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A TECHNIQUE FOR PREDICTING
NONLINEAR WIND COMPENSATION
OF BALLISTIC ROCKET SYSTEMS

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SANDIA LABORATORIES

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A Sandia Corporation Monograph

A TECHNIQUE FOR PREDICTING NONLINEAR WIND
COMPENSATION OF BALLISTIC ROCKET SYSTEMS

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ABSTRACT

Factors affecting the nonlinearities associated with wind compensation of ballistic rocket systems are discussed. A linear wind compensation method, used by Sandia Laboratory since 1959, is presented and extended to account for nonlinear effects. Comparisons between the two methods are made to show expected improvements in accuracy.

SUMMARY

A theoretical development of wind compensation equations is presented and discussed. The coefficients in the equations are assumed to be a function of the ballistic wind magnitude and direction, and the change in launch elevation angle. The proposed method of uniquely defining and determining the coefficients is evaluated through the use of a digital six-degree-of-freedom trajectory program that incorporates the physical, aerodynamic, thrust, and fuel flow characteristics of a typical ballistic rocket system.

The launcher set-correction technique is evaluated by assuming values of the ballistic wind and its direction. These values are used to determine from the launcher set chart the azimuth and elevation corrections required to compensate for the ballistic wind. Fourteen six-degree-of-freedom trajectories were computed to evaluate the proposed correction technique. These trajectories, using two magnitudes of ballistic wind impinging on the rocket system from seven different directions, show that the flight path of the rocket is corrected to within a maximum error of 3 mils, based on the slant range, for the ballistic wind environments tested. Fourteen comparison trajectories, using azimuth and elevation corrections determined from linear coefficients, give a maximum error of 19 mils.

LIST OF SYMBOLS

D	Deflection, meters
h	Altitude, meters
R	Range, meters
R_N	Nominal range, meters
V	Ballistic wind velocity, meters per second
V_N	Normal component of ballistic wind, meters per second
V_P	Parallel component of ballistic wind, meters per second
X	Distance along X-axis, meters
Y	Distance along Y-axis, meters
Δ	Denotes incremental quantity
$\frac{\Delta R}{\Delta \theta}$	Change in range due to a change in elevation angle, meters per degree
η	Azimuth angle, degrees
η_o	Nominal azimuth angle, degrees
θ	Elevation angle, degrees
θ_o	Nominal elevation angle, degrees
λ	Ballistic wind direction, degrees

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A TECHNIQUE FOR PREDICTING NONLINEAR WIND COMPENSATION OF BALLISTIC ROCKET SYSTEMS

Introduction

Statistical variations in ballistic rocket parameters that cause deviation from the nominal trajectory have been of increasing concern to the ballisticians as the altitude and range of rocket systems have increased. It is necessary from a range safety point of view that the impact point of each stage and its related hardware be accurately predicted. Also, accurate impact prediction is desirable to facilitate airborne and surface vessel payload recovery and data acquisition. More recently, scientific experimenters have required greater accuracy of payload placement at a designated point in space. To assist in the attainment of these goals, this paper presents a linear wind compensation method* used very successfully by Sandia Laboratory since 1959. The linear technique is extended to account for all nonlinear effects, regardless of their origin.

Development of Equations

The equations being developed assume that the altitude-wind profile obtained near the time of launch is effectively reduced by a wind-weighting system to a ballistic wind of magnitude, V , that is blowing at

*The original equations using linear (or constant) coefficients were developed by Mr. H. A. Wente, Staff Member, Aero-Thermodynamics Department, Sandia Laboratory.

an angle, λ . The right-hand orthogonal axis system used to describe the physical quantities is located at the launch point on the earth with the X-axis northward, the Y-axis eastward, and the Z-axis downward. All angles are measured by the right-hand rule from the X-axis, with the exception of the elevation angle, θ , which is measured positive-upward from the X, Y-plane. Three coordinates are required to fully specify the position of the vehicle in space at any given time. Instead of using the X, Y, Z coordinates, three generalized coordinates: elevation angle, θ , deflection from the corrected azimuth, D, and range, R, are used to specify the vehicle location.

Figure 1 shows the projection of ballistic rocket trajectories onto the X, Y-plane. When no ballistic wind is present, the range, R, for a new launcher setting located at azimuth angle, η , and elevation angle, θ , may be obtained by

$$R = R_N + \left(\frac{\Delta R}{\Delta \theta} \right) \cdot \Delta \theta, \quad (1)$$

where R_N and θ_0 are the nominal range and nominal elevation angle, respectively ($\Delta \theta = \theta - \theta_0$). The quantity $\Delta R / \Delta \theta$ is negative for a positive $\Delta \theta$ at large elevation angles.*

When a ballistic wind acts on the system, as shown in Figure 2, the launcher must be moved through incremental changes in azimuth and elevation angles to compensate for the wind. The correct changes will cause the vehicle to pass through the nominal aiming point, A. Expressions for ΔD and ΔR are obtained from Figure 2 and combined with Equation (1) to give

$$\Delta D = - R_N \sin \Delta \eta, \quad (2)$$

and

$$\Delta R = R_N (\cos \Delta \eta - 1) - \left(\frac{\Delta R}{\Delta \theta} \right) \cdot \Delta \theta. \quad (3)$$

* For elevation angles below the maximum range elevation angle, $(\Delta R / \Delta \theta)$ changes sign.

