyn Smith, H. Stern and M. Veltman.

REFERENCES

1) L.M. Lederman - CRISP 72-13.

2) See, e.g. :
S.D. Drell and T.-M. Yan - Ann.Phys. 66, 578-623 (1971) ;
P.V. Landshoff and J.C. Polkinghorne - Nuclear Phys. B33, 221 (1971) ;
S. Berman, J.D. Bjorken and J.B. Kogut - Phys.Rev. D4, 3388 (1971).

3) M.S. Chen, I.J. Muzinich and H. Terazawa - Preprint BNL 17161 (1972).

4) J.F. Davis, S. Hayes, R. Imlay, P.C. Stein and P.J. Wanderer - Phys. Rev.Letters 29, 1356 (1972).

5) S. Weinberg - Phys.Rev. D5, 1412 (1972).

6) B.W. Lee - Proceedings XVI International Conference in High Energy Physics, Chicago-Batavia, Vol. 2, p. 190 (1972).

7) The effects of varying the parton distributions within experimental constraints, two-photon and higher order weak exchange, and possible strong interaction effects (parity violation) are being studied in collaboration with N. Stanko, H. Stern and H. Strubbe.

FIGURE CAPTIONS

Figure 1 Parton model diagram for $pp \to \ell^+ \ell^- + X$.

Figure 2 Dominant two-photon exchange contribution.

Figure 3 Two-photon exchange diagram giving asymmetry in parton model.

Figure 4 Parton distribution asymmetry over entire solid angle and with lepton transverse momentum cut-off.

Figure 5 Charge asymmetry at ISR energies as a function of $r = Q^2/s$.

Figure 6 Charge asymmetry at ISABELLE energies.

