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Internal Report

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SERVO SYSTEM FOR MAGNETIC CONTROLLED CONSTANT INTENSITY
FLAT TOP BEAM SPILL-OUT

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A uniform intensity flat top spill-out cannot be obtained by manual control for two reasons:

1. The horizontal density of the internal beam of the Cosmotron is far from uniform. As a result, a manually controlled linear motion of the internal beam into a target will result in a non-uniform spill-out intensity.
2. Stability requirements of the Cosmotron magnet voltage are not easily met without feedback because of inherent component stability..

The proposed servo system will sense the external beam intensity, and correct the magnet voltage to keep this intensity constant. This servo must operate through the transfer function of the main ignitron system and the flat top filter. Both of these transfer functions impose special problems. The ignitron system is discontinuous in frequency space, in that it samples at finite time intervals. The sampling rate during flat top is approximately 698 cps. The first troublesome frequency occurs at 349 cps (half 698). The cross modulation products at $1/3$ the sampling frequency, and below, are small. The Nyquist diagram of any servo system operating through the ignitrons must be complete before reaching 349 cps to avoid the discontinuity. The transfer function of the flat top filter has a resonance with $Q = 22$ and its accompanying 180° phase shift. The half width of this phase change occurs during a frequency factor of $1/22$. A system of networks is required to reduce the loop gain to 0.5 before 349 cps and correct the 180° phase shift to a smaller chosen value. Variation in loop gain magnitude, resulting from changes in internal beam intensity, must also be accommodated.

Before going further with the detailed solution of this problem let us review the transfer function of two networks.

Network 1

Network 2

Facsimile Price \$ 1.60
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Available from the
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Their analytical transfer function normalized to unity at $w = 0(\text{dc})$ gives

$$A = A_0 \sqrt{\frac{\left(\frac{w}{w_0}\right)^2 + \alpha^2}{\left(\frac{w}{w_0}\right)^2 + (1 + \alpha)^2}}$$

$$\theta_1 = \tan^{-1} \frac{1 + \alpha}{w/w_0} - \tan^{-1} \frac{\alpha}{w/w_0}$$

Where R_2
 $\alpha = \frac{R_2}{R_1}$ and $w_0 = \frac{1}{R_2 C}$

θ_1 maximum occurs when

$$\frac{w}{w_0} = \sqrt{\alpha(\alpha+1)}$$

$$A = \sqrt{\frac{\left(\frac{w_0}{w}\right)^2 + \alpha^2}{\left(\frac{w_0}{w}\right)^2 + (1 + \alpha)^2}}$$

$$\theta_2 = \tan^{-1} \frac{w}{w_0} \alpha - \tan^{-1} \frac{w}{w_0} (1 + \alpha)$$

Where
 $\alpha = \frac{R_2}{R_1}$ and $w_0 = \frac{1}{R_1 C}$

θ_2 maximum occurs when

$$\frac{w_0}{w} = \sqrt{\alpha(\alpha+1)}$$

These networks are summarized in fig. 1.

If we introduce a change of parameters we can make the two amplitude expressions reciprocal of each other and $\theta_1 = -\theta_2$ for the same parameter α .

Let: $w = kw_0 \sqrt{\alpha(\alpha+1)}$

and $y = \sqrt{\frac{\alpha+1}{\alpha}}$

$$A = y \sqrt{\frac{k^2 y^2 + 1}{k^2 + y^2}}$$

$$\theta_1 = \tan^{-1} \frac{y}{k} - \tan^{-1} \frac{1}{ky}$$

Let: $w = \frac{kw_0}{\sqrt{\alpha(\alpha+1)}}$

and $y = \sqrt{\frac{\alpha+1}{\alpha}}$

$$A = \frac{1}{y} \sqrt{\frac{k^2 + y^2}{k^2 y^2 + 1}}$$

$$\theta_2 = - \left[\tan^{-1} \frac{y}{k} - \tan^{-1} \frac{1}{ky} \right]$$

Now, with the above symmetry in mind, we need only analyze one network type.