Figure 1. Schematic of Rocket Trajectories for No Wind Condition

Figure 2. Schematic of Rocket Trajectories for a Ballistic Wind Impinging on the Rocket System

To determine ΔR and ΔD , it is necessary to consider these quantities as functions of $\Delta\theta$ and the ballistic wind components V_N and V_P , which are the normal and parallel components, respectively. Figures 3 and 4 show range and deflection variations with ballistic wind components for a given elevation angle. These figures show range and deflection for a typical ballistic, fin-stabilized rocket system. For a vehicle launched at high elevation angles, having large launch accelerations and comparatively linear aerodynamics, the curves in these figures tend to coalesce and approach the same slope about the origin for small changes in elevation angle. Under these conditions the changes in range and deflection can be assumed to be independent of normal and parallel ballistic wind components, respectively. The expressions for ΔR and ΔD can then

be written

$$\Delta R = \left(\frac{\partial R}{\partial V_P} \right) \cdot V_P = \left(\frac{\partial R}{\partial V} \right) \cdot V \cdot \cos (\lambda - \eta_o - \Delta\eta) \quad (4)$$

and

$$\Delta D = \left(\frac{\partial D}{\partial V_N} \right) \cdot V_N = \left(\frac{\partial D}{\partial V} \right) \cdot V \cdot \sin (\lambda - \eta_o - \Delta\eta). \quad (5)$$

The subscript notation has been dropped for convenience and the expressions for V_N and V_P are obtained from Figure 2. Equations (3) and (4), and (2) and (5) are combined to yield

$$\left(\frac{\partial R}{\partial V} \right) \cdot V \cdot \cos (\lambda - \eta_o - \Delta\eta) = R_N (\cos \Delta\eta - 1) - \left(\frac{\partial R}{\partial \theta} \right) \cdot \Delta\theta, \quad (6)$$

and

$$\left(\frac{\partial D}{\partial V} \right) \cdot V \cdot \sin (\lambda - \eta_o - \Delta\eta) = - R_N \sin \Delta\eta. \quad (7)$$

Equations (6) and (7) contain the four unknowns λ , $\Delta\eta$, $\Delta\theta$, and V . It is possible to assume values of two of the unknowns and solve for the remaining unknowns. For launcher set chart construction, values of $\Delta\theta$

Figure 3. Variation of Range with Ballistic Wind Components

Figure 4. Variation of Deflection with Ballistic Wind Components

and $\Delta \eta$ are assumed and the equations are solved for λ and V . Polar coordinates are used to locate λ and V along the angular and radial coordinates, respectively. Lines of constant $\Delta \eta$ and $\Delta \theta$ are constructed graphically to facilitate locating the correct launcher setting, as shown in Figure 5. The equations can be programmed on relatively small digital computers to aid in determining launcher settings at the launch site. The value of the ballistic wind and its direction are known and the equations are solved for the wind compensation settings, $\Delta \theta$ and $\Delta \eta$.

The impact point of any portion of the rocket system may be determined from Figure 2 by summing the components in the X and Y directions to give

$$X = X_O + (R + \Delta R) \cos \eta - \Delta D \sin \eta \quad (8)$$

and

$$Y = Y_O + (R + \Delta R) \sin \eta + \Delta D \cos \eta. \quad (9)$$

Substitution for the expressions in these equations gives

$$\begin{aligned} X = X_O + \left[\left(\frac{\partial R}{\partial \theta} \right) \cdot \Delta \theta + R_N \right] \cos \eta + \left[\left(\frac{\partial R}{\partial V} \right) \cdot V \cdot \cos (\lambda - \eta_O - \Delta \eta) \right] \cos \eta \\ - \left[\left(\frac{\partial D}{\partial V} \right) \cdot V \cdot \sin (\lambda - \eta_O - \Delta \eta) \right] \sin \eta, \end{aligned} \quad (10)$$

and

$$\begin{aligned} Y = Y_O + \left[\left(\frac{\partial R}{\partial \theta} \right) \cdot \Delta \theta + R_N \right] \sin \eta + \left[\left(\frac{\partial R}{\partial V} \right) \cdot V \cos (\lambda - \eta_O - \Delta \eta) \right] \sin \eta \\ + \left[\left(\frac{\partial D}{\partial V} \right) \cdot V \cdot \sin (\lambda - \eta_O - \Delta \eta) \right] \cos \eta. \end{aligned} \quad (11)$$

The appropriate values of the constants and coefficients are provided for each stage or component part to yield its respective impact location.

Figure 5. Launcher Set Chart Based on Linear Coefficients

When the rocket system is accelerating slowly and/or has a low velocity at launch, the changes in range and deflection are not linear. The assumption that these changes are independent of the ballistic wind components is not valid, and the resulting large changes in elevation angle cause changes in the slopes of Figures 3 and 4. Nonlinear aerodynamic coefficients and all other nonlinear phenomena help compound the nonlinearity associated with these figures. If accurate wind compensation is to be realized, these nonlinearities must be fully accounted for.