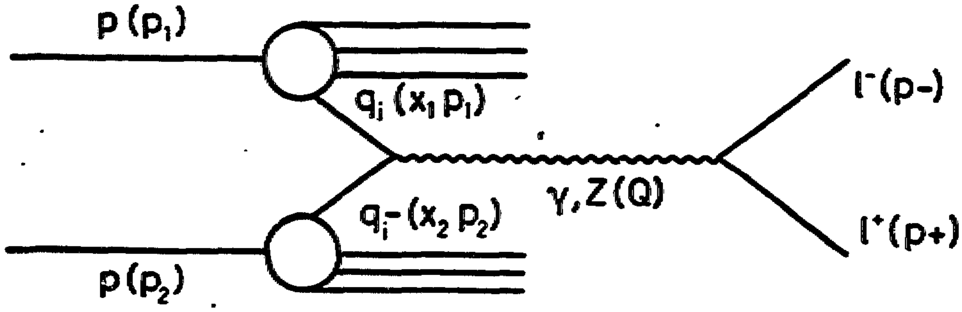


FIG.1

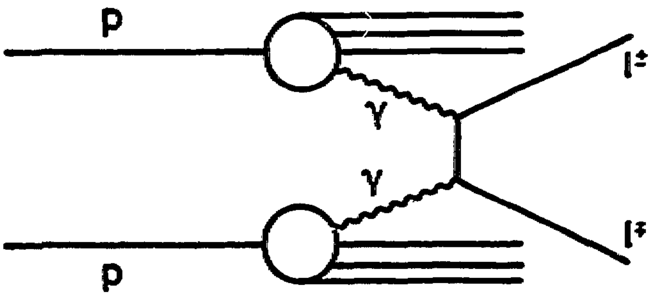


FIG. 2

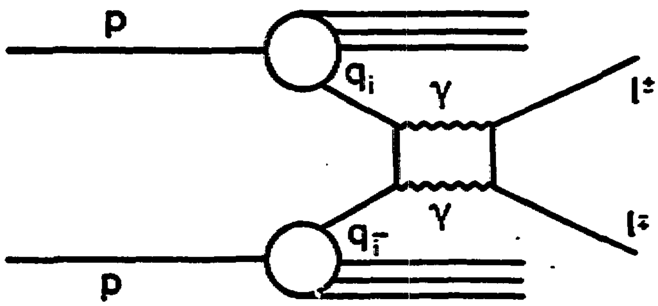


FIG.3

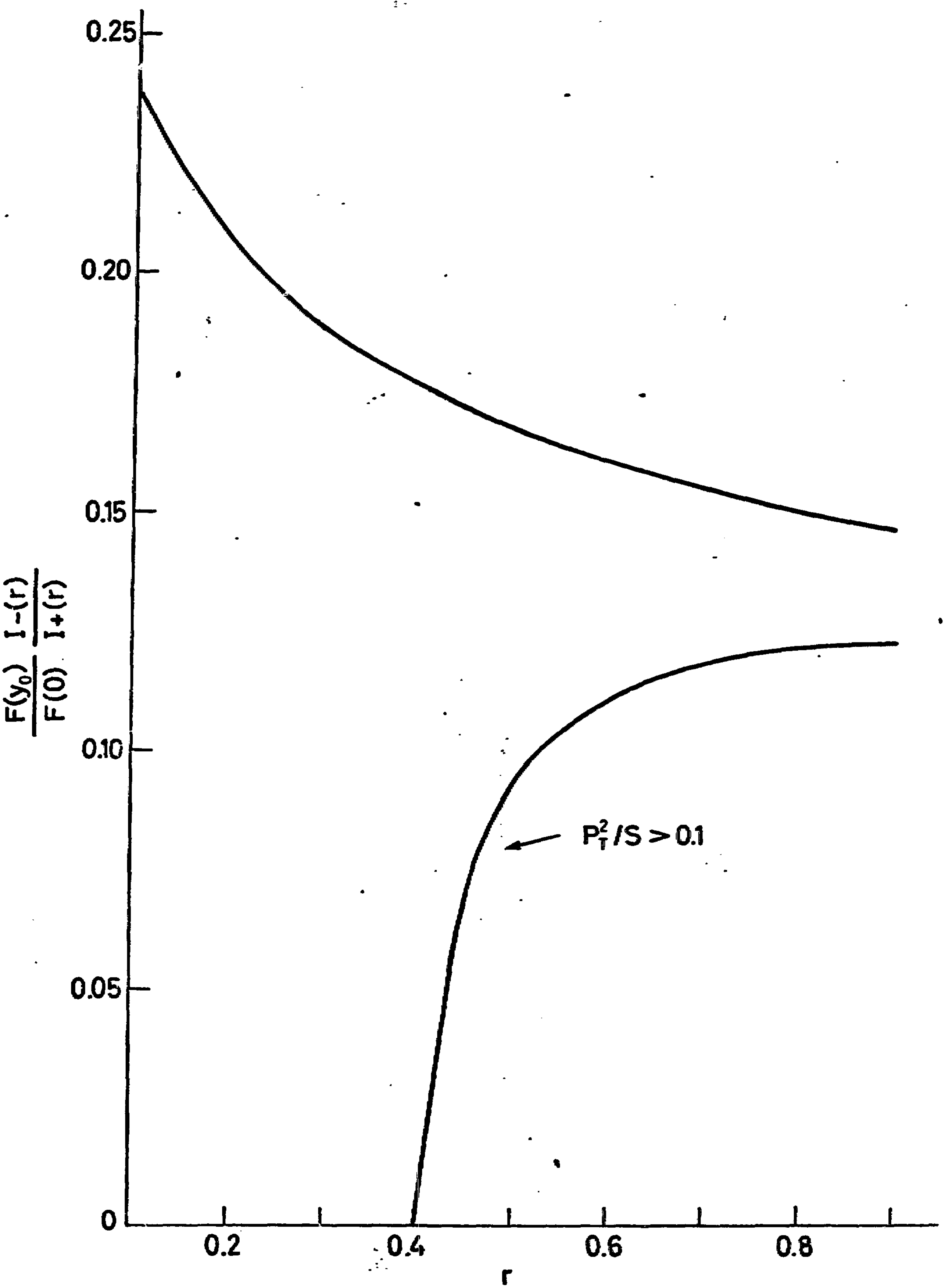
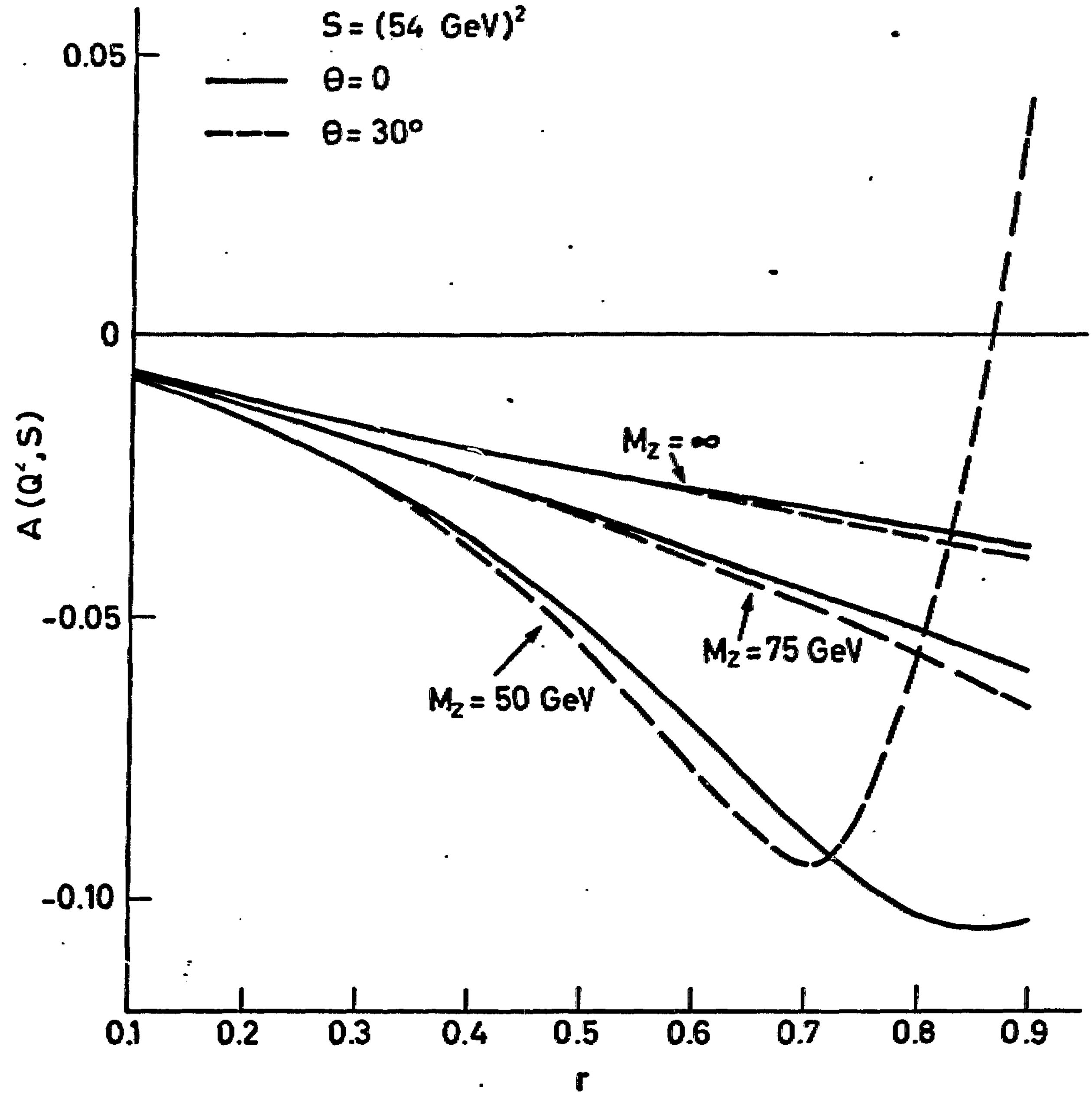