Let us find the relationship between A and β for the general case of n electrical section in cascade, where $\beta = n\theta$ max. (θ max occurs where $k = 1$). β becomes a desired angle of phase angle correction.

$$\beta = n \left[\tan^{-1} y - \tan^{-1} \frac{1}{y} \right] = n \left[2 \tan^{-1} y - \frac{\pi}{2} \right]$$

$$y = \tan \left(\frac{\beta}{n} + \frac{\pi}{2} \right) / 2$$

$$A_n = \left(\frac{\alpha + 1}{\alpha} \right)^n = y^{2n}$$

$$\therefore A_n = \left[\tan \left(\frac{\beta}{2n} + \frac{\pi}{4} \right) \right]^{2n}$$

Fig. 2 is a plot of constant β line with n as a variable. It is clear from fig. 2 that the constant β lines approach limiting values of A_n while n is still small enough to be practical. For convenience, constant α lines are also shown on fig. 2. The limiting value of A_n as n approaches infinity is

$$A_n \xrightarrow{n \rightarrow \infty} e^{2\beta} \quad \text{where } \beta \text{ is in radians}$$

reduced to practical units

$$A_n \xrightarrow{n \rightarrow \infty} \text{anti log}_{10} (.015159\beta) \text{ with } \beta \text{ in degrees.}$$

If staggered sections are to be employed, it would be desirable to overlap them at the point where n^θ becomes

$$\frac{n\theta_{\max}}{2} \text{ or } \theta = \frac{\theta_{\max}}{2}$$

$$\theta = \tan^{-1} \frac{y}{k} - \tan^{-1} \frac{1}{ky} = \frac{\theta_{\max}}{2} = 2\tan^{-1} y - \frac{\pi}{2}$$

The solution of this equation gives

$$k^2 = \frac{1 + \sqrt{1 - \frac{4y^2}{(1+y)^4}}}{1 - \sqrt{1 - \frac{4y^2}{(1+y)^4}}}$$

$$\text{where } y^2 = \frac{\alpha + 1}{\alpha}$$

In review, k is the half width factor on the frequency axis.

A folded log-log plot of k^2 versus α is shown in fig. 3. From fig. 3 we see that for $A_n \approx A_n$ (limit $n \rightarrow \infty$), α is approximately equal to or greater than unity. For α thus restricted, k^2 is approximately its limiting value, 13.9282032

$$k^2 \xrightarrow{\alpha \rightarrow \infty} \frac{2 + \sqrt{3}}{2 - \sqrt{3}} = 13.9282032$$

The amplitude at the $\frac{\theta_{\max}}{2}$ points can be expressed as follows, defining new parameters r and s

$$(A \text{ upper})^n = y^n \left[\frac{\frac{k^2 y^2 + 1}{2}}{k + y} \right]^{n/2} = A_n^r$$

$$(A \text{ lower})^n = y^n \left[\frac{k^2 y^2 + 1}{k^2 + y^2} \right]^{n/2} \approx A_n^s$$

Plots of r and s are shown in fig. 4. Again for α approximately equal to or greater than unity both r and s are very close to their limiting values of

$$\begin{array}{ll} r \rightarrow 0.9330127 & s \rightarrow 0.0669873 \\ \alpha \rightarrow \infty & \alpha \rightarrow \infty \end{array}$$

With the network constants normalized in this way a general method for satisfying Nyquist requirements is possible.

Let us assume a general problem with the following information and restrictions:

1. The amplitude and phase functions of all components in the loop other than correction networks are known. $[A(f) \text{ --- etc.}]$ and $[\theta(f) \text{ --- etc.}]$
2. An upper frequency limit exists such that the Nyquist diagram must be complete (gain less than 1/2) before this frequency is reached. If this limit does not exist solutions may be found by simpler methods.
3. The phase angle for all frequencies up to the frequency limit must be less than a chosen value (θ_L). This limitation is usually set by the desired transient response which requires that the Nyquist plot lie outside a certain "M" circle. If loop gain is a dynamic variable the Nyquist diagram must lie to the right of the polar line tangent to this "M" circle.
4. The desired dc loop gain is G .