One method of accounting for the nonlinearities is to use a six-degree-of-freedom digital computer program to determine ΔR and ΔD in Figure 2. If the computer program incorporates a rotating oblate-spheroid earth, along with the vehicle's aerodynamic, physical, thrust, and fuel flow characteristics, then all the nonlinear effects are incorporated in the determination of ΔR and ΔD . It is then possible to determine these changes in range and deflection as functions of the ballistic wind components and changes in elevation angle.

Equations (2) and (3) retain their original form where ΔD and ΔR are written

$$\Delta D = f_1 (V_N, V_P, \Delta\theta), \quad (12)$$

and

$$\Delta R = f_2 (V_N, V_P, \Delta\theta), \quad (13)$$

to show functional dependence. It should also be noted that $(\Delta R/\Delta\theta)$ in Equation (1) must show functional dependence on $\Delta\theta$ for moderate-to-large changes in elevation angle.

Equations (12) and (13) are equated to Equations (2) and (3) to provide two equations in the two unknowns, $\Delta\theta$ and $\Delta\eta$, and must be solved simultaneously. These equations do not represent an explicit

closed-form solution as was obtained in Equations (6) and (7). Solution of these equations requires that an iterative technique be used and proceeds in the following manner:

1. Assume incremental values of λ and V .
2. For an assumed value of V , compute V_P and V_N using Equations (4) and (5), assuming $\Delta\eta = 0$.
3. For $\Delta\theta = 0^\circ$, and the computed values of V_P and V_N , find $\Delta R/\Delta\theta$, $\Delta R(V_P, V_N, \Delta\theta)$, and $\Delta D(V_P, V_N, \Delta\theta)$.
4. Use Equation (12) to compute $\Delta\eta$.
5. Use Equation (13) to compute $\Delta\theta$ using the value of $\Delta\eta$ from Step (4).
6. Recompute V_P and V_N using the results of Step (4).
7. Using the results of Steps (5) and (6), find new values $\Delta R/\Delta\theta$, $\Delta R(V_P, V_N, \Delta\theta)$, and $\Delta D(V_P, V_N, \Delta\theta)$.
8. Repeat Steps (4) and (5).
9. Continue the iteration procedure till convergence occurs for both $\Delta\theta$ and $\Delta\eta$.

A launcher set chart can be graphically constructed by plotting $\Delta\theta$ as a function of $\Delta\eta$. Lines of constant $\lambda - \eta_0$ and constant V are located on the chart to form a grid that aids in locating the proper launcher correction.

The nonlinear wind compensation method was evaluated using a typical rocket system. A six-degree-of-freedom program⁽¹⁾ incorporating the aerodynamic, physical, thrust, and fuel flow characteristics of the vehicle was used to determine ΔR , ΔD , and $\Delta R/\Delta\theta$ as

functions of the ballistic wind components and changes in elevation angle. A Fortran IV program, containing the necessary two- and four-dimensional tables needed to describe accurately the coefficients, was used to solve Equations (12) and (13). The results of the computations and the launcher set chart are shown in Figure 6.

Ballistic wind velocities of 4.572 and 9.144 meters per second for impingement angles of 0, 30, 60, 90, 120, 150 and 180 degrees were selected for evaluation purposes. The corresponding azimuth and elevation angle corrections, along with the appropriate ballistic wind components, were used in the six-degree-of-freedom trajectory program and evaluated at 87.0 seconds trajectory time. This is the time the rocket system reaches the aiming point in space for a no-wind condition. The results of the fourteen trajectories were compared with the nominal trajectory. The maximum error at the target point is 238.1 meters or 3 mils* based on the slant range distance from launch to aiming point.

For comparison purposes, fourteen trajectories were computed using azimuth and elevation angle corrections based on linear coefficients and the same ballistic wind velocities. These results were compared to the nominal trajectory. The maximum error at the target point is 1551.7 meters or 19 mils.

Concluding Remarks

Evaluation of the correction technique validates the basic launcher set correction equations, and shows that the coefficients in the equation, which incorporate the vehicle characteristics, can be accurately determined as functions of the ballistic wind velocity components and changes in elevation angle.

* Mil error = Miss distance x 1000/Slant range

Figure 6. Launcher Set Chart for a Typical Ballistic Rocket System

The 3 mil error using nonlinear coefficients is a significant improvement over the 19 mil error using linear coefficients for the low-launch velocity of the typical rocket system considered. Rocket systems having large accelerations and velocities at launch would not show the same improvement in accuracy, but would maintain the same degree of accuracy.

Inaccuracies encountered during reduction of the altitude-wind profile to a ballistic wind have not been evaluated. This particular source of error should not be overlooked when applying any wind-compensation method.

REFERENCE

1. "Six-Degree-of-Freedom Flight-Path Study Generalized Computer Program," FDL-TDR-64-1, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, October 1964.