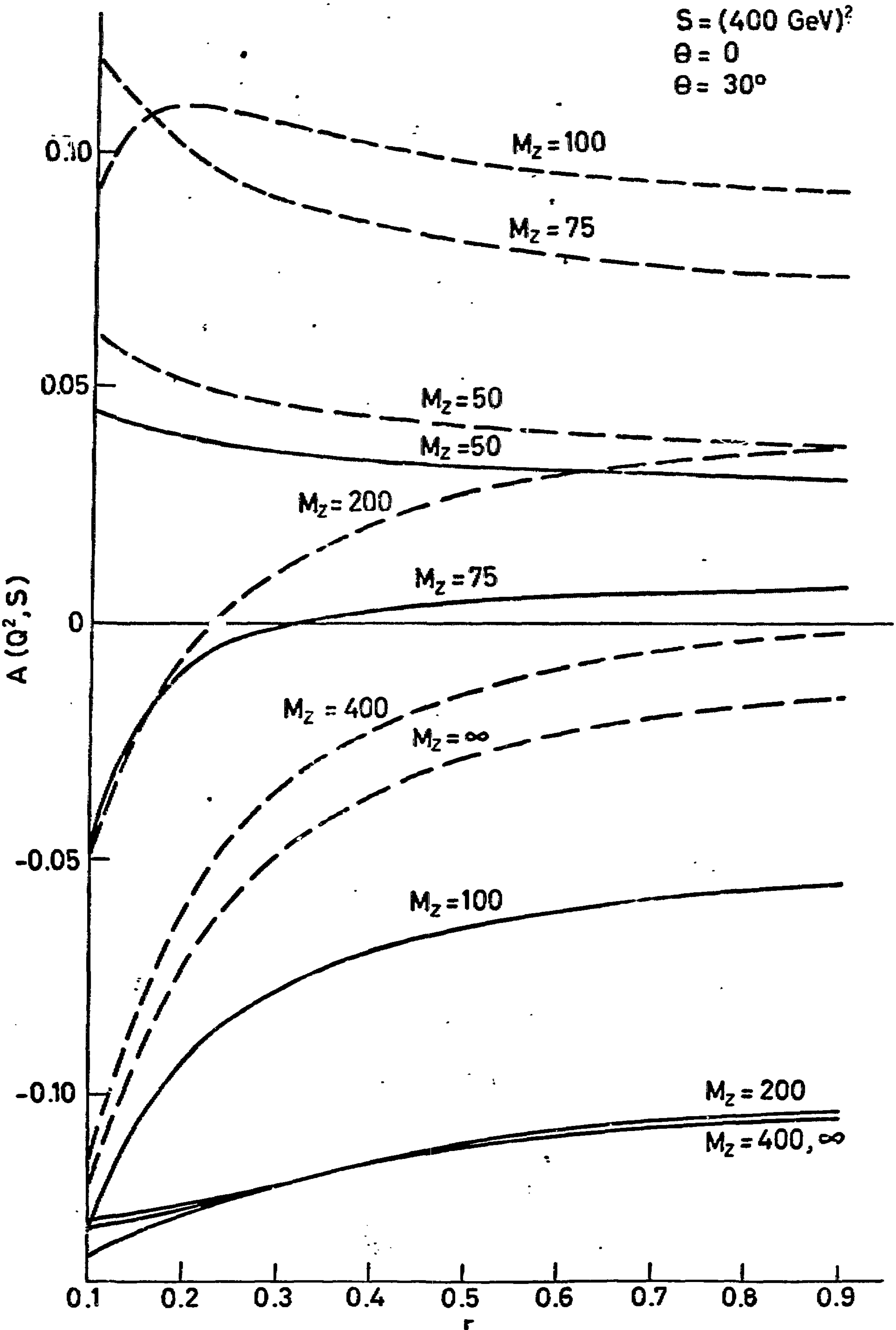


FIG.4



S = (54 GeV)^2
θ = 0
θ = 30°
0.05
0
-0.05
-0.10
A (Q², S)
$M_Z = \infty$
M_Z = 75 GeV
M_Z = 50 GeV
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
r

FIG. 5



S = (400 GeV)²
θ = 0
θ = 30°
M_Z = 100
M_Z = 75
M_Z = 50
M_Z = 50
M_Z = 200
M_Z = 75
M_Z = 400
M_Z = ∞
M_Z = 100
M_Z = 200
M_Z = 400, ∞
A (Q², S)
0.10
0.05
0
-0.05
-0.10
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
r

FIG.6