A general solution consists of staggering P networks separated by the factor k in frequency space (call these frequencies $f_1, f_3 \text{ --- } f_p$). The first network is centered on the frequency limit. The $(P - 1)^{\text{th}}$ network is centered in frequency near the frequency where the uncorrected transfer function crosses the 90 degree lag vector. One additional network (th P^{th}) is permissible. The following set of linear simultaneous equations with known constants results:

$$\beta_1 + \frac{\beta_2}{2} \cong \theta(f_1) - \theta_L$$

$$\frac{\beta_1}{2} + \beta_2 + \frac{\beta_3}{2} \cong \theta(f_2) - \theta_L$$

$$\frac{\beta_2}{2} + \beta_3 + \frac{\beta_4}{2} \cong \theta(f_3) - \theta_L$$

$$\frac{\beta_{(P-2)}}{2} + \beta_{(P-1)} + \frac{\beta_P}{2} \cong \theta[f_{(P-1)}] - \theta_L$$

This results in $(P - 1)$ equations with P parameters. The last equation comes from amplitude consideration.

$$GA(f_1) e^{2(0.5 \beta_1 + 0.933 \beta_2 + \beta_3 + \dots + \beta_p)} = 0.5$$

$$\text{or } 2(0.5 \beta_1 + 0.933 \beta_2 + \beta_3 + \dots + \beta_p) = \text{natural log } \frac{0.5}{GA(f_1)}$$

Solutions of the above will give all values of β and in fact will automatically determine the type of network (network 1 or network 2). If network 2 is required, the value of β will simply be negative. Negative β in the last equation produces the necessary amplitude reciprocal.

The constant intensity spill out servo has exactly the requirements outlined above. In review, the known constants are:

$$f(\text{max}) = 349 \text{ cps}$$

$$\theta_L = 130^\circ \text{ (chosen to be tangent to the 120\% "M" circle.)}$$

Total transfer is (from ripple filter):

$$A(f) = \frac{1}{\sqrt{\frac{w^2}{w_0^2 \alpha^2} + \left(\frac{w^2}{w_0^2} - 1\right)^2}}$$

$$\theta(f) = -\frac{\pi}{2} - \tan^{-1} \frac{\alpha \left(\frac{w}{w_0} - 1\right)}{w/w_0}$$

$$\text{where } w = 2\pi f$$

$$w_0 = \frac{1}{\sqrt{LC}}$$

$$L = 8.25 \text{ millhenries}$$

$$C = 10^{-2} \text{ farads}$$

$$\alpha = 22$$

Loop gain at dc = 40 (chosen to produce 2-1/2% variation in external beam intensity.

All other time constants must be small compared with 400 microseconds. This condition is necessary to make the transfer function above valid through 350 cps.

Solutions for the above are:

$$\beta_1 = 83 \text{ degrees}$$

$$\beta_2 = -41 \text{ "}$$

$$\beta_3 = +124 \text{ degrees}$$

$$\beta_4 = -82 \text{ "}$$

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A check of loop gain at 349 cps yields 0.606 instead of 0.5 because of accumulated slide rule errors. This difference is not significant.

The network parameter becomes:

	1	2	3	4
Network type	1	2	1	2
β	83	-41	124	-82
A_0	19	1 / 4.2	90	1/17
Number of sections	4	2	5	4
α	.92	.95	.69	.97
Frequency of maximum phase shift	349 cps	78 cps	17.5 cps	4 cps.
w_0	2195	490	110	35

Fig. 5 shows the resulting Nyquist diagram. This diagram predicts 15% overshoot in response to step error. The servo should be completely stable independent of loop gain reduction resulting from drop in internal beam intensity. The dc loop gain must be manually adjusted to be 40 at the instant of maximum beam intensity.